The following publication Song-Man Wu, Felix T.S. Chan \& S.H. Chung (2022) A study on green supply chain under capital constraint considering time-varying salvage value, International Journal of Production Research, 60:1, 8-24, is available at http:// doi.org/10.1080/00207543.2021.1982153.

# A Study on Green Supply Chain Under Capital Constraint <br> Considering Time-Varying Salvage Value 


#### Abstract

To reduce environmental pollution and promote sustainable development, more and more suppliers are committed to producing more environmentally friendly products such as green electrical appliances through a green supply chain (GSC) system. However, some suppliers are often limited by a lack of funds in the production and supply process. In this paper, buyer-supported purchase order financing (BPOF) and advance payment discount (APD) are adopted to help the supplier successfully produce green products in a GSC system consisting of a financially constrained supplier and a reputable retailer. Moreover, the salvage values of unsold inventory in most traditional models are fixed, but in real life, the salvage value tends to be time dependent. Therefore, how the time-varying salvage value affects the operation and financing decisions as well as the profit risks is studied. We find that the clearance time of the unsold items affects the optimal order quantity and the supplier's discount rate and the financial institution's optimal interest rate. In addition, the financing equilibrium is BPOF under certain conditions, and the profit risks of the retailer and the entire supply chain are increasing with the order quantity and the clearance time. Finally, our results are verified through numerical analysis.


Keywords: green supply chain; financial management; supply chain management; time-varying salvage value; capital constraint

## 1. Introduction

In modern society, daily products such as air conditioners, refrigerators, washing machines, televisions, and computers are an indispensable part of every family. Over the years, these products have undergone tremendous changes and companies in this field are becoming more environmentally conscious. Hence, increasing suppliers have focused on developing more energy-efficient and environmentally friendly green products, and downstream retailers are also more willing to sell greener products to meet the demands of eco-conscious consumers. In the electrical appliance industry, for instance, Sony has long been environmentally conscious, having developed the Handycam DCR-TRV30 with $45 \%$ more pixels to improve image quality while maintaining almost the same power consumption (Schvaneveldt 2003). Inverter air conditioners developed in recent years can save $44 \%$ more energy than traditional non-inverter air conditioners while reducing carbon dioxide emission by $49 \%$ (Almogbel et al. 2020) making a big contribution to energy conservation and environmental protection. Further, because of the continuous renewal of green products, increasing off-season items that are less environmentally friendly cannot be sold during the selling season. In many retail industries, clearance sales at salvage value after the normal sales quarter is the most common form in clearing stock, and can affect the scale of the enterprise's cost reduction and thus net profit (Wang and Webster 2009). In most research, the salvage value is assumed to be constant. However, in real life, the salvage value varies with the clearance time. The later the clearance, the lower the salvage value of the product. Therefore, the management of out-of-season items with time-varying salvage value has become a significant issue that can not only reduce the profit loss of the retailers' remaining inventory but also benefits the development of global sustainable development and environmental
protection. Such products can be resold to remote areas of the country or other economically poorer countries. As a result, the products in these regions are also upgraded to more environmentally friendly products than before, thus promoting sustainable development and reducing environmental pollution.

Besides, due to the large investment in green product research and development and the high production costs, some suppliers are often restricted by limited initial capital in the production and supply of products. For example, German TV maker Loewe, founded in Berlin in 1923, launched a full range of high-end energy-efficient televisions based on OLED technology in 2013. However, due to insufficient funds, it initiated bankruptcy proceedings and ceased operations in July 2019 (Flatpanelshd, 2019). Toshiba TV was also cash-strapped due to operating losses and 95 percent of the company was acquired by Chinese Hisense in November 2017 (Theverge, 2017). Furthermore, Chigo Air Conditioner used to be one of the largest private air conditioner companies in China, however, it also filed for bankruptcy due to continued losses and a lack of capital to support its operations (DayDayNews. 2020). Hence, without sufficient financial support, vulnerable suppliers may encounter many problems, ranging from supply delays or cutbacks to bankruptcies and acquisitions, which will ultimately affect the normal operation of their downstream enterprises and even the entire supply chain.

To solve such problems, practitioners and scholars are increasingly paying attention to supply chain finance (SCF), which can help suppliers obtain funds effectively through supply chain cooperation. Therefore, in the green supply chain (GSC), to derive funds for producing more green products, suppliers can solve the problem of funding constraints via buyer-supported purchase order financing (BPOF) and advance payment discount (APD). The former means that a financial institution
provides a loan to the supplier before shipment which is guaranteed by the downstream retailer. The latter refers to the up-front payment of the downstream retailer to the upstream supplier, who provides the wholesale price with a discount to the retailer. Both APD and BPOF can solve the problem in which some suppliers may be unable to obtain bank loans due to the poor credit levels and lack of asset mortgages. Although APD and BPOF are common in practice, most of the current research is based on bank financing, trade credit, and some other post-shipment financing tools, and there is sparse literature on pre-shipment financings like APD and BPOF, which contains higher risks because pre-shipment financing is based on purchase orders rather than invoices and relies on the trust relationship between buyers and sellers. Furthermore, APD can accelerate the recovery of the supplier's receivables, thus improving the supplier's cash conversion cycle, while BPOF can reduce the financial burden of the retailer and strengthen the connection of the participants. Both of them can better address the supplier's financial problem effectively and reduce the financing costs. Therefore, APD and BPOF are the focuses of this study here. Moreover, how to measure potential risks in the GSC has attracted attention from researchers. A well-known method is the mean-variance risk measurement approach, which was proposed by Acerbi (2002) and has been widely applied in the literature (Chiu and Choi 2016; Wen and Siqin 2020; Choi et al. 2019). Simply put, it is the expected profit and the trade-off between profit changes.

Inspired by the above facts, we are interested in a GSC in which the retailer sells unsold products with time-varying salvage values, and the supplier can adopt financing strategies for the smooth production of new, greener products. We investigate the following research problems: 1) How to build a GSC model and what is the effect of the time-vary salvage value on the supply chain participants'
decisions under financing strategies? 2) Which financial strategy, APD or BPOF, should the retailer select to help the supplier obtain funds to produce green products? 3) How to measure the retailer's risk level under different financing schemes?

To solve these problems, a model with a cash-strapped supplier and a creditworthy retailer is developed. The supplier can obtain funds via BPOF and APD for producing more green products. The closest work to our study was that of Zhao and Huchzermeier (2019), but our paper mainly makes the following new contributions: 1) In this paper, the same two pre-shipment financing instruments (APD and BPOF) as in Zhao and Huchzermeier (2019) are used, but the difference is that we focus on environmental issues and apply them specifically to the industry, such as the electrical appliance industry, to help the suppliers produce green products smoothly; 2) Zhao and Huchzermeier (2019) assumed that the salvage value of unsold products was set to zero. However, the salvage value varies according to the clearance time, and we analyze the impact of the time-varying salvage value on the optimal decisions of the participants under APD and BPOF. We find that the clearance time affects the retailer's optimal order quantity, the supplier's decision on the discount rate, and the financial institution's decision on the interest rate; 3) We characterize the financing equilibrium in this paper, which is BPOF when the clearance time is below a certain threshold or when the retailer's internal asset level is below a certain level and the threshold of the asset level is inversely proportional to the clearance time. However, the financing equilibrium study discussed by Zhao and Huchzermeier (2019) is based on the retailer's internal capital level and demand variability; 4) A profit-risk analysis of different financing schemes is carried out to obtain more insights, which was not undertaken by Zhao and Huchzermeier (2019). The results
show that profit risks of the retailer under APD and BPOF are increasing with the order quantity and the clearance time, and the profit risks of the retailer and the entire supply chain via two financing instruments are equal, which implies that the retailer's profit risks determine the whole supply chain's risks; 6) Some novel managerial insights are presented and contribute to the current literature.

The rest of this paper is arranged as follows. In the second section, the relevant literature is reviewed. In Section 3, we explain the problem and present our models concerning the time-varying salvage value under BPOF and APD in Section 4. In Section 5, we analyze profit risks by a mean-variance method. We further verify our results via numerical analysis in Section 6 and generate some managerial sights in Section 7. The final section summarizes the paper and provides some directions for future research.

## 2. Literature review

Three main literature streams are related to our study: green supply chain management, integrating financial consideration with operations, and perishable goods with salvage value.

### 2.1. Green supply chain management

Green supply chain management aims to encourage suppliers and retailers to consider energy conservation and emission reduction when making business decisions, and to solve the pollution problems caused by industrial development (Bhatia and Gangwani 2020). Recent research introduced the concepts of sustainability and environmental concerns into green supply chain management (Sheu, Chou, and Hu 2005). For example, Cheah and Phau (2011) observed that consumers have an environmentally
friendly attitude when purchasing products. Cheah and Phau (2011) investigated the relationship between consumers' knowledge of environmental pollution and their purchase intention regarding green products. Subsequently, Zhang and Liu (2013) established a GSC coordination model where customer demand depended on product greenness. Gao et al. (2020) investigated the dual-channel GSC with the eco-label policy. Zou, Farnoosh, and McNamara (2021) put forward a risk standard evaluation system of GSC, and the correlation degree among supply chain risk factors was clarified by using the grey correlation analysis method. Peng, Pang, and Cong (2018) proposed a novel contract to better coordinate the GSC and reduce carbon emissions through combining yield uncertainty and consumers' low-carbon preference. Cong, Pang, and Peng (2020) considered a capital-constrained GSC with yield uncertain, carbon cap-and-trade, emission reduction subsidies, and consumers' low-carbon preference under green finance and studied how these factors affected the optimal strategy and profit.

### 2.2. Integrating financial consideration with operations

Integrating financial consideration with operations is primarily related to our research. The existing studies show that financial constraints significantly affect business decisions (Beranek 1967; Zhang, Xu, and Chen 2020; Chen et al. 2020; Huang, Wu, and Chiang 2018). Therefore, many researchers use SCF to solve the financial constraint problems when establishing models. Xu and Birge (2004) were the first to introduce capital constraints into the newsvendor model and studied how capital structure affected the retailer's decisions. Kouvelis and Zhao (2012) analyzed a supplier funded a capital-constrained retailer in a newsvendor setting with trade credit. Cao et al. (2020) found that trade credit could increase the product ordering
quantity compared with bank loaning, and the increase of unit carbon tax and bank interest rate had a negative impact on the optimal order quantity. Other models with financial constraints can also be found in the study of Cao and Yu (2018) and Li , An, and Song (2018) et al. In contrast to the above studies, our research aims to mitigate the impact of the upstream supplier's financing constraints. Recently, in solving the supplier's lack of funds, the advance payment policy was studied by Li et al. (2019), Qin et al. (2019) and Khan et al. (2019). Additionally, Reindorp, Tanrisever, and Lange (2018) assumed a newsvendor model and presented the effect of a purchase order financing (POF) scheme. Tang, Yang, and Wu (2018) analyzed an inventory model with financial constraints and considered information asymmetry in both buyer direct financing and POF. Zhao and Huchzermeier (2019) examined APD and BPOF and studied how the retailer adopted financing tools depending on the internal asset level and demand variability. In our study, we use the same two financing tools as Zhao and Huchzermeier (2019), which are less studied. Moreover, some studies focused on the integration of SCF into GSC. For instance, Xu and Fang (2020) developed an emission-dependent model with partial credit guarantee and trade credit. Other studies can be found in Zhan, Li, and Chen 2018; Lin and He 2019; Tiwari, Ahmed, and Sarkar 2019. In summary, previous literature mainly reports the models with a single financing strategy or post-shipment financing tools, and there has not been much research on two pre-shipment financing models such as early-payment financing and POF. Besides, although the above-mentioned scholars have studied the financing problem of the GSC, none of the researchers have proposed to apply financing strategies to deal with the capital constraints of the supplier based on APD and BPOF for producing more green products.

### 2.3. Perishable goods with salvage value

The last stream is the operation management of perishable goods with salvage value. Perishable products are characterized by a short life cycle, fast price reduction, large uncertainty in market demand, and other features. After the new product is launched, the market share of the old product drops rapidly and may not be fully sold in the normal selling season. Retailers often clear excess inventory through salvage value (Wang and Webster 2009). Yu and Xiang (2016) studied the optimal yield and expected profit of a manufacturer with different salvage value channel structures. Yang et al. (2014) studied two competing retailers with revenue targets and found that positive and non-positive salvage values could bring different outcomes. A salvage value contract was designed by Castañeda, Brennan, and Goentzel (2019) under which the United Nations World Food Programme would purchase the remaining stock from the manufacturer at salvage value. According to the existing literature, it is clear that most of the research assumed that the salvage value was fixed. However, some scholars and empirical evidence indicated that the salvage value was variable (Cachon and Kök 2007; Alan and Gaur 2018), and no researchers have yet investigated the time-varying salvage value in a capital constrained supply chain.

In summary, differing from the above studies, especially the study of Zhao and Huchzermeier (2019) which also examined APD and BPOF, our study focuses on the application in the industry, such as the electric appliance industry, and helps the suppliers with financial constraints to smoothly produce green products. Second, in contrast to the assumption that the salvage value is set to zero, our model considers a time-varying salvage value to better capture the practical problem. Moreover, instead of investigating the retailer's internal capital level and demand variability on the operational and financing strategies, we investigate the effect of the time-varying
salvage value on the optimal decisions of the participants under APD and BPOF. Furthermore, we conduct a profit-risk analysis with different financing schemes to obtain more insights, which was not conducted by Zhao and Huchzermeier (2019).

## 3. Models

### 3.1 Model description

There are three key participants: a supplier with financial constraints, a retailer with high credibility, and a financial institution, all of whom are risk neutral. Such parties in this supply chain collaborate to deal with the financial stress by using two financing strategies (APD and BPOF). The BPOF strategy means that the supplier obtains funds from the financial institution based on a purchase order with the established retailer's guarantee to deliver products on time. By using APD, the retailer can finance the supplier by pre-paying for products at a discount before delivery. That is, the supplier obtains early payment from the retailer for production. Figure 1 displays the sequence of events of the two financing instruments (Zhao and Huchzermeier 2019).

## BPOF




Figure 1. The sequence of events of two financing instruments.

Figure 1 Alt Text: Participants in this supply chain collaborate to deal with the financial stress by using APD and BPOF financing strategies. Under BPOF, the supplier obtains funds from the financial institution based on a purchase order with the established retailer's guarantee to deliver products on time. Under APD, the retailer can finance the supplier by pre-paying for products at a discount before delivery.

Moreover, there are three stages in the process: 1) financing, in which the retailer chooses a financing strategy to reduce the supplier's financial stress; 2) executing: both parties complete the business; 3) salvage processing stage: unsold items are salvaged. The timeline of events is shown in Figure 2, which is based on the study of Zhao and Huchzermeier (2019). Subscripts "S" and "R", and "F" represent the supplier, the downstream retailer, and the financial institution. To be specific, our model for executing the inventory problem has two periods: a normal sales period (executing stage) where green products are sold at a predetermined price $p$ (Pasternack 1985; Wang and Webster 2009), and a clearance sales period (salvage processing stage) where items are settled at price $s$. The salvage value is determined based on time.


Figure 2. Timeline of events.

Figure 2 Alt Text: There are three stages in the process: 1) financing, in which the retailer chooses a financing strategy to reduce the supplier's financial stress; 2) executing: both parties complete the business; 3) salvage processing stage: unsold items are salvaged.

Furthermore, $D$ is the uncertain demand. The cumulative distribution function (CDF) is $F(D)$ which is absolutely continuous with probability density function (PDF) $f(D)$. The complementary CDF is $\bar{F}(D)=1-F(D)$, and the hazard rate is $h(D)=f(D) / \bar{F}(D)$, increasing in $D$. Let $H(D)=D h(D)$ represent the generalized failure rate. We assume that the distribution function includes a strictly increasing generalized failure rate, and $h(D)=D f(D) /(1-F(D)($ Lariviere and Porteus 2001). In this paper, we assume that the demand for surplus stock is clearance time independent. After the sales season ends, any leftover inventory $[\min (q, K)-D]^{+}$is salvaged at $s$. In practice, the unit salvage value is not fixed, but depends on clearance time decision: the shorter time duration of the product in the warehouse, the higher the value. Therefore, in this paper, it is assumed that the salvage value for the surplus is a function of the clearance time, that is $s(t)=a$ - $b t$ where the values of $a$ and $b$ are
known and constant. Here $a$ is the initial (maximum) salvage value, and $b$ is the sensitivity of price to the time ( $b \geq 0$ ). Muyldermans, Van Wassenhove, and Guide (2019) stated that the life cycle of some products was very short, the value would be lost rapidly over time, and when the new generation of products was launched, the resale income would drop sharply. Typically, companies will set a low salvage value for products that have been obsolete for a long time and are of low use value and look outdated, are relatively cheap, or for products that are rapidly becoming obsolete (printers, laptops, etc.).

In our paper, the supplier is experiencing financial distress in the business, and has short-term debt $L_{s}$ and stochastic internal assets $A_{s} \in\left[\underline{A}_{s}, \bar{A}_{s}\right], 0 \leq \underline{A}_{s}<\bar{A}_{s} \leq \infty$ (Babich 2010). Besides, a supplier failing to pay off debts can either be liquidated or reorganized. The liquidation cost is a ratio $1-\gamma$ of the enterprise value, and the reorganization cost is a ratio $1-\alpha$ of the raised funds (Gamba and Triantis 2014). In BPOF, the supplier will receive a $\lambda w q$ loan through a $\operatorname{contract}(\bar{\lambda}, r)$ from the financial institution. Here $\lambda \in[0, \bar{\lambda}]$ represents the borrowing level and $r$ is the interest rate. In the case of liquidation, the retailer, as the credit guarantor, pays the previously defined part of the loss of financial institutions $(\delta)$. Although the retailer provides financing to the supplier, he/she may also experience financial distress, and if he/she can't meet his debt obligations, the company can be liquidated or reorganized. The lending market is highly competitive and the financial institution's expected profit is zero (Zhao and Huchzermeier 2019). For convenience, the notations are summarized in Table 1.

Table 1. Summary of notations.

| Notation | Description |
| :--- | :--- |
| $\pi$ | Expected profit |
| $A$ | Asset level |
| $L$ | Short-term debt |
| $D$ | Product demand |
| $p$ | Unit selling price |
| $w$ | Unit wholesale price |
| $\alpha$ | Proportional distress cost |
| $\gamma$ | Proportional liquidation cost |
| $\delta$ | The portion of financial institution's loss reimbursed by retailer |
| $c_{p}$ | Unit production cost |
| $c_{k}$ | Unit capacity cost |
| $r$ | The interest rate of BPOF |
| $s(t)$ | Unit salvage value of the item at time $t$ |
| $K$ | Supplier's capacity level, decision variable |
| $q$ | Order quantity, decision variable |
| $d$ | Discount rate under APD, decision variable |
| $\lambda$ | The portion of purchase order value financed by a financial institution |

### 3.2 Financing with APD or BPOF

### 3.2.1. APD with a time-varying salvage value

By adopting APD financing, the retailer will pay the supplier before product delivery.
The supplier should first offer a discount rate $d$ in APD financing, and then the retailer will decide the order quantity $q$. Next, the supplier will determine the capacity level $K$ which depends on $q$ and how much support the working capital can provide. For any outstanding pre-paid orders, the retailer will refund the supplier. Hence, we can obtain that the supplier's profit function as:

$$
\begin{equation*}
\pi_{s}^{a p d}(K, d)=\left[w(1-d)-c_{p}\right] \min (q, K)-c_{k} K-(1-\alpha)\left[L_{s}-A_{s}+c_{k} K-w(1-d) q\right]^{+} . \tag{1}
\end{equation*}
$$

When $L_{s}-A_{s}+c_{k} K-w(1-d) q \leq 0$, it means that the supplier's current assets can meet liabilities and can continue to operate. Otherwise, the supplier enters the reorganization or liquidation stage, and will go into liquidation only if the cost of the financial distress exceeds the operating profit of the restructured company (Yang,

Birge, and Parker 2015). For liquidation, the optimal capacity $K^{a p d *}=0$ otherwise $K^{\text {apd }{ }^{*}}$ satisfies the first-order conditions:

$$
\begin{array}{ll}
(1-d)\left[p-(p-a+b t) F\left(K^{a p d^{*}}\right)\right]\left[1-h\left(K^{a p d^{*}}\right)\right]=c_{p}+c_{k} & \text { Continuation } \\
(1-d)\left[p-(p-a+b t) F\left(K^{a p d^{*}}\right)\right]\left[1-h\left(K^{a p d^{*}}\right)\right]=c_{p}+(2-\alpha) c_{k} \text { Reorganization } \tag{2}
\end{array}
$$

The profit function of the retailer under the APD scenario is found as follows:

$$
\begin{align*}
\pi_{r}^{a p d}(q) & =p \operatorname{E} \min [D, \min (q, K)]-w(1-d) \min (q, K)  \tag{3}\\
& +(a-b t)[\min (q, K)-D]^{+}-(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q\right]^{+}
\end{align*}
$$

Here $(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q\right]^{+}$is the cost of financial distress in reorganization, and the retailer needs to estimate his/her operating conditions based on the initial order quantity $q$ before financing. Once in liquidation, the retailer will not adopt APD financing. Furthermore, if the supplier's working capital is sufficient to support APD, the retailer's optimal order quantity $q^{a p d *}$ is:

$$
\begin{array}{ll}
p-(p-a+b t) F\left(q^{a p d^{*}}\right)=w(1-d) & \\
\text { Continuation }  \tag{4}\\
p-(p-a+b t) F\left(q^{a p d^{*}}\right)=w(1-d)(2-\alpha) & \\
\text { Reorganization. }
\end{array}
$$

The retailer's expected profit function is:

$$
\begin{align*}
E\left(\pi_{r}^{a p d}\right)= & {[p-w(1-d)] \min (q, K)-(p-a+b t) \int_{0}^{\min (q, K)} F(x) d x }  \tag{5}\\
& -(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q\right]^{+}
\end{align*}
$$

Proposition 1. Under APD, $\pi_{r}^{a p d}$ and $q^{b s a p d *}$ are relevant to $s$, and $\frac{\partial \pi_{r}^{a p d}}{\partial t}<0$,

$$
\frac{\partial q^{a p d^{*}}}{\partial t}<0 .
$$

This proposition shows that the retailer's expected profit and the optimal order quantity are affected by the salvage value, while they are negatively affected by $t$, since an item with a high salvage value is more attractive to the retailer, allowing the
ordering more products and make more money. In contrast, for a product with a low salvage value, the order quantity and the profit of the retailer are less than those with a high salvage value.

Proposition 2. Given $w, p$ and $K, \frac{\partial d}{\partial t}<0$.
Proposition 2 shows that in APD, the later the products are sold during the clearance period, the lower the discount rate offered by the supplier. This is because the salvage value is greatly affected by the clearance time. The later the time, the lower the salvage value of unsold products will be. When the time-varying salvage value is low, it harms the retailer's profit, that is, the retailer's financial risks become high. Therefore, in the face of the high financial risks of the retailer, the supplier should provide the retailer with a lower discount rate to maintain a higher retailer's purchase price (wholesale price), thus reducing the financial risks. Some retailers such as Wal-Mart have taken this discount rate into account when offering APD to the cash-constrained suppliers through C2FO financing platforms (Qin et al. 2019).

Theorem 1. To ensure that the downstream retailer benefits from APD and chooses it, the time of the time-varying salvage value is met $t<\eta_{r}^{a p d^{*}}$ where $\eta_{r}^{a p d^{*}}=\frac{a}{b}-\frac{w(1-d) \min (q, K)-p \mathrm{E} \min [D, \min (q, K)]+(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q\right]^{+}}{b[\min (q, K)-D]^{+}}$.

Theorem 1 shows that to ensure that participants benefit from APD financing, the clearance time should be within a certain range. The financing mode decision for the retailer is related to the clearance time. A higher return at a clearance time below a certain value would prompt the retailer to apply APD financing. Otherwise, a late
clearance time will lead to a low salvage value, affecting the profit of the retailer which brings great financial risks.

### 3.2.2. BPOF with a time-varying salvage value

Under BPOF financing, the retailer will offer a loan guarantee to the financial institution to help the supplier obtain funds. The retailer will first decide on $q$ and provide a loan guarantee via BPOF, and then the financial institution will fund the supplier at an interest rate $r$. The supplier will next determine the capacity level $K$ which depends on $q$ and the working capital. We can derive that the supplier's profit under BPOF as:

$$
\begin{equation*}
\pi_{s}^{b p o f}(K)=\left(w-c_{p}\right) \min (q, K)-c_{k} K-\lambda w q r-(1-\alpha)\left(L_{s}-A_{s}+c_{k} K-\lambda w q\right)^{+} . \tag{6}
\end{equation*}
$$

If the supplier repays the short-term debt before shipment (that is $L_{s}-A_{s}+c_{k} K-\lambda w q \leq 0$ ), the supplier continues to operate during the selling season. Conversely, the supplier goes into bankruptcy. If the supplier's internal assets fall below the threshold $\mathscr{F}_{s}=L_{s}+c_{k} K-\lambda w q-\frac{\left(w-c_{p}\right) \min (q, K)-c_{k} K-\lambda w q r}{1-\alpha}$, the supplier liquidates, and the probability of liquidation is $\operatorname{Pr}(l)=\Phi\left(\mathscr{R}_{s}\right)$. For liquidation, $K^{\text {bpof* }}=0$. Otherwise $K^{\text {bpof* }}$ satisfies:

$$
\begin{array}{lll}
{\left[p-(p-a+b t) F\left(K^{\text {bpof** }}\right)\right]\left[1-h\left(K^{\text {bpof** }}\right)\right]=c_{p}+c_{k}} & \text { Continuation }  \tag{7}\\
{\left[p-(p-a+b t) F\left(K^{\text {bpof** }}\right)\right]\left[1-h\left(K^{\text {bpof** }}\right)\right]=c_{p}+(2-\alpha) c_{k}} & \text { Reorganization }
\end{array}
$$

In the scenario of BPOF, the retailer's profit is:

$$
\begin{align*}
\pi_{r}^{b p o f}(q)= & p E \min [D, \min (q, K)]-w \min (q, K)+(a-b t)[\min (q, K)-D]^{+}- \\
& \int_{A_{s}}^{q_{s}} \delta\left\{\lambda w q-\gamma\left[\left(w-c_{p}\right) \min (q, K)-c_{k} K-\lambda w q r+A_{s}-L_{s}\right]\right\} \phi\left(A_{s}\right) d A_{s} \tag{8}
\end{align*}
$$

The expected profit of the retailer is:

$$
\begin{align*}
E\left(\pi_{r}^{b p o f}\right)= & (p-w) \min (q, K)-(p-a+b t) \int_{0}^{\min (q, K)} F(x) d x- \\
& \int_{A_{s}}^{q_{s}} \delta\left\{\lambda w q-\gamma\left[\left(w-c_{p}\right) \min (q, K)-c_{k} K-\lambda w q r+A_{s}-L_{s}\right]\right\} \phi\left(A_{s}\right) d A_{s} \tag{9}
\end{align*}
$$

Hence, we can obtain the optimal order quantity as $q^{b p o f} *=F^{-1}\left(\frac{p-w-\left(\delta \lambda w-\delta \gamma w+\delta \gamma c_{p}+\delta \gamma \lambda w r\right)\left[\Phi\left(\mathscr{F}_{s}^{O}\right)-\Phi\left(\underline{A}_{s}\right)\right]}{p-a+b t}\right)$.

Proposition 3. Under BPOF, both $\pi_{r}^{\text {bpof }}$ and $q^{b p o f^{*}}$ are affected by $s$, and $\frac{\partial \pi_{r}^{b p o f}}{\partial t}<0$,

$$
\frac{\partial q^{b p o f^{*}}}{\partial t}<0 .
$$

Proposition 3 suggests that if there is surplus stock, a shorter clearance time incentivizes the retailer to place more orders, reducing the losses and maximizing the profit because the salvage value of the unsold inventory is greater. Besides, regardless of whenever the item is sold during the clearance period, the supplier's capacity level and profit are not correlated with the time-varying salvage value because all unsold items are handled by the retailer.

Since the supplier and the retailer can repay the entire debt, the financial institution's profit is

$$
\pi_{f}=\lambda w q r \bar{\Phi}\left(\mathbb{R}_{o_{s}}\right)+\int_{\mathbb{A}_{s}}^{\mathbb{K}_{s}}(1-\delta)\left\{\gamma\left[\begin{array}{l}
\left(w-c_{p}\right) \min (q, K)-c_{k} K  \tag{10}\\
-\lambda w q r+A_{s}-L_{s}
\end{array}\right]-\lambda w q\right\} \phi\left(A_{s}\right) d A_{s} .
$$

Since the capital market is competitive, the financial institution has zero expected returns. Hence, we can obtain the financial institution's interest rate:

Proposition 4. Under BPOF, the financial institution's interest rate $r$ is positively affected by $s$.

Proposition 4 suggests that if there is surplus stock, the time-dependent salvage value will affect the financial institution's interest rate. The reason is that the shorter the clearance time, the higher the salvage value, and the retailer is willing to order more products because the inventory risk of products with high salvage value is low. Moreover, since the retailer has collateral liability in BPOF financing, he/she may also face financial distress. As a result, as the retailer orders more, the retailer faces greater financial risks, and the financial institution should offer a higher interest rate to avoid possible losses and reduce financing risks when the clearance time is shorter. In practice, Jingdong, a Chinese e-commerce company, launched its financing service for cash-strapped suppliers by considering the interest rate in late 2013. By 2015, loans exceeded 4 billion USD, serving more than 2,000 enterprises (Yang et al. 2019).

Theorem 2. To ensure that the downstream retailer benefits from BPOF and chooses it, the time of the time-varying salvage value is met for $t<\eta_{r}^{\text {bpof }}$ * where $\eta_{r}^{b \text { pof* }}=\frac{a}{b}-\frac{w \min (q, K)-p \min [D, \min (q, K)]+\int_{A_{s}}^{\tau_{s}} \delta\left\{\lambda w q-\gamma\left[\begin{array}{l}\left(w-c_{p}\right) \min (q, K)- \\ c_{k} K-\lambda w q r+A_{s}-L_{s}\end{array}\right]\right\} \phi\left(A_{s}\right) d A_{s}}{b[\min (q, K)-D]^{+}}$.

Theorem 2 ensures that the retailer benefits from the BPOF financing model, and the clearance time should be within a certain range. An early clearance time will lead to high salvage value which brings more profits to the retailer and few financial risks.

## 4. Financing equilibrium with a time-varying salvage value

When either of the financing instruments is sufficient to meet the working capital demand of the supplier, the retailer will choose one of the two financing strategies.

Theorem 3. Under a single financing condition, a unique threshold of the retailer's internal asset level $\omega_{r}$ exists, which makes the retailer more inclined to BPOF if $A_{r}<\omega_{r}$. Otherwise, APD is preferred. In addition, $\frac{\partial \omega_{r}}{\partial t}<0$.

Theorem 3 reveals that considering the time-varying salvage value, $\omega_{r}$ exists to ensure that the retailer prefers to apply BPOF. The reason is that the earlier the clearance time, the greater the salvage value which promotes the retailer to order more, while the increase of order quantity increases the inventory risk and makes the inventory tend to be overstocked. Therefore, the retailer should have a higher asset level if the clearance period is too long so that the retailer has sufficient funds to provide APD financing for the supplier.

Theorem 4. Under a single financing condition, a unique threshold of the clearance time $\omega_{t}$ exists, which makes the retailer more inclined to BPOF if $t<\omega_{t}$. Otherwise, APD is preferred.

Theorem 4 reveals that the threshold of clearance time ensures that the retailer prefers to apply BPOF. The reason is that the later the clearance time, the smaller the salvage value of the unsold goods which decreases the retailer's order quantity and reduces the risk of excess inventory. Under BPOF, the retailer only needs to pay part of the losses of the financial institution as compensation when the supplier is in a liquidation state, while under APD, the retailer needs to have sufficient funds to finance the supplier. Therefore, if the clearance time is late (at the time when the
salvage value is low), the retailer will order less and suffer fewer losses, and should choose APD due to having more funds to finance the supplier. If the clearance time is early, BPOF has a relative advantage. This is because the retailer may order more and therefore might not have sufficient funds to finance the supplier, and BPOF is the better choice.

## 5. Profit risk analysis

The retailer's profit risks under the time-varying salvage value scenarios are analyzed by conducting a mean-variance analysis (Choi, Li, and Yan 2008). The variance of the retailer's profit under APD is:

$$
\begin{equation*}
V\left(\pi_{r}^{a p d}\right)=\mathrm{E}\left[\left(\pi_{r}^{a p d}\right)^{2}\right]-\left[\mathrm{E}\left(\pi_{r}^{a p d}\right)\right]^{2} \tag{12}
\end{equation*}
$$

Putting (3) \& (5) into (12), we have
$V\left(\pi_{r}^{a p d}\right)=(p-a+b t)^{2}\left[2 \min (q, K) n-2 \int_{0}^{\min (q, K)} x F(x) d x-n^{2}\right]$
where $n=\int_{0}^{\min (q, K)} F(x) d x$. For $\frac{d V\left(\pi_{r}^{a p d}\right)}{d q}=2(p-a+b t)^{2}[1-F(\min (q, K))] n \geq 0 \quad$ and $\frac{d V\left(\pi_{r}^{a p d}\right)}{d t}=2(p-a+b t) b \geq 0$. Thus, $V\left(\pi_{r}^{a p d}\right)$ is a monotone increasing function of $q^{a p d}$ and $t$.

The variance of the retailer's profit under BPOF is:

$$
\begin{equation*}
V\left(\pi_{r}^{b p o f}\right)=\mathrm{E}\left[\left(\pi_{r}^{b p o f}\right)^{2}\right]-\left[\mathrm{E}\left(\pi_{r}^{b p o f}\right)\right]^{2} \tag{14}
\end{equation*}
$$

Putting (8) \& (9) into (14), we have

$$
\begin{equation*}
V\left(\pi_{r}^{\text {bpof }}\right)=(p-a+b t)^{2}\left[2 \min (q, K) n-2 \int_{0}^{\min (q, K)} x F(x) d x-n^{2}\right] \tag{15}
\end{equation*}
$$

where $n=\int_{0}^{\min (q, K)} F(x) d x$. For $\frac{d V\left(\pi_{r}^{b p o f}\right)}{d q}=2(p-a+b t)^{2}[1-F(\min (q, K))] n \geq 0$ and $\frac{d V\left(\pi_{r}^{b p o f}\right)}{d t}=2(p-a+b t) b \geq 0$. Thus, $V\left(\pi_{r}^{b p o f}\right)$ is a monotone increasing function of $q^{\text {bpof }}$ and $t$.

The profit of the entire supply chain in the APD financing mode is

$$
\begin{align*}
\pi_{s c}^{a p d}= & p \operatorname{E} \min [D, \min (q, K)]+(a-b t)[\min (q, K)-D]^{+}-c_{p} \min (q, K)-c_{k} K  \tag{16}\\
& -(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q\right]^{+}-(1-\alpha)\left[L_{s}-A_{s}+c_{k} K-w(1-d) q\right]^{+} .
\end{align*}
$$

The entire supply chain's expected profit under APD is:

$$
\begin{align*}
E\left(\pi_{s c}^{a p d}\right) & \left.=p\left[\min (q, K)-\int_{0}^{\min (q, K)} F(x) d x\right)\right]+(a-b t) \int_{0}^{\min (q, K)} F(x) d x-c_{p} \min (q, K) \\
& -c_{k} K-(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q\right]^{+}-(1-\alpha)\left[L_{s}-A_{s}+c_{k} K-w(1-d) q\right]^{+} .  \tag{17}\\
& =\left(p-c_{p}\right) \min (q, K)-(p-a+b t) \int_{0}^{\min (q, K)} F(x) d x \\
& -(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q\right]^{+}-(1-\alpha)\left[L_{s}-A_{s}+c_{k} K-w(1-d) q\right]^{+} .
\end{align*}
$$

Therefore, the variance of the entire supply chain's profit under APD is:
$V\left(\pi_{s c}^{a p d}\right)=(p-a+b t)^{2}\left[2 \min (q, K) n-2 \int_{0}^{\min (q, K)} x F(x) d x-n^{2}\right]$
where $n=\int_{0}^{\min (q, K)} F(x) d x$.
The profit of the entire supply chain in the BPOF financing mode is:

$$
\begin{align*}
\pi_{s c}^{b p o f}= & p \operatorname{E} \min [D, \min (q, K)]+(a-b t)[\min (q, K)-D]^{+}-c_{p} \min (q, K) \\
& -c_{k} K-\lambda w q r-\int_{A_{s}}^{\alpha_{s}} \delta\left\{\lambda w q-\gamma\left[\begin{array}{l}
\left(w-c_{p}\right) \min (q, K) \\
-c_{k} K-\lambda w q r+A_{s}-L_{s}
\end{array}\right]\right\} \phi\left(A_{s}\right) d A_{s}  \tag{19}\\
& -(1-\alpha)\left(L_{s}-A_{s}-\lambda w q+c_{k} K\right)^{+}
\end{align*}
$$

The entire supply chain's expected profit under BPOF is:

$$
\begin{align*}
E\left(\pi_{\mathrm{sc}}^{\text {bpof }}\right)= & \left(p-c_{p}\right) \min (q, K)-(p-a+b t) \int_{0}^{\min (q, K)} F(x) d x-\lambda w q r-c_{k} K \\
& -\int_{A_{s}}^{q_{s}} \delta\left\{\lambda w q-\gamma\left[\begin{array}{l}
\left(w-c_{p}\right) \min (q, K)-c_{k} K \\
-\lambda w q r+A_{s}-L_{s}
\end{array}\right]\right\} \phi\left(A_{s}\right) d A_{s}  \tag{20}\\
& -(1-\alpha)\left(L_{s}-A_{s}-\lambda w q+c_{k} K\right)^{+}
\end{align*}
$$

Therefore, the variance of supply chain profit under BPOF is:

$$
\begin{equation*}
V\left(\pi_{\mathrm{sc}}^{b p p o f}\right)=(p-a+b t)^{2}\left[2 \min (q, K) n-2 \int_{0}^{\min (q, K)} x F(x) d x-n^{2}\right] \tag{21}
\end{equation*}
$$

where $n=\int_{0}^{\min (q, K)} F(x) d x$.

Comparing the variances of the retailer's profits and the entire supply chain's profits under both APD and BPOF, we conclude the finding in Proposition 5.

Proposition

$$
\text { 5. (a) } \frac{\partial V\left(\pi_{r}^{a p d}\right)}{q^{\text {apd }}}>0, \frac{\partial V\left(\pi_{r}^{\text {bpof }}\right)}{q^{\text {bpof }}}>0 \text {; }
$$

(b).
$\frac{\partial V\left(\pi_{r}^{a p d}\right)}{t}>0, \frac{\partial V\left(\pi_{r}^{b p o f}\right)}{t}>0$; (c) $V\left(\pi_{r}^{a p d}\right)=V\left(\pi_{s c}^{a p d}\right), V\left(\pi_{r}^{b p o f}\right)=V\left(\pi_{s c}^{b p o f}\right)$.
Proposition 5 identifies the relationship of the variance of the retailer's profit with the order quantity and the clearance time. To be specific, Proposition 5(a) illustrates that the variance of profit risks of the retailer under APD and BPOF are increasing with the order quantity, that is, the higher order quantity leads to higher profit risks. Proposition 5(b) implies that the retailer's profit risks under APD and BPOF are related to the clearance time, that is, the earlier the retailer clears the inventory after the normal selling season, the lower the profit risk. Proposition 5(c) shows that the profit risks of the retailer and the profit risks of the entire supply chain under APD and BPOF are equal, which implies the retailer's profit risks determine the risks of the whole supply chain. Increasing numbers of enterprises are beginning to weigh between profit and risk. For example, Hewlett-Packard company has realized the importance of supply chain risk management and established a procurement risk management system to evaluate and control supply chain risks. Through this system, Hewlett-Packard saved at least \$100 US million in procurement costs in 2008 (Nagali et al. 2008).

## 6. Numerical analysis

In the following numerical study, information on the parameter values refers to an existing numerical study (Ranjbar et al. 2020) and the actual situation of the current appliance market, such as the price of an LG TV (55UP8000PUA Alexa Built-in 55" 4K Smart UHD TV) on Amazon. We approximate the parameters to satisfy the hypothesis of the problem so that using these parameters to solve problems is not far from reality. Therefore, the basic parameter settings are listed below: (1) the demand $D$ is normally distributed and can be expressed as $\mathrm{N}(1000,100)$. (2) for the cost-revenue parameter: $p=580$ USD, $c_{k}=c_{p}=120$ USD. (3) $\alpha=0.85$, and $\gamma=0.9$ (Gamba and Triantis 2014; Dedrick, Kraemer, and Linden 2011). (4) The time-varying salvage value is $s(t)=a-b t$ where initial (maximum) salvage value $a$ is 300 , and the sensitivity of price to the time $b$ is 10 . When adjusting the clearance time, we allow it to vary between 0 and 30 .

### 6.1. Impact of t on retailer's optimal order quantity under APD and BPOF

Figure 3 demonstrates that the clearance time influences the retailer's optimal order quantity. For example, when $t$ is relatively small (i.e., $t=3$ ), there is a larger order quantity ( $q=1001$ under APD financing, $q=1010$ under BPOF financing). When $t$ takes a large value (i.e., $t=27$ ), the two order quantities under APD ( $q=995$ ) and BPOF $(q=999)$ are smaller than when taking a small $t$ (i.e., $t=3$ ). Thus, under both financing strategies, the retailer prefers to order more goods with an earlier clearance time. The reason is that the products with a high salvage value (when the clearance time is early) can help the retailer avoid excessive losses and have higher inventory levels because the remaining items are still valuable. This can encourage more items to be ordered from the supplier. Thus, an earlier clearance time lowers the risks related to
random customer demand. The retailer does not have to worry about losing a lot of inventory, which allows the retailer to increase the number of orders to meet market demand.


Figure 3. Impact of $t$ on retailer's optimal order quantity under APD and BPOF.

Figure 3 Alt Text: The clearance time of the unsold products influences the retailer's optimal order quantity under APD and BPOF financing strategies. That is, under both financing strategies, the retailer prefers to order more goods with an earlier clearance time.

### 6.2. Impact of t on supplier's setting on discount rate under APD

We now examine how the salvage value affects the supplier's setting on the discount rate under APD financing. We show the results in Figure 4 which indicate that in APD, the discount rate provided by the supplier to the downstream retailer decreases as $t$ increases. For instance, when $t$ is relatively small (i.e., $t=6$ ), $d=0.41$, while as $t$ increases (i.e., $t=24, t=27$ ), the discount rate is significantly reduced ( $d=0.23$ and
0.19 , respectively). This reflects the fact that the later the clearance time, the more adverse the effect on the retailer, that is, the greater the financial risks of the retailer; hence, in the face of high financial risks of the retailer, the supplier should offer a lower discount rate to maintain a higher wholesale price in the order, and thereby reduce the financial risks and maintain a higher return.


Figure 4. Impact of $t$ on supplier's discount rate.

Figure 4 Alt Text: The clearance time of the unsold products influences the discount rate provided by the supplier under APD, and the discount rate decreases as the clearance time increases.

### 6.3. Impact of t on financing equilibrium

We obtain the relationship between $\omega_{r}$ and $t$ (see Figure 5), as well as $\omega_{t}$ which is shown in Figure 6. In Figure 5, we can see that $\omega_{r}$ is more than $5.32 \times 10^{5}$ when $t=9$, while for $t=24, \omega_{r}$ is about $5.25 \times 10^{5}$ which is lower than that in the case of early
clearance time. It's clear that the clearance time has a significant effect on the threshold $\omega_{r}$. The underlying reason is that the earlier the clearance time, the greater the salvage value. This promotes the retailer to order more, while the increase of the order quantity increases the inventory risk and makes the inventory tend to be overstocked, thereby increasing the cost of the inventory backlog. Therefore, the retailer should have a higher asset level if the clearance period is too long so that the retailer has sufficient funds to provide APD financing for the supplier.


Figure 5. Impact of $t$ on $\omega_{r}$.

Figure 5 Alt Text: Considering the time-varying salvage value, a unique threshold of the retailer's internal asset level $\omega_{r}$ exists to ensure that the retailer prefers to apply BPOF, and the retailer should have a higher asset level if the clearance period is too long so that the retailer has sufficient funds to provide APD financing.

Figure 6 illustrates that there is a unique threshold of clearance time which ensures the retailer prefers BPOF mode. The retailer prefers BPOF when the clearance time is relatively early (i.e., if $\omega_{t}<12.97$ ); otherwise, APD is preferred. If the clearance time is sufficiently late, APD dominates BPOF. Therefore, BPOF dominates APD in financing equilibrium when the clearance time is below a certain threshold. The reason is that the later the clearance time, the smaller the salvage value, thus reducing the retailer's order quantity, reducing the risk of excess inventory, and reducing the losses. Therefore, APD should be chosen because of having more funds to finance the supplier. If clearance is early, BPOF has a comparative advantage, as the retailer is likely to order more and therefore does not have sufficient funds to subsidize the capital-constrained supplier, hence the supplier needs to obtain the funds from the financial institution.


Figure 6. The unique threshold of clearance time.

Figure 6 Alt Text: There is a unique threshold of clearance time which ensures the retailer prefers BPOF mode, that is, BPOF dominates APD in financing equilibrium when the clearance time is below a certain threshold.

### 6.4. Impact of the threshold of the retailer's asset level on retailer's financing

 channel decisionIn Figure 7, under APD (blue curve), the retailer's expected profit increases with the increase of the retailer's internal asset level; however, the red BPOF curve is stable. By comparing APD and BPOF (blue and red curves), Theorem 3 is verified in that BPOF dominates APD in financing equilibrium when the internal asset level of the retailer is below a certain threshold $\mathrm{Ar}<\omega$ (i.e., $\mathrm{Ar}<5.49 \times 10^{5}$ ). When the internal asset level of the retailer exceeds a certain level (i.e., $\mathrm{Ar}>5.54 \times 10^{5}$ ), APD dominates BPOF in financing equilibrium, which is due to the increase of the asset level alleviating the financial difficulties. These results are consistent with Theorem 3. Therefore, when the retailer has a high level of internal assets and does not have to worry about financial issues, the retailer prefers to provide APD via using internal sources of capital for the supplier instead of applying BPOF.


Figure 7. Impact of the threshold of the retailer's asset level on retailer's financing channel decision.

Figure 7 Alt Text: BPOF dominates APD in financing equilibrium when the internal asset level of the retailer is below a certain threshold. When the internal asset level of the retailer exceeds a certain level, APD dominates BPOF.

### 6.5. Impact of different factors ( $q$ \& $t$ ) on the variance of retailer profit

From Figure 8, the variances of the retailer's profit under APD and BPOF are related to the order quantity $q$. To be specific, the variances of the profit risks of the retailer are increasing with the order quantity. It is obvious that the variance of the retailer's profit is more than $5.45 \times 10^{6}$ when $q=996$ under APD. When $q$ increases (i.e., $q$ $=1004$ ), the variance of the retailer's profit is about $1.32 \times 10^{7}$, which is higher than the variance of the retailer's profit when $q=996$. This phenomenon also exists in BPOF financing. This is consistent with Proposition 5(a). Hence, we can verify that under
both APD and BPOF, the higher order quantity leads to the higher variances of profit risks, which brings greater financial risks to the retailer.


Figure 8. Impact of $q$ on the variance of retailer profit under APD and BPOF.
Figure 8 Alt Text: The variances of the retailer's profit under APD and BPOF are increasing with the retailer's order quantity.

The clearance time has a significant effect on the variance of the retailer's profits under both APD and BPOF, as shown in Figure 9. By examining the impact, we find that $V\left(\pi_{r}^{\text {apd }}\right)=6.48 \times 10^{6} V\left(\pi_{r}^{\text {bpof }}\right)=1.95 \times 10^{7}$ when $t=3$; as $t$ becomes larger, such as when $t=21$, the variances of the retailer's profit under APD and BPOF $\left(V\left(\pi_{r}^{a p d}\right)=1.62 \times 10^{7}, V\left(\pi_{r}^{\text {bpof }}\right)=4.87 \times 10^{7}\right)$ is greater than the variances of the retailer's profit when $t=3$. This suggests that under both APD and BPOF, the earlier the retailer clears the inventory after the normal selling season, the lower the profit risks. Therefore, if the retailer wants to clear unsold inventory later, then he/she should also bear a higher risk, which is consistent with Proposition 5(b).


Figure 9. Impact of $t$ on the variance of retailer profit under APD and BPOF.

Figure 9 Alt Text: The clearance time has a significant effect on the variance of the retailer's profits under both APD and BPOF, and the earlier the retailer clears the inventory after the normal selling season, the lower the profit risks.

## 7. Managerial insights

Nowadays, most industries attach great importance to green environmental protection.
By addressing the financial constraints of the supplier via APD and BPOF, more environmentally friendly products can be produced. This study provides the following management insights in Table 2, which are briefly summarized.

Table 2. Summary of the managerial insights

| Managerial <br> insights | Descriptions |
| :--- | :--- |
| From the supplier's <br> perspective | 1) Selecting an appropriate financing decision can help the supplier <br> successfully produce more green products. <br> 2) The supplier's capacity level and expected profits are not affected by <br> time-varying salvage values. <br> 3) Under APD, it is suggested that the supplier should provide a lower |


|  | discount rate for products with lower salvage values to obtain higher returns <br> and lower the financial risks. |
| :--- | :--- |
|  | 1) By helping the supplier with financing instruments, the retailer can <br> smoothly obtain as many green products as possible to meet the order <br> requirements. <br> 2) Under APD and BPOF, the time-varying salvage value affects the <br> optimal order quantity and the retailer's expected profits. The retailer should <br> deal with those unsold inventories as soon as possible to reduce the losses <br> caused by overordering. <br> From the retailer' |
| perspective retailer needs to pay attention to the order quantity and the clearance |  |
| time. When the order quantity is larger and the clearance time is later, the |  |
| retailer's profit risk level becomes higher, and the profit risks of the retailer |  |
| determine the profit risks of the whole supply chain. |  |
| 4) The retailer should have a higher asset level if the clearance period is |  |
| long so that the retailer has sufficient funds to provide APD, otherwise, |  |
| BPOF is preferred, and the threshold is inversely proportional to the |  |
| clearance time. |  |$|$| 1) Under BPOF, the financial institution can help the supplier solve funding |
| :--- |
| problems without increasing credit risk and establish a relationship among |
| participants. |
| 2) The financial institution is suggested to offer a higher interest rate to |
| avoid excessive losses and financing risks when the clearance time is |
| shorter. |

## 8. Conclusions

In the face of the new challenges and requirements of consumers in regard to environmental protection, enterprises all over the world are adopting "Green" and " Energy conservation and emission reduction" as their themes and carrying out appropriate research and development. Since some products such as electrical appliances are innovative products with a short life cycle, suppliers are constantly introducing new environmentally friendly products. After the selling season, there are unsold items, which can be sold at salvage value to remote areas of the country or exported to less economically developed countries. Such approaches can promote environmental sustainability by replacing items that are more environmentally friendly than before. It's worth noting that the salvage value varies over time, with a lower value the later they are cleared. In addition, many suppliers face financial
constraints. Therefore, this paper can help the supplier with capital constraints obtain financing to produce more green products, with our established model via APD and BPOF, following Zhao and Huchzermeier (2019)'s study. However, the difference is that our study focuses on the impact of the time-dependent salvage value on the operation and financing decisions of supply chain participants. In addition, the supply chain system's level of risk was modelled.

Some significant outcomes result from this study are: 1) A supply chain system consisting of a cash-strapped supplier and a retailer is presented; 2) A financially constrained supplier can obtain funds via two effective financing instruments (APD and BPOF) to produce green items smoothly. 3) The salvage value of the product varies according to clearance time, and the clearance time affects the retailer's optimal order quantity, the supplier's decision on the discount rate under APD, and the financial institution's decision on the interest rate under BPOF; 4) the financing equilibrium is BPOF when the clearance time is below a certain threshold or the retailer's internal asset level is below a certain level and the threshold of the asset level is inversely proportional to the clearance time; 4) The profit risks of the retailer under APD and BPOF increase with the order quantity and the clearance time, and the profit risks of the retailer and the entire supply chain are equal, which implies that the retailer's profit risks determine the risks of the whole supply chain.

This research has some limitations. Firstly, we assume that the demand for surplus inventory is independent of the clearance time, while in real life, the clearance time of many products will impact on the demand for surplus inventory. In the future, it would be appropriate to study the situation where the residual demand depends on the clearance time. Secondly, this research is mainly focused on the relationship between the time-varying salvage value and the supply chain strategy with different
financing modes for producing green products smoothly. However, we do not mention the role of green (low carbon) technology and green preferences in the supply chain. Hence, in the future, we will investigate how these two key green factors play a role in the capital contrasted supply chain with financing strategies (Peng, Pang, and Cong 2018; Cong, Pang, and Peng 2020). Finally, to further verify the feasibility and practicability of the presented model, empirical research on specific companies will be investigated, and more complex studies on other products carried out to obtain more significant results.

## Funding

The work described in this paper was supported by grants from The Natural Science Foundation of China (Grant No. 71971143); The Research Committee of Hong Kong Polytechnic University (Project Numbers G-UADM; G-UAFS; Student account code RK2C).

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## Appendix Proofs

Proof of Proposition 1 From Eq. (4), we can obtain that $\frac{\partial \pi_{r}^{a p d}}{\partial q}=p-w(1-d)-[p-(a-b t)] F(q)>0$. The optimal order quantity $q^{a p d^{*}}$ in the case of continuation satisfies the first-order condition, and can be derived from: $p-(p-a+b t) F\left(q^{a p d^{*}}\right)=w(1-d)$.

Hence, $q^{a p d *}=F^{-1}\left(\frac{p-(1-d) w}{p-a+b t}\right)$.
In the same way, we can also derive that in the reorganization,

$$
\begin{aligned}
& p-(p-a+b t) F\left(q^{a p d^{*}}\right)=w(1-d)(2-\alpha) \\
& q^{a p d^{*}}=F^{-1}\left(\frac{p-(2-\alpha)(1-d) w}{p-a+b t}\right)
\end{aligned}
$$

To sum up, we have

$$
q^{a p d^{*}}=\left\{\begin{array}{cc}
F^{-1}\left(\frac{p-(1-d) w}{p-a+b t}\right) & \text { Continuation } \\
F^{-1}\left(\frac{p-(2-\alpha)(1-d) w}{p-a+b t}\right) & \text { Reorganization }
\end{array}\right.
$$

Moreover, we can derive that

$$
\begin{aligned}
& \frac{\partial^{2} \pi_{r}^{a p d}}{\partial q^{2}}=-[p-(a-b t)] f(q)<0 \\
& \frac{\partial^{2} \pi_{r}^{a p d}}{\partial q \partial t}=-b F(q)<0 \\
& \frac{d q}{d t}=\frac{-b F(q)}{[p-(a-b t)] f(q)}<0
\end{aligned}
$$

Hence, $q^{a p d d^{*}}$ and $\pi_{r}{ }^{a p d}$ decrease with time.

Proof of Proposition 2 Since $d=1-\frac{c_{p}+c_{k}}{\left[p-(p-a+b t) F\left(K^{\text {apd* }}\right)\right]\left[1-h\left(K^{\text {apd** }}\right)\right]}$, and $\frac{\partial d}{\partial t}<0$. The discount rate $d$ is decreasing with time.

Proof of Theorem 1 To satisfy both $\pi_{r}{ }^{\text {apd }}>0$, we can derive that: $(a-b t)[\min (q, K)-D]^{+}>w(1-d) \min (q, K)-p \mathrm{E} \min [D, \min (q, K)]$ $+(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q\right]^{+}$ $w(1-d) \min (q, K)-p E \min [D, \min (q, K)]$
$t<\eta_{r}^{a p d^{*}}=\frac{a}{b}-\frac{+(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q\right]^{+}}{b[\min (q, K)-D]^{+}}$.

Proof of Proposition 3 Taking the first-order and second-order partial derivative of $\pi_{r}^{\text {bpof }}$ we have $\frac{\partial \pi_{r}^{b p o f}}{\partial q}=(p-a+b t) \bar{F}(q)-\left[w-a+b t+\int_{\underline{A}_{s}}^{q_{s}}\left(\delta \lambda w-\delta \gamma w+\delta \gamma c_{p}+\delta \gamma \lambda w r\right) \phi\left(A_{s}\right) d A_{s}\right]$
$\frac{\partial^{2} \pi_{r}^{b p o f}}{\partial q^{2}}=-(p-a+b t) f(q)<0$
$\frac{\partial^{2} \pi_{r}^{\text {bpof }}}{\partial q \partial t}=-b f(q)<0$
Accordingly,

$$
\frac{d q}{d t}=\frac{\partial^{2} \pi_{r}^{\text {bpof }} / \partial q \partial t}{-\partial^{2} \pi_{r}^{\text {bpof }} / \partial q^{2}}=\frac{-b f(q)}{(p-a+b t) f(q)}<0
$$

Hence, $q^{\text {bpof }} *$ and $\pi_{r}^{\text {bpof }}$ increase with the salvage value.
From the first-order condition: $\frac{\partial \pi_{r}^{\text {bpof }}}{\partial q^{\text {bpof** }}}=0$, we have

$$
\begin{aligned}
& (p-a+b t) \bar{F}(q)=w-a+b t+\left(\delta \lambda w-\delta \gamma w+\delta \gamma c_{p}+\delta \gamma \lambda w r\right)\left[\Phi\left(\mathscr{A}_{S}^{o}\right)-\Phi\left(\underline{A}_{S}\right)\right] \\
& \bar{F}(q)=\frac{w-a+b t+\left(\delta \lambda w-\delta \gamma w+\delta \gamma c_{p}+\delta \gamma \lambda w r\right)\left[\Phi\left(\mathscr{A}_{S}\right)-\Phi\left(\underline{A}_{S}\right)\right]}{p-a+b t} \\
& F(q)=\frac{p-w-\left(\delta \lambda w-\delta \gamma w+\delta \gamma c_{p}+\delta \gamma \lambda w r\right)\left[\Phi\left(\mathscr{A}_{S}\right)-\Phi\left(\underline{A}_{S}\right)\right]}{p-a+b t}
\end{aligned}
$$

Therefore, $q^{\text {bpof** }}=F^{-1}\left(\frac{p-w-\left(\delta \lambda w-\delta \gamma w+\delta \gamma c_{p}+\delta \gamma \lambda w r\right)\left[\Phi\left(\AA_{s}^{0}\right)-\Phi\left(\underline{A}_{s}\right)\right]}{p-a+b t}\right)$.

Proof of Theorem 2 To satisfy $\pi_{r}^{\text {bpof }}>0$, we can derive that:

Proof of Theorem 3 The financing strategy chosen by the retailer will bring a greater expected profit: $\pi_{r}^{\text {bpof }}>\pi_{r}^{a p d *}$ if and only if

$$
p \operatorname{Emin}\left(D, q^{b p o f^{*}}\right)-w q^{b p o f *}+(a-b t)\left(q^{b p o f^{*}}-D\right)^{+}
$$

$$
-\int_{A_{s}}^{q_{s}} \delta\left\{\lambda w q^{\text {bpof }^{*}}-\gamma\left[\left(w-c_{p}\right) q^{\text {bpof }}-c_{k} K-\lambda w q^{\text {bpof }} r+A_{s}-L_{s}\right]\right\} \phi\left(A_{s}\right) d A_{s}
$$

$$
>p \operatorname{Emin}\left(D, q^{a p d^{*}}\right)-w(1-d) q^{a p d^{*}}+(a-b t)\left(q^{a p d^{*}}-D\right)^{+}-(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q^{a p d^{*}}\right]^{+}
$$

where the threshold value of the retailer's internal assets:

Since there exist $(a-b t)\left[\left(q^{a p d^{*}}-D\right)^{+}-\left(q^{b p p f^{*}}-D\right)^{+}\right]$and $q^{b p o f^{*}}>q^{a p d^{*}}>D$, the value $\omega_{r}$ is in inverse proportion to the clearance time.

$$
\begin{aligned}
& \omega_{r}=L_{r}+w(1-d) q^{a p d^{*}}- \\
& \frac{p \operatorname{Emin}\left(D, q^{a p d *}\right)-w(1-d) q^{\text {apd } d^{*}}+(a-b t)\left(q^{\text {apd } d^{*}}-D\right)^{+}-p \operatorname{E} \min \left(D, q^{\text {bpof** }}\right)+w q^{\text {bpof* }}-(a-b t)\left(q^{\text {bpof** }}-D\right)^{+}}{1-\alpha}
\end{aligned}
$$

$$
\begin{aligned}
& =L_{r}+w(1-d) q^{a p d^{*}}- \\
& \frac{p \operatorname{Emin}\left(D, q^{a p d^{*}}\right)-w(1-d) q^{a p d d^{*}}-p \operatorname{Emin}\left(D, q^{\text {bpof** }}\right)+w q^{b p p f^{*}}+(a-b t)\left[\left(q^{\text {apd** }}-D\right)^{+}-\left(q^{\text {bpof }}{ }^{*}-D\right)^{+}\right]}{1-\alpha} \\
& +\frac{\int_{\boldsymbol{A}_{s}}^{q_{5}} \delta\left\{\lambda w q^{\text {bpof }}-\gamma\left[\left(w-c_{p}\right) q^{\text {bpof }}{ }^{*}-c_{k} K-\lambda w q^{\text {bpof }} r+A_{s}-L_{s}\right]\right\} \phi\left(A_{s}\right) d A_{s}}{1-\alpha} .
\end{aligned}
$$

$$
\begin{aligned}
& \pi_{r}^{b p o f}(q, w)=p \min [D, \min (q, K)]-w \min (q, K)+(a-b t)[\min (q, K)-D]^{+}- \\
& \int_{A_{s}}^{\alpha_{s}} \delta\left\{\lambda w q-\gamma\left[\left(w-c_{p}\right) \min (q, K)-c_{k} K-\lambda w q r+A_{s}-L_{s}\right]\right\} \phi\left(A_{s}\right) d A_{s}>0 \\
& >\frac{w \min (q, K)-p \min [D, \min (q, K)]+\int_{{\underset{A}{s}}^{s}}^{\mathcal{K}_{s}} \delta\left\{\lambda w q-\gamma\left[\begin{array}{l}
\left(w-c_{p}\right) \min (q, K)- \\
c_{k} K-\lambda w q r+A_{s}-L_{s}
\end{array}\right]\right\} \phi\left(A_{s}\right) d A_{s}}{[\min (q, K)-D]^{+}} \\
& t<\eta_{r}^{\text {bpof** }}=\frac{a}{b}-\frac{w \min (q, K)-p \min [D, \min (q, K)]+\int_{A_{s}}^{\chi_{s}} \delta\left\{\lambda w q-\gamma\left[\begin{array}{l}
\left(w-c_{p}\right) \min (q, K)- \\
c_{k} K-\lambda w q r+A_{s}-L_{s}
\end{array}\right]\right\} \phi\left(A_{s}\right) d A_{s}}{b[\min (q, K)-D]^{+}} .
\end{aligned}
$$

Proof of Theorem 4 The financing strategy chosen by the retailer will bring a greater expected profit: $\pi_{r}^{\text {bpof* }}>\pi_{r}{ }^{\text {apd }}{ }^{*}$ iff
$p \operatorname{Emin}\left(D, q^{\text {bpof** }^{*}}\right)-w q^{\text {bpof }^{*}}+\left(a-b t^{\text {ppof* }^{*}}\right)\left(q^{\text {bpof }^{*}}-D\right)^{+}$
$-\int_{A_{s}}^{T_{5}} \delta\left\{\lambda w q^{b p o f^{*}}-\gamma\left[\left(w-c_{p}\right) q^{b p o f^{*}}-c_{k} K-\lambda w q^{b p o f f^{*}} r+A_{s}-L_{s}\right]\right\} \phi\left(A_{s}\right) d A_{s}$
$>p \operatorname{Emin}\left(D, q^{a p d^{*}}\right)-w(1-d) q^{a p d^{*}}+\left(a-b t^{a p d^{*}}\right)\left(q^{a p d^{*}}-D\right)^{+}-(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q^{a p d^{*}}\right]^{+}$.
where the threshold value of the clearance time:
$\left.(a-b t)\left[q^{\text {bpof }}-D\right)^{+}-\left(q^{a q d^{*}}-D\right)^{+}\right]>$
$p \mathrm{E} \min \left(D, q^{\text {apd** }}\right)-w(1-d) q^{\text {apd } d^{*}}-(1-\alpha)\left[L_{r}-A_{r}+w(1-d) q^{\text {apd } d^{*}}\right]^{+}-p \mathrm{E} \min \left(D, q^{\text {bopf** }}\right)+w q^{\text {bpop }}$




