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Distributed Transactive Energy Trading Framework in Distribution Networks

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***Abstract*—In this paper, we propose a novel transactive energy trading (TET) framework to deal with the economic issues in energy trading and the technical issues in distribution system operation in a holistic manner. In particular, we innovatively integrate a bilateral energy trading mechanism with the optimal power flow (OPF) technique to increase economic benefits to individual participants and meanwhile ensure the reliability and security of the system operation. In order to resolve the inherent conflict of interests, Nash bargaining theory is used to model the TET problem which is further decomposed into a multi-period OPF problem and a payment bargaining problem. Moreover, we develop an efficient distributed algorithm for solving the TET problem base on alternating direction method of multipliers (ADMM). Instead of directly solving optimization subproblems like most ADMM based distributed algorithms, we derive closed form solutions to all subproblems to significantly improve the computational efficiency. Finally, numerical tests on IEEE 37-bus and 123-bus distribution systems demonstrate the effectiveness of our proposed framework and the efficiency of our distributed algorithm.**

***Index Terms*—Transactive energy, bilateral energy trading, distribution networks, photovoltaic system, Nash bargaining, alternating direction method of multipliers, distributed algorithm.**

NOMENCLATURE

Indices and sets:

i	Index of nodes/lines/agents
t	Index of time slots
$\mathcal{N}$	Set of buses
$\mathcal{E}$	Set of distribution lines
C_i	Set of children nodes of node i
$\mathcal{T}$	Set of time slots
k	Index of iterations

Parameters:

r_i	Resistance of i-th line
x_i	Reactance of i-th line
$\overline{d}_{i,t}/\underline{d}_{i,t}$	Upper/Lower bound of agent i's load demand at time t
E_i	Minimum required energy of agent i over the time horizon
$d_{i,t}$	Preferred load demand of agent i at time t
α_i	Unit discomfort cost for load deviation of agent i
$\overline{Q}_{i,t}/\underline{Q}_{i,t}$	Upper/Lower bound of reactive power output of agent i's PV system at time t
S_i	Rated apparent power of agent i's PV system
θ	Power factor angle associated with the minimum allowed power factor
$p^g_{i,t}$	Active power output of agent i's PV system at time t
λ^b_t/λ^s_t	Prices for buying/selling energy from/to the utility company at time t
$q^d_{i,t}$	Reactive load of agent i at time t
$\overline{v}/\underline{v}$	Upper/Lower bound of squared voltage magnitude at each node
$\overline{p}_i/\underline{p}_i$	Upper/Lower bound of nodal active power injection at node i
$\overline{q}_i/\underline{q}_i$	Upper/Lower bound of nodal reactive power injection at node i

Variables:

$v_{i,t}$	Squared voltage magnitude of node i at time t
$l_{i,t}$	Squared line current magnitude of line i at time t
$P_{i,t}/Q_{i,t}$	Active/Reactive power flow on line i at time t
$p_{i,t}/q_{i,t}$	Active/Reactive power injection at node i at time t
$p^d_{i,t}$	Scheduled load demand of agent i at time t
$q^g_{i,t}$	Reactive power output of agent i's PV system at time t
$e^j_{i,t}$	Energy amount agent i purchases from agent j at time t
ϕ^j_i	Bilateral payment from agent i to agent j

Functions:

$D_{i,t}(\cdot)$	Discomfort cost function of agent i for load deviation at time t
$\tilde{C}_i(\cdot)$	Total cost function of agent i without BET
$\tilde{B}_{i,t}(\cdot)$	Utility bill function of agent i at time t without BET
$C_i(\cdot)$	Total cost function of agent i with BET
$B_{i,t}(\cdot)$	Utility bill function of agent i at time t with BET
$W_i(\cdot)$	Cost function of agent i excluding bilateral payments

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I. Introduction

RECENTLY, electric distribution networks (DNs) are undergoing a dramatic change due to the increasing installations of residential solar photovoltaic (PV) systems driven by the reduced installation cost and encouraging feed-in-tariffs [1]. However, the proliferation of distributed PV systems poses significant challenges to DN operation due to not only the intermittent and volatile nature of PV generations but also the complexity in coordinating a large number of new entities, namely, PV prosumers. Prosumers are end-use consumers who also have local generation sources, e.g. PV panels, and thus are able to feed electricity back to the power grid. On the other hand, the high entry threshold to traditional electricity markets, either wholesale market or retail market, inhibits active engagement of end-use customers, thus reducing economic efficiency. Lately, the emerging transactive energy (TE) concept sheds new lights on addressing the aforementioned concerns [2]. In this regard, we develop a novel transactive energy framework to increase economic benefits to customers and to enhance the security and reliability of DN operation.

According to the Gridwise Architecture Council (GWAC), transactive energy system is defined as a set of mechanisms that use economic based instruments to achieve the dynamic balance between the generation and consumption while considering operation constraints of a power system [2]. It is a multi-agent system that enables active participation of customers to contribute to the enhancement of the system reliability, security and efficiency. There are several works related with transactive energy system. In [3], a transactive control strategy with a double-auction market is proposed for commercial buildings to coordinate the internal electric appliances. In [4], a transactive energy framework is presented for the decision making of virtual power plants. However, these works overlook the impact of TE on the power system operation. In [5], a day-ahead transactive market model is proposed for the distribution system operator (DSO) to manage the distribution level operation and participate in wholesale market. Nonetheless, it does not consider the energy trading within the distribution network. Ref. [6] conducts a cost-benefit analysis for transactive energy sharing within a microgrid. Ref. [7] carries out a case study of a transactive energy trading in distribution systems. However, these works only focus on conceptual discussion and preliminary study without detailed designs. Up to now, the research on applying TE to distribution system operation is still at its very early stage.

Economic and system operating issues are two major concerns of a TE based framework. Current policies of most countries encourage self-consumption of solar PV energy so as to mitigate its adverse influence [8]. But it is not preferable to PV prosumers who may have excess PV generation after meeting their own load especially during the peak irradiance period. In order to improve the economic benefit to PV prosumers and other consumers, some energy sharing and trading mechanisms have been developed in recent research works. Ref. [9] presents an energy sharing mechanism with an internal pricing model for prosumers within a microgrid. Ref. [10] investigates the interconnected microgrids and proposes a holistic model for energy scheduling and trading. Ref. [11] develops a distributed model for energy trading among multiple microgrids. In [12], a study on energy exchange is carried out using DC based interconnected nanogrids. There are also some works [13], [14] focusing on game-theoretic approach based energy trading in smart grid, which are summarized in [14]. Nevertheless, few existing works have jointly studied the economic issues of energy trading and the technical issues of distribution system operation.

Optimal power flow (OPF) technique plays an important role in dealing with practical issues in power systems, such as transmission expansion planning [15], stability analysis [16], congestion management [17], volt/var control [18], etc. In wholesale market, the market clearing problem is often modelled as an OPF problem whose objective is to maximize the overall social welfare [19]. Consequently, the market outcome is highly efficient and compatible with the transmission system operation. Inspired by this, we incorporate the OPF technique to the energy trading in distribution systems.

Traditionally, distribution systems are managed by DSOs in a centralized manner. However, with the proliferation of distributed energy resources (DERs) and household automation products, it becomes challenging to centrally control customer-owned assets due to privacy concerns and complex communication and control requirements. In this regard, distributed operation and control schemes have been extensively studied in recent years [20]–[23]. In [20], a distributed dispatch method is proposed based on primal-dual subgradient algorithm. In [21], an ADMM based distributed algorithm is developed for the OPF problem in distribution systems. In [22], a distributed method is used to optimize the active and reactive power set-points of DER inverters. In [23], distributed approaches are applied to voltage regulation.

In this paper, we propose a novel transactive energy trading (TET) framework with detailed designs to accommodate high PV penetration in distribution networks. Nash bargaining theory is used to model the TET problem. Then we develop an efficient distributed algorithm based on ADMM for solving the TET problem so that the autonomy and privacy of individual entities can be preserved. The main contributions are threefold.

- Different from most TE based works (e.g. [10]) that only focus on addressing economic issues in energy trading, our proposed TET framework is able to deal with both economic and technical issues in distribution systems in a holistic manner. In particular, we innovatively integrate a bilateral energy trading mechanism with the distribution system OPF technique, which has not been studied before. In this way, we can improve economic benefits to individual participants and meanwhile ensure the reliability and security of the distribution system.
- By applying Nash bargaining theory to our problem, we not only resolve the conflict of interests among different participants, but also align their interests with the need of the entire distribution system. Consequently, we can decompose the TET problem into a multi-period OPF problem and a payment bargaining problem to reduce computational complexity.

- Most ADMM based distributed algorithms require to solve optimization subproblems iteratively to update the variables. In contrast, we derive the closed-form solutions to all subproblems, which results in a significant improvement in computational efficiency.

The remainder of this paper is organized as follows. In Section II, we introduce the branch flow model, TET agent model and bilateral energy trading model. In Section III, we introduce some backgrounds on Nash bargaining theory and use it to model our TET problem. Then we develop an advanced distributed algorithm for solving it in Section IV. Numerical results are demonstrated in Section V. Finally, we conclude our paper in Section VI.

II. System Model

A. Branch Flow Model

Consider a distribution network $\mathcal{G} := (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} := \{0, 1, \ldots, N\}$ represents the node set and $\mathcal{E}$ represents the line set. Each node except the substation node (indexed as 0) has a unique parent node A_i and a set of child nodes, denoted by C_i. We assume each directed line points from a node i to its unique parent node A_i. Thus, we uniquely label the line from i to A_i as i and have $\mathcal{E} := \{1, \ldots, N\}$. Given a radial distribution network, the branch flow model [24] is given as

$$v_{i,t} - v_{A_i,t} = 2(r_i P_{i,t} + x_i Q_{i,t}) - l_{i,t}(r_i^2 + x_i^2) \quad i \in \mathcal{E} \tag{1a}$$

$$\sum_{j \in C_i} (P_{j,t} - l_{j,t} r_j) + p_{i,t} = P_{i,t} \quad i \in \mathcal{N} \tag{1b}$$

$$\sum_{j \in C_i} (Q_{j,t} - l_{j,t} x_j) + q_{i,t} = Q_{i,t} \quad i \in \mathcal{N} \tag{1c}$$

$$l_{i,t} = \frac{P_{i,t}^2 + Q_{i,t}^2}{v_{i,t}} \quad i \in \mathcal{N} \tag{1d}$$

Since (1d) is nonconvex, a second order cone relaxation [24] can be applied as

$$\|(2P_{i,t}, 2Q_{i,t}, v_{i,t} - l_{i,t})\|_2 \leq v_{i,t} + l_{i,t} \quad i \in \mathcal{N} \tag{2}$$

B. TET Agent Model

For a better energy trading coordination, TET agent is introduced to represent the aggregation of customers on the same node, as illustrated in Fig. 1. Each agent is allowed to trade energy with other agents on behalf of its local customers. Moreover, TET agents are also responsible for ensuring the reliable and secure operation of the distribution system through cooperation with each other.

Since each agent only concerns about the aggregated load demand in energy trading, we only need to model it in this paper. For conciseness, we simply term the aggregated demand as the demand. We assume each demand is flexible and can be scheduled across time as long as it satisfies the following two constraints.

$$\underline{d}_{i,t} \leq p_{i,t}^d \leq \overline{d}_{i,t} \quad t \in \mathcal{T} \tag{3}$$

$$\sum_{t \in \mathcal{T}} p_{i,t}^d \geq E_i \tag{4}$$

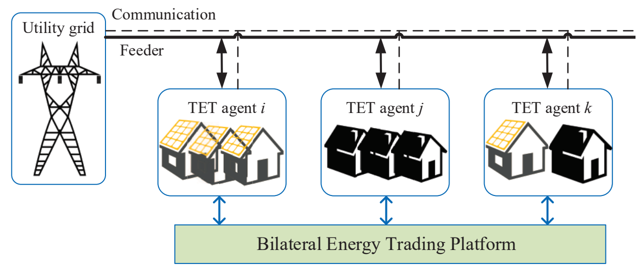


Fig. 1. Transactive energy trading framework in a distribution system, where the solid black line represents the distribution feeder line and the dashed black line represents communication line

We assume each customer has a preference on its power usage due to its living habit. Hence, each agent has a preferred demand in each time slot, which represents the actual demand only when no incentives are provided to customers. However, when bilateral energy trading is introduced, rescheduling demand may give rise to a great cost reduction higher than the compensation of discomfort cost incurred by the load deviation. In this paper, the discomfort cost is modeled as

$$D_{i,t}(p_{i,t}^d) = \alpha_i \left(p_{i,t}^d - d_{i,t}\right)^2 \tag{5}$$

We assume each PV system operates at the maximum power point so as to harvest as much solar energy as possible. Thus, the aggregated PV generation of agent i, denoted as $p_{i,t}^g$, is a predicted parameter. Note if agent i does not have PV installations, then we simply set $p_{i,t}^g$ as 0. In addition, we assume PV systems can provide reactive power support to the distribution system. The range of reactive power output is given as

$$\underline{Q}_{i,t} \leq q_{i,t}^g \leq \overline{Q}_{i,t} \tag{6}$$

where $\underline{Q}_{i,t} = -\overline{Q}_{i,t}$ and $\overline{Q}_{i,t} = \min\left(\sqrt{S_i^2 - (p_{i,t}^g)^2},\ p_{i,t}^g \times tan\theta\right)$.

C. Bilateral Energy Trading and Cost Functions

1) Bilateral energy trading: Consider a bilateral energy trading between agent i and agent j. Let $e_{i,t}^j$ denote the amount of energy that agent i buys from (if $e_{i,t}^j > 0$) or sells to (if $e_{i,t}^j < 0$) agent j in time slot t. Denote ϕ_i^j as the total payment of agent i to (if $\phi_i^j > 0$) or from (if $\phi_i^j < 0$) agent j over the day. Then, we have the following clearing constraints.

$$e_{i,t}^j + e_{j,t}^i = 0 \quad i \in \mathcal{N},\ j \in \mathcal{N} \backslash i,\ t \in \mathcal{T} \tag{7}$$

$$\phi_i^j + \phi_j^i = 0 \quad i \in \mathcal{N},\ j \in \mathcal{N} \backslash i \tag{8}$$

2) Cost functions: The cost function of each agent i without bilateral energy trading (BET) is composed of utility bill and discomfort cost, as shown below.

$$\widetilde{C}_i(\mathbf{p}_i^d) = \sum_{t \in \mathcal{T}} \left(\widetilde{B}_{i,t}(p_{i,t}^d) + D_{i,t}(p_{i,t}^d)\right) \tag{9}$$

where $\mathbf{p}_i^d := \{p_{i,t}^d\}_{t \in \mathcal{T}}$ and $\widetilde{B}_{i,t}(p_{i,t}^d)$ is the utility bill without BET, defined as follows,

$$\widetilde{B}_{i,t}(p_{i,t}^d) = \lambda_t^b \left[p_{i,t}^d - p_{i,t}^g\right]^+ - \lambda_t^s \left[p_{i,t}^g - p_{i,t}^d\right]^+$$

where $[\cdot]^+$ denotes the projection operator onto the non-negative orthant, i.e. $[x]^+ = \max(x, 0)$; λ_t^b denotes the price for buying energy from the utility company, which is typically a fix value over the time; λ_t^s denotes the feed-in tariff representing the price for selling energy to the uitlity company. Generally, λ_t^b is higher than λ_t^s [9].

The cost of agents i with BET is defined as follows,

$$C_i(\mathbf{p}_i^d, \mathbf{e}_i, \boldsymbol{\phi}_i) = \sum_{t\in\mathcal{T}} \left(B_{i,t}(p_{i,t}^d, \mathbf{e}_{i,t}) + D_{i,t}(p_{i,t}^d)\right) + \sum_{j\in\mathcal{N}\backslash i} \phi_i^j \tag{10}$$

where $\mathbf{e}_i := \{e_{i,t}^j\}_{j\in\mathcal{N}\backslash i, t\in\mathcal{T}}$, $\mathbf{e}_{i,t} := \{e_{i,t}^j\}_{j\in\mathcal{N}\backslash i}$ and $\boldsymbol{\phi}_i := \{\phi_i^j\}_{j\in\mathcal{N}\backslash i}$; The last term on the right hand side of (10) represents the total payment of agent i to other agents; $B_{i,t}(p_{i,t}^d, \mathbf{e}_{i,t})$ is the utility bill with BET, defined as

$$B_{i,t}(p_{i,t}^d, \mathbf{e}_{i,t}) = \lambda_t^b \Delta_{i,t}^+ - \lambda_t^s \Delta_{i,t}^-$$

where $\Delta_{i,t}^+ := \left[p_{i,t}^d - p_{i,t}^g - \sum_{j\in\mathcal{N}\backslash i} e_{i,t}^j\right]^+$ and $\Delta_{i,t}^- := \left[p_{i,t}^g - p_{i,t}^d + \sum_{j\in\mathcal{N}\backslash i} e_{i,t}^j\right]^+$, representing the net energy exchange with the utility company.

III. Nash Bargaining Approach

The goal of this paper is to design a transactive energy framework to facilitate bilateral energy trading among TET agents and to ensure the reliability and security of the distribution system operation. One biggest challenge is to resolve the conflict of interests among different agents and to align the interests of individual agents with the need of the distribution system operation. Therefore, a cooperative game theory, namely Nash bargaining theory [25] is used to study the transactive energy trading problem as it has the potential to achieve a mutually beneficial outcome through negotiation and coordination.

In the following two subsections, we first present some background on Nash bargaining theory and then develop a novel transactive energy framework by applying Nash bargaining theory to an OPF problem with bilateral energy trading.

A. Nash Bargaining Theory

For simplicity, we only introduce Nash bargaining theory for two-player bargaining problem, but will extend to multi-player situation in our application. In a bargaining problem, self-interested players negotiate with each other to either achieve a mutually beneficial agreement or end up with a disagreement. Suppose there are two players in a bargaining problem. Let A denote the set of feasible agreements and u_i over A denote the payoff function of player i, that reflects its preferences. We denote the payoffs of two players from the disagreement by $d = (d_1, d_2)$ and the set of possible payoffs from an agreement by U, defined as follows.

$$U := \left\{\left(u_1(a), u_2(a)\right) | a \in A\right\}$$

A bargaining problem is to find an agreement upon which the payoff of each player is no less than the payoff from the disagreement. However, the challenges lie in modelling the detailed bargaining process and deciding which agreement is reasonable when there exist multiple agreements. To address these issues, Nash [25] proposed an axiomatic approach that abstracts away the details of bargaining process and uniquely determines a bargaining solution named as Nash bargaining solution (NBS) if U is a convex and compact set and there exists at least one $u \in U$ such that $u_i > d_i$, $\forall i$. NBS satisfies the following four "reasonable" axioms. (1) Pareto efficiency: it is impossible to find another solution that makes some players better off without making at least one player worse off. (2) Symmetry: if the players are indistinguishable in terms of payoff functions and disagreement, they will receive the same payoffs. (3) Invariant to affine transformation: the solution is invariant if an affine transformation is applied to the payoff and disagreement point. (4) Independence of irrelevant alternatives: if the bargaining solution chosen from a feasible agreement set A is an element of a subset $B \subseteq A$, then replacing set A with set B will not affect the solution. Mathematically, NBS is defined by

Definition 1. *A pair of payoffs (u_1^*, u_2^*) is a Nash bargaining solution only if it solves the following optimization problem:*

$$\max_{u_1,u_2} \quad (u_1 - d_1)(u_2 - d_2) \tag{11a}$$
$$\text{s.t.} \quad (u_1, u_2) \in U \tag{11b}$$
$$(u_1, u_2) \geq (d_1, d_2) \tag{11c}$$

where $(u_1 - d_1)(u_2 - d_2)$ is so called Nash product.

B. Nash Bargaining based Transactive Energy Trading

Nash bargaining theory is applied to model the bilateral energy trading negotiation like [10]. Here, the payoffs of each agent for the agreement and disagreement are defined as the minus cost with and without BET, i.e. $u_i = -C_i(\mathbf{p}_i^d, \mathbf{e}_i, \boldsymbol{\phi}_i)$ and $d_i = -\widetilde{C}_i(\widetilde{\mathbf{p}}_i^d)$. Different from [10] that overlooks the distribution system operation issues, we take the AC power flow constraints into account to ensure the trading outcome is technically implementable. The mathematical formulation of our proposed transactive energy trading problem is given as

$$\max \prod_{i=1}^{N} \left(\widetilde{C}_i(\widetilde{\mathbf{p}}_i^d) - C_i(\mathbf{p}_i^d, \mathbf{e}_i, \boldsymbol{\phi}_i)\right) \tag{12a}$$

$$\text{over } \big\{p_{i,t}^d, q_{i,t}^g, \{e_{i,t}^j, \phi_i^j\}_{j\in\mathcal{N}\backslash i}, p_{i,t}, q_{i,t}, P_{i,t}, Q_{i,t}, l_{i,t}, v_{i,t}\big\}_{i\in\mathcal{N}, t\in\mathcal{T}}$$

$$\text{s.t. } (3), (4) \text{ and } (6) \tag{12b}$$
$$(7) \text{ and } (8) \tag{12c}$$
$$(1a)\text{-}(1c) \text{ and } (2) \tag{12d}$$
$$p_{i,t} = p_{i,t}^g - p_{i,t}^d \quad i \in \mathcal{N}\backslash 0,\ t \in \mathcal{T} \tag{12e}$$
$$q_{i,t} = q_{i,t}^g - q_{i,t}^d \quad i \in \mathcal{N}\backslash 0,\ t \in \mathcal{T} \tag{12f}$$
$$\underline{v} \leq v_{i,t} \leq \overline{v} \quad i \in \mathcal{N},\ t \in \mathcal{T} \tag{12g}$$
$$\widetilde{C}_i(\widetilde{\mathbf{p}}_i^d) \geq C_i(\mathbf{p}_i^d, \mathbf{e}_i, \boldsymbol{\phi}_i) \quad i \in \mathcal{N} \tag{12h}$$

where constraints (12b)-(12g) correspond to (11b) and constraint (12h) corresponds to (11c). (12b) summarizes local scheduling constraints for each agent; (12c) is associated with bilateral energy trading; (12d)-(12f) are power flow equations and (12g) is voltage constraint.

In order to reduce the computational complexity, problem (12) is decomposed into two subproblems by employing the

Pareto efficiency of the Nash bargaining solution. The detailed illustration is presented in the following theorem.

Theorem 1. *The proposed TET problem (12) can be equivalently decomposed into the following two subproblems. The first subproblem **S1** solves a multi-period OPF problem with energy sharing and the second subproblem **S2** determines the corresponding bilateral payments.*

S1: multi-period OPF problem

$$\begin{aligned}\min\ & \sum_{i\in\mathcal{N}} W_i(\mathbf{p}_i^d,\mathbf{e}_i)\\ \text{over}\ & \{p_{i,t}^d, q_{i,t}^g, \{e_{i,t}^j\}_{j\in\mathcal{N}\setminus i}, p_{i,t}, q_{i,t}, P_{i,t}, Q_{i,t}\\ & \quad l_{i,t}, v_{i,t}\}_{i\in\mathcal{N},t\in\mathcal{T}}\\ \text{s.t.}\ & \text{(1a)-(1c)}, \text{(2)-(4)}, \text{(6)}, \text{(7)}, \text{(12e)-(12g)}\end{aligned}$$

where $W_i(\mathbf{p}_i^d,\mathbf{e}_i) := \sum_{t\in\mathcal{T}}\big(B_{i,t}(p_{i,t}^d,\mathbf{e}_{i,t}) + D_{i,t}(p_{i,t}^d)\big)$

S2: payment bargaining problem

$$\begin{aligned}\max\ & \prod_{i=1}^{N}\Big(\widetilde{C}_i(\widetilde{\mathbf{p}}_i^d) - W_i(\mathbf{p}_i^{d*},\mathbf{e}_i^*) - \sum_{j\in\mathcal{N}\setminus i}\phi_i^j\Big)\\ \text{over}\ & \{\phi_i^j\}_{j\in\mathcal{N}\setminus i, i\in\mathcal{N}}\\ \text{s.t.}\ & \text{(8)}\end{aligned}$$

where $(\mathbf{p}_i^{d*},\mathbf{e}_i^*)$ is the optimal solution of **S1**.

Proof. Let $U_i(\mathbf{p}_i^d,\mathbf{e}_i,\boldsymbol{\phi}_i) := \widetilde{C}_i(\widetilde{\mathbf{p}}_i^d) - C_i(\mathbf{p}_i^d,\mathbf{e}_i,\boldsymbol{\phi}_i)$. Plugging in (10) and $W_i(\mathbf{p}_i^d,\mathbf{e}_i)$, we have

$$U_i(\mathbf{p}_i^d,\mathbf{e}_i,\boldsymbol{\phi}_i) = \widetilde{C}_i(\widetilde{\mathbf{p}}_i^d) - W_i(\mathbf{p}_i^d,\mathbf{e}_i) - \sum_{j\in\mathcal{N}\setminus i}\phi_i^j$$

Since $\phi_i^j + \phi_j^i = 0$, we have $\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\setminus i}\phi_i^j = 0$. Thus, the following equality holds.

$$\sum_{i\in\mathcal{N}} U_i(\mathbf{p}_i^d,\mathbf{e}_i,\boldsymbol{\phi}_i) = \sum_{i\in\mathcal{N}}\widetilde{C}_i(\widetilde{\mathbf{p}}_i^d) - \sum_{i\in\mathcal{N}} W_i(\mathbf{p}_i^d,\mathbf{e}_i)$$

where $\widetilde{C}_i(\widetilde{\mathbf{p}}_i^d)$ represents the cost of agent i at the disagreement and is known to agent i as it is the optimal value of the following problem.

$$\begin{aligned}\min_{\mathbf{p}_i^d}\ & \widetilde{C}_i(\mathbf{p}_i^d)\\ \text{s.t.}\ & \text{(3) and (4)}\end{aligned}$$

Note $\sum_{i\in\mathcal{N}} W_i(\mathbf{p}_i^d,\mathbf{e}_i)$ is the total cost of the entire system at an agreement. We hence claim that the Nash bargaining solution also minimizes problem **S1**. Otherwise, there exists a solution that makes some U_i larger without making at least one U_i smaller by reducing $\sum_{i\in\mathcal{N}} W_i(\mathbf{p}_i^d,\mathbf{e}_i)$, which contradicts the property of Pareto efficiency. Moreover, since problem **S1** is a strictly convex optimization problem, its optimal solution is unique. Therefore, we can obtain the optimal $\{p_{i,t}^d,$ $q_{i,t}^g, \{e_{i,t}^j\}_{j\in\mathcal{N}\setminus i}, p_{i,t}, q_{i,t}, P_{i,t}, Q_{i,t}, l_{i,t}, v_{i,t}\}_{i\in\mathcal{N},t\in\mathcal{T}}$ by solving problem **S1** and then obtain the optimal $\{\phi_i^j\}_{j\in\mathcal{N}\setminus i,i\in\mathcal{N}}$ by solving problem **S2**. □

Remark 1: Theorem 1 indicates that the solution to the TET problem (12) also minimizes the overall social cost considering the practical operating constraints, which implies the most efficient operation outcome like traditional locational marginal price (LMP) based market framework. The key difference is that TET framework enables peer to peer energy trading, and LMP based framework does not. The Pareto efficiency and convexity ensure there exists a unique Nash bargaining solution to problem (12) and it represents an agreement that is beneficial to all agents. We assume all agents are rational. Thus, no one is willing to break the agreement.

IV. Distributed Algorithm for TET Problem

Although both problem **S1** and **S2** can be solved in a centralized fashion by the cutting-edge solvers, it will violate the privacy and autonomy of individual agents since the centralized optimization requires the complete information of the entire system. In this regard, we develop a distributed algorithm for solving the TET problem in this section. Specifically, we first decompose the multi-period OPF problem **S1** into multiple single-period OPF subproblems using Lagrangian relaxation. Then, we develop distributed algorithms for the single-period OPF problem and payment bargaining problem **S2** by applying a consensus version of ADMM [26]. More importantly, we derive closed form solutions to each optimization subproblems so as to significantly improve the computational efficiency.

A. Decoupling of Temporally Coupled Constraint

In **S1**, the constraint (4) couples the decision variables of all time slots, which inhibits **S1** to be solved separately at each time slot. We hence relax it by introducing a Lagrangian multiplier $\pi_i \geq 0$ for each agent i as

$$\begin{aligned}\mathcal{L} &= \sum_{i\in\mathcal{N}} W_i(\mathbf{p}_i^d,\mathbf{e}_i) + \sum_{i\in\mathcal{N}}\pi_i(E_i - \sum_{t\in\mathcal{T}} p_{i,t}^d)\\ &= \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} F_{i,t}\big(p_{i,t}^d,\mathbf{e}_{i,t},\pi_i\big) + \sum_{i\in\mathcal{N}} E_i\pi_i \end{aligned} \tag{16}$$

where $F_{i,t}\big(p_{i,t}^d,\mathbf{e}_{i,t},\pi_i\big) := B_{i,t}(p_{i,t}^d,\mathbf{e}_{i,t}) + D_{i,t}(p_{i,t}^d) - \pi_i p_{i,t}^d$.

Since **S1** is a convex optimization problem with zero duality gap, it can be equally transformed to problem **S1′**.

$$\textbf{S1}'\textbf{:}\ \max_{\boldsymbol{\pi}\geq\mathbf{0}} \sum_{t\in\mathcal{T}}\Big(\min_{\mathbf{y}_t\in\mathcal{Y}_t}\sum_{i\in\mathcal{N}} F_{i,t}\big(p_{i,t}^d,\mathbf{e}_{i,t},\pi_i\big)\Big) + \sum_{i\in\mathcal{N}} E_i\pi_i$$

where $\mathbf{y}_t := \{p_{i,t}^d, q_{i,t}^g, \mathbf{e}_{i,t}, p_{i,t}, q_{i,t}, P_{i,t}, Q_{i,t}, l_{i,t}, v_{i,t}\}_{i\in\mathcal{N}}$ and $\boldsymbol{\pi} := \{\pi_i\}_{i\in\mathcal{N}}$. $\mathcal{Y}_t$ represents the feasible region of $\mathbf{y}_t$, which is defined by the constraints (1a)-(1c), (2), (3), (6), (7) and (12e)-(12g).

Note that the inner level problem is decoupled into several single-period OPF problems that can be solved in parallel. Next, we drop the subscript t for simplicity and reformulate the single-period OPF problem as below by eliminating p_i^d and q_i^g.

$$\min\ \sum_{i\in\mathcal{N}} g_i\big(p_i, \{e_i^j\}_{j\in\mathcal{N}\setminus i}\big) \tag{17a}$$

$$\text{over}\ \big\{\{e_i^j\}_{j\in\mathcal{N}\setminus i}, p_i, q_i, P_i, Q_i, l_i, v_i\big\}_{i\in\mathcal{N}}$$

$$\text{s.t.}\ v_i - v_{A_i} = 2(r_iP_i + x_iQ_i) - l_i(r_i^2 + x_i^2) \quad i\in\mathcal{E} \tag{17b}$$

$$\sum_{j\in C_i}(P_j - l_jr_j) + p_i = P_i \qquad i\in\mathcal{N} \tag{17c}$$

$$\sum_{j\in C_i}(Q_j - l_j x_j) + q_i = Q_i \qquad i\in\mathcal{N} \tag{17d}$$

$$e^j_{i,t} + e^i_{j,t} = 0 \qquad j\in\mathcal{N}\backslash i,\ i\in\mathcal{N} \tag{17e}$$

$$\|(2P_i, 2Q_i, v_i - l_i)\|_2 \le v_i + l_i \qquad i\in\mathcal{N} \tag{17f}$$

$$\underline{v} \le v_i \le \overline{v} \qquad i\in\mathcal{N} \tag{17g}$$

$$\underline{p}_i \le p_i \le \overline{p}_i \qquad i\in\mathcal{N} \tag{17h}$$

$$\underline{q}_i \le q_i \le \overline{q}_i \qquad i\in\mathcal{N} \tag{17i}$$

where $g_i\big(p_i, \{e^j_i\}_{j\in\mathcal{N}\backslash i}\big) := \lambda^b\big[-p_i - \sum_{j\in\mathcal{N}\backslash i} e^j_i\big]^+$

$$-\lambda^s\big[p_i + \sum_{j\in\mathcal{N}\backslash i} e^j_i\big]^+ + \alpha_i(p_i - p^g_i + d_i)^2 + \pi_i p_i$$

Let $p^d_{i,t}[k]$ denote the optimal solution of (17) for a given $\boldsymbol{\pi}[k]$. Then the Lagrangian multiplier π_i can be iteratively updated as

$$\pi_i[k+1] = \Big[\pi_i[k] + \gamma\big(E_i - \sum_{t\in\mathcal{T}} p^d_{i,t}[k]\big)\Big]^+ \quad i\in\mathcal{N} \tag{18}$$

where $\gamma > 0$ is the step size.

B. Distributed Algorithms with Closed-form Updates

In this subsection, we develop distributed algorithms with closed-form updates for solving problem (17) and **S2**. Towards this end, both problems are first expressed in a uniform compact matrix form for mathematical conciseness, as shown below.

$$\min_{\mathbf{x}} \ \sum_{i\in\mathcal{N}} f_i(\mathbf{x}_i) \tag{19a}$$

$$\text{s.t.} \ \sum_{j\in\mathcal{N}} \mathbf{A}_{ij}\mathbf{x}_j = 0 \qquad i\in\mathcal{N} \tag{19b}$$

$$\mathbf{x}_i \in \mathcal{X}_i \qquad i\in\mathcal{N} \tag{19c}$$

where $\mathbf{x} := \{\mathbf{x}_i\}_{i\in\mathcal{N}}$ and $\mathbf{x}_i$ is a vector of decision variables associated with agent i. For instance, $\mathbf{x}_i := \{\phi^j_i\}_{j\in\mathcal{N}\backslash i}$ in problem **S2** and $\mathbf{x}_i := \big\{\{e^j_i\}_{j\in\mathcal{N}\backslash i}, p_i, q_i, P_i, Q_i, l_i, v_i\big\}$ in problem (17). $f(\mathbf{x}_i)$ is a convex function. $\mathbf{A}_{ij}$ is a constant matrix and (19b) represents a group of linear constraints that couple all local variables $\mathbf{x}_i$ together, such as (8) in problem **S2** and (17b)-(17e) in problem (17). $\mathcal{X}_i$ is a convex set and (19c) corresponds to the local constraints of each agent, such as (17f)-(17i) in problem (17).

In (19), each agent's decision variables $\mathbf{x}_i$ are strongly coupled with others' via the constraint (19b). In order to apply a consensus version of ADMM [26], problem (19) is reformulated as the problem below by introducing a set of auxiliary variables $\mathbf{z}_{(j)i}$ for each agent i, that represents the duplicate of $\mathbf{x}_j$ on the side of agent i.

$$\min_{\mathbf{x},\mathbf{z}} \ \sum_{i\in\mathcal{N}} f_i(\mathbf{x}_i) \tag{20a}$$

$$\text{s.t.} \ \sum_{j\in\mathcal{N}} \mathbf{A}_{ij}\mathbf{z}_{(j)i} = 0 \qquad i\in\mathcal{N} \tag{20b}$$

$$\mathbf{x}_i \in \mathcal{X}_i \qquad i\in\mathcal{N} \tag{20c}$$

$$\mathbf{x}_j - \mathbf{z}_{(j)i} = 0 \qquad j\in\mathcal{N},\ i\in\mathcal{N} \tag{20d}$$

where $\mathbf{z} := \{\mathbf{z}_i\}_{\in\mathcal{N}}$ and $\mathbf{z}_i := \{\mathbf{z}_{(j)i}\}_{j\in\mathcal{N}}$. The corresponding explicit formulations of problem (17) and **S2** are illustrated in Appendix A.

Let $\boldsymbol{\mu}_{(j)i}$ denote the vector of Lagrangian multipliers associated with the consensus constraint (20d). For a given penalty parameter $\rho > 0$, the augmented Lagrangian is defined as

$$\begin{aligned}\mathcal{L}_\rho(\mathbf{x},\mathbf{z},\boldsymbol{\mu}) &= \sum_{i\in\mathcal{N}} \mathcal{L}^x_i\big(\mathbf{x}_i, \{\mathbf{z}_{(i)j}, \boldsymbol{\mu}_{(i)j}\}_{j\in\mathcal{N}}\big)\\ &= \sum_{i\in\mathcal{N}} \mathcal{L}^z_i\big(\{\mathbf{x}_j\}_{j\in\mathcal{N}}, \mathbf{z}_i, \boldsymbol{\mu}_i\big)\end{aligned} \tag{21}$$

where $\mathcal{L}^x_i\big(\mathbf{x}_i, \{\mathbf{z}_{(i)j}, \boldsymbol{\mu}_{(i)j}\}_{j\in\mathcal{N}}\big)$

$$:= f_i(\mathbf{x}_i) + \sum_{j\in\mathcal{N}}\big(\langle\boldsymbol{\mu}_{(i)j}, \mathbf{x}_i - \mathbf{z}_{(i)j}\rangle + \frac{\rho}{2}\|\mathbf{x}_i - \mathbf{z}_{(i)j}\|^2\big)$$

$$\mathcal{L}^z_i\big(\{\mathbf{x}_j\}_{j\in\mathcal{N}}, \mathbf{z}_i, \boldsymbol{\mu}_i\big)$$

$$:= f_i(\mathbf{x}_i) + \sum_{j\in\mathcal{N}}\big(\langle\boldsymbol{\mu}_{(j)i}, \mathbf{x}_j - \mathbf{z}_{(j)i}\rangle + \frac{\rho}{2}\|\mathbf{x}_j - \mathbf{z}_{(j)i}\|^2\big)$$

where $\langle\cdot,\cdot\rangle$ denotes the operation of inner product and $\boldsymbol{\mu}_i := \{\boldsymbol{\mu}_{(j)i}\}_{j\in\mathcal{N}}$.

Owing to the decomposability of the augmented Lagrangian (21) as well as the constraints (20b) and (20c), problem (20) can be solved in a fully distributed manner. The overall iterative procedure is given as follows.

(A1) Upon receiving the latest updated $\mathbf{z}^k_{(i)j}$ and $\boldsymbol{\mu}^k_{(i)j}$ from other agents, each agent i updates $\mathbf{x}_i$ as (22) and broadcasts it to other agents.

$$\mathbf{x}^{k+1}_i := \arg\min_{\mathbf{x}_i\in\mathcal{X}_i} \ \mathcal{L}^x_i\big(\mathbf{x}_i, \{\mathbf{z}^k_{(i)j}, \boldsymbol{\mu}^k_{(i)j}\}_{j\in\mathcal{N}}\big) \tag{22}$$

(A2) Upon receiving the latest updated $\mathbf{x}^{k+1}_j$ from other agents, each agent i updates $\mathbf{z}_i$ as (23) and broadcasts $\mathbf{z}_{(j)i}$ to agent j.

$$\mathbf{z}^{k+1}_i := \arg\min_{\mathbf{z}_i\in\mathcal{Z}_i} \ \mathcal{L}^z_i\big(\{\mathbf{x}^{k+1}_j\}_{j\in\mathcal{N}}, \mathbf{z}_i, \boldsymbol{\mu}^k_i\big) \tag{23}$$

where $\mathcal{Z}_i := \{\mathbf{z}_i | \sum_{j\in\mathcal{N}} \mathbf{A}_{ij}\mathbf{z}_{(j)i} = 0\}$.

(A3) Each agent i updates $\boldsymbol{\mu}_{(j)i}$ as (24) and broadcasts it to agent j.

$$\boldsymbol{\mu}^{k+1}_{(j)i} := \boldsymbol{\mu}^k_{(j)i} + \rho(\mathbf{x}^{k+1}_j - \mathbf{z}^{k+1}_{(j)i}) \quad j\in\mathcal{N} \tag{24}$$

Proposition 1.

(a) *There exist closed-form expressions for the updates of* $\mathbf{x}_i$ *and* $\mathbf{z}_i$ *in problem (17).*

(b) *There exist closed-form expressions for the updates of* $\mathbf{x}_i$ *and* $\mathbf{z}_i$ *in problem* ***S2***.

Proof. See Appendices B and C. □

C. Implementation

Algorithm 1 summarizes the distributed algorithm for solving **S1′**, where the outer and inner loops represent dual ascent method and ADMM based iterations, respectively. Note that the algorithm for solving **S2** is similar to the inner loop of Algorithm 1. Thus, for conciseness, it is not presented here. Furthermore, the individual privacy is preserved since each $\mathbf{z}_{(j)i}$ only contains partial information of $\mathbf{x}_j$, i.e. power flow

Algorithm 1: Distributed Algorithm for Solving **S1′**

Initialize all $\mathbf{z}_{i,t}$, $\boldsymbol{\mu}_{i,t}$ and π_i. Set the inner and outer loop tolerance levels, ε_1 and ε_2. Initialize the iteration indices, $k=0$ and $m=0$;
repeat
  for $t=1$ **to** T **do**
    while $\|\mathbf{x}_t^k-\mathbf{z}_t^k\|>\varepsilon_1$ **or** $\rho\|\mathbf{z}_t^k-\mathbf{z}_t^{k-1}\|>\varepsilon_1$ **do**
      Each agent i updates $\mathbf{x}_{i,t}$ according to (22);
      Each agent i updates $\mathbf{z}_{i,t}$ according to (23);
      Each agent i updates $\boldsymbol{\mu}_{(j)i,t}$ according to (24);
      Update inner loop iteration index $k=k+1$;
    end
  end
  Each agent i updates π_i according to (18);
  Update outer loop iteration index $m=m+1$;
until $\|\boldsymbol{\pi}^m-\boldsymbol{\pi}^{m-1}\|\le\varepsilon_2$;

information and energy trading information as illustrated in Appendix A. The information exchange among TET agents can be facilitated by the advanced information and communication technologies (ICTs), e.g. LTE technology, which is designed for high-speed wireless communication.

V. Numerical results

In this section, we test our proposed TET framework on the modified IEEE 37-bus and 123-bus distribution systems. Detailed information of the systems can be found in [27]. We consider three-phase balanced scenarios for simplicity. In addition, we classify the distribution nodes into residential nodes and commercial nodes based on their load patterns. We assume each residential node is installed with a PV system with the rated capacity being 200 kVA. The data of the load is simulated using the same technique as our previous work [28] and the PV generation data is calculated using the actual solar irradiance data provided by [29]. Without loss of generality, the purchasing price λ_t^b is set as \$0.8/kWh during the off-peak periods (12:00 a.m.-6:00 a.m.) and \$1/kWh during other periods, and the selling price λ_t^s is set as \$0.4/kWh. All the costs and prices are presented in HK dollars. The length of the entire horizon $|\mathcal{T}|$ is 24 hours and the duration of each time slot is 1 hour. The tolerance level for the convergence of the distributed algorithm is 10^{-4}. Other parameters are summarized as follows: $\alpha_i=500$, $\rho=1$, $\underline{v}=0.95^2$ and $\overline{v}=1.05^2$. All tests are implemented using MATLAB on a computer with an Intel Core i5 of 2.4GHz and 12GB memory.

A. Case Study 1: IEEE 37-bus Distribution System

The nominal voltage value of the 37-bus distribution system is 4.8 kV and the network topology is shown in Fig. 2. Per unit value is used in the case studies.

1) From the Perspective of the Distribution System: Table I lists the total operating cost of the distribution system with and without TET. We can see that TET can bring about 24.8% cost saving resulting from the emergence of competitive bilateral energy trading among TET agents. Fig. 3 shows

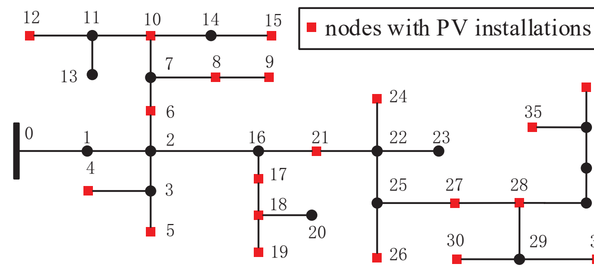


Fig. 2. IEEE 37-bus distribution system with PV installations

TABLE I
TOTAL OPERATING COST OF 37-BUS SYSTEM WITH AND WITHOUT TET

Cost without TET (\$)	Cost with TET (\$)	Cost reduction (\$)	Relative cost reduction
18812	14138	4674	24.8%

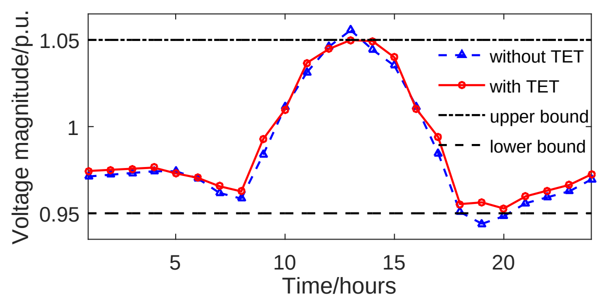
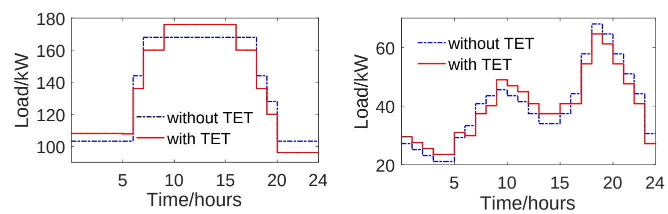


Fig. 3. Voltage magnitudes of node 30 in 37-bus system with and without TET

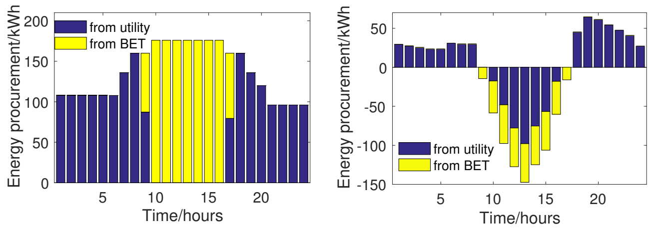


(a) agent 1 (b) agent 4

Fig. 4. load schedule of two agents in 37-bus system with and without TET, (a) agent 1: commercial load, (b) agent 4: residential load

the voltage profile of node 30 with and without TET. We only demonstrate the voltage profile of node 30 because it is a terminal node and is more likely to experience voltage violations. Without TET, overvoltage violation is observed at noon when peak PV generation occurs and undervoltage violation is observed at early night when peak load occurs. However, all voltage violations are removed when TET is introduced. The reason is that without TET each agent merely schedules its local power consumptions without systematic coordination with other agents. By contrast, TET takes the system operating constraints into account and thus enables coordinated management of the distribution system. Therefore, TET can not only facilitate bilateral energy trading but also improve the system performance in terms of economic efficiency and voltage security.

2) From the Perspective of TET Agents: We select two representative agents to demonstrate the results. Particularly,

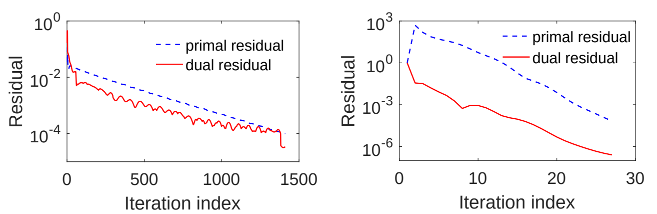


(a) agent 1 (b) agent 4

Fig. 5. Hourly net load and procurement of two representative agents in 37-bus system

TABLE II
COST COMPARISON WITH AND WITHOUT TET FOR AGENT 1 AND 4 IN 37-BUS SYSTEM ($)

Items	Agent 1	Agent 4
Cost without TET	3163.7	222.9
Utility bill plus discomfort cost	1781.6	373.0
Payment to other agents	1209.0	−323.2
Cost with TET	2990.6	49.7
Cost reduction	173.1	173.1



(a) **S1′** (b) **S2**

Fig. 6. Convergence result for 37-bus system, (a) subproblem **S1′**, (b) subproblem **S2**

agent 1 represents a commercial node and agent 4 represents a residential node. Fig. 4(a) and 4(b) show the load schedule of two agents with and without TET, respectively. We can see that both agents shift part of their power consumption from the peak load periods (18:00-20:00) to the periods of peak PV generation (10:00-16:00). As a result, the energy utilization efficiency is improved as more PV power is consumed locally instead of feeding back to the grid during daytime and less power is consumed at early night. Fig. 5(a) and 5(b) demonstrate the energy procurements of two agents in TET framework, respectively. The negative values in Fig. 5(b) mean agent 4 sells energy to the utility company and other agents. We can observe that when the PV power is unavailable, both agents are supplied only by the utility company. When the PV power is sufficiently high, agent 1 procures all the required energy from other agents through bilateral energy trading (BET) and agent 4 sells its excess energy to the utility company after meeting its own need and the needs of other agents. Thus, both the prosumers and the consumers prefer BET over energy trading with the utility company as they can procure energy with lower costs or sell energy with higher profits. Table II shows the cost comparison with and without TET for agent 1 and 4. Both agents are awarded with an equal benefit for participating in TET. Therefore, TET is economically feasible for individual agents.

3) Computational efficiency: Fig. 6(a) and 6(b) semi-logarithmically plot the convergence results for solving problem **S1′** and **S2** using our proposed distributed algorithm, respectively. It is observed that solving problem **S1′** takes more iterations due to its great complexity. However, the total computation time is relatively short for both problems, as shown in Table III where the comparison of computational efficiency using Algorithm 1 and an off-the-shelf solver (SDPT3) [30] are also demonstrated. Significant time reduction (more than 3000 times faster) is achieved by using Algorithm 1 because of the employment of closed form solutions. Note that all simulations are run on a single computer. The computation time in Table III is obtained by dividing the computation time on a single computer with the number of agents.

TABLE III
COMPUTATION TIME FOR IEEE 37-BUS DISTRIBUTION SYSTEM

Problem	Iteration	Total Time (s)	Time/iteration (s) (Algorithm 1)	Time/iteration (s) (SDPT3)
S1′	1411	4.32	3.1×10^{-3}	12.19
S2	27	0.01	3.8×10^{-4}	2.82

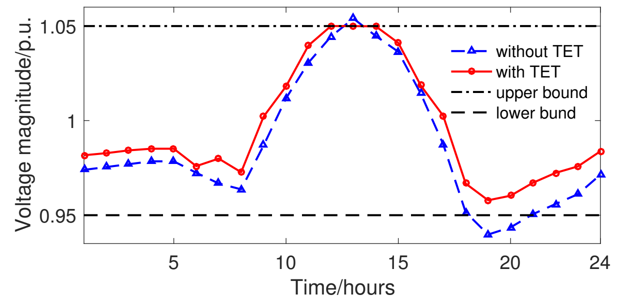
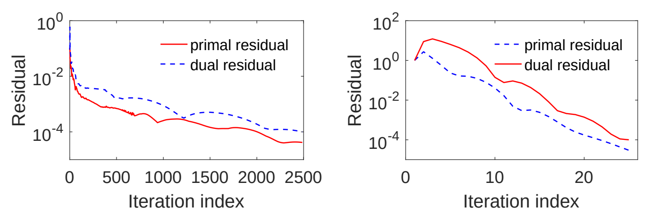


Fig. 7. Voltage magnitudes of node 122 in 123-bus system with and without TET

TABLE IV
COST COMPARISON WITH AND WITHOUT TET FOR AGENT 1 AND 5 IN 123-BUS SYSTEM ($)

Items	Agent 1	Agent 5
Cost without TET	1883.1	39.7
Utility bill plus discomfort cost	1071.3	218.7
Payment to other agents	690.8	−300
Cost with TET	1762.1	−81.3
Cost reduction	121	121



(a) **S1′** (b) **S2**

Fig. 8. Convergence result for 123-bus system, (a) subproblem **S1′**, (b) subproblem **S2**

B. Case Study 2: IEEE 123-bus Distribution System

In this subsection, we test our proposed TET framework and advanced distributed algorithm on the modified IEEE 123-bus distribution system to verify their effectiveness on large systems. The nominal voltage is 4.16 kV. Forty distributed PV systems are installed with the capacity of each being 200k VA. The network topology is not presented here due to page limits.

The total operating costs of the 123-bus system are \$40579 and \$29681 without and with TET, respectively. Just like we observed in the 37-bus system, the introduction of TET significantly lowers the operating cost in the 123-bus system as well. Fig.7 shows the voltage profile of a terminal node without and with TET. We can observe overvoltage and undervoltage violations over the day without TET due to the lack of coordination among different agents. In contrast, when TET is introduced, all voltage violations are eliminated.

The costs and benefits of two representative agents are demonstrated in Table IV, where agent 1 represents a commercial node and agent 5 represents a residential node. We can see both agents are better off in participating in TET. Actually, all agents are better off and their economic gains are equal. Therefore, the proposed TET framework is beneficial to the individual agents and the distribution system.

Fig. 8(a) and 8(b) depict the convergence result for solving **S1′** and **S2**, respectively. The total computation time is 20.9s and 0.05s for **S1′** and **S2**, respectively, if the proposed algorithm is implemented in a distributed manner. Thus, our proposed algorithm is effective and efficient in solving the TET problem.

VI. Conclusions

In this paper, we propose a novel transactive energy trading framework in DNs aiming at dealing with economic and technical issues in a holistic manner. In particular, we innovatively integrate a bilateral energy trading mechanism with the OPF technique. The Nash bargaining theory is used to model our problem in order to align the interests of individual agents with the interest of the entire system. We decompose our problem into a multi-period OPF subproblem and a payment bargaining subproblem by utilizing the Pareto efficiency of the Nash bargaining solution. Moreover, we develop an efficient distributed algorithm with closed-form updates for solving our problem based on ADMM. Case studies on IEEE 37-bus and 123-bus distribution feeders demonstrate the effectiveness of our proposed framework and the efficiency of our distributed algorithm.

In this work, we assume the DNs are three-phase balanced for simplicity. We will extend our proposed framework to three-phase unbalanced distribution systems in our future work by considering the coupling between phases.

Appendix A

For problem (17), $\mathbf{z}_{(j)i}$ only contains partial information of $\mathbf{x}_j$. $\mathbf{x}_i$ and $\mathbf{z}_{(j)i}$ are defined as

$$\mathbf{x}_i := \{p_i^x, q_i^x, P_i^x, Q_i^x, l_i^x, v_i^x, \{e_i^{j,x}\}_{j\in\mathcal{N}\backslash i}\}$$

$$\mathbf{z}_{j(i)} := \begin{cases} \{p_{(i)i}^z, q_{(i)i}^z, P_{(i)i}^z, Q_{(i)i}^z, l_{(i)i}^z, v_{(i)i}^z, \{e_{(i)i}^{k,z}\}_{k\in\mathcal{N}\backslash i}\} & \\ & j = i \\ \{P_{(j)i}^z, Q_{(j)i}^z, l_{(j)i}^z, e_{(j)i}^{i,z}\} & j \in C_i \\ \{v_{(j)i}^z, e_{(j)i}^{i,z}\} & j = A_i \\ \{e_{(j)i)}^{i,z}\} & \text{otherwise} \end{cases}$$

where the superscript x and z is used to differentiate the categories of variables. Then, the consensus form of (17) is given as

$$\min_{\mathbf{x},\mathbf{z}} \sum_{i\in\mathcal{N}} g_i\big(p_i^x, \{e_i^{j,x}\}_{j\in\mathcal{N}\backslash i}\big) \tag{25a}$$

$$\text{s.t. } v_{(i)i}^z - v_{(A_i)i}^z = 2(r_i P_{(i)i}^z + x_i Q_{(i)i}^z) - l_{(i)i}^z(r_i^2 + x_i^2) \quad i \in \mathcal{E} \tag{25b}$$

$$\sum_{j\in C_i}(P_{(j)i}^z - l_{(j)i}^z r_j) + p_{(i)i}^z = P_{(i)i}^z \quad i \in \mathcal{N} \tag{25c}$$

$$\sum_{j\in C_i}(Q_{(j)i}^z - l_{(j)i}^z x_j) + q_{(i)i}^z = Q_{(i)i}^z \quad i \in \mathcal{N} \tag{25d}$$

$$e_{(i)i}^{j,z} + e_{(j)i}^{i,z} = 0 \quad j \in \mathcal{N}\backslash i,\ i \in \mathcal{N} \tag{25e}$$

$$\|(2P_i^x, 2Q_i^x, v_i^x - l_i^x)\|_2 \le v_i^x + l_i^x \quad i \in \mathcal{N} \tag{25f}$$

$$\underline{v}_i \le v_i^x \le \overline{v}_i \quad i \in \mathcal{N} \tag{25g}$$

$$\underline{p}_i \le p_i^x \le \overline{p}_i \quad i \in \mathcal{N} \tag{25h}$$

$$\underline{q}_i \le q_i^x \le \overline{q}_i \quad i \in \mathcal{N} \tag{25i}$$

$$\mathbf{x}_j - \mathbf{z}_{j(i)} = 0 \quad j \in \mathcal{N},\ i \in \mathcal{N} \tag{25j}$$

where the consensus constraint (25j) is written explicitly as below with a slight abuse of notations.

$$0 = \mathbf{x}_j - \mathbf{z}_{(j)i}$$

$$:= \begin{cases} \{p_i^x - p_{(i)i}^z, q_i^x - q_{(i)i}^z, P_i^x - P_{(i)i}^z, Q_i^x - Q_{(i)i}^z, \\ l_i^x - l_{(i)i}^z, v_i^x - v_{(i)i}^z, \{e_i^{k,x} - e_{(i)i}^{k,z}\}_{k\in\mathcal{N}\backslash i}\} & j = i \\ \{P_j^x - P_{(j)i}^z, Q_j^x - Q_{(j)i}^z, l_j^x - l_{(j)i}^z, e_j^{i,x} - e_{(j)i}^{i,z}\} & j \in C_i \\ \{v_j^x - v_{(j)i}^z, e_j^{i,x} - e_{(j)i}^{i,z}\} & j = A_i \\ \{e_j^{i,x} - e_{(j)i}^{i,z}\} & \text{otherwise} \end{cases}$$

For problem **S2**, $\mathbf{x}_i$ and $\mathbf{z}_{(j)i}$ for each agent i is defined as

$$\mathbf{x}_i := \{\phi_i^{j,x}\}_{j\in\mathcal{N}\backslash i}$$

$$\mathbf{z}_{(j)i} := \begin{cases} \{\phi_{(i)i}^{k,z}\}_{k\in\mathcal{N}\backslash i} & j = i \\ \phi_{(j)i}^{i,z} & \text{otherwise} \end{cases}$$

The consensus form of **S2** is given as

$$\min_{\mathbf{x},\mathbf{z}} \sum_{i\in\mathcal{N}} -\ln\Big(\xi_i - \sum_{j\in\mathcal{N}\backslash i}\phi_i^{j,x}\Big) \tag{26a}$$

$$\text{s.t. } \phi_{(i)i}^{j,z} + \phi_{(j)i}^{i,z} = 0 \quad j \in \mathcal{N}\backslash i,\ i \in \mathcal{N} \tag{26b}$$

$$\phi_{(i)i}^{j,z} - \phi_i^{j,x} = 0 \quad j \in \mathcal{N}\backslash i,\ i \in \mathcal{N} \tag{26c}$$

$$\phi_{(j)i}^{i,z} - \phi_j^{i,x} = 0 \quad j \in \mathcal{N}\backslash i,\ i \in \mathcal{N} \tag{26d}$$

where $\xi_i = \widetilde{C}_i(\tilde{\mathbf{p}}_i^d) - W_i(\mathbf{p}_i^{d*}, \mathbf{e}_i^*)$ is a parameter.

Appendix B

Proof of part (a) of Proposition 1: We first prove that $\mathbf{z}_i$-update problem (23) has a closed-form solution. To this end, it is reformulated as

$$\min_{\mathbf{z}_i} \ G_i(\mathbf{z}_i) \tag{27a}$$

$$\text{s.t. } \sum_{j\in\mathcal{N}} \mathbf{A}_{ij}\mathbf{z}_{(j)i} = 0 \tag{27b}$$

where $G_i(\mathbf{z}_i) := \sum_{j\in\mathcal{N}} \big(-\langle \boldsymbol{\mu}_{(j)i}, \mathbf{z}_{(j)i}\rangle + \frac{\rho}{2}\|\mathbf{x}_j - \mathbf{z}_{(j)i}\|^2\big)$ is convex quadratic function. Hence, problem (27) can be generalized as

$$\min_{\mathbf{z}_i} \ \frac{1}{2}\mathbf{z}_i^T\mathbf{Q}\mathbf{z}_i + \mathbf{c}^T\mathbf{z}_i \tag{28a}$$
$$\text{s.t. } \mathbf{B}\mathbf{z}_i = 0 \tag{28b}$$

where $\mathbf{Q}$ and $\mathbf{B}$ are constant matrices, and $\mathbf{c}$ is a constant vector. (28) has a unique solution that can be expressed as

$$\mathbf{z}_i = \big(\mathbf{Q}^{-1}\mathbf{B}^T(\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}^T)^{-1}\mathbf{B}\mathbf{Q}^{-1} - \mathbf{Q}^{-1}\big)\mathbf{c}$$

Then we will show the $\mathbf{x}_i$-update in problem (17) can also be solved in a closed-form. The recent proposed distributed OPF algorithm in [21] is utilized to prove the proposition. To this end, we first specify the related Lagrangian multipliers, as shown in Table V. Through expansion and recollection of quadratic terms, the $\mathbf{x}_i$-update problem (22) can be further decomposed into three subproblems. The tedious process is not elaborated here for clarity. Interested readers may refer to the subsection III-B of [21].

The first subproblem solves the optimal $(P_i^x, Q_i^x, l_i^x, v_i^x)$, i.e.

$$\min \ \big(P_i^x - \widehat{P}_i\big)^2 + \big(Q_i^x - \widehat{Q}_i\big)^2 + \big(l_i^x - \widehat{l}_i\big)^2 + \frac{|C_i|+1}{2}\big(v_i^x - \widehat{v}_i\big)^2 \tag{29a}$$
$$\text{over } P_i^x, Q_i^x, l_i^x, v_i^x$$
$$\text{s.t. (25f) and (25g)} \tag{29b}$$

where

$$\widehat{P}_i = \frac{P^z_{(i)i} + P^z_{(i)A_i}}{2} - \frac{\mu^{(1)}_{(i)i} + \mu^{(1)}_{(i)A_i}}{2\rho}$$
$$\widehat{Q}_i = \frac{Q^z_{(i)i} + Q^z_{(i)A_i}}{2} - \frac{\mu^{(2)}_{(i)i} + \mu^{(2)}_{(i)A_i}}{2\rho}$$
$$\widehat{l}_i = \frac{l^z_{(i)i} + l^z_{(i)A_i}}{2} - \frac{\mu^{(3)}_{(i)i} + \mu^{(3)}_{(i)A_i}}{2\rho}$$
$$\widehat{v}_i = \frac{v^z_{(i)i} + \sum_{j\in C_i} v^z_{(i)j}}{|C_i|+1} - \frac{\mu^{(4)}_{(i)i} + \sum_{j\in C_i}\mu^{(4)}_{(i)j}}{\rho(|C_i|+1)}$$

According to [21], (29) has a closed form solution.

The second subproblem solves the optimal q_i^x, i.e.

$$\min_{q_i^x} \ \big(q_i^x - \widehat{q}_i\big)^2 \tag{30a}$$
$$\text{s.t. } \underline{q}_i \le q_i^x \le \overline{q}_i \tag{30b}$$

where $\widehat{q}_i = q^z_{(i)i} - \frac{\mu^{(6)}_{(i)i}}{\rho}$. The optimal solution can be easily obtained as $\big[\widehat{q}_i\big]_{\underline{q}_i}^{\overline{q}_i}$, where $[\cdot]_a^b$ represents the projection operator onto the range $[a, b]$.

TABLE V
MULTIPLIERS ASSOCIATED WITH CONSENSUS CONSTRAINTS

$\mu^{(1)}_{(j)i}$: $P_j^x = P^z_{(j)i}$ $j \in i\cup C_i$	$\mu^{(2)}_{(j)i}$: $Q_j^x = Q^z_{(j)i}$ $j \in i\cup C_i$
$\mu^{(3)}_{(j)i}$: $l_j^x = l^z_{(j)i}$ $j \in i\cup C_i$	$\mu^{(4)}_{(j)i}$: $v_j^x = v^z_{(j)i}$ $j \in i\cup A_i$
$\mu^{(5)}_{(i)i}$: $p_i^x = p^z_{(i)i}$	$\mu^{(6)}_{(i)i}$: $q_i^x = q^z_{(i)i}$
$\mu^{j,(7)}_{(i)i}$: $e_i^{j,x} = e^{j,z}_{(i)i}$ $j \in \mathcal{N}\backslash i$	$\mu^{i,(7)}_{(j)i}$: $e_j^{i,x} = e^{i,z}_{(j)i}$ $j \in \mathcal{N}\backslash i$

The third subproblem solves the optimal $\big(p_i^x, \{e_i^{j,x}\}_{j\in\mathcal{N}\backslash i}\big)$, i.e.

$$\min \ (\alpha_i + \frac{\rho}{2})\big(p_i^x - \widehat{p}_i\big)^2 + \sum_{j\in\mathcal{N}\backslash i}\rho\big(e_i^{j,x} - \widehat{e}_i^j\big)^2 \tag{31a}$$
$$\text{over } p_i^x, \{e_i^{j,x}\}_{j\in\mathcal{N}\backslash i}$$
$$\text{s.t. } \underline{p}_i \le p_i^x \le \overline{p}_i \tag{31b}$$
$$\underline{e}_i \le \sum_{j\in\mathcal{N}\backslash i} e_i^{j,x} \le \overline{e}_i \tag{31c}$$

where

$$\widehat{p}_i = \frac{1}{2\alpha_i + \rho}\big(2\alpha_i(p_i^g - d_i) + \rho p^z_{(i)i} + \lambda - \pi_i - \mu^{(5)}_{(i)i}\big)$$
$$\widehat{e}_i^j = \frac{e^{j,z}_{(i)i} + e^{j,z}_{(i)j}}{2} + \frac{\lambda - \mu^{j,(7)}_{(i)i} - \mu^{j,(7)}_{(i)j}}{2\rho}$$

The values of λ, $\underline{e}_i$ and $\overline{e}_i$ depend on the sign of p_i^x. If $p_i^x \le 0$, then $\lambda = \lambda^b$, $\underline{e}_i = 0$, $\overline{e}_i = -p_i^x$; otherwise $\lambda = \lambda^s$, $\underline{e}_i = -p_i^x$, $\overline{e}_i = 0$.

As long as the problem (31) can be solved in closed form, we would complete the proof. Indeed, we derive the closed form solution to (31). Since (31) is a convex quadratic optimization problem with linear inequalities, it has a unique global minimizer. Without loss of generality, suppose $p_i^x \le 0$ and update $\overline{p}_i$ as $\min(\overline{p}_i, 0)$. Then (31) is transformed to

$$\min \ (\alpha_i + \frac{\rho}{2})\big(p_i^x - \widehat{p}_i\big)^2 + \sum_{j\in\mathcal{N}\backslash i}\rho\big(e_i^{j,x} - \widehat{e}_i^j\big)^2 \tag{32a}$$
$$\text{over } p_i^x, \{e_i^{j,x}\}_{j\in\mathcal{N}}$$
$$\text{s.t. } \underline{p}_i \le p_i^x \le \overline{p}_i \tag{32b}$$
$$0 \le \sum_{j\in\mathcal{N}\backslash i} e_i^{j,x} \le -p_i^x \tag{32c}$$

In the following, we will derive its closed form solution by enumerating the activeness of the inequality constraints.

Case 1: (32c) is inactive.

Then $p_i^{x*} = \big[\widehat{p}_i\big]_{\underline{p}_i}^{\overline{p}_i}$ and $e_i^{j,x*} = \widehat{e}_i^j,\ j \in \mathcal{N}\backslash i$.

Case 2: (32c) is active and $\sum_{j\in\mathcal{N}\backslash i} e_i^{j,x} = 0$.

Then (32) can be decomposed into two subproblems. One subproblem only involves variable p_i^x, i.e.

$$\min_{p_i^x} \ (\alpha_i + \frac{\rho}{2})\big(p_i^x - \widehat{p}_i\big)^2$$
$$\text{s.t. } \underline{p}_i \le p_i^x \le \overline{p}_i$$

whose optimal solution is $p_i^{x*} = \big[\widehat{p}_i\big]_{\underline{p}_i}^{\overline{p}_i}$. The other subproblem only involves variables $\{e_i^{j,x}\}_{j\in\mathcal{N}\backslash i}$, i.e.

$$\min \ \sum_{j\in\mathcal{N}\backslash i}\rho\big(e_i^{j,x} - \widehat{e}_i^j\big)^2$$
$$\text{over } \{e_i^{j,x}\}_{j\in\mathcal{N}}$$
$$\text{s.t. } \sum_{j\in\mathcal{N}\backslash i} e_i^{j,x} = 0$$

It can be generalized as problem (28) and thus can be solved in closed form.

Case 3: (32c) is active and $\sum_{j\in\mathcal{N}\backslash i} e_i^{j,x} = -p_i^x$.

Then (32) can be reformulated as the problem below by eliminating p_i^x.

$$\min \; F_i\big(\{e_i^{j,x}\}_{j\in\mathcal{N}\backslash i}\big) \quad (35a)$$
$$\text{over} \;\; \{e_i^{j,x}\}_{j\in\mathcal{N}}$$
$$\text{s.t.} \; -\overline{p}_i \le \sum_{j\in\mathcal{N}\backslash i} e_i^{j,x} \le -\underline{p}_i \quad (35b)$$

where $F_i\big(\{e_i^{j,x}\}_{j\in\mathcal{N}\backslash i}\big) = (\alpha_i + \frac{\rho}{2})\Big(\sum_{j\in\mathcal{N}\backslash i} e_i^{j,x} + \widehat{p}_i\Big)^2 + \sum_{j\in\mathcal{N}\backslash i} \rho\big(e_i^{j,x} - \widehat{e}_i^{j}\big)^2$. Similarly, we solve (35) in closed form by enumerating the activeness of the constraint (35b).

- **Subcase 3.1:** (35b) is inactive .
  Then the optimal solution can be obtained by solving the following linear system:

  $$\frac{\partial F_i\big(\{e_i^{j,x}\}_{j\in\mathcal{N}\backslash i}\big)}{\partial e_i^{j,x}} = 0 \qquad j \in \mathcal{N}\backslash i$$

  Denote $X := \sum_{j\in\mathcal{N}\backslash i} e_i^{j,x}$. Then, we have

  $$X^* = \frac{-(N-1)(\rho+2\alpha_i)\widehat{p}_i + 2\rho\sum_{j\in\mathcal{N}\backslash i}\widehat{e}_i^{j}}{(N-1)(\rho+2\alpha_i)+2\rho}$$
  $$e_i^{j,x*} = \widehat{e}_i^{j} - \frac{\rho+2\alpha_i}{2\rho}(X^* + \widehat{p}_i)$$

- **Subcase 3.2:** (35b) is active and $\sum_{j\in\mathcal{N}\backslash i} e_i^{j,x} = -\overline{p}_i$.
  Then, the problem (35) can be converted to the following problem.

  $$\min \sum_{j\in\mathcal{N}\backslash i} \rho\big(e_i^{j,x} - \widehat{e}_i^{j}\big)^2 \quad (37a)$$
  $$\text{over} \;\; \{e_i^{j,x}\}_{j\in\mathcal{N}}$$
  $$\text{s.t.} \sum_{j\in\mathcal{N}\backslash i} e_i^{j,x} = -\overline{p}_i \quad (37b)$$

  With a slight abuse of notation, (37) can be generalized as

  $$\min_{\mathbf{y}} \; \frac{1}{2}\mathbf{y}^T\mathbf{D}\mathbf{y} + \mathbf{b}^T\mathbf{y} \quad (38a)$$
  $$\text{s.t.} \; \mathbf{E}\mathbf{y} = \mathbf{f} \quad (38b)$$

  where $\mathbf{y}$ is the vector of decision variables constituted by $e_i^{j,x}$; $\mathbf{D} := 2\rho\mathbf{I}_{N-1}$; $\mathbf{b}$ is a vector constituted by $-2\rho\widehat{e}_i^{j}$; $\mathbf{E} = \mathbf{1}^T$ and $\mathbf{f} = -\overline{p}_i$; $\mathbf{I}_{N-1}$ is a $N-1$ dimensional identity matrix. (38) has a closed form solution given by

  $$\mathbf{y}^* = \big(\mathbf{D}^{-1}\mathbf{E}^T(\mathbf{E}\mathbf{D}^{-1}\mathbf{E}^T)^{-1}\mathbf{E}\mathbf{D}^{-1} - \mathbf{D}^{-1}\big)\mathbf{b} + \mathbf{D}^{-1}\mathbf{E}^T(\mathbf{E}\mathbf{D}^{-1}\mathbf{E}^T)^{-1}\mathbf{f}$$

- **Subcase 3.3:** (35b) is active and $\sum_{j\in\mathcal{N}\backslash i} e_i^{j,x} = -\underline{p}_i$.
  Then the optimal $\{e_{ij}^x | j \in \mathcal{N}\backslash i\}$ can be obtained similarly as the subcase 3.2.

Appendix C

Proof of part (b) of Proposition 1: $\mathbf{z}_i$-update can be solved in closed form as shown in Appendix B. In this Appendix, we show the closed-form expression for updating $\mathbf{x}_i$ in problem **S2**. Let $\mu_{(i)i}^j$ and $\mu_{(j)i}^i$ denote the multipliers for (26c) and (26d), respectively. Then the explicit form of $\mathcal{L}_i^x\big(\mathbf{x}_i, \{\mathbf{z}_{(i)j}, \boldsymbol{\mu}_{(i)j}\}_{j\in\mathcal{N}}\big)$ is given as

$$\mathcal{L}_i^x\big(\mathbf{x}_i, \{\mathbf{z}_{(i)j}, \boldsymbol{\mu}_{(i)j}\}_{j\in\mathcal{N}}\big) := -\ln\big(\xi_i - \sum_{j\in\mathcal{N}\backslash i}\phi_i^{j,x}\big) + \sum_{j\in\mathcal{N}\backslash i}\Big(\mu_{(i)i}^j(\phi_i^{j,x} - \phi_{(i)i}^{j,z}) + \frac{\rho}{2}(\phi_i^{j,x} - \phi_{(i)i}^{j,z})^2\Big) + \sum_{j\in\mathcal{N}\backslash i}\Big(\mu_{(i)j}^j(\phi_i^{j,x} - \phi_{(i)j}^{j,z}) + \frac{\rho}{2}(\phi_i^{j,x} - \phi_{(i)j}^{j,z})^2\Big)$$

Note that in problem **S2** there is no local constraints. Hence, the optimality condition for the $\mathbf{x}_i$-update problem is given by

$$\frac{\partial\mathcal{L}_i^x\big(\mathbf{x}_i, \{\mathbf{z}_{(i)j}, \boldsymbol{\mu}_{(i)j}\}_{j\in\mathcal{N}}\big)}{\partial\phi_{ij}^x} = 0 \quad j \in \mathcal{N}\backslash i$$

Denote $Y_i := \sum_{j\in\mathcal{N}\backslash i}\phi_i^{j,x}$. Then the optimal Y_i at $(k+1)$-th iteration, denote as $Y_i[k+1]$, can be obtained by solving the following quadratic equation.

$$Y_i^2 + \Big(\frac{a_i[k+1]}{2\rho} - \xi_i\Big)Y_i - \frac{a_i[k+1]\xi_i}{2\rho} - \frac{N-1}{2\rho} = 0$$

where

$$a_i[k+1] = \sum_{j\in\mathcal{N}\backslash i}\Big(\mu_{(i)i}^j[k] + \mu_{(i)j}^j[k] - \rho\phi_{(i)i}^{j,z}[k] - \rho\phi_{(i)j}^{j,z}[k]\Big)$$

Finally, we obtain the closed-form expression for updating $\mathbf{x}_i$, as shown below.

$$\phi_i^{j,x}[k+1] = \frac{\phi_{(i)i}^{j,z}[k] + \phi_{(i)j}^{j,z}[k]}{2} - \frac{\mu_{(i)i}^j[k] + \mu_{(i)j}^j[k]}{2\rho} - \frac{1}{2\rho(\xi_i - Y_i[k+1])}$$

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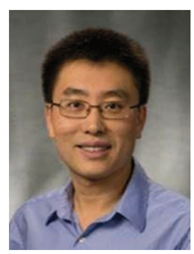

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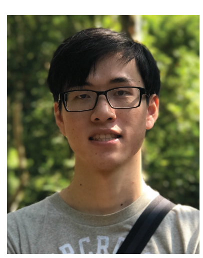

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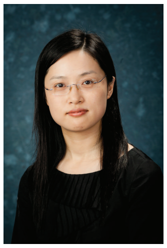

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