A fleet deployment model to minimize the covering time of maritime rescue missions

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Abstract

This paper investigates a covering time minimization problem of the maritime rescue missions that arise in practical rescue operations in the context of Hong Kong waters. In this problem, a fleet of heterogeneous vessels is deployed at marine police bases to deal with emergencies. Once an emergency rescue request is received, the marine police should send sufficient vessels to arrive at the incident site as soon as possible to provide critical medical service to the injured or the sick and then transport patients to emergency medical facilities. A basic question to the rescue missions is that what is the minimal covering time that marine police could promise to arrive at any incident site in its responsible water district. The shorter time the water district can be covered, the more likely lives and properties can be saved and the better the rescue service is. To address this problem, this paper formulates a mixed-integer programming model. Considering the expensive computational effort associated with solving this model, a two-stage method is proposed. Extensive numerical experiments and a case study are performed to demonstrate the efficiency of the proposed algorithm and illustrate how our model can be applied to solve practical problems. Our study contributes to the stream of research on maritime rescue problem that is gaining increasing concern in recent years. The proposed method can help rescue agency to provide faster response to rescue missions.

Keywords: maritime rescue; minimal covering time; fleet deployment; mixed-integer programming problem

1 Introduction

The maritime industry, including navigation, shipping, and marine engineering, is one of the largest industries on the planet and has a direct impact on much of our everyday life. Nowadays, around 90% of world trade is transported by the international shipping industry. Meanwhile, the offshore oil exploration and production industry and the cruise sector have experienced a significant qualitative and quantitative expansion (Northeast Maritime Institute, 2018; Qi et al., 2021).

To run a successful and sustainable maritime industry (Wang et al., 2021), ensuring maritime safety is one of the most essential prerequisites and requires determination and substantial effort (Jiang et al., 2020; Nguyen, 2020). The International Maritime Organization (IMO) developed and adopted various conventions and standards regarding maritime safety, including, but not limited to, the International Convention for the Safety of Life at Sea, the International Convention on Maritime Search and Rescue, the Convention on the International Regulations for Preventing Collisions at Sea, the International Convention for Safe Containers, and the Convention for the Suppression of Unlawful Acts Against the Safety of Maritime Navigation (IMO, 2021; Yan et al., 2020, 2021a; Yan et al., 2021b). These conventions can be summarized into two categories. The first category is for reduction of incidents risk/probability by setting specific codes and standards, and providing necessary guidelines, such as codes for cargo stowage and securing, and for ships carrying timber deck cargoes. The second category is for coping with the aftermath of an incident, i.e., maritime search and rescue (MSAR). Besides the IMO, many coastal countries around the world also developed their arrangements to fulfill their responsibilities related to ports, shipping, and maritime emergencies.

Although a variety of efforts have been made to minimize the risks and threats at sea, maritime casualties and incidents still cannot be eliminated at the sea. The European Maritime Safety Agency reported that 3174 occurrences were reported in the year 2018, and over 23,000 marine casualties or incidents were recorded in their database during 2011–2018 (European Maritime Safety Agency, 2019). The U.S. Coast Guard recorded a total of 1067 injury/death cases and 15,097 vessel events between 2002–2015 (The U.S. Coast Guard, 2015). The Hong Kong (HK) Maritime Department reported 2915 reported accidents during 2012–2020 which resulted in a total of 73 lives lost and 648 lives injured within HK waters (HK Marine Department, 2020). Facing a large number of casualties occurring at sea, quick response to an incident is crucial for MSAR which receives much attention from the public. For instance, in the sinking of MV Sewol, a disaster occurred on 16 April 2014 when the ferry MV Sewol sunk during its trip from Incheon towards Jeju in South Korea, the heavy casualties were in part due to a slow and badly coordinated coast guard response. It is believed that more can be saved if the Korea Coast Guard had provided better response because even fishing boats and other commercial vessels that arrived later but put rescue into action earlier saved more than the coast guard did (The Chosun Ilbo, 2014). Besides responding quickly, rescue vessels should be capable of providing pre-hospital emergency medical service to the sick or injured at the scene before transporting them to an emergency medical facility (Cheng and Liang, 2014; Wu et al., 2020). In brief, efficient and effective marine rescue operation is of great significance to provide instantaneous help for victims in danger and reduces the fatality rate at sea.

Hong Kong is one of the world's most important maritime hubs, which is home to the fourth-largest ship register, with a total of about 130 million gross tonnes, and the eighth-busiest container port (World Shipping Council, 2021). Maritime and port industry in HK is playing a significant role in contributing to achieving sustainable development, economic growth, livelihoods, and has traditionally been one of the key pillars of HK's vibrant economy. The HK waters have one of the heaviest maritime traffic flow around the world due to a wide range of vibrant maritime transportation between the Pearl River Delta and the rest of the world, including shipping, fishing, cruise tours, and tugging, *etc*. The location and movement of ships in HK waters can be found on an online website called MarineTraffic[*], which shows the whole HK waters are covered by various types of ships. The large traffic volume and narrow waters

[*] https://www.marinetraffic.com/

bring about risks to maritime safety. Therefore, providing effective management of MSAR resources and prompt response to an incident in HK waters are meaningful to increasing the confidence for carrying out maritime economic activities, providing a secure environment for tourism, commerce, transportation, preventing sea pollution, and protecting maritime resources.

Hong Kong maritime rescue coordination center (MRCC), a branch of HK Marine Department, is a coordination center to coordinate resources (e.g., Marine Police, and Government Flying Service) from HK government departments to perform MSAR missions in HK territorial waters and an extensive area of the South China Sea. Since rescue vessels are the primal vehicles used to respond to maritime rescue requests and HK waters are the central district where the maritime traffic demands are concentrated, this study focuses on the vessel assets and their arrangement to fulfill MSAR missions within HK territorial waters**.**

The HK MRCC has a standard operating procedure and guideline to deal with various maritime incidents. When a vessel encounters an emergency, a message should be sent to MRCC which should contain the ship's name, position, number of injuries and their condition, the extent of ship's damage, *etc*. Once receiving an emergency message from a vessel, marine officers will initially investigate and verify the reported distress to determine if an MSAR response is needed. If the need is validated, the officers will evaluate the nature of the distress and consider the availability of MSAR resources. Then, an MSAR plan will be developed and the officers will coordinate resources to execute this plan. Officers will be fully engaged in the rescue mission, updating information on the distress situation, coordinating support requests, and documenting all activities associated with the mission. When all rescue activity has terminated, a report will be prepared and submitted to the Director of Marine (HK MRCC, 2020).

The overall rescue process suggests that the marine officers play the key role in a rescue mission as they are responsible for evaluating the emergency, developing an MSAR plan, engaging in tracking the progress of each resource responding to the rescue, preparing and submitting mission report. The marine officers should be experienced persons who hold seagoing master mariner class I certificate and have undergone intensive training in MSAR techniques. During a rescue mission, the marine officers will use their professional judgment to coordinate resources. However, decisions based on only experience and intuition may be suboptimal since rescue decision process is usually dynamic and very complicated because of lots of contingencies and the large number of factors to take into account. Green and Kolesar (2004) pointed out that management science (MS)/operations research (OR) applications play an important role in enhancing the efficiency of minimizing the impact of emergencies, which is particularly applicable to MSAR operations in HK which has one of the heaviest maritime traffic flow around the world.

The response time to arrive at an incident spot, which is defined as the time from the dispatch of rescue force until the arrival of the rescued at the destination spot, is a useful performance measurement to evaluate the efficiency of rescue operations. Considering that the incidents may happen at any spot in waters, the response times to rescue requests are variable and random. Therefore, a conservative and robust performance indicator to measure the service of the MSAR missions is the maximal time that the HK rescue force needs to arrive at any requested spot, which is termed *covering time* of the water district. Note that in practice it is very common for emergency services to set a service pledge for this covering time. Once promising a limit of the covering time, rescue force needs to arrive at the scene within this limit no matter where the incident happens. The MRCC's service level is higher if it can give a stricter service pledge and is capable of achieving it. While there are many factors that can affect actual response time, rescue fleet deployment is in no doubt one of the most important decisions that play crucial roles in determining the covering time a priori. In this study we propose a mixed-integer programming model for a tactical vessel deployment problem, where the objective is to minimize the covering time. We analyze the properties of the optimal decision. Considering the computation time of off-the-shelf solvers increases rapidly as the size of the problem grows, a two-stage method is proposed to solve this problem.

1.1 Literature review

In our literature review, we specifically address two areas of research. First, we treat related optimization problems of other fields with a similar orientation. Then, we further address the literature dedicated to MSAR problems.

There exist quite a few studies that treat similar coverage optimization problems in other domains. For instance, the facility location covering problem. The seminar model is the set covering location problem which objective is minimizing the facility location cost such that a certain coverage level is obtained. In other words, the set covering problem addresses the problem of deciding the facility location to ensure a certain level of coverage to all demands. Further studies investigate facility location decision under a variety of extensions, such as the maximal covering (Church and ReVelle, 1974; White and Case, 1974), multiple coverage (Daskin and Stern, 1981), backup coverage (Batta and Mannur, 1990), P-center or minimax problem (Lin and Lin, 2018), etc. Another line of studies relates the covering problem to the vehicle touring (Laporte, 2009), location routing (Nagy and Salhi, 2007), traveling salesman (Bektas, 2006), where the visiting path is variable and the possible objective is minimizing the travel distance/time. The covering problem has been widely investigated in the domain of safety and emergency management (Bao et al., 2021; Huang et al., 2021), such as the maximum expected covering model to locate and dispatch ambulances (Ansari et al., 2017), the hierarchical objective set covering model for ambulances deployment (Daskin and Stern, 1981), the probabilistic location model (Daskin, 1983), to name a few.

While there is extensive literature related to emergency management, few studies consider optimization and evaluation of MSAR problems. The topics in MSAR can be classified into three categories. The first category integrates search and rescue operation which is suitable for the scenarios that incident location is uncertain (Abi-Zeid and Frost, 2005; Abi-Zeid et al., 2011). The second category aims to evaluate the performance of fleet deployment plan (Karatas et al., 2020; Tozan and Karatas, 2018), and the last category is to manage the MSAR resources to improve the search and rescue services (Akbari et al., 2018a; Akbari et al., 2018b; Azofra et al., 2007; Hornberger et al., 2021; Karatas, 2021; Razi and Karatas, 2016; Wagner and Radovilsky, 2012). Our research falls within this third category.

Table 1 summarizes the literature on MSAR resources management (i.e., third category). Among them, the studies that try to improve the response time include Pelot et al. (2015), Razi and Karatas (2016), Akbari et al. (2018a), Akbari et al. (2018b), Karatas (2021), and Hornberger et al. (2021). All of them consider multi-objective and multiple criteria to optimize and evaluate their MSAR resource deployment plan. Response time reduction is one objective among them. To calculate the response time between a boat station and accident spot, the studied water area has to be discretized, i.e., partitioned the water area into a set of zones/grids (details will be described in Section 2.2). In theory, to obtain an accurate estimate of response time, the size of zones should be sufficient small. However, the computational cost grows dramatically as the size of zone set increases. To address this issue, the common idea is to control the problem scale. Two general approaches are adopted in the abovementioned studies. Razi and Karatas (2016) and Hornberger et al. (2021) apply the zonal distribution (ZD) model which is originally proposed by Azofra et al. (2007). The spirit of ZD model is to cluster/aggregate the accident spots into a small number of zones and use a centroid called "super-accident" to represent a group of accidents. Due to the rough representation of water zones, the estimated response time is inaccurate. Other studies, i.e., Pelot et al. (2015), Akbari et al. (2018a), Akbari et al. (2018b), and Karatas (2021), reduce the size of problem by adjusting or controlling the size of grids/zones. Typically, they use various grid cell size: small size (0.25×0.25 degree), medium size (0.5×0.5 degree), and large size (1.0×1.0 degree) to control the problem size. The selection criteria of grid size are the density of accidents and the distance from coastline. The area with higher accident density or closer to the coastline will be represented by smaller grid. Even with the small size grid (0.25×0.25 degree), it is at least a 20×20 km^2 area, let alone the large size grid which is 16 times as large.

Insert Table 1 here

1.2 Objectives and contributions

In summary, existing MSAR resource management models are usually formulated as a multi-objective problem, thus, they can only give a rough estimation of the response time. Moreover, due to the complexity of these models, their responsible water districts (which are mainly open waters) were either clustered into a small number of zones according to the historical accident records or represented by oversize grids, which make their problem easy to tackle but the estimated rescue time is inaccurate. By comparison, this study focuses on the covering time minimization problem and considers features such as heterogeneous vessel capacity and speeds, rescue capacity requirement, and geographical shapes. By an in-depth analysis of the properties of the proposed model, we design an efficient solution approach to addressing large-scale problems and provide much more accurate estimation of the maritime rescue time. To the best of our knowledge, our study is the first to focus on the optimization of covering time for MSAR.

The remainder of this paper is organized as follows. In Section 2, we provide a detailed description of the problem considered and formulates a mixed-integer programming model for the problem. In Section 3, we analyze the model. In Section 4, we present a two-stage method to deal with the expensive computation effort associated with the large-scale variables. To test the performance of the proposed algorithm, we conduct extensive numerical experiments and a case study in Section 5. Finally, we present the conclusions and future research directions in Section 6.

2 Problem statement

The MSAR operation in HK waters is coordinated by the HK MRCC and undertaken by the Marpol. Figure 1 shows the overview of this area, which is a 1,651 km^2 area enclosed by the border (dash-dot ring) and coastline (solid rings). The rescue requests could happen in any spot in this area. When a rescue request is received, the Marpol should send sufficient rescue vessels to arrive at the scene as soon as possible. Currently, the Marpol operates a fleet of over 110 vessels to fulfill its responsibilities in a wide range of operational environments. However, not all of these vessels are suitable to fulfill the rescue missions. Generally, the vessels that are suitable for rescue operations should be large enough to accommodate rescue equipment and injured and be fast enough to arrive at the scene. Therefore, we select three types of vessels to carry out the missions: (i) MSAR launches with a length of more than 30 meters, (ii) MSAR launches between 15-30 meters, and (iii) MSAR boats under 15 meters in length. A total of nine Marpol bases are located on the coastline of HK's islands, which are represented by hexagon nodes in Figure 1. The rescue operation of Marpol in this problem involves decisions of two levels, both tactical and operational. At the first level (tactical one), the Marpol assigns vessels to Marpol bases. Following this, at the second level (operational one), the vessels to execute MSAR mission should be selected according to the rescue demands, location of the bases, and the event spot. These two-level decisions are interconnected and thus require formulating a mathematical optimization problem to search for the optimal solution.

Insert Figure 1 here

2.1 An illustration of the covering problem

In this section, we illustrate the covering problem and its solution by using several simplified examples. Suppose that the map is represented by a two-dimensional plane with axes u and v. The water district is the area enclosed by a circle with center (0, 0) and radius r. There are two bases, located at (-1, 0) and (1, 0) respectively. The operator currently has four small vessels, two medium vessels, and two large vessels, with capacity 2, 3, and 4, respectively. A rescue team with total capacity no less than $\overline{c}$ is required at each base. At first glance, the problem seems easy, and it is intuitive to split the fleet equally between two bases because the water area is symmetric and accidents occur independently. However, it may not be optimal to do so. First, we consider a case (Case 1 in Table 2 and Figure 2) where the radius of the water area is 1.5, the speeds of the small, medium, and large vessels are 4, 3, and 1, respectively, and a qualified rescue team requires a total capacity no less than 7 (referred to as the rescue capacity requirement and denoted by $\overline{c}$). Then, we find by simple analysis that the optimal vessel allocation plan is symmetric: allocate two small vessels and one medium vessel to each base to carry out rescue missions. The large vessels are not used and can be allocated anywhere. Each base is assigned a half circle area. We further consider three other cases in which the parameters change a little. We find that the optimal solution becomes asymmetric when the radius of the water area increases to 3 (in Case 2), or the speed of the originally unused large vessels increases to 2 (in Case 3), or the rescue capacity requirement increases to 8 (in Case 4). The setting and solutions of these four examples are summarized in Table 2. Figure 2 illustrates the service regions assigned to each base: the red region is covered by the base at (-1, 0) and the green region is covered by the base at (1, 0). The points that have the largest covering time (annotated by CT) are marked by a "*" on the graphs.

Insert Table 2 here

Insert Figure 2 here

The results of these simplified examples demonstrate how sensitive the solution is to input information. Even with a small change in the parameters, the optimal vessel allocation and dispatch plans may entirely change. The shapes of the assigned service regions may be irregular (e.g., non-convex) even when the water area and problem parameters are symmetric. Therefore, solving the covering problem is can be very challenging and hence requires not only intuitive practices but also a systematic approach to find out the optimal solution.

2.2 Water district discretization

The covering time minimization problem belongs to the category of covering problem. An intuitive idea is to assume each Marpol base as the center of a circle and to use these circles to cover the water district regulated by the Marpol. Then, this problem can be formulated as a linear programming problem. However, the real-world situation is more complicated. As shown in Figure 1, there are many islands in HK waters and the coastlines of these islands are irregular. The vessels cannot arrive at all area with a straight path, and in most situations, the voyage trajectory is a polyline. Considering this constraint, we first propose a discretization modeling approach. Specifically, we partition the water district into a set of small square zones, as illustrated in Figure 3, and then bypass the obstacles. After partition, the water district is discretized in the way that each square zone is represented by its centroid. The centroids are those “destination” nodes where the rescue force is destined. Once the set of centroids is defined, the rescue path, i.e., the shortest path between a Marpol base to the event point, over the water area can be derived. Theoretically, smaller zones (finer partition) lead to a more accurate rescue path. However, it also increases the size of the problem and the computation time. In practice, the size of zones depends on specific requirement such as how accurate we want the discrete approximation of the map be, how complicated the costal line is, the obstacles in waters, etc. Once the set of centroids is defined, the shortest travel distance between these centroids and Marpol bases can be derived.

Insert Figure 3 here

2.3 The mathematical formulation

After the discretization of waters, the covering time minimization (MCT) problem can be formulated as a mixed-integer programming problem, where the objective is to minimize the covering time to the event spot and the constraints are to specify the interconnections between the fleet deployment plan, the mission assignment plan, the vessel attributions, Marpol base locations, and rescue capacity requirements. Before formulating the mixed-integer programming problem, we list the notation used in the model as follows:

Sets

I	The set of marine police bases
J	The set of zones
$\tilde{J}$	The restricted set of zones
K	The set of marine police vessels

Indices

i	The index of marine bases
j	The index of zones

k	The index of marine police vessels
Parameters	
s_k	The speed of marine police vessel k
c_k	The rescue capacity of marine police vessel k
$l_{i,j}$	The travel distance from marine base i to zone j
$t_{i,j,k}$	The travel time of vessel k from marine base i to zone j
$\overline{c}$	The number of casualties in an event, which is also the rescue capacity requirement
Decision Variables	
$x_{i,k}$	A binary decision variable which equals 1 if and only if vessel k is deployed in base i
$y_{k,j}$	A binary decision variable which equals 1 if and only if vessel k is assigned to sail to zone j
$z_{i,j,k}$	A binary decision variable which equals 1 if and only if vessel k is deployed in base i and assigned to sail to zone j
$\tau_{i,j,k}$	An intermediate decision variable
t	A covering time promised by the marine police that they can arrive at any event scene with this time

2.3.1 The mixed-integer programming model

According to the report of HK Marine Department (2020), the number of recorded incidents/cases are 312, 302, 358, 319, 319, 368, 335, 321, and 281, respectively, between 2012 and 2020. On average, 0.89 case happen every day, which means the frequency of doing the rescue missions is rare in a day. Therefore, we assume only one event happens during the rescuing process and all the vessels are standing by in their marine bases. Let I denote the set of Marpol bases, and i denote the index of a Marpol base. The HK water district is partitioned into a set of zones J and j denotes the index of one zone. The travel distance from Marpol base i to zone j is denoted as $l_{i,j}$. We assume the rescue vessels always choose the shortest travel path whose length is denoted by $l_{i,j}$. Let K denote the fleet of vessels operated by the Marpol and $k \in \{1, 2, \cdots, \mathrm{card}(K)\}$ denote the index of one vessel, where $\mathrm{card}(\cdot)$ denotes the cardinality of a set. The speed and rescue capacity of vessel k is denoted as s_k and c_k, respectively. The travel time of vessel k from marine base i to zone j is determined by $l_{i,j}$ and s_k, i.e., $t_{i,j,k} = l_{i,j} / s_k$. For a zone j, the response time and rescue capacity of fleet depend on the selection of vessels. Namely, the selection of vessels determines the travel time and rescue capacity for zone j. Appropriate deployment and selection of vessels determine the minimal covering time. Then, we have the following optimization problem:

[MCT]

$$\min_{X,Y,Z} t \tag{1}$$

subject to

$$\tau_{i,j,k} = t_{i,j,k} \cdot z_{i,j,k}, \forall i \in I, j \in J, k \in K, \tag{2}$$

$$\tau_{i,j,k} \le t, \forall i \in I, j \in J, k \in K, \tag{3}$$

$$\sum_{k \in K} c_k \cdot y_{k,j} \ge \overline{c}, \forall j \in J, \tag{4}$$

$$\sum_{i \in I} x_{i,k} = 1, \forall k \in K, \tag{5}$$

$$z_{i,j,k} \ge x_{i,k} + y_{k,j} - 1, \forall i \in I, j \in J, k \in K, \tag{6}$$

$$z_{i,j,k} \le x_{i,k}, \forall i \in I, j \in J, k \in K, \tag{7}$$

$$z_{i,j,k} \le y_{k,j}, \forall i \in I, j \in J, k \in K, \tag{8}$$

$$x_{i,k} = \{0,1\}, \forall i \in I, k \in K, \tag{9}$$

$$y_{k,j} = \{0,1\}, \forall k \in K, j \in J, \tag{10}$$

$$z_{i,j,k} = \{0,1\}, \forall i \in I, j \in J, k \in K, \tag{11}$$

where $x_{i,k}$, $y_{k,j}$, and $z_{i,j,k}$ are decision variables. $x_{i,k}$ decides whether deploy vessel k to base i, $y_{k,j}$ decides whether assign the task of sailing from zone j to vessel k, and $z_{i,j,k}$ decides whether vessel k starts from base i and sails to zone j. For ease of notation, we let X denote the matrix $\left[x_{i,k}\right]$, Y denote the matrix $\left[y_{k,j}\right]$, and Z denote the three-dimensional array $\left[z_{i,j,k}\right]$, where $i \in I$, $j \in J$, $k \in K$. The objective function (1) is to minimize the covering time t. Constraint (2) defines a type of intermediate variable $\tau_{i,j,k}$ which equals $t_{i,j,k}$ if vessel k starts from base i and sails to zone j. Otherwise, $\tau_{i,j,k}$ equals 0. In other words, if $z_{i,j,k}$ equals 1, we concern about the specific cruising time $t_{i,j,k}$, and wish to select the smallest one from all the possible combinations. Otherwise, as no task is assigned to vessel k, it will not sail to zone j, and thus $\tau_{i,j,k}$ is set to 0. Constraint (3) requires the covering time t is no less than $\tau_{i,j,k}, \forall i \in I, j \in J, k \in K$. Constraints (4) requires the rescue capacity provided by the marine policy is no less than a predetermined requirement, which is an important indicator of service level in practice. If a single vessel is not able to satisfy the predetermined rescue capacity requirement, more vessels at bases would be selected to satisfy this constraint. The rescue capacity of this task is defined as the sum of rescue capacities of selected vessels. Constraint (5) ensures each vessel is deployed to a base. Constraints (6)–(8) define the relationship between $x_{i,k}$, $y_{k,j}$, and $z_{i,j,k}$. Constraints (9)–(11) ensure the domain of decision variables.

The proposed model can be solved by off-the-shelf solvers. However, to guarantee the solution accuracy, the water district should be partitioned into many small zones. According to our experiment, although the off-the-shelf solvers can efficiently handle small-scale problems, the computation time and requirement of computer memory grow rapidly or even exponentially as the number of zones, i.e., $|J|$, increases. Take the HK water district as an example, there exist nine Marpol bases and approximately 30 vessels can fulfill the rescue missions. However, to ensure the solution accuracy, the number of zones should be large enough, e.g., at least thousands of zones. The off-the-shelf solvers fail to find the optimal fleet deployment plan within an acceptable time range. In fact, Kouvelis and Yu (1997) show in Chapter 3 of their book that most classic robust discrete optimization problems belong to the NP-hard class. Our problem, with a minimax objective in essence as discussed in the next section, is a robust discrete optimization problem and computationally complex. In our study, to handle a large number of variables generated by a realistic discrete approximation, we first simplify the problem based on several properties of the optimal solution.

3 Problem simplification

In this section, we analyze the problem and show several important properties of the optimal solution. These properties allow us to simplify the problem by reducing its scale and provide intuitions for the design of a two-stage algorithm in the next section.

Note that the decisions of the [MCT] problem consist of a vessel allocation plan specified by X and a vessel dispatch plan specified by Y. The variables $z_{i,j,k}$'s can be uniquely determined by X and Y according to constraints (6)–(8). By separating the decisions X and Y, we can transform the [MCT] problem into a two-stage problem: a problem to determine the optimal allocation of vessels across bases (referred to as the *[MCT-allocation] problem*) and then a sub-problem to optimally dispatch the allocated vessels to cover all zones (referred to as the *[MCT-dispatch] sub-problem*). We analyze this two-stage problem backward.

First, the optimal dispatch sub-problem at the second stage given a specific allocation plan X can be formulated as follows.

[MCT-dispatch]

$$\min_{Y,Z} t$$

subject to constraints (2)–(4), (6)–(8), (10), and (11), where the value of X is given and Y and Z are decision variables. We let $CT(X, J)$ denote the optimal objective value of the [MCT-dispatch] sub-problem on the set of zones J for a given allocation plan X and let it be infinity if the sub-problem has no feasible solution. Likewise, let $CT(X, \tilde{J})$ be the optimal value of the problem on a subset of zones $\tilde{J}$. In particular, $CT(X, j)$ is the optimal value of the problem restricted to a single zone j and hence is the *shortest response time* in which sufficient rescue vessel(s) is/are deployed to zone j given X, which is referred to as the *covering time of the zone* under a vessel allocation plan. Then, $CT(X, J)$ is referred to as the *covering time of an area* consisting of all zones $j \in J$. It is equal to the maximum of all the covering times of the zones in the area, i.e., $CT(X, J) = \max_{j \in J}\{CT(X, j)\}$. Furthermore, $CT(X, J_1 \cup J_2) = \max\{CT(X, J_1), CT(X, J_2)\}$.

Second, knowing the allocated vessels will be optimally dispatched by solving [MCT-dispatch], the vessel allocation problem at the first stage is to find out the minimum covering time of the area, which can be formulated as follows.

[MCT-allocation]

$$\min_{X} CT(X, J)$$

subject to constraints (5) and (9).

According to the above decomposition, we can see that the [MCT] problem is in fact a min-max-min problem[†] where the objective is to minimize the maximum of the shortest response times of all zones. Since we need to take into account all zones in the problem, it is intuitive that only those far away from the bases are effective in determining the optimal solution. The following lemma and propositions substantiate this intuition.

For two zones j and j', we define $j \prec_I j'$ if $l_{i,j} \leq l_{i,j'}$ for all $i \in I$ and there exists one i' such that the inequality is strict, i.e., $l_{i',j} < l_{i',j'}$. This relationship means that a zone j is *uniformly closer* than a zone j' with regard to the set of bases I. With the notation $\prec$, we can

[†] The first "min" is over all vessel allocation plans; the "max" is over all zones; the second "min" is over all bases.

define the "inner" part of J with regard to bases I as $J^{\circ} := \{ j : \exists\, j' \in J \text{ s.t. } j \prec_I j' \}$, which is the set of all interior zones. Let $J^b := J \setminus J^{\circ}$ denote the set of all boundary zones.

Lemma 1. *Given any vessel allocation plan X,* $CT(X, J^{\circ}) \leq CT(X, J^b) = CT(X, J)$*, that is,*

$$\max_{j \in J^{\circ}} \{ CT(X, j) \} \leq \max_{j \in J^b} \{ CT(X, j) \} = \max_{j \in J} \{ CT(X, j) \}.$$

This lemma shows that given any vessel allocation plan X, solving the [MCT-dispatch] sub-problem involves only zones in J^b and is independent of zones in J°. By this result, we have the following proposition for the [MCT] problem.

Proposition 1. *A vessel allocation plan* X^* *is optimal for [MCT] on* J *if and only if it is optimal for [MCT] on* J^b*. That is, the [MCT] problem on* J *and the [MCT] problem on* J^b *have the same optimal solution(s) and optimal objective value.*

For ease of exposition, we call that two optimization problems are equivalent if they have the same optimal solution(s) and optimal objective value. Proposition 1 states that the [MCT] problem on J is equivalent to the problem restricted to J^b, with all interior zones removed from the problem. Thus, if we eliminate all constraints regarding $j \in J^{\circ}$ in [MCT], the problem can be simplified without sacrificing accuracy. The reduction of computation efforts can be very substantial since there are usually a lot more interior zones than boundary zones.

Besides eliminating interior zones, we further examine how the distance of a zone affects whether a zone is effective or ineffective in solving [MCT]. Let $l_{\cdot,j}^{min} := \min_i \{ l_{i,j} \}$ denote the distance from a zone j to its closest base and $\bar{l} := \max_j \{ l_{\cdot,j}^{min} \}$ be the largest of these values among all zones. Let $J^{near} := \left\{ j \in J : \frac{l_{\cdot,j}^{\min}}{\min\{s_k\}} \leq \frac{\bar{l}}{\max\{s_k\}} \right\}$. Under an allocation plan, we call a base is capacitated if the total capacity of vessels assigned to it is no less than $\bar{c}$ and is incapacitated otherwise. Let an *admissible allocation plan* be defined as an allocation plan such that if $\sum_{k \in K} c_k x_{i,k} < \bar{c}$ for some base i, then $c_{k'} > \sum_{k \in K} c_k x_{i',k} - \bar{c}$ for any $x_{i',k'} = 1$. That is, in an admissible plan, if a base i is incapacitated (i.e., the total capacity of vessels assigned to it is less than $\bar{c}$), then the other bases should not be assigned redundant vessel(s). Intuitively, it does not make sense and is not optimal to consider inadmissible allocation plan in solving the [MCT] problem.

Proposition 2. *Suppose there are sufficient vessels that all bases are capacitated in any admissible allocation plan. Then, if* $\frac{l_{\cdot,j}^{min}}{min\{s_k\}} \leq \frac{\bar{l}}{max\{s_k\}}$ *for a zone* j*, the [MCT] problem on* J *is equivalent to the [MCT] problem on* $J \setminus \{j\}$*. Furthermore, the [MCT] problem on* J *is equivalent to the [MCT] problem on* $J \setminus J^{near}$*.*

Note that the prerequisite condition of Proposition 2 can be verified by showing that the set of admissible allocation plan(s) with incapacitated base(s) is empty, which can be done by formulating an auxiliary mixed-integer programming problem and showing it does not have any feasible solution. Note that there is a fixed fleet for the HK Marpol's problem we consider in the case study in Section 5.3. The fleet is sufficient to satisfy this condition if the rescue capacity requirement $\bar{c}$ is small enough. Therefore, we are also interested in finding out the largest $\bar{c}$ such that all bases are capacitated in any admissible allocation plan, or the smallest $\bar{c}$ such that there exists incapacitated base(s) in some admissible allocation plan. We refer to the latter as the critical rescue capacity $\tilde{c}$ (CRC) in our paper. Assume that all c_k's and $\bar{c}$ are integers and let $M = \sum_{k \in K} c_k$ be a "large" constant. To address this CRC optimization (CRCO) problem, we formulate a mixed-integer programming problem using a Big-M method, which is expressed as follows:

[CRCO]

$$\min_{X,c,p,p_i} c \tag{12}$$

subject to

$$M \cdot p_i \geq \sum_{k \in K} c_k x_{i,k} - c + 1, \forall i \in I, \tag{13}$$

$$M \cdot p_i \leq M + \sum_{k \in K} c_k x_{i,k} - c, \forall i \in I, \tag{14}$$

$$p \leq p_i, \forall i \in I, \tag{15}$$

$$p \geq \sum_{i \in I} p_i - card\left(I\right) + 1, \tag{16}$$

$$c_k x_{i,k} + M \cdot p + M\left(1 - p_i\right) + M\left(1 - x_{i,k}\right) \geq \sum_{k \in K} c_k x_{i,k} - c + 1, \forall i \in I, k \in K, \tag{17}$$

$$c \geq M \cdot p, \tag{18}$$

$$p, p_i \in \{0,1\}, \forall i \in I,$$

$$c \geq 0,$$

and constraints (5) and (9),

where p_i's and p are binary variables that denote whether a base is or all bases are capacitated, and c is the capacity requirement. Given an allocation plan X, constraints (13) and (14) help to mark all capacitated bases with $p_i = 1$ and all incapacitated bases with $p_i = 0$. Specifically, constraints (13) require that $p_i = 1$ if the capacity allocated to base i is no smaller than c. Constraints (14) require that $p_i = 0$ if the capacity allocated to base i is less than c. By constraints (15) and (16), the variable p is assigned a value 1 if all bases are capacitated and 0 otherwise. Constraints (17) guarantees that the allocation plan X is admissible. That is, any capacitated base should not be assigned redundant vessel(s) if all bases are not capacitated under the plan X. For example, if $p = 1$, or $p_i = 0$, or $x_{i,k} = 0$, constraint (17) is trivial since M is large. Otherwise, if $p = 0$, $p_i = 1$, and $x_{i,k} = 1$, then constraint (17) reduces to $c_k \geq \sum_{k \in K} c_k x_{i,k} - c + 1$, which ensures that any vessel k assigned to a capacitated base i is not redundant if not all bases are capacitated. Constraint (18) prompts the solver to find an allocation plan with incapacitated base(s). The objective (12) of [CRC] problem is to find the smallest capacity requirement such that there exists incapacitated base(s) in some admissible allocation plan. Let $\tilde{c}$ be the optimal objective value of (12) and hence be defined as the critical rescue capacity value. Then, if the predetermined rescue capacity requirement $\bar{c} < \tilde{c}$, there are sufficient vessels that all bases are capacitated in any admissible allocation plan. In other words, Proposition 2 can be applied to further reduce the size of J_0 when $\bar{c} < \tilde{c}$.

Besides, we further provide in the following proposition some sufficient conditions that are easier to verify. Rank the rescue capacities of all vessels in descending order and let $c_{(k)}^{\downarrow}$ be the *k*th-largest value.

Proposition 3. *All bases are capacitated in any admissible allocation plan if any of the following conditions hold.*

(i). $\mathrm{card}(\{k \in K: c_k \geq \bar{c}\}) \geq \mathrm{card}(I)$.

(ii). All c_k*'s and* $\bar{c}$ *are integer-valued and* $\sum_{k\in K} c_k \geq \mathrm{card}(I) \cdot \bar{c} + \sum_{k=1}^{\mathrm{card}(I)-1}(c_{(k)}^{\downarrow} - 1)$.

The first condition means that there are more vessels with sufficient capacity than bases. The second condition requires that the total capacity of all vessels should be larger than $\mathrm{card}(I) \cdot \bar{c}$ plus a redundant term $\sum_{k=1}^{\mathrm{card}(I)-1}(c_{(k)}^{\downarrow} - 1)$ that is needed because a vessel cannot be divided. Put it differently, this condition can be rewritten as $\mathrm{card}(I) - 1 + \sum_{k=\mathrm{card}(I)}^{\mathrm{card}(K)} c_{(k)}^{\downarrow} \geq \mathrm{card}(I) \cdot \bar{c}$, which means that the total rescue capacity is sufficient to satisfy the total requirement $(\mathrm{card}(I) \cdot \bar{c})$ even if the largest $\mathrm{card}(I) - 1$ vessels are replaced with unit-capacity vessels.

According to Propositions 1 and 2, one can solve the [MCT] problem with all interior zones and near zones eliminated from the problem and still obtain the optimal solution to the original one on the whole area J. The insights of these two results are as follows. The zones lying in inner part of the region can be automatically covered if the boundary zones are covered. Zones that are close enough to the bases can always be covered within a reasonable time no matter what covering plans are used.

4 The solution procedure

In this section, we introduce our approach to solving the [MCT] problem, which splits the solution procedure into two stages. In the first stage, we eliminate all zones in J° and J^{near} and restrict the problem to a smaller effective zone set. Specifically, let $J_0 := J^b \setminus J^{near}$ if the condition of Proposition 2 holds and let $J_0 := J^b$ otherwise. Then, in the second stage, we solve the [MCT] problem for the restricted zone set J_0 by the off-the-shelf solver. The proposed approach is then termed two-stage (TS) method. Note that in the step of deriving $l_{\cdot,j}^{min}$, i.e., in line 13, a slight modification is made to further simplify this operation. Specifically, we replace J with J^b to further restrict the feasible space. It is easy to see that $\bar{l} := \max_{j \in J}\{ l_{\cdot,j}^{min}\} = \max_{j \in J^{\circ} \cup J^b}\{ l_{\cdot,j}^{min}\} = \max_{j \in J^b}\{ l_{\cdot,j}^{min}\}$, since $j \prec_I j'$ for $\forall j \in J^{\circ}, j' \in J^b$.

Algorithm 1: The TS method

1 **Input:** Read travel distance $l_{i,j}$ between each Marpol base and zone, original zone set J, and a fleet of Marpol vessels K. Obtain the required rescue capacity requirement $\bar{c}$. Calculate the critical rescue capacity $\tilde{c}$ required by Proposition 2.
2 **Stage 1 (Lines 3–18):**
3 **Let** $J^b = J$.
4 **For** each zone j in zone set J^b
5   **For** each zone j' in J
6   **If** $j \neq j'$
7   **If** $l_{i,j} \leq l_{i,j'}$ for all $i \in I$
8   **If** $l_{i,j} < l_{i,j'}$ for one $i \in I$
9   Remove j from the zone set J^b.
10 Let $J_0 = J^b$.
11 **If** $\bar{c} < \tilde{c}$
13   **For** each zone j in zone set J^b
14   Let $l_{\cdot,j}^{min}$ equals the travel distance from a zone j to its closest base.
15   Let $\bar{l}$ equals the maximum travel distance among $\{l_{\cdot,j}^{\min}, \forall j \in J^b\}$.
16   **For** each zone j in zone set J^b
17   **If** $\frac{l_{\cdot,j}^{\min}}{\min\{s_k\}} \leq \frac{\bar{l}}{\max\{s_k\}}$
18   Remove j from J_0.
19 **Stage 2 (Line 20):**
20 Solve the [MCT] problem for the restricted zone set J_0 by the off-the-shelf solver.

We also present another greedy algorithm to solve the [MCT] problem. Our plan is to compare the TS with the off-the-shelf solver in small instances, and then compare the TS with the greedy algorithm in large instances to demonstrate the superiority of the TS algorithm in both aspects of solving velocity and accuracy. Particularly, the greedy algorithm makes the locally optimal choice at each stage. In many problems, a greedy strategy, although optimality is not guaranteed, may yield locally optimal solutions that well approximate the globally optimal solution. Detailed steps of the greedy algorithm to solve the [MCT] problem is presented as follows.

Algorithm 2: The greedy algorithm for the [MCT] problem

Input: Travel distance between each Marpol base and zone, $l_{i,j}$, and a fleet of Marpol vessels K.
Sort all the $k \in K$ according to the decreasing order of cruising speed s_k.
For each zone $j \in J$
    Choose the nearest Marpol base i for zone j, $Base(j) = i$.
    Record the travel distance between the selected i and zone j, $l_{\cdot,j}^{min}$.
For each Marpol $i \in I$
    Choose the longest distance among zones linked to i, i.e., $Base(j) = i$ and record the corresponding travel distance, $\overline{l}_i$.
Sort all the $i \in I$ according to the decreasing order of $\overline{l}_i$.
For each Marpol $i \in I$
    If $K \neq \varnothing$
        Let $sa = 0$
        While $sa < \overline{c}$ and $K \neq \varnothing$
            Assign the first vessel k in K to Marpol base i.
            Remove k from K.
            $sa = sa + c_k$.
While $K \neq \varnothing$
    Select a k from K and randomly assign it to a base.
    Remove k from K.

5 Numerical experiments

In this section, we perform extensive computational experiments to demonstrate the applicability and effectiveness of the proposed model and the solution approach. A real case study is also conducted based on the HK water district which is divided into 19785 zones. The experiments are performed on two sets of instances, i.e., sets A and B, with different input parameters. The instances in set A are small-scale instances solved directly by the off-the-shelf solver, i.e., Gurobi, and indirectly by TS method. The instances in set B are large/practical scale instances that are solved by the TS and the greedy algorithm. The experiments are coded in Python calling Gurobi of version 9.03 and implemented on a computer with AMD 8 Cores 2.9 GHz central processing unit (CPU) and 16GB RAM.

5.1 Instance generation

To test the performance of the proposed algorithm, we study 28 instances in set A and 24 instances in set B. The input parameters related to the model are described as follows. To begin with, the number of zones $|J|$ are set as 100 and 600 in set A and as 3000 and 4500 in set B, respectively. For each input of $|J|$, we generate 5 random instances in set A and 3 random instances in set B. Here, we let l denote the index of an instance. The number of Marpol bases $|I|$ is set to be 3 and 8 in set A and set B, respectively. The zones and Marpol bases in these instances are randomly selected from the HK waters, within a real-world background. The corresponding travel distance between Marpol base i and zone j is also derived based on their real location. The number of rescue vessels $|K|$ is randomly chosen from $\{5, 10, 15, 20\}$ in set A and $\{20, 25, 30\}$ in set B. As abovementioned, the rescue vessels can be categorized into three categories: small size, medium size, and large size. Vessels with different sizes have different cruising speeds and rescue abilities: smaller vessels have faster speed, while larger vessels have higher rescue capacity. Table 3 shows the specific parameters used in the numerical experiments in detail. We assume 20% vessels are large size, 40% vessels are medium size, and 40% vessels are small size in the fleet. We also vary the rescue capacity requirement $\bar{c}$ to see how it affects the minimal covering times and the performance of our algorithm. For notational simplicity, we denote an instance by $|J| - l - |K|(\bar{c})$, where $|J|$ is the number of zones, l is the index for the instance in a group of instances that share the same $|J|$, $|K|$ is the number of rescue vessels, and $\bar{c}$ is the rescue capacity requirements.

Insert Table 3 here

5.2 Computational results

Instances in set A are solved by the Gurobi and TS methods. The results obtained by the two solution methods for these instances are shown in Table 4. In this table, the performance of the Gurobi and TS for solving each instance is measured by the objective function value, i.e., MCT, and the CPU time. In addition, the relative difference of the CPU time (TRD) required by the Gurobi and TS is reported in the last column, which is defined as $100\% \times (CT_{TS}/CT_G)$. In this equation, CT_{TS} is the TS's CPU time and CT_G is the Gurobi's CPU time.

Insert Table 4 here

As expected, the proposed TS method obtains the optimal solution and is superior to Gurobi in the aspect of computation time in all testing instances. Even in the smallest testing instances, the CPU time of TS is much smaller than Gurobi. In addition, Gurobi is more sensitive to the number of variables (zones). When the number of zones increases from 100 to 600, the average computation time of Gurobi grows from 5.78 seconds to 1055.78 seconds, about 182.66 times larger. By comparison, the average computation time of TS changes from 0.037 seconds to 0.066 seconds, about 1.78 times larger. Moreover, the computation time of Gurobi is also sensitive to the rescue capacity requirement, i.e., constraint (4) in the [MCT] model. We take the instances 600-1-20($\bar{c}$) for example, when the rescue capacity requirement varies between 2 and 50, the computation time of Gurobi various between 172.25 seconds and 1518.62 seconds, the ratio of the maximal to the minimal computation time is 8.82. By comparison, the TS's ratio between the maximal and minimal computation time is 3.00 which is much smaller than Gurobi. Other instances show a similar phenomenon. Overall, the TS outperforms Gurobi in terms of the solution speed for solving the instances in set A. Meanwhile, the TS is able to solve the problem to optimum in all instances.

We proceed to investigate the effects of Proposition 1 and 2 in reducing the size of zones. As shown in Table 5, the original zone set is first eliminated by Proposition 1, and then further reduced by Proposition 2. The remaining number of zones after zone elimination (Proposition 1 or 2) is reported in the column of “Number of zones”, and the ratio between the remaining number of zones and original number of zones is reported in the column of “Reduced ratio”. The critical capacity value for the condition of Proposition 2 is provided in the last column.

Table 5 reflects that most zones can be eliminated by applying Proposition 1. Especially for the case of 100-3-15, 600-4-10, and 600-5-5, only one zone is left in the reduced zone set. Even in the worst case, i.e., 100-2-10, 90% of zones are identified and eliminated. Proposition 2 further largely restricts the zone size in the case of 100-2-10, 600-1-20, and 600-2-15. After applying Proposition 1 and 2, average 98.05% zones and 98.62% zones are eliminated respectively, in the testing instances, and consequently, a very limited number of zones are identified as effective in solving the problem. Considering the powerful effect of Proposition 1 and 2, it is not surprising the TS method achieves such huge performance improvement.

Insert Table 5 here

Instances in set B are solved by the TS and greedy algorithm. The results obtained by the two solution methods for these instances are reported in Table 6. The performance of the TS and greedy algorithm for solving each instance is measured by the objective function value, i.e., MCT, and the CPU time. The difference of the solved MCT between the TS and greedy algorithm is reported in the column of “Gap”.

Insert Table 6 here

As shown in Table 6, the greedy algorithm can return near-optimal solutions when the rescue capacity requirement is not large. For example, the greedy algorithm returns the same objective value generated by TS in instances 3000-1-25(2), 3000-1-25(20), 3000-2-20(2), 3000-3-25(2), and 4500-1-25(2). However, the gap rapidly increases when the rescue capacity requirement rises. We then derive the relative gap (RG) of the solution obtained by the TS against the solution obtained by the greedy algorithm, which is defined as $100\% \times (obj_G - obj_{TS})/obj_{TS}$. In this equation, obj_G is the objective value obtained by the greedy algorithm, and obj_{TS} is the objective value obtained by the TS. The results are RG are reported in the last column of Table 6. It can be seen that the errors resulted from the greedy algorithm is significant. As the rescue capacity requirement rises, the RG can be larger than 60. This indicates that although the greedy algorithm can obtain the near-optimal solutions at specific scenarios, e.g., when the rescue capacity requirement is low, its performance drops rapidly at other scenarios, e.g., when increasing the requirement of rescue capacity. When it comes to the computation time, the greedy method is more stable than the TS. This is mainly because the greedy algorithm makes one locally optimal choice at each stage, reducing each given problem into a smaller one, rather than making decisions based on all the feasible space as in the TS. Therefore, it fails to produce the optimal solution with various rescue capacity requirements in the [MCT] problem.

Table 7 shows the effects of Proposition 1 and 2 in reducing the size of zones in larger instances. Similar to Table 5, the performance of Propositions 1 and 2 are reflected by the "Number of zones" and "Reduced ratio". The critical capacity value is shown in the last column. Generally, the performance of Proposition 1 and 2 is better in larger instances. In the first round, after applying Proposition 1, average 99.16% zones are identified as ineffective and removed. Moreover, even in the worst case, i.e., 3000-2-20, 98.73% zones are identified which reflects the efficient and stable performance of Proposition 1. In the second round, after applying Proposition 2, more ineffective zones are identified and removed which further reduces the size of zone sets. The critical capacity values are obtained by solving the [CRCO] problem and reported in the last column. The effectiveness of zone elimination operation provides a solid foundation for TS to tackle large/huge scale problems.

Insert Table 7 here

We proceed to perform a comparison before and after applying Proposition 2 and report the results in Table 8. In this table, the objective value is reported in column "MCT" and the computation time is reported in column "CPU time". To meet the condition of Proposition 2, the rescue capacity requirement should below the critical capacity value reported in the last column of Table 7. Consistent with the theory, TS method returns the optimal solution in all testing instances in set B. Although TS is already very efficient by applying Proposition 1, its performance can be further enhanced by applying both Propositions 1 and 2. In majority of testing instances, the CPU time can be significantly reduced. Only one exception is the case of 3000-3-25(5), in which solution time is a bit longer than the strategy of only applying Proposition 1.

Insert Table 8 here

5.3 Case study

The HK Marpol is responsible for the HK water district with 1651 km^2. To obtain the minimal covering time of this district by solving the [MCT] problem. We divide the HK water district into 19785 discrete squared zones. Each zone is represented by its centroid, as shown in Figure 4. In Figure 4, small blue points represent the zones that are eliminated by Proposition 1; larger orange dots represent the zones that are further eliminated by Proposition 2; orange dots with a cross in them represent the remaining zones. Obviously, the majority of zones are eliminated. Specifically, 105 zones are left after applying Proposition 1 and 94 zones are left after applying Propositions 1 and 2. 99.47% zones are removed in the first round of zone elimination and 99.52% zones are removed in the second round of zone elimination.

A total of nine Marpol bases are located on the coastlines of islands, which is indicated by the red square nodes in Figure 4. The shortest cruising distance from a Marpol base to each zone is already derived and taken as input of this problem. The vessels that can fulfill the rescue missions are summarized in Table 9, which can be classified into three groups (large, medium, and small) according to their sizes. The numbers of large, medium, and small vessels are set to be 6, 17, and 5, respectively, to obtain some insights. The vessels are deployed to different Marpol bases. Once a rescue request is received, the Marpol should send sufficient vessels to the accident site. Therefore, it is of significant responsibility for the HK Marpol to appropriately deploy the fleet and provide a reliable minimal covering time that they are able to arrive at any accident site within this period. More detailed description of the case is presented in Appendix A.1.

Insert Figure 4 here

Insert Table 9 here

We solve this problem by TS. The detailed setting of this algorithm is the same as the above instances. The optimal vessel deployment plan and minimal covering time are obtained and shown in this section and Appendix A.2. It should be mentioned that addressing this real-world case by Gurobi is computationally expensive. According to our experiment, the 16GB RAM computer encounters the "out of memory" issue even on a simplified HK water district with 4922 discretized zones, not to mention the fine-partitioned case with 19785 zones we consider. Then, a workstation with 64GB RAM and Intel 24 Cores 2.19 GHz CPU solve this simplified case in 107127.87 seconds (over 29 hours). By comparison, the proposed TS can find the optimal solution in a much shorter time than Gurobi, which is 31.00 seconds on the computer. A rescue vessel deployment plan obtained in one run is shown in Table 10, and the corresponding minimal covering time is 19.28 minutes. We note that the fleet deployment plan can be varied among different runs since the optimal solutions are not unique, but the minimal covering time is identical at 19.28 minutes.

Insert Table 10 here

We proceed to study the effects of rescue capacity requirements and report them in Table 11. Specifically, we set five different rescue capacity requirements, which are 2, 5, 10, 15, and 20, referred to as level i to level v. The critical capacity value is found to be 16 by solving the [CRCO]. The objective value of MCT gradually increases when the rescue capacity requirement increases. It can be seen that the fleet deployment plan is various under different rescue capacity settings. The Marpol can select the most suitable deployment plan based on their demands in practice and adjust their deployment plan, when adopting the proposed model.

Insert Table 11 here

We also study the effect of the fineness of the discretization. Specifically, we examine the covering time solution in a coarse-grained discrete zone set that contains 4922 zones. The size of zones is four times larger than the size of zones in the fine-grained setting (19785 discrete zones). The results are shown in Table 12. The relative difference in the third row is defined as $RD = 100\% \times \left(CT_f - CT_c\right) / CT_f$ where CT_f is the covering time concerning the fine-grained setting and CT_c is the covering time concerning the coarse-grained setting.

Insert Table 12 here

Generally, although the size of zones in the coarse-grained setting is four times large as the size of zones in the fine-grained setting, the difference between covering time solutions is marginal. Therefore, we can see that the model is not sensitive to the fineness of the discretization.

6 Conclusions

Maritime safety is an essential prerequisite of running a successful maritime industry. MSAR operations, which provide promptly help for victims in danger, is of great importance for building a safe maritime hub. We study in this paper a covering time minimization problem in which the MSAR operator aims to minimize the maximal response time to arrive at any incident spot in a water district by appropriately deploying the rescue fleet to existing Marpol bases. We formulate a mixed-integer programming model, [MCT] model, for the considered problem and analyze the properties of the optimal solution. Considering the expensive computation cost of using off-the-shelf solvers for the problem, a tailored two-stage algorithm is proposed based on the special characteristics of the problem. Extensive numerical experiments are conducted, and the results demonstrate that the algorithm outperforms other solution methods for solving the [MCT] problem with different sizes. We also apply the algorithm to solve a practical problem faced by HK Marpol, which further demonstrates the efficiency of the algorithm.

Our study can be of great value to HK MRCC, the agency that is responsible for MSAR missions in HK waters, and to MSAR operators in other areas. By using our approach, the MSAR operator can obtain the following deliverables: (1) a tactical vessel allocation plan (e.g., made quarterly, biannually, or yearly), (2) an operational vessel dispatch plan when an emergency occurs, (3) the covering times of the whole water district and the sub-region responsible by each Marpol base. Knowing the last one, the MSAR operator can also set a service pledge of the response time, by which it can drive frontline rescuers to work at their best to meet this achievable optimal goal (thereby better accomplishing the ultimate goal of saving life) and to publicize its superior service level (thereby to gain advantage in global maritime centers competition).

Considering that the consequence of mishandling an incident is disastrous, we adopt a robust optimization approach to model the maritime search and rescue problem and to solve a conservative solution. For future studies, it is interesting to consider location-dependent incident chances and model the [MCT] problem as one that minimizes the expected response time under uncertainties.

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The first three authors contributed equally to the paper and are co-first author.

Appendix

Appendix A. Supplementary data in the case study

We present the supplementary data in the case study in this appendix.

A.1. Input data of the real case faced by HK Marpol

The input data of the real case faced by HK Marpol is listed as follows. The HK water district is divided into 19785 zones with each zone representing a 300×300 square meters water area. The centroid of each zone is taken as the zone's representative. Nine maritime bases are located along the coastline in the Kowloon Peninsula, Hong Kong Island, and its offshore islands (Peng Chau island, Cheung Chau island, and Lamma Island). The specific longitude and latitude of each maritime base are provided in Table A.1 and are shown in Figure 4. The shortest travel distances between each base and zone are precalculated and taken as input parameters.

Insert Table A.1 here

A.2. Fleet deployment results and the minimal covering time

Table A.2 reports the fleet deployment plan in the HK water district when different service levels are desired. The service levels are reflected by the minimum rescue capacity guaranteed for each rescue mission. The solutions are generated by the TS method. The corresponding minimal covering time is presented in Table 11. The fleet deployment plan at level (i) is reported in Table 10.

Insert Table A.2 here

Appendix B. Proofs

In this section, provide proofs for our results.

Proof of Lemma 1. First, it follows directly from the definition of J° and J^{b} that

$$\max_{j\in J^{\circ}}\{\,CT(X,j)\,\} \leq \max_{j\in J^{b}}\{\,CT(X,j)\,\}.$$

Second, since $J = J^{\circ} \cup J^{b}$, we must have

$$\max_{j\in J}\{\,CT(X,j)\,\} = \max\left\{\max_{j\in J^{b}}\{\,CT(X,j)\,\}, \max_{j\in J^{\circ}}\{\,CT(X,j)\,\}\right\} = \max_{j\in J^{b}}\{\,CT(X,j)\,\}.$$

Proof of Proposition 1. Note that an allocation plan X^{*} is an optimal solution to [MCT] on J iff $CT(X^{*},J) \leq CT(X,J)$, $\forall X$. Similarly, X^{*} is an optimal solution to [MCT] on J^{b} iff

$CT(X^*, J^b) \leq CT(X, J^b), \forall X$. Thus, the proposition immediately holds by Lemma 1 which states that $CT(X, J) = CT(X, J^b), \forall X$.

Proof of Proposition 2. If the condition of the proposition holds, on the one hand, the response time to a zone j under any admissible allocation plan is at most $\frac{l_{\cdot,j}^{min}}{\min\{s_k\}}$, where $\min\{s_k\}$ is a lower bound of the speeds of the vessels used to serve zone j. On the other hand, the covering time of the whole water district is no smaller than the covering time of the "farthest" served by fastest vessel(s), which is $\frac{\bar{l}}{\max\{s_k\}}$.

Therefore, if $\frac{l_{\cdot,j}^{min}}{\min\{s_k\}} \leq \frac{\bar{l}}{\max\{s_k\}}$ for some zone j, then this zone is ineffective in determining the covering time of the water district, which is the maximum response time of all zones. The [MCT] problem is unaffected with or without this zone. The same argument applies when all zones in J^{near} are eliminated.

Proof of Proposition 3. We prove by contradiction. Suppose that there exists a base *i* such that it is incapacitated, i.e., $\sum_{k \in K} c_k x_{i,k} < \bar{c}$ under an admissible allocation plan *X*. We discuss the two conditions separately.

(i) Noting that base *i* is incapacitated, any vessel assigned to it should have capacity less than $\bar{c}$. Thus, all vessels that have capacity larger than $\bar{c}$ are assigned to other bases, i.e., those in $I \backslash \{i\}$. If the first condition of the proposition holds, i.e., $\text{card}(\{k \in K: c_k \geq \bar{c}\}) \geq \text{card}(I)$, there must exist one base (e.g., i') that is assigned more than one vessels with capacity larger than $\bar{c}$. Any one of these vessels is redundant at base i', which contradicts the assumption that *X* is admissible.

(ii) Since *X* is admissible, there should be no redundant vessel at other bases. That is, for any $i' \in I \backslash \{i\}$, we have $\sum_{k \in K} c_k x_{i',k} < \bar{c} + c_{\kappa(i')}$, where $\kappa(i')$ denote the index of an arbitrary vessel that is assigned to base i'. Because all c_k's and $\bar{c}$ are integer-valued, the inequality reduces to $\sum_{k \in K} c_k x_{i',k} \leq \bar{c} + c_{\kappa(i')} - 1$. Therefore,

$$\sum_{i' \neq i} \sum_{k \in K} c_k x_{i',k} \leq [\text{card}(I) - 1](\bar{c} - 1) + \sum_{i' \neq i} c_{\kappa(i')} \leq [\text{card}(I) - 1](\bar{c} - 1) + \sum_{k=1}^{\text{card}(I)-1} c_{(k)}^{\downarrow},$$

where $\kappa(i')$'s in the first inequality are arbitrarily selected. Combining the second inequality and the incapacitated condition $\sum_{k\in K} c_k x_{i,k} < \bar{c}$ yields

$$\sum_{k\in K} c_k < \text{card}(I)\cdot\bar{c} - [\text{card}(I) - 1] + \sum_{k=1}^{\text{card}(I)-1} c_{(k)}^{\downarrow},$$

which contradicts the second condition of the proposition.

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Table 1 Summary of literature on the MSAR resources management

Paper	Problem and major considerations	Approach
Azofra et al. (2007)	Sea rescue resource location; characteristics of the accident, the vessel and the damage produced; cost-effectiveness	Gravitational models (Individual distribution model, Zonal distribution model)
Wagner and Radovilsky (2012)	The United States Coast Guard boat fleet allocation; risk-management capability;	Integer linear programming; value-at-risk; robust optimization
Pelot et al. (2015)	Locating maritime search and rescue vessels; boat range; bi-objective	Maximal covering location problem (MCLP); capacity limits MCLP; stochastic factors CLP
Razi and Karatas (2016)	Locating search and rescue boats; multi-objective; density and types of incidents; resource capabilities, geographical factors; governments' business rules	Analytical Hierarch Process; zonal distribution model; multi-objective mixed integer program
Akbari et al. (2018a)	Location of maritime search and rescue resources; simulated demand; multiple criteria	Maximal covering location problem; p-median problem;
Akbari et al. (2018b)	Location-allocation of maritime search and rescue vessels; multiple criteria; multi-objective	Goal programming multi-objective model;
Karatas (2021)	Location and allocation of search and rescue boats and helicopters; demand uncertainty; seasonally relocation of vessels	Dynamic multi-objective mixed integer linear programming model; simulated incident scenarios
Hornberger et al. (2021)	Heterogeneous search and rescue asset location;	Stochastic zonal distribution; bi-objective integer linear program

Table 2 Worded examples of covering problem

Vessel size	Rescue capacity	Number of vessels	Speed
Small (S)	2	4	v_s
Medium (M)	3	2	v_m
Large (L)	4	2	v_l

	Base 1 at (-1, 0)	Base 2 at (1, 0)
Case 1: $(v_s, v_m, v_l) = (4, 3, 1)$, $r = 1.5$, $\bar{c} = 7$.		
Optimal allocation	S×2, M×1	S×2, M×1
Dispatched capacity	7	7
Bottleneck speed	3	3
Assigned service region	$\{u \leq 0\}$	$\{u > 0\}$
Covering time of the assigned region	$\boldsymbol{\sqrt{13}/6}$	$\boldsymbol{\sqrt{13}/6}$
Case 2: $(v_s, v_m, v_l) = (4, 3, 1)$, $r = 2$, $\bar{c} = 7$.		
Optimal allocation	M×1, L×1	S×4
Dispatched capacity	7	8
Bottleneck speed	1	4
Assigned service region	$\{(u + 17/15)^2 + v^2 \leq (8/15)^2\}$	remaining area
Covering time of the assigned region	$2/3$	$\boldsymbol{3/4}$
Case 3: $(v_s, v_m, v_l) = (4, 3, 2)$, $r = 1.5$, $\bar{c} = 7$.		
Optimal allocation	M×1, L×1	S×4
Dispatched capacity	7	8
Bottleneck speed	2	4
Assigned service region	$\{(u + 5/3)^2 + v^2 \leq (4/3)^2\}$	remaining area
Covering time of the assigned region	$\boldsymbol{\sqrt{13/40}}$	$\boldsymbol{\sqrt{13/40}}$
Case 4: $(v_s, v_m, v_l) = (4, 3, 1)$, $r = 1.5$, $\bar{c} = 8$.		
Optimal allocation	M×2, L×1	S×4
Dispatched capacity	10	8
Bottleneck speed	1	4
Assigned service region	$\{(u + 17/15)^2 + v^2 \leq (8/15)^2\}$	remaining area
Covering time of the assigned region	$\boldsymbol{\sqrt{13/34}}$	$\boldsymbol{\sqrt{13/34}}$

Table 3 Marpol vessels for rescue missions

Vessel Type	Speed (knot)	Service Capacity (passengers)
Large Marpol Launch	25	10
Medium Marpol Launch	45	5
Small Marpol Launch	60	2

Table 4 Results of instances in Set A

Instances	Gurobi		TS method		TRD (%)
	MCT (min)	CPU Time (sec)	MCT (min)	CPU Time (sec)	
100-1-5(2)	30.84	0.36	30.84	0.02	5.556
100-1-5(20)	74.06	0.03	74.06	0.01	33.333
100-2-10(2)	29.53	1.81	29.53	0.08	4.420
100-2-10(20)	39.50	4.68	39.50	0.09	1.923
100-2-10(40)	91.70	0.99	91.70	0.04	4.040
100-3-15(2)	30.29	1.89	30.29	0.01	0.529
100-3-15(20)	40.39	7.56	40.39	0.02	0.265
100-3-15(50)	72.70	8.85	72.70	0.02	0.226
100-3-15(70)	72.70	0.77	72.70	0.02	2.597
100-4-20(2)	28.53	3.27	28.53	0.02	0.612
100-4-20(20)	38.04	6.62	38.04	0.05	0.755
100-4-20(50)	40.53	28.16	40.53	0.04	0.142
100-4-20(70)	68.48	22.92	68.48	0.04	0.175
100-4-20(90)	73.09	3.51	73.09	0.03	0.855
100-5-15(2)	29.02	2.25	29.02	0.03	1.333
100-5-15(20)	38.70	5.05	38.70	0.05	0.990
100-5-15(50)	69.66	9.47	69.66	0.04	0.422
100-5-15(70)	73.09	0.69	73.09	0.02	2.899
600-1-20(2)	30.46	172.25	30.46	0.08	0.046
600-1-20(20)	40.62	1192.07	40.62	0.27	0.023
600-1-20(50)	42.42	1518.62	42.42	0.24	0.016
600-1-20(70)	73.11	1789.10	73.11	0.21	0.012
600-1-20(90)	101.82	12.85	101.82	0.08	0.623
600-2-15(2)	30.17	123.13	30.17	0.03	0.024
600-2-15(20)	40.23	989.64	40.23	0.04	0.004
600-2-15(50)	72.41	692.69	72.41	0.05	0.007
600-2-15(70)	76.45	0.84	76.45	0.02	2.381
600-3-20(2)	30.38	300.72	30.38	0.03	0.010
600-3-20(20)	40.51	1242.21	40.51	0.05	0.004
600-3-20(50)	41.49	1428.45	41.49	0.04	0.003
600-3-20(70)	72.93	926.90	72.93	0.04	0.004
600-3-20(90)	74.68	2.75	74.68	0.03	1.091
600-4-10(2)	32.08	26.64	32.08	0.02	0.075
600-4-10(20)	42.78	194.10	42.78	0.03	0.015
600-4-10(40)	77.00	8.01	77.00	0.02	0.250
600-5-5(2)	42.40	29.65	42.40	0.02	0.067
600-5-5(20)	101.77	0.23	101.77	0.01	4.348

Table 5 Remaining number of zones after applying Proposition 1 and 2 for instances in set A

Instances	Original number of zones	Remaining zones after applying Proposition 1		Remaining zones after applying Proposition 2		
		Number of zones	Reduced ratio	Number of zones	Reduced ratio	Critical capacity
100-1-5	100	2	98.00%	2	98.00%	−
100-2-10	100	10	90.00%	6	94.00%	10
100-3-15	100	1	99.00%	1	99.00%	−
100-4-20	100	2	98.00%	1	99.00%	−
100-5-15	100	2	98.00%	2	98.00%	−
600-1-20	600	8	98.67%	5	99.17%	26
600-2-15	600	3	99.50%	2	99.67%	18
600-3-20	600	2	99.67%	2	99.67%	−
600-4-10	600	1	99.83%	1	99.83%	−
600-5-5	600	1	99.83%	1	99.83%	−

Table 6 Results of instances in Set B

Instances	Greedy algorithm		TS method		Gap	RG(%)
	MCT (min)	CPU Time (sec)	MCT (min)	CPU Time (sec)		
3000-1-25(2)	28.74	3.63	28.74	0.08	0.00	0.00
3000-1-25(20)	28.74	3.39	28.74	0.35	0.00	0.00
3000-1-25(50)	45.04	3.62	43.11	0.54	1.93	4.48
3000-1-25(80)	68.98	3.56	68.98	0.19	0.00	0.00
3000-1-25(100)	106.14	3.41	77.61	0.22	28.53	36.76
3000-2-20(2)	19.09	3.45	19.09	1.79	0.00	0.00
3000-2-20(20)	30.79	3.44	27.37	22.13	3.42	12.50
3000-2-20(50)	45.58	3.47	43.54	12.15	2.04	4.69
3000-2-20(70)	73.89	3.43	65.70	10.40	8.19	12.47
3000-2-20(90)	109.39	3.42	78.37	4.93	31.02	39.58
3000-3-25(2)	19.09	3.54	19.09	1.39	0.00	0.00
3000-3-25(20)	24.93	3.43	24.93	12.81	0.00	0.00
3000-3-25(50)	44.87	3.47	43.54	11.57	1.33	3.05
3000-3-25(80)	73.89	3.69	45.81	13.18	28.08	61.30
3000-3-25(100)	109.39	3.42	78.37	6.22	31.02	39.58
4500-1-25(2)	28.74	5.16	28.74	0.13	0.00	0.00
4500-1-25(20)	28.74	5.10	28.74	0.36	0.00	0.00
4500-1-25(50)	45.04	5.01	43.11	0.54	1.93	4.48
4500-1-25(80)	68.98	5.08	68.98	0.18	0.00	0.00
4500-1-25(100)	106.14	5.41	77.61	0.24	28.53	36.76
4500-2-30(2)	19.09	5.07	19.09	6.94	0.00	0.00
4500-2-30(20)	24.71	5.13	24.71	51.70	0.00	0.00
4500-2-30(50)	44.47	5.22	36.50	61.67	7.97	21.84
4500-2-30(80)	45.58	5.25	43.54	52.23	2.04	4.69
4500-2-30(100)	73.89	5.01	46.89	78.39	27.00	57.58
4500-2-20(120)	109.39	5.25	78.37	28.27	31.02	39.58
4500-3-25(2)	19.09	5.15	19.09	4.73	0.00	0.00
4500-3-25(20)	24.93	5.05	24.93	28.01	0.00	0.00
4500-3-25(50)	44.87	5.07	43.54	54.67	1.33	3.05
4500-3-25(80)	73.89	5.09	45.81	42.99	28.08	61.30
4500-3-25(100)	109.39	5.23	78.37	36.74	31.02	39.58

Table 7 Remaining number of zones after applying Proposition 1 and 2 for instances in set B

Instances	Original number of zones	Remaining zones after applying Proposition 1		Remaining zones after applying Proposition 2		
		Number of zones	Reduced ratio	Number of zones	Reduced ratio	Critical capacity
3000-1-25	3000	5	99.83%	2	99.93%	8
3000-2-20	3000	38	98.73%	33	98.90%	5
3000-3-25	3000	33	98.90%	29	99.03%	8
4500-1-25	4500	7	99.84%	4	99.91%	8
4500-2-30	4500	53	98.82%	48	98.93%	11
4500-3-25	4500	52	98.84%	46	98.98%	8

Table 8 Performance comparison between Proposition 1 and 2 for instances in set B

Instances	Gurobi	Zone elimination by Proposition 1		Zone elimination by Proposition 1 and 2	
	MCT (min)	MCT (min)	CPU Time (sec)	MCT (min)	CPU Time (sec)
3000-1-25(2)	28.74	28.74	0.21	28.74	0.08
3000-1-25(5)	28.74	28.74	0.23	28.74	0.09
3000-1-25(8)	28.74	28.74	0.32	28.74	0.16
3000-2-20(2)	19.09	19.09	2.13	19.09	1.79
3000-2-20(5)	23.06	23.06	3.62	23.06	2.90
3000-3-25(2)	19.09	19.09	2.70	19.09	1.39
3000-3-25(5)	19.09	19.09	3.80	19.09	**4.51**
3000-3-25(8)	23.06	23.06	10.73	23.06	6.66
4500-1-25(2)	28.74	28.74	0.21	28.74	0.13
4500-1-25(5)	28.74	28.74	0.23	28.74	0.18
4500-1-25(8)	28.74	28.74	0.31	28.74	0.27
4500-2-30(2)	19.09	19.09	9.43	19.09	6.94
4500-2-30 (5)	19.09	19.09	9.73	19.09	8.20
4500-2-30 (10)	23.06	23.06	22.30	23.06	14.87
4500-2-30 (11)	24.71	24.71	39.82	24.71	36.93
4500-3-25(2)	19.09	19.09	5.45	19.09	4.73
4500-3-25(5)	19.09	19.09	12.80	19.09	6.64
4500-3-25(8)	23.06	23.06	20.08	23.06	14.81

Table 9 Rescue vessels regulated by the HK Marpol

Vessel size	Vessel speed (Knot)	Rescue capacity (Passengers)	Number of vessels
Large	25	10	6
Medium	45	5	17
Small	60	2	5

Table 10 A developed deployment plan

Base Index	Fleet deployment plan (level i)		
	Large	Medium	Small
1	1	0	1
2	1	1	0
3	1	2	0
4	0	3	0
5	1	1	0
6	1	2	0
7	1	1	1
8	0	5	2
9	0	2	1

Table 11 Effects of rescue capacity requirements on the minimal covering time

Rescue capacity requirement	level i	level ii	level iii	level iv	level v
Covering time (min)	19.28	25.10	25.10	25.70	25.70
CPU time with Proposition 1 (sec)	37.00	39.68	63.09	59.43	110.09
CPU time with Propositions 1 and 2 (sec)	21.77	17.01	59.79	61.67	–

Table 12 Covering time with respect to the fineness of the discretization

Rescue capacity requirement	level i	level ii	level iii	level iv	level v
Fine-grained setting (min)	19.28	25.10	25.10	25.70	25.70
Coarse-grained setting (min)	19.09	23.64	23.64	25.45	25.45
Relative difference (*RD*)	0.99%	5.82%	5.82%	0.97%	0.97%

Table A.1 Maritime bases in HK water district

Base Index	Base Name	Longitude	Latitude
1	Marine West Division	114.017	22.358
2	Peng Chau Police Post	114.037	22.282
3	Cheung Chau Division	114.032	22.200
4	Lamma Island Police Post	114.113	22.216
5	Sok Kwu Wan Police Post	114.137	22.211
6	Marine South Division	114.166	22.235
7	Marine Harbour Division	114.230	22.284
8	Marine East Division	114.280	22.377
9	Marine North Division	114.219	22.407

Table A.2 Fleet deployment plans in HK water district with various service levels

Base Index	Fleet deployment plan (level ii)			Fleet deployment plan (level iii)		
	Large	Medium	Small	Large	Medium	Small
1	0	3	3	0	0	5
2	0	3	1	1	1	0
3	1	1	0	1	0	0
4	1	1	0	1	0	0
5	1	1	0	1	0	0
6	1	1	0	1	1	0
7	0	3	0	1	3	0
8	1	1	0	0	10	0
9	1	3	1	0	2	0

Base Index	Fleet deployment plan (level iv)			Fleet deployment plan (level v)		
	Large	Medium	Small	Large	Medium	Small
1	0	3	0	0	4	0
2	1	1	1	1	2	0
3	1	1	0	2	0	0
4	1	1	0	0	2	0
5	1	0	3	1	0	0
6	1	1	0	1	1	1
7	0	5	0	1	1	2
8	0	4	1	0	3	2
9	1	1	0	0	4	0

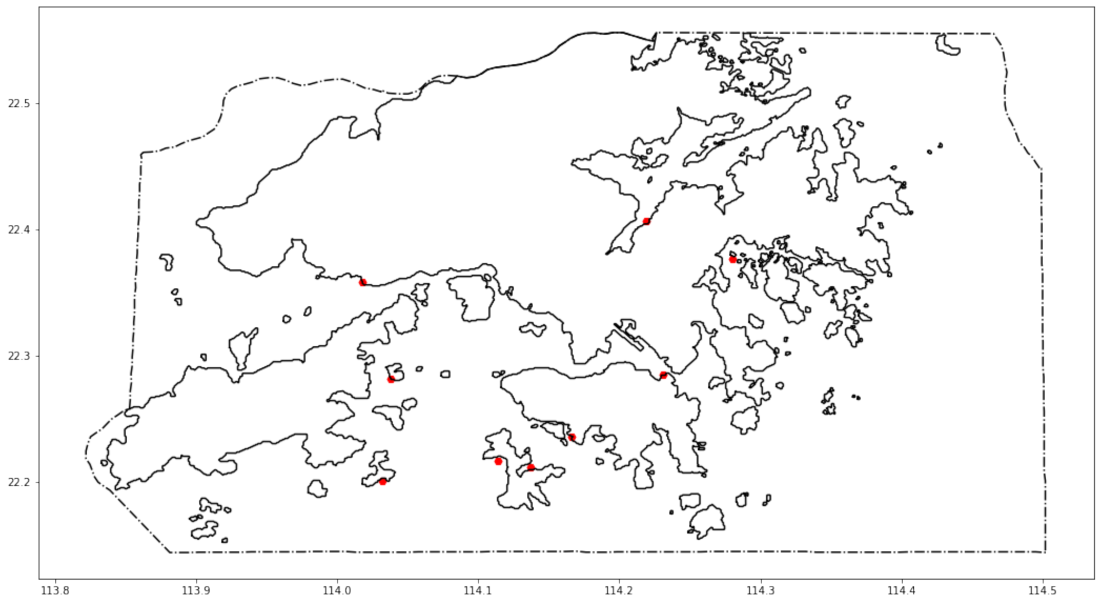

Figure 1 Water district regulated by the Hong Kong Marine Police

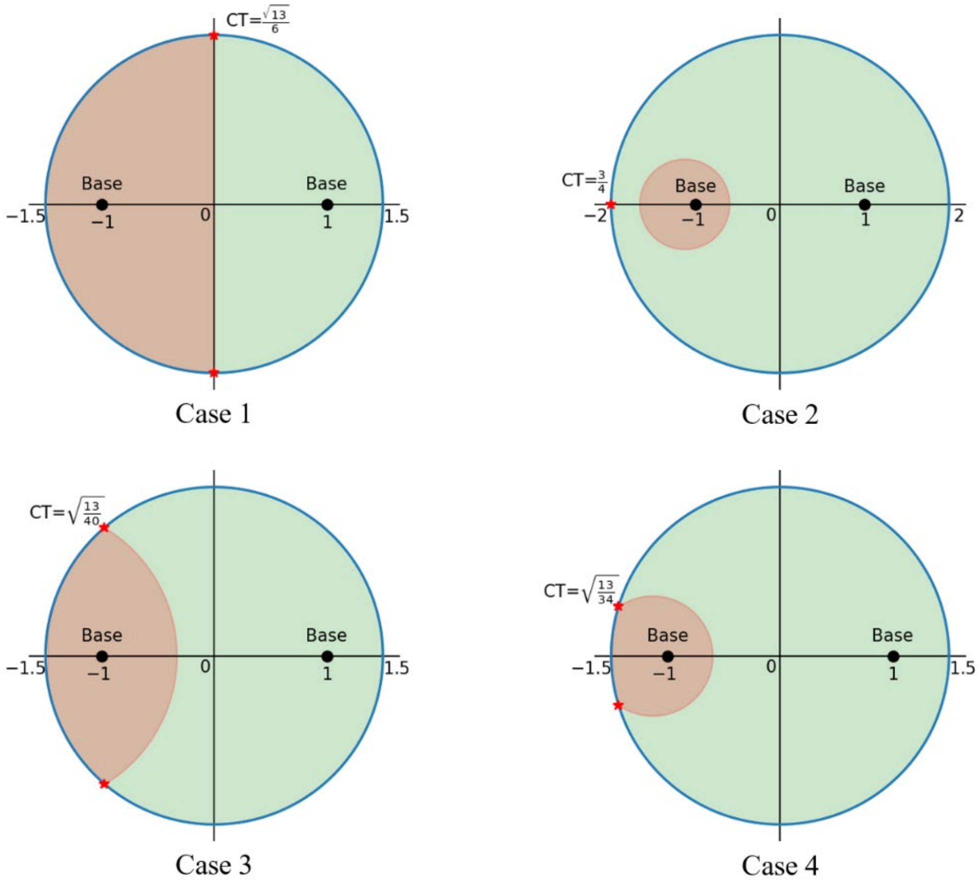

Figure 2 The assigned service regions in the four worked examples

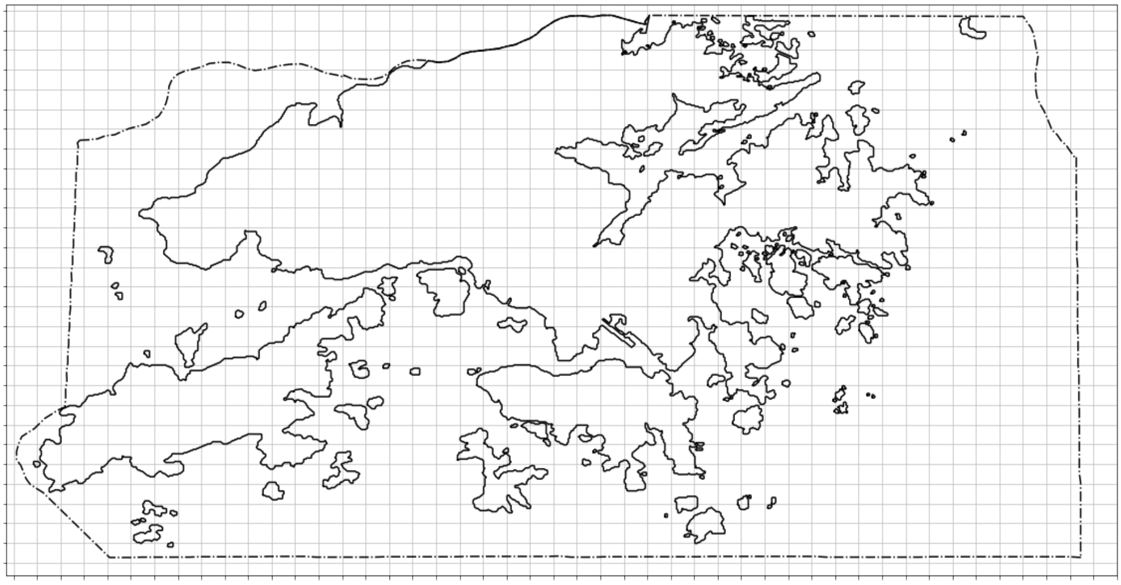

Figure 3 Illustration of water district discretization

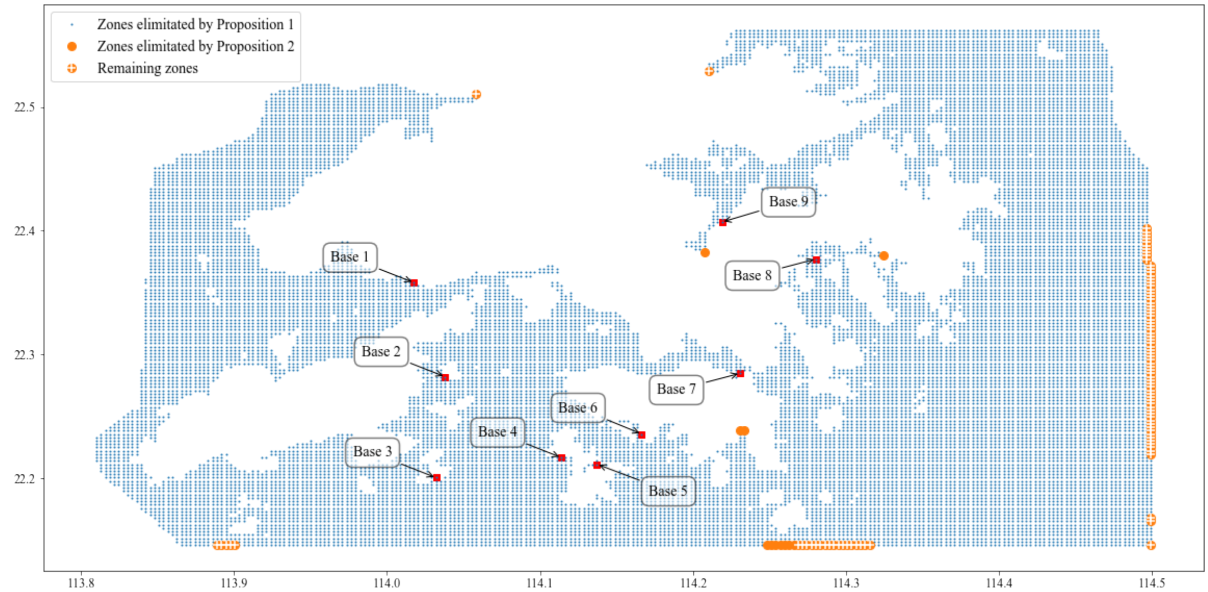

Figure 4 Water district discretization and representation

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