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Unlock the Sharing Economy: the Case of the Parking Sector for Recurrent Commuting Trips

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Abstract

This study examines the pricing strategy of a parking sharing platform that rents the daytime-usage rights of private parking spaces from parking owners and sells them to parking users. In an urban area with both shared parking and curbside parking, a choice equilibrium model is proposed to predict the number of shared parking users under any given pricing strategy of the platform. We analytically analyze how the pricing strategy of the platform (price charged on users and rent paid to parking owners or sharers) would affect the parking choice equilibrium and several system efficiency metrics. It is shown that the platform is profitable when some parking owners have a relatively small inconvenience cost from sharing their spaces, but its profit is always negative at minimum social cost. Numerical studies are conducted to illustrate the analytical results and provide further understanding. *Keywords:* Shared parking; two-sided market; parking choice equilibrium; revenue maxi-mization; social cost minimization.

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1 Introduction

In many large cities, finding a suitable and vacant parking space is a headache for many private car drivers, and constitutes an non-negligible proportion of the trip travel time. Moreover, the cruising for parking can add to the problems of chronic congestion and choking pollution (Arnott and Inci, 2006; Shoup, 2006; Van Ommeren et al., 2012; Liu and Geroliminis, 2016). For instance, Ayala et al. (2011) found that each year in Chicago, there are 63 million vehicle miles traveled due to looking for vacant spaces to park, which generate 48,000 tons of carbon dioxide.

However, in the context of growing car ownership and shrinking usable land in cities, it is infeasible to solve the parking problem by simply continuing the construction of new parking facilities. Instead, many studies have proposed and evaluated parking pricing, parking reservation, parking permit systems or mechanisms to effectively manage parking supply and traffic congestion (Arnott et al., 1991, 2015; Zhang et al., 2011; Qian et al., 2012; Yang et al., 2013; Liu et al., 2014; Inci and Lindsey, 2015; Chen et al., 2015; Mackowski et al., 2015; Chen et al., 2016; Lei and Ouyang, 2017). A latest review of economic analysis and modeling of parking was provided by Inci (2015). Some recent studies looked at the parking spot allocation and pricing problem using game theory or incorporating the parking searching process as part of the network traffic equilibrium (He et al., 2015; Boyles et al., 2015). Besides, there is a branch of studies looking into park-and-ride facilities in a multi-modal system (Wang et al., 2004; Liu et al., 2009; Liu and Geroliminis, 2017).

Peer-to-peer markets, collectively known as the sharing economy, have emerged as alternative suppliers of goods and services traditionally provided by long-established industries (e.g., Airbnb, Uber, Didi Chuxing). Parking sharing emerges as a new way of more efficiently utilizing existing parking facilities. It utilizes existing gaps intended for parking cars when the parking owner is not using it. Many parking spaces are only used part time by the parking owners who live in one location and work in another. Furthermore, the parking utilization and availability patterns follow predictable daily, weekly and annual cycles. For example, in the Randwick City Council (a Local Government Area in Sydney that covers a number of suburbs), there are 85,138 employed residents but only 57,355 local jobs as of 2019 (`http://economy.id.com.au/randwick`). Since some local jobs are taken by residents in other local government areas, there must be more than 27,783 local residents working elsewhere, where some of them might be able to share their private parking spaces. In this context, Randwick City Council is examining the potential to introduce parking sharing programs in the area (`https://www.infrastructure.gov.au/cities/smart-cities/collaboration-platform/integrated-smart-parking-system.aspx`), especially in the high parking demand suburbs, such as the popular beachside suburb Coogee and the suburb Randwick (i.e., the administrative centre of Randwick City Council).

With the rapid development of information and communications technology and especially the latest rise of the mobile internet, private parking sharing is enabled through an "e-parking platform" to help match the supply with the demand. In fact, some e-parking platforms (or shared parking apps) have already appeared in the smartphone era, and are reshaping the parking industries and our daily lives. For example, Divvy (https://www.divvyparking.com/) and Oscar (https://www.sharewithoscar.com.au/) both operate in main cities in Australia (e.g., Sydney, Melbourne, and Brisbane), J-Park (http://www.jiepark.com/) operates in Shanghai, China, and Moby (https://www.mobypark.com) operates in The Netherlands and France. The emergence of e-parking platforms and/or apps not only help alleviate the aforementioned shortage of parking spaces, but also provide the owners a way to make additional money from their idle parking spaces.

Guo et al. (2016) was among the earliest to develop a simulation based approach for decision making of repurchasing private parking spots and selling them to public users. Shao et al. (2016) then explored shared parking reservation and allocation problems and proposed integer linear programming models to optimize the allocation of parking requests to specific parking spots in order to maximize the parking utilization or the number of accepted parking requests. Recently, Xu et al. (2016) modeled the private parking spot sharing problem during working hours by using the market design theory where money flow is allowed in the matching mechanisms. These studies often focused on the optimization of platform's user-space matching strategies under given demand and supply of parking sharing. However, the platform's pricing strategies, which have strong impacts on the volume of parking demand and supply, have not been fully examined. More recently, Xiao et al. (2020) and Zhang et al. (2020) examined the parking sharing pricing problem, where Xiao et al. (2020) focused on auction mechanisms and Zhang et al. (2020) emphasized the spatial distribution of parking. The models in these studies have limited analytical tractability for generating insights and policy implications.

This study makes an attempt to optimize the pricing strategies of parking sharing platforms, considering the reaction of parking space users and private parking space owners. In particular, we consider an urban area with both public curbside parking and private parking that can be potentially shared. The rent offered by the platform determines the number of private parking space owners who would like to rent their space out for sharing, and the platform's parking charge as well as the level of competition for both shared parking and free curbside parking determine the parking demand for both types of parking. All users are assumed to be rational, seeking to minimize their individual travel costs. They have

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full information regarding the parking pricing and capacities, which allows them to make rational decisions. Given the parking sharing platform's pricing strategy, a parking choice equilibrium of travelers is reached when no traveler can further reduce his or her travel cost by changing his/her parking choice. By modeling and analyzing the parking choice equilibrium under any pricing strategies of the parking platform, we examine the properties of pricing strategies that are best for the platform's revenue or social welfare. The pricing strategies of the platform can substantially affect the parking demand and supply, as well as the platform's performance. This highlights the importance to appropriately explore the pricing strategies of the parking sharing platform operator. In summary, this study improves and enhances our understanding of the parking sharing, pricing, and management, and also enriches the literature on the two-sided markets, where ride sharing/sourcing has attracted much more attention (Agatz et al., 2012; Wang et al., 2016; Zha et al., 2016; Chen et al., 2017; Liu et al., 2017; Zhang et al., 2017; Wang et al., 2018; Bai et al., 2019).

The remainder of the paper is structured as follows. Section 2 describes the parking sharing problem involving private car travelers, parking owners, and parking sharing platform operator, and presents the basic formulations. Section 3 formulates travelers' parking choice equilibrium under given supplies of public curbside parking and private shared parking, and graphically illustrates the choice equilibrium in the two dimensional domain of shared parking fee and shared parking supply (or rent). Moreover, the pricing strategies of the parking platform operator to achieve two different objectives, i.e., platform revenue maximization (a private operator) and social cost minimization (a public operator), are analyzed. Section 4 numerically examines the models in this paper and also provides further understanding. Section 5 discusses the analysis and results. Finally, Section 6 concludes the paper.

2 Problem description and basic formulation

In this section, we start with a description of the rush-hour commuting problem in the presence of a parking sharing e-platform, and then describe behaviors of three parties involved: drivers (or travelers) who require a parking space; individual parking owners (or sharers) who may share their private parking spaces; and the operator of the parking sharing platform.

Consider an urban area with a mixed land use of residential and business functionalities (typically this will be suburbs or sub-centers in cities or metropolitan areas). Suppose there is a total number of n users who need to park their cars every day after the commuting trips in the area. A number of m_f public curbside parking spaces are available in the area. Besides the curbside parking spaces involving cruising-for-parking, there are private residential parking in the area. The residents who own private parking are potential private parking suppliers or sharers. Their spaces are vacant during a certain period of time when they drive to work elsewhere. The total number of such potential parking sharers is M_s. If sufficient rents are paid to these potential parking sharers, they will share their private parking spaces.

2.1 Travelers

Travelers who require a parking space have two options: (i) search and park at curbside parking spaces; and (ii) park at the shared but guaranteed (and reserved) parking space through parking sharing service. Note that while we only model two parking options here, the parking choice equilibrium model in this study can be readily extended to incorporate other travel alternatives, such as garage parking and public transit, where the cost formulations of these alternatives should also be formulated then. Here we do not try to provide an exhaustive modelling of all travel alternatives, but aim to derive analytical insights on the parking sharing problem where the insights can be well traced back to the modelling framework. We adopted a tractable and stylized model with only two parking options. This is sufficient to serve the goal of this study, i.e., we model shared parking while we account for the fact that users will shift to other modes when shared parking is more costly. It is also noteworthy that we did not exclude the possibility that the parking sharing operator may integrate local garage parking into its platform, where the garage parking operator/owner can be regarded as an owner of a certain number of parking spaces.

Under the first option (curbside parking), travelers' individual parking cost is

$$c_f = c_f\left(n_f\right) = \alpha \cdot t\left(n_f\right) + q, \tag{1}$$

where n_f is the number of drivers that choose curbside parking, $t\left(n_f\right)$ is the expected cruising time for finding a vacant curbside parking space given the curbside flow n_f, α is the value of driving time, and q is the public curbside parking fee.

More specifically, the cruising time for parking is assumed to be an increasing and convex function of the parking occupancy rate $\frac{n_f}{m_f}$, i.e., $t\left(n_f\right) = \kappa\left(\frac{n_f}{m_f}\right)$, where $\kappa\left(\cdot\right)$ is the cruising time function. This is similar to a number of parking studies in the literature (Anderson and De Palma, 2004; Calthrop and Proost, 2006; Qian and Rajagopal, 2014, 2015; Liu and Geroliminis, 2016; Arnott and Williams, 2017; Gu et al., 2020). Observe that this study models the recurrent parking sharing problem at the strategic level rather than the operational level. Therefore, the approach is aggregate and static. The cruising time here is an "expected" cruising time, which is calculated based on the curbside flow at the parking choice equilibrium (which can be regarded a long-term "expected" flow). From day to day, an individual traveler might experience a different cruising time, where the average should approach this "expected" cruising time.

Moreover, we assume that when $\frac{n_f}{m_f} \geq 1$, $\kappa \to +\infty$ (e.g., if $\frac{n_f}{m_f} = 1.1$, $\kappa \to +\infty$); and when $\frac{n_f}{m_f} < 1$, $\kappa < +\infty$. This treatment can be regarded as a soft parking capacity constraint, which ensures that $n_f < m_f$ is automatically satisfied at the parking choice equilibrium to be introduced in Section 3.1 (so the curbside parking occupancy rate will never go beyond ONE and $\kappa \to +\infty$ never occurs). The treatment of "$\kappa \to +\infty$ if $\frac{n_f}{m_f} \geq 1$" does not correspond to any real situation but is a technique to implicitly enforce a capacity constraint. The cruising time formulation captures the trend that if parking occupancy rate is higher, it is more difficult to find a vacant parking space (takes a longer time). Also, the "convexity of cruising time function" captures that on average, when the parking occupancy rate is higher, a further increase in the occupancy yields a larger increase in cruising time. We also let $t^{-1}(\cdot)$ denote the inverse function of $t(\cdot)$.

Under the second option (shared parking), travelers can drive to the area and then park at the shared (and reserved) parking space without searching. However, he or she has to pay a shared parking fee for using the shared parking. Parking cost for him or her is

$$c_s = p, \tag{2}$$

where p is the fee for a shared space. The users pay this price to the e-platform operator for the shared parking space. As mentioned earlier, the platform operator has to pay a rent to the parking owner, i.e., "repurchase" in Guo et al. (2016), which will be further discussed in Section 2.3.

Travelers have to make choices between curbside and shared parking spaces to minimize their travel cost. This results in a parking choice equilibrium (formally formulated in Section 3.1), at which no one can reduce individual travel cost through unilaterally changing his or her parking choice. It is assumed that the parking choice is purely based on the costs formulated in Eq. (1) and Eq. (2) for the two parking options and parking at any available space is feasible and acceptable. This treatment simplifies the spatial dimension and other heterogeneous features of parking, as well as individual-specific user preference over parking. This treatment will be more accurate for small regions such as suburbs (for instance, the area size of a popular suburb Coogee in Sydney is less than 2 km^2). However, this treatment tends to overestimate the shareability of parking spaces and the matching between demand and supply (even for small regions). A user-parking matching function similar to driver-rider matching in taxi or ride-sourcing studies might be incorporated in a future study to accommodate this. The current study can be regarded as an optimistic situation and provides an efficiency upper bound in terms of parking utilization.

When we consider the parking choice equilibrium, the difference between the two prices, i.e., q and p, matters. Thus, to economize the notation and presentation, we let $q = 0$, and p then represents the relative price of shared parking against public curbside parking. However, this can be readily extended to the case with a non-zero curbside parking fee.

Given the numbers of travelers that choose curbside and shared parking, i.e., n_f and n_s, where $n_f + n_s = n$, the total user cost is

$$TC = c_f \cdot n_f + c_s \cdot n_s. \tag{3}$$

2.2 Parking owners (or sharers)

There are M_s residents who are parking owners as well as potential parking sharers. These residents may drive to other areas to work and leave their parking spaces vacant during the daytime. When sharing their spaces, they will incur an inconvenience cost δ. It is assumed that δ is distributed over $[\delta_l, +\infty)$ with a probability distribution function $f(\delta)$ and cumulative distribution function $F(\delta)$, where $\delta_l \geq 0$. It is noteworthy that the inconvenience of individual parking owners to share their spaces is private information. However, the platform may conduct surveys to calibrate the distribution of inconvenience among the parking owners (potential parking sharers). This paper assumes that the platform operator knows the distribution $F(\delta)$. This information can be utilized to determine its pricing strategies. For analytical purpose, we adopt the following assumption for $F(\delta)$ and $f(\delta)$.

Assumption 1. *The cumulative distribution function $F(\delta)$ is twice differentiable such that $f'(\delta)$ exists, and the probability distribution function satisfies $f(\delta) > 0$ on $[\delta_l, +\infty)$.*

Assumption 1 indicates that the probability distribution function $f(\cdot)$ is smooth and $F(\cdot)$ is strictly increasing before F reaches ONE. We further let $F^{-1}(\cdot)$ denote the inverse function of $F(\cdot)$ for $0 \leq F \leq 1$, where $F^{-1}(0) = \delta_l$.

If a rent r is paid to the parking owners by the platform operator, the parking owners with an inconvenience cost $\delta \leq r$ (who can benefit from sharing their spaces) will "sell the right-of-use" of their parking spaces to the platform operator. The realized shared parking supply under a given rent r is

$$m_s = F(r) \cdot M_s. \tag{4}$$

It is obvious that $m_s \leq M_s$. Moreover, $n_s \leq m_s$ also holds, i.e., the total number of drivers using the shared parking spaces should be less than or equal to the total number of available ones (or the "expected" shared parking occupancy rate is no greater than 100%). This study focuses on $r \geq \delta_l$ since $r < \delta_l$ yields the same outcome as $r = \delta_l$, where $m_s = 0$.

Given the rent $r \geq \delta_l$, the total net benefit of parking owners equals the total parking rent minus the total inconvenience cost caused, i.e.,

$$R_s(r) = \left[F(r) \cdot r - \int_{\delta_l}^{r} \delta \cdot f(\delta) \cdot d\delta\right] \cdot M_s. \tag{5}$$

It can be readily verified that the net benefit of parking owners R_s is non-decreasing with respect to r.

2.3 Parking sharing platform operator

The shared parking operator charges the parking users a fee of p for a shared parking space, and compensates a private parking sharer a fee of r or we call it "rent". The interactions among the operator, the parking users and owners are illustrated in Figure 1. The pricing strategies of the parking sharing platform, i.e., the combinations of p and r, will influence the shared parking demand and supply, as well as the platform's performance. In Section 3, we will explore the pricing strategies of a private platform operator for revenue maximization and those of a public operator for social cost minimization or system optimum. We will also compare the pricing strategies and system performance metrics under private and public operators.

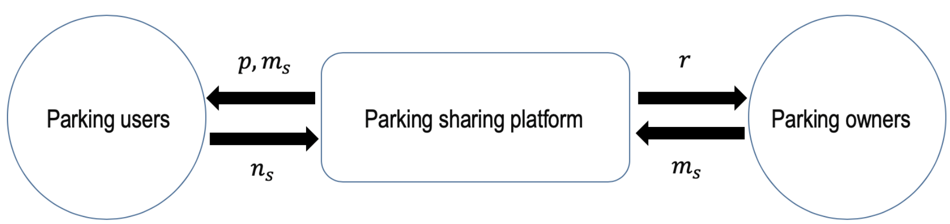

Figure 1: Interactions among the platform operator, parking users and owners

A private parking sharing platform operator decides its pricing strategy, i.e., p and r, to maximize its net revenue, which is

$$R_p(p, r) = n_s \cdot p - m_s \cdot r, \tag{6}$$

subject to users' parking choices and parking owners' sharing decisions, i.e., n_s will depend on p; and m_s will depend on r; and furthermore n_s is constrained by m_s.

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A public (or regulated) platform operator concerns the total social cost of the three parties (i.e., travelers, owners, and the operator), which is

$$TSC = TC - R_s - R_p \tag{7}$$

where TC is defined in Eq. (3), and R_s is defined in Eq. (5), and R_p is defined in Eq. (6). The social cost excludes money transfers between different parties involved (those between the platform operator and the parking users and those between the platform operator and the parking owners or sharers).

3 Parking choice equilibrium and pricing strategies

In this section, we firstly describe and identify the parking choice equilibrium under given parking prices and capacities, and then discuss the pricing strategies for revenue maximization (a private operator) and social cost minimization (a public operator).

3.1 Parking choice equilibrium

In this section, we formulate travelers' parking choice problem given the pricing strategy (p, r) of the platform operator and the curbside parking supply m_f. Note that, once r is given, the shared parking supply m_s is also given. The parking choice equilibrium is a variant of the classic User Equilibrium traffic assignment problem with capacity constraints (Nie et al., 2004), where a traveler minimizes his or her cost when making parking choice decisions subject to the parking capacity constraints.

We begin with listing Assumption 2, which means that there is always sufficient parking supply for the n travelers and guarantees that the static parking choice problem has one feasible solution at least.

Assumption 2 (Sufficient Parking)**.** *It is assumed that* $m_f + m_s > n$.

Observe that we consider a long-term equilibrium of parking choice. The equilibrium parking occupancy rate is no greater than ONE, i.e., we have $n_f < m_f$ and $n_s \leq m_s$. To have a feasible solution we should have $n = n_f + n_s < m_f + m_s$. The above assumption helps reflect that parking has capacity limits and parking occupancy does not go beyond ONE, which is appropriate for a static equilibrium problem. If one incorporates the time dimension and considers dynamics of parking inflow and outflow, certainly the total number of parking users can go beyond the number of parking spaces. However, that does not conflict with the current formulation, where the current study focuses on an average state for a given duration within the day and parking occupancy rate never goes beyond ONE. Generally speaking, in the long-term, the number of travelers that choose to drive is governed by the parking supply as travelers will shift to other travel modes (e.g., public transit, shared-ride) when parking is costly (either costly searching time or costly fees for reserved spaces).

At the parking choice equilibrium, no parking user can reduce individual cost by unilaterally changing his or her parking choice. The underlying assumption of the parking choice equilibrium is that parking users are rational, and they try to minimize their individual travel costs. Moreover, they have information regarding the parking pricing and capacities, which allow them to make rational decisions.

The User Equilibrium conditions for the parking choice problem with parking capacity constraint reads in the following form, given $n_f + n_s = n$, $n_f \geq 0$ and $n_s \geq 0$:

$$n_f > 0 \Rightarrow c_f = c^* \tag{8a}$$

$$n_f = 0 \Rightarrow c_f \geq c^* \tag{8b}$$

$$n_s > 0 \Rightarrow c_s + \lambda = c^* \tag{8c}$$

$$n_s = 0 \Rightarrow c_s + \lambda \geq c^*. \tag{8d}$$

where $c^* = \min \{c_f, c_s + \lambda\}$, and λ is a multiplier associated with the shared parking capacity constraint defined as follows:

$$\lambda \left(m_s - n_s\right) = 0; m_s - n_s \geq 0; \lambda \geq 0. \tag{9}$$

$\lambda > 0 \Rightarrow m_s = n_s$ and $\lambda = 0 \Rightarrow m_s \geq n_s$. The multiplier λ is the shadow price that reflects the additional indirect cost experienced by the shared parking users to ensure the chosen option (e.g., book the shared space in advance). The multiplier λ will also appear in Eq. (12), where detailed derivations are provided in Appendix A.1. Similar observations (indirect cost due to, e.g., advance booking) have been made for high-speed railway ticket booking in the study of Xu et al. (2018). Future studies can optimize the parking reservation schemes for parking sharing services at the operational level (Shao et al., 2016).

Following the well-known Beckmann's formulation (Beckmann et al., 1956), the minimization problem below can be constructed to obtain the parking choice equilibrium, i.e.,

$$\min z\left(n_f, n_s\right) = \int_0^{n_f} c_f\left(w\right) dw + \int_0^{n_s} c_s dw \tag{10}$$

subject to

$$n_f + n_s = n; \tag{11a}$$

$$n_s \leq m_s; \tag{11b}$$

$$n_f \geq 0, n_s \geq 0. \tag{11c}$$

where Eq. (11a) is the flow conservation condition, Eq. (11b) is the shared parking capacity constraint, Eq. (11c) ensures non-negativity of the parking flows. Note that the curbside parking capacity constraint $n_f < m_f$ will be automatically satisfied after solving the above minimization problem since the cruising time $t\left(n_f\right) \to +\infty$ when $n_f \geq m_f$ (this can be regarded as a soft capacity constraint) and the total parking supply is sufficient, i.e., $m_f + m_s > n$. Since the cost functions c_f and c_s are continuous and c_f is strictly increasing, one can readily verify that there exists a unique solution to the above minimization problem, i.e., the parking choice equilibrium is unique.

For the minimization problem in Eq. (10) subject to the constraints in Eq. (11), we can write down its Lagrangian function (refer to Appendix A.1 for details), where u and λ denote the Lagrange multipliers associated with flow conservation in Eq. (11a) and the shared parking capacity constraint in Eq. (11b), respectively. We then can derive the following optimality conditions (similar Karush-Kuhn-Tucker (KKT) conditions can be found in, e.g., Hearn, 1980):

$$n_f \cdot (c_f - u) = 0 \tag{12a}$$

$$c_f - u \geq 0 \tag{12b}$$

$$n_f \geq 0 \tag{12c}$$

$$n_s \cdot (c_s + \lambda - u) = 0 \tag{12d}$$

$$c_s + \lambda - u \geq 0 \tag{12e}$$

$$n_s \geq 0 \tag{12f}$$

$$\lambda \cdot (m_s - n_s) = 0 \tag{12g}$$

$$m_s - n_s \geq 0 \tag{12h}$$

$$\lambda \geq 0 \tag{12i}$$

$$n_f + n_s = n \tag{12j}$$

At equilibrium, u is equal to c^* in Eq. (8), and the multiplier λ associated with the shared parking capacity is identical to that defined in Eq. (9). One can readily verify the equivalence between KKT conditions and User Equilibrium conditions. The proof of equivalence

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is standard in the User Equilibrium traffic assignment literature and thus is omitted. It is noteworthy that on the left-hand sides of conditions in Eq. (12), the costs (e.g., c_f) are functions. These functions should be evaluated when we identify possible equilibrium solutions below. Moreover, these functions should be evaluated when we analyze the revenue maximization in Section 3.2 since it is subject to the parking choice equilibrium.

While the parking choice equilibrium has a unique solution under a given pair of shared parking price and supply (p, m_s) (or a pair of price and rent (p, r)), the solution varies with (p, m_s) (or (p, r)). Four possible scenarios of equilibrium solutions can arise under different (p, m_s):

Scenario I: $n_s = n, n_f = 0$, i.e., all travelers choose shared parking, and no traveler choose curbside parking;

Scenario II: $n_s = m_s, n_f = n - m_s$, i.e., travelers choose both shared parking and curbside parking, and all realized shared parking spaces are utilized;

Scenario III: $0 < n_s < m_s, n_f > 0$, i.e., travelers choose both shared parking and curbside parking, but the realized shared parking is not fully utilized;

Scenario IV: $n_s = 0, n_f = n$, i.e., all travelers choose curbside parking and no travelers choose shared parking.

From Eq. (12), it is not difficult to see that Scenario I occurs when $m_s \geq n$ and $p \leq \alpha t\left(0\right)$. In this case, the shared parking price is sufficiently low, and the shared parking supply is sufficiently large to meet all parking demand, so all travelers choose shared parking. Scenario II occurs when $m_s < n$ and $p \leq \alpha t\left(0\right)$, and $m_s < n - t^{-1}\left(\frac{p}{\alpha}\right)$, $\alpha t\left(0\right) < p < \alpha t\left(n\right)$, in which case the shared parking price is low, but the shared parking supply is insufficient to satisfy all demand. Scenario III corresponds to an interior parking choice equilibrium when $c_s = c_f$. Such interior equilibrium occurs when $\alpha t\left(0\right) < p < \alpha t\left(n\right)$, $m_s \geq n - t^{-1}\left(\frac{p}{\alpha}\right)$, i.e., the shared parking supply is sufficient given the price. And travelers parking choice in this case satisfies $(n_s, n_f) = \left(n - t^{-1}\left(\frac{p}{\alpha}\right), t^{-1}\left(\frac{p}{\alpha}\right)\right)$. Finally, Scenario IV occurs when $p \geq \alpha t\left(n\right)$, in which case the shared parking price is always higher than curbside parking cost, so no one choose shared parking. A graphical illustration of the above four scenarios is provided in Figure 2. The Origin in Figure 2 is $(p = 0, m_s = 0)$.

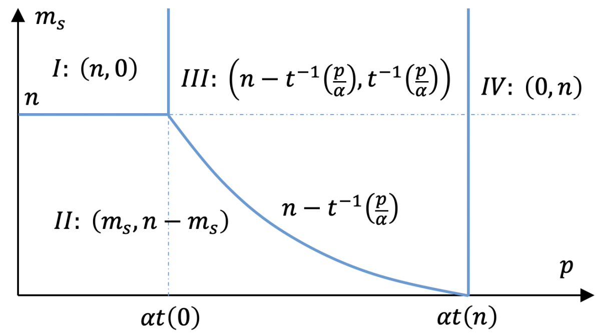

Figure 2: Parking choice equilibrium in the domain of (p, m_s)

As shared parking supply m_s is endogenously determined by parking rent r, Figure 3 further converts all the terms related to m_s in Figure 2 into r (through replacing m_s with $m_s = F(r) \cdot M_s$ or equivalently $r = F^{-1}\left(\frac{m_s}{M_s}\right)$), to display the possible equilibrium solutions under given (p, r). Note that $F^{-1}(0) = \delta_l$, so we only need to focus on $r \geq \delta_l$. Furthermore, as mentioned earlier, the feasible domain for the parking sharing problem is constrained by $m_s \leq M_s$, so Scenario I in Figure 2 may or may not exist, depending on $M_s > n$ or $M_s \leq n$.

When $M_s \geq n$, one may let $r \geq F^{-1}\left(\frac{n}{M_s}\right)$, and thus $m_s \geq n$ may occur. In this case, all the four categories of equilibrium solutions in Figure 2 could arise, as shown in Figure 3(a). The Origin in Figure 3(a) is $(p = 0, r = \delta_l)$. The four curves dividing the domain of (p, r) into four regions are: (i) $r = F^{-1}\left(\frac{n}{M_s}\right)$ and $p \in [0, \alpha t(0)]$, and (ii) $p = \alpha t(0)$ and $r \geq F^{-1}\left(\frac{n}{M_s}\right)$, and (iii) $p = \alpha t(n)$ and $r \geq \delta_l$, and (iv) $r = F^{-1}\left(\frac{n - t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right)$ and $p \in [\alpha t(0), \alpha t(n)]$. These four curves in Figure 3(a) are the counterparts of the four curves marked in Figure 2.

When $M_s < n$, m_s will always be less than n even if we set a very large r. As shown in Figure 3(b), the equilibrium solution $(n, 0)$ in Figure 2 can never occur. Similarly, the Origin in Figure 3(b) is $(p = 0, r = \delta_l)$. The two curves, i.e., "$p = \alpha t(n)$ and $r \geq \delta_l$" and "$r = F^{-1}\left(\frac{n - t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right)$ and $p \in (\alpha t(n - M_s), \alpha t(n)]$", divide the domain of (p, r) into three regions (Region I does not exist in this case and $(n_s, n_f) = (n, 0)$ can never arise).

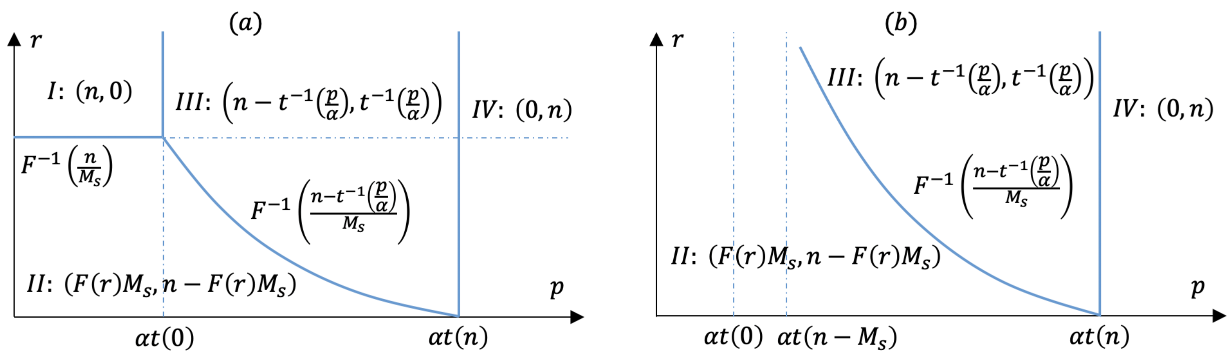

Figure 3: Parking choice equilibrium in the domain of (p, r): (a) $M_s \geq n$; (b) $M_s < n$

Figure 3 clearly shows that the parking choice equilibrium varies significantly in the domain of (p, r). A platform operator can vary (p, r) (decision variables or pricing strategy) to achieve revenue maximization (a private operator) or social cost minimization (a public operator). These are discussed in the next two subsections.

3.2 Revenue maximization

We now consider a revenue-maximizing operator. It aims to maximize its net revenue (or profit), i.e.,

$$\max : R_p(p, r) \tag{13}$$

subject to travelers' parking choice equilibrium formulated in Section 3.1, and the parking owners' supplying equilibrium introduced in Section 2.2, where $R_p(p, r)$ is given in Eq. (6). In the following, Proposition 3.1 and Proposition 3.2 discuss the optimal price and rent to maximize net revenue, the corresponding optimal parking supply and flows, and the resultant system efficiency metrics (i.e., user cost, owners' benefit, platform revenue). Proposition 3.3 discusses how the inconvenience cost distribution of parking owners and curbside parking cost conditions/parameters (cruising time function and value of time) may affect the profitability of the platform operator.

Let (p^{rm}, r^{rm}) be the solution to the revenue maximization problem in Eq. (13). Observe that we also use the subscript 'rm' for other notations to indicate the correspondence to revenue maximization. It is noteworthy that we have $p^{rm} \geq 0$ and $r^{rm} \geq \delta_l$ for a revenue-maximizing operator. Firstly, as mentioned earlier $r < \delta_l$ would simply yield zero parking owners to share their parking spaces and the platform operator has zero net revenue. This is never better than letting $r = \delta_l$. Secondly, if $p < 0$, one can readily verify that the operator must be better off by increasing p to zero.

Proposition 3.1. *The optimal pricing strategy to maximize the platform operator's net revenue, i.e.,* (p^{rm}, r^{rm})*, satisfies the following:*
(i) when $M_s \geq n$, $r^{rm} = F^{-1}\left(\frac{n-t^{-1}\left(\frac{p^{rm}}{\alpha}\right)}{M_s}\right)$ *and* $p^{rm} \in [\alpha t(0), \alpha t(n)]$*;*
(ii) when $M_s < n$, $r^{rm} = F^{-1}\left(\frac{n-t^{-1}\left(\frac{p^{rm}}{\alpha}\right)}{M_s}\right)$ *and* $p^{rm} \in (\alpha t(n - M_s), \alpha t(n)]$*.*

The proof of Proposition 3.1 is relayed to Appendix A.2. Proposition 3.1 defines a subset of the domain (p, r), which contains the revenue-maximizing solution (p^{rm}, r^{rm}). This subset is exactly the curve "$r = F^{-1}\left(\frac{n-t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right)$ and $p \in [\alpha t(0), \alpha t(n)]$" in Figure 3(a) for the case with $M_s \geq n$, and is the curve "$r = F^{-1}\left(\frac{n-t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right)$ and $p \in (\alpha t(n - M_s), \alpha t(n)]$" in Figure 3(b) for the case with $M_s < n$. The property that $r^{rm} = F^{-1}\left(\frac{n-t^{-1}\left(\frac{p^{rm}}{\alpha}\right)}{M_s}\right)$ is later utilized to further examine (p^{rm}, r^{rm}) and the profitability of the sharing platform operator.

One can readily verify from Proposition 3.1 the following two implications (more detailed discussions provided in Appendix A.3). First, under revenue maximization, all shared parking spaces rented from parking owners should be used, and thus no rent from the operator is wasted, i.e., $n_s = m_s = F(r)M_s$. Second, the shared parking fee is equal to the cruising time cost of curbside parking users, i.e., $p = \alpha t\left(n_f\right)$ or $c_s = c_f$, where a lower fee fails to fully utilize the profitability from a shared parking space and a higher fee will push users away.

The above results for revenue maximization can be summarized as: $n_s = m_s = F(r)M_s$, $r = F^{-1}\left(\frac{n-t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right)$ or equivalently $p = \alpha t\left(n - F\left(r\right)M_s\right)$. By embedding these conditions into Eq. (6), we can obtain R_p as a function of r, i.e.,

$$R_p = F(r)M_s \cdot \left(\alpha t\left(n - F\left(r\right)M_s\right) - r\right) \tag{14}$$

As stated earlier, $F(\cdot)$ is assumed differentiable to allow analytical tractability. By looking at the first derivative of Eq. (14) with respect to r, we can further explore the properties of (p^{rm}, r^{rm}) and the corresponding shared parking supply and system efficiency metrics (i.e., the total user cost TC, the net benefit of parking owners R_s, and the platform operator's net revenue R_p). Note that R_p in Eq. (14) may not be concave with respect to r and the first-order optimality condition (i.e., $\frac{dR_p}{dr} = 0$) is only a necessary condition for an interior optimum, as will be discussed shortly. For comparison purpose, we let $TC^0 = n \cdot \alpha \cdot t(n)$, $R_s^0 = 0$, and $R_p^0 = 0$, which are the total user cost, net benefit of parking owners, and platform operator's net revenue before introducing the parking sharing, respectively.

Proposition 3.2. *Given the inconvenience distribution* $F(\delta)$ *where* $f(\delta) > 0$ *for* $\delta \in [\delta_l, +\infty)$*, under revenue maximization:*
(i) if $\delta_l < \alpha t(n)$*,* $p^{rm} > r^{rm} > \delta_l$*, and the shared parking supply is positive, i.e.,* $m_s^{rm} > 0$*, and the net revenue of the platform operator is positive, i.e.,* $R_p(p^{rm}, r^{rm}) > 0$*;*
(ii) if $\delta_l < \alpha t(n)$*, the revenue-maximizing pricing strategy, i.e.,* (p^{pm}, r^{rm})*, yields a Win-Win-Win situation, i.e.,* $TC < TC^0$*,* $R_s > R_s^0$*, and* $R_p > R_p^0$*;*
(iii) the optimal shared parking supply satisfies the following: $m_s^{rm} < M_s < n$ *when* $M_s < n$ *and* $m_s^{rm} < n \leq M_s$ *when* $M_s \geq n$ *and* $t(0)$ *is sufficiently small, i.e., some but not all potential shared parking supply should be rented.*

The proof of Proposition 3.2 is relayed to Appendix A.4. In Proposition 3.2, δ_l is the minimum inconvenience cost experienced by some parking owners if they choose to share their spaces and $\alpha t(n) = c_f(n)$ is the curbside parking user cost when all travelers choose curbside parking. Proposition 3.2(i) indicates that as along as there exist a group of parking owners with sufficiently small inconvenience to share their parking spaces, i.e., $\delta_l < \alpha t(n)$, the platform can gain a positive net revenue by buying their parking and "sell the right of use" to parking users. The operator can empirically evaluate whether $\delta_l < \alpha t(n)$ holds in an urban area to assess the potential profitability of running parking sharing in the area. It is evident that if δ_l is small (e.g., equals zero) and α or $t(n)$ is large, parking sharing platform is more likely to earn positive net revenue.

Proposition 3.2(ii) indicates that with a revenue-maximizing operator, a Win-Win-Win situation for the parking users, parking owners (or sharers), and the platform operator can be achieved. This is further discussed as follows. For the travelers, after parking sharing is introduced, equilibrium travel cost is reduced due to more parking options, which results in less cruising cost ($-TC$ can be regarded as the total utility). For the M_s parking owners, some of them (m_s owners) with the smallest inconvenience will share their parking, and they are strictly better off and others (i.e., $M_s - m_s$ owners) are at least not worse off. For the platform operator, it is already shown in Proposition 3.2(i) that the platform can earn a positive revenue.

Proposition 3.2(iii) indicates the potential shared parking supply M_s should not be fully utilized in order to maximize revenue. This is because to fully utilize M_s it is too costly (the rent paid is too large). It is worth mentioning that $t(0)$ is very likely to be small, since when no travelers using curbside parking, finding a vacant space costs almost zero cruising time (e.g., travelers may find a vacant space immediately after they enter the parking area, note that access time to parking area is not included in t).

We now further look at the solution (p^{rm}, r^{rm}). Based on Eq. (26) in Appendix A.4, i.e., the first derivative of Eq. (14) with respect to r, and letting $\frac{dR_p}{dr} = 0$, one can further verify that an interior optimal rent r^{rm} solves:

$$f(r)M_s\left[\alpha t - r\right] = F(r)M_s\left[\alpha t' f(r)M_s + 1\right], \tag{15}$$

where $t = t\left(n - F\left(r\right)M_s\right)$ and $t' = t'\left(n - F\left(r\right)M_s\right)$. Observe that R_p is not necessarily concave with respect to r, which is subject to the shapes of $F(\cdot)$ and $t\left(\cdot\right)$. The above first-order optimality condition is only necessary for an interior optimum but not sufficient. In Eq. (15), $f(r)M_s\left[\alpha t - r\right]$ is the marginal revenue change due to the marginal change in the number of shared-parking users, and $F(r)M_s\left[\alpha t' f(r)M_s + 1\right]$ is the change due to the marginal change in the shared parking price and rent. Furthermore, we have $p^{rm} = \alpha t\left(n - F\left(r^{rm}\right)M_s\right)$. Given the $\left(p^{rm}, r^{rm}\right)$, one can readily obtain that $n_s^{rm} = m_s^{rm} = F(r^{rm})M_s$ and $n_f^{rm} = n - F(r^{rm})M_s$.

Proposition 3.2(i) also shows that $p^{rm} > r^{rm}$. One can further quantify the ratio of p^{rm} and r^{rm} given a specific distribution of inconvenience among the potential parking suppliers M_s. In particular, suppose the inconvenience of parking sharers is uniformly distributed over $[\delta_l, \delta_u]$, and δ_u is sufficiently large ($r^{rm} < \delta_u$), the optimal pricing strategy $\left(p^{rm}, r^{rm}\right)$ to maximize R_p given in Eq. (30) should satisfy

$$\frac{p^{rm}}{r^{rm}} > 2. \tag{16}$$

The derivation of Eq. (16) is discussed in Appendix A.5. The price-to-rent ratio indeed reflects the return on unit rent paid. Eq. (16) says that given the assumed inconvenience cost distribution in the above, at revenue maximization, one dollar rent should generate more than two dollars income.

While r^{rm} can occur in the interior, multiple optimal solutions may exist and $\frac{dR_p}{dr} = 0$ is only a necessary condition. If $\frac{d^2R_p}{dr^2} < 0$ (i.e., R_p in Eq. (14) is strictly concave with respect to r), we have a unique r^{rm}. For example, if the inconvenience is quite evenly distributed, i.e., $f'(r) \to 0$, and $t' \geq 0.5M_s t''$ ($t = t\left(\cdot\right)$ is expected cruising time, t' and t'' are the first and second derivatives, respectively), one can verify that $\frac{d^2R_p}{dr^2} < 0$, and we have a unique optimal pricing strategy.

We proceed to examine how the inconvenience cost distribution of parking owners and cost of choosing curbside parking (cruising time function of curbside parking and value of time) may affect the profitability of the platform operator.

Proposition 3.3. *Ceteris paribus, (i) if two distributions of the inconvenience δ satisfy $F_1\left(\delta\right) \leq F_2\left(\delta\right)$ for any δ, the maximum net revenue of the platform operator under $F_2\left(\delta\right)$ will be no less than that under $F_1\left(\delta\right)$; (ii) if two potential shared parking supplies satisfy*

$M_{s,1} \leq M_{s,2}$, the maximum revenue of the platform operator under $M_{s,2}$ will be no less than that under $M_{s,1}$; (iii) if two cruising time function satisfy $t_1(n_f) \leq t_2(n_f)$, the maximum revenue of the platform operator under $t_2(n_f)$ will be no less than that under $t_1(n_f)$; (iv) if two values of time satisfy $\alpha_1 \leq \alpha_2$, the maximum revenue of the platform operator under α_2 will be no less than that under α_1.

The proof of Proposition 3.3 is relayed to Appendix A.6. Proposition 3.3(i) and Proposition 3.3(ii) indicate that when more parking owners have smaller inconvenience due to parking sharing, i.e., $F_2(\delta) \leq F_1(\delta)$ or $M_{s,1} \leq M_{s,2}$, it is more likely for the platform to gain a larger net revenue (mainly due to cheaper shared parking). Proposition 3.3(iii) and Proposition 3.3(iv) indicate that when curbside parking is more costly, i.e., $t_2 \geq t_1$ or $\alpha_2 \geq \alpha_1$, it is more likely for the platform to gain a larger revenue (mainly due to a less competitive travel alternative, i.e., more costly curbside parking).

3.3 Social cost minimization (or system optimum)

We now discuss a public operator to minimize social cost, and the system optimal parking flow and shared parking supply pattern (i.e., system optimum). The problem is to minimize the total social cost:

$$\min : TSC \tag{17}$$

subject to the travelers' parking choice equilibrium, and the parking owners' sharing equilibrium. Based on Eq. (3), Eq. (5), Eq. (6), and Eq. (7), we have

$$TSC = c_f n_f + \int_{\delta_l}^{r} \delta f(\delta) M_s d\delta. \tag{18}$$

One can see from the above that the total social cost contains two parts: the parking searching cost of travelers choosing curbside parking, and the inconvenience cost of the parking sharers who supplied a private parking space.

In the following, Proposition 3.4 discusses the system optimal parking flow pattern and its implications. Proposition 3.5 discusses the optimal price and rent to minimize the total social cost and the corresponding system efficiency metrics, and further compares system optimum with revenue maximization. Proposition 3.6 discusses how the inconvenience cost distribution of parking owners and curbside parking cost conditions/parameters (cruising time function and value of time) may affect the minimum total social cost. Note that discussions below on these propositions also emphasize the difference between system optimum and revenue maximization.

Similar to revenue maximization (see Remark A.1), under system optimum, we should have $m_s = n_s$. This is explained as follows. Firstly, the capacity constraint $m_s \geq n_s$ always holds. Secondly, if $m_s > n_s$, by reducing m_s to n_s, while travelers' parking choice does not change as well as their costs, the inconvenience cost of parking owners can be saved (for $m_s - n_s$ parking owners). The saving amount is equal to $\int_{F^{-1}\left(\frac{n_s}{M_s}\right)}^{F^{-1}\left(\frac{m_s}{M_s}\right)} \delta f(\delta) M_s d\delta$.

Since $m_s = n_s$ holds under system optimum, r in Eq. (18) can be replaced by $r = F^{-1}\left(\frac{n_s}{M_s}\right)$, and the total social cost can be written as a function of n_f and n_s, subject to $n_f + n_s = n$. We can then define the following Lagrangian:

$$L(n_f, n_s, \nu) = c_f n_f + \int_{\delta_l}^{F^{-1}\left(\frac{n_s}{M_s}\right)} \delta f(\delta) M_s d\delta + \nu \cdot (n - n_f - n_s), \tag{19}$$

where we further have

$$\frac{\partial L}{\partial n_f} = c_f + n_f \frac{dc_f}{dn_f} - \nu. \tag{20}$$

$$\frac{\partial L}{\partial n_s} = F^{-1}\left(\frac{n_s}{M_s}\right) - \nu. \tag{21}$$

$$\frac{\partial L}{\partial \nu} = n - n_f - n_s. \tag{22}$$

We are now ready to examine the system optimal parking flows and shared parking supply, which are denoted by (n_f^{so}, n_s^{so}) and m_s^{so}, where $n_s^{so} = m_s^{so}$.

Proposition 3.4. *The parking supply and choice pattern under system optimum satisfies the following:*
(i) if $\alpha\,(t(n) + nt'(n)) \leq \delta_l$, no shared parking should be rented, i.e., $n_s^{so} = m_s^{so} = 0$, and if $\alpha\,(t(n) + nt'(n)) > \delta_l$, some shared parking should be rented and used, i.e., $n_s^{so} = m_s^{so} > 0$;
(ii) for an interior optimal solution where $n_f^{so} > 0$ and $n_s^{so} > 0$, $\alpha\left(t(n_f^{so}) + n_f^{so} t'(n_f^{so})\right) = F^{-1}\left(\frac{n_s^{so}}{M_s}\right)$ holds, i.e., the marginal cost of an additional cruising driver for curbside parking is equal to the inconvenience cost of an additional parking owner who supplies a shared space.

Proof. If $\alpha\,(t(n) + nt'(n)) \leq \delta_l$, based on Eq. (20) and Eq. (21) one can readily verify that $\frac{\partial L}{\partial n_f} \leq \frac{\partial L}{\partial n_s}$ always holds, which mean that we should set $n_f^{so} = n$ and $n_s^{so} = 0$ (m_s^{so} should also be zero in order to save inconvenience cost). This verifies Proposition 3.4(i).

When we consider an interior optimal solution where $n_f^{so} > 0$ and $n_s^{so} > 0$, based on Eq. (20) and Eq. (21), we have $\frac{\partial L}{\partial n_f} = \frac{\partial L}{\partial n_s} = 0$ and thus $\alpha\left(t(n_f^{so}) + n_f^{so} t'(n_f^{so})\right) = F^{-1}\left(\frac{n_s^{so}}{M_s}\right)$. This verifies Proposition 3.4(ii). □

In Proposition 3.4(i), δ_l is the minimum inconvenience cost and $\alpha\,(t(n) + nt'(n)) =$

$c_f(n) + n\frac{dc_f(n)}{dn}$ is the marginal cost when all travelers choose curbside parking. Proposition 3.4(i) indicates that when there exists a group of parking owners with sufficiently small inconvenience to share their parking spaces, i.e., $\delta_l < \alpha\left(t(n) + nt'(n)\right)$, the platform should rent some private parking and "sell the right of use" to parking users in order to minimize total social cost. The operator can empirically evaluate whether $\delta_l < \alpha\left(t(n) + nt'(n)\right)$ holds in an urban area to assess the potential of parking sharing to reduce social cost.

Proposition 3.4(ii) states that, when we have an interior system optimum parking flow pattern, the marginal cost of an additional cruising driver, i.e., $\alpha \cdot (t + n_f t')$, should equal the marginal cost of an additional driver that chooses shared parking, which is exactly the inconvenience cost due to supplying an additional shared space (incurred by the additional parking owner), i.e., $F^{-1}\left(\frac{n_s}{M_s}\right)$. For the sharing problem, the social cost includes those of both parking users and parking sharers (a two-sided market), and the system optimum balances the marginal costs from both sides. An interior system optimum (n_f^{so}, n_s^{so}) can be determined by simultaneously solving $\alpha\left(t(n_f) + n_f t'(n_f)\right) = F^{-1}\left(\frac{n_s}{M_s}\right)$ and $n_f + n_s = n$, the solution of which is unique given that $t(\cdot)$ is increasing and convex and $F^{-1}(\cdot)$ is increasing or at least non-decreasing.

From the condition $\alpha\left(t(n_f) + n_f t'(n_f)\right) = F^{-1}\left(\frac{n_s}{M_s}\right)$ in Proposition 3.4(ii), we can further verify that a more costly curbside parking (i.e., a larger α or a larger $t(n_f)$ given the same n_f) will yield a smaller n_f^{so} and a larger n_s^{so}, and more parking owners with a smaller inconvenience cost will yield a smaller n_f^{so} and a larger n_s^{so}.

Proposition 3.5. *Given the inconvenience distribution $F(\delta)$ where $f(\delta) > 0$ for $[\delta, +\infty)$, under system optimum,*
(i) if $\delta_l < \alpha\left(t(n) + nt'(n)\right)$, we have $r^{so} = F^{-1}\left(\frac{n_s^{so}}{M_s}\right) > \delta_l$ and $p^{so} \leq \alpha t(n_f^{so}) < r^{so}$, and the net revenue of the platform operator is negative or more specifically $R_p\left(p^{so}, r^{so}\right) \leq -n_f^{so}\left[\alpha n_f^{so} t'(n_f^{so})\right] < 0$;
(ii) if $\delta_l < \alpha\left(t(n) + nt'(n)\right)$, the social-cost-minimizing pricing strategy, i.e., (p^{so}, r^{so}), yields a Win-Win-Lose situation, i.e., $TC < TC^0$, $R_s > R_s^0$, and $R_p < R_p^0$;
(iii) suppose we have interior parking flow solutions under both revenue maximization and system optimum, then $r^{so} > r^{rm}$ and $p^{so} < p^{rm}$, and at the same time $n_s^{so} > n_s^{rm}$ and $n_f^{so} < n_f^{rm}$.

The proof of Proposition 3.5 is relayed to Appendix A.7. Proposition 3.5(i) follows Proposition 3.4(i). Besides illustrating the potential of parking sharing to reduce total social cost, Proposition 3.5(i) says that $p^{so} < r^{so}$ and to minimize social cost the platform experiences a loss no less than $n_f^{so}\left[\alpha n_f^{so} t'(n_f^{so})\right]$. It is noteworthy that p^{so} is not unique and can be any value less than $\alpha t(n_f^{so})$.

By comparing Proposition 3.4(i) and Proposition 3.5(i) for system optimum and Proposition 3.2(i) for revenue maximization, we can see that the condition $\delta_l < \alpha\left(t(n) + nt'(n)\right)$ to ensure $m_s^{so} > 0$ is more relaxed than the condition $\delta_l < \alpha t(n)$ to ensure $m_s^{rm} > 0$. This is because, the platform's profitability relies on cheap shared parking at the individual user level, i.e., $\delta_l < r^{rm} < p^{rm} = \alpha t(n_f^{rm}) \leq \alpha t(n)$; and the capability to reduce social cost relies on cheap shared parking at the system level (marginal cost $\alpha\left(t(n) + nt'(n)\right)$ appears). It is obvious that $\delta_l < \alpha t(n)$ is a subset of $\delta_l < \alpha\left(t(n) + nt'(n)\right)$. This implies that when introducing parking sharing is profitable for a private operator, it can reduce social cost under a public operator for sure. However, being able to reduce social cost under a public operator does not guarantee the profitability of parking sharing even with a private operator to maximize its net revenue. In short, a public operator is more likely to be incentivized to introduce parking sharing than a private operator.

In comparison with the Win-Win-Win situation in Proposition 3.2(ii) under revenue maximization, Proposition 3.5(ii) establishes the existence of a Win-Win-Lose situation under social cost minimization. This implies that subsidy is required to persuade a private operator to manage the parking sharing system in a social-cost-minimizing way. The required subsidy amount is no less than $n_f^{so}\left[\alpha n_f^{so} t'(n_f^{so})\right]$ (as presented in Proposition 3.5(i)) to ensure non-negative net revenue for the platform and there is no need to provide a subsidy greater than $R_p\left(p^{rm}, r^{rm}\right) + n_f^{so}\left[\alpha n_f^{so} t'(n_f^{so})\right]$. We will more systematically discuss the implementation of system optimum and the regulation of a revenue-maximizing operator to reduce total social cost in Section 5.

Proposition 3.5(iii) further compares the parking flows, i.e., n_f and n_s, shared parking supply m_s, and shared parking price and rent, i.e., p and r, under revenue maximization and system optimum. Proposition 3.5(iii) clearly indicates that (considering interior solutions) more shared parking should be rented under system optimum in order to reduce parking cruising externality and a lower shared parking price should be set to attract travelers to use shared parking, while less shared parking should be rented under revenue maximization which results in a higher cruising time cost as well as a higher shared parking price.

It should be noted that lower parking cost due to parking sharing may induce additional car traffic. Therefore, whether or not such a platform should be subsidized has to take into account how much additional demand will be induced and the generated negative externality. This should be further examined in a framework with other travel and parking alternatives.

Proposition 3.6. *Ceteris paribus, (i) if two distributions of the inconvenience δ satisfy $F_1(\delta) \leq F_2(\delta)$ for any δ, the minimum total social cost under $F_2(\delta)$ will be no larger than that under $F_1(\delta)$; (ii) if two potential shared parking supplies satisfy $M_{s,1} \leq M_{s,2}$, the minimum total social cost under $M_{s,2}$ will be no larger than that under $M_{s,1}$; (iii) if two*

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cruising time function satisfy $t_1(n_f) \geq t_2(n_f)$*, the minimum total social cost under* $t_2(n_f)$ *will be no greater than that under* $t_1(n_f)$*; (iv) if two values of time satisfy* $\alpha_1 \geq \alpha_2$*, the minimum total social cost under* α_2 *will be no greater than that under* α_1*.*

The proof of Proposition 3.6 is relayed to Appendix A.8. Proposition 3.6 examines how the inconvenience cost distribution of parking owners and curbside parking cost (cruising time function and value of time) may affect the minimum total social cost involving the parking users and owners, and the platform operator.

Proposition 3.6(i) and Proposition 3.6(ii) indicate that when more parking owners have smaller inconvenience due to parking sharing, i.e., $F_2(\delta) \leq F_1(\delta)$ or $M_{s,1} \leq M_{s,2}$, it is more likely for the system to achieve a smaller minimum total social cost. These are in line with Proposition 3.3(i) and Proposition 3.3(ii) for revenue maximization, where smaller inconvenience indicates more profitability. Together these results indicate when more parking owners have smaller inconvenience due to parking sharing, profitability of the platform and the capability of introducing parking sharing to reduce social cost are both better.

Proposition 3.6(iii) and Proposition 3.6(iv) indicate that when curbside parking is less costly, i.e., $t_2 \leq t_1$ or $\alpha_2 \leq \alpha_1$, it is more likely for the system to achieve a smaller total social cost. These are different from Proposition 3.3(iii) and Proposition 3.3(iv) for revenue maximization, where more costly curbside parking (i.e., $t_2 \geq t_1$ or $\alpha_2 \geq \alpha_1$) yields better profitability. This is because, less costly curbside parking means less externality from using curbside parking as well as potential smaller social cost. However, less costly curbside parking implies that a revenue-maximizing operator has to set a lower shared parking fee in order to attract users (curbside parking is competing with shared parking) and thus ends up with less net revenue.

4 Numerical studies

This section presents numerical results to illustrate the proposed model and analysis. We examine the platform's net revenue, parking owners' net benefit, total social cost, and total user cost under different pricing strategies of the platform operator. We summarize the basic numerical setting in Table 1. AUD represents Australian Dollar, which is the monetary unit used in this paper. 37.60 AUD/hr is the average of private and business time values based on recommended data by Transport for New South Wales (TfNSW), i.e., TfNSW Economic Parameter Values (https://www.transport.nsw.gov.au/projects/project-delivery-requirements/evaluation-and-assurance/resources). The total demand, the total curbside parking supply, the total potential shared parking supply and the distribution of inconvenience cost are all assumed. The demand and supply values are in the suburb-level rather than large downtown level. For the distribution of inconvenience cost, Section 4.2 conducts sensitivity analysis and examines how the distribution variations might affect the results. Section 4.1 discusses the difference between revenue maximization, system optimum, and original user equilibrium without parking sharing under the benchmark numerical setting. Section 4.3 discusses a revenue-maximizing operator under pricing regulation.

Table 1: Basic numerical settings

Parameters or Functions	Specification
Total demand	$N = 4000$
Value of driving (cruising) time	$\alpha = 37.60$ (AUD per hour)
Total curbside parking supply	$m_f = 4200$
Total potential shared parking supply	$M_s = 4200$
Distribution of inconvenience cost	$F(\delta) = 1 - e^{-0.1\delta}$ where $\delta \geq \delta_l = 0$

The cruising time function (in minutes) is as follows: when $\frac{n_f}{m_f} \leq 0.6$, $\kappa\left(\frac{n_f}{m_f}\right) = 0.5/\left(1 - \frac{n_f}{m_f}\right)$ and otherwise $\kappa\left(\frac{n_f}{m_f}\right) = \kappa_0 exp\left(\kappa_1 \frac{n_f}{m_f}\right)$ where $\kappa_1 = 2.907 \times 10^{-4}$ and $\kappa_2 = 7.12$. Note that κ is continuous at $\frac{n_f}{m_f} = 0.6$. Figure 4 displays the cruising time against parking occupancy rate. As can be seen, when the parking occupancy rate exceeds 80%, the parking cruising time starts to grow more sharply. While the cruising time setting is assumed, it is comparable to (but slightly larger than) the cruising time in Gu et al. (2020) based on survey data and parking occupancy information.

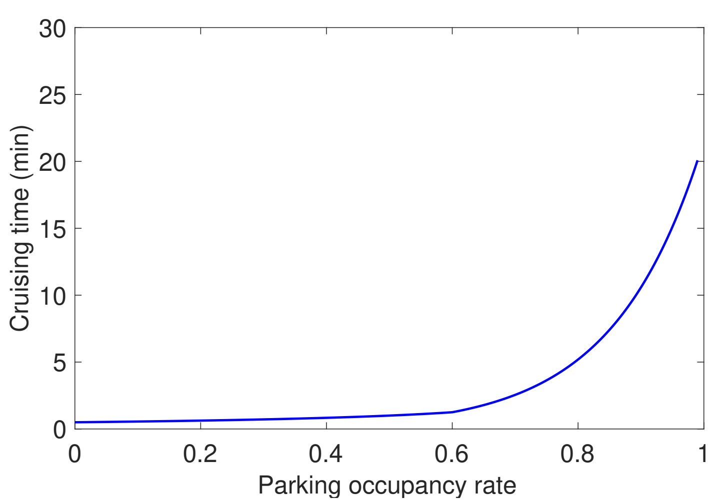

Figure 4: The cruising time function: cruising time vs. occupancy rate

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4.1 Benchmark Case

This subsection compares the difference between revenue maximization, system optimum, and original user equilibrium without parking sharing under the benchmark numerical setting.

In Table 2, we summarize the pricing strategies, i.e., shared parking fee p and the rent r, the operator's net revenue, i.e., R_p in Eq. (6), the net benefit of parking sharers, i.e., R_s in Eq. (5), the total social cost, i.e., TSC in Eq. (7), and the total user cost, i.e., TC in Eq. (3) under Revenue Maximization (RM) and System Optimum (SO), respectively. Also, we compare them with those under the Original User Equilibrium (OUE), where parking sharing is not introduced.

As can be seen in Table 2, in the RM case, the platform operator has a substantial net revenue, while in the SO case, the platform operator has to be subsidized (a negative revenue of -4.661×10^3). However, while the RM case can yield a positive revenue for the operator, the social cost is increased by $(1.846 - 0.517) \times 10^4$, and the total user cost is increased by $(2.020 - 0.346) \times 10^4$ as against the SO case. This is mainly due to that there are less shared parking users and more curbside parking users in the RM case than the SO case, and there are more cruising for parking. It is also noteworthy that in the SO case, the parking owners or sharers as a whole are also better off when compared with the RM case ($2.934 > 0.181$). The above results indicate that parking sharing should be regulated to achieve higher social efficiency. However, subsidies are needed to sustain the SO.

Besides, if we compare the RM and SO cases with the OUE case without parking sharing, we see that both the total social cost and the total user cost are reduced through parking sharing. A Win-Win-Win situation is achieved for RM when compared with OUE, and a Win-Win-Lose situation is achieved for SO when compared with OUE. These results are consistent with the analysis in Section 3.

Table 2: Comparison of three cases: RM, SO, and OUE (monetary unit: AUD)

Cases	(p, r)	$R_p(10^3)$	$R_s(10^3)$	$TSC(10^4)$	$TC(10^4)$
RM	$(5.05, 0.95)$	1.562	0.181	1.846	2.020
SO	$(0.86, 4.13)$	-4.661	2.934	0.517	0.346
OUE	$(0, 0)$	0.000	0.000	3.821	3.821

Note: R_p: parking operator's net revenue in Eq. (6); R_s: net benefit of parking sharers in Eq. (5); TSC: total social cost in Eq. (7); TC: total user cost in Eq. (3) (these four metrics are also evaluated in Table 3 and Table 4.)

4.2 Inconvenience cost distribution

In Section 4.1, the inconvenience cost distribution is assumed as follows: $F(\delta) = 1 - e^{-0.1\delta}$ where $\delta \geq 0$ (the benchmark case: RM and SO). To examine the effect of inconvenience cost distribution, we consider two additional cases, i.e., $F(\delta) = 1 - e^{-0.05\delta}$ and $F(\delta) = 1 - e^{-0.2\delta}$, and compare the results. Table 3 summarizes the pricing strategies, the operator's net revenue, the net benefit of parking sharers, the total social cost, and the total user cost under RM and SO for the two cases (the same metrics as those in Table 2).

Table 3: Comparison of RM and SO under different distributions of inconvenience cost (monetary unit: AUD)

Cases	(p, r)	$R_p(10^3)$	$R_s(10^3)$	$TSC(10^4)$	$TC(10^4)$
RM-1 ($F(\delta) = 1 - e^{-0.05\delta}$)	$(5.89, 1.43)$	1.293	0.207	2.207	2.357
SO-1 ($F(\delta) = 1 - e^{-0.05\delta}$)	$(1.27, 6.68)$	-6.452	3.983	0.756	0.509
RM-2 ($F(\delta) = 1 - e^{-0.2\delta}$)	$(4.53, 0.56)$	1.766	0.125	1.623	1.812
SO-2 ($F(\delta) = 1 - e^{-0.2\delta}$)	$(0.78, 2.18)$	-2.078	1.618	0.358	0.312

When compared with the benchmark case with $F(\delta) = 1 - e^{-0.1\delta}$ associated with RM and SO in Table 2, $F(\delta) = 1 - e^{-0.05\delta}$ indicates that more parking owners have a larger inconvenience cost, and is associated with RM-1 and SO-1 in Table 3, and $F(\delta) = 1 - e^{-0.2\delta}$ indicates that more parking owners have a smaller inconvenience cost, and is associated with RM-2 and SO-2 in Table 3.

We have the following main observations from the results in Table 2 and Table 3. First, the difference between revenue maximization and system optimum remains with alternative inconvenience cost distributions.

Second, in terms of both p and r, RM-1 $>$ RM $>$ RM-2 and SO-1 $>$ SO $>$ SO-2. This means that when more parking owners have a smaller inconvenience cost, a revenue-maximizing or social-cost-minimizing operator should set a smaller shared parking rent as the same rent attracts more supply, and a smaller shared parking price in order to attract more demand. The combined effects of reduced price and rent when more parking owners have a smaller inconvenience cost yield either larger net revenue or smaller social cost, i.e., in terms of platform revenue, RM-1 $<$ RM $<$ RM-2, and in terms of total social cost, SO-1 $>$ SO $>$ SO-2. In short, a revenue-maximizing operator can take advantage of the profitability from the low-cost shared parking while a social-cost-minimizing operator saves inconvenience cost associated with shared parking. Furthermore, in terms of total social cost, RM-1 $>$ RM $>$ RM-2; and in terms of operator's net revenue, SO-1 $<$ SO $<$ SO-2. This means that when more parking owners have a smaller inconvenience cost, while the objective is to maximize revenue, the total social cost is smaller; and while the objective is to minimize total social cost, the net revenue is larger (however, it is still negative).

Third, in terms of the total net benefit of the parking owners, RM-1 $>$ RM $>$ RM-2 and SO-1 $>$ SO $>$ SO-2. This means that while more parking owners have a smaller inconvenience cost, due to a reduced rent (as discussed in the above) for both revenue maximization and system optimum, the net benefit of parking owners decreases, i.e., the reduction in rent cannot be recovered by the reduction in inconvenience cost.

Fourth, in terms of the price-to-rent ratio, i.e., p/r, RM-1 $<$ RM $<$ RM-2 and SO-1 $<$ SO $<$ SO-2. This means that when more parking owners have a smaller inconvenience cost, while optimal prices and rents decrease (for both revenue maximization and system optimum), the relative magnitude of optimal price against the optimal rent increases. This reflects a better return on unit rent paid when more parking owners have a smaller inconvenience cost.

Fifth, as discussed in the above, when more parking owners have a smaller inconvenience cost, the revenue-maximizing or social-cost-minimizing shared parking price is smaller. This further results in a lower user cost as users pay less, i.e., in terms of total user cost, RM-1 $>$ RM $>$ RM-2 and SO-1 $>$ SO $>$ SO-2.

4.3 Price-constrained revenue maximization

We now further discuss a revenue-maximizing platform operator's while the shared parking price is regulated by the transport authority through setting an upper bound for the price, p_u.

Table 4 summarizes the operator's net revenue, the net benefit of the parking sharers, the total social cost, and the total user cost under four different price bounds, 4.00, 3.34, 2.00 and 0.86, respectively (the same metrics as those in Table 2). A price bound p_u of 3.34 yields the minimum total social cost under a revenue-maximizing operator, and 0.86 equals the optimal price at system optimum. The other value is randomly selected for comparison. For illustration purpose, the revenue maximization and system optimum without pricing regulation in Table 2 are included in Table 4.

We have several observations from Table 4. First, the revenue-maximizing shared parking price under pricing regulation is equal to the pricing bound p_u, i.e., when the bound is less than the optimal price under no regulation, the revenue-maximizing operator sets the price to the allowed maximum.

Second, an appropriate pricing regulation is able to help reduce total social cost, e.g., $TSC = 1.178 \times 10^4$ under $p \leq 3.34$ is much less than 1.846×10^4 under no regulation.

However, a lower pricing bound does not necessarily reduce total social cost, i.e., in terms of total social cost, RM ($p \leq 0.86$) > RM > RM ($p \leq 2.00$) > RM ($p \leq 4.00$) > RM ($p \leq 3.34$). This is because, a too small bound will drive the revenue-maximizing operator to set a very low rent and thus does not take full advantage of the shared parking in order to reduce parking cruising and make parking easier. Indeed, when p_u is reduced from a larger value to the critical value3.34, a smaller pricing bound helps reduce social cost; and if we further reduce p_u from 3.34, a lower pricing bound will increase the total social cost and total user cost.

Third, the observation on total social cost is consistent with that on the optimal rent (as well as the sharers' total net benefit) where SO > RM ($p \leq 3.34$) > RM ($p \leq 4.00$) > RM ($p \leq 2.00$) > RM > RM ($p \leq 0.86$).

Fourth, pricing regulation may also help reduce total user cost, i.e., in terms of total user cost, RM ($p \leq 3.34$) < RM ($p \leq 4.00$) < RM ($p \leq 2.00$) < RM. However, a too low pricing bound may indeed increase total user cost since shared parking is not well utilized to make parking easier, i.e., in terms of total user cost, RM ($p \leq 0.86$) > RM.

Fifth, while pricing regulation might help reduce total social cost, the effectiveness of the pricing regulation is limited to a certain extent where an efficiency gap exists, i.e., $TSC = 1.178 \times 10^4$ under RM ($p \leq 3.34$) and 0.517×10^4 under SO.

Table 4: RM under pricing regulation (monetary unit: AUD)

Cases	(p, r)	$R_p(10^3)$	$R_s(10^3)$	$TSC(10^4)$	$TC(10^4)$
RM	$(5.05, 0.95)$	1.562	0.181	1.846	2.020
SO	$(0.86, 4.13)$	-4.661	2.934	0.517	0.346
RM ($p \leq 4.00$)	$(4.00, 1.32)$	1.387	0.343	1.427	1.600
RM ($p \leq 3.34$)	$(3.34, 1.61)$	1.081	0.503	1.178	1.336
RM ($p \leq 2.00$)	$(2.00, 0.98)$	0.400	0.192	1.807	1.866
RM ($p \leq 0.86$)	$(0.86, 0.43)$	0.076	0.038	2.732	2.744

5 Managerial insights

This section discusses the main results in this paper and their implications. In summary, this study examines the parking sharing problem in an urban area with mixed land use, where the parking sharing platform operator can temporarily purchase a certain number of parking spaces from parking owners and rent them to parking users.

It is found that a private revenue-maximizing operator should set a shared parking price equal to the curbside parking cost such that the two parking options (shared parking and curbside parking) are indifferent to users. Under-pricing does not take full advantage of the profitability from the shared spaces and over-pricing will push parking users away (Proposition 3.1 and Lemma A.2). We also found that the revenue-maximizing operator should set the rent such that the rented shared parking spaces from owners are fully utilized by users. Over-supply will simply waste rent from the operator's point of view (Proposition 3.1 and Lemma A.1). For social cost minimization, over-supply is not beneficial either, since it simply increases the inconvenience cost of parking owners while has no positive effect.

We further establish a condition $\delta_l < c_f(n)$, i.e., the minimum inconvenience cost is smaller than the curbside parking cost when all users choose curbside parking, which ensures that a revenue-maximizing operator will yield a Win-Win-Win situation, i.e., total user cost is strictly reduced, total net benefit of parking owners is strictly positive, and the operator's net revenue is strictly positive (Proposition 3.2). This result highlights that the profitability of introducing parking sharing relies on the existence of potential sharers, the inconvenience cost of whom is smaller than curbside parking cost when there is no shared parking.

For a public social-cost-minimizing operator, we establish a condition $\delta_l < c_f(n) + n\frac{dc_f(n)}{dn}$ that yields a Win-Win-Lose situation, i.e., the total user cost is strictly reduced, total net benefit of parking owners is strictly positive, while the operator's net revenue must be negative (Proposition 3.5). Moreover, the total social cost is also strictly reduced against the case without parking sharing. This result indicates that the possibility of parking sharing to reduce social cost relies on the existence of parking owners whose inconvenience cost is smaller than the marginal curbside parking cost when there is no shared parking. It is noteworthy that $\delta_l < c_f(n) + n\frac{dc_f(n)}{dn}$ is more relaxed than $\delta_l < c_f(n)$. This implies that (i) a public social-cost-minimizing operator is more likely to be incentivized to introduce parking sharing than a private revenue-maximizing operator; (ii) introducing parking sharing is more likely to help reduce social cost than producing profit for the platform operator.

Moreover, for a public social-cost-minimizing operator, if introducing parking sharing can help reduce total social cost, at the system optimal parking flow pattern, the marginal cost of an additional cruising driver for curbside parking is equal to the inconvenience cost of an additional parking owner who supplies a shared space (Proposition 3.4). This result is unique for the two-sided market problem, where the social cost includes those of parking users and owners (i.e., the demand and supply sides in the two-sided market), the system optimum should balances the marginal costs from the two sides. We also found that, under a social-cost-minimizing operator, a more costly curbside parking (i.e., a larger α or a larger $t(n_f)$ given the same n_f) will yield a smaller n_f^{so} and a larger n_s^{so}, and more parking owners with a smaller inconvenience cost will yield a smaller n_f^{so} and a larger n_s^{so}.

We also examine how the inconvenience cost distribution of parking owners and curbside parking cost (cruising time function and value of time) may affect the profitability and the total social cost after introducing parking sharing. When more parking owners have smaller inconvenience due to parking sharing, profitability of the platform and the capability to reduce social cost are both better. When curbside parking is less costly (smaller cruising time under the same flow or smaller value of time), the shared parking is less profitable since the parking alternative is more competitive. However, less costly curbside parking is socially preferable.

The implementation of system optimum (social cost minimization) with either subsidy or pricing bound is discussed. For a social-cost-minimizing platform operator which is directly controlled by the transport authority, it can set the system optimal pricing strategy, i.e., (p^{so}, r^{so}), but then requires a subsidy of at least $n_f^{so}\left[\alpha n_f^{so} t'(n_f^{so})\right]$ to maintain break-even. For a private profit-driven operator without any regulations, a subsidy of $R_p\left(p^{rm}, r^{rm}\right) + n_f^{so}\left[\alpha n_f^{so} t'(n_f^{so})\right]$ is required to persuade the operator to set the system optimal pricing strategy. For a private profit-driven operator under shared parking pricing regulation (from local authorities), we numerically show in Section 4.3 that an appropriate shared parking price bound can help reduce total social cost while the operator may still obtain a positive revenue. However, an over-small pricing bound may indeed not only reduce the platform operator's net revenue, but also increase the total social cost and total user cost. This is because, the over-small pricing bound drives the private profit-driven operator to reduce its rent and thus parking supply significantly, resulting in inefficient utilization of existing parking facilities.

We also numerically examine the impact of the inconvenience cost distribution in Section 4.2 (the numerical findings do not exclude other inconvenience cost distribution dependent possibilities). We found that, when more parking owners have a smaller inconvenience cost, a revenue-maximizing or social-cost-minimizing operator should set a smaller shared parking rent as the same rent attracts more supply, and should also set a smaller shared parking price in order to attract more demand. The combined effects of reduced price and rent when more parking owners have a smaller inconvenience cost yield either a larger net revenue or a smaller social cost. However, even if more parking owners have a smaller inconvenience cost, the impact of the reduced optimal rent is more significant than the smaller inconvenience cost, and thus total net benefit of the parking owners becomes smaller. Moreover, while the optimal prices and rents decrease (for both revenue maximization and system optimum) when more parking owners have a smaller inconvenience cost, the price-to-rent ratio, i.e., p/r, will increase, which reflects a better return on unit rent paid to parking owners.

6 Conclusion

This paper studies the optimal pricing strategies of a parking sharing platform to either maximize its net revenue or minimize the total social cost, considering the reactions of parking users and private parking owners to the pricing strategy. In an urban area with both shared parking and curbside parking, the rent offered by the platform determines the number of private parking space owners who would like to rent their space out for sharing, and the platform's charge as well as the level of competition between alternative parking options determines the parking demand for both types of parking. A parking choice equilibrium is proposed to determine the number of shared parking users under any given pricing strategy of the platform.

We analytically identified how the parking choice equilibrium of users will vary with the shared parking fee and the shared parking capacity. We further established the parking sharing platform operator's pricing strategies under revenue maximization and social cost minimization and compared different efficiency metrics.

While the introduction of parking sharing platform holds the potential to yield positive outcomes for the platform operator and the society, it relies on several conditions. First, there are such urban areas with mixed land use, where some residents' parking spaces are near the workplace of others. Second, some of these residents have a relatively small inconvenience and safety concern about sharing their spaces. Third, the additional car traffic induced by the additional parking supply from the sharing platform and its potential negative externality should be managed (elastic demand is not considered in this paper). Otherwise, the road may be much more congested due to more parking options, and users are worse off on the road.

This study improves and enhances our understanding of the parking sharing, pricing, and management. Beyond this, the study also further enriches the literature on the two-sided markets, where ride sharing/sourcing has attracted much more attention (Wang et al., 2016; Zha et al., 2016; Chen et al., 2017). Given the fast growth of the sharing economy worldwide, this paper delivers insightful information to both parking business stakeholders and policy makers.

This study can be further extended in several directions. Firstly, this study adopts an aggregate model for an urban area with mixed land use to examine the parking sharing problem and the spatial dimension of parking is ignored. In future studies, a general road and parking network with spatial heterogeneity such as those in Boyles et al. (2015) or a number of parking clusters distributed over space can be considered. Secondly, this study does not model other travel alternatives for travelers. Future research may examine the parking sharing problem in the context of a multi-modal system, especially when the public transit service is responsive to roadway conditions and should be optimized simultaneously (Zhang et al., 2014, 2016). Thirdly, the current study assumes a dominating parking sharing platform operator. This can be extended to the cases with multiple operators where competition exists among different operators. Fourthly, the current study adopts steady-state or static parking flow analysis, i.e., the time dimension of parking is not considered. A future study can further explore the parking sharing and parking pricing problem in a time-dependent context such as those in Zhang et al. (2008), and might also incorporate micro-level traveler behaviors, e.g., a traveler looking for vacant curbside spaces might change his or her mind and shift to shared parking during the parking search process.

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A Appendices

A.1 Derivations of optimality conditions in Section 3.1

We can write down the following Lagrangian function for the minimization problem in Eq. (10)

$$L_z\left(n_f, n_s, \lambda, u\right) = \int_0^{n_f} c_f\left(w\right) dw + \int_0^{n_s} c_s dw - \lambda\left(m_s - n_s\right) + u\left(n - n_s - n_f\right) \tag{23}$$

where u and λ denote the Lagrange multipliers associated with the flow conservation constraint in Eq. (11a) and the shared parking capacity constraint in Eq. (11b), respectively.

The optimality conditions can be derived as follows:

$$\frac{\partial L_z}{\partial n_f} \geq 0, n_f \geq 0, \frac{\partial L_z}{\partial n_f} n_f = 0; \tag{24a}$$

$$\frac{\partial L_z}{\partial n_s} \geq 0, n_s \geq 0, \frac{\partial L_z}{\partial n_s} n_s = 0; \tag{24b}$$

$$\frac{\partial L_z}{\partial \lambda} \geq 0, \lambda \geq 0, \frac{\partial L_z}{\partial \lambda} \lambda = 0; \tag{24c}$$

$$\frac{\partial L_z}{\partial u} = 0. \tag{24d}$$

or in detailed form as follows:

$$c_f - u \geq 0, n_f \geq 0, (c_f - u) n_f = 0; \tag{25a}$$

$$c_s + \lambda - u \geq 0, n_s \geq 0, (c_s + \lambda - u) n_s = 0; \tag{25b}$$

$$m_s - n_s \geq 0, \lambda \geq 0, (m_s - n_s) \lambda = 0; \tag{25c}$$

$$n - n_s - n_f = 0. \tag{25d}$$

which are identical to the optimality conditions in Eq. (12).

A.2 Proof of Proposition 3.1

Proof. We start with the case when $M_s \geq n$. It suffices to show that for any pricing strategy (p, r) not satisfying the conditions: $r = F^{-1}\left(\frac{n - t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right)$ and $p \in [\alpha t(0), \alpha t(n)]$, we can find a pair (p, r) satisfying these conditions that yields a larger net revenue or at least the same revenue.

Firstly, we consider (p, r) in Region I in Figure 3. As shown Figure 3(a), $p \leq \alpha t(0)$ and $r \geq F^{-1}\left(\frac{n}{M_s}\right)$. The net revenue $np - rF(r) M_s$ is then less than or equal to $n\alpha t(0) - F^{-1}\left(\frac{n}{M_s}\right) F\left(F^{-1}\left(\frac{n}{M_s}\right)\right) M_s$, where the equality holds only when $p = \alpha t(0)$ and $r = F^{-1}\left(\frac{n}{M_s}\right)$. This means that for Region I the revenue is maximized at the boundary with $p = \alpha t(0)$ and $r = F^{-1}\left(\frac{n}{M_s}\right)$, which satisfies the conditions given in Proposition 3.1(i).

Secondly, we consider (p, r) in Region II, where for given r, we have $p \leq \alpha t\left(n - F(r) M_s\right)$, or equivalently $r \leq F^{-1}\left(\frac{n - t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right)$. The net revenue is $pF(r)M_s - rF(r)M_s$, which is less than or equal to $\alpha t\left(n - F(r)M_s\right) F(r)M_s - rF(r)M_s$, where the equality hols only when $p = \alpha t\left(n - F(r)M_s\right)$. This means that for any (p, r) in Region II, we can improve the revenue by setting $p = \alpha t\left(n - F(r)M_s\right)$, which satisfies the conditions given in Proposition 3.1(i).

We now consider (p, r) in Region III, where for given p, we have $r \geq F^{-1}\left(\frac{n-t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right)$. The revenue is $p \cdot \left(n - t^{-1}\left(\frac{p}{\alpha}\right)\right) - rF(r)M_s$, which is less than or equal to $p \cdot \left(n - t^{-1}\left(\frac{p}{\alpha}\right)\right) - F^{-1}\left(\frac{n-t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right) F\left(F^{-1}\left(\frac{n-t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right)\right) M_s$. This means that for any (p, r) in Region III, we can improve the revenue by setting $r = F^{-1}\left(\frac{n-t^{-1}\left(\frac{p}{\alpha}\right)}{M_s}\right)$, which satisfies the conditions given in Proposition 3.1(i).

Lastly, we consider (p, r) in Region IV, where for any r, we have $p \geq \alpha t(n)$. The revenue is equal to $p \cdot 0 - rF(r)M_s$, which is less than or equal to $p \cdot 0 - 0 \cdot F(0)M_s$, where the equality only holds when $r = 0$. This means that there is no need to consider the whole Region IV when examining optimal pricing strategy, where the maximum revenue is zero, which is achieved by letting $r = 0$ and $p = \alpha t(n)$. The point of $(r = 0,\ p = \alpha t(n))$ satisfies the conditions given in Proposition 3.1(i).

For the case with $M_s < n$, the proof is similar to that for the case with $M_s \geq n$, which is omitted here. This completes the proof. □

A.3 Further Implications of Proposition 3.1

Remark A.1. *At the parking choice equilibrium under the revenue-maximizing pricing strategy (p^{rm}, r^{rm}), we should have $n_s = m_s$, i.e., all shared parking spaces will be used.*

Remark A.1 says that the shared parking spaces repurchased by the platform should be fully utilized under a profit-maximizing sharing platform operator. This can be readily derived from Proposition 3.1. Proposition 3.1 indicates that any (p, r) in the interior of Region I, Region III, and Region IV will be non-optimal in terms of maximizing revenue. Therefore, $n_s = F(r)M_s = m_s$ when $r > \delta_l$; and $n_s = 0 = m_s$ when $r = \delta_l$ (as discussed earlier we do not consider $r < \delta_l$), i.e., we should always have $n_s = m_s$.

We further explain Remark A.1 as follows. Firstly, if $m_s = 0$, then we must have $n_s = m_s = 0$. For $m_s > 0$, if $n_s < m_s$ under (p^{rm}, r^{rm}), we can reduce the rent by a small amount Δr where $[F(r^{rm}) - F(r^{rm} - \Delta r)] \cdot M_s \leq m_s - n_s$. Doing so, the operator will save a rent in the amount of $r^{rm} \cdot m_s - (r^{rm} - \Delta r) \cdot (m_s - [F(r^{rm}) - F(r^{rm} - \Delta r)] \cdot M_s)$ (reduced rent per shared parking space, and reduced number of rented shared parking). However, as $m_s - [F(r^{rm}) - F(r^{rm} - \Delta r)] \cdot M_s \geq n_s$ and parking price p^{rm} does not change, the operator will receive the same parking fees from the travelers, i.e., $n_s \cdot p^{rm}$. Therefore, the original r^{rm} must not be optimal.

Remark A.2. *The revenue-maximizing pricing strategy (p^{rm}, r^{rm}) satisfies $p^{rm} = \alpha t\left(n_f\right)$, i.e., the parking fee is equal to the cruising time cost of curbside parking users.*

Remark A.2 says that a revenue-maximizing sharing platform operator would set the price of parking spaces to the level thus travelers will be indifferent between the two parking options: shared and free curbside parking. This can be readily derived from Proposition 3.1. Based on Proposition 3.1, we have $r^{rm} = F^{-1}\left(\frac{n-t^{-1}\left(\frac{p^{rm}}{\alpha}\right)}{M_s}\right)$, which is equivalent to $p^{rm} = \alpha t\left(n - F\left(r^{rm}\right)M_s\right)$, where $F\left(r^{rm}\right)M_s = m_s = n_s$ following Remark A.1. Therefore, $p^{rm} = \alpha t\left(n - n_s\right) = \alpha t\left(n_f\right)$.

Remark A.2 is further explained as follows. Firstly $p^{rm} > \alpha t\left(n_f\right)$ cannot occur if $n_f > 0$. This is because if $p^{rm} > \alpha t\left(n_f\right)$ then $c_s > c_f$, and no users will choose shared parking spaces, i.e., $n_s = 0$, and the operator will earn zero amount parking fees while it has to pay rents to the parking owners. However, by reducing p^{rm} until $n_s > 0$, the operator can earn a positive amount. Secondly, if $p^{rm} < \alpha t\left(n_f\right)$, the platform operator can increase the price to $\alpha t\left(n_f\right)$ where no users will shift his or her choice, but additional profit of $\left[\alpha \cdot t\left(n_f\right) - p^{rm}\right] \cdot n_s$ can be gained.

A.4 Proof of Proposition 3.2

Proof. We first prove Proposition 3.2(i). The first derivative of R_p in Eq. (14) with respect to r is

$$\frac{dR_p}{dr} = f(r)M_s\left[\alpha t - r\right] - F(r)M_s\left[\alpha t' f(r)M_s + 1\right] \tag{26}$$

where $t = t\left(n - F\left(r\right)M_s\right)$ and $t' = t'\left(n - F\left(r\right)M_s\right)$.

We examine the first-order derivative of R_p with respect to r around $r = \delta_l$. Based on Eq. (26), we have

$$\left.\frac{dR_p}{dr}\right|_{r=\delta_l} = M_s f(0)\alpha t\left(n\right) > 0 \tag{27}$$

Since $\frac{dR_p}{dr}$ is continuous over r, Eq. (27) means that there must exist a sufficiently small ε such that for $r \in [\delta_l, \delta_l + \varepsilon]$, we have $\frac{dR_p}{dr} > 0$. Therefore, we can always set $r = \delta_l + \varepsilon$ (then $p = \alpha t\left(n - F\left(\delta_l + \varepsilon\right)M_s\right)$) such that $R_p(p, r) > 0$. This implies that $r^{rm} > \delta_l$, and $R_p(p^{rm}, r^{rm}) \geq R_p(\alpha t\left(n - F\left(\delta_l + \varepsilon\right)M_s\right), \delta_l + \varepsilon) > 0$.

Since $f(\delta) > 0$ over $[\delta_l, +\infty)$, $r^{rm} > \delta_l$ leads to a supply $m_s^{rm} > 0$. Furthermore, $0 < R_p(p^{rm}, r^{rm}) = p^{rm}n_s - r^{rm}m_s \leq p^{rm}m_s - r^{rm}m_s^{rm}$ and thus $p^{rm} > r^{rm}$. This completes the proof for Proposition 3.2(i).

We now prove Proposition 3.2(ii). We first look at R_p. Based on Proposition 3.2(i), we have $r^{rm} > \delta_l$ and $R_p(p^{rm}, r^{rm}) > 0 = R_p^0$. Also note that $p^{rm} = \alpha t\left(n - F\left(r^{rm}\right)M_s\right)$. We further look at TC. Given $r^{rm} > \delta_l$, we have $n_f^{rm} = n - F\left(r^{rm}\right)M_s < n$ and $n_s^{rm} = F\left(r^{rm}\right)M_s = m_s^{rm} > 0$. Also, under revenue maximization, $p^{rm} = \alpha t\left(n_f^{rm}\right)$. The total cost of users is $TC = p^{rm}n_s^{rm} + \alpha t\left(n_f^{rm}\right)n_f^{rm} = \alpha t\left(n - F\left(r^{rm}\right)M_s\right)n$. It can be readily verified that $TC < TC^0$ since $n - F\left(r^{rm}\right)M_s < n$. We finally look at R_s. Since $r^{rm} > \delta_l$, $R_s = r^{rm}\int_{\delta_l}^{r^{rm}} f\left(\delta\right)M_s d\delta > 0$, i.e., $R_s > R_s^0$. This completes the proof for Proposition 3.2(ii).

We now further prove Proposition 3.2(iii). For the case with $M_s < n$, it is obvious that to achieve $m_s = M_s$, we should let $r \to \infty$, and

$$\left.\frac{dR_p}{dr}\right|_{r=\infty} = M_s\left[f(\infty)\left(\alpha t\left(n - M_s\right) - \infty\right) - \left(\alpha t'\left(n - M_s\right)f(\infty)M_s + 1\right)\right] < 0 \tag{28}$$

indicating $r \to \infty$ is non-optimal, and we should have $r^{rm} < +\infty$ and thus $m_s^{rm} < M_s$.

For the case with $M_s \geq n$, when $r = F^{-1}\left(\frac{n}{M_s}\right)$, we have

$$\left.\frac{dR_p}{dr}\right|_{r=F^{-1}\left(\frac{n}{M_s}\right)} = M_s\left[f\left(F^{-1}\left(\frac{n}{M_s}\right)\right)\left(\alpha t\left(0\right) - F^{-1}\left(\frac{n}{M_s}\right)\right) - \frac{n}{M_s}\left(\alpha t'\left(0\right)f\left(F^{-1}\left(\frac{n}{M_s}\right)\right)M_s + 1\right)\right] \tag{29}$$

When $t(0)$ is sufficiently small, i.e., $t(0) < t'(0)n + \frac{F^{-1}\left(\frac{n}{M_s}\right) + \frac{n}{M_s}\frac{1}{f\left(F^{-1}\left(\frac{n}{M_s}\right)\right)}}{\alpha}$, one can verify that $\frac{dR_p}{dr} < 0$. Therefore, it is non-optimal to set $r \geq F^{-1}\left(\frac{n}{M_s}\right)$. It follows that to maximize revenue we should have $r^{rm} < F^{-1}\left(\frac{n}{M_s}\right)$ and thus $m_s^{rm} < n \leq M_s$. This completes the proof for Proposition 3.2(ii). □

A.5 The price-to-rent ratio under revenue maximization

Suppose the inconvenience of parking sharers is uniformly distributed over $[\delta_l, \delta_u]$ and $\delta_l < \delta_u$. We now briefly explain the result in Eq. (16). In the revenue maximization regime, based on previous analysis in Section 3, we have the following results. Firstly, $p = \alpha t\left(n_f\right)$ and thus $n_f = t^{-1}\left(\frac{p}{\alpha}\right)$ and $n_s = n - t^{-1}\left(\frac{p}{\alpha}\right)$. Secondly, given parking rent r, the number of sharers is equal to $m_s = r\frac{M_s}{\delta_u - \delta_l}$ based on the uniform distribution for δ. Thirdly, to maximize net revenue, we should have $n_s = m_s$. Therefore, $n - t^{-1}\left(\frac{p}{\alpha}\right) = r\frac{M_s}{\delta_u - \delta_l}$, and thus $r = [n - t^{-1}\left(\frac{p}{\alpha}\right)] \cdot \frac{\delta_u - \delta_l}{M_s}$, or alternatively, $p = \alpha t\left(n - r\frac{M_s}{\delta_u - \delta_l}\right)$. The net revenue of the platform operator under a uniform distribution for δ can then be written as follows:

$$R_p = r\frac{M_s}{\delta_u - \delta_l} \cdot \left[\alpha t\left(n - r\frac{M_s}{\delta_u - \delta_l}\right) - r\right]. \tag{30}$$

The first derivative of Eq. (30) with respect to r is

$$\frac{dR_p}{dr} = \frac{M_s}{\delta_u - \delta_l} \cdot (\alpha t - r) + r\frac{M_s}{\delta_u - \delta_l} \cdot \left(\alpha t' \cdot \left(-\frac{M_s}{\delta_u - \delta_l}\right) - 1\right), \tag{31}$$

where $t = t\left(n - r\frac{M_s}{\delta_u - \delta_l}\right)$ and $t' = \frac{dt\left(n - r\frac{M_s}{\delta_u - \delta_l}\right)}{d\left(n - r\frac{M_s}{\delta_u - \delta_l}\right)}$. At the equilibrium, a marginal change in r will lead to a marginal change in the corresponding shared parking price $p = \alpha t\left(n - r\frac{M_s}{\delta_u - \delta_l}\right)$ and in the number of shared parking users $n_s = m_s = r\frac{M_s}{\delta_u - \delta_l}$.

A necessary condition for an interior optimal solution for r is $\frac{dR_p}{dr} = 0$, and we immediately have

$$r^{rm} = \frac{\alpha t}{\alpha t' \frac{M_s}{\delta_u - \delta_l} + 2}, \tag{32}$$

Note that in Eq. (32) t and t' both depend on r^{rm}, and $p^{rm} = \alpha t$. Therefore, we have $\frac{p^{rm}}{r^{rm}} = \alpha t' \frac{M_s}{\delta_u - \delta_l} + 2 > 2$.

A.6 Proof of Proposition 3.3

Proof. For Proposition 3.3(i), it suffices to show that for a given pair (p_1, r_1) and an inconvenience distribution of F_1 with a net revenue of $R_{p,1}$, we can always find a pair of (p_2, r_2) thus the net revenue $R_{p,2}$ under an inconvenience distribution of F_2 will be no less than $R_{p,1}$.

Under F_1, the total net revenue is $R_{p,1} = p_1 n_s - r_1 m_s$, where $m_s = F_1(r_1)M_s$. We can define r_2 by solving the equation $F_1(r_1)M_s = F_2(r_2)M_s = m_s$. Since $F_1(\delta) \leq F_2(\delta)$, we have $r_1 \geq r_2$. We can set $p_2 = p_1$. It follows that the shared parking supply m_s and the total number of shared parking users at the parking choice equilibrium n_s will remain the same, where the total net revenue is $R_{p,2} = p_2 n_s - r_2 m_s$. The difference between the two revenues is $R_{p,2} - R_{p,1} = (r_1 - r_2)\, m_s \geq 0$.

The proof for Proposition 3.3(ii) is similar to that for Proposition 3.3(i), which is omitted.

For Proposition 3.3(iii), it suffices to show that for a given pair (p_1, r_1) and a cruising time function of t_1 with a net revenue of $R_{p,1}$, we can always find a pair of (p_2, r_2) thus the net revenue $R_{p,2}$ under a cruising time function of t_2 will be no less than $R_{p,1}$.

Under t_1, the total net revenue is $R_{p,1} = p_1 n_s - r_1 m_s$, where $n_s \leq m_s = F(r_1)M_s$ and $p_1 \leq \alpha t_1(n - n_s)$. Under t_2, we can set $r_2 = r_1$ thus m_s remain identical. Moreover, we can set $p_2 = \alpha t_2(n - n_s) \geq \alpha t_1(n - n_s) \geq p_1$ where n_s remain unchanged at the parking choice equilibrium. Thus, $R_{p,2} = p_2 n_s - r_2 m_s$ and $R_{p,2} - R_{p,1} = (p_2 - p_1)m_s \geq 0$.

The proof for Proposition 3.3(iv) is similar to that for Proposition 3.3(iii), which is omitted. □

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A.7 Proof of Proposition 3.5

Proof. Based on Proposition 3.4(i), if $\delta_l < \alpha\left(t(n) + nt'(n)\right)$, we have $n_s^{so} = m_s > 0$. It follows that $r^{so} = F^{-1}\left(\frac{n_s^{so}}{M_s}\right) > F^{-1}(0) = \delta_l$. Moreover, to support the system optimum as an equilibrium solution, $p^{so} \leq \alpha t(n_f^{so})$. Based on Proposition 3.4(ii), we have $\alpha\left(t(n_f^{so}) + n_f^{so}t'(n_f^{so})\right) = F^{-1}\left(\frac{n_s^{so}}{M_s}\right)$. Thus, $p^{so} + \alpha n_f^{so}t'(n_f^{so}) \leq r^{so}$. It follows that $R_p\left(p^{so}, r^{so}\right) = n_s^{so}\left(p^{so} - r^{so}\right) \leq -n_s^{so}\left[\alpha n_f^{so}t'(n_f^{so})\right] < 0$. This completes the proof for Proposition 3.5(i).

The proof for Proposition 3.5(ii) on system optimum is similar to that for Proposition 3.2(ii) on revenue maximization, which is omitted.

We now verifies Proposition 3.5(iii) by contradiction. Suppose Proposition 3.5(iii) is not true and $n_s^{so} \leq n_s^{rm}$ and $n_f^{so} \geq n_f^{rm}$. It follows that $m_s^{so} \leq m_s^{rm}$, $r^{so} \leq r^{rm}$, and $F^{-1}\left(\frac{n_s^{so}}{M_s}\right) \leq F^{-1}\left(\frac{n_s^{rm}}{M_s}\right)$.

For an interior system optimum solution, based on Proposition 3.4(ii), we have $\alpha\left(t(n_f^{so}) + n_f^{so}t'(n_f^{so})\right) = F^{-1}\left(\frac{n_s^{so}}{M_s}\right)$. For an interior revenue maximization solution, based on Proposition 3.2(i), we have $F^{-1}\left(\frac{n_s^{rm}}{M_s}\right) = r^{rm} < p^{rm} = \alpha t(n_f^{rm})$. $F^{-1}\left(\frac{n_s^{so}}{M_s}\right) \leq F^{-1}\left(\frac{n_s^{rm}}{M_s}\right)$ then leads to $\alpha\left(t(n_f^{so}) + n_f^{so}t'(n_f^{so})\right) < \alpha t(n_f^{rm})$, and thus $n_f^{so} < n_f^{rm}$ since $t(\cdot)$ is increasing and convex. This contradicts with $n_s^{so} \leq n_s^{rm}$ and $n_f^{so} \geq n_f^{rm}$. Therefore, we should have $n_s^{so} > n_s^{rm}$ and $n_f^{so} < n_f^{rm}$. It follows that $r^{so} > r^{rm}$ and $p^{so} \leq \alpha t(n_f^{so}) < \alpha t(n_f^{rm}) = p^{rm}$. This completes the proof. □

A.8 Proof of Proposition 3.6

Proof. For Proposition 3.6(i), it suffices to show that for a given pair (p_1, r_1) and an inconvenience distribution of F_1 with a total social cost of TSC_1, we can always find a pair of (p_2, r_2) thus the net revenue TSC_2 under an inconvenience distribution of F_2 will be no greater than TSC_1.

Under F_1, the total social cost is $TSC_1 = n_f\alpha t\left(n_f\right) + \int_{\delta_l}^{r_1}\delta f(\delta)M_s d\delta$, where $m_s = F_1(r_1)M_s$. We can define r_2 by solving the equation $F_1(r_1)M_s = F_2(r_2)M_s = m_s$. Since $F_1\left(\delta\right) \leq F_2\left(\delta\right)$, we have $r_1 \geq r_2$. We can set $p_2 = p_1$. It follows that the shared parking supply m_s and the total number of curbside parking users at the parking choice equilibrium n_f will remain the same, where the total social cost is $TSC_2 = n_f\alpha t\left(n_f\right) + \int_{\delta_l}^{r_2}\delta f(\delta)M_s d\delta$. The difference between the two total social costs is $TSC_2 - TSC_1 = -\int_{r_2}^{r_1}\delta f(\delta)M_s d\delta \leq 0$ since $r_1 \geq r_2$.

The proof for Proposition 3.6(ii) is similar to that for Proposition 3.6(i), which is omitted.

For Proposition 3.6(iii), it suffices to show that for a given pair (p_1, r_1) and a cruising time function of t_1 with a total social cost of TSC_1, we can always find a pair of (p_2, r_2) thus the total social cost T_2 under a cruising time function of t_2 will be no greater than TSC_1.

Under t_1, the total social cost is $TSC_1 = n_f \alpha t_1(n_f) + \int_{\delta_l}^{r_1} \delta f(\delta) M_s d\delta$, where $n_s \leq m_s = F(r_1)M_s$ and $p_1 \leq \alpha t_1(n_f)$. Under t_2, we can set $r_2 = r_1$ thus m_s remain identical. Moreover, we can set $p_2 \leq \alpha t_2(n_f)$, and n_f (as well as n_s) remain unchanged. Thus, $TSC_2 = n_f \alpha t_2(n_f) + \int_{\delta_l}^{r_2} \delta f(\delta) M_s d\delta$ and $TSC_2 - TSC_1 = n_f \alpha (t_2(n_f) - t_1(n_f)) \leq 0$ since $t_2(n_f) \leq t_1(n_f)$.

The proof for Proposition 3.6(iv) is similar to that for Proposition 3.6(iii), which is omitted.

□

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