AUGMENTED BALANCE POINT DIAGRAMS FOR MATCHING SITE AND CONCRETE SUPPLY RESOURCES

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ABSTRACT

Matching site concrete placing resources with the appropriate number of concrete delivery truckmixers depends upon good site and concrete plant coordination, if good concreting productivity is to be achieved. In general practice, however, the placing crew is usually idle for some of the pour time waiting for deliveries and at other times, truckmixers are idle on site waiting to be emptied. In the case of concrete supplied by a circulating fleet of truckmixers, a "balance point" process, two new diagram models were developed relating fleet size to the parameters, placing crew idle time, truckmixer idle time, truckmixer unloading time, round trip time, and concrete placing production rate. The new models augment classical balance point theory. To illustrate practical application, (i) the diagrams were developed and used to reveal system behaviour insights for the case of three circulating truckmixers and (ii) the relevance of the new model to a real pour of forty-six deliveries was examined, in relation to the balancing of site and plant resources for better coordination and system productivity.

KEYWORDS

Balance point theory; concrete delivery scheduling; supply and demand resources coordination; site productivity optimisation; concreting system behaviour.

INTRODUCTION

Study background

For a particular day, a construction site manager estimates the amount of concrete required on site and typically places an order with a concrete supplier on the preceding day. The concrete supplier may allocate a set of truckmixers, *N*, to the site, each of which will circulate between site and concrete plant until the necessary number of deliveries has been made. If *N* is too large (i.e., above the optimum *N* value, see below), truckmixers will sometimes queue, idle, on site. If *N* is too small, the concrete placing crew will sometimes be idle waiting for the next delivery to arrive. Thus, the productivity of the site concreting process as a whole (taking into account placing plant and crew site resources as well as truckmixer vehicles) is governed by the number of truckmixers in the circulating group as it relates to the values of particular pour parameters.

Balance point theory, developed by Halpin and Woodhead (1976), is already available to help determine the optimum number of units cycling around in support of a potentially continuous central construction process. Importantly, they identified and named the "balance point" class of construction processes and derived associated theory. The placing of concrete on site in formwork is one of these processes, as long as the site is served by a set of truckmixers each of which circulates between plant and site until the required number of deliveries has been made. The fundamental aim is to "balance" the production rate of the central process with the optimum number of supporting "server" units.

In Halpin and Woodhead (1976), an earthmoving example illustrates balance point theory, whereby earth is loaded into a scraper by a pusher. When the scraper is fully loaded, it moves off to empty its spoil before cycling back to rejoin the pusher for refilling. The size of the scraper fleet needed to support continuous pusher operation depends on (a) the duration of a scraper emptying and return round trip, and (b) the time taken for the pusher to fill up a scraper and be ready for the next scraper. Only if, by coincidence, duration (a) is an integer multiple of duration (b), can there be continuous employment of both the pusher and all scrapers. In general, a well-chosen fleet will either be one too few or one too many. If too few, the pusher will sometimes be idle waiting for a scraper to arrive and no scraper is ever idle. If too many, the pusher will never be idle, and central production will not suffer, but all scrapers will sometimes be idle waiting to be loaded. The theory assumes constant activity durations.

This classical analysis is focussed on the productivity of the central production process, not on the productivity also of the cycling server units and does not directly relate to the coordination required, therefore, when the two types of resources are managed by different companies, as is the case with ready-mixed concreting. Anson et al. (2002), in fact, benchmarked the very considerable degree of seemingly poor coordination existing in Hong Kong between the needs of the concrete placing crew and the actual supply of the concrete. The study described here is concerned with modelling coordination performance.

In Bernold and AbouRizk (2010), a different balance point process example is given to illustrate balance point theory, whereby aggregates being quarried are delivered by a circulating fleet of trucks to a concrete plant consuming those aggregates.

Classic balance point theory in the context of a concrete delivery system

Fig. 1, after Fig. 6.4 of Halpin and Woodhead (1976), is drawn for the case of a crew on site capable of unloading 3 truckmixers per hour (i.e., unloading time, *UL*, equals 20 minutes), being served by a set of *N* circulating trucks, each with *RT* durations of 45 minutes, where *RT* is the round trip time, leave site to return site, including the refilling with fresh concrete at the plant. Thus, one truck is capable of delivering 60/(45+20)=0.923 truckloads per hour. At the balance point, as seen in Fig. 1, N=3.25, so the concrete supplier must choose either *N*=3 or *N*=4, but notably, and usefully, not any other number. With *N*=3, the crew works below capacity at 2.769 loads per hour. With *N*=4, the crew works at 3 loads per hour capacity but trucks have to wait to be unloaded. Such a "lack of fit" type of productivity loss is a property of the system. It is not to be confused with the inefficiencies arising from the fact that *RT* and *UL* are essentially stochastic variables, not constant as assumed in balance point theory. If, referring to Fig. 1, *RT* had instead been 40 minutes, this time an integer multiple of *UL*, the dotted line of Fig. 1 would apply, intersecting the crew unloading capacity line at the perfect balance point where *N*=3 and neither crew nor truckmixers are ever idle. Nonetheless, the classic balance point theory does not relate system productivity with supply and demand resources coordination performance.

[Insert Fig. 1 here].

The purpose of this study

As a result of an investigation by Anson et al. (2019), into the underlying reasons for the apparently poor levels of coordination existing between ready-mixed concrete suppliers and placing crews on site in Hong Kong, the research question was raised as to whether it would be possible to model the site concreting process, for sites served by a fleet of circulating truckmixers, as an augmented balance point process. The augmentation to provide a measure of the coordination achieved.

Study of this question led to two new balance point diagrams which complement the classical balance point theory. The new diagrams, it was discovered, also provide additional insights into system behaviour and do indeed augment the theory in providing productivities of the circulating units also. The classical theory, as stated above, concentrates on productivity of the central process only. Of course, in the case of site concreting, non-productive truckmixer time is clearly of material concern to the concrete plant manager, who usually has several other sites requiring deliveries that day.

The methodology of the balance point diagrams derivation process was essentially one of discovery. The revelation of the diagrams arose from much scrutiny of concrete delivery and placing time flow charts based on the concrete delivery records collected in 1998 (Anson and Wang 1998; Anson et al. 2002), their expression in mathematical terms relating resource idle time parameters to pour characteristics parameters inspired by balance point theory and simulation of concrete delivery schedules to examine the effects on system productivity of changing the system parameters. The crystallisation of the new diagrams was a gradual process of increasing comprehension as the research studies progressed.

This paper is primarily focused on introducing these new balance point diagrams, their augmenting of classical balance point theory with coordination metrics and their practical significance and application. These diagrams relate the size of the fleet to the same classical theory parameters of truckmixer unloading time, round trip time, and production rate of concrete placing. Apart from also revealing circulating server unit productivities, the diagrams provide additional insights into the nature and performance trends of this routine, every day, world-wide, construction industry process as affected by changes in the values of the system parameters. A subsidiary purpose of the paper is to draw the attention of researchers and practitioners studying concreting productivities and those studying the truckmixer dispatch scheduling problem, to the relevance of site and plant coordination measures.

Immediately below, a research literature review is given of productivity benchmarking, simulation, and optimisation studies relating to the site concreting and delivery process. Researchers involved in the concreting process, usually fit within one of those three categories. The review illustrates the wider research context within which this study sits and represents a contribution relevant to all those interested in the difficult concreting scheduling process. The newly derived balance point diagrams relating RT, UL, N, P, $\%TM$, and $\%W$, the core of this paper, are then presented.

LITERATURE REVIEW RELATED TO CONCRETE DELIVERY AND PLACING STUDIES

Particularly within the past thirty years, the productivity of site concreting has been much studied. Samples of the various studies made, for reader convenience purposes are placed under the three headings of (i) productivity benchmarking of a concrete delivery system, (ii) process simulation for characterising a concrete delivery system, and (iii) optimisation of concrete delivery schedules.

Productivity benchmarking of a concrete delivery system

Benchmarking quantifies productivities actually being achieved for reference and targeting purposes. Benchmarking has proved useful, amongst other things, for quantifying the coordination being achieved between site and concrete plant managements. Anson and Cooke (1988) benchmarked concreting productivities in a mainly rural area of the UK. Anson et al. (1996) and Anson and Wang (1998) benchmarked the productivities of different concrete placing methods on Hong Kong buildings, obtaining $\%W$ and $\%TM$ pairs for each pour by direct observation. They suggested that "good matching" of concrete supply with site placing resources might be arbitrarily defined as $100<\%TM<150$ and $0<\%W<10$. Anson et al. (2002) found that 83% of pours failed to achieve that arbitrary measure of good matching. Lu and Anson (2004) utilised quality control records to benchmark the production rate of the different concrete placing methods in terms of m^3/hour and m^3/truckmixer-hour. Aziz (2017) formulated stepwise regression models to benchmark system performance using the metric actual work units/hour divided by expected work units/hour related to concrete batching plant, travel and site variables.

Nevertheless, although researchers have used different metrics to quantify system productivity, the metric ($\%W$, $\%TM$) of Anson and Wang (1998) seems the only one directly measuring resource coordination. It must be stated, however, that although benchmarking metrics can be used to compare performances, such comparisons are of little direct help in understanding the reasons behind the differences. Benchmarking of itself, therefore, provides little guidance on the measures to take which might improve coordination.

Process simulation for characterising a concrete delivery system

Simulation mimics a concrete delivery system by modelling the workflows of individual truckmixers and site placing crews, allowing for uncertainties in such as journey times and concrete placing times. Given a simulation event list, queuing time and resource utilisation rates statistics can be modelled. Dawood (1995) and Misir et al. (2011) focused on the development of heuristic rules for truckmixer allocations to improve concrete plant utilisation. Using simulation models, "what-if" scenarios can be explored, by varying the values of system parameters and analysing the consequent performance. To minimise project cost, Smith (1998, 1999) and Dunlop and Smith (2002) determined optimum truckmixer site inter-arrival times and Zayad and Halpin (2001) studied the maximisation of plant system productivity given values for various parameters including number of truckmixers, pumping and conveyor space and hopper loads. Sawhney et al. (1999) determined the optimum number of truckmixers needed. Also developed, have been simulation platforms, such as HKCONSIM (Lu et al., 2004), RMCDiSO (Feng et al., 2004), and RMCSIM (Tang et al., 2005), to help simulate concrete delivery schedules.

Parameters critical to system productivity can be identified via simulation study. For example, Wang et al. (2001), as others above, discovered that truckmixer inter-arrival times affect efficiency significantly. Lu et al. (2004) and Tang et al. (2005) discovered that the number of truckmixers involved relates to %*W* and %*TM*. Park et al. (2011) found that durations of loading, unloading, positioning on site, and slump tests are critical factors correlated with system productivity. Although some of these researchers discovered that the number of truckmixers deployed, relates to pour efficiency, no research study has made the link also with *RT* and *UL* in the case of circulating fleet delivery.

Thus, to date, the overarching fundamental system relationships connecting *P*, *N*, %*TM*, %*W*, *RT*, and *UL*, in the case of a circulating delivery fleet, seem not to have been uncovered. It might prove helpful for those simulating at more detail to relate their variables to these broader system variables. Duration of loading at plant, for example, is a component of *RT*. Truckmixer positioning onto the unloading point on a congested site can be seen as a component of *UL*.

Optimisation of concrete delivery schedules

Optimisation is used to generate optimal solutions in terms of some chosen criterion, given either a simulation model base augmented by one of those optimisation algorithms which have evolved, such as genetic algorithms (Naso et al., 2004, 2007; Maghrebi et al., 2014), particle swarm algorithms (Lu and Lam, 2005) and bee colony optimisation (Srichandum and Rujirayanyong, 2010), or else a mathematical model (Yan and Lai, 2007; Yan et al., 2008; Asbach et al., 2009; Hertz et al., 2012; Liu et al., 2014) that sufficiently represents a concrete delivery system. Most researchers have focused on providing optimum truckmixer schedules. For example, Naso et al. (2004, 2007), Yan and Lai (2007), Yan et al. (2008), and Liu et al. (2014) produced minimum delivery cost schedules by varying the number of truckmixers, and Albayrak and Albayrak (2016) by changing the concrete volumes supplied by the multiple plants involved. Lu and Lam (2005) produced schedules minimising truckmixer idle time and concreting crew idle time based on appropriate truckmixer inter-arrival times on site. Hertz et al. (2012) formulated delivery routes to minimise truckmixer travelling time.

These studies add to understanding of what is essentially a process possessing competing multi-criteria performance objectives in practice. The site coordination criterion, the (%*W*, %*TM*) pair, central to this paper, also recognised by Lu and Lam (2005), above, is proposed as a criterion of relevance to all modellers. No specific pour is likely to be efficient, whatever the performance criteria measured, unless there is good coordination between plant and site. Efficiency, here, is seen in the context of the system as a whole, not just the efficient use of the crew only or the utilisation of the truckmixers only.

NEWLY DERIVED BALANCE POINT DIAGRAMS

Key parameters of a concrete delivery system

The pour characteristic parameters consist of (i) the size of the circulating fleet of truckmixers, (ii) the time it takes the site placing crew to unload and empty a truckmixer, and (iii) the truckmixer site-plant round trip journey time. The resulting performance parameters consist of (iv) the time that the crew is idle waiting for concrete to arrive on site, (v) the time that truckmixers are kept on site, idle, waiting to be emptied and (vi) the production rate achieved, expressed as the number of truckmixers emptied per hour. As with Fig. 1, the classical balance point theory diagram, the pour parameters are assumed fixed for any particular pour.

RT (round trip time leave site to return site), and *UL* (unloading time of a truckmixer on site), usefully, are independent variables in that the site controls *UL*, but has no influence on *RT*. The plant location and other factors control *RT* but have no influence on *UL*. Relating to classical balance point theory, *RT* and *UL* relate respectively to the production capabilities of the circulating transit units and of the central production process.

Unless *RT* is an integer multiple of *UL*, there is a "lack of fit" in the ability to match resources in that: either, the pour will be continuous but the truckmixers will sometimes be idle on site waiting to be unloaded, or, the concrete placing crew will sometime be idle, waiting for the next concrete delivery to arrive, but no truckmixer will wait to be unloaded. Perfect matching of resources only occurs, or "perfect balance", if RT/UL happens to be an integer and the truckmixer fleet size is also then correctly chosen as $1+RT/UL$ (see below). Apart from the fixed *RT* and *UL*, the process also assumes that a truckmixer is refilled as soon as it arrives back at plant and that truckmixer emptying (*UL*) starts as soon as the delivery arrives on site unless the crew is, at that moment, still unloading the previous delivery.

For the purpose of analysis below, some definitions, in addition to *RT* and *UL* have been adopted after those provided by Anson and Wang (1998). TM is an abbreviation for "truckmixer".

- "Pour duration" equals "time finish unloading final TM delivery minus time start unloading first delivery".
- "TM waiting on site" associated with any single delivery equals "time start TM unloading minus time of TM arrival on site" (such waiting includes any quality control check and TM maneuvering on to the unloading point, not only time strictly idle. Essentially, it is time spent under the control of the site, before actual unloading).
- "Total time on site" for that single TM equals "waiting time on site plus unloading time" (usually after unloading, a truckmixer will washout before travelling to the site exit. These activities are seen as components of *RT*, as site control of the truckmixer effectively ceases as soon as unloading is completed).
- "Waiting by the placing crew" equals "time start unloading of a TM minus time finish unloading of the previous TM". The crew waiting time for the whole pour is the sum of such individual waits.

Resource utilisation measures, normalised to pour duration are:

- %*TM*=100×[total time spent on site by all TMs/pour duration]
  Thus, if %*TM*=100, there is no queuing and truckmixers are fully utilised.
- %*W*=100×[total of crew waiting times/pour duration]
  Thus, if %*W*=0, the placing crew is never idle.

The new models are formally stated below. To re-iterate, they illustrate graphically the effects on the %*TM* and %*W* resource utilisation measures and the concrete placing production rate, *P*, of the pour parameters *N*, *RT* and *UL*.

The new balance point diagrams relating *RT*, *UL*, *N*, *P*, %*TM* and %*W*

As stated, the literature review revealed no research related to the theoretical relationships connecting *P*, *N*, %*TM*, %*W*, *RT*, and *UL* and only one study which explicitly recognised site and plant coordination as a desirable objective. As such, the authors propose the two balance point diagrams, Figs. 2 and 3, which complement that of the classical theory. They help, too, in understanding the nature of the system, valuable to those actively working on software to improve system productivity. The term "balance point", reflects the nature of the problem type, which is to strike an appropriate balance between the provision of production and server resources.

Fig. 2 is a general balance point diagram relating *RT*, *UL*, *N*, and *P* from which %*W* and %*TM* can be directly deduced. Any particular pour has its own unique *RT* and *UL* values, related to the distance between plant and site, the shape and size of the pour, the site placing plant being used (e.g., mobile pump, crane and skip) and the degree of site congestion. The diagram is constructed on *RT*, *UL* axes therefore. For any given pour, a point on the diagram represented by the pour parameter coordinates (*RT*, *UL*) is known as the pour "operating point" (Although Fig. 2 relates to site concreting, it would be applicable to any balance point process. *RT* and *UL* have parallel equivalents whatever comprises the circulating fleet of servers and whatever the central production operation).

[Insert Fig. 2 here].

For all operating points on the bold cranked operating line, concrete is delivered at a constant rate, *P* (See Proof 1). The crank point, C, where *RT* is an integer multiple of *UL* and $RT/UL=N-1$, represents the perfect balance point, where %*W*=0 and %*TM*=100 for *N* circulating truckmixers. Balance point analysis tells us that operating points on the horizontal part of the crank, where *RT* is less than at C, gives rise to continuous production with %*W*=0, but truckmixers have to sometimes queue. Operating points on the sloping part of the crank, where *RT* is greater than at C, give rise to intermittent delays to concrete placing in the forms and to the immediate emptying of truckmixers as they arrive on site.

<u>Proof 1</u>

RT values lying along EC are clearly all associated with the constant *P* of $60/UL_C$. Along the slope CD, where $RT>RT_C$, trucks are unloaded immediately on arrival. Thus, for operating point A, each truck makes a delivery every (RT_A+UL_A) minutes. For N circulating trucks, therefore, there is a delivery every $[(RT_A+UL_A)/N]$ minutes and the production at A, P_A, is $[60N/(RT_A+UL_A)]$ truckloads per hour. For P_A to equal P, $[60/UL_C=60N/(RT_A+UL_A)]$ or $[UL_A=60N/P–RT_A]$. This is the equation of the sloping line shown on Fig. 2, applying to any general operating point A on CD.

The values for %W and %TM are derived directly from Fig. 2 as follows: For operating points, A, on the crank slope, the ratio K=CA/CD, expressed as a percentage, gives %W directly (**see Proof 2**). It follows that %TM=100–%W since no truckmixers wait to be unloaded under this scenario.

Proof 2

If the pour receives D deliveries, the pour time applying to point A is $(D\times UL_C)$, since productivity is constant along the sloping line, on which C is also located. The slower arrivals of trucks at A, causing the crew to wait, is compensated by the faster UL at A. The crew waiting time CW_A plus the unloading time UL_A must equal UL_C for equal production rates at A and C. Thus, $[D\times(CW_A+UL_A)=D\times UL_C]$ or $[D\times CW_A=D\times(UL_C–UL_A)]$. But pour time is $(D\times UL_C)$, so %W, which is [100×total CW_A/pour time] becomes $[100\times(UL_C–UL_A)/UL_C]$. This is the ratio K, or CA/CD, from similar triangles.

For all operating points, B, on the horizontal part of the crank, %TM is given directly by (N–EB/EO) expressed as a percentage (see **Proof 3**). %W=0 under this scenario.

Proof 3

Because of the early arrival at B, rather than the ideal arrival at C, $(RT_C–RT_B$=BC) represents the waiting time for each truckmixer. Each TM is on site therefore for (BC+UL) minutes, which makes TM, as a percentage of pour time, equal to [100×(BC+UL)/UL]. But [BC=(N–1)×UL–EB], so %TM is [100×(N–EB/EO)], since UL is represented by the length, EO. Note, this is also expressed, equally usefully, as $(N–RT_B/UL_B)$, for any general point B along EC.

Also to be noted is that all operating points along OC are points of perfect balance, since RT/UL is always the integer (N–1), but in relation to a different value of P in each case. The further the operating point from O, the lower the rate of concrete placing, P.

Similarly, all points on OA give rise to identical (%W, %TM) pairs. The same applies to all points along OB. All lines similar to OA, rotated clockwise from OC, are associated with delays to pours, with %W>0. All lines rotated anti-clockwise from OC are associated with truckmixers waiting idle on site but no delays to placing in the forms, so %W=0.

A diagram of the same form as Fig. 2 could be constructed for any balance point process.

Fig. 3, the second balance point diagram, covers a range of N values all on the one diagram and provides a direct read-off of the values %W and %TM on the left hand axis for a pour with a given N and a given ratio RT/UL. Note that Fig. 3 combines the variables RT and UL into the one parameter RT/UL. In fact, resource wastage behaviour is dependent on only the two parameters RT/UL and N (see Proof 4). Unlike Fig. 2, however, Fig. 3 provides no direct information on the production rate P. The equations underlying these curves are derived in **Proof 4**.

[Insert Fig. 3 here].

The region of curved lines applies to truckmixer under-supply conditions where the site has to wait for deliveries. The upper region of the chart applies to truckmixer over-supply conditions where truckmixers have to wait to be unloaded. For any given values of RT/UL and N, $\%W$ and $\%TM$ are simply read off from the left hand axis. The points of discontinuity, where the straight lines join the curved lines, represent points of perfect balance, where crew and truckmixer resources are perfectly matched. These points, as they should, coincide with integer values of RT/UL. Fig. 3, by inspection, makes clear from the slopes of the lines, the degree of sensitivity which exists to changes in the value RT/UL, of the degree of over-provision or under-provision of truckmixer time on site as represented by the $\%W$ and $\%TM$ values. The slopes of the lines and curves of Fig. 3 also provide the important insight that, on the whole, truckmixers are more adversely affected by small changes in RT/UL than are placing crews. For the sake of completeness, the sensitivity of $\%W$ and $\%TM$ is also given by the derivatives of $\%W$ and $\%TM$ with respect to (RT/UL), using the equations of the curves of Fig. 3. In the case of truckmixer oversupply, that equation is $\%TM/100=N–RT/UL$ and the derivative of $\%TM$, therefore, is -100. Thus $\%TM$ increases by 10% if RT/UL decreases by 0.1. In the case of undersupply, the governing equation is $\%W/100=1–N/(1+RT/UL)$, and the $\%W$ derivative is $N/(1+RT/UL)^2$. Thus, $\%W$ increases by $N/(1+RT/UL)^2$ for a unit increase in RT/UL. To further illustrate the degree of inherent sensitivity of the system, for a pour where RT=50 minutes and UL=25 minutes, say, the ideal number of circulating truckmixers would be 3 (i.e., $1+RT/UL$). Referring to Fig. 3, the N=3 curve and the RT/UL=2 ordinate do indeed intersect at the perfect balance point where $\%W$=0 and $\%TM$=100. If RT/UL were to be a little less at 1.8, however, (e.g., UL is only 3 minutes slower at about 28 minutes, or RT is 5 minutes faster at 45 minutes), $\%TM$ is seen to be about 120, meaning that truckmixers queue on site for 20% of the pour duration, rather than the 0% if RT/UL had remained at 2.0. If RT/UL were to be a little greater at 2.2, (UL is 3 minutes faster at just over 22 minutes, or RT is only 5 minutes slower at 55 minutes), $\%W$ is seen to be 6%, meaning that the placing crew becomes idle for 6% of pour time.

<u>Proof 4</u>

The equations underlying these curves were derived from delivery truckmixer time flow diagrams and are given as below.

- For the perfect balance ($N=1+RT/UL$) scenario, $\%W=0$ and $\%TM=100$.
- For TM over-supply ($N>1+RT/UL$) scenario, $\%W=0$ and $\%TM=100\times(N–RT/UL)$.
- For TM under-supply ($N<1+RT/UL$) scenario, $\%W=100\times[1–N/(1+RT/UL)]$ and $\%TM=100–\%W$.

The N deliveries of the 1st set are assumed dispatched at intervals of UL to achieve perfect matching for those first N deliveries, a sensible strategy for a plant manager.

i. The case of $N<1+RT/UL$ or $RT>(N–1)\times UL$:

Considering the 2nd set, delivery $N+1$ will arrive on site later than the finish of the N^{th} unloading by the amount of time $RT–(N–1)\times UL$. Delivery $N+2$ arrives exactly as delivery $N+1$ finishes unloading. Subsequent deliveries follow the same pattern until the delivery $2N$ (the last of the 2nd set of deliveries). The same pattern exists for the 3rd set of deliveries. To calculate the steady state $\%W$ and $\%TM$ for the N deliveries of all sets, it is only necessary to consider one set. The concrete placing crew waits only once for the time $RT-(N-1)\times UL$ and the pour time, therefore, is $\{N\times UL+[RT-(N-1)\times UL]\}$. Thus, $\%W=100\times\{\{[RT-(N-1)\times UL]\}/\{N\times UL+[RT-(N-1)\times UL]\}$. This simplifies to $\%W=100\times\{1-[N/(1+RT/UL)]\}$, $\%TM$ equals $100-\%W$.

ii. The case of $N>1+RT/UL$ or $RT<(N-1)\times UL$:
Apart from the N deliveries of the 1st set, every individual truckmixer will wait to be unloaded for the time period $[(N-1)\times UL-RT]$. Thus, the steady state total time of every truckmixer on site is $[(N-1)\times UL-RT+UL]$ and $\%TM=100\times\{[(N-1)\times UL-RT+UL]/UL\}$. This simplifies to $\%TM=100\times(N-RT/UL)$; $\%W=0$ for over-supply conditions.

An interesting link with classical balance point theory

Halpin and Woodhead (1976) derived productivity index values for 3, 5, and 7 circulating vehicles (transit units) serving a single production plant unit. For given ratios of "central plant production rate" to "transit unit production rate" on the horizontal axis, curves for different numbers of transit units in the circulating fleet, give the productivity index being achieved, on the vertical axis. These curves are identical in principle to those given above in the under-supply section of Fig. 3 making a direct connection between the two pieces of work. The classical productivity index is equivalent to $(100-\%W)$ in terms of Fig. 3 and the classical ratio of plant/server production rates is equivalent to UL/RT rather than the RT/UL of Fig. 3. On converting for that inverse ratio effect, the two sets of curves for $N=3$, 5 and 7 are compared in Fig. 4.

[Insert Fig. 4 here].

Fig. 4 serves, in part, to validate the form of Fig. 3. Even through the two sets of curves are not identical in position, the same trends are evident. The curves are not identical because the classical curves of Halpin and Woodhead (1976) are based on random transit unit arrivals, whereas Fig. 3 is based on regular arrivals, the very opposite of random. The two sets of curves become closer as RT/UL gets larger for a given N and when the fleet size gets smaller for a given RT/UL. The greater the degree of truckmixer under-provision, the greater are the inter-arrival times on site and the patterns of random arrivals look more like regular under provision arrivals. Observations reported in Halpin and Riggs (1992) found that the random arrival assumption was not representative of reality, in general, but hardly is the constant RT of Fig. 3.

PRACTICAL USE OF THE NEW BALANCE POINT DIAGRAMS

Illustrating the use of the new balance point diagrams for practical application

Fig. 5 is the general case diagram of Fig. 2, transformed for the specific case of *N*=3. The topmost heavy cranked line, in this example, highlights the case of *P*=3 truckmixer loads placed in the forms per hour, the planned production rate, say.

[Insert Fig. 5 here].

Point E, (60/*P*), is therefore plotted at *UL*=20 minutes, since three truckmixers per hour are to be emptied. C is the perfect balance point calculated as (40, 20). *RT*/*UL* at C has the integer value of 2, which means that the 3 circulating truckmixers never have to queue and the crew never lacks for concrete. Point D, 60*N*/*P*, is at *RT*=60 minutes. The cranked line ECD is drawn accordingly.

Since all operating points on EC, correspond to unloadings of 20 minutes, a production of 3 truckmixer loads per hour, all operating points on the slope CD similarly represent a production of 3 truckmixer loads per hour.

For the operating point A, at (45, 15), %*W* is given by 100×(CA/CD) which is 25% since (CA/CD) is $[(UL_C–UL_A) /(UL_C–UL_D)]$, or (5/20). Thus, %*TM* is 75%, since %*TM*=100–%*W*.

For the operating point B at (30, 20), %*TM*, at 100×(*N*–EB/EO), is calculated as 150%. %*W* is always 0% for points on the horizontal part of the crank.

Thus for 3 truckloads per hour production and *N*=3 circulating truckmixers, the amount of time spent idle by both crew and truckmixers is readily deduced directly from the proposed diagram for any (*RT*, *UL*) combination located on the topmost cranked line ECD.

Fig. 5, however, can be of much further use as long as *N* remains at 3 truckmixers. Operating point Y, for example, lies on OA. The sloping part of the Y-relevant cranked line can be constructed to pass through Y, with a gradient of -1, to intersect OC. If the coordinates of Y are (27, 9), the intersection with OC is at (24, 12). The horizontal part of that new crank indicates an unloading time of 12 minutes. Thus, if Y were to be the operating point, the productivity *P* is rapidly deduced as 5 truckloads per hour. The value of %*W* remains at 25%, and %*TM* at 75%, since the ratio *K* is the same for all points along OA.

Likewise, X lies on OB. Because OB is anti-clockwise from OC, the cranked line is constructed by first drawing a horizontal line through X as far as line OC, the location of the X-relevant crank point. The sloping part is then constructed from that crank point with a gradient of -1. If the coordinates of X are (21, 14), the productivity *P* is deduced as 4.28 (i.e., 60/14) truckloads per hour, the value of %*TM* is 150 and %*W*=0, since %*TM* and %*W* values are identical for all operating points on OB.

Thus, an operating point can be selected anywhere on the diagram to suit the estimated (*RT*, *UL*) values of a particular pour. The cranked line is then constructed through that point, paying attention only to whether the point is below OC or above it and the values of %*W* and %*TM* and *P* can then be calculated using only geometry as immediately above, as long as *N*=3.

Fig. 5 illustrates the power of this diagram to provide insights into system behaviour. Simple inspection study of the diagram allows coordination performance trends and production rates to be assessed for all feasible operating points (*RT*, *UL*), as long as *N*=3.

Fig. 5 is applicable to any value of productivity *P*. It is restricted only to 3 circulating truckmixers. As stated, Fig. 5 derives from Fig. 2 for the specific case of *N*=3. A similar diagram, drawn for *N*=4, would be used for system study and understanding when four truckmixers are circulating.

For the sake of completeness, operating points A, B, C, X and Y in Fig. 5, are also plotted on the *N*=3 curve of Fig. 3. The %*TM* and %*W* values correspond to those deduced above from the Fig. 5 diagram. B and X are coincident on Fig. 3, as are A and Y, even though the production rate at B is different to that at X. The balance point diagrams of Fig. 2 and Fig. 5 are insightful then, also in respect of production rates but Fig. 3 is not. To embrace both coordination and production metrics across the practical range of values for *N*, five more balance point diagrams (Fig. 5 type) are needed, say, for *N*=4, 5, 6, 7 and 8. The curves of Fig. 3, however do have the advantage of covering the range of practical values for *N* on the one diagram, as long as only the coordination metrics are of principal interest.

A case study illustrating a use of the newly derived balance point diagrams

In line with one of the objectives above, this section demonstrates the potential application value of the new balance point diagrams making use of concrete delivery data for a Hong Kong housing project in 2018. This case study pour was observed as a component of a completely separate study. It is included here because (i) it neatly illustrates the model accuracy limitation when predicting resource idle times, as a result of not allowing for the stochastic nature of *RT* and *UL* and (ii) because it also illustrates the practical usefulness of the model in spite of that limitation.

A concrete superstructure of forty floors is under construction. The concrete is delivered to the site by circulating truckmixers from a remote concrete batching plant. Table 1 gives the concrete supply data for a one day pour of 368m^3, delivered in 46 truckmixer batches of 8m^3. Note that this reliable dataset was obtained from direct study of plant and site videos and purchase orders. Data analysis gives *W*=57 minutes, *TM*=857 minutes, and pour duration=617 minutes. The coordination performance measures, therefore, are %*W*=9 and %*TM*=139.

Since the order was placed only one day before delivery, the normal situation for pours in Hong Kong, there was insufficient time to plan to maximise resource coordination. The new balance point diagrams could be used, however, to improve system productivity by determining the appropriate number of circulating truckmixers for future similar pours on the site concerned. Based on Table 1, the average *RT* duration was 60 minutes, all within the range of 40 minutes to 80 minutes. The average *UL* duration was 12 minutes. Thus, given that *RT*/*UL* averages 5, an integer, a fleet of 6 truckmixers ought to produce %*W*=0 and %*TM*=100 since *N*=1+*RT*/*UL*.

Thus, a set of 6 circulating truckmixers would theoretically maximise system productivity, representing excellent site and plant coordination.

The actual performance of %*W*=9, %*TM*=139 differs from model prediction, probably mainly because of the stochastic nature of *RT* and *UL* and it is not known whether, or to what extent, a circulating fleet policy was strictly observed in practice. A study by the authors of the effect on predictions of *RT* and *UL* variability, is now under way for a wide range of pour parameter values. Regardless, the derivatives of the curves of Fig. 3 (or, simple inspection of its slopes) make clear, as discussed above, the degree of sensitivity of the coordination performance metrics to small changes in *RT*/*UL*. This sensitivity property of the system is especially marked in the case of truckmixer oversupply, i.e., when *N*>1+*RT*/*UL*.

Nevertheless, the insight and guidance given by the simple model, including that degree of sensitivity, are also relevant to those undertaking research into practical scheduling solutions, in adding to their understanding of the nature of the problem they are dealing with, and to site and plant managers in selecting the sizes of circulating fleets. By taking care to estimate the most likely average *RT* and *UL* values, as accurately as possible the value of *N* can be chosen accordingly, if to minimise crew and truckmixer down time and maximise coordination is the main objective.

[Insert Table 1 here].

CONCLUSIONS

For the case of a concrete pour on site, served by a circulating fleet of *N* truckmixers, new balance point diagrams have been successfully derived, the principle purpose of the study, which predict the rate of concrete placing in the forms, the placing crew time spent on a pour, idle, in waiting for deliveries, %*W*, and the provision of truckmixer time on site, %*TM* (the use of percentages normalises these measures to pour durations). These predictions depend only on *N* and the pour parameter *RT*/*UL*, where *RT* is the truckmixer round trip duration, finish unloading to return to site, assumed the same for every delivery, and *UL* is the assumed constant time taken to unload each truckmixer. The process is a balance point process, whereby it is impossible to attain continuous employment of all resources involved, unless the rate of production of the central process is exactly an integer multiple of the rate of production of each service vehicle. That is, *RT*/*UL* must be an integer in the concrete placing case.

The two new balance point diagrams make a contribution to balance point theory given by Halpin and Woodhead (1976). The first new diagram, Fig. 2, links *RT*, *UL* and system production rate, *P*, with lost times %*W* and %*TM*–100, for any general value of *N*. For practical use, it is demonstrated how diagrams applying to specific values of *N* are readily derived from this general diagram. The second new balance point diagram, Fig. 3, links *RT*/*UL* and a range of specific values of *N*, with %*W* and %*TM*, but contains no information on *P*. The provision of %*TM* parameter values, augments classical theory, which focuses on the values for %*W*. Only by providing both %*TM* and %*W* values, is a measure obtained of the resource coordination being achieved between the two parties involved in the process.

The new balance point diagrams are insightful in that, by their simple inspection, users are able to relate the production rate of concrete placing on site, *P*, to the pour parameters *RT* and *UL*, the size, of the circulating fleet of truckmixers *N*, and the site and plant coordination metrics: (i) times spent idle by the placing crew in waiting for deliveries (%*W*), and (ii) times spent idle by truckmixers in waiting on site to be unloaded (%*TM*). The diagrams are also of practical value therefore. A plant manager can estimate *RT* and *UL* for a given pour and derive that optimum fleet size which minimises crew and truckmixer idle times. As the diagrams augment existing balance point theory in specifically highlighting the extent of loss of productivity of truckmixers, the concrete plant manager receives information of importance to him. The second balance point diagram makes clear the degree of sensitivity of the system to small changes in *RT*/*UL*, by direct study of the curves themselves in Fig. 3, or more formally, by study of the mathematical derivatives of the curve equations, for %*W* and %*TM* with respect to the pour parameter *RT*/*UL*. The lost production of crew and service vehicles are significantly affected by quite small changes, in practical terms for this industry, of just a few minutes in the values of *RT* and *UL*. The fact of this sensitivity, a property of the system, is relevant to those researchers studying the productivity, simulation, and optimisation of concrete delivery systems, not only to construction managers.

In practice, of course, the pour parameters *RT* and *UL* are stochastic variables, not fixed values as assumed in the balance point diagrams. By adding a simulation capability to the model, so that *RT* and *UL* are recognised as the stochastic variables they actually are, it becomes possible to explore the effects of variability on the simple model predictions. Such further study is on-going with respect to the patterns of distributions and magnitudes of *RT* and *UL* values, based on the variabilities seen in practice in Hong Kong. The fixed, or averaged, *RT* and *UL* values for any pour, for use with the models presented above, do limit, of course, the accuracy of the prediction modelling of system behaviour. The on-going study will establish the effects of this limitation.

It remains the case, however, that the fundamental balance point diagrams themselves, aid in understanding the coordination performance of the site concreting process when supported by a circulating fleet of truckmixers, both on the productivities achieved by placing crews and by truckmixers and on overall concrete placing production rates for various *RT*/*UL* and *N*. Like the classical diagram, these new balance point diagrams are easy to understand and easy to apply assisting planners to select the optimum number of circulating truckmixers needed, having first carefully estimated values for both *RT* and *UL*. Indeed, in the case of successive similar pours being completed over a lengthy period on a large project, the concrete delivery records themselves, could be used to obtain good estimates for *RT* and *UL*.

Finally, the generality of the balance point diagrams means that the parameters (*RT*, *UL*) and (%*W*, %*TM*) have equivalent parallels in the case of other balance point processes involving a central production process supported by a set of circulating servers.

DATA AVAILABILITY STATEMENT

All data generated and analysed during the study are included in the published paper.

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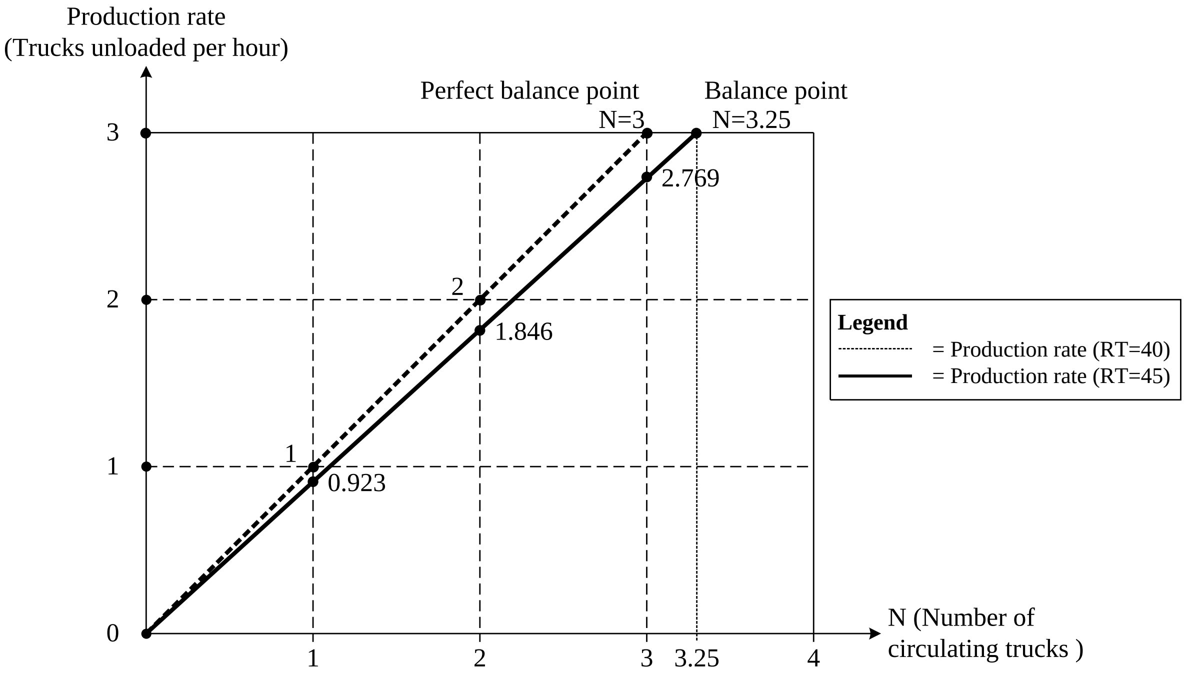

Fig. 1. Production rate by deploying 1, 2, 3, 4 circulating trucks

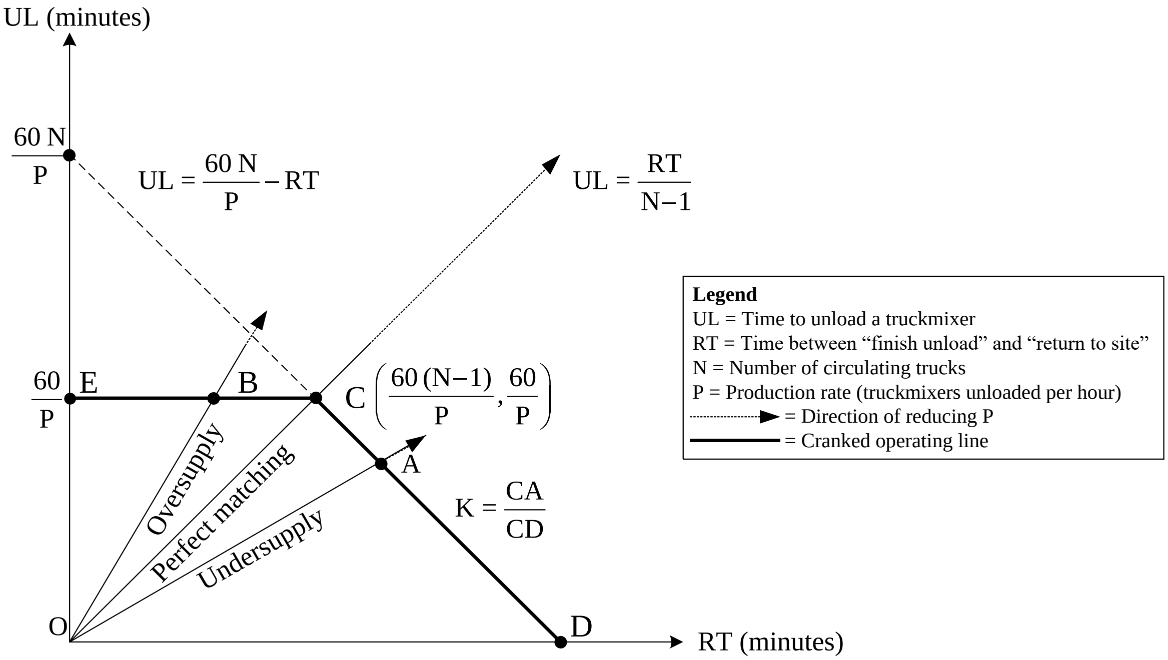

Fig. 2. Balance point diagram relating UL, RT, N, P, %W, and %TM

(Complementary to that of Halpin and Woodhead 1976)

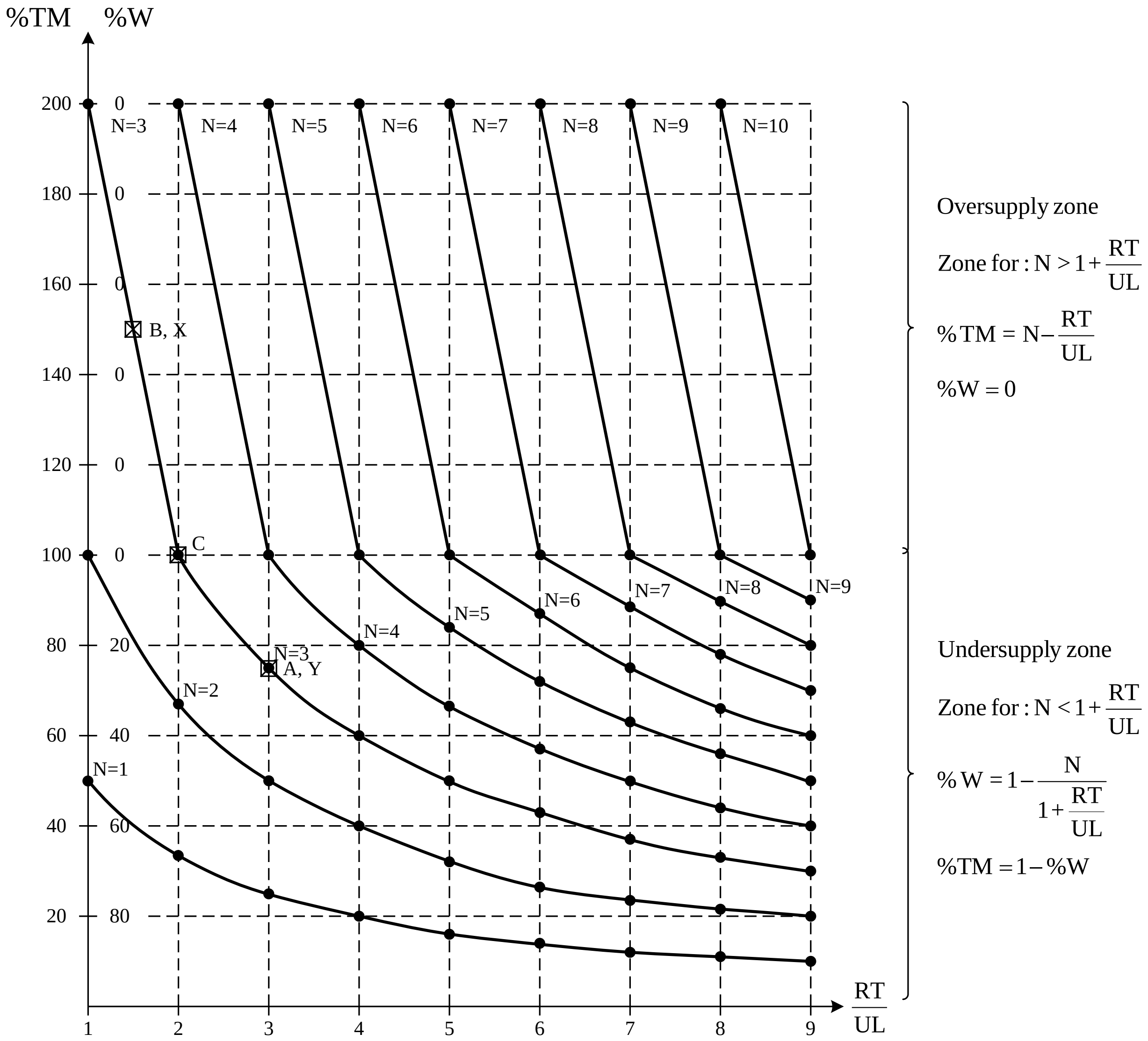

Fig. 3. Curves showing the relationships between (RT/UL), N, %W, and %TM (modified from Anson, Ying, and Siu* 2019)

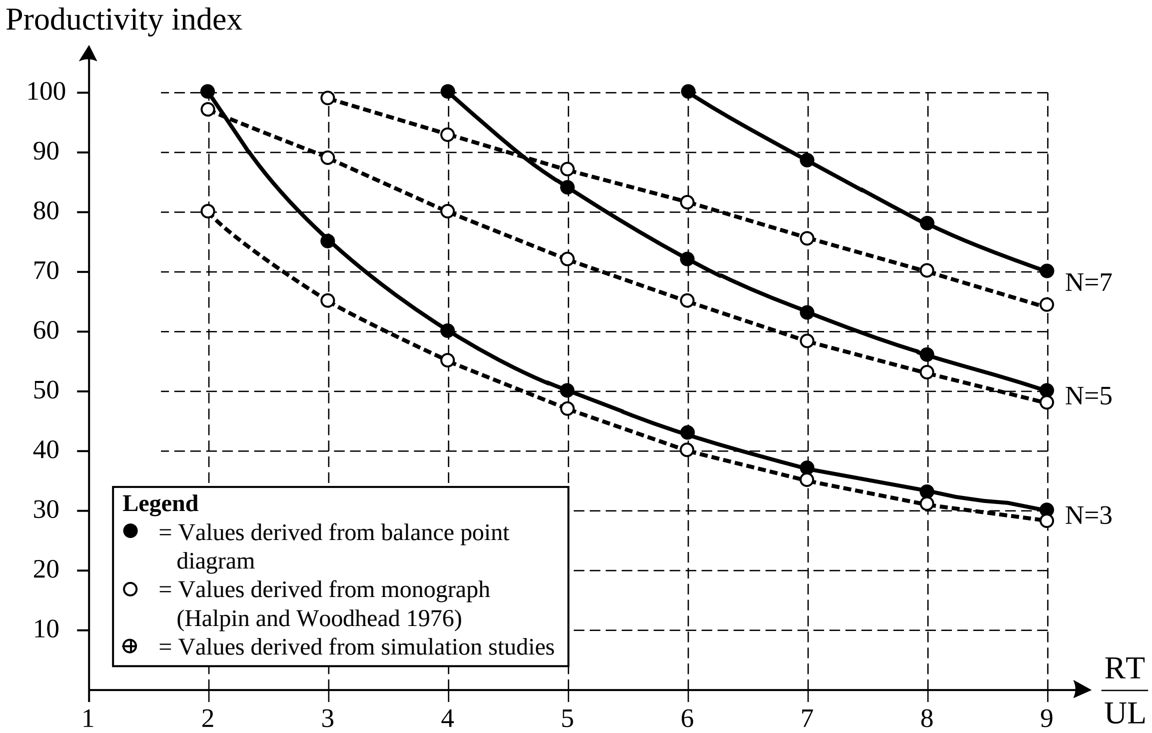

Fig. 4. Complementary balance point diagram theory compared with curves in Halpin and Woodhead (1976)

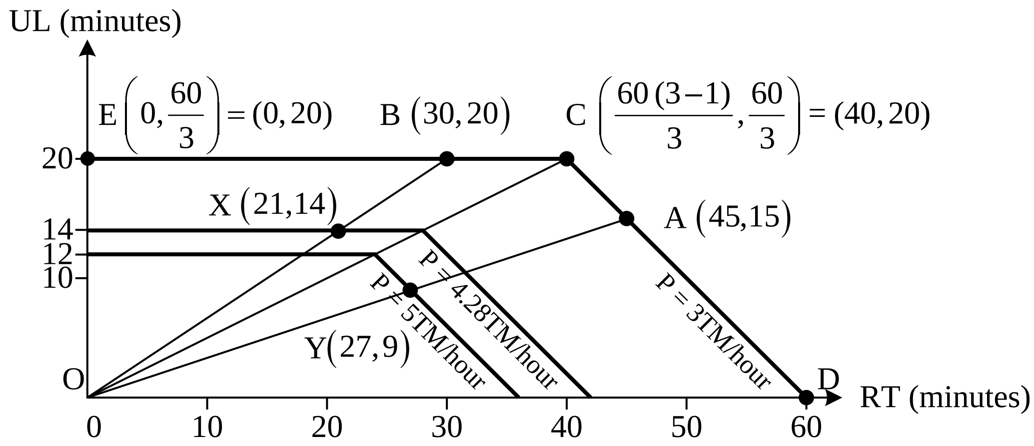

Fig. 5. Specific balance point diagram for N=3

(Cranked lines represent P=3, 4.28, and 5 TM/hour)