A General Simple Method for Calculating Consolidation Settlements of Layered Clayey Soils with Vertical Drains under Staged Loadings

by

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Abstract:

It is well known that the calculation of consolidation settlements of clayey soils shall consider creep compression in both "primary" consolidation and so-called "secondary" consolidation periods. Rigorous Hypothesis B method is a coupled method and can consider creep compression in the two periods. But this method needs to solve a set of non-linear partial differential equations with a proper Elastic Visco-Plastic (EVP) constitutive model so that this method is not easy to be used by engineers. Recently, Yin and his co-workers have proposed a simplified Hypothesis B method for single and two layers of soils. But this method cannot consider complicated loadings such as loading, unloading, and reloading. This paper proposes and verifies a general simple method with a new logarithmic function for calculating consolidation settlements of viscous clayey soils without or with vertical drains under staged loadings such as loading, unloading, and reloading. This new logarithmic function is suitable to cases of zero or very small initial effective stress. Equations of this simple method are derived for complicated loading conditions. This method is then used to calculate consolidation settlements of clayey soils in three typical cases: Case 1 is a single soil layer without vertical drains under loading only, Case 2 is a two-layered soil profile with vertical drains subjected to loading, unloading, and reloading, and Case 3 is a real case of a test embankment on seabed of four soil layers installed with vertical drains under three stages of loading. Settlements of all three cases using the new general simple methods are compared with values calculated using rigorous fully coupled finite element method (FEM) with an Elastic Visco-Plastic (EVP) constitutive model (Cases 1 and 2) and measured data for Case 3. It is found that the calculated settlements are in good agreement with values from FEM and/or measured data. It is concluded that the general simple method is suitable for calculating consolidation settlements of layered viscous clayey soils without or with vertical drains under complicated loading conditions with good accuracy and also easy to use by engineers using spread-sheet calculation.

Keywords: clayey soil, settlement, consolidation, time-dependent, creep, elastic visco-plastic

1. Introduction

In recent decades, many geotechnical structures have been constructed on clayed soil ground, especially on seabed with layered clayey soils and other soil types in many coastal cities in the world. One typical example is two artificial islands (5.10 km^2 for runway one and 5.45 km^2 for runway 2) of Kansai International Airport in Osaka, Japan. Runway one was constructed starting in December 1986 and was open in September 1994. Runway two was constructed in May 1999 and was open in August 2007. The excessive settlements have been a problematic issue (Akai and Tanaka 1999). In Hong Kong, a total area of 1100 km^2 was reclaimed on seabed since 1980's to 2003. Recently, three large artificial islands were constructed on seabed as part of Hong Kong-Zhuhai-Macao Link project. In near future, more marine reclamations will be constructed on seabed in Hong Kong waters. Excessive settlements, especially long-term settlements have been and will be a big concern. It is well known that settlements of saturated clayey soils are caused by dissipation of excessive pore water pressure in voids of soils and also by viscous deformation of soil skeleton. The stress-strain behaviour of the skeleton of clayey soils is time-dependent due to the viscous nature of the skeleton (Bjerrum 1967; Graham *et.al.* 1983; Leroueil *et.al.* 1985; Olson 1998). Methods for calculating settlements of saturated clayey soils shall consider the coupling process of dissipation of excessive pore water pressure and viscous deformation of soil skeleton.

Terzaghi (1943) first presented a theory and equations for analysis of the consolidation of soil in one-dimensional (1D) straining (oedometer condition). But this theory cannot consider viscous deformation of soil skeleton. Later, improved methods were proposed, including methods based on Hypothesis A (Ladd *et. al.* 1977; Mesri and Godlewski 1977) and other methods based on Hypothesis B (Gibson and Lo 1961; Barden 1965, 1969; Bjerrum 1967; Garlanger 1972; Leroueil *et.al.* 1985; Hinchberger and Rowe 2005; Kelln, *et al.* 2008). Hypothesis A method assumes no creep compression during the "primary" consolidation period and the creep compression occurs only in the "secondary" compression starting at t_{EOP} which is the time at End-Of-Primary consolidation. Yin and Feng (2017) and Feng and Yin (2017) pointed out that Hypothesis A method normally underestimates the total settlements due to ignoring creep compression in the "primary" consolidation period.

Hypothesis B is a coupled consolidation analysis using a proper constitutive relationship for the time-dependent stress-strain behaviour of clayey soils. Hypothesis B method needs to solve a set of two partial equations: (i) an equation derived based on mass continuity condition using Dancy's law and (i) a constitutive equation such Yin and Graham's (1994) 1D Elastic Visco-Plastic model (1D EVP) (Yin and Graham 1996). Yin and Graham (1996) used a finite difference method to solve this set of equations. The computed settlements and excessive pore water pressures were in good agreement with measured data from tests done by Berre and Iversen (1972). Yin and Graham (1996) also found that Hypothesis A method underestimated total settlements. Nash and Ryde (2000, 2001) also used Hypothesis B method adopting 1D EVP model (Yin and Graham 1994) to analyze the consolidation settlement of an embankment on soft ground with vertical drains. Their computed settlements were in good agreement with measured values.

Hypothesis B method needs to solve a set of non-linear partial different equations and a computer program is needed. This method is difficult to be used by practicing engineers without such computer program and without a good knowledge of non-linear constitutive model. To overcome this limitation, Yin and Feng (2017) and Feng and Yin (2017) proposed a de-coupled simplified Hypothesis B method for calculating settlements due to both excessive porewater pressure dissipation and also due to creep compression during and after the "primary" consolidation period. The calculated settlements are in close agreement with measured data and computed values using the fully coupled Hypothesis B method with aid of computer software. However, this simplified method is neither suitable for complicated loading such as staged unloading and reloading, nor for multiple layers of soils with vertical drains. In this paper, authors propose and verify a general simplified Hypothesis B method (also called a general simple method) for calculating consolidation settlements of layered clayey soils with or without vertical drains under staged loadings including loading, unloading, and reloading. Such loading process is commonly used in practice. In addition, a new logarithmic function which has definition at zero stress is used in this method for calculating settlements of soils at very small vertical effective stress.

2. Formulation of a General Simple Method for Calculating Consolidation Settlements of Multi-layered Soils Exhibiting Creep under Staged Loading

2.1. Formulation of a general simplified Hypothesis B method

Figure 1 shows a soil profile with *n*-layers of soils with corresponding thicknesses $(H_1, H_2, ...H_n)$ and depths $(z_1, z_2, ...z_n)$. The total thickness of this profile is H. A vertical drain with smear zone is shown in Figure 1, where $d_d = 2r_d$ is diameter of a drain equal to twice radius r_d of the drain, $d_s = 2r_s$ is diameter of a smear zone equal to twice radius r_s of the smear zone, $d_e = 2r_e$ is diameter of an equivalent unit cell equal to twice radius r_e of the cell. It is noted that vertical drains are installed all in the same triangular pattern or the same square pattern and are subjected a uniform surcharge over all vertical drains. Therefore, deformation of soils in all unit cells is approximately in vertical direction. Thus, soils in each unit cell are assumed to be in 1D straining in average. 1D straining constitutive models can be used, for example 1D EVP model (Yin and Graham 1989, 1994). If a horizonal soil profile has no vertical drains, then $d_d = d_s = 0$ and $d_e = \infty$ in Figure 1, which is also suitable for multi-layered soils without vertical drains.

Authors propose a general simplified Hypothesis B method for calculating consolidation settlement of multi-layered viscous soils with or without vertical drains under any loading condition for the soil profile under uniform surcharge *q(t)* in Figure 1. Formulation of this general simple method is presented below:

$$\begin{aligned} S_{totalB} &= S_{primary} + S_{creep} = \sum_{j=1}^{j=n} U_j S_{fj} + \sum_{j=1}^{j=n} S_{creepj} \\ &= U \sum_{j=1}^{j=n} S_{fj} + \sum_{j=1}^{j=n} [\alpha U_j^{\beta} S_{creep,fj} + (1-\alpha U_j^{\beta}) S_{creep,dj}] \\ & \qquad for \quad all \quad t \geq t_{EOP,lab} \quad (t \geq t_{EOP,field} \; for \; S_{creep,dj}) \end{aligned} \tag{1}$$

The formulation in Eq.(1) is a de-coupled simplified Hypothesis B method. The "de-coupled" means that "primary" consolidation settlement $S_{primary}$ is separated from creep settlement S_{creep}. The separation of "primary" consolidation from "secondary" compression for a lab test is shown in Figure 2. A normal soil specimen in oedometer test has 20 mm in thickness with double drainage so that the value of $t_{EOP,lab}$ in Figure 2 is small with tens of minutes only. $t_{EOP,field}$ in Eq.(1) is the End-Of-Primary (EOP) time for soil layers in the field. The value of $t_{EOP,field}$ may vary from a few years to tens of years depending on the thickness and permeability of soils in the field. t_{24hrs} in Figure 2 is the time with duration of 24 hours in an oedometer test, normally larger than $t_{EOP,lab}$ with $t_{EOP,lab} < t_{24hrs} < t_{EOP,field}$ normally true. In practical application, $t_{EOP,lab}$ will be replaced by the time t_0, which is conveniently adopted as 24 hours with conventional oedometer tests. The compression indices are calculated using test data from the same duration of 24 hours as t_0. It shall be pointed out that in Eq.(1), the items of $S_{creep,dj}$ will be zero for $t \leq t_{EOP,field}$ and will become positive $t > t_{EOP,field}$.

In Eq.(1), "primary" consolidation settlement $S_{primary}$ shall be calculated for multiple soil layers with or without a vertical drain:

$$S_{primary} = \sum_{j=1}^{j=n} U_j S_{fj} = U \sum_{j=1}^{j=n} S_{fj} \quad (2)$$

where U_j is combined average degree of consolidation for *j*-layer and U is combined average degree of consolidation for all multiple soil layers with or without a vertical drain:

$$U_j = 1-(1-U_{vj})(1-U_{rj}) \quad (3a)$$

$$U = 1-(1-U_v)(1-U_r) \quad (3b)$$

Eq.(3) is called Carrillo's (1942) formula where U_{vj} and U_{rj} or U_v and U_r are average degree of vertical consolidation and radial consolidation for *j*-layer or multiple soil layers. If there is no vertical drain, $U_{rj} = U_r = 0$, from (3), $U_j = U_{vj}$ or $U = U_v$. For multiple soil layers, the superposition of the average degree of consolidation for each layer is not valid since the continuation condition at each interface of two layers must be satisfied. S_{fj} is the final "primary" consolidation at End-Of-Primary (EOP) consolidation for *j*-layer. S_{fj} can be calculated using the coefficient of volume compressibility m_v or compression indexes C_c, C_r of *j*-layer. More details on calculations of S_{fj} and U are presented in next section.

In Eq.(1), S_{creepj} is creep settlement of soil skeleton in *j*-layer and is equal to:

$$S_{creepj} = \alpha U_j^{\beta} S_{creep,fj} + (1-\alpha U_j^{\beta}) S_{creep,dj} \quad for \;\; all \;\; t \geq t_{EOP,lab} \;\; (t \geq t_{EOP,field} \; for \; S_{creep,dj}) \quad (4a)$$

Eq.(4a) can also be written as:

$$S_{creepj} = \begin{cases} \alpha U_j^{\beta} S_{creep,fj} & for \quad t \geq t_{EOP,lab} \\ \alpha U_j^{\beta} S_{creep,fj} + (1-\alpha U_j^{\beta}) S_{creep,dj} & t \geq t_{EOP,field} \end{cases} \quad (4b)$$

where U_j is from Eq.(3a) with value from 0 to 1 only and β is a power index with value from 0 to 1. Yin (2011) used a parameter $\alpha = 1$ without U_j^{β}. But this over predicted total consolidation settlement. Yin and Feng (2017) and Feng and Yin (2017) used $\alpha = 0.8$ without

U_j^{β} and gave results in close agreement with measured data and values from rigorous fully coupled consolidation modelling. In this paper, a general term of αU_j^{β} is suggested. See more examples later in this paper on more accurate prediction results.

$S_{creep,fj}$ in Eq.(1) or Eq.(4) is creep settlement of *j*-layer under the "final" vertical effective stress after load increased, ignoring the excess porewater pressure. $S_{creep,dj}$ in Eq.(1) or Eq.(4) is "delayed" creep settlement of *j*-layer under the "final" vertical effective stress ignoring the excess porewater pressure. $S_{creep,dj}$ starts for $t \geq t_{EOP,field}$, in other words, is "delayed" by time of $t_{EOP,field}$ to occur. $t_{EOP,field}$ is the End-Of-Primary (EOP) of consolidation for field condition of *j*-later. More discussion on $S_{creep,fj}$ and $S_{creep,dj}$ are in later section.

2.2. Calculation of S_{fj}

In Eq.(2), the total primary consolidation settlement $S_{primary}$ is sum of settlements S_{fj} of all sub-layers multiplied by an over-all average degree of consolidation U. This section presents methods and solutions for calculating S_{fj}. In the following calculations, in order make all equations and text in following paragraphs concise, the layer index "*j*" is removed, keeping in minds that these equations are for one soil layer.

If the coefficient of volume compressibility m_v is used, vertical effective stress increment $\Delta\sigma_z^{'}$ and thickness *H* are known for a soil *j*-layer. S_f for *j*-layer is:

$$S_f = m_v \Delta\sigma_z^{'} H \tag{5}$$

It is noted that m_v is not a constant, depending on vertical effective stress, and shall be used with care. For clayey soils or soft soils, it is better to use C_c and C_r to calculate S_f for higher accuracy. An oedometer test is normally done on the same specimen in multi-stages.

According to British Standard 1377 (1990), the standard duration for each load shall normally last for 24 hours. In this paper, the indexes C_r, C_c and pre-consolidation stress point $(\sigma'_{zp}, \varepsilon_{zp})$ are all determined from the standard oedometer test with duration of 24 hours (1 day), that is, $t_{24hrs} = t_{1day} = 24 \text{ hours} = 1 \text{ day}$, for each load and for each layer. The idealized relationship between the vertical strain and the log (effective stress) is shown in Figure 3 with loading, unloading and reloading states.

Yin and Graham (1989, 1994) and Yin (1990) pointed out limitations of using a logarithmic function for fitting creep curve of log(time) and strain, when time is zero. In 1D EVP model, Yin and Graham (1989, 1994) introduced a time parameter t_o in a logarithmic function to care creep starting from time zero. In many real cases, the vertical effective stress σ'_z is zero or very near zero, for example, σ'_z at surface or near surface of seabed soils or soil ground. If a normal logarithmic function is used for fitting compression curve of log(effective stress) and strain, when the stress is zero, the strain is infinite. To overcome this problem, a unit stress σ'_{unit} is added to the logarithmic function in this paper and was also in Yin's a non-linear logarithmic-hyperbolic function in (Yin 1999). Adding σ'_{unit} in linear logarithmic stress function is particularly necessary for very soft soils in a soil ground with initial effective stress zero at the top of the surface. For example, the initial vertical effective stress at the top surface in soft Hong Kong Marine Clay (HKMC) in seedbed is zero.

Based on Figure 3 and assuming stresses in each layer are uniform, the final settlements S_f for *j*-layer in Eq.(2) for six cases are calculated as follows by adding σ'_{unit1} and σ'_{unit2} in a new logarithmic stress function for elastic compression and elastic-plastic (NCL) compression separately.

(i) Loading from point 1 to point 2 with $OCR = \sigma'_{zp} / \sigma'_{z1}$ and point 2 in OCL:

$$S_{f,1-2} = \varepsilon_{z,1-2} H = \frac{C_r}{1+e_o} \log(\frac{\sigma'_{z2} + \sigma'_{unit1}}{\sigma'_{z1} + \sigma'_{unit1}}) H \quad (6a)$$

The $\varepsilon_{z,1-2}$ is the vertical strain increase due to stress increases from σ'_{z1} to σ'_{z2}. The OCR is over-consolidation ratio and OCL is an over-consolidation line. If σ'_{unit1} is zero, (6a) goes back to conventional logarithmic stress function. The value of σ'_{unit1} is from 0.001 kPa to 1 kPa. For very soft soils, σ'_{unit1} takes values close to 0.01 kPa. Similar strain increase symbols are used in the following equations. Eq.(6a) can avoid singularity problem at initial stress zero ($\sigma'_{z1} = 0$) and is good for very soft soils, such as slurry under self-weight consolidation.

(ii) Loading from point 1 to point 4 with $OCR = \sigma'_{zp} / \sigma'_{z1} > 1$ and point 4 in NCL:

$$S_{f,1-4} = \varepsilon_{z,1-4} H = [\frac{C_r}{1+e_o} \log(\frac{\sigma'_{zp} + \sigma'_{unit1}}{\sigma'_{z1} + \sigma'_{unit1}}) + \frac{C_c}{1+e_o} \log(\frac{\sigma'_{z4} + \sigma'_{unit2}}{\sigma'_{zp} + \sigma'_{unit2}})] H \quad (6b)$$

NCL is a normal consolidation line. Adding σ'_{unit1} and σ'_{unit2} in Eq.(6b) can avoid singularity problem at initial stress zero ($\sigma'_{z1} = \sigma'_{zp} = 0$).

(iii) Loading from point 3 to point 4 with $OCR = \sigma'_{zp} / \sigma'_{z3} = 1$ and point 4 in NCL:

$$S_{f,3-4} = \varepsilon_{z,3-4} H = \frac{C_c}{1+e_o} \log(\frac{\sigma'_{z4} + \sigma'_{unit2}}{\sigma'_{zp} + \sigma'_{unit2}}) H \quad (6c)$$

(iv) Unloading from point 4 to point 6:

$$S_{f,4-6} = \varepsilon_{z,4-6} H = \frac{C_r}{1+e_o} \log(\frac{\sigma'_{z6} + \sigma'_{unit1}}{\sigma'_{z4} + \sigma'_{unit1}}) H \quad (6d)$$

(v) Reloading from point 6 to point 5:

$$S_{f,6-5} = \varepsilon_{z,6-5} h = \frac{C_r}{1+e_o} \log(\frac{\sigma'_{z5} + \sigma'_{unit1}}{\sigma'_{z6} + \sigma'_{unit1}}) h \quad (6e)$$

(vi) Reloading from point 6 to point 7:

$$S_{f,6-7} = \varepsilon_{z,6-7} H = [\frac{C_r}{1+e_o}\log(\frac{\sigma'_{z4}+\sigma'_{unit1}}{\sigma'_{z6}+\sigma'_{unit1}}) + \frac{C_c}{1+e_o}\log(\frac{\sigma'_{z7}+\sigma'_{unit2}}{\sigma'_{z4}+\sigma'_{unit2}})]H \quad (6f)$$

However, the initial stresses and stress increments in a clayey soil layer are not uniform, Eq.(6) cannot be used. There are two approaches to consider this non-uniform stress as below.

(a) Dividing *j*-layer into sub-layers

A general method is to divide this soil layer into sub-layers with smaller thickness, say, 0.25 m to 0.5m, which has been adopted by previous studies (Yin and Feng 2017; Yin and Zhu 2020). The stresses and parameters in each sub-layer are considered uniform and constant. The final settlement S_f for *j*-layer is sum of settlements of all sub-layers (Yin and Feng 2017, Feng and Yin 2017). For each sub-layer with uniform stresses, equations in Eqs.(6a) to (6f) can be used depending on the initial and final stress points. This method is flexible and valid for complicated cases in which vertical stress and pre-consolidation pressure may not be uniform.

(b) Special case of constant parameters C_c, C_r and linear changes of initial stresses, stress increments, and pre-consolidation pressure for *j*-layer

For a clayey soil layer of thickness *H*, C_c, C_r are often constant, but stresses may vary with depth *z*. Figure 4 shows linear changes of initial vertical effective stress, total vertical effective stress, vertical pre-consolidation stress for a soil layer. Linear changes are in following equations:

$$\sigma'_{z1} = \sigma'_{z1,0} + \frac{z}{H}(\sigma'_{z1,H} - \sigma'_{z1,0}) \quad (7a)$$

$$\sigma'_{zp} = \sigma'_{zp,0} + \frac{z}{H}(\sigma'_{zp,H} - \sigma'_{zp,0}) \quad (7b)$$

$$\sigma'_{z} = \sigma'_{z4,0} + \frac{z}{H}(\sigma'_{z4,H} - \sigma'_{z4,0}) \quad (7c)$$

where σ'_{z1} is the initial vertical effective stress. It is noted that the increase of pre-consolidation stress (or pressure) σ'_{zp} may not be as fast as the total vertical effective stress σ'_z as shown in Figure 4. Therefore, there is a point which $\sigma'_{zp} = \sigma'_z$ at depth z_p. Let us consider a general case of loading form point 1 to point 4, the calculation of settlements of *j*-layer for four different cases are in followings.

(i) Normal consolidation case: $OCR = \sigma'_{zp} / \sigma'_{z1} = 1$

In this case, initial effective stress σ'_{z1} and pre-consolidation stress σ'_{zp} are the same, after the stress increase, $\sigma'_z > \sigma'_{zp} = \sigma'_{z1}$. In this case, $S_{f,1-4}$ is:

$$S_{f,1-4} = \int_{z=0}^{z=H} \varepsilon_{z,1-4} dz = \int_{z=0}^{z=H} \frac{C_c}{1+e_o} log(\frac{\sigma'_z + \sigma'_{unit2}}{\sigma'_{zp} + \sigma'_{unit2}}) dz \tag{8a}$$

Substituting Eq.(6) into above equation:

$$S_{f,1-4} = \int_{z=0}^{z=H} \frac{C_c}{(1+e_o)\ln(10)} \ln[\frac{\sigma'_{z4,0} + \frac{z}{H}(\sigma'_{z4,H} - \sigma'_{z4,0}) + \sigma'_{unit2}}{\sigma'_{z1,0} + \frac{z}{H}(\sigma'_{z1,H} - \sigma'_{z1,0}) + \sigma'_{unit2}}] dz = \frac{C_c}{(1+e_o)\ln(10)}$$
$$\left\{ \int_{z=0}^{z=H} \ln[\sigma'_{z4,0} + \frac{z}{H}(\sigma'_{z4,H} - \sigma'_{z4,0}) + \sigma'_{unit2}] dz - \int_{z=0}^{z=H} \ln[\sigma'_{z1,0} + \frac{z}{H}(\sigma'_{z1,H} - \sigma'_{z1,0}) + \sigma'_{unit2}] dz \right\}$$

Let us introduce a new variable $x = \sigma'_{z4,0} + \frac{z}{H}(\sigma'_{z4,H} - \sigma'_{z4,0}) + \sigma'_{unit2}$ and $y = \sigma'_{z1,0} + \frac{z}{H}(\sigma'_{z1,H} - \sigma'_{z1,0}) + \sigma'_{unit2}$, we have $dz = [H / (\sigma'_{z4,H} - \sigma'_{z4,0})] dx$ and $dz = [H / (\sigma'_{z1,H} - \sigma'_{z1,0})] dy$. Noting that for $z = 0 \text{ and } H$, we have $x_{z=0} = \sigma'_{z4,0} + \sigma'_{unit2}$ and $x_{z=H} = \sigma'_{z4,H} + \sigma'_{unit2}$; $y_{z=0} = \sigma'_{z1,0} + \sigma'_{unit2}$ and $y_{z=H} = \sigma'_{z1,H} + \sigma'_{unit2}$. The above equation can be written as:

$$S_{f,1-4} = \frac{C_c}{(1+e_o)\ln(10)} \left\{ \int_{x=\sigma'_{z4,0}+\sigma'_{unit2}}^{x=\sigma'_{z4,H}+\sigma'_{unit2}} \frac{H}{(\sigma'_{z4,H} - \sigma'_{z4,0})} \ln x dx - \int_{y=\sigma'_{z1,0}+\sigma'_{unit2}}^{y=\sigma'_{z1,H}+\sigma'_{unit2}} \frac{H}{(\sigma'_{z1,H} - \sigma'_{z1,0})} \ln y dy \right\}$$

Since $\int \ln x dx = x\ln x - x$ and $\int \ln y dy = y\ln y - y$, the above equation becomes:

$$S_{f,1-4} = \frac{C_c}{(1+e_o)\ln(10)}\left\{\frac{H}{(\sigma'_{z4,H}-\sigma'_{z4,0})}\left[x\ln x - x\right]_{x=\sigma'_{z4,0}+\sigma'_{unit2}}^{x=\sigma'_{z4,H}+\sigma'_{unit2}} - \frac{H}{(\sigma'_{z1,H}-\sigma'_{z1,0})}\left[y\ln y - y\right]_{y=\sigma'_{z1,0}+\sigma'_{unit2}}^{y=\sigma'_{z1,H}+\sigma'_{unit2}}\right\}$$

From above, we have:

$$\begin{aligned} S_{f,1-4} = \frac{C_c}{(1+e_o)\ln(10)}\Bigg\{&\frac{H}{(\sigma'_{z4,H}-\sigma'_{z4,0})}[(\sigma'_{z4,H}+\sigma'_{unit2})\ln(\sigma'_{z4,H}+\sigma'_{unit2})-(\sigma'_{z4,H}+\sigma'_{unit2})- \\ &((\sigma'_{z4,0}+\sigma'_{unit2})\ln(\sigma'_{z4,0}+\sigma'_{unit2})-(\sigma'_{z4,0}+\sigma'_{unit2}))]-\frac{H}{(\sigma'_{z1,H}-\sigma'_{z1,0})}[(\sigma'_{z1,H}+\sigma'_{unit2})\ln(\sigma'_{z1,H}+\sigma'_{unit2}) \\ &-(\sigma'_{z1,H}+\sigma'_{unit2})-((\sigma'_{z1,0}+\sigma'_{unit2})\ln(\sigma'_{z1,0}+\sigma'_{unit2})-(\sigma'_{z1,0}+\sigma'_{unit2}))]\Bigg\} \end{aligned}$$

(8b)

(ii) Over-consolidation case: $OCR = \sigma'_{zp}/\sigma'_{z1} > 1$ and $\sigma'_z \geq \sigma'_{zp}$ for $0 \leq z \leq H$

Figure 4 shows a case commonly encountered in the field. Initially, the soil is over-consolidated with $OCR = \sigma'_{zp}/\sigma'_{z1} > 1$. After increased loading $\Delta\sigma'_z = \sigma'_z - \sigma'_{z1}$, we have and $\sigma'_z \geq \sigma'_{zp}$ for $0 \leq z \leq H$. In this case, we have:

$$S_{f,1-4} = \int_{z=0}^{z=H} \varepsilon_{z,1-4} dz = \int_{z=0}^{z=H} [\frac{C_r}{1+e_o} log(\frac{\sigma'_{zp}+\sigma'_{unit1}}{\sigma'_{z1}+\sigma'_{unit1}}) + \frac{C_c}{1+e_o} log(\frac{\sigma'_z+\sigma'_{unit2}}{\sigma'_{zp}+\sigma'_{unit2}})]dz \quad (8c)$$

Substituting equations in (6) for $\sigma'_{z1}, \sigma'_{zp}, \sigma'_z$ into (8c) and using the same method in (i), the integration of above equation is:

$$\begin{aligned} S_{f,1-4} = \frac{C_r}{(1+e_o)\ln(10)}\Bigg\{&\frac{H}{(\sigma'_{zp,H}-\sigma'_{zp,0})}[(\sigma'_{zp,H}+\sigma'_{unit1})\ln(\sigma'_{zp,H}+\sigma'_{unit1})-(\sigma'_{zp,H}+\sigma'_{unit1})- \\ &((\sigma'_{zp,0}+\sigma'_{unit1})\ln(\sigma'_{zp,0}+\sigma'_{unit1})-(\sigma'_{zp,0}+\sigma'_{unit1}))]-\frac{H}{(\sigma'_{z1,H}-\sigma'_{z1,0})}[(\sigma'_{z1,H}+\sigma'_{unit1})\ln(\sigma'_{z1,H}+\sigma'_{unit1}) \\ &-(\sigma'_{z1,H}+\sigma'_{unit1})-((\sigma'_{z1,0}+\sigma'_{unit1})\ln(\sigma'_{z1,0}+\sigma'_{unit1})-(\sigma'_{z1,0}+\sigma'_{unit1}))]\Bigg\}+ \\ \frac{C_c}{(1+e_o)\ln(10)}\Bigg\{&\frac{H}{(\sigma'_{z4,H}-\sigma'_{z4,0})}[(\sigma'_{z4,H}+\sigma'_{unit2})\ln(\sigma'_{z4,H}+\sigma'_{unit2})-(\sigma'_{z4,H}+\sigma'_{unit2})- \\ &((\sigma'_{z4,0}+\sigma'_{unit2})\ln(\sigma'_{z4,0}+\sigma'_{unit2})-(\sigma'_{z4,0}+\sigma'_{unit2}))]-\frac{H}{(\sigma'_{zp,H}-\sigma'_{zp,0})}[(\sigma'_{zp,H}+\sigma'_{unit2})\ln(\sigma'_{zp,H}+\sigma'_{unit2}) \\ &-(\sigma'_{zp,H}+\sigma'_{unit2})-((\sigma'_{zp,0}+\sigma'_{unit2})\ln(\sigma'_{zp,0}+\sigma'_{unit2})-(\sigma'_{zp,0}+\sigma'_{unit2}))]\Bigg\} \end{aligned}$$

(8d)

(iii) Over-consolidation case: $OCR=\sigma'_{zp}/\sigma'_{z1}>1$ and $\sigma'_{z}<\sigma'_{zp}$ $for\ 0\le z\le z_p$

Figure 4 shows a case in which $OCR=\sigma'_{zp}/\sigma'_{z1}>1$, but $\sigma'_{z}<\sigma'_{zp}$ $for\ 0\le z\le z_p$ and $\sigma'_{z}\ge\sigma'_{zp}$ $for\ z_p\le z\le H$. In this case, the settlement calculation shall consider depth z_p:

$$S_{f,1-4}=\int_{z=0}^{z=H}\varepsilon_{z,1-4}dz=\begin{cases}\displaystyle\int_{z=0}^{z=z_p}\frac{C_r}{1+e_o}log(\frac{\sigma'_z+\sigma'_{unit1}}{\sigma'_{z1}+\sigma'_{unit1}})dz & for\ 0\le z\le z_p\\ \displaystyle\int_{z=z_p}^{z=H}[\frac{C_r}{1+e_o}log(\frac{\sigma'_{zp}+\sigma'_{unit1}}{\sigma'_{z1}+\sigma'_{unit1}})+\frac{C_c}{1+e_o}log(\frac{\sigma'_z+\sigma'_{unit2}}{\sigma'_{zp}+\sigma'_{unit2}})]dz & for\ z_p\le z\le H\end{cases}\tag{8e}$$

Linear equations in Eq.(6) for $\sigma'_{z1},\sigma'_{zp},\sigma'_{z}$ can be substituted into Eq.(8e). Analytical integration solution can be obtained using the same method in (i) and is not presented here. Equations like Eq.(8) can be obtained for other loading, unloading, and reloading cases with linear changes of stresses and are not discussed here.

In many calculations, m_v is needed, for example in Eq.(5) and $c_v=k_v/(m_v\gamma_w)$ and $c_r=k_r/(\gamma_w m_v)$ in order to calculate U_v and U_r. If indexes C_r, C_c and pre-consolidation stress point $(\sigma'_{zp},\varepsilon_{zp})$ are used to calculate final settlements in Eqs.(6), (7), and (8), the coefficient of vertical volume compressibility m_v can be back-calculated as

(i) for the case of Eq. (6b) in normal loading:

$$m_{v,1-4}=\frac{S_{f,1-4}}{H\left(\sigma'_{z4}-\sigma'_{z1}\right)}\tag{9a}$$

(ii) for the case of Eq.(6d) in unloading:

$$m_{v,4-6}=\frac{S_{f,4-6}}{H\left(\sigma'_{z4}-\sigma'_{z6}\right)}\tag{9b}$$

In Eqs.(9a) and (9b), settlements and stress increments are known so that m_v corresponding to the same stress increment can be calculated. In Eq.(9b), $S_{f,4-6}$ and $(\sigma'_{z4}-\sigma'_{z6})$ are both negative so that $m_{v,4-6}$ is positive. The calculation method for m_v in Eqs.(9a) and (9b) can be applied to other different loading stages.

2.3. Calculation of U_j and U

In Eqs.(1) and (2), an average degree of consolidation U_j for *j*-layer or over-all average degree of consolidation U is needed. The basic definition of U_j for *j*-layer is:

$$U_j = \frac{S_j(t)}{S_{fj}} = \frac{\int_{z=0}^{z=H_j} m_{vj}\Delta\sigma'_{zj}(t)dz}{\int_{z=0}^{z=H_j} m_{vj}\Delta\sigma'_{zjf}dz} = \frac{\int_{z=0}^{z=H_j} [u_{eij} - u_{ej}(t)]dz}{\int_{z=0}^{z=H_j} u_{eij}dz} = 1 - \frac{\int_{z=0}^{z=H_j} u_{ej}(t)dz}{\int_{z=0}^{z=H_j} u_{eij}dz} \tag{10a}$$

where S_{fj} is the final settlement for j-layer using m_{vj} and $\Delta\sigma'_{zjf}$, calculated using Eq(5). It is noted that the final vertical effective stress increment $\Delta\sigma'_{zjf}$ is equal to the initial excess pore water pressure u_{eij} for *j*-layer. $u_{ej}(t)$ is the excess pore water pressure at time *t* for *j*-layer. Eq.(10a) can be written:

$$U_j = 1 - \frac{\frac{1}{H_j}\int_{z=0}^{z=H_j} u_{ej}(t)dz}{\frac{1}{H_j}\int_{z=0}^{z=H_j} u_{eij}dz} = 1 - \frac{\overline{u}_{ej}(t)}{\overline{u}_{eij}} \tag{10b}$$

where $\overline{u}_{eij}$ and $\overline{u}_{ej}$ are the average initial and current excess porewater pressures respectively *t* for *j*-layer. The over-all average degree of consolidation U is:

$$U = \frac{S_{primary}}{S_f} = \frac{\sum_{j=1}^{j=n} S_j(t)}{\sum_{j=1}^{j=n} S_{fj}} = \frac{\sum_{j=1}^{j=n}\int_{z=0}^{z=H_j} m_{vj}\Delta\sigma'_{zj}(t)dz}{\sum_{j=1}^{j=n}\int_{z=0}^{z=H_j} m_{vj}\Delta\sigma'_{zjf}dz} = 1 - \frac{\sum_{j=1}^{j=n} m_{vj}\int_{z=0}^{z=H_j} u_{ej}(t)dz}{\sum_{j=1}^{j=n} m_{vj}\int_{z=0}^{z=H_j} u_{eij}dz} \tag{11a}$$

From Eq.(10a), $\overline{u}_{ej}(t) = (1 - U_j)\overline{u}_{eij}$. Using this relation, (11a) can be written:

$$U = 1 - \frac{\sum_{j=1}^{j=n} m_{vj} \frac{H_j}{H_j} \int_{z=0}^{z=H_j} u_{ej}(t) dz}{\sum_{j=1}^{j=n} m_{vj} \frac{H_j}{H_j} \int_{z=0}^{z=H_j} u_{eij} dz} = 1 - \frac{\sum_{j=1}^{j=n} m_{vj} H_j \overline{u}_{ej}(t)}{\sum_{j=1}^{j=n} m_{vj} H_j \overline{u}_{eij}} = 1 - \frac{\sum_{j=1}^{j=n} m_{vj} H_j (1-U_j) \overline{u}_{eij})}{\sum_{j=1}^{j=n} m_{vj} H_j \overline{u}_{eij}} =$$
$$= 1 - \frac{\sum_{j=1}^{j=n} m_{vj} H_j \overline{u}_{eij} - \sum_{j=1}^{j=n} m_{vj} H_j U_j \overline{u}_{eij}}{\sum_{j=1}^{j=n} m_{vj} H_j \overline{u}_{eij}} = \frac{\sum_{j=1}^{j=n} m_{vj} H_j \overline{u}_{eij} U_j}{\sum_{j=1}^{j=n} m_{vj} H_j \overline{u}_{eij}} \quad (11b)$$

Attention shall be paid to the definition and differences of U_j and U. The following paragraphs summarize existing solutions for U_v, U_r, U_j and U.

The early analytical solutions were obtained by Terzaghi (1925, 1943) for a single soil layer with thickness H under suddenly applied load for 1-D straining. Charts of these solutions can be found in Craig's Soil Mechanics (Knappett 2019). For double drainage with linear excess pore water pressure u_e distribution or one way drainage with uniform u_e distribution, following appreciate equation is good and simple to calculating U_v:

$$\begin{cases} For\ U_v < 0.6: & T_v = \frac{\pi}{4} U_v^2, \quad U_v = \sqrt{\frac{4T_v}{\pi}} \\ For\ U_v \geq 0.6: & T_v = -0.944 \log(1-U_v) - 0.085, \quad U_v = 1 - 10^{-\frac{T_v + 0.085}{0.933}} \end{cases} \quad (12a)$$

If we assume that when $U_v = 98\%$, $u_e \approx 0$; time at $U_v = 98\%$ is selected as time at EOP in the field $t_{EOP,field}$. We have:

$$T_v = -0.944 \log(1-U_v) - 0.085 = 0.150 \quad (12b)$$

$$t_{EOP,field} = \frac{T_v d^2}{c_v} = \frac{1.50 d^2}{c_v} \quad (12c)$$

where d is the maximum drainage path of a soil layer, if double drainage, $d = H/2$, c_v is the coefficient of vertical consolidation.

To consider ramp loading as shown in Figure 4, a simple correction method for U_v

proposed by Terzaghi (1943) can be used. Solutions to 1-D consolidation under depth-dependent ramp load and to special 1-D consolidation problems can be found in Zhu and Yin (1998, 2012). Solutions to double soil layers without vertical drains under ramp load can be found in Zhu and Yin (1999b). Solutions to 2-D consolidation of a single soil layer with vertical drains under ramp load were obtained by Zhu and Yin (2001a,b; 2004). Solutions to 2-D consolidation of a single soil layer with vertical drains without well resistance under suddenly applied load were obtained by Barron (1948). Hansbo (1981) presented analytical solution to consolidation problem of a soil with vertical drains considering both smear zone and well resistance under suddenly applied load under equal vertical strain assumption.

Solutions to consolidation problem of a stratified soil with vertical and horizontal drainage under ramp loading were obtained by Walker and Indraratna (2009) and Walker *et al.* (2009) using a spectral method. The main partial differential equation for the average excess pore water pressure $\bar{u}$ using spectral method is:

$$\frac{m_v}{\overline{m}_v}\frac{\partial \bar{u}}{\partial t} = -\left[dT_r\frac{\eta}{\bar{\eta}}\bar{u} - dT_v\left(\frac{\partial}{\partial Z}\left(\frac{k_v}{\overline{k}_v}\right)\frac{\partial \bar{u}}{\partial Z} + \frac{k_v}{\overline{k}_v}\frac{\partial^2 \bar{u}}{\partial Z^2}\right)\right] + \frac{m_v}{\overline{m}_v}\frac{\partial \bar{\sigma}}{\partial t} + dT_r\frac{\eta}{\bar{\eta}}w \tag{13a}$$

where, $\eta = \frac{k_r}{r_e^2\mu}, \mathrm{d}T_v = \frac{\bar{c}_v}{H^2}, \mathrm{d}T_r = \frac{2\bar{\eta}}{\gamma_w \overline{m}_v}, \bar{c}_v = \frac{\overline{k}_v}{\gamma_w \overline{m}_v}, Z = \frac{z}{H}$. Vertical and horizontal drainages are considered simultaneously in Eq.(13a). All parameters are explained below:

$\bar{u}$: averaged excess pore water pressure (averaged along radial coordinate r) at depth *Z,* a function of time *t* and *Z*.

$\bar{\sigma}$: average total stress (averaged along r) at depth *Z*, a function of time *t* and *Z*.

w: water pressure applied on the vertical drains, varying with depth *Z*, which is zero without vacuum pre-loading pressure.

r_w: unit weight of water.

k_r : the horizontal permeability coefficient of the undisturbed soil, a function of *Z*.

m_v : coefficient of volume compressibility (assumed the same in smear and undisturbed zone), calculated using total incremental strain resulted from primary consolidation under total stress increment, and a function of *Z*.

Parameters k_v , m_v and η can be depth-dependent in a piecewise linear way or kept constant within each layer. $\bar{k}_v$, $\bar{m}_v$ and $\bar{\eta}$ are convenient reference values at certain depth, for example values $k_{v,j=1}$, $m_{v,j=1}$ and $\eta_{j=1}$ of layer 1. If so, $\bar{c}_v = \bar{k}_v / \gamma_w \bar{m}_v = k_{v,j=1} / \gamma_w m_{v,j=1} = c_{v,j=1}$. $\bar{\eta} = \bar{k}_r / r_e^2 \mu = k_{r,j=1} / r_e^2 \mu = \eta_{j=1}$. All the parameters in Eq.(13a) have been normalized and may be different for different soil layers (no layer index is used here to make presentation concise). Normalized parameters in Eq.(13a) are: $m_v / \bar{m}_v$, $\eta / \bar{\eta}$, $k / \bar{k}_v$.

The parameter $\eta = k_r / (r_e^2 \mu)$ is related to radial permeability k_r, equivalent radius r_e of cylinder cell, and μ . If there is no horizonal drainage in a soil layer, $k_r = 0$ to that $\eta = 0$. This is useful for consolidation analysis of soils with partially penetrating vertical drains. All soil layers below vertical drains have $\eta = 0$. Walker and Indraratna (2009) and Walker *et al.* (2009) discussed that their method can also simulate the effect of using long and short drains in unison. For example, in lower soil layers where only long drains are installed, η shall have smaller value than that of upper soil layers where both short and long drains are present.

μ inside η is a dimensionless drain geometry/smear zone parameter. Expressions for μ can be taken as the following by considering effects of smear zone, well resistance, or approximation (Hansbo 1981):

$$\mu = \frac{n^2}{n^2-1}\left(\ln\frac{n}{s} - \frac{3}{4} + \frac{k_r}{k_s}\ln s\right) + \frac{s^2}{n^2-1}\left(1 - \frac{s^2}{4n^2}\right) + \frac{k_r}{k_s}\frac{1}{n^2-1}\left(\frac{s^4-1}{4n^2} - s^2 + 1\right) + \pi z(2l - z)\frac{k_r}{q_w}\left(1 - \frac{1}{n^2}\right) \quad (13b)$$

In (13b), $T_r = \frac{c_r t}{r_e^2}, n = \frac{r_e}{r_d}, s = \frac{r_s}{r_d}$. $q_w = k_w \pi r_w^2$ is specific discharge capacity of drain (vertical hydraulic gradient *i*=1). *z* is the vertical coordinate in Figure 1 and *l* is length of drain when closed at bottom or a half of drain when bottom is open. If hydraulic resistance of vertical drains is zero, this means $q_w = k_w \pi r_d^2 \Rightarrow \infty$. (13b) can be simplified:

$$\mu = \frac{n^2}{n^2-1}(\ln\frac{n}{s} - \frac{3}{4} + \frac{k_r}{k_s}\ln s) + \frac{s^2}{n^2-1}(1-\frac{s^2}{4n^2}) + \frac{k_r}{k_s}\frac{1}{n^2-1}(\frac{s^4-1}{4n^2} - s^2 + 1) \tag{13c}$$

Walker and Indraratna (2006) also provided an expression for μ considering parabolic smear zone permeability but ingoring smear zone:

$$\mu = \ln\frac{n}{s} - \frac{3}{4} + \frac{\kappa(s-1)^2}{(s^2-2\kappa s+\kappa)}\ln\frac{s}{\sqrt{\kappa}} - \frac{s(s-1)\sqrt{\kappa(\kappa-1)}}{2(s^2-2\kappa s+\kappa)}\ln\left(\frac{\sqrt{\kappa}+\sqrt{\kappa-1}}{\sqrt{\kappa}-\sqrt{\kappa-1}}\right) \tag{13d}$$

where κ is ratio of undisturbed horizontal permeability k_r to smear zone permeability k_{so} at the drain/soil interface, (at $r = r_d$, $k_s = k_{s0}$). At $r = r_s$, $k_s = k_r$.

Walker and Indraratna (2009) provided an Excel spreadsheet calculation program implemented with VBA program named SPECCON to enable convenient adoption of this method for consolidation analysis of multiple soils layers with or without vertical drains. After inputted all parameters and load $\bar{\sigma}$, this program gives excess pore water pressure at time *t* $\bar{u}_{ej}(t)$ for *j*-layer and $\bar{u}_e(t)$ for all layers together. The combined average degree of consolidation U_j for each *j*-layer is calculated using Eq.(10a). The overall combined average degree of consolidation U for all layers shall be calculated using Eq.(11a) or Eq.(11b). Once U_j and U with time *t* are known, total "primary" consolidation settlement $S_{primary}$ can be calculated using Eq.(2).

2.4. Calculation of S_{creep}, S_{creepj}, $S_{creep,fj}$ *and* $S_{creep,dj}$

In Eq.(1) or Eq.(4b), the total creep settlement S_{creep} of all layers together is sum of S_{creepj} for all layers. This is a simple superposition. The key items for calculating S_{creepj} are $S_{creep,fj}$

and $S_{creep,dj}$.

(a) Calculation of $S_{creep,fj}$ for different stress-strain states

Creep settlement $S_{creep,fj}$ of *j*-layer is calculated as creep compression under the "final" vertical effective stress ignoring coupling of excess porewater pressure nor any ramp loading process. This is an ideal case in order to de-couple this consolidation problem. To consider creep compression occurred in "primary" consolidation starting from time zero, the void ratio e due to creep is (Yin and Graham 1989, 1994):

$$e = e_o - C_{\alpha e} \log \frac{t_o + t_e}{t_o} \tag{14}$$

where $C_{\alpha e}$ is a creep parameter; t_o is another creep parameter; t_e is "equivalent time" defined by Yin and Graham (1989, 1994); and e_o is the initial void ratio at $t_e = 0$. In this study, $C_{\alpha e}$ is considered constant as a common practice in engineering. Yin (1999) proposed a nonlinear creep model which considers the creep limit with time and the decreasing trend of $C_{\alpha e}$ with effective stress, which shows advantages in very long-term settlement calculations (Chen et al. 2021). For settlement calculation of settlements of most soft soils in a normal service life (say 50 years) of a geotechnical structure, it is still reliable and convenient to adopt constant values of $C_{\alpha e}$ to avoid lengthy equations as much as possible (Yin and Feng 2017; Feng and Yin 2017). According to the "equivalent time" concept (Yin and Graham 1989, 1994, Yin 1990, 2011, 2015), the total strain ε_z at any stress-strain state in Figure 3 can be calculated by the following equation:

$$\varepsilon_z = \varepsilon_{zp} + \frac{C_c}{V} \log \frac{\sigma_z^{'}}{\sigma_{zp}^{'}} + \frac{C_{\alpha e}}{V} \log \frac{t_o + t_e}{t_o} \tag{15}$$

where $\varepsilon_{zp} + \frac{C_c}{V}\log\frac{\sigma_z^{'}}{\sigma_{zp}^{'}}$ is the strain on the normal consolidation line (NCL) under stress $\sigma_z^{'}$ (also called "reference time line" and noting specific volume $V = 1 + e_o$); and $\frac{C_{\alpha e}}{V}\log\frac{t_o + t_e}{t_o}$ is the creep strain occurring from the NCL under the same stress $\sigma_z^{'}$. The above equation is valid for any 1-D loading path. The calculation of $S_{creep,fj}$ is dependent on the final stress-strain state $(\sigma_z^{'}, \varepsilon_z)$. To make presentation concise, in the following equations, layer index j is removed.

(i) The final stress-strain point is on an NCL line, for example at point 4

The final creep settlement for any point on NCL line is:

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o + t_e}{t_o})H \qquad for \quad t_e \geq 0 \tag{16a}$$

For a suddenly applied load kept for a duration time t, we have $t_e = t - t_o$. Submitting the above relation into (16a), we have:

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t}{t_o})H \qquad for \quad t \geq 1\,day \tag{16b}$$

Noting $t_o = 1$ day since C_r and C_c are determined using data with 1 day duration. In (16a), if $t = 1$ day, from $t_e = t - t_o$, $t_e = 1 - 1 = 0$. This means that at time $t = 1$ day, creep settlement $S_{creep,f}$ on NCL is zero. According to Elastic Visco-Plastic (EVP) modelling theory (Yin and Graham 1989, 1994), the compression strain rate is sum of elastic strain rate and visco-plastic strain rate. The NCL line in Figure 3, in fact, has included both elastic strain and visco-plastic strain (or creep strain). The creep settlement in (16a) is additional creep compression starting from 1 day or below NCL.

(ii) The final stress-strain point is on an OCL line, for example at point 2

Consider a sudden load increase from point 1 to point 2 which is kept unchanged with a duration time *t*. The final creep settlement for any point, for example point 2, on Over-Consolidation Line (OCL) line is:

$$S_{creep,f}=\frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o+t_e}{t_o+t_{e2}})H \quad for \quad t_e\geq t_{e2} \tag{16c}$$

(16c) can be re-written with $\Delta\varepsilon_{zcreep}$ included:

$$S_{creep,f}=[\frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o+t_e}{t_o})-\frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o+t_{e2}}{t_o})]H=\Delta\varepsilon_{zcreep}H$$

Referring to Figure 3, it is seen that $\frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o+t_{e2}}{t_o})$ is the strain from point $2'$ to point 2; while $\frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o+t_e}{t_o})$ is the strain from point $2'$ to a point below point 2 downward. The increased strain for further creep done from point 2 is $\Delta\varepsilon_{zcreep}$, which is used to calculate creep settlement $S_{creep,f}$ under loading at point 2. It is noted that the relationship between t_e and the creep duration time *t* under the stress σ'_{z2} is $t_e=t_{e2}+t-t_o$. t_{e2} here or in (16c) can be calculated below. Using Eq.(15), at point 2 of $(\sigma'_{z2},\varepsilon_{z2})$, we have:

$$\varepsilon_{z2}=\varepsilon_{zp}+\frac{C_c}{V}\log\frac{\sigma'_{z2}}{\sigma'_{zp}}+\frac{C_{\alpha e}}{V}\log\frac{t_o+t_{e2}}{t_o}$$

From the above, we have:

$$\log\frac{t_o+t_{e2}}{t_o}=(\varepsilon_{z2}-\varepsilon_{zp})\frac{V}{C_{\alpha e}}-\frac{C_c}{C_{\alpha e}}\log\frac{\sigma'_{z2}}{\sigma'_{zp}}$$

$$t_{e2}=t_o\times 10^{(\varepsilon_{z2}-\varepsilon_{zp})\frac{V}{C_{\alpha e}}}(\frac{\sigma'_{z2}}{\sigma'_{zp}})^{-\frac{C_c}{C_{\alpha e}}}-t_o \tag{16d}$$

It is seen from (16d) that the equivalent time t_{e2} at point 2 is uniquely related to the stress-strain state point $(\sigma'_{z2},\varepsilon_{z2})$. Substituting $t_e=t_{e2}+t-t_o$ into (16c), we have:

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t+t_{e2}}{t_o+t_{e2}})H \qquad t \geq 1\text{ day} \tag{16e}$$

If we consider unloading from point 4 to point 6 in Figure 3, using the same approach above, we can derive the following equations:

$$t_{e6} = t_o \times 10^{(\varepsilon_{z6}-\varepsilon_{zp})\frac{V}{C_{\alpha e}}}(\frac{\sigma'_{z6}}{\sigma'_{zp}})^{-\frac{C_c}{C_{\alpha e}}} - t_o \tag{16f}$$

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o+t_e}{t_o+t_{e6}})H = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t+t_{e6}}{t_o+t_{e6}})H \qquad t \geq 1\text{ day} \tag{16g}$$

Reloading from point 6 to point 5:

$$t_{e5} = t_o \times 10^{(\varepsilon_{z5}-\varepsilon_{zp})\frac{V}{C_{\alpha e}}}(\frac{\sigma'_{z5}}{\sigma'_{zp}})^{-\frac{C_c}{C_{\alpha e}}} - t_o \tag{16h}$$

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o+t_e}{t_o+t_{e5}})H = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t+t_{e5}}{t_o+t_{e5}})H \qquad t \geq 1\text{ day} \tag{16i}$$

(b) Calculation of $S_{creep,dj}$ for different stress-strain states

$S_{creep,dj}$ is called "delayed" creep settlement of *j*-layer under the "final" vertical effective stress ignoring the excess porewater pressure. $S_{creep,dj}$ starts for $t \geq t_{EOP,field}$, that is, is "delayed" by time of $t_{EOP,field}$. The selection of time at EOP is subjective since the separation of "primary" consolidation from "secondary" compressions is not scientific and subjective. In the general simple method, the time at $U_j = 98\%$ is considered to be the time at EOP, that is, $t_{EOP,field}$ for field condition for *j*-layer (Yin and Feng 2017). Eq.(12c) or other solutions for U_j can be used to calculate $t_{EOP,field}$ for a single layer case. Equations for calculating $S_{creep,dj}$ for different "final" stress-strain state are presented below. The layer index *j* is removed in following equations.

(i) The final stress-strain point is on an NCL line, for example at point 4

Eq.(16a) is the final creep settlement for any point on NCL line for $t_e \geq 0$ or $t \geq 1$ day:

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o + t_e}{t_o})H \quad t_e \geq 0$$

$S_{creep,d}$ is delayed by $t_{EOP,field}$:

$$\begin{aligned} S_{creep,d} &= \frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o + t_e}{t_o})H - \frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o + t_{e,EOP,field}}{t_o})H \\ &= \frac{C_{\alpha e}}{1+e_o}\log(\frac{t_o + t_e}{t_o + t_{e,EOP,field}})H \qquad for \quad t_e \geq t_{e,EOP,field} \end{aligned} \tag{17a}$$

Noting $\because t_e = t - t_o \quad \therefore t_{e,EOP,field} = t_{EOP,field} - t_o$. Substituting these time relations into (17a):

$$S_{creep,d} = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t}{t_{EOP,field}})H \qquad for \quad t \geq t_{EOP,field} \tag{17b}$$

In (17b) $S_{creep,d}$ is calculated for $t \geq t_{EOP,field}$, that is, “delayed” by time $t_{EOP,field}$.

(ii) The final stress-strain point is on an OCL line, for example at point 2

The final creep settlement at point 2 is:

$$S_{creep,f} = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t + t_{e2}}{t_o + t_{e2}})H \quad for \;\; t_e \geq t_{e2} \; or \; t \geq t_o = 1 \text{ day}$$

$S_{creep,d}$ is delayed by $t_{EOP,field}$:

$$\begin{aligned} S_{creep,d} &= \frac{C_{\alpha e}}{1+e_o}\log(\frac{t + t_{e2}}{t_o + t_{e2}})H - \frac{C_{\alpha e}}{1+e_o}\log(\frac{t_{EOP,field} + t_{e2}}{t_o + t_{e2}})H \\ S_{creep,d} &= \frac{C_{\alpha e}}{1+e_o}\log(\frac{t + t_{e2}}{t_{EOP,field} + t_{e2}})H \qquad for \quad t \geq t_{EOP,field} \end{aligned} \tag{18a}$$

When $t = t_{EOP,field}$, $S_{creep,d}$ in (18a) is zero.

Using the same approach, at point 6:

$$S_{creep,d} = \frac{C_{\alpha e}}{1+e_o}\log(\frac{t + t_{e6}}{t_{EOP,field} + t_{e6}})H \qquad for \quad t \geq t_{EOP,field} \tag{18b}$$

at point 5:

$$S_{creep,d} = \frac{C_{\alpha e}}{1+e_o}\log\left(\frac{t+t_{e5}}{t_{EOP,field}+t_{e5}}\right)H \qquad for \quad t \geq t_{EOP,field} \tag{18c}$$

3. Consolidation Settlements of a Clay Layer with OCR=1 or 1.5 from General Simple Method and Fully Coupled Consolidation Analyses

In this section, consolidation settlements of an idealized horizontal layer of Hong Kong Marine Clay (HKMC) are calculated using the simplified Hypothesis B method and two fully coupled Finite Element (FE) consolidation models. This HKMC layer has 4 m in thickness and is free drained on the top surface and impermeable at the bottom. Over-Consolidation Ratio (OCR) is OCR=1 or 1.5. Two FE programs are used for fully coupled consolidation analysis of the HKMC layer: one is software "Consol" developed by Zhu and Yin (1999a, 2000), and the other one is Plaxis software (2D 2015 version) (Plaxis 2015). In the "Consol" analysis, a 1D EVP model (Yin and Graham 1989, 1994) is used for the consolidation modelling. In Plaxis software (2D 2015 version), a soft soil creep (SSC) model is adopted in the FE simulations. SSC model is in fact a 3D EVP model (Vermeer and Neher 1999). The structure and parameters of this SSC model is almost the same as a 3D EVP model proposed by Yin (1990) and Yin and Graham (1999).

Values of all parameters used in FE consolidation simulation are listed in Table 2. In all FE simulations, a vertical stress of 20 kPa is assumed to be instantly applied on the top surface and kept constant for a period of 18250 days (50 years). Since HKMC layer is in seabed, the initial vertical effective stress is zero at the top of the HKMC layer surface. Therefore, the unit stress σ'_{unit1} or σ'_{unit2} in Eq.(6) cannot be zero. The best value of σ'_{unit1} or σ'_{unit2} shall be determined by oedometer compression test data at very small vertical effective stress. Here we may assume that σ'_{unit1} or σ'_{unit2} takes values from 0.0l kPa to 1 kPa and discuss difference of calculated settlement values.

(a) Normally Consolidated HKMC Layer with *H*=4m and OCR=1

The integrated Eq.(8b) is used to calculate the final "primary" settlement $S_{f,1-4}$. The values of all parameters are listed in Table 1. The values of all stresses are $\sigma'_{z1,0} = 0, \sigma'_{z1,H} = 20.76kPa$, $\sigma'_{z4,0} = 20, \sigma'_{z4,H} = 40.76kPa$. $S_{f,1-4}$ is:

$$S_{f,1-4} = \frac{1.4624}{3.65\ln(10)}\left\{\frac{4}{(40.76-20)}[(40.76+\sigma'_{unit2})\ln(40.76+\sigma'_{unit2})-(40.76+\sigma'_{unit2})-\right.$$
$$((20+\sigma'_{unit2})\ln(20+\sigma'_{unit2})-(20+\sigma'_{unit2}))]-\frac{4}{(20.76-0)}[(20.76+\sigma'_{unit2})\ln(20.76+\sigma'_{unit2})$$
$$\left.-(20.76+\sigma'_{unit2})-((0+\sigma'_{unit2})\ln(0+\sigma'_{unit2})-(0+\sigma'_{unit2}))]\right\}$$

Using above equation with σ'_{unit2}=0.01 kPa, it is found that $S_{f,1-4}$ =0.944 m; if σ'_{unit2}=0.1 kPa, $S_{f,1-4}$ =0.928 m; if σ'_{unit2}=0.5 kPa, $S_{f,1-4}$ =0.879 m; if σ'_{unit2}=1 kPa, $S_{f,1-4}$ =0.834 m. This means that the final "primary" settlement $S_{f,1-4}$ is sensitive to the value of σ'_{unit2}. In this example, we select σ'_{unit2}=0.1 kPa so that the final "primary" settlement $S_{f,1-4}$ is 0.928 m.

The calculation of average m_v and c_v is below:

$$\Delta\varepsilon_{z,1-4} = S_{f,1-4} = 0.928/4 = 0.232$$
$$m_v = \Delta\varepsilon_{z,1-4} / \Delta\sigma'_{z,1-4} = 0.232/20 = 0.0116 \quad (1/\text{kPa})$$

$$c_v = k/(\gamma_w m_v) = 1.9\times10^{-4}/(9.81\times0.0116) = 1.670\times10^{-3}\ (\text{m}^2/\text{day}) = 0.610\ (\text{m}^2/\text{year})$$

As explained, a thick layer can be divided into small sub-layers. The stresses and values of soil parameters in each sub-layer are assumed be constant. In this case, simple equations in Eq.(6b) can be used to calculate that the final "primary" settlement $S_{f,1-4}$ for each sub-layer. This layer of 4m can be divided into 2, 4, or 8 sub-layers with thickness of 2m, 1m, and 0.5m respectively. The final "primary" settlement $S_{f,1-4}$ calculated is 0.743m, 0.831m, 0.881m, and 0.910 m sub-layers with thickness of 4m, 2m, 1m, and 0.5m respectively. Values of S_f, m_v and c_v

for sub-layer thickness of 0.5 m for OCR=1 are listed in Table 2. ε_{zp} in Table 2 is the vertical strain in each sub-layer (0.5 m here) from the initial effective stress $\sigma_{zi}^{'}$ to pre-consolidation pressure $\sigma_{zp}^{'}$ and $\bar{\varepsilon}_{zp}$ is average of all ε_{zp} values. Since OCR=1, $\sigma_{zp}^{'}=\sigma_{zi}^{'}$, the strain ε_{zp} and $\bar{\varepsilon}_{zp}$ are zero. $\Delta\varepsilon_z$ is the strain increase in each sub-layer for loading from $\sigma_{zp}^{'}$ to current stress $\sigma_z^{'}$. $\Delta\bar{\varepsilon}_z$ is average of all $\Delta\varepsilon_z$ values. $(\bar{\varepsilon}_{zp}+\Delta\bar{\varepsilon}_z)$ is total vertical strain. Summary of values of S_f, m_v and c_v for different number of sub-layers for OCR=1 are listed in Table 3 including S_f obtained by more accurate integration method. It is seen from Table 3 that the more sub-layers (or the smaller thickness of the sub-layers), the more accurate are these S_f, m_v and c_v. A thickness of 0.5 m is considered as appropriate since the relative error of S_f is only $\frac{0.928-0.910}{0.928}\times 100\%=1.9\%$ of the integrated one.

In this example, the general simplified Hypothesis B method in Eq.(1) together with other equations on relevant parameters is used to calculate the total settlement S_{totalB} using $\alpha=0.8$ and $\beta=0$ (denoted B Method 1) , $\beta=0.3$ (denoted B Method 2), and $\beta=1$ (denoted B Method 3). B Method 1 using $\alpha=0.8$ and $\beta=0$ is in fact the method published by Yin and Feng (2017). The calculated curves of settlements with log(time) from the simplified Hypothesis B method are shown in Figure 5(a) for time up to 100 years. At the same time, Hypothesis A method and two fully coupled finite element models are used to calculate the curves of settlements with log(time) which are also shown in Figure 5(a) for comparison. It is seen from Figure 5(a) that, when $\alpha=0.8$ and $\beta=0.3$ m, B Method 2 gives curves much closer to the curves from the two finite element models of "Consol" by Zhu and Yin (1999a, 2000) and Plaxis software (2D 2015 version). Values of parameters used in Consol software are listed in Table 1(b) and those of Plaxis in Table 1(c). As shown in Figure 5(a), again, Hypothesis A method underestimates the total settlement for the time period.

(b) Over-Consolidated HKMC Layer with *H*=4m and OCR=1.5

Eq.(8d) from integration is used to calculate the final "primary" settlement $S_{f,1-4}$. Values of all parameters are listed in Table 1. Values of all stresses are $\sigma'_{z1,0}=0, \sigma'_{z1,H}=20.76$ kPa, $\sigma'_{zp,0}=0, \sigma'_{zp,H}=31.14$ kPa $\sigma'_{z4,0}=20, \sigma'_{z4,H}=40.76$ kPa. $S_{f,1-4}$ is:

$$
\begin{aligned}
S_{f,1-4} = {} & \frac{0.0913}{3.656\ln(10)}\left\{\frac{4}{(31.14-0)}[(31.14+\sigma'_{unit1})\ln(31.14+\sigma'_{unit1})-(31.14+\sigma'_{unit1})-\right. \\
& ((0+\sigma'_{unit1})\ln(0+\sigma'_{unit1})-(0+\sigma'_{unit1}))]-\frac{4}{(20.76-0)}[(20.76+\sigma'_{unit1})\ln(20.76+\sigma'_{unit1}) \\
& \left.-(20.76+\sigma'_{unit1})-((0+\sigma'_{unit1})\ln(0+\sigma'_{unit1})-(0+\sigma'_{unit1}))]\right\}+ \\
& \frac{1.4624}{3.65\ln(10)}\left\{\frac{4}{(40.76-20)}[(40.76+\sigma'_{unit2})\ln(40.76+\sigma'_{unit2})-(40.76+\sigma'_{unit2})-\right. \\
& ((20+\sigma'_{unit2})\ln(20+\sigma'_{unit2})-(20+\sigma'_{unit2}))]-\frac{4}{(40.76-0)}[(31.14+\sigma'_{unit2})\ln(31.14+\sigma'_{unit2}) \\
& \left.-(31.14+\sigma'_{unit2})-((0+\sigma'_{unit2})\ln(0+\sigma'_{unit2})-(0+\sigma'_{unit2}))]\right\}
\end{aligned}
$$

Using above equation with $\sigma'_{unit1}=\sigma'_{unit2}$=0.01 kPa, we find $S_{f,1-4}$=0.681 m; if $\sigma'_{unit1}=\sigma'_{unit2}$ =0.1 kPa, $S_{f,1-4}$=0.669 m; if $\sigma'_{unit1}=\sigma'_{unit2}$=0.5 kPa, $S_{f,1-4}$=0.635 m; if $\sigma'_{unit1}=\sigma'_{unit2}$=1 kPa, $S_{f,1-4}$=0.604 m. This means that the final "primary" settlement $S_{f,1-4}$ is sensitive to the value of σ'_{unit1} and σ'_{unit2}. In this example, we select $\sigma'_{unit1}=\sigma'_{unit2}$=0.1 kPa so that the final "primary" settlement $S_{f,1-4}$ is 0.669 m. The calculation of average m_v and c_v is below:

$$
\begin{aligned}
& \Delta\varepsilon_{z,1-4} = S_{f,1-4} = 0.669/4 = 0.167 \\
& m_v = \Delta\varepsilon_{z,1-4}/\Delta\sigma'_{z,1-4} = 0.167/20 = 0.00837 \quad (1/\text{kPa})
\end{aligned}
$$

$$
c_v = k/(\gamma_w m_v) = 1.9\times10^{-4}/(9.81\times0.00837) = 2.316\times10^{-3}\ (\text{m}^2/\text{day}) = 0.845\ (\text{m}^2/\text{year})
$$

This 4m thick layer can be divided into small sub-layers. The stresses and values of soil parameters in each sub-layer are assumed be constant. In this case, simple equations in (6b) can be used to calculate that the final "primary" settlement $S_{f,1-4}$ in each sub-layer. This layer of 4m can divided into 2, 4, or 8 sub-layers with thickness of 2m, 1m, and 0.5m respectively. The final "primary" settlement $S_{f,1-4}$ calculated is 0.487m, 0.573m, 0.625m, and 0.646 m sub-layers with thickness of 4m, 2m, 1m, and 0.5m respectively. Values of S_f, m_v and c_v for sub-layer thickness of 0.5 m for OCR=1.5 are listed in Table 4. The meanings of ε_{zp}, $\bar{\varepsilon}_{zp}$, $\Delta\varepsilon_z$, $\Delta\bar{\varepsilon}_z$, and $(\bar{\varepsilon}_{zp}+\Delta\bar{\varepsilon}_z)$ are the same as those in Table 2. Summary of values of S_f, m_v and c_v for different number of sub-layers for OCR=1.5 are listed in Table 5 including S_f obtained by more accurate integration method. It can be seen that the relative error of S_f with sub-layer thickness of 0.5 m is only $\frac{0.669-0.646}{0.669}\times 100\% = 3.4\%$.

The simplified Hypothesis B method in (1) together with other equations on relevant parameters is used to calculate the total settlement S_{totalB} using $\alpha = 0.8$ and $\beta = 0$ (denoted B Method 1) , $\beta = 0.3$ (denoted B Method 2), and $\beta = 1$ (denoted B Method 3) for OCR=1.5. The calculated curves of settlements with log(time) from the simplified Hypothesis B method are shown in Figure 5(b) for time up to 100 years. At the same time, Hypothesis A method and two fully coupled finite element models are used to calculate the curves of settlements with log(time) which are also shown in Figure 5(b) for comparison. It is seen from Figure 5(b) that when $\alpha = 0.8$ and $\beta = 0.3$ m, B Method 2 gives curves much closer to the curves from the two finite element models of "Consol" by Zhu and Yin (1999, 2000) and Plaxis software (2D 2015 version). Again, Hypothesis A method underestimates the total settlement.

4. Consolidation Settlements of Layered Soils with Vertical Drains under Staged Loading-Unloading-Reloading from General Simple Method and Fully Coupled Consolidation Analysis

4.1. Description of soil conditions

In this section, easy use and accuracy of the general simple method is demonstrated through calculation of consolidation settlements of a multiple-layered soil under multi-staged loadings with comparison with values from fully coupled FE simulations. The soil profile is modified from a real case in Hong Kong (Koutsoftas *et al*. 1987; Zhu *et al*. 2001) as shown in Figures 6 and 7. This section only studies the first two layers, namely upper marine clay of 6.22 m thick and upper alluvium of 5.80 m thick. To make the consolidation analysis more accurate and to record accumulated settlement at different depths, the upper marine clay layer is divided into two layers by Sondex anchor 3, forming a total of three layers of soils. Properties of upper marine clay and upper alluvium can be found in Table 6.

Prefabricated vertical drains (PVDs) with a spacing of 1.5m in triangular pattern were inserted in the soils. The radius of influence zone of each PVD was $r_e = 0.525d = 0.7875\text{m}$ for triangular pattern. The width of PVD was b = 100mm, thickness was t = 7mm, and equivalent radius is calculated as $r_d = \left(b+t\right)/4 + t/10 = 27.45$ mm (Yin and Zhu 2020). The installation of PVDs normally causes a smear zone around the vertical drains as shown in Figure 7. We assume that radius of this smear zone $r_s = 5r_d = 137.25$ mm, in which the soils were disturbed and the horizontal permeability k_r became k_s with values listed in Table 6. Other properties such as OCR and compression indices of the smear zone remain the same as the undisturbed region.

There are four stages of loadings to be applied on top of the soils, including two stages of loading, one stage of unloading and the final stage of reloading. The magnitude of vertical load (p_1, p_2, p_3, p_4), construction time ($t_{c1}, t_{c2}, t_{c3}, t_{c4}$) and loading stage duration (t_1, t_2, t_3, t_4) are shown in Figure 8(a). This type of staged loading is very close the real case of reclamation process from loading (filling to a designed level), surcharging fill (added pressure), unloading by removing part of surcharging fill, and re-loading again due to construction of superstructures on reclaimed land. The final stage of loading (superstructures) may last for 50 years (18250 days) after completion of reclamation construction. To validate the general simplified Hypothesis B method, a fully coupled finite element (FE) analysis is conducted in Plaxis 2D (2015) for this case. A Soft Soil Creep (SSC) model (Vermeer and Neher 1999), which is mostly similar to the 3D EVP model by Yin and Graham (1999), is adopted as the constitutive model for the two clayey soils in Figure 7. The parameters used in the FE model for the two soils are the same as those in Table 6. Accumulated settlements at settlement monitoring points 1, 3 and 5 (depths of 0 m, 3 m, and 6m respectively) are calculated using the general simple method and the FE model and are plotted with total elapsed time. Excess pore pressures at the center of each layer and at the middle between r_d and r_e are calculated by the FE model during the whole consolidation process.

4.2. Consolidation settlement calculation by general simple method under staged loadings

This section shows details with steps how to use the general simple method to calculate consolidation settlements of Case 2 under staged loading-unloading-reloading. The total consolidation settlements are summation of "primary" consolidation settlement and creep settlements in Eq.(1). For four stages of loading, details of calculations are presented below.

Stage 1

As shown in Figure 8(a), for Stage 1 under $p_1 = 52\text{kPa}$, the stress-strain state will move from point i $(\sigma_{zi}^{'}, \varepsilon_{zi})$ to point 1 $(\sigma_{z1}^{'}, \varepsilon_{z1})$ as in Figure 8(b). The calculation method of S_f is similar to the case of load increment from point 1 to 2 or point 1 to 4 in Figure 3. Due to the nonlinear strain-stress relationship of soils and non-uniform stress distribution, each *j*-layer (*j* = 1, 2, 3 for 3 layers) is divided into several sub-layers (say *N* sub-layers) with a thickness of

h_n (0.5 m or less) to calculate S_f and m_v. Within each sub-layer, initial effective stress σ'_{zi} can be considered as constant. The final vertical effective stress at Stage 1 is calculated as $\sigma'_{z1} = \sigma'_{zi} + p_1$ for each sub-layer. The settlement for each *j*-layer will be the superposition of settlements of all sub-layers ($n = 1 \ldots N$). Therefore, S_{fj1} and m_{vj1} for *j*-layer with thickness H_j in Stage 1 (sub-index "$_1$" for Stage 1; later "$_2$", "$_3$", "$_4$" for Stages 2, 3, and 4) are calculated in the following equations:

$$S_{fj1} = \sum_{n=1}^{n=N} \begin{cases} \dfrac{C_r}{1+e_o} \log\left(\dfrac{\sigma'_{z1} + \sigma'_{unit1}}{\sigma'_{zi} + \sigma'_{unit1}}\right) h_n, & \text{if } \sigma'_{z1} \le (\sigma'_{zi} + POP) \\ \left[\dfrac{C_r}{1+e_o} \log\left(\dfrac{\sigma'_{zi} + POP + \sigma'_{unit1}}{\sigma'_{zi} + \sigma'_{unit1}}\right) + \dfrac{C_c}{1+e_o} \log\left(\dfrac{\sigma'_{z1} + \sigma'_{unit2}}{\sigma'_{zi} + POP + \sigma'_{unit2}}\right)\right] h_n, & \text{if } \sigma'_{z1} > (\sigma'_{zi} + POP) \end{cases} \tag{19a}$$

$$m_{vj1} = \frac{\varepsilon_{z1} - \varepsilon_{zi}}{p_1} = \frac{S_{fj1}}{H_j p_1} \tag{19b}$$

where *n* is index for sub-layers within *j*-layer ($n = 1 \ldots N$), h_n is thickness of a sub-layer ($h_n \le 0.5\text{m}$), POP in Table 6 is called Pre-Overconsolidation Pressure (after Zhu *et al.* 2001) and $\text{POP} = \sigma'_{zp} - \sigma'_{zi}$. Eq.(19) is valid for the final state $(\sigma'_{z1}, \varepsilon_{z1})$ in either over-consolidation (OC) state or Normal consolidation (NC) state.

Values of S_{f1} and m_{v1} for three layers $(H_j, j = 1,2,3)$ under Stage 1 are calculated using Eq.(19) and listed in Table 7. After this, a Microsoft Excel spreadsheet with macros based on a spectral method developed by Walker and Indraratna (2009) is used to calculated the average excess porewater pressure $\bar{u}_{ej}$ for *j*-layer using known values of k_v, k_r, and k_s in Table 6 and the calculated m_{vj1} in Table 7. The average degree of consolidation U_{j1} for *j*-layer for Stage 1 is then calculated using Eq.(10b). Using calculated values of U_{j1} and S_{fj1}, the "primary" consolidation settlement $S_{primary1}$ in Stage 1 is calculated as:

$$S_{primary} = \sum_{j=1}^{3} S_{primary,j} = \sum_{j=1}^{3} U_{j1} S_{fj1} \tag{20}$$

To calculate creep settlement S_{creep1} during Stage 1, the equivalent time in Yin and Graham's 1D EVP model should be determined according to the final stress-strain state $(\sigma_{z1}^{'}, \varepsilon_{z1})$ for each sub-layer. If the soil is in normal consolidation state (*i.e.* $\sigma_{z1}^{'} \geq (\sigma_{zi}^{'} + POP)$), equivalent time t_{e1} at the "final" effective stress $\sigma_{z1}^{'}$ in Stage 1 is zero. If the soil is in OC state (*i.e.* $\sigma_{z1}^{'} < (\sigma_{zi}^{'} + POP)$), t_{e1} should be calculated as:

$$t_{e1} = t_o \times 10^{(\varepsilon_{z1} - \varepsilon_{zp})\frac{V}{C_{\alpha e}}} (\frac{\sigma_{z1}^{'}}{\sigma_{zp}^{'}})^{-\frac{C_c}{C_{\alpha e}}} - t_o \tag{21}$$

where $\varepsilon_{z1} = \varepsilon_{zi} + S_{fj1} / H_j$ is the "final" strain without creep at Stage 1. In fact, Eq.(21) is also valid for NC state. The value of t_{e1} is calculated for each sub-layer h_n. Therefore, $S_{creep,fj1}$, $S_{creep,dj1}$, $S_{creepj1}$ and total settlement $S_{totalBj1}$ for each *j*-layer, no mater the "final" stress-strain point is in OC or NC state, can be calculated using the following equations:

$$S_{creep,fj1} = \sum_{n=1}^{n=N} \frac{C_{\alpha e}}{1+e_o} \log \frac{t_{e1}+t}{t_{e1}+t_o} h_n \quad \text{for } t_o \leq t \leq t_1 \tag{22a}$$

$$S_{creep,dj1} = \sum_{n=1}^{n=N} \frac{C_{\alpha e}}{1+e_o} \log \frac{t_{e1}+t}{t_{e1}+t_{EOP,field}} h_n \quad \text{for } t_{EOP,field} \leq t \leq t_1 \tag{22b}$$

$$S_{creepj1} = \alpha U_{j1}^{\beta} S_{creep,fj1} + \left(1 - \alpha U_{j1}^{\beta}\right) S_{creep,dj1} \tag{22c}$$

$$S_{totalBj1} = U_{j1} S_{fj1} + \left[\alpha U_{j1}^{\beta} S_{creep,fj1} + \left(1 - \alpha U_{j1}^{\beta}\right) S_{creep,dj1}\right] \tag{22d}$$

In this case, Eq.(22d) is used to calculate $S_{totalBj1}$ for *j*-layer in Stage 1 with $\alpha = 0.8$ and $\beta = 0.3$. Using Eq.(1), the total settlement $S_{totalB1}$ of 3 layers in Stage 1 is

$$S_{totalB1} = \sum_{j=1}^{j=3} S_{totalBj1} = \sum_{j=1}^{j=3} U_{j1} S_{fj1} + \left[\alpha U_{j1}^{\beta} S_{creep,fj1} + \left(1 - \alpha U_{j1}^{\beta}\right) S_{creep,dj1}\right] \tag{22e}$$

Stage 2

For Stage 2 with $p_2 = 100\text{kPa}$, the final vertical effective stress σ'_{z2} is $\sigma'_{z2} = \sigma'_{z1} + p_2 = \sigma'_{zi} + p_1 + p_2$ as in Figure 8(b) at point 2 $(\sigma'_{z2}, \varepsilon_{z2})$. The calculation of S_{fj} is dependent on the soil stress-strain state before and after loading increment as below:

$$S_{fj2} = \sum_{n=1}^{n=N} \begin{cases} \dfrac{C_r}{1+e_o} \log\left(\dfrac{\sigma'_{z2} + \sigma'_{unit1}}{\sigma'_{z1} + \sigma'_{unit1}}\right) h_n, & \text{if } \sigma'_{z1} < \sigma'_{z2} \le (\sigma'_{zi} + POP) \\ \left[\dfrac{C_r}{1+e_o} \log\left(\dfrac{\sigma'_{zi} + POP + \sigma'_{unit1}}{\sigma'_{z1} + \sigma'_{unit1}}\right) + \dfrac{C_c}{1+e_o} \log\left(\dfrac{\sigma'_{z2} + \sigma'_{unit2}}{\sigma'_{zi} + POP + \sigma'_{unit2}}\right)\right] h_n, & \text{if } \sigma'_{z1} < (\sigma'_{zi} + POP) < \sigma'_{z2} \\ \dfrac{C_c}{1+e_o} \log\left(\dfrac{\sigma'_{z2} + \sigma'_{unit2}}{\sigma'_{z1} + \sigma'_{unit2}}\right) h_n, & \text{if } (\sigma'_{zi} + POP) \le \sigma'_{z1} < \sigma'_{z2} \end{cases} \quad (23a)$$

$$m_{vj2} = \frac{\varepsilon_{z2} - \varepsilon_{z1}}{p_2} = \frac{S_{fj2}}{H_j p_2} \quad (23b)$$

where $\varepsilon_{z2} = \varepsilon_{z1} + S_{fj2} / H_j$ is the final accumulated vertical strain without creep strain at Stage 2. Values of S_{f2} and m_{v2} for three layers $(H_j, j = 1,2,3)$ under Stage 2 are listed in Table 7. In this stage, average degree of consolidation for each layer U_{j1} under p_1 and U_{j2} under p_2 should be calculated independently using Walker and Indraratna (2009)'s spectral method. For U_{j1}, the staged-consolidation time at Stage 2 should be from t_1 to $(t_1 + t_2)$. For U_{j2}, the staged-consolidation time at Stage 2 should be from 0 to t_2. Total $S_{primary}$ should include the settlements produced by p_1 and p_2 with total time below:

$$S_{primary} = \sum_{j=1}^{3} S_{primary,j} = \sum_{j=1}^{3} (U_{j1} S_{fj1} + U_{j2} S_{fj2}) \quad (24)$$

$S_{creepj2}$ only includes the creep settlement at the current loading stage under p_2 (*i.e.* $t_o < t < t_2$ for $S_{creep,f}$ and $t_{EOP,field} < t < t_2$ for $S_{creep,d}$). To calculate $S_{creepj2}$, the actual stress-strain state at Stage 2 and its corresponding equivalent time t_{e2} should be determined.

First of all, the final creep strain $\varepsilon_{z,creep1}$ shown in Figure 8(b) and accumulated total strain ε_{z1end} at the end of Stage 1 (point $1''$) should be calculated by the following equations:

$$\varepsilon_{z,creep1} = \frac{S_{creepj,1}\left(t_1\right)}{H_j} \tag{25a}$$

$$\varepsilon_{z1end} = \varepsilon_{z1} + \varepsilon_{z,creep1} \tag{25b}$$

The new apparent pre-consolidation pressure σ'_{zp1} and the corresponding strain ε_{zp1} at the end of Stage 1 shown in Figure 8(b) due to previous creep (or ageing) should be calculated by solving the following two equations:

$$\varepsilon_{zp1} = \varepsilon_{z1end} + \frac{C_r}{1+e_o}\log\frac{\sigma'_{zp1}+\sigma'_{unit1}}{\sigma'_{z1}+\sigma'_{unit1}} \tag{26a}$$

$$\varepsilon_{zp1} = \varepsilon_{zp} + \frac{C_c}{1+e_o}\log\frac{\sigma'_{zp1}+\sigma'_{unit2}}{\sigma'_{zp}+\sigma'_{unit2}} \tag{26b}$$

From Eqs.(26a)-(26b), the apparent pre-consolidation pressure σ'_{zp1} can be solved as:

$$\sigma'_{zp1} = \frac{\left(\sigma'_{zp}+\sigma'_{unit2}\right)^{\frac{C_c}{C_c-C_r}}}{\left(\sigma'_{z1}+\sigma'_{unit1}\right)^{\frac{C_r}{C_c-C_r}}} \times 10^{\left(\varepsilon_{z1end}-\varepsilon_{zp}\right)\frac{1+e_o}{C_c-C_r}} - \sigma'_{unit1} \tag{26c}$$

where σ'_{unit1} is assumed equal to σ'_{unit2} here. If σ'_{zp1} is known, ε_{zp1} can be calculated using Eq.(26a) or (26b). With p_2 applied, if $(\sigma'_{z1}+p_2)=\sigma'_{z2}\geq\sigma'_{zp1}$ (*i.e.* the soil is in NC state at point 2_{NC}) as in Figure 8(b), the equivalent time $t_{e2}=0$. Otherwise, $\sigma'_{z2}\leq\sigma'_{zp1}$, as the case of point 2_{OC} in OC state as shown in Figure 8(b), t_{e2} at σ'_{z2} should be calculated as:

$$t_{e2} = t_o \times 10^{(\varepsilon_{z2}-\varepsilon_{zp})\frac{V}{C_{\alpha e}}} \left(\frac{\sigma'_{z2}}{\sigma'_{zp}}\right)^{-\frac{C_c}{C_{\alpha e}}} - t_o \tag{27}$$

where $\varepsilon_{z2} = \varepsilon_{z1end} + \frac{C_r}{1+e_o}\log\frac{\sigma'_{z2} + \sigma'_{unit1}}{\sigma'_{z1} + \sigma'_{unit1}}$ is the vertical strain at point 2_{OC} in OC state, before creep at the beginning of Stage 2 loading. The value of t_{e2} is calculated for each sub-layer with thickness h_n for the point in either NC state or OC state. Therefore, $S_{creep,fj2}$ and $S_{creep,dj2}$ for each *j*-layer can be calculated using the following equations:

$$S_{creep,fj2} = \sum_{n=1}^{n=N} \frac{C_{\alpha e}}{1+e_o}\log\frac{t_{e2}+t}{t_{e2}+t_o} h_n \quad \text{for } t_o \le t \le t_2 \tag{28a}$$

$$S_{creep,dj2} = \sum_{n=1}^{n=N} \frac{C_{\alpha e}}{1+e_o}\log\frac{t_{e2}+t}{t_{e2}+t_{EOP,field}} h_n \quad \text{for } t_{EOP,field} \le t \le t_2 \tag{28b}$$

However, since $S_{creepj2}$ is calculated from the current stress-strain state under $(p_1 + p_2)$ loading, U_j in Eq.(22c) should be replaced by the accumulated average degree of consolidation $U_{multi,j2}$ for multi-stages of loadings, which is calculated by:

$$U_{multi,j2} = \frac{(U_{j2}p_1 + U_{j2}p_2)}{p_1 + p_2} \tag{29}$$

Finally, the total consolidation settlements for *j*-layer and for all three layers in the period of Stage 2 are calculated by:

$$S_{totalBj2} = U_{multi,j2}S_{fj2} + \left[\alpha U^{\beta}_{multi,j2}S_{creep,fj2} + \left(1-\alpha U^{\beta}_{multi,j2}\right)S_{creep,dj2}\right] \tag{30a}$$

$$S_{totalB2} = \sum_{j=1}^{j=3} S_{totalBj2} = \sum_{j=1}^{j=3} U_{multi,j2}S_{fj2} + \left[\alpha U^{\beta}_{multi,j2}S_{creep,fj2} + \left(1-\alpha U^{\beta}_{multi,j2}\right)S_{creep,dj2}\right] \tag{30b}$$

Stage 3

For Stage 3 of unloading $p_3 = -116\text{kPa}$, S_{fj3}, m_{vj3} and U_{j3} are calculated using the same procedures as Stages 1 and 2. It should be noted that, for this unloading stage, S_{fj3} is simply calculated by:

$$S_{fj3} = \sum_{n=1}^{n=N} \frac{C_r}{1+e_o}\log(\frac{\sigma'_{z3} + \sigma'_{unit1}}{\sigma'_{z2} + \sigma'_{unit1}})h_n \tag{31}$$

where $\sigma'_{z2} = \sigma'_{zi} + p_1 + p_2$, $\sigma'_{z3} = \sigma'_{zi} + p_1 + p_2 + p_3$, and $\sigma'_{z3} < \sigma'_{z2}$. As shown in Figure 8(b), point 3_{OC} must be in an OC state, but point 3_{OC} may be reached by from the end of creep at point 2_{NC}. However, point 2 could be at point 2_{OC} in an OC state. If this case, Eq.(31) can still be used. Under unloading condition, as both S_{fj3} and p_3 are negative, m_{v3} is still positive, and therefore the spectral method can be normally used to compute the degree of consolidation. The calculation of $S_{primary,j}$ should contain the settlements produced in the previous stages during the current stage period in the following equation:

$$S_{primary,j} = U_{j1}S_{fj1} + U_{j2}S_{fj2} + U_{j3}S_{fj3} \tag{32}$$

where U_{j1}, U_{j2} and U_{j3} are the average degree of consolidation under (i) p_1 from $(t_1 + t_2)$ to $(t_1 + t_2 + t_3)$, (ii) p_2 from t_2 to $(t_2 + t_3)$, (ii) p_3 from 0 to t_3 respectively. Calculation of $S_{creepj3}$ follows similar procedures as in Stage 2, not be elaborated here. $U_{multi,j3}$ for calculating creep settlement should be calculated as:

$$U_{multi,j3} = \frac{(U_{j1}p_1 + U_{j2}p_2 + U_{j3}p_3)}{p_1 + p_2 + p_3} \tag{33}$$

Stage 4

For Stage 4 with $p_4 = 74\text{kPa}$, similar procedures as those for Stages 1 and 2 are used for calculations of S_{fj4}, U_{j4} and $S_{creepj4}$. But $S_{primary,j}$ and $U_{multi,j4}$ should be calculated as:

$$S_{primary,j} = U_{j1}S_{fj1} + U_{j2}S_{fj2} + U_{j3}S_{fj3} + U_{j4}S_{fj4} \tag{34}$$

$$U_{multi,j4} = \frac{(U_{j}p_1 + U_{j2}p_2 + U_{j3}p_3 + U_{j4}p_4)}{p_1 + p_2 + p_3 + p_4} \tag{35}$$

where U_{j1}, U_{j2}, U_{j3}, and U_{j4} is the average degree of consolidation under (i) p_1 from $(t_1+t_2+t_3)$ to $(t_1+t_2+t_3+t_4)$, (ii) p_2 from (t_2+t_3) to $(t_2+t_3+t_4)$, (iii) p_3 from t_3 to (t_3+t_4), and (iv) p_4 from 0 to t_4.

The values of S_{fj} and m_{vj} for Stages 1 to 4 are listed in Tables 7. Using the spectral method and Eqs. (10b), (24), (26) and (28), the average degree of consolidation degree $U_{multi,j}$ for each layer during four stages is calculated and plotted with time in Figure 9.

4.3. Comparisons of results from the general simple method and fully coupled FE analysis

Figure 10 shows the computed settlements at three measurement points (0, 3 and 6m) by both simplified Hypothesis B method and FE analysis. It can be found that the settlements at three different depths are close to those computed by FE analysis under four stages of loading, unloading and re-loading. The settlements in 50 years in Stage 4 are very small. This is because the soils are in over-consolidation state in Stage 4 due to the surcharge in Stage 2. The results demonstrated that surcharge loading before construction will significantly reduce long-term post-construction settlements.

Figure 11 shows the average excess porewater pressure $\bar{u}_j$ at the center of each soil layer, compared with the computed excess porewater pressure at the abovementioned measurement points in the FE model. It is found that excess porewater pressure computed by the spectral method adopted in the general simple method fit well with the one simulated by FE model. In conclusion, the proposed simplified Hypothesis B method is close to fully coupled FE analysis for the case with multiple layered soils under multi-staged loading conditions.

5. Consolidation Settlements of Test Embankment on Layered Soils with Vertical Drains under Staged Loading from General Simple Method, Fully Coupled Consolidation Analyses, and Measurement

5.1. General descriptions of the test embankment

In this section, Test Embankment at Chek Lap Kok for Hong Kong International Airport (HKIA) project in 1980s is used as an example to demonstrate the validity of the new general simple method. Consolidation settlements of this HKIA Chek Lap Kok Test Embankment are calculated using the new general simplified Hypothesis B method and are compared with measured data and values from the simplified finite element (FE) method reported by Zhu, *et al.* (2001). Details of the site conditions, properties of soils, parameters of vertical drains, construction process, parameters used in the FE model can be found in Koutsoftas *et al*. (1987) and Zhu *et al*. (2001). The calculations of S_{fj}, $\Delta\sigma_z^{'}$, $\Delta\varepsilon_z$, m_{vj} and c_{vj} for each layer under three stages are listed in Table 8.

Figure 6 shows soil profile and settlement monitoring points of Chek Lap Kok Test Embankment (Handfelt *et al.* 1987; Koutsoftas *et al.* 1987). Elevation in mPD (meter in Principal Datum), depth coordinate, thickness values of four major layers, 8 settlement monitoring points by Sondex anchors, and 9 pore water pressure measurement points are all shown in Figure 6. In this section, only 4 points at depths 0 m, 3 m, 6 m, and 14.5 m are selected to calculate settlements for comparison with measured data.

Figure 7 shows soil profile and vertical drain with smear zone. It is noted that the vertical drain penetrated only 5.1 m into “Lower marine clay”. Therefore, “Lower marine clay” is divided into two layers: “Lower marine clay 1” with thickness of 5.1 m and “Lower marine clay 2” with thickness of 0.72 m in order to calculate the average degree of consolidation of each layer better.

Values of parameters of soils and vertical drains for HKIA Chek Lap Kok Test Embankment are listed in Table 6. For more accurate calculate of settlements and the average degree of consolidation, as well as convenient calculation at settlement monitoring points, “Upper marine clay” is divided into two main layers of $H_j = 3.01\ \text{m}$ and $3.21\ \text{m}$, “Lower marine clay 1” is divided into $H_j = 2.47\ \text{m}$ and $2.63\ \text{m}$, “Lower alluvium” is divided into two layers with

$H_j = 4.165$ m each. There are a total of 8 layers (j=1…8).

Figure 12 shows construction time ($t_{c1,}$ $t_{c2,}$ or t_{c3}), loading stage times ($t_{1,}$ $t_{2,}$ or t_3), and stage vertical pressures ($p_{1,}$ $p_{2,}$ or p_3) for each of three staged loadings. It should be noted that in-situ monitoring of settlements by Sondex anchors were started 65 days after the construction began. The in-situ settlement data from 65th to 909th day of total construction time were recorded and used for comparisons in this study.

5.2. Comparisons of results from the general simple method, fully coupled FE analyses, and measurement

In the general simplified Hypothesis B method, calculations of S_{fj}, m_v, U_j and S_{creepj} for each j-layer under three loading stages are completed in a Microsoft Excel spreadsheet in the same way as that in Section 4. In this case, $\alpha = 0.8$ and $\beta = 0.3$ are used, which is also the same as in previous sections.

The total consolidation settlements S_{totalB} at depths of 0m, 3m, 6 m, and 14.5 m are calculated using the general simplified Hypothesis B method for three stages of loading. Comparison of curves of settlements with accumulated time at depths of 0 m, 3 m, 6 m, and 14.5 m from the general simplified Hypothesis B method, fully coupled finite element modelling, and measurement are shown in Figure 13. It is found that the values from the general simplified Hypothesis B method are in good agreement with measured data and values from fully coupled finite element modelling (Zhu *et al.* 2001) using a 1-D Elastic Visco-Plastic (1-D EVP) model (Yin and Graham 1989, 1994).

6. Summary and Conclusions

A new general simplified Hypothesis B method, also called general simple method, is proposed and verified for calculating consolidation settlements of layered clayey soils exhibiting creep without or with vertical drains under complicated staged loadings. This method is a new un-coupled method compared with fully coupled consolidation methods. Equations of this general simple method incorporating a new logarithmic stress function which avoid singularity problem are rigorous derived. Excess pore water pressure in "primary consolidation" is calculated by using a spectral method implemented in a Microsoft Excel spreadsheet. Two parameters, namely α and β, are introduced in this method. All other parameters in this method are convectional parameters which can be easily determined from multi-staged oedometer tests. It is worthy to note that the two creep parameters $C_{\alpha e}$ and t_o are determined from a creep test under a vertical effective stress in a normal consolidation (NC) state. But, using the "equivalent time" (t_e) concept and theory of Yin and Graham (1989, 1994), the creep function using $C_{\alpha e}$ and t_o as well t_e can be used to calculate creep settlements in OC state and also in unloading/reloading states. Verification studies are carried out by comparing calculated values of settlements by this general simple method with values from fully coupled finite element analysis for Cases 1 and 2 as well as in-situ measured data for Case 3. Based on these works, following conclusions can be made.

(a) From the case study of a single soil layer with OCR = 1 or 1.5 under instantaneous vertical loading, calculated settlements by using the new general simple method agree well with values from fully coupled finite element (FE) analyses by Plaxis and Consol. Selection of $\alpha = 0.8$ and $\beta = 0.3$ is found to have the best performance compared to other selections. It is also clearly revealed that Hypothesis A method underestimates the total settlements.

(b) From the case study for double layered soils under multi-staged loading-unloading-reloading, consolidation settlements in either short-term or long-term period are very close to values from an FE analysis. It can be concluded that the proposed general simplified Hypothesis B method has a stable performance and good accuracy for layered soils under complicated staged loading schemes.

(c) The general simple method is applied to calculate consolidation settlements in a real case in HKIA Chek Lap Kok Test Embankment with multi-layered soils and vertical drains under multi-staged loading. Calculated settlements by this new simple method are in good agreement with in-situ measured data and also values from an FE analysis.

(d) Based on the above comparisons and validations, it is found that the new general simple method is accurate and easy to use for calculating consolidation settlements of single or layered soils with and without vertical drains under multi-staged loading, unloading and reloading using parameters from conventional oedometer tests.

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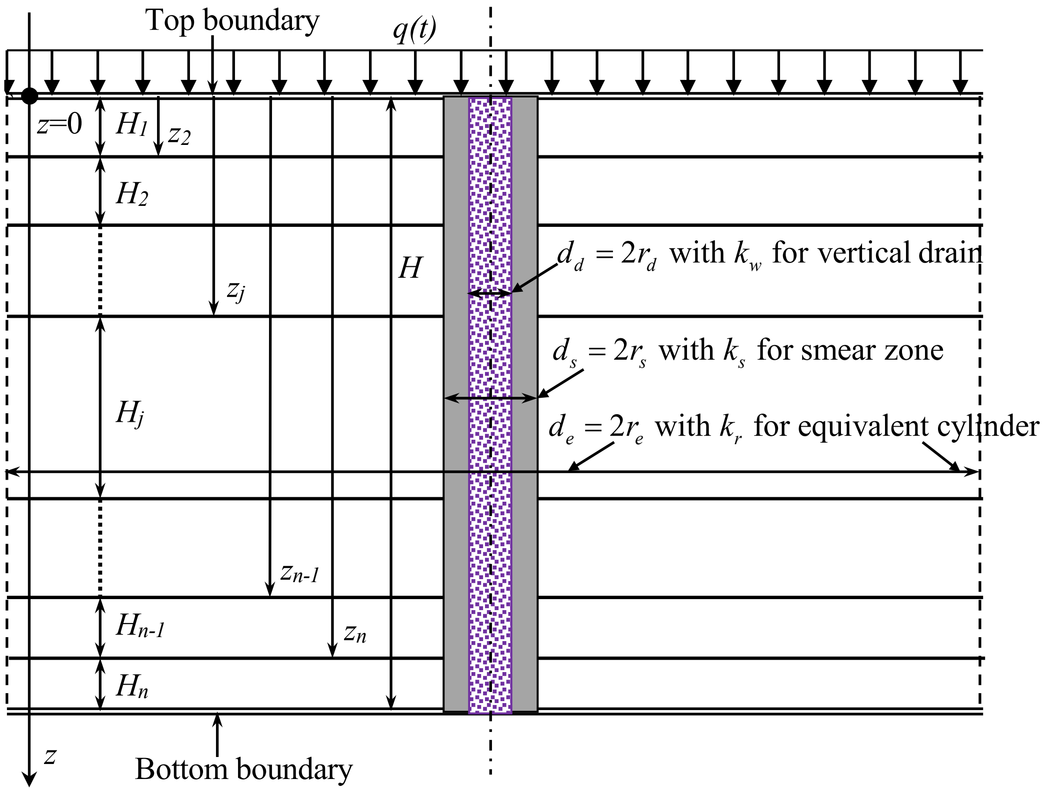


Figure 1. A soil profile of *n*-layers with vertical drain subjected to uniform surcharge *q(t)* with time

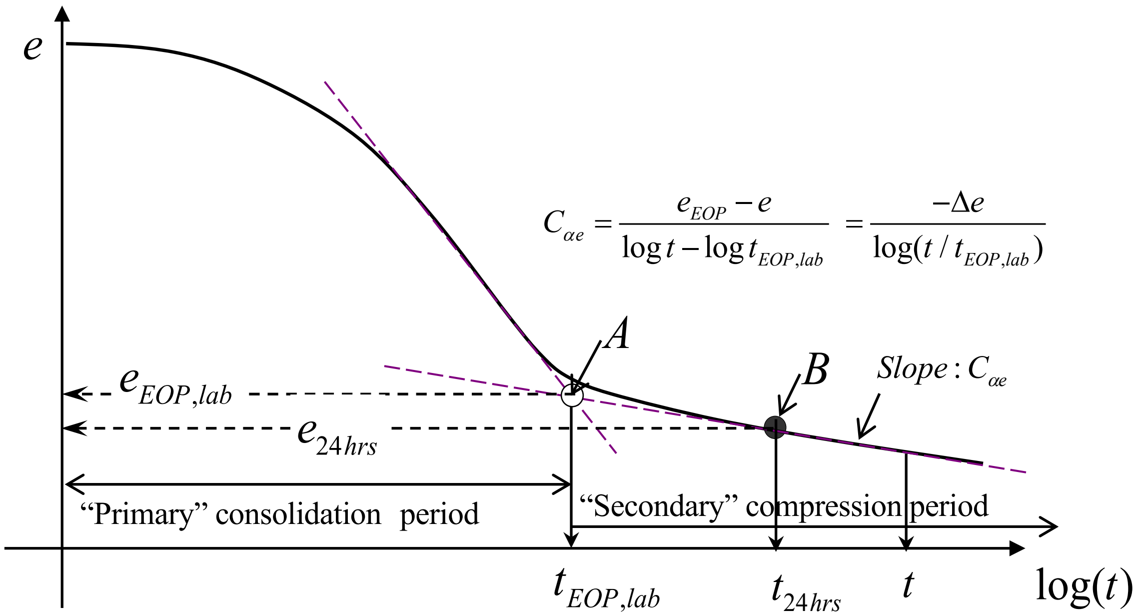


Figure 2. Curve of void ratio *versus* log (time) and "secondary" compression coefficient

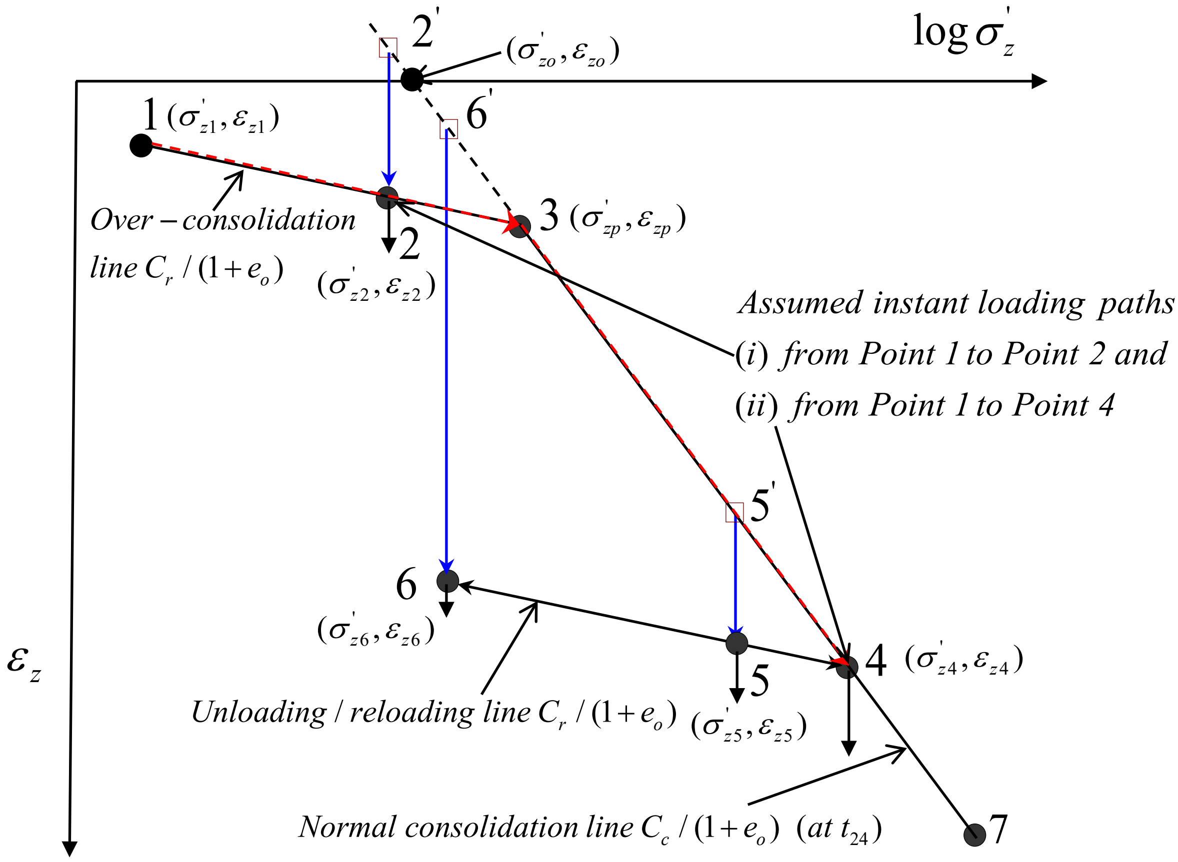


Figure 3. Relationship of strain (or void ratio) and log(effective stress) with different consolidation states

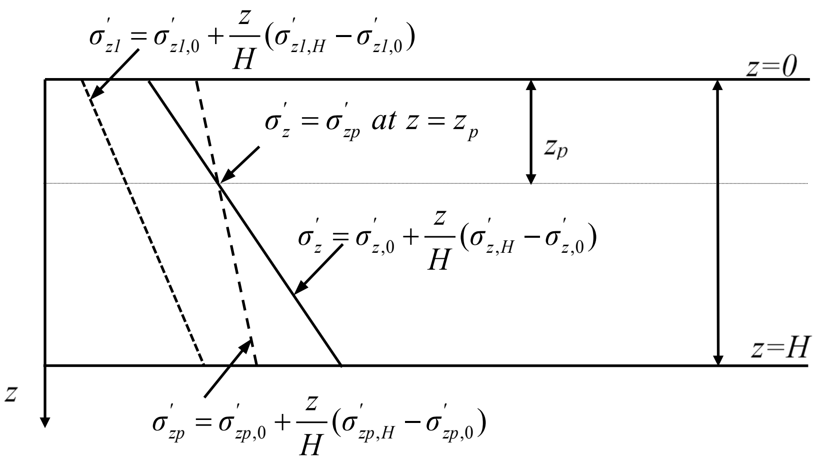


Figure 4. Linear changes of initial vertical effective stress (σ'_{z1}), total vertical effective stress (σ'_z), vertical pre-consolidation stress (σ'_{zp}) for a soil layer

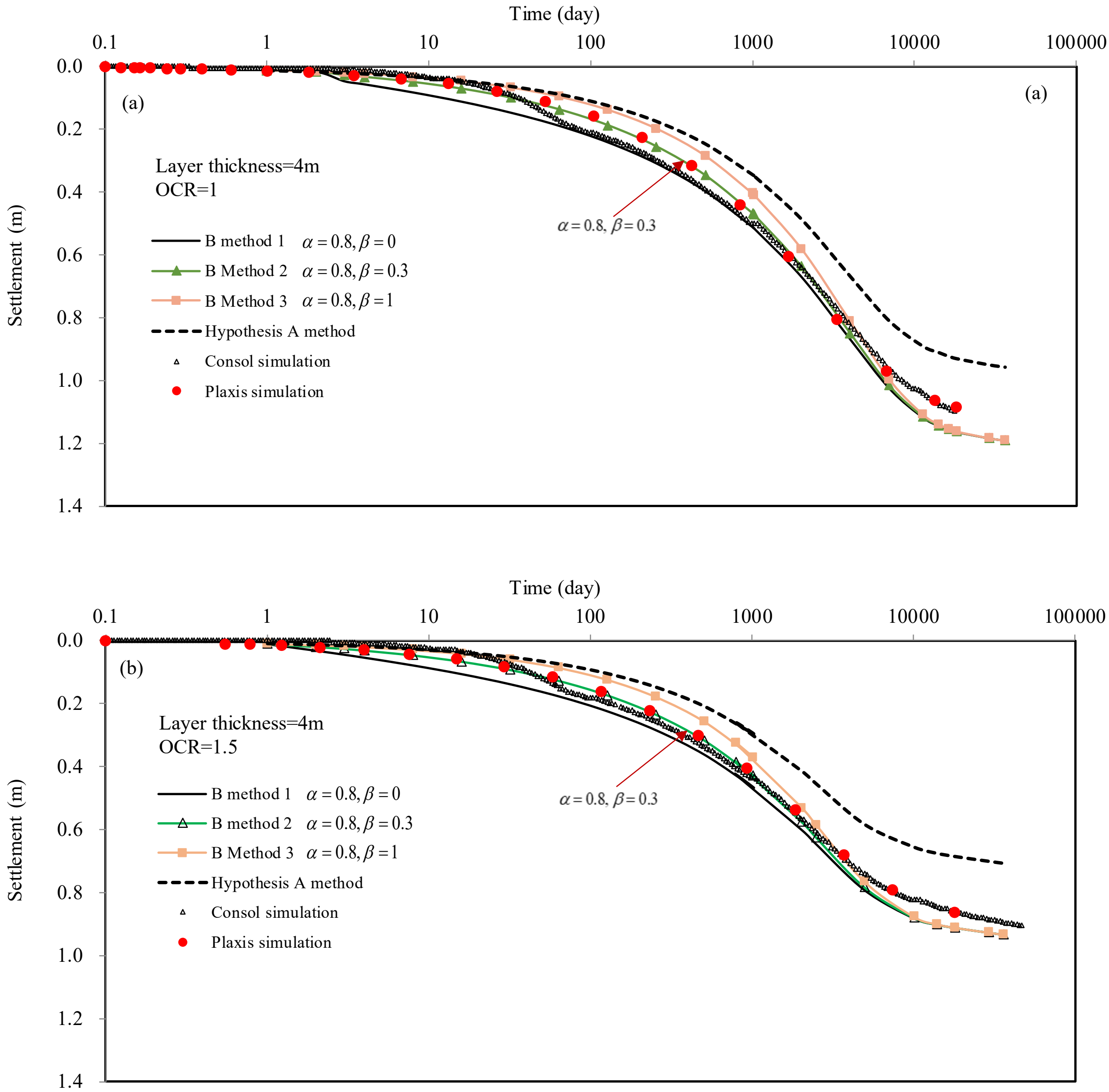


Figure 5. Comparison of curves of settlements with log(time) from the simplified Hypothesis B method, Hypothesis A method, and two fully coupled finite element modellings – (a) *h*=4m and OCR=1 and (b) *h*=4m and OCR=1.5

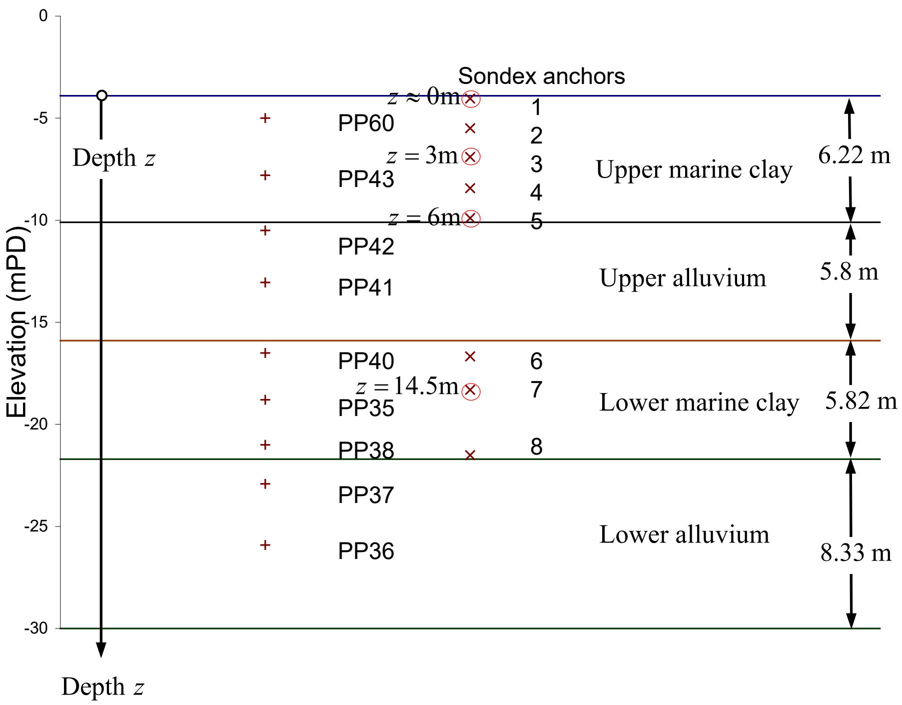
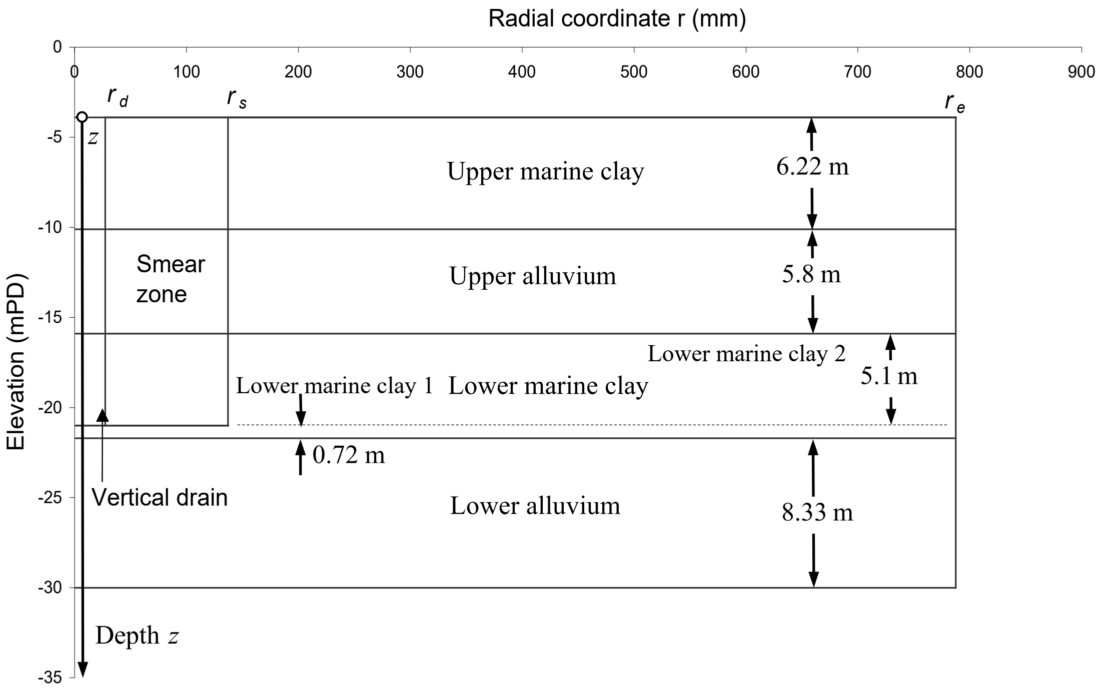
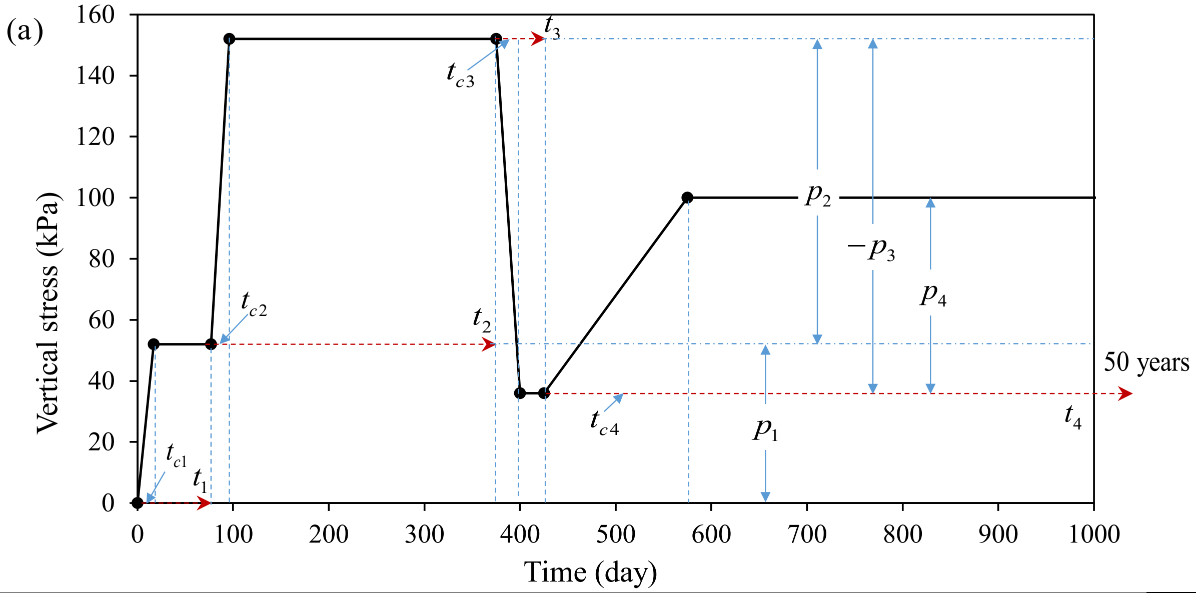


Figure 6. Soil profile and settlement monitoring points of a test embankment at Chek Lap Kok for Hong Kong International Airport project in 1980s



1023 Figure 7. Soil profile including a vertical drain and a smear zone of a test embankment at Chek Lap Kok for Hong Kong International Airport project in 1980s



	Stage 1	Stage 1	Stage 1	Stage 2	Stage 2	Stage 3	Stage 3	Stage 4	Stage 4
Time (day)	0	17	77	96	375	400	425	575	18675
Stage pressure (kPa)	0	52	52	152	152	36	36	100	100
t_c and stage ending time t_1, t_2, t_3, t_4 (day)	$t_{c1} = 17$, $t_1 = 77$			$t_{c2} = 19$, $t_2 = 298$		$t_{c3} = 25$, $t_3 = 50$		$t_{c4} = 150$, $t_4 = 18520$	

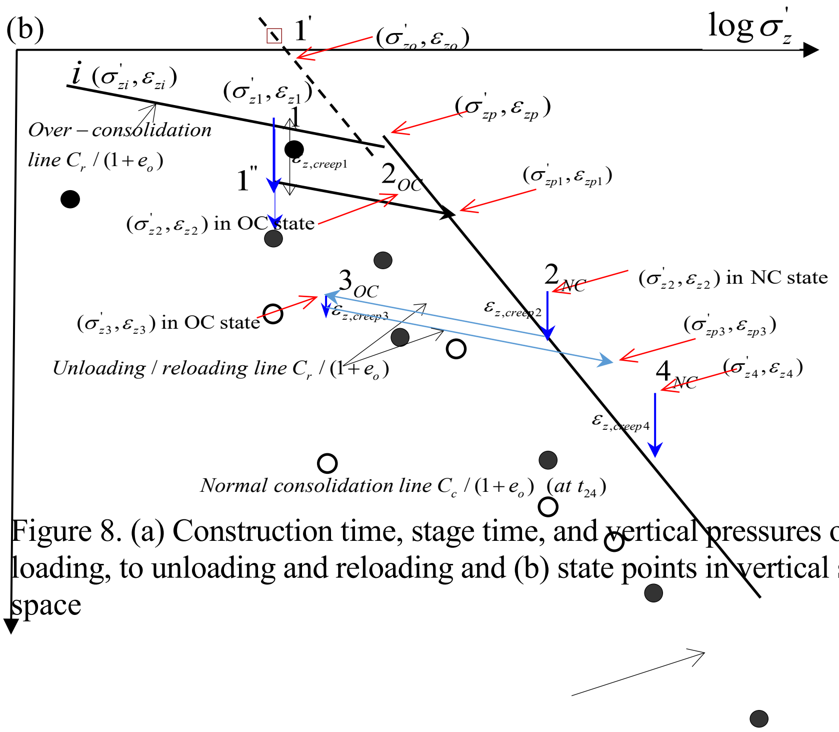


Figure 8. (a) Construction time, stage time, and vertical pressures of four staged loadings from loading, to unloading and reloading and (b) state points in vertical strain-log(effective stress) space

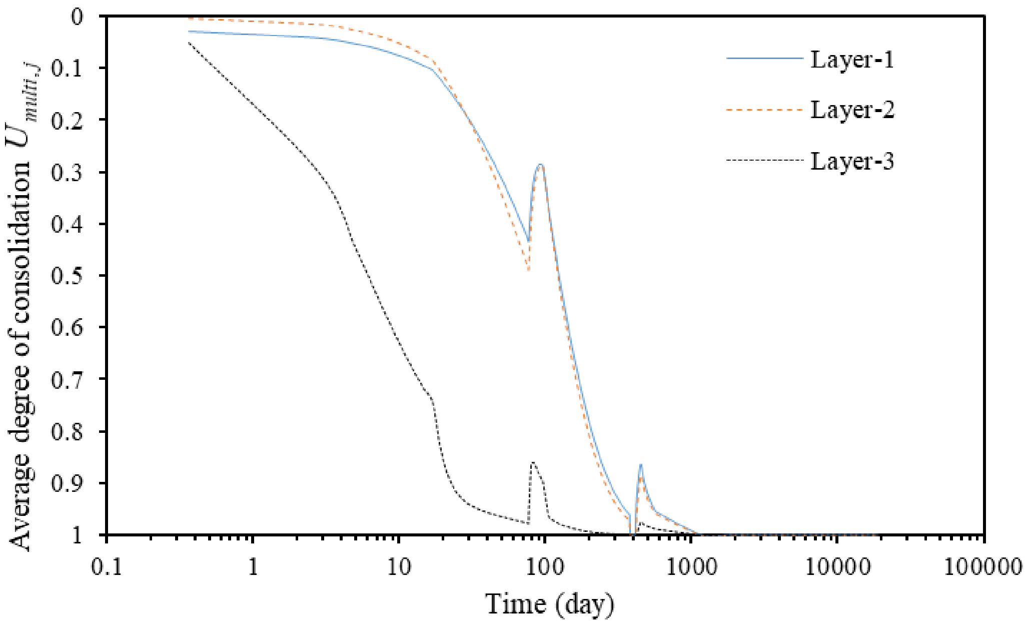


Figure 9. Calculated curves of $U_{multi,j}$ and total loading time in logarithmic scale for each *j*-layer under multi-staged four loadings

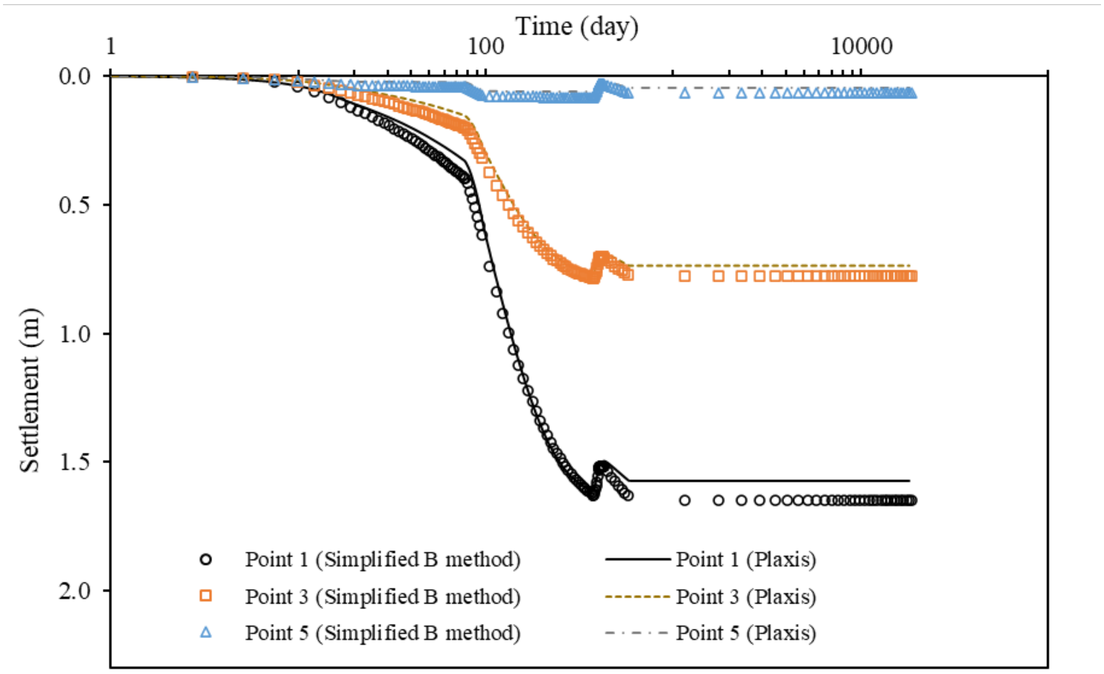


Figure 10. Comparison of settlements with accumulated total loading time in logarithmic scale at three settlement monitoring points at z=0 m, 3 m, and 6 m from the simplified Hypothesis B method and fully coupled finite element modelling

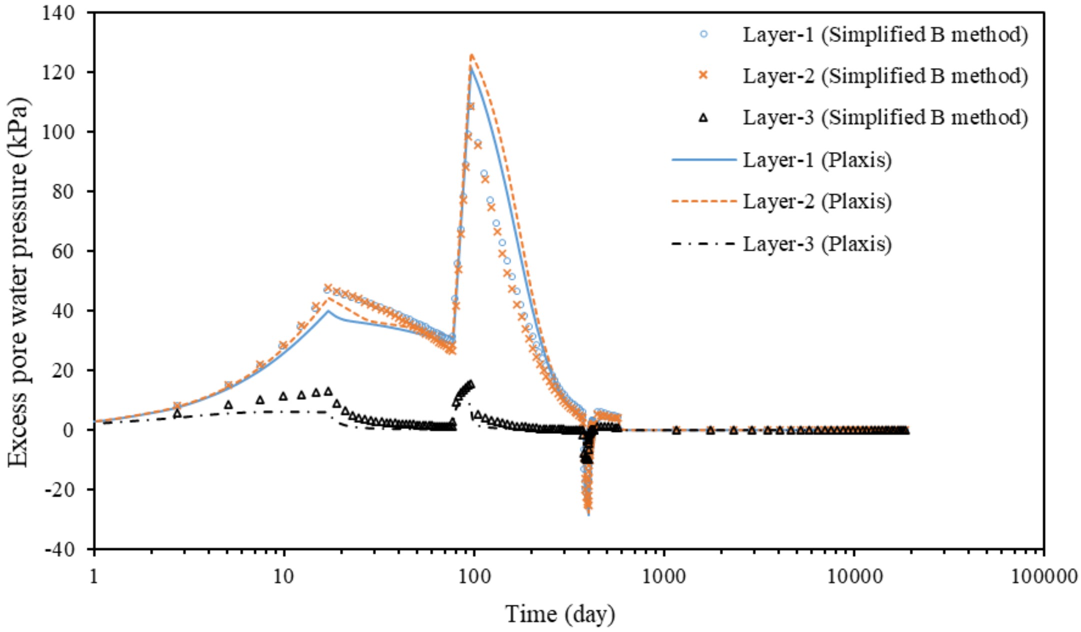
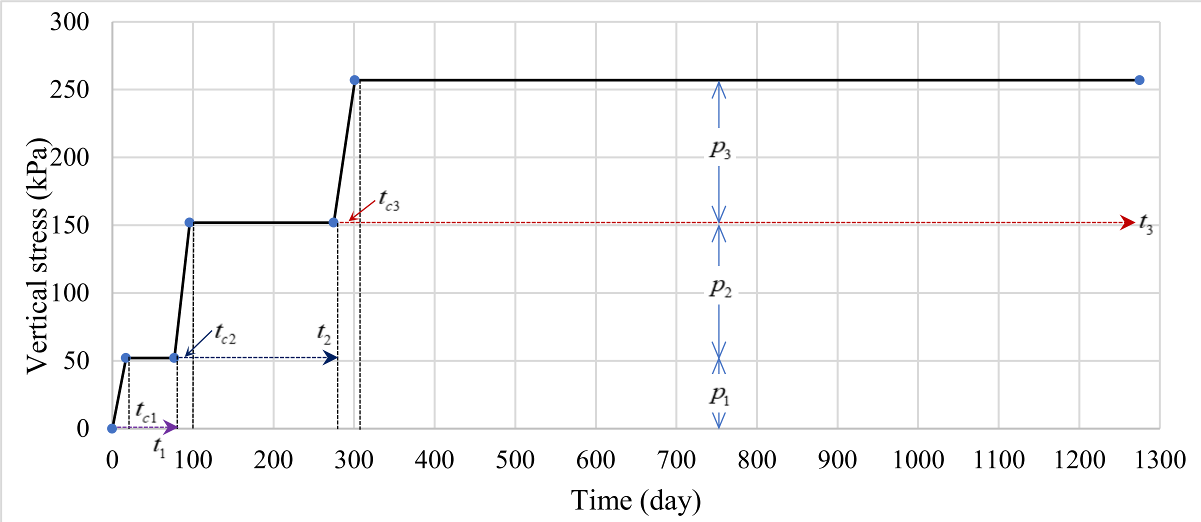


Figure 11. Comparison of excess porewater pressure with log(total loading time) for three layers from the general simplified Hypothesis B method and fully coupled finite element modelling



Stage pressure (kPa)	0	52	52	100	100	105	105
Total pressure (kPa)	0	52	52	152	152	257	257
t_c and stage ending time t_1, t_2, t_3 (day)	$t_{c1} = 17$, $t_1 = 77$			$t_{c2} = 19$, $t_2 = 198$		$t_{c3} = 26$, $t_3 = 1000$	

Figure 12. Construction time, stage time, and vertical pressures of three staged loadings in HKIA Chek Lap Kok Test Embankment

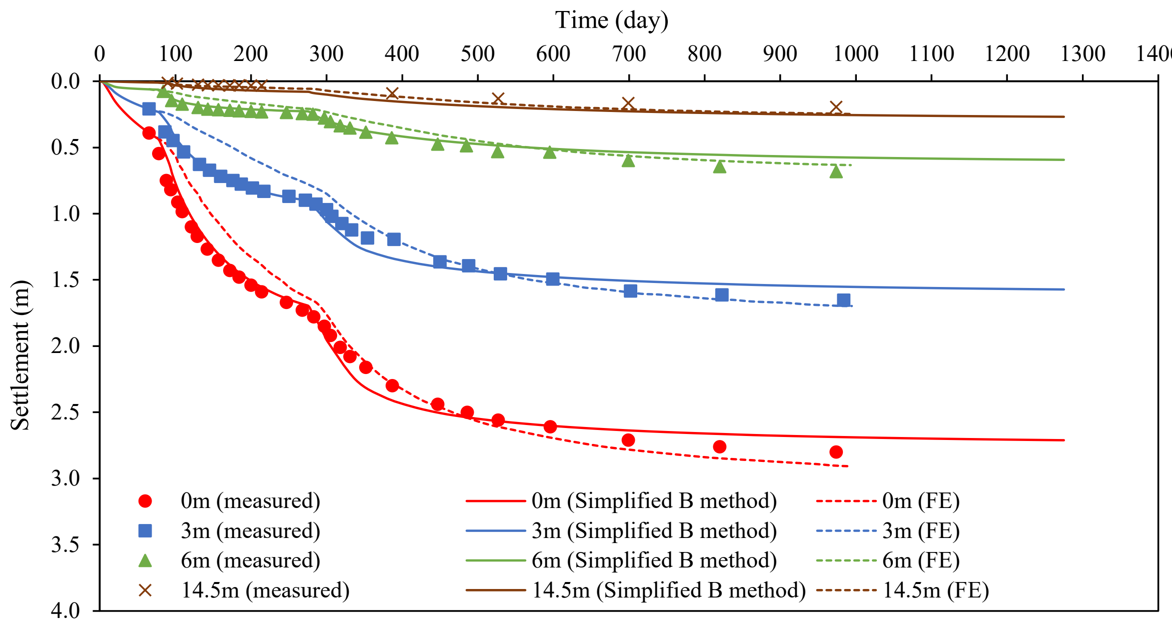


Figure 13. Comparison of curves of settlements with total loading time at depths 0 m, 3 m, 6 m, and 14.5 m from the general simplified Hypothesis B method, fully coupled finite element modelling, and measurement