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TWO-AGENT INTEGRATED SCHEDULING OF PRODUCTION AND DISTRIBUTION OPERATIONS WITH FIXED DEPARTURE TIMES

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Abstract. We consider integrated scheduling of production and distribution operations associated with two customers (agents). Each customer has a set of orders to be processed on the single production line at a supplier on a competitive basis. The finished orders of the same customer are then packed and delivered to the customer by a third-party logistics (3PL) provider with a limited number of delivery transporters. The number of orders carried in a delivery transporter cannot exceed its delivery capacity. Each transporter incurs a fixed delivery cost regardless of the number of orders it carries, and departs from the 3PL provider to a customer at fixed times. Each customer desires to minimise a certain optimality criterion involving simultaneously the customer service level and the total delivery cost for its orders only.

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The customer service level for a customer is related to the times when its orders are delivered to it. The problem is to determine a joint schedule of production and distribution to minimise the objective of one customer, while keeping the objective of the other customer at or below a predefined level. Using several optimality criteria to measure the customer service level, we obtain different scenarios that depend on optimality criterion of each customer. For each scenario, we either devise an efficient solution procedure to solve it or demonstrate that such a solution procedure is impossible to exist.

1. **Introduction.** Consider a make-to-order supply chain involving one supplier producing time-sensitive products, e.g., fashion apparel, or drugs (see, e.g., Johnson [11]), and two customers, e.g., a group of closely located retailers or consumers.

At the beginning of the planning horizon, the supplier receives a set of orders from two customers with service requirements for the timely delivery their orders. The orders are first processed on a single dedicated production line at the supplier, after which they are packed into batches and delivered to their respective customers by a third-party logistics (3PL) provider with a limited number of delivery transporters. Each transporter has a limited delivery capacity, can take up a delivery assignment at a fixed departure time and delivers the finished orders of the same customer directly from the 3PL provider to the customer. The delivery cost of each transporter is fixed regardless of the number of orders it carries.

In the make-to-order setting, the customer service level, which depends the delivery times of the finished jobs of each customer, is the major concern of the customers. Since only limited production time is available, the two customers' orders have to compete for the use of the production line. Each customer desires to minimise a certain optimality criterion, which depends on its orders only and is measured by the sum of the customer service level and total delivery cost for its orders only. The optimality criteria to measure the customer service level are the maximum value of a regular optimality criterion, the total weighted order lead time, and the weighted number of late orders. The goal is to determine jointly a production schedule at the supplier, i.e., the order sequence scheduled on the single production line, and a distribution schedule, i.e., the number of delivery transporters to use, the orders carried by each delivery transporter, and the departure time of each delivery transporter from the supplier, to minimise the objective of one customer subject to a restriction on the objective of the other customer.

The goal of this study is twofold. One is to introduce a novel model that addresses the practically relevant and theoretically important integrated scheduling of production and distribution operations in the multi-agent setting. The other is to ascertain the computational complexity of different cases of the model under study by either proving that the case is $\mathcal{NP}$-hard, i.e., intractable, or devising an exact solution procedure based on the derived structural properties to solve the case in pseudo-polynomial time.

We organize the remaining part of this paper as follows: In the second section we provide a brief review of the related studies. In the third section we present the basic definitions and notation, derive some results about the structures of the problems under study, and provide an overview of our results. We analyze the problems in the fourth to seventh sections by providing proofs of their $\mathcal{NP}$-hardness or designing efficient solution procedures. In the last section we conclude the paper and suggest possible extensions for future research.

2. **Literature review.** Many integrated production and distribution models have been presented in the literature on supply chain management (see, e.g., Chen [4],

Fathollahi-Fard et al. [6], Hall and Potts [8], Potts and Kovalyov [21], Safaeian et al. [22], Tian et al. [25, 26], Wang et al. [30], and Wang et al. [31]). However, our model differs from most of these models in the following ways.

First, most of the existing models assume that a delivery transporter (or an order) is dispatched to a customer at the instant when all the orders it carries complete their processing without any transport delay (or the order's processing is finished). This assumption neglects the issue in real practice that delivery is a costly operation that can only be performed within a fixed time interval (Hall et al. [7]). In such situations, a set of departure times are usually stipulated before any orders are processed. In our model, we rely on a 3PL provider to perform the delivery function, which has a set of delivery transporters that deliver the finished orders to the customers at fixed departure times. Chen [4] reported that over 70% of the companies worldwide now rely on 3PL providers for their daily distribution and other logistics needs, and many 3PL providers have daily departure times. Second, in most of the existing models, there is only one optimality criterion. In the model we consider, however, there are two competing customers, each with an optimality criterion depending on its orders only to optimise. Accordingly, the supplier needs to identify an integrated optimal schedule that takes each customer's optimality criterion into account, which renders the model more intractable to solve. Table 1 summarizes the recent studies on integrate production and distribution in terms of the corresponding problem characteristics.

3. **Problem and preliminary analysis.**

This section introduces the problems under study mathematically and provides some optimality properties that simplify the subsequent analysis.

3.1. **Problem description.**

The supplier receives from two customers, to which we refer as customers A and B, a set of orders to be processed on a single production line (machine). The machine and orders all are available at time zero, and the machine can process at most one order at a time, and order preemption is not permitted. Throughout the paper, we let $X \in \{A, B\}$. Customer X wants to process the order set $J^X = \{J_1^X, \cdots, J_{n_X}^X\}$, the orders in which are referred to as the X-orders. Each order $J_j^X \in J^X$ has a processing time p_j^X, a weight w_j^X denoting the importance of order J_j^X relative to other X-orders, and a due date d_j^X before or at which order J_j^X is expected to leave the supplier for its customer. The finished orders from the same customer need to be packed to form batches and delivered to the customer by a 3PL provider with a limited number of transporters, each of which departs at a predefined time point. Specifically, let $T_1, \cdots, T_s$ denote the fixed departure times with $0 = T_0 < T_1 < \cdots < T_s$. At each time point $T_k, k = 1, \cdots, s$, there are v_k^X transporters available for delivering the X-orders to customer X. The transporters for delivering the X-orders are referred to as the X-transporters, and each X-transporter owns a capacity limit, i.e., it can carry up to q_X X-orders per delivery. A fixed delivery cost c_X per delivery for the X-orders is incurred regardless of the number of orders it carries, since the distances from the supplier to the two customers are different.

For a given schedule, we define the following variables:

D_j^X: the delivery time of order J_j^X equal to the time when the transporter containing order J_j^X departs from the 3PL provider to customer X;

U_j^X: a lateness indictor equal to 1 iff order J_j^X is late, i.e., $D_j^X > d_j^X$.

Since all the orders are available at time 0, D_j^X also denotes the lead time of order J_j^X. Each customer has a certain optimality criterion consisting of the customer service level and total delivery cost desired to optimize, which depends on its orders only. We use $H^X(D_1^X, \cdots, D_{n_X}^X)$ to measure the customer service level related to customer X, which is a nondecreasing function and depends on the lead time of the X-orders only, and use TC^X to denote the total delivery cost for delivering the X-orders. Thus, customer X desires to minimize $H^X(D_1^X, \cdots, D_{n_X}^X) + TC^X$. In this study, we address the following particular forms of the customer service function:

$f_{\max}^X = \max_{J_j^X \in J^X} \{f_j^X(D_j^X)\}$: the maximum value of a regular optimality criterion, where each $f_j^X(.)$ is a nondecreasing function of the lead time of order J_j^X;

$\sum(w_j^X)D_j^X = \sum_{J_j^X \in J^X} (w_j^X)D_j^X$: the total (weighted) lead time of the X-orders;

$\sum w_j^X U_j^X = \sum_{J_j^X \in J^X} w_j U_j^X$: the weighted number of late X-orders.

Note that all these optimality criteria are regular, so any two optimality criteria, in which one corresponds to customer A and another corresponds to customer B, are conflicting. To address the two optimality criteria, we adopt the constrained optimization approach. The problem is thus to identify jointly a production schedule and a distribution schedule to minimise the objective of one customer, while keeping the objective of the other customer at or below a predefined level.

Following the five-field notation system for integrated scheduling of production and distribution operations by Chen [4], and the notation for multi-agent scheduling by Agnetis et al. [2], we denote the problems under consideration by $1||V(v_X, c_X), fedp|1|(\gamma^A, \gamma^B \leq V^B)$, where in the third field, $V(v_X, c_X)$ indicates that the number of the X-transporters and the delivery capacity of each X-transporter are both limited, and $fedp$ indicates that the departure times are fixed and specified, and in the fifth field, γ^A and γ^B are the optimality criteria of customers A and B, respectively, and V^B is a predefined upper limit on the optimality criterion γ^B. However, since we only investigate this kind of problems, for ease of presentation, we denote this problem by $(\gamma^A, \gamma^B \leq V^B)$. In addition, if the equality $s = \overline{s}$ appears in the bottom right corner of $(\gamma^A, \gamma^B \leq V^B)$, i.e, $(\gamma^A, \gamma^B \leq V^B)_{s=\overline{s}}$, we mean that the number of fixed departure times is a constant; otherwise, the number of fixed departure times is arbitrary.

For notational convenience, we let $n = n_A + n_B$, $P^X = \sum_{j=1}^{n_X} p_j^X$, $P = P^A + P^B$, $v^X = \sum_{r=1}^{s} v_r^X$, and $n_{\max d}^X = \min\{n_X, \max_{r=1}^s \{q_X v_r^X\}\}$. We assume that the parameters are all integer valued, and $D_s \geq P$ since otherwise not all the orders can be delivered to their customers.

Table 2 summarizes the computational complexity results of the problems we obtain, where "ONP", "SNP", and "PS" represent that a problem is binary $\mathcal{NP}$-hard, strongly $\mathcal{NP}$-hard, and polynomially solvable, respectively, and "Open" indicates that the computational complexity of a problem is still unknown.

3.2. **Preliminary analysis.** This section provides some preliminary results about the structure of the problems, which will be used in the remaining part of this study.

In the sequel, we briefly review research on integrated scheduling of production and distribution operations with fixed departure times or competing agents.

TABLE 1. Computational complexity results

Problem	Complexity
$(\sum w_j^A D_j^A + TC^A, \gamma^B \leq V^B)$	SNP, even if there is no capacity constraint on the delivery transporters, Theorems 5.1 and 7.2
$(\sum D_j^A + TC^A, f_{\max}^B + TC^B \leq V^B)$	PS, $O(s^2 n_A n_B^2 n_{\max d}^A n_{\max d}^B \min\{v^B, n_B\})$, Theorem 4.4
$(f_{\max}^B + TC^B, \sum D_j^A + TC^A \leq V^A)$	PS, $O(s^2 n_A n_B^2 n_{\max d}^A n_{\max d}^B \min\{v^B, n_B\} \log(Q_u^B - Q_l^B))$, Theorem 4.5
$(\sum w_j^A D_j^A + TC^A, f_{\max}^B + TC^B \leq V^B)_{s=\overline{s}}$	ONP, $O(n_A n_B^2 (n_{\max d}^A)^{\overline{s}-1} n_{\max d}^B P^{\overline{s}-1} \min\{v^B, n_B\})$, Theorem 7.2
$(f_{\max}^B + TC^B, \sum w_j^A D_j^A + TC^A \leq V^A)_{s=\overline{s}}$	ONP, $O(n_A n_B^2 (n_{\max d}^A)^{\overline{s}-1} n_{\max d}^B P^{\overline{s}-1} \min\{v^B, n_B\} \log(Q_u^B - Q_l^B))$, Theorem 7.2
$(\sum w_j^A D_j^A + TC^A, \sum D_k^B + TC^B \leq V^B)_{s=\overline{s}}$	ONP, $O(n_A n_B (n_{\max d}^A)^{\overline{s}-1} n_{\max d}^B P^{\overline{s}-1} V^B)$, Theorem 5.6
$(\sum w_j^A D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)_{s=\overline{s}}$	ONP, $O(n_A n_B (n_{\max d}^A)^{\overline{s}-1} n_{\max d}^B P^{\overline{s}-1} P^B V^B)$, Theorem 7.2
$(\sum D_j^A + TC^A, \sum D_k^B + TC^B \leq V^B)$	Open, $O(s^2 n_A n_B (n_{\max d}^A) n_{\max d}^B V^B)$, Theorem 7.2
$(\sum D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$	ONP, $O(s^2 n_A n_B (n_{\max d}^A) n_{\max d}^B P^B V^B)$, Theorem 6.5
$(\sum w_j^A U_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$	ONP, $O(s^2 n_A n_B (n_{\max d}^A) n_{\max d}^B P V^B)$, Theorem 7.1

TABLE 2. Overview of the problem characteristics in recent recent studies on integrate production and distribution

Article	Number of agents	Delivery capacity	Delivery cost	Delivery mode	Departure times
Agnetis et al. [1]	One	Bounded	Yes	Non-splittable	Fixed
Hall et al. [7]	One	Unbounded	No	Non-splittable	Fixed
Han et al. [9]	One	Bounded	Yes	Non-splittable	Fixed
Kovalyov et al. [12]	Two	Unbounded	No	Non-splittable	Fixed
Leung and Chen [14]	One	Bounded	No	Non-splittable	Fixed
Li et al. [15]	One	Bounded	No	Splittable or Non-splittable	Fixed
Melo and Wolsey [18]	One	Bounded	Yes	Non-splittable	Fixed
Mor and Mosheiov [19]	Two	Unbounded	No	Non-splittable	Fixed
Seddik et al. [23]	One	Not involve	No	Not involve	Fixed
Stecke and Zhao [24]	One	Bounded	Yes	Splittable or Non-splittable	Fixed
Yin et al. [35]	Two	Unbounded	Yes	Non-splittable	Fixed
Yin et al. [37, 34]	Two	Unbounded	Yes	Non-splittable	No
Our paper	Multiple	Bounded	Yes	Non-splittable	Yes

The pioneering work on models that integrate production and distribution with fixed delivery times can be traced to Matsuo [17], who pointed out that the number of departure times is far less than that of orders in many practical settings. Considering the problem of determining an optimal schedule that leads to a good trade-off between the late delivery penalty and the overtime cost with fixed delivery times, the author developed a heuristic solution procedure for it. Hall et al. [7]

considered a different set of models that depend on the optimality criteria, and on the structure of the processing system, i.e., the single-machine, parallel-machine, and shop system, where the number of fixed departure times is either constant or arbitrary. However, in their problems, the delivery capacity of each transporter is unlimited. For each of problems considered, The authors either provided an efficient solution procedure or proved that the problem is intractable. Leung and Chen [13] considered the problems with the following objectives: (i) minimizing the maximum lateness of the orders; (2) minimizing the number of delivery transporters used under the condition that the maximum lateness is minimum; and (iii) minimizing the weighted sum of the maximum lateness and number of delivery transporters used, and the authors showed that they are all polynomial time solvable. Seddik et al. [23] considered a model with unequal release dates to maximize the cumulative number of orders scheduled before each departure time. The authors proved that the general problem is strongly $\mathcal{NP}$-hard, and provided a pseudo-polynomial-time solution procedure for the two-delivery-dates problem. However, none of the above works consider the delivery cost. Stecke and Zhao [24] considered a model with evenly spaced fixed departure times and order deadlines, where both cases of non-splittable and splittable delivery are allowed. For the former case, the authors proved that the nonpreemptive schedule where jobs are sequenced in the "Earliest Due-Date" (EDD) order is optimal. For the latter case, the authors proved that it is $\mathcal{NP}$-hard and provided a heuristic solution procedure for it. Melo and Wolsey [18] further studied the non-splittable delivery problem considered in Stecke and Zhao [24] by developing an integer programming model with very tight dual bounds that can solve large-scale instances. Some studies also investigate integrated models with fixed departure times and optimality criteria comprised of the inventory cost and delivery cost. Agnetis et al. [1] addressed the problem to minimise the total delivery and inventory cost. The authors proved that it is $\mathcal{NP}$-hard and devised polynomial time solution procedures for two special cases. Li et al. [15] studied a different set of models with fixed delivery time windows, where the delivery of orders can be either splittable or nonsplittable. The authors either proved that it is $\mathcal{NP}$-hard or devising an exact solution procedure for each of the problems they consider. Han et al. [9] investigated a model in a three-stage supply chain, where the orders are first partly processed by the supplier, then processed with identical processing times by the manufacturer, and the finished orders are delivered to their respective customers by delivery transporters, each of which has a set of fixed departure times. They derived the computational complexity results and solution solution procedures for several variants of the model.

However, all the above-cited studies either involve a single customer with an optimality criterion, or multiple customers with an integrated optimality criterion. Recently, multi-agent scheduling, which refers to the process of allocating services over time to perform a set of orders (jobs) from two or more competing customers whereby each customer desires to optimize its own optimality criterion on its orders only, has attracted increasing interest from the scheduling community. Pioneering multi-agent scheduling research, Agnetis et al. [2] and Baker and Smith [3] investigated the two-agent scheduling problems, where Baker and Smith [3] desired to minimise the weighted sum of the optimality criteria of the two customers, and Agnetis et al. [2] focused on the constrained optimization problems (determining the optimal solution for one agent subject to a restriction on the objective of the other agent) and the Pareto-optimization problems (identifying all nondominated schedules). Since then, multi-agent scheduling has been extensively investigated. For more results on this line of research, see Gerstl and Mosheiov [5], Hermelinn et al. [10], Leung et al. [14], Li and Yuan [16], Wan et al. [27], Wang et al. [28, 31], Yin et al. [32, 36, 33] etc, and the excellent survey by Perez-Gonzalez and Framinan [20]. Among these works, there are a few studies focusing on the integrated scheduling of production and distribution operations. For example, Mor and Mosheiov [19] studied a two-agent scheduling model, where the orders are first processed on a machine, and the finished orders are then packed to form batches and delivered to the customers in batches immediately after all the orders in a batch finish their processing. The authors focused on the case that the processing times and setup times of the orders of the same customer are identical, and the batches of the second customer must be processed continuously. Kovalyov et al. [12] considered the general model studied in Mor and Mosheiov [19], and devised pseudo-polynomial-time or polynomial-time solution procedures for several problems depending on the optimality criteria of the two customers. Yin et al. [35] further generalized the model of Kovalyov et al. [12] by adding delivery cost to the optimality criterion, and devised alternative solution procedures for the problems they consider and developed fully polynomial-time approximation schemes for some problems. Yin et al. [37, 34] focused on a set of similar models, except that the due dates of some orders are part of the decision process rather than input parameters. In Yin et al. [37], the orders of the first customer share a common due date that is part of the decision process, while the due dates of the orders of the second customer are predefined. The optimality criterion of the first customer is related to the earliness penalty, weighted number of late orders, inventory cost, due date assignment cost, and delivery cost, whereas the second customer wants to minimise the sum of one of the following optimality criteria and delivery cost: the maximum value of a regular optimality criterion, the total completion time, and the weighted number of late orders. In Yin et al. [34], the due dates of all the orders are part of the decision process, which are determined by two commonly used due date assignment models, i.e., the common due date assignment and unrestricted due date assignment models. The objective of each customer is to minimise an integrated cost of its orders that consists of the earliness, tardiness, or weighted number of late orders, order holding, due date assignment, and delivery costs. In both studies, the authors provided the computational complexity results and devised efficient solution procedures for the problems they consider. It is noted that none of the above studies investigates the case with limited delivery capacity of the transporters, which is more intractable than the case with unlimited delivery capacity. In addition, compared with our study, a delivery transporter is dispatched to a customer immediately when all the orders in it finish their processing in all the above studies.

4. Problem and preliminary analysis.

This section introduces the problems under study mathematically and provides some optimality properties that simplify the subsequent analysis.

4.1. Problem description.

The supplier receives from two customers, to which we refer as customers A and B, a set of orders to be processed on a single production line (machine). The machine and orders all are available at time zero, and the machine can process at most one order at a time, and order preemption is not permitted. Throughout the paper, we let $X \in \{A, B\}$. Customer X wants to process the order set $J^X = \{J_1^X, \cdots, J_{n_X}^X\}$, the orders in which are referred to as the X-orders. Each order $J_j^X \in J^X$ has a processing time p_j^X, a weight w_j^X denoting the importance of order J_j^X relative to other X-orders, and a due date d_j^X before or at which order J_j^X is expected to leave the supplier for its customer. The finished orders from the same customer need to be packed to form batches and delivered to the customer by a 3PL provider with a limited number of transporters, each of which departs at a predefined time point. Specifically, let $T_1, \cdots, T_s$ denote the fixed departure times with $0 = T_0 < T_1 < \cdots < T_s$. At each time point $T_k, k = 1, \cdots, s$, there are v_k^X transporters available for delivering the X-orders to customer X. The transporters for delivering the X-orders are referred to as the X-transporters, and each X-transporter owns a capacity limit, i.e., it can carry up to q_X X-orders per delivery. A fixed delivery cost c_X per delivery for the X-orders is incurred regardless of the number of orders it carries, since the distances from the supplier to the two customers are different.

For a given schedule, we define the following variables:

D_j^X: the delivery time of order J_j^X equal to the time when the transporter containing order J_j^X departs from the 3PL provider to customer X;

U_j^X: a lateness indictor equal to 1 iff order J_j^X is late, i.e., $D_j^X > d_j^X$.

Since all the orders are available at time 0, D_j^X also denotes the lead time of order J_j^X. Each customer has a certain optimality criterion consisting of the customer service level and total delivery cost desired to optimize, which depends on its orders only. We use $H^X(D_1^X, \cdots, D_{n_X}^X)$ to measure the customer service level related to customer X, which is a nondecreasing function and depends on the lead time of the X-orders only, and use TC^X to denote the total delivery cost for delivering the X-orders. Thus, customer X desires to minimize $H^X(D_1^X, \cdots, D_{n_X}^X) + TC^X$. In this study, we address the following particular forms of the customer service function:

$f_{\max}^X = \max_{J_j^X \in J^X} \{f_j^X(D_j^X)\}$: the maximum value of a regular optimality criterion, where each $f_j^X(.)$ is a nondecreasing function of the lead time of order J_j^X;

$\sum(w_j^X)D_j^X = \sum_{J_j^X \in J^X} (w_j^X)D_j^X$: the total (weighted) lead time of the X-orders;

$\sum w_j^X U_j^X = \sum_{J_j^X \in J^X} w_j U_j^X$: the weighted number of late X-orders.

Note that all these optimality criteria are regular, so any two optimality criteria, in which one corresponds to customer A and another corresponds to customer B, are conflicting. To address the two optimality criteria, we adopt the constrained optimization approach. The problem is thus to identify jointly a production schedule and a distribution schedule to minimise the objective of one customer, while keeping the objective of the other customer at or below a predefined level.

Following the five-field notation system for integrated scheduling of production and distribution operations by Chen [4], and the notation for multi-agent scheduling by Agnetis et al. [2], we denote the problems under consideration by $1||V(v_X, c_X), fedp|1|(\gamma^A, \gamma^B \leq V^B)$, where in the third field, $V(v_X, c_X)$ indicates that the number of the X-transporters and the delivery capacity of each X-transporter are both limited, and $fedp$ indicates that the departure times are fixed and specified, and in the fifth field, γ^A and γ^B are the optimality criteria of customers A and B, respectively, and V^B is a predefined upper limit on the optimality criterion γ^B. However, since we only investigate this kind of problems, for ease of presentation, we denote this problem by $(\gamma^A, \gamma^B \leq V^B)$. In addition, if the equality $s = \overline{s}$ appears in the bottom right corner of $(\gamma^A, \gamma^B \leq V^B)$, i.e, $(\gamma^A, \gamma^B \leq V^B)_{s=\overline{s}}$, we mean that the number of fixed departure times is a constant; otherwise, the number of fixed departure times is arbitrary.

For notational convenience, we let $n = n_A + n_B$, $P^X = \sum_{j=1}^{n_X} p_j^X$, $P = P^A + P^B$, $v^X = \sum_{r=1}^{s} v_r^X$, and $n_{\max d}^X = \min\{n_X, \max_{r=1}^s\{q_X v_r^X\}\}$. We assume that the parameters are all integer valued, and $D_s \geq P$ since otherwise not all the orders can be delivered to their customers.

Table 2 summarizes the computational complexity results of the problems we obtain, where "ONP", "SNP", and "PS" represent that a problem is binary $\mathcal{NP}$-hard, strongly $\mathcal{NP}$-hard, and polynomially solvable, respectively, and "Open" indicates that the computational complexity of a problem is still unknown.

Table 3. Computational complexity results

Problem	Complexity
$(\sum w_j^A D_j^A + TC^A, \gamma^B \leq V^B)$	SNP, even if there is no capacity constraint on the delivery transporters, Theorems 5.1 and 7.2
$(\sum D_j^A + TC^A, f_{\max}^B + TC^B \leq V^B)$	PS, $O(s^2 n_A n_B^2 n_{\max d}^A n_{\max d}^B \min\{v^B, n_B\})$, Theorem 4.4
$(f_{\max}^B + TC^B, \sum D_j^A + TC^A \leq V^A)$	PS, $O(s^2 n_A n_B^2 n_{\max d}^A n_{\max d}^B \min\{v^B, n_B\} \log(Q_u^B - Q_l^B))$, Theorem 4.5
$(\sum w_j^A D_j^A + TC^A, f_{\max}^B + TC^B \leq V^B)_{s=\overline{s}}$	ONP, $O(n_A n_B^2 (n_{\max d}^A)^{\overline{s}-1} n_{\max d}^B P^{\overline{s}-1} \min\{v^B, n_B\})$, Theorem 7.2
$(f_{\max}^B + TC^B, \sum w_j^A D_j^A + TC^A \leq V^A)_{s=\overline{s}}$	ONP, $O(n_A n_B^2 (n_{\max d}^A)^{\overline{s}-1} n_{\max d}^B P^{\overline{s}-1} \min\{v^B, n_B\} \log(Q_u^B - Q_l^B))$, Theorem 7.2
$(\sum w_j^A D_j^A + TC^A, \sum D_k^B + TC^B \leq V^B)_{s=\overline{s}}$	ONP, $O(n_A n_B (n_{\max d}^A)^{\overline{s}-1} n_{\max d}^B P^{\overline{s}-1} V^B)$, Theorem 5.6
$(\sum w_j^A D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)_{s=\overline{s}}$	ONP, $O(n_A n_B (n_{\max d}^A)^{\overline{s}-1} n_{\max d}^B P^{\overline{s}-1} P^B V^B)$, Theorem 7.2
$(\sum D_j^A + TC^A, \sum D_k^B + TC^B \leq V^B)$	Open, $O(s^2 n_A n_B (n_{\max d}^A) n_{\max d}^B V^B)$, Theorem 7.2
$(\sum D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$	ONP, $O(s^2 n_A n_B (n_{\max d}^A) n_{\max d}^B P^B V^B)$, Theorem 6.5
$(\sum w_j^A U_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$	ONP, $O(s^2 n_A n_B (n_{\max d}^A) n_{\max d}^B P V^B)$, Theorem 7.1

4.2. **Preliminary analysis.**

This section provides some preliminary results about the structure of the problems, which will be used in the remaining part of this study.

Lemma 4.1. *For each problem we consider, an optimal schedule exists, if any, such that all of the following hold:*

(1) *All the orders are processed continuously from time zero;*

(2) *The orders processed earlier are delivered no later than those from the same customer processed later;*

(3) *The departure time of each transporter is one of the fixed departure times that are no earlier than the time when all the orders in it finish their processing;*

(4) *If not all the v_i^X, $i = 1, \cdots, s$, delivery transporters with departure time T_i are used, there are fewer than q_X orders that are finished by T_i but delivered at a later departure time;*

(5) *At each departure time T_i, $r = 1, \cdots, s$, all the X-transporters, except possibly one transporter, are full-load; and if there exists an X-transporter with departure time T_i that is not full-load, all the X-orders finished by T_i are delivered by T_i.*

Proof. Property (3) is straightforward. The correctness of Properties (1), (2), and (4) stems from the fact that each optimality criterion we consider is nondecreasing in the order the lead times. The proof of (5) is similar to that of Lemma 8 in Chen [4]. □

Lemma 4.2. *When the optimality criterion $\sum D_j^X$ is addressed, an optimal schedule exists such that the X-orders are processed in the "Shortest Processing Time" (SPT) order.*

Proof. Consider an optimal schedule ρ^* where the X-orders are not processed in SPT order. Let J_j^X and J_k^X be the first pair of orders in ρ^*, such that J_j^X is processed prior to J_k^X with $p_j^X > p_k^X$, and let π be the set of the Y-orders with $\{A,B\} \setminus \{X\}$ that are processed between orders J_j^X and J_k^X. Consider another schedule ρ constructed by interchanging the processing positions and delivery transporters of orders J_j^X and J_k^X, while leaving the other orders unchanged. It is evident that the completion times of order J_k^X and the orders in set π in schedule ρ are less than those in ρ^*, and the completion times of the other orders in schedule ρ^* are identical to those in ρ^*. It follows that schedule ρ is feasible and no worse than ρ^* since all the optimality criteria we consider are regular, as required. □

Lemma 4.3. *When the optimality criterion $\sum w_j U_j^X$ is considered, an optimal schedule exists such that the X-orders are processed in the EDD order.*

Proof. The proof is similar to that of Lemma 3.2 with the following difference: when an optimal schedule exits such that J_j^X and J_k^X are the first pair of orders such that J_j^X is processed prior to J_k^X with $d_j^X > d_k^X$, we construct another schedule from this schedule by extracting order J_j^X, inserting it in the processing position just after order J_k^X, and interchanging the delivery transporters of orders J_j^X and J_k^X. □

5. **Problems ($\sum D_j^A + TC^A, f_{\max}^B + TC^B \le V^B$) and ($f_{\max}^B + TC^B, \sum D_j^A + TC^A \le V^A$).** This section first considers problem ($\sum D_j^A + TC^A, f_{\max}^B + TC^B \le V^B$). Given the number of B-transporters m_B, $m_B = \lceil \frac{n_B}{q_B} \rceil, \cdots, n_B$, for each job J_k^B, we define an induced deadline $\overline{d}_k^{m_B}$ such that $f_k^B(D_k^B) \le V^B - c_B m_B$ for $D_k^B \le \overline{d}_k^{m_B}$ and $f_k^B(D_k^B) \ge V^B - c_B m_B$ for $D_k^B \ge \overline{d}_k^{m_B}$. It is assumed that each inverse function $(f_k^B)^{-1}(.)$ is available, implying that the deadlines can be calculated in constant time.

The following result states the structure properties of an optimal schedule for problem ($\sum D_j^A + TC^A, f_{\max}^B + TC^B \le V^B$).

Lemma 5.1. *For problem ($\sum D_j^A + TC^A, f_{\max}^B + TC^B \le V^B$) with a given m_B, an optimal schedule exists, if any, such that all of the following hold:*

(1) *The A-orders are processed in SPT order;*
(2) *The B-orders are processed in non-decreasing order of $\overline{d}_k^{m_B}$.*

Proof. Property (1) is an immediate consequence of Lemma 3.2 and the proof of property (2) is analogous to that of Lemma 3.3. □

In view of Lemma 4.1, we re-number the A-orders in SPT order, and the B-orders in non-decreasing order of $\overline{d}_k^{m_B}$ for any given m_B in this section. For any given j and k, $j = 0, 1, \cdots, n_A$, $k = 0, 1, \cdots, n_B$, we let $P(j,k)$ be the total processing time of the first j A-orders and the first k B-orders. Our solution procedure for problem ($\sum D_j^A + TC^A, f_{\max}^B + TC^B \le V^B$), to which we refer as Algorithm $DP1$, is a forward dynamic programming solution procedure that iteratively appends a single A-order or B-order to a previously generated partial schedule of orders, and is based on the results described in Lemmas 3.1 and 4.1.

In Algorithm $DP1$, we use a state variable m_B to enumerate the number of B-transporters used in the final complete schedule. For any given m_B, j, and k, where $m_B = \lceil \frac{n_B}{q_B} \rceil, \cdots, n_B$, $j = 0, \cdots, n_A$, and $k = 0, \cdots, n_B$, let $\mathcal{L}_{m_B}(j,k)$ be a state set in which any state is a vector $(l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A)$ that stands for a feasible partial schedule for the orders in $\{J_1^A, \cdots, J_j^A, J_1^B, \cdots, J_k^B\}$, where

- l^A (resp., l^B) denotes that the last processed A-order J_j^A (resp., B-order J_k^B) is delivered at departure time T_{l^A} (resp., T_{l^B});
- n_{l^A} (resp., n_{l^B}) represents that there are n_{l^A} A-orders (resp., n_{l^B} B-orders) delivered at departure time T_{l^A} (resp., T_{l^B}) with $\lceil \frac{n_l^A}{q_A} \rceil \le v_{l^A}^A$ (resp., $\lceil \frac{n_l^B}{q_B} \rceil \le v_{l^B}^B$);
- α_B measures the number of B-transporters used;
- f^A denotes the sum of the total lead time and total delivery cost of the A-orders.

The state sets $\mathcal{L}_{m_B}(j,k)$ are generated iteratively, which is initialized with $\mathcal{L}_{m_B}(0,0) = \{(\underbrace{0, \cdots, 0}_{6})\}$. In the (j,k)-th phase with $j + k \ge 1$, a state set $\mathcal{L}_{m_B}(j,k)$ is generated from $\mathcal{L}_{m_B}(j-1,k)$ and $\mathcal{L}_{m_B}(j,k-1)$. To be precise, for each state $(l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A) \in \mathcal{L}_{m_B}(j-1,k)$, to append order J_j^A to the corresponding partial schedule, three decisions need to be considered.

Decision FTA1: place order J_j^A as the last processed order, and add it to the last used A-transporter with departure time T_{l^A}. This is valid only if $P(i,j) \le T_{l^A}$ and $\frac{n_{l^A}+1}{q_A} < \lceil \frac{n_{l^A}+1}{q_A} \rceil \le v_{l^A}^A$, in which the second inequality indicates that the last used A-transporter is not full-load. The contribution of order J_j^A in this case to the objective of customer A is T_{l^A}. Therefore, if $P(i,j) \le T_{l^A}$ and $\frac{n_{l^A}+1}{q_A} < \lceil \frac{n_{l^A}+1}{q_A} \rceil \le v_{l^A}^A$, add the state $(l^A, l^B, n_{l^A}+1, n_{l^B}, \alpha_B, f^A + T_{l^A})$ to $\mathcal{L}_{m_B}(j,k)$.

Decision FTA2: place order J_j^A as the last processed order, and add it to a new A-transporter with departure time T_{l^A}. This is valid only if $P(i,j) \le T_{l^A}$ and $\frac{n_{l^A}}{q_A} = \lceil \frac{n_{l^A}}{q_A} \rceil$. The contribution of order J_j^A in this case to the objective of customer A is $T_{l^A} + c_A$. Therefore, if $P(i,j) \le T_{l^A}$ and $\frac{n_{l^A}}{q_A} = \lceil \frac{n_{l^A}}{q_A} \rceil$, add the state $(l^A, l^B, n_{l^A}+1, n_{l^B}, \alpha_B, f^A + T_{l^A} + c_A)$ to $\mathcal{L}_{m_B}(j,k)$.

Decision FTA3: place order J_j^A as the last processed order, and add it to a new A-transporter with departure time T_{o^A}, where o^A is the minimum subscript such that $l^A < o^A$, $v_{o^A}^A > 0$, and $P(i,j) \le T_{o^A}$. This is valid only if $\frac{n_{l^A}}{q_A} = \lceil \frac{n_{l^A}}{q_A} \rceil$. The contribution of order J_j^A in this case to the objective of customer A is $T_{o^A} + c_A$. Therefore, if there exists such a subscript o^A and $\frac{n_{l^A}}{q_A} = \lceil \frac{n_{l^A}}{q_A} \rceil$, add the state $(o^A, l^B, 1, n_{l^B}, \alpha_B, f^A + T_{o^A} + c_A)$ to $\mathcal{L}_{m_B}(j,k)$.

For each state $(l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A) \in \mathcal{L}_{m_B}(j,k-1)$, to append order J_k^B to the corresponding partial schedule, three decisions need to be considered.

Decision FTB1: place order J_k^B as the last processed order, and add it to the last used B-transporter with departure time T_{l^B}. This is valid only if $P(i,j) \le T_{l^B} \le \overline{d}_k^{m_B}$ and $\frac{n_{l^B}+1}{q_B} < \lceil \frac{n_{l^B}+1}{q_B} \rceil \le v_{l^B}^B$. The contribution of order J_k^B in this case to the objective of customer A is 0. Therefore, if $P(i,j) \le T_{l^B} \le \overline{d}_k^{m_B}$ and $\frac{n_{l^B}+1}{q_B} < \lceil \frac{n_l^B+1}{q_B} \rceil \le v_{l^B}^B$, add the state $(l^A, l^B, n_{l^A}, n_{l^B}+1, \alpha_B, f^A)$ to $\mathcal{L}_{m_B}(j,k)$.

Decision FTB2: place order J_k^B as the last processed order, and add it to a new B-transporter with departure time T_{l^B}. This is valid only if $P(i,j) \le T_{l^B} \le \overline{d}_k^{m_B}$ and $\frac{n_{l^B}}{q_B} = \lceil \frac{n_{l^B}}{q_B} \rceil$. The contribution of order J_k^B in this case to the objective of customer A is 0. Therefore, if $P(i,j) \le T_{l^B} \le \overline{d}_k^{m_B}$ and $\frac{n_l^B}{q_B} = \lceil \frac{n_l^B}{q_B} \rceil$, add the state $(l^A, l^B, n_{l^A}, n_{l^B}+1, \alpha_B+1, f^A)$ to $\mathcal{L}_{m_B}(j,k)$.

Decision FTB3: place order J_k^B as the last processed order, and add it to a new B-transporter with departure time T_{o^B}, where o^B is the minimum subscript such that $l^B < o^B$, $v_{o^B}^B > 0$, and $P(i,j) \le T_{o^B} \le \overline{d}_k^{m_B}$. This is valid only if $\frac{n_{l^B}}{q_B} = \lceil \frac{n_{l^B}}{q_B} \rceil$. The contribution of order J_k^B in this case to the objective of customer A is 0. Therefore, if there exists such a subscript o^B and $\frac{n_{l^B}}{q_B} = \lceil \frac{n_{l^B}}{q_B} \rceil$, add the state $(l^A, o^B, n_{l^A}, 1, \alpha_B+1, f^A)$ to $\mathcal{L}_{m_B}(j,k)$.

The following results show how to delete some dominated states which will not lead to a complete optimal schedule.

Lemma 5.2. *For any two states $(l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A)$ and $(l'^A, l'^A, n'_{l^A}, n'_{l^B}, \alpha'_B, f'^A)$ in $\mathcal{L}_{m_B}(j,k)$, if $l^A \le l'^A, l^B \le l'^B, n_{l^A} \le n'_{l^A}, n_{l^B} \le n'_{l^B}, \alpha_B \le \alpha'_B$, and $f^A \le f'^A$, the latter state can be deleted from $\mathcal{L}_{m_B}(j,k)$.*

Proof. The proof is straightforward and we omit it. □

For any state $(l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A)$ in $\mathcal{L}_{m_B}(j,k)$, we store the number of X-transporters n_{s^X} with departure time T_{s^X} used in the partial schedule corresponding to the state $(l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A)$, where T_{s^X} is the time instant immediately prior to T_{l^A}.

Lemma 5.3. *For any state $(l^A, l^B, n_l^A, n_l^B, \alpha_B, f^A)$ in $\mathcal{L}_{m_B}(j,k)$, if $(\frac{n_{s^A}}{q_A} < \lceil \frac{n_{s^A}}{q_A} \rceil$, $P(i,j) \le T_{s^A}$ and $n_{l^A} \ge q_A)$ or $(\frac{n_{s^B}}{q_B} < \lceil \frac{n_{s^B}}{q_B} \rceil$, $P(i,j) \le T_{s^B}$ and $n_{l^B} \ge q_B)$, the state can be deleted from $\mathcal{L}_{m_B}(j,k)$.*

Proof. The conditions $\frac{n_{s^A}}{q_A} < \lceil \frac{n_{s^A}}{q_A} \rceil$, $P(i,j) \le T_{s^A}$, and $n_{l^A} \ge q_A$ indicate that the last used A-transporter with departure time T_{s^A} is not full-load, the completion time of the last A-order is not larger than T_{s^A}, and the number of A-orders finished processing by T_{s^A} but delivered at a later departure time is larger or equal to q_X. As a consequence, by property (5) in Lemma 4.1, any extension of $(l^A, l^B, n_l^A, n_l^B, \alpha_B, f^A)$ cannot lead to a complete optimal schedule, so the state can be deleted from $\mathcal{L}_{m_B}(j,k)$. The analysis of the conditions $\frac{n_{s^B}}{q_B} < \lceil \frac{n_{s^B}}{q_B} \rceil$, $P(i,j) \le T_{s^B}$, and $n_{l^B} \ge q_B$ is analogous, which completes the proof. □

Algorithm $DP1$ can be formally depicted as follows:

Algorithm $DP1$
Step 1. Re-number the A-orders in SPT order.
Step 2. Set $\mathcal{L}_{m_B}(0,0) = (\underbrace{0, \cdots, 0}_{6})$ and $\mathcal{L}_{m_B}(j,k) = +\infty$ with $j = -1$ or $k = -1$,

for $\lceil \frac{n_B}{q_B} \rceil \le m_B \le n_B$.
Step 3.

For $m_B = \lceil \frac{n_B}{q_B} \rceil$ to n_B, do

Calculate $\overline{d}_k^{m_B}$ from $f_k^B(\overline{d}_k^{m_B}) = V^B - c_B m_B$ for $k = 1, \cdots, n_B$;

Re-number the B-orders according to non-decreasing order of $\overline{d}_j^{m_B}$ such that $\overline{d}_1^{m_B} \le \cdots \le \overline{d}_{n_B}^{m_B}$;

For each combination of (j,k) with $0 \le j \le n_A$, $0 \le k \le n_B$, and $j+k \ge 1$, do

Set $\mathcal{L}_{m_B}(j,k) = \emptyset$;

For each $(l^A, l^B, n_l^A, n_l^B, \alpha_B, f^A) \in \mathcal{L}_{m_B}(j-1,k)$, do

/**Decision FTA1**/

If $P(i,j) \le T_{l^A}$ and $\frac{n_{l^A}+1}{q_A} < \lceil \frac{n_{l^A}+1}{q_A} \rceil \le v_{l^A}^A$, then

$\mathcal{L}_{m_B}(j,k) \leftarrow \mathcal{L}_{m_B}(j,k) \cup \{(l^A, l^B, n_{l^A}+1, n_{l^B}, \alpha_B, f^A + T_{l^A})\}$;

Endif

/**Decision FTA2**/

If $P(i,j) \le T_{l^A}$ and $\frac{n_{l^A}}{q_A} = \lceil \frac{n_{l^A}}{q_A} \rceil$, then

$\mathcal{L}_{m_B}(j,k) \leftarrow \mathcal{L}_{m_B}(j,k) \cup \{(l^A, l^B, n_{l^A}+1, n_{l^B}, \alpha_B, f^A + T_{l^A} + c_A)\}$;

Endif

/**Decision FTA3**/

If there exists a minimum subscript o^A such that $l^A < o^A$, $v_{o^A}^A > 0$, and $P(i,j) \le T_{o^A}$, and $\frac{n_{l^A}}{q_A} = \lceil \frac{n_{l^A}}{q_A} \rceil$, then

$\mathcal{L}_{m_B}(j,k) \leftarrow \mathcal{L}_{m_B}(j,k) \cup \{(o^A, l^B, 1, n_{l^B}, \alpha_B, f^A + T_{o^A} + c_A)\}$;

Endif

Endfor

For each $(l^A, l^B, n_l^A, n_l^B, \alpha_B, f^A) \in \mathcal{L}_{m_B}(j,k-1)$, do

/**Decision FTB1**/

If $P(i,j) \le T_{l^B} \le \overline{d}_k^{m_B}$ and $\frac{n_{l^B}+1}{q_B} < \lceil \frac{n_l^B+1}{q_B} \rceil \le v_{l^B}^B$, then

$\mathcal{L}_{m_B}(j,k) \leftarrow \mathcal{L}_{m_B}(j,k) \cup \{(l^A, l^B, n_{l^A}, n_{l^B}+1, \alpha_B, f^A)\}$;

Endif

/**Decision FTB2**/

If $P(i,j) \le T_{l^B} \le \overline{d}_k^{m_B}$ and $\frac{n_l^B}{q_B} = \lceil \frac{n_l^B}{q_B} \rceil$, then

$\mathcal{L}_{m_B}(j,k) \leftarrow \mathcal{L}_{m_B}(j,k) \cup \{(l^A, l^B, n_{l^A}, n_{l^B}+1, \alpha_B+1, f^A)\}$;

Endif

/**Decision FTB3**/

If there exists a minimum subscript o^A such that $l^B < o^B$, $v_{o^B}^B > 0$, and $P(i,j) \le T_{o^B} \le \overline{d}_k^{m_B}$, and $\frac{n_{l^B}}{q_B} = \lceil \frac{n_{l^B}}{q_B} \rceil$, then

$\mathcal{L}_{m_B}(j,k) \leftarrow \mathcal{L}_{m_B}(j,k) \cup \{(l^A, o^B, n_{l^A}, 1, \alpha_B+1, f^A)\}$;

Endif

Endfor

/**Elimination**/

(1) For any two states $(l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A)$ and $(l'^A, l'^A, n'_{l^A}, n'_{l^B}, \alpha'_B, f'^A)$ in $\mathcal{L}_{m_B}(j,k)$, if the conditions in Lemma 4.2 are valid, delete the latter state from $\mathcal{L}_{m_B}(j,k)$;

(2) For any state $(l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A)$ in $\mathcal{L}_{m_B}(j,k)$, if the conditions in Lemma 4.3 are valid, delete the state from $\mathcal{L}_{m_B}(j,k)$;

Endfor

Endfor

Step 4. The optimal solution value is $\min\{f^A | (l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A) \in \mathcal{L}_{m_B}(n_A, n_B), \lceil \frac{n_B}{q_B} \rceil \le m_B \le n_B\}$.

Theorem 5.4. *Algorithm DP1 solves problem* $(\sum D_j^A + TC^A, f_{\max}^B + TC^B \le V^B)$ *in* $O(s^2 n_A n_B^2\ n_{\max d}^A n_{\max d}^B \min\{v^B, n_B\})$ *time.*

Proof. Algorithm $DP1$ implicitly enumerates all the feasible schedules fulfilling the properties described in Lemmas 3.1 and 4.1, so it will find an optimal solution. We next study the time complexity, which is mainly consumed in Step 3. In Step 3, there are at most $(s+1)$ possible values for l^A and l^B, $n^A_{\max d}$ possible values for n_{l^A}, $n^B_{\max d}$ possible values for n_{l^B}, and $\min\{v^B, n_B\}$ possible values for α_B. Due to the elimination rule, the number of different combinations of $(l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A)$ is at most $O(s^2 n^A_{\max d} n^B_{\max d} \min\{v^B, n_B\})$. Thus, the number of new states generated in $\mathcal{L}_{m_B}(j,k)$ is upper-bounded by $O(s^2 n^A_{\max d} n^B_{\max d} \min\{v^B, n_B\})$ after the elimination process. Therefore, after at most $n_B(n_A+1)(n_B+1)$ iterations, Step 3 takes $O(s^2 n_A n_B^2 n^A_{\max d} n^B_{\max d} \min\{v^B, n_B\})$ time, as required. □

Now, we consider problem $(f^B_{\max}+TC^B, \sum D^A_j+TC^A \le V^A)$. To solve it, we first design a solution procedure with a slight modification of Algorithm $DP1$ by deleting the state $(l^A, l^B, n_{l^A}, n_{l^B}, \alpha_B, f^A)$, if $f^A > V^A$, to identify whether or not a feasible schedule exists for the the decision problem $(f^B_{\max}+TC^B \le V^B, \sum D^A_j+TC^A \le V^A)$ with $V^B \ge 0$. We then use the modified solution procedure as a subroutine to solve problem $(f^B_{\max}+TC^B, \sum D^A_j \le V^A)$. Specifically, we need to enumerate all the possible thresholds V^B to determine the optimal V^{B*}, which can be achieved by conducting a binary search on $V^B \in [Q^B_l, Q^B_u]$, where Q^B_l and Q^B_u are the lower and upper bounds on $f^B_{\max}+TC^B$. We can set $Q^B_l = \max\{f^B_k(D_r) : 1 \le k \le n^B\} + c_B\lceil \frac{n_B}{q_B}\rceil$ and $Q^B_l = \max\{f^B_k(D_s) : 1 \le k \le n^B\} + c_B\lceil \frac{n_B}{q_B}\rceil$ since the delivery time of the last finished order must be no earlier than D_r and no later than D_s, where r and s are the subscripts such that $D_{r-1} < P^B \le D_r$ and $D_{s-1} < P \le D_s$, respectively. Thus, by enumerating the value of $V^B \in [Q^B_l, Q^B_u]$ via a bisection search with $O(\log(Q^B_u - Q^B_l))$ iterations, problem $(f^B_{\max}+TC^B, \sum D^A_j \le V^A)$ can be solved in $O(s^2 n_A n_B^2 n^A_{\max d} n^B_{\max d} \min\{v^B, n_B\} \log(Q^B_u - Q^B_l))$ time. As consequence, the following result is valid.

Theorem 5.5. *Problem $(f^B_{\max}+TC^B, \sum D^A_j+TC^A \le V^A)$ can be solved in $O(s^2 n_A n_B^2 n^A_{\max d} n^B_{\max d} \min\{v^B, n_B\} \log(Q^B_u - Q^B_l))$ time.*

6. **Problems $(\sum w^A_j D^A_j + TC^A, \sum D^B_k + TC^B \le V^B)$ and $(\sum w^A_j D^A_j + TC^A, \sum D^B_k + TC^B \le V^B)_{s=\widetilde{s}}$.** This section addresses problems $(\sum w^A_j D^A_j+TC^A, \sum D^B_k+TC^B \le V^B)$ and $(\sum w^A_j D^A_j + TC^A, \sum D^B_k + TC^B \le V^B)_{s=\widetilde{s}}$. The following result states the computational complexity of the considered problems.

Theorem 6.1. *The problem $(\sum w^A_j D^A_j + TC^A, \sum D^B_k + TC^B \le V^B)$ is strongly $\mathcal{NP}$-hard, even if there is no capacity constraint on the delivery transporters.*

Proof. For sufficiently large V^B, the problem under consideration reduces to the single-agent problem with an arbitrary number of fixed departure times to minimize $\sum w^A_j D^A_j$, which has been proven to be strongly $\mathcal{NP}$-hard (see Theorem 3.4 in Hall et al. [8]), so the result follows. □

Theorem 6.2. *Problem $(\sum w^A_j D^A_j + TC^A, \sum D^B_k + TC^B \le V^B)_{s=\widetilde{s}}$ is $\mathcal{NP}$-hard, even if there is no capacity constraint on the delivery transporters.*

Proof. Similar to the proof Theorem 5.1, the $\mathcal{NP}$-harness of the problem under consideration comes from the NP-harness of the single-agent problem with a given number of fixed departure times to minimize $\sum w^A_j D^A_j$ (see Theorem 3.3. in Hall et al. [8]), as required. □

In what follows, we focus on problem $(\sum w_j^A D_j^A + TC^A, \sum D_k^B + TC^B \leq V^B)_{s=\widetilde{s}}$ and devise a dynamic programming solution procedure that runs in pseudo-polynomial time, establishing that it is binary $\mathcal{NP}$-hard. We first introduce a result on the structure of the problem, analogous to that stated in Lemma 5.1.

Lemma 6.3. *For problem $(\sum w_j^A D_j^A + TC^A, \sum D_k^B + TC^B \leq V^B)_{s=\widetilde{s}}$, an optimal schedule exists, if any, such that the B-orders are processed in SPT order.*

In view of Lemma 5.3, we re-number the B-orders in SPT order in this section. Our dynamic programming solution procedure for problem $(\sum w_j^A D_j^A + TC^A, \sum D_k^B + TC^B \leq V^B)_{s=\widetilde{s}}$, denoted by Algorithm $DP2$, is a forward algorithm strongly depending on the properties stated in Lemmas 3.1 and 5.3. For any given j and k, where $j = 0, \cdots, n_A$ and $k = 0, \cdots, n_B$, let $\mathcal{H}(j,k)$ be a state space, in which each state is a vector $(l, n_1^A, \cdots, n_{\overline{s}}^A, t_1, \cdots, t_{\overline{s}}, n_l^B, f^A, f^B)$ that stands for a feasible partial schedule for the orders in $\{J_1^A, \cdots, J_j^A, J_1^B, \cdots, J_k^B\}$, where

- l denotes that the last processed A-order J_j^A is delivered at departure time T_l;
- $n_i^A, i = 1, \cdots, \overline{s}$, gives the number of A-orders delivered at time T_i with $\lceil \frac{n_i^A}{q_A} \rceil \leq v_l^A$ and $\sum_{r=1}^{\overline{s}} n_r^A = j$;
- $t_i, i = 1, \cdots, \overline{s}$, measures the total processing time of the orders delivered at time T_k with $\sum_{r=1}^{i} t_r \leq T_i$ and $\sum_{r=1}^{\overline{s}} t_r = P(j,k)$;
- n_l^B gives the number of B-orders delivered at time T_l with $\lceil \frac{n_l^B}{q_B} \rceil \leq v_l^B$;
- f^A and f^B denote the total weighted lead time of the A-orders and the total lead time of the B-orders, respectively.

For ease of presentation, we simplify $(n_1^A, \cdots, n_{\overline{s}}^A)$, $(t_1, \cdots, t_{\overline{s}})$, $(n_1^A, \cdots, n_{r-1}^A, x, n_{r+1}^A, \cdots, n_{\overline{s}}^A)$, and $(t_1, \cdots, t_{r-1}, y, t_{r+1}, \cdots, t_{\overline{s}})$ to $\mathfrak{n}^A$, $\mathfrak{t}$, $(\cdots, x, \cdots)$, and $(\cdots, y, \cdots)$, respectively.

The state sets $\mathcal{H}(j,k)$ are generated iteratively, which is initialized with $\mathcal{H}(0,0) = \{(\underbrace{0, \cdots, 0}_{4+2\overline{s}})\}$. In the (j,k)-th phase with $j + k \geq 1$, a state set $\mathcal{H}(j,k)$ is generated from $\mathcal{H}(j-1,k)$ and $\mathcal{H}(j,k-1)$. To be precise, for each state $(j, k, l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B) \in \mathcal{H}(j-1,k)$, to append order J_j^A to the corresponding partial schedule, the following $\overline{s}$ decisions need to be considered.

Decision WTTAi: deliver order J_j^A at time $T_i, i = 1, \cdots, \overline{s}$. This is valid only if $\lceil \frac{n_{lA}+1}{q_A} \rceil \leq v_i^A$ and $p_j^A + \sum_{r=1}^{o} t_r \leq T_o$ for $o = i, \cdots, \overline{s}$, in which the latter conditions ensure that the insertion of order J_j^A into the partial schedule would not lead the resulting completion time of each order assigned to be delivered at time T_o, $o = i, \cdots, \overline{s}$, in the partial schedule that stands for the state $(j, k, l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A)$ to exceed T_o. In this case, the contributions of order J_j^A to the objectives of customer A and customer B are $w_j^A T_i + c_A(\lceil \frac{n_{lA}+1}{q_A} \rceil - \lceil \frac{n_{lA}}{q_A} \rceil)$ and 0, respectively. Therefore, if $\lceil \frac{n_{lA}+1}{q_A} \rceil \leq v_i^A$ and $p_j^A + \sum_{r=1}^{o} t_r \leq T_o$ for $o = i, \cdots, \overline{s}$, add the state $(l, \cdots, n_i^A + 1, \cdots, t_i + p_j^A, \cdots, n_l^B, f^A + w_j^A T_i + c_A(\lceil \frac{n_i^A+1}{q_A} \rceil - \lceil \frac{n_i^A}{q_A} \rceil), f^B)$ to $\mathcal{H}(j,k)$.

For each state $(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B) \in \mathcal{H}(j, k-1)$, to append order J_k^B to the corresponding partial schedule, three decisions similar to *Decision FTA1* to *Decision FTA3* in Algorithm $DP1$ need to be considered.

The following results demonstrate how to delete the dominated states generated in $\mathcal{H}(j, k)$.

Lemma 6.4. *For any two states* $(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B)$ *and* $(l', \mathfrak{n}'^A, \mathfrak{t}', n_l'^B, f'^A, f'^B)$ *in* $\mathcal{H}(j, k)$, *if* $l \le l', \cdots, n_i^A \le n_i'^A, \cdots, t_i \le t_i', \cdots, n_l^B \le n_l'^B$, $f^A \le f'^A$, *and* $f^B \le f'^B$, *the latter state can be deleted from* $\mathcal{H}(j, k)$.

Proof. The proof is similar to that of Lemma 4.2. □

For any state $(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B)$ in $\mathcal{H}(j, k)$, we store the number of B-transporters n_s with departure time T_s used in the partial schedule encoding the state $(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B)$, where T_s is the time instant immediately prior to T_l.

Lemma 6.5. *For any state* $(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B)$ *in* $\mathcal{H}(j, k)$, *if* $\frac{n_s}{q_A} < \lceil \frac{n_s}{q_A} \rceil$, $\sum_{r=1}^{l} t_r \le T_s$, *and* $n_l^B \ge q_A$, *the state can be deleted from* $\mathcal{H}(j, k)$.

Proof. The proof is similar to that of Lemma 4.3. □

Algorithm $DP2$ can be formally depicted as follows:

Algorithm $DP2$
Step 1. Re-number the B-orders in SPT order.
Step 2. Set $\mathcal{H}(0,0) = \{(\underbrace{0, \cdots, 0}_{4+2\bar{s}})\}$ and $\mathcal{H}(j,k) = +\infty$ with $j = -1$ or $k = -1$.
Step 3.
For each combination of (j,k) with $0 \le j \le n_A$, $0 \le k \le n_B$, and $j + k \ge 1$, do
  Set $\mathcal{H}(j,k) = \emptyset$;
  For each $(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B) \in H(j-1, k)$, do
    For $i = 1$ to $\bar{s}$, do
      /**Decision WTTAi*/
      If $\lceil \frac{n_{lA}+1}{q_A} \rceil \le v_i^A$ and $p_j^A + \sum_{r=1}^{o} t_r \le T_o$ for $o = i, \cdots, \bar{s}$, then
      $\mathcal{H}(j,k) \leftarrow \mathcal{H}(j,k) \cup \{(l, \cdots, n_i^A + 1, \cdots, t_i + p_j^A, \cdots, n_l^B, f^A + w_j^A T_i + c_A(\lceil \frac{n_r^A+1}{q_A} \rceil - \lceil \frac{n_r^A}{q_A} \rceil), f^B)\}$;
      Endif
    Endfor
  Endfor
  For each $(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B) \in \mathcal{H}(j, k-1)$, do
    /**Decision FTA1*/
    If $\sum_{r=1}^{l} t_r + p_k^B \le T_l$, $\frac{n_l^B+1}{q_B} < \lceil \frac{n_l^B+1}{q_B} \rceil \le v_l^B$ and $f^B + T_l \le v_B$, then
      $\mathcal{H}(j,k) \leftarrow \mathcal{H}(j,k) \cup \{(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B + 1, f^A, f^B + T_l)\}$;
    Endif
    /**Decision FTA2*/
    If $\sum_{r=1}^{l} t_r + p_k^B \le T_l$, $\frac{n_l^B}{q_B} = \lceil \frac{n_l^B}{q_B} \rceil$ and $f^B + T_l + c_B \le V^B$, then
      $\mathcal{H}(j,k) \leftarrow \mathcal{H}(j,k) \cup \{(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B + 1, f^A, f^B + T_l + c_B)\}$;

Endif
/**Decision FTA3**/
If there exists a minimum subscript o such that $l < o$, $v_o^B > 0$, and $\sum_{r=1}^{o} t_r + p_k^B \leq T_o)$, $\frac{n_l^B}{q_B} = \lceil \frac{n_l^B}{q_B} \rceil$, and $f^B + T_o + c_B \leq V^B$, then
$\mathcal{H}(j,k) \leftarrow \mathcal{H}(j,k) \cup \{(o, \mathfrak{n}^A, \mathfrak{t}, 1, f^A, f^B + T_o + c_B)\}$;
Endif
Endfor
/**Elimination**/
(1) For any two states $(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B)$ and $(l', \mathfrak{n}'^A, \mathfrak{t}', n_l'^B, f'^A, f'^B)$ in $\mathcal{H}(j,k)$, if the conditions in Lemma 5.4 are valid, delete the latter state $\{romH(j,k)$;
(2) For any state $(l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B)$ in $\mathcal{H}(j,k)$, if the conditions in Lemma 5.5 are valid, delete the state from $\mathcal{H}(j,k)$;
Endfor
Step 4. The optimal solution value is $\min\{f^A | (l, \mathfrak{n}^A, \mathfrak{t}, n_l^B, f^A, f^B) \in \mathcal{H}(n_A, n_B)$.

Theorem 6.6. *Algorithm DP2 solves problem* $(\sum w_j^A D_j^A + TC^A, \sum D_k^B + TC^B \leq V^B)$ *in* $O(n_A n_B (n_{\max d}^A)^{\bar{s}-1} P^{\bar{s}-1} n_{\max d}^B V^B)$ *time.*

Proof. The proof is similar to that of Theorem 4.4, where the difference lies in that the number of states in $\mathcal{H}(j, x_1, \cdots, x_r)$ is at most $O(\bar{s}(n_{\max d}^A)^{\bar{s}-1} P^{\bar{s}-1} n_{\max d}^B V^B)$ due to the fact that there are at most $(\bar{s}+1)$ possible values for l, $(n_{\max d}^A + 1)$ and $(P+1)$ possible values for n_r^A and $t_r, r = 1, \cdots, \bar{s}$, respectively, $(n_{\max d}^B + 1)$ possible values for n_l^B, and $(V^B + 1)$ possible values for f^B, and we only consider those $\mathfrak{n}^A$ and $\mathfrak{t}$ such that $\sum_{r=1}^{\bar{s}} n_r^A = j$ and $\sum_{r=1}^{\bar{s}} t_r = P(j,k)$. □

7. **Problem** $(\sum D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$**.** This section studies problem $(\sum w_j^A D_j^A + TC^A, \sum D_k^B + TC^B \leq V^B)$. The following results state the computational complexity and the structure of an optimal schedule for the considered problem.

Theorem 7.1. *Problem* $(\sum D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$ *is* $\mathcal{NP}$*-hard, even if there is no capacity constraint on the delivery transporters.*

Proof. For sufficiently large V^B, the problem reduces to the single-agent problem with an arbitrary number of fixed departure times to minimize $\sum w_j^A U_j^A$, which has been proven to be $\mathcal{NP}$-hard (see Theorem 2.9 in Hall et al. [8]), so the result follows. □

Lemma 7.2. *For problem* $(\sum D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$*, an optimal schedule exists, if any, such that all the following hold:*
(1) *The A-orders are processed in SPT order;*
(2) *The B-orders are processed in EDD order.*

In view of Lemma 6.2, we re-number the A-orders and B-orders in SPT order and EDD order, respectively, in this section. In what follows, based on Lemmas 3.1 and 6.2, we devise a forward dynamic programming solution procedure, to which we refer as Algorithm $DP3$, for problem $(\sum D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$ with pseudo-polynomial running time, indicating that it is binary $\mathcal{NP}$-hard.

For any given j and k, where $j = 0, \cdots, n_A$, and $k = 0, \cdots, n_B$, let $\mathcal{S}(j,k)$ the set of states, in which each state is a vector $(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B)$ that stands for a feasible partial schedule for the orders in $\{J_1^A, \cdots, J_j^A, J_1^B, \cdots, J_k^B\}$, where

- t is the total processing time of the early B-orders;
- l^B represents that the last early B-order is delivered at time T_{l^B};
- n_{l^B} indicates that there are n_{l^B} early B-orders delivered at time T_{l^B} with $\lceil \frac{n_l^B}{q_B} \rceil \le v_{l^B}^B$;
- f^B gives the weighted number of late B-jobs;
- l^A, n_{l^A}, and f^A are defined as those in Algorithm $DP1$.

The state sets $\mathcal{G}(j,k)$ are generated iteratively, which is initialed with $\mathcal{G}(0,0) = \{(\underbrace{0, \cdots, 0}_{7})\}$. In the (j,k)-th phase with $j+k \ge 1$, a state set $\mathcal{G}(j,k)$ is generated from $\mathcal{G}(j-1,k)$ and $\mathcal{G}(j,k-1)$. To be precise, for each state $(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B) \in \mathcal{G}(j-1,k)$, to append order J_j^A to the corresponding partial schedule, three decisions similar to *Decision FTA1* to *Decision FTA3* in Algorithm $DP1$ need to be considered.

For each state $(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B) \in \mathcal{G}(j,k-1)$, to append order J_k^B to the corresponding partial schedule, four decisions need to be considered.

Decision TWL1: schedule order J_k^B as early, and add it to the last used B-transporter with departure time T_{l^B}. This is valid only if $P(j)+t+p_k^B \le T_{l^B} \le d_k^B$ and $\frac{n_{l^B}+1}{q_B} < \lceil \frac{n_{l^B}+1}{q_B} \rceil \le v_{l^B}^B$, in which the first two inequalities ensure that order J_k^B can be delivered at time T_{l^B} prior to its due date, and $P(j)$ represents the total processing time of the first j A-orders. The contributions of order J_k^B in this case to the objectives of customers A and B are both 0. Therefore, if $P(j)+t+p_k^B \le T_{l^B} \le d_k^B$ and $\frac{n_{l^B}+1}{q_B} < \lceil \frac{n_{l^B}+1}{q_B} \rceil \le v_{l^B}^B$, add the state $(l^A, l^B, n_{l^A}, n_{l^B}+1, t+p_k^B, f^A, f^B)$ to $\mathcal{G}(j,k)$.

Decision TWL2: schedule order J_k^B as early, and add it to a new B-transporter with departure time T_{l^B}. This is valid only if $P(j)+t+p_k^B \le T_{l^B} \le d_k^B$ and $\frac{n_{l^B}}{q_B} = \lceil \frac{n_{l^B}}{q_B} \rceil$. The contributions of order J_k^B in this case to the objectives of customers A and B are 0 and c_B, respectively. Therefore, if $P(j)+t+p_k^B \le T_{l^B} \le d_k^B$, $\frac{n_{l^B}}{q_B} = \lceil \frac{n_{l^B}}{q_B} \rceil$ and $f^B + c_B \le V^B$, add the state $(l^A, l^B, n_{l^A}, n_{l^B}+1, t+p_k^B, f^A, f^B+c_B)$ to $\mathcal{G}(j,k)$.

Decision TWL3: schedule order J_k^B as early, and add it to a new B-transporter with departure time T_{o^B}, where o^B is the minimum subscript such that $l^B < o^B$, $v_{o^B}^B > 0$, and $P(j)+t+p_k^B \le T_{o^A} \le d_k^B$. This is valid only if $\frac{n_{l^B}}{q_B} = \lceil \frac{n_{l^B}}{q_B} \rceil$. The contributions of order J_k^B in this case to the objectives of customers A and B are 0 and c_B, respectively. Therefore, if there exists such a subscript o^B, $\frac{n_{l^B}}{q_B} = \lceil \frac{n_{l^B}}{q_B} \rceil$, and $f^B + c_B \le V^B$, add the state $(l^A, o^B, n_{l^A}, 1, t+p_k^B, f^A, f^B+c_B)$ to $\mathcal{G}(j,k)$.

Decision TWL4: schedule order J_k^B as a late order. By our assumption, order J_k^B is neither processed nor delivered in this case. The contributions of order J_k^B in this case to the objectives of customers A and B are 0 and w_k^B, respectively. Therefore, if $f^B + w_k^B \le V^B$, add the state $(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B + w_k^B)$ to $\mathcal{G}(j,k)$.

Analogous to Lemmas 4.2 and 4.3, we provide the following results to reduce the state set $\mathcal{G}(j,k)$.

Lemma 7.3. *For any two states* $(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B)$ *and* $(l'^A, l'^A, n'_{l^A}, n'_{l^B}, t', f'^A, f'^B)$ *in* $\mathcal{G}(j,k)$*, if* $l^A \leq l'^A, l^B \leq l'^B, n_{l^A} \leq n'_{l^A}, n_{l^A} \leq n'_{l^A}, t \leq t'$*,* $f^A \leq f'^A$*, and* $f^B \leq f'^B$*, the latter state can be deleted from* $\mathcal{G}(j,k)$*.*

Lemma 7.4. *For any state* $(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B)$ *in* $\mathcal{G}(j,k)$*, if* $(\frac{n_{s^A}}{q_A} < \lceil \frac{n_{s^A}}{q_A} \rceil$*,* $P(j) + t + p_k^B \leq T_{s^A}$ *and* $n_{l^A} \geq q_A)$ *or* $(\frac{n_{s^B}}{q_B} < \lceil \frac{n_{s^B}}{q_B} \rceil$*,* $P(j) + t + p_k^B \leq T_{s^B}$ *and* $n_{l^B} \geq q_B)$*, the state can be deleted from* $\mathcal{G}(j,k)$*.*

Algorithm $DP3$ can be formally depicted as follows:

Algorithm $DP3$

Step 1. Re-number the A-orders and B-orders in SPT order and EDD order, respectively.

Step 2. Set $\mathcal{G}(0,0) = (\underbrace{0, \cdots, 0}_{7})$ and $\mathcal{G}(j,k) = +\infty$ with $j = -1$ or $k = -1$.

Step 3.

For each combination of (j,k) with $0 \leq j \leq n_A$, $0 \leq k \leq n_B$, and $j + k \geq 1$, do

Set $\mathcal{G}(j,k) = \emptyset$;

For each $(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B) \in \mathcal{G}(j-1,k)$, do

/**Decision FTA1**/

If $P(j) + t \leq T_{l^A}$ and $\frac{n_{l^A}+1}{q_A} < \lceil \frac{n_{l^A}+1}{q_A} \rceil \leq v_{l^A}^A$, then

$\mathcal{G}(j,k) \leftarrow \mathcal{G}(j,k) \cup \{(l^A, l^B, n_{l^A}+1, n_{l^B}, t, f^A + T_{l^A}, f^B)\}$;

Endif

/**Decision FTA2**/

If $P(j) + t \leq T_{l^A}$ and $\frac{n_{l^A}}{q_A} = \lceil \frac{n_{l^A}}{q_A} \rceil$, then

$\mathcal{G}(j,k) \leftarrow \mathcal{G}(j,k) \cup \{(l^A, l^B, n_{l^A}+1, n_{l^B}, t, f^A + T_{l^A} + c_A, f^B)\}$;

Endif

/**Decision FTA3**/

If there exists a minimum subscript o^A such that $l^A < o^A$, $v_{o^A}^A > 0$, and $P(j) + t \leq T_{o^A}$, and $\frac{n_{l^A}}{q_A} = \lceil \frac{n_{l^A}}{q_A} \rceil$, then

$\mathcal{G}(j,k) \leftarrow \mathcal{G}(j,k) \cup \{(o^A, l^B, 1, n_{l^B}, t, f^A + T_{o^A} + c_A, f^B)\}$;

Endif

Endfor

For each $(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B) \in \mathcal{G}(j,k-1)$, do

/**Decision TWL1**/

If $P(j) + t + p_k^B \leq T_{l^B} \leq d_k^B$ and $\frac{n_{l^B}+1}{q_B} < \lceil \frac{n_{l^B}+1}{q_B} \rceil \leq v_{l^B}^B$, then

$\mathcal{G}(j,k) \leftarrow \mathcal{G}(j,k) \cup \{(l^A, l^B, n_{l^A}, n_{l^B}+1, t + p_k^B, f^A, f^B)\}$;

Endif

/**Decision TWL2**/

If $P(j) + t + p_k^B \leq T_{l^B} \leq d_k^B$, $\frac{n_{l^B}}{q_B} = \lceil \frac{n_{l^B}}{q_B} \rceil$ and $f^B + c_B \leq V^B$, then

$\mathcal{G}(j,k) \leftarrow \mathcal{G}(j,k) \cup \{(l^A, l^B, n_{l^A}, n_{l^B}+1, t + p_k^B, f^A, f^B + c_B)\}$;

Endif

/**Decision TWL3**/

If there exists a minimum subscript o^A such that $l^B < o^B$, $v_{o^B}^B > 0$, and $P(j) + t + p_k^B \leq T_{o^A} \leq d_k^B$, $\frac{n_{l^B}}{q_B} = \lceil \frac{n_{l^B}}{q_B} \rceil$, and $f^B + c_B \leq V^B$, then

$\mathcal{G}(j,k) \leftarrow \mathcal{G}(j,k) \cup \{(l^A, o^B, n_{l^A}, 1, t + p_k^B, f^A, f^B + c_B)\}$;

Endif

/**Decision TWL4**/

If $f^B + w_k^B \leq V^B$, then
$\mathcal{G}(j,k) \leftarrow \mathcal{G}(j,k) \cup \{(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B + w_k^B)\}$;
Endif
Endfor
/*Elimination*/
(1) For any two states $(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B)$ and $(l'^A, l'^A, n'_{l^A}, n'_{l^B}, t',$ $f'^A, f'^B)$ in $\mathcal{G}(j,k)$, if the conditions in Lemma 6.3 are valid, delete the latter state from $\mathcal{G}(j,k)$;
(2) For any state $(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B)$ in $\mathcal{G}(j,k)$, if the conditions in Lemma 6.4 are valid, delete the state from $\mathcal{G}(j,k)$;
Endfor
Step 4. The optimal solution value is $\min\{f^A|(l^A, l^B, n_{l^A}, n_{l^B}, t, f^A, f^B) \in \mathcal{G}(n_A, n_B)\}$.

Theorem 7.5. *Algorithm DP3 solves problem* $(\sum D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$ *in* $O(s^2 n_A n_B (n_{\max d}^A) n_{\max d}^B P^B V^B)$ *time.*

Proof. The proof is similar to that of Theorem 4.4, where the difference lies in that the number of states in $\mathcal{G}(j,k)$ is at most $O(s^2 n_{\max d}^A n_{\max d}^B P^A V^B)$ due to the fact that there are at most $(s+1)$ possible values for l^A and l^B, $(n_{\max d}^A + 1)$ and $(n_{\max d}^B + 1)$ possible values for n_{l^A} and n_{l^B}, respectively, P^B possible values for t, and $(V^B + 1)$ possible values for f^B. □

8. **Extensions.** This section analyzes the computational complexity of the other combinations of the considered optimality criteria of the two customers by borrowing the idea for designing the algorithms in Sections 3 through 6.

We first consider problem $(\sum w_j^A U_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$. By the proof of Theorem 6.1, we know that any problem involving the optimality criterion $\sum w_j^X U_j^X + TC^X, X \in \{A, B\}$ is $\mathcal{NP}$-hard, implying that the problem is $\mathcal{NP}$-hard too. Note that for the problem, property (1) in Lemma 6.2 is valid for both the A-orders and B-orders. Based on this, we can re-number both the A-orders and B-orders in EDD order, and define $(l^A, l^B, n_{l^A}, n_{l^B}, \mathrm{t}, f^A, f^B)$ that stands for a feasible partial schedule for the orders in $\{J_1^A, \cdots, J_j^A, J_1^B, \cdots, J_k^B\}$, where

- t is the total processing time of the early orders;
- l^A represents that the last early A-order is delivered at time T_{l^A};
- n_{l^A} indicates that there are n_{l^A} early A-orders delivered at time T_{l^A} with $\lceil \frac{n_l^A}{q_A} \rceil \leq v_{l^A}^A$;
- f^A gives the weighted number of late A-jobs;
- l^B, n_{l^B}, and $f^A B$ are defined as those in Algorithm $DP3$.

We devise a dynamic programming solution procedure with running time $O(s^2 n_A n_B$ $n_{\max d}^A n_{\max d}^B P V^B)$ to solve problem $(\sum w_j^A U_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$ by borrowing the idea for Algorithm $DP3$. It follows that the problem $(\sum w_j^A U_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$ is binary $\mathcal{NP}$-hard. As a consequence, the following result is valid.

Theorem 8.1. *Problem* $(\sum w_j^A U_j^A + TC^A, \sum w_k^B U_k^B + TC^B \leq V^B)$ *is binary* $\mathcal{NP}$*-hard and can be solved in* $O(s^2 n_A n_B n_{\max d}^A n_{\max d}^B P V^B)$ *time.*

Similarly, for other combinations of the considered optimality criteria of the two customers, we conclude with the following results.

Theorem 8.2. (1) *Problem* $(\sum w_j^B D_j^A + TC^A, f_{\max}^B + TC^B \le V^B)$ *is strongly* $\mathcal{NP}$*-hard; and problem* $(\sum w_j^B D_j^A + TC^A, f_{\max}^B + TC^B \le V^B)_{s=\bar{s}}$ *is binary* $\mathcal{NP}$*-hard and can be solved in* $O(n_A n_B^2 (n_{\max d}^A)^{\bar{s}-1} n_{\max d}^B P^{\bar{s}-1} \min\{v^B, n_B\})$ *time.*

(2) *Problem* $(f_{\max}^B + TC^B, \sum w_j^B D_j^A + TC^A \le V^A)$ *is binary* $\mathcal{NP}$*-hard; and problem* $(f_{\max}^B + TC^B, \sum w_j^B D_j^A + TC^A \le V^A)_{s=\bar{s}}$ *is binary* $\mathcal{NP}$*-hard and can be solved in* $O(n_A n_B^2 (n_{\max d}^A)^{\bar{s}-1} n_{\max d}^B\ P^{\bar{s}-1} \min\{v^B, n_B\} \log(Q_u^B - Q_l^B))$ *time.*

(3) *Problem* $(\sum w_j^B U_j^A + TC^A, f_{\max}^B + TC^B \le V^B)$ *is binary* $\mathcal{NP}$*-hard and can be solved in* $O(s^2 n_A n_B^2 n_{\max d}^A n_{\max d}^B P^B \min\{v^B, n_B\})$ *time.*

(4) *Problem* $(f_{\max}^B + TC^B, \sum w_j^B U_j^A + TC^A \le V^A)$ *is binary* $\mathcal{NP}$*-hard and can be solved in* $O(s^2 n_A n_B^2 n_{\max d}^A n_{\max d}^B P^B \min\{v^B, n_B\} \log(Q_u^B - Q_l^B))$ *time.*

(5) *Problem* $(\sum w_j^A D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \le V^B)$ *is strongly* $\mathcal{NP}$*-hard; and problem* $(\sum w_j^A D_j^A + TC^A, \sum w_k^B U_k^B + TC^B \le V^B)_{s=\bar{s}}$ *is binary* $\mathcal{NP}$*-hard and can be solved in* $O(n_A n_B (n_{\max d}^A)^{\bar{s}-1} n_{\max d}^B P^{\bar{s}-1} P^B V^B)$ *time.*

(6) *Problem* $(\sum D_j^A + TC^A, \sum D_k^B + TC^B \le V^B)$ *can be solved in* $O(s^2 n_A n_B n_{\max d}^A n_{\max d}^B V^B)$ *time.*

9. Conclusions.

This study considers several problems related to integrated scheduling of production and distribution operations with two competing customers, where the number of delivery transporters and the delivery capacity of each transporter are both limited, and the departure time of each delivery transporter is fixed and specified. Each customer desires to minimise a certain optimality criterion that takes into account both the customer service level and total delivery cost of its orders only. The overall goal is to determine a joint schedule of production and distribution to minimise the objective of one customer, subject to a limit on the objective of the other customer. We analyze the computational complexity of various problems and develop pseudo-polynomial-time solution procedures, if viable. However, the complexity status of problem $(\sum D_j^A + TC^A, \sum D_k^B + TC^B \le V^B)$ is still open.

Our model can be extended in various different directions. First, future research may analyze special cases of the $\mathcal{NP}$-hard problems, e.g., assuming the orders have identical processing times, the number or capacity of delivery transporters is unlimited etc. Second, it would be valuable to extend our model to more general machine environments, e.g., parallel-machine, flow shop etc. Third, it is of interest to investigate the model with more than two customers, e.g., one can consider the model of minimizing the objective of one customer, while keeping the value of each of the other customers' objective functions at or below a predefined value. This extension will not alter the model structure and the solutions we provide in our study remain valid. Fourth, it is interesting to perform sensitivity analyses of the key parameters of the model. Finally, it is challenging to study the model in the stochastic or dynamic setting.

Our research findings reveal that the problems under study are very difficult to solve. Thus, it is of great interest to design efficient and effective solution algorithms using various mixed integer linear programming methods, such as branch-and-price, Benders decomposition etc, or develop approximation algorithms and polynomial approximation schemes to deal with the computationally intractable cases.

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