Scalability and Latency Analysis of the Centralized 3D Mapping Aided GNSS-Based Collaborative Positioning

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BIOGRAPHY

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ABSTRACT

The recent development of the collaborative positioning (CP) algorithm enables a new possibility to improve the positioning performance, by making use of the measurements from neighboring road agents. Among different CP algorithms, the 3D mapping aided (3DMA) GNSS based CP has shown great potential to improve the GNSS positioning performance for the road agents located in the urban canyon. However, due to the difficulty of conducting real experiments, the 3DMA GNSS CP is usually only validated by the collaboration between few agents. The influence of the collaboration network size on the CP performance is worth to be investigated before its practical applications. Besides, the communication latency during CP may also degrade the positioning performance in real applications. In this study, based on a realistic GNSS measurement simulator, the impacts of the network size and the communication latency on the performance of the centralized 3DMA GNSS CP algorithm are investigated. The scalability analysis result shows that, simply enlarging the network size may not always improve the CP performance, or even worse, degrades the performance by involving additional error sources. On the other hand, the CP performance can be sensitively degraded by communication latency, which needs to be mitigated in practical applications.

1. INTRODUCTION

GNSS is essential for personal navigation and location-based service (LBS), which directly provides the absolution positioning solution without expensive or complicated instruments. The accuracy of GNSS positioning is a critical factor that guarantees the effectiveness of LBS applications. Due to the recent progress on wireless communication technology, it is feasible to improve the GNSS performance by collaborative positioning (CP) with neighboring users, besides relying on the techniques only from a single receiver aspect. A direct benefit of the CP is eliminating the systematic errors shared among the neighboring users in the same region. The double difference (DD) technique has been employed between different users during CP to achieve better relative position estimations by eliminating the atmospheric delay and satellite/receiver clock biases [1, 2]. Another benefit of collaborating the neighboring users for positioning is its potential to average and reduce the estimation variance due to measurement noise [3]. However, it is still challenging to achieve satisfactory GNSS positioning accuracy in dense urban areas, where most people and applications are located, especially for the users only equipping low-cost devices. The sky-view in the urban area can be significantly blocked by the buildings, resulting in a limited amount of visible satellites. Even worst, the reception of satellite signals reflected from surrounding buildings could introduce severe delay and attenuation, namely the multipath and non-line-of-sight (NLOS) receptions, leading to enormous positioning errors. The conventional CP methods are incapable of mitigating these interferences related to the surrounding environment and unique to individual users. Various methods have been explored to improve CP accuracy in the urban scenario. The consistency check technique has been employed during CP to detect and isolate the faulty measurements in urban areas [4]. On the other hand, the inter-agent ranging (IAR) technique has been developed to improve the positioning performance of the user lacks satellite visibility by the aid of neighboring users [5].

An effective approach to improve the CP performance in the urban area is to integrate with the 3DMA GNSS, which has shown excellent contributions to multipath/NLOS error mitigation. The shadow matching [6], one of the popular 3DMA GNSS techniques improving positioning accuracy by the building-model-predicted satellite visibility, has been extended with CP to reduce its solution ambiguous along a particular direction [7]. On the other hand, the satellite visibility prediction during shadow matching can be shared with neighboring users to exclude unreliable measurements [8]. Recently, another 3DMA GNSS method, GNSS ray-tracing [9], has been proposed to integrate with the DD technique for CP in the urban environment [10]. The 3DMA GNSS ray-tracing can correct the NLOS delay in pseudorange measurements and maintain a sufficient amount of satellite measurements, which guarantees the effectiveness of DD-based CP. Conversely, the DD-based CP technique can mitigate or even eliminate those shared unpredictable systematic errors or noises that degrade the performance of 3DMA GNSS ray-tracing. Therefore, the integration of CP with the 3DMA GNSS is complementary and significantly improves the GNSS positioning performance in the urban area.

Although the 3DMA GNSS CP has shown significant benefits, several issues remain to be addressed before its implementation. First, the scalability of the 3DMA GNSS CP needs to be analyzed. Many of the improved CP algorithms for the urban scenario are only validated by a network with a few collaborators, while the number of the available collaborators could be over ten or more in reality, especially for the cases including smartphone users. Usually, an extensive network size could help to scale down the variance of noisy factors. However, it may also incorporate the road agent with a very different error characteristic that degrades overall performance. Moreover, the collaboration of a larger network may also exponentially enhance the computation burden, which has already been massive for 3DMA GNSS based algorithms. On the other hand, the communication between different collaborators and the central server may experience latencies. Such latencies may cause some of the constraints are unavailable or even degenerate the factor graph of collaborating agents insufficient for the positioning optimization. The analysis of these effects needs the measurements and ground truth from a large number of receivers, and the capability to model communication latencies, which are very difficult to be achieved from a real experiment. Hence, a realistic multi-agent GNSS simulator for the urban scenario is also required to be developed beforehand.

In this study, the scalability and the latency-related degradation on the 3DMA GNSS CP algorithm in the urban canyon are analysed based on simulation. Firstly, the trajectories of multiple road agents in the urban canyon are simulated based on the SUMO (Simulation of Urban MObility) [11] to consider the dynamic behavior of vehicular or pedestrian agents. Based on the trajectory of each road agent, the corresponding GNSS measurements, including the carrier-to-noise ratio (C/N_0), pseudorange, and Doppler frequency, are simulated by the GNSS realistic urban multi-agent simulator (GNSS RUMS) [12] developed by our team. During the GNSS measurement simulation, the measurement degradation from buildings is searched using the ray-tracing technique, and further simulated by the GNSS-reflectometry (GNSS-R) technique [13] and the uniform geometrical theory of diffraction (UTD) [14] for the reflection effect and the diffraction effect, respectively. Moreover, the multipath interferences are simulated based on the superposition of electromagnetic fields, and the multipath noise envelope [15]. The simulated measurements from multiple road agents are then applied with the 3DMA GNSS based CP method proposed in [10] to evaluate the performance. For the scalability analysis, different numbers of road agents are randomly selected for the CP algorithm, in order to evaluate the positioning performance with different network sizes. Meanwhile, the positioning computation load corresponding to different networks size are evaluated. On the other hand, the performance degradations due to the communication latency are analysed by assigning different simulated upload delay based on the half-normal distribution []. The contribution of this study is twofold: 1) The impact of network size on the CP performance is analysed, which provides a potential direction to further improves the effectiveness of the CP algorithm; 2) The impact of communication latency is evaluated, exploring the requirement for practical CP applications.

The remainder of this paper is structured as follows. The structure and methodology of our developed GNSS realistic urban measurement simulator are briefly introduced in Section 2. The overview of the centralized 3DMA GNSS collaborative positioning algorithm employed in this study is given in Section 3. In Section 4, the detailed setups and results of the scalability analysis and the latency analysis are presented with discussions. Finally, the conclusion is drawn with suggested future works in Section 6.

2. GNSS REALISTIC URBAN MULTI-AGENT SIMULATOR

Since the GNSS measurements suffer complicated interferences from the buildings in the urban canyon, it is essential to employ a sophisticated GNSS measurement simulator to obtain the measurements with realistic error behaviors. Therefore, we employ our developed GNSS RUMS [12] to simulate realistic GNSS measurements for the performance analysis of the collaborative positioning algorithm. The overall flowchart of the employed simulator is given in Fig. 1. Firstly, the true trajectories $\mathbf{x}_R$ of multiple road agents, including pedestrians and vehicles, are generated from SUMO, which also simulates the agent dynamic behavior $\mathbf{v}_R$ by considering urban mobility. Based on the satellite position $\mathbf{x}_R$ and the 3D building model $\mathbf{X}_R$ (locations of the building corners), the ray-tracing technique is applied to trace the possible reflection or diffraction interferences in the located environment. Then, the GNSS measurement is categorized into 4 types with different simulation approaches. The LOS measurement is simulated based on the open-sky C/N_0 regression model [16] and the direct propagation geometrical parameters. The diffracted measurement is simulated based on the geometrical parameter from ray-tracing and the UTD model. Similarly, the reflected measurement is simulated based on the reflection geometrical parameters from ray-tracing and the GNSS-R model. For the multipath measurement with multiple available interferences, it is simulated based on the superposition of each signal, and the multipath noise envelope. After applying different simulation approaches, the GNSS C/N_0, interfered ranging measurement s, and the Doppler shift Δf are obtained, which will be further combined with the receiver-parameter-related systematic errors and noises as the final simulated GNSS measurements. The detailed simulation steps can be found in [12]. The simulated GNSS measurements of multiple road agents will be applied with the 3DMA GNSS collaborative positioning algorithm to evaluate the positioning performance.

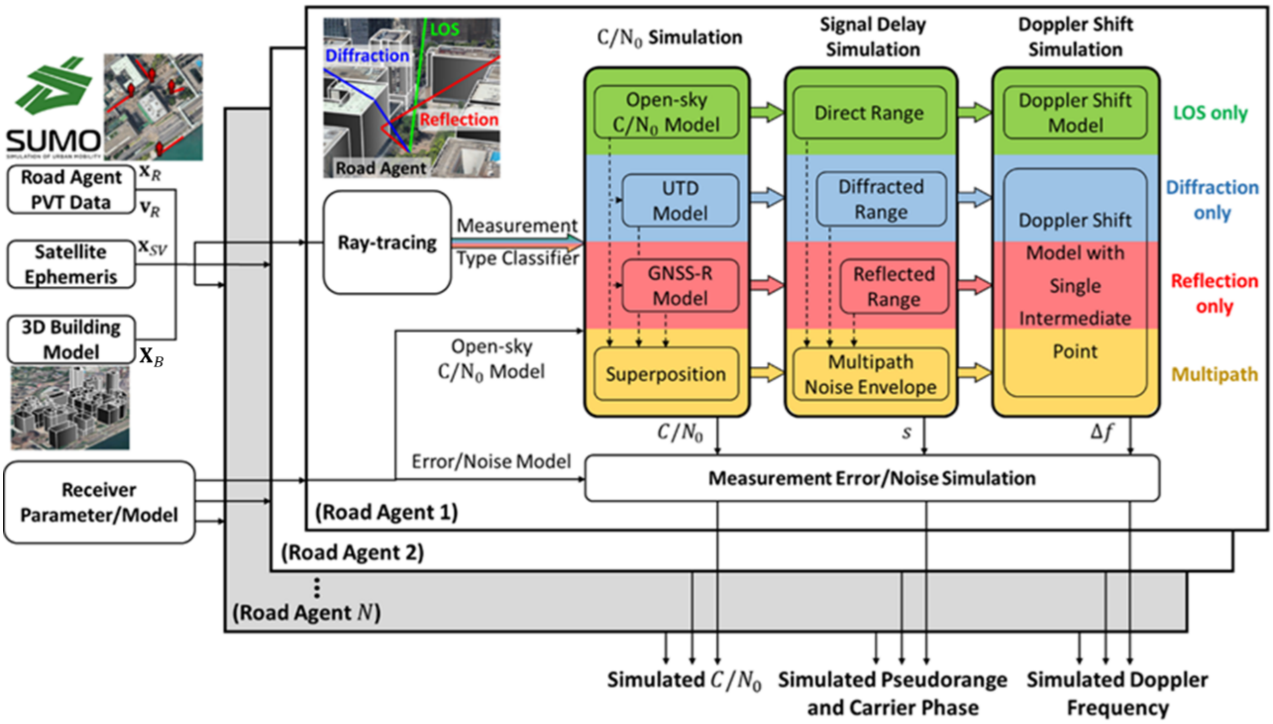

Fig. 1 The flowchart of the employed GNSS realistic urban multi-agent simulator, GNSS RUMS [12].

3. CENTRALIZED 3DMA GNSS COLLABORATIVE POSITIONING ALGORITHM

The simulated GNSS measurements of different road agents will be collaborated to apply our previously developed 3DMA GNSS collaborative positioning algorithm in [10], which is effective in the dense urban area. During this collaborative positioning, the ray-tracing technique is first employed to evaluate the measurement status and the reflection delay on the candidate positions around the agent's initial location estimated from raw measurements. Based on the ray-tracing result, each agent's absolute position can be preliminarily determined by searching for the candidate position with the interfered measurements best matching the ray-tracing results. On the other hand, the pseudorange measurements of each agent will be applied with different reflection delay compensation based on the ray-tracing result on its different candidate positions. Each compensated measurement sets from one agent will be paired with those from another agent, and further applied with the DD technique with consistency-check, in order the obtain the corresponding relative positioning estimations. Then, by searching for the relative positioning estimation best consistent with the corresponding candidate positions, the relative position between agents can be determined. After that, the positions of all the involving agents form a factor graph, where the preceding absolute and relative positioning solutions are employed as the constraints. Moreover, the Doppler shift measurements of each agent are used to obtain the inter-epoch dynamic constraint, which connects and extends the factor graph along the operation time. The uncertainty of each constraint is evaluated based on the predicted positioning error map, which can be simulated from the preceding ray-tracing results. Finally, by optimizing the factor graph, a more accurate and robust positioning solution can be obtained for each road agent in the urban canyon.

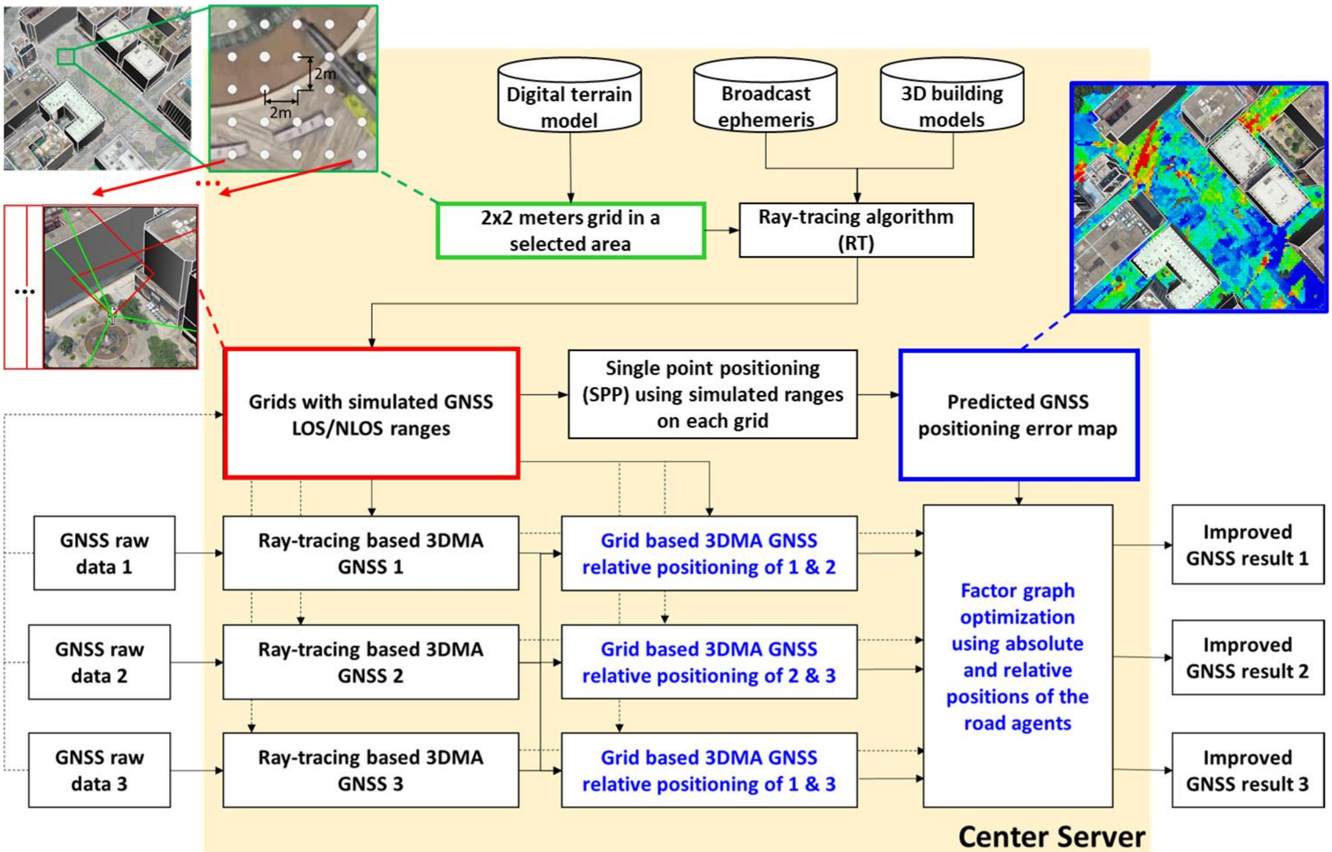

Fig. 2 The system architecture of the employed 3DMA GNSS collaborative positioning algorithm [10].

4. SCALABILITY ANALYSIS FOR 3DMA GNSS CP

4.1 Analysis setup

The collaborative positioning algorithm may achieve different performances on different networks, especially the networks with different collaborator amount, namely the network size. To evaluate the influence of network size on the collaborative positioning performance, 30 seconds of the GNSS measurements from 30 road agents are simulated, as Fig. 3 shows, including 8 vehicles and 22 pedestrians. A pedestrian agent in the urban canyon with enormous least squares positioning error is selected as the target agent for evaluation. Then, for each network size N, $(N-1)$ agents are randomly selected from the available 29 agents other than the target agent. Those randomly chosen agents collaborate with the target agent to form a network with size N, which will be applied with the 3DMA GNSS CP technique introduced in Section 3. For different network size N, the Monte Carlo method with 20 samples is conducted to evaluate the 3DMA GNSS CP performance with different collaborator combinations. Finally, the averaged positioning root-mean-square-error (RMSE) of the target agent among 20 Monte Carlo method samples denotes the positioning performance on a specific network size N. Meanwhile, the computation load of the centralized 3DMA GNSS CP during the test are recorded to compare with the accuracy improvement.

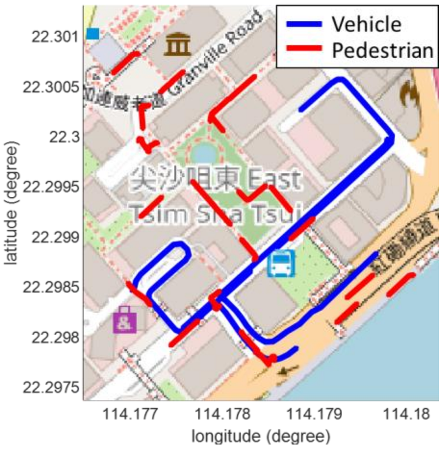

Fig. 3 The simulation setup for the scalability analysis.

4.2 Positioning performance analysis

By conducting the scalability analysis based on the simulated measurements, the positioning performances of the target agent from the centralized 3DMA GNSS CP (RT-FGO) in Section 3 with different network sizes are shown in Fig. 4, compared with the performances from the stand-alone least squares method (LS), the stand-alone 3DMA GNSS ray-tracing positioning method (RT), and the anchor-based 3DMA GNSS CP with shadow matching (SM-CP) [17]. The stand-alone LS and RT approach for reference have around 50 meters and 20 meters RMSE, respectively. For the SM-CP approach, it can achieve a similar positioning accuracy compared to the RT approach. By enlarging the network size, the SM-CP has a higher possibility to select a more reliable anchor from the participating agents to conduct collaborative positioning, resulting in a better positioning performance. For the RT-FGO, the overall positioning accuracy is better than other methods. However, its positioning performance does not show a straightforward relationship with the network size.

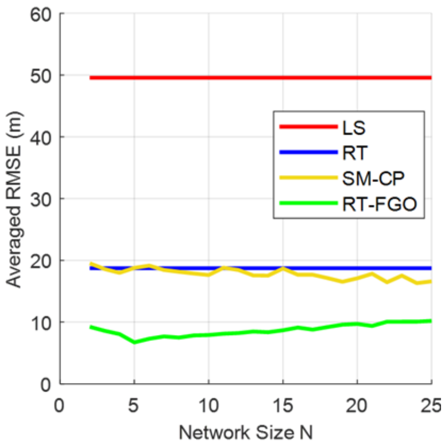

Fig. 4 The employed centralized 3DMA GNSS CP performance (RT-FGO) for the target agent with different network size, compared with the stand-alone least squares positioning (LS), stand-alone 3DMA GNSS ray-tracing positioning (RT), and the anchor-based 3DMA GNSS CP with shadow matching (SM-CP).

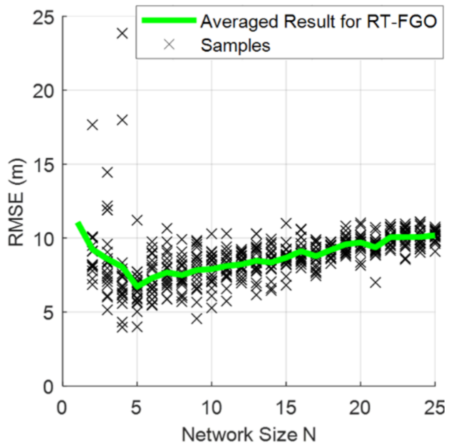

Fig. 5 The RMSE of each sample and the averaged value from the Monte Carlo method on the 3DMA GNSS CP performance with different networks size.

The detailed positioning performance of RT-FGO with Monte Carlo method samples is shown in Fig. 5. When the network size $N = 1$, only the inter-epoch dynamic constraints (derived from the Doppler shift measurements) are available for the target agent in the factor graph, whereas no inter-agent constraint is available for the target agent. After optimizing this degenerated factor graph, the corresponding positioning RMSE is 11.1 meters. For the network size $N = 2$, the target agent starts to collaborate with another randomly chosen agent, where the inter-agent constraints are available in the factor graph. With the additional information and constraints from the neighboring agent, the RT-FGO positioning performance is slightly improved. However, since the network size is limited and has only one collaborator, it has a high possibility that the target agent is only collaborating with a severely degraded agent with poor inter-agent constraints, which introduce significant positioning error after optimization. As a result, the averaged performance improvement with $N = 2$ is limited. When the network size increases, more inter-agent constraints from different agents become available to the target agent. Therefore, for a larger network, the final optimization performance will be less dominated by unreliable collaborators with poor constraints, achieving more robust performance improvements. This can be verified by the Monte Carlo method samples in Fig. 5, where the samples with extreme errors are less likely to occur for a larger network.

However, in this simulation analysis, after the network size reaching five, increasing the networks will degrade the positioning optimization performance. Although the result becomes more robust with less variance among Monte Carlo method samples, the capability of the network achieving less positioning error is also reduced. This may be because most of the collaborating road agents are located in the urban canyon with complicated measurement interferences, involving signal reflection, diffraction, or multipath. These interferences are more likely to introduce a significant bias on the measurements. The collaborative positioning is famous for reducing the measurement-variance-related errors with similar behavior between agents, but less effective for the bias-related error that is usually unique to different agents. By employing the ray-tracing, the RT-FGO can mitigate many of the bias-related degradations with NLOS delay correction, but it cannot effectively mitigate the multipath error. Therefore, when involving more agents to reduce the noise, in the meantime, more unique errors from different agents are also involved, resulting in accuracy degradation after the network is enlarged to a certain size.

It is worth mentioning that, even though the NLOS reflection and the multipath error are uniquely introduced based on the building geometries surrounding the road agents, these errors could still have correlations for some of the agents in a network. The study from [18] investigates the NLOS reflection correlations within a certain area of the urban trench. It has great potential to make use of such correlations to build up special constraints in the collaborative positioning, which may also be used to mitigate the aforementioned bias-related degradations. Moreover, by applying the collaborator selection before CP based on the error correlation behavior, the collaborating agents could be guaranteed to have less unique measurement error, which enables the CP to achieve better performance (e.g. the network sample with minimum RMSE in Fig. 5). A good starting point is to investigate the subset of networks that able to achieve outstanding performance in the simulation.

4.3 Computation load analysis

Besides the positioning accuracy, the computation load of the centralized 3DMA GNSS CP is also important for practical applications. Fig. 6 shows the averaged computation cost during the scalability analysis with different network sizes in Section 4.2. The total computation load can be divided into three main parts: RT-SPP) the stand-alone 3DMA GNSS ray-tracing positioning to derive the NLOS delay corrections and the absolution positioning constraints in the factor graph; RTDDCC) the ray-tracing based NLOS-delay-corrected DD positioning with consistency check, which derives the relative positioning constraints collaborating different agents in the factor graph; and FGO) the optimization of factor graph which determines the final positioning solution of each agent. As the result shows, the computation load is dominated by the RTDDCC process. Since the factor graph has been famous for autonomous driving applications with real-time operation requirements, it already employed advanced enhancements on computation load reduction, which cost the lowest computation. The RT-SPP has a large computation load by applying ray-tracing to search for available interferences, while the RTDDCC employs the same idea but with the need of $\frac{N(N-1)}{2}$ running times for the network with size N (applied on all different agent combinations in the network). Therefore, the computation increment of the centralized 3DMA GNSS CP follows $O(N^2)$ for the network with size N.

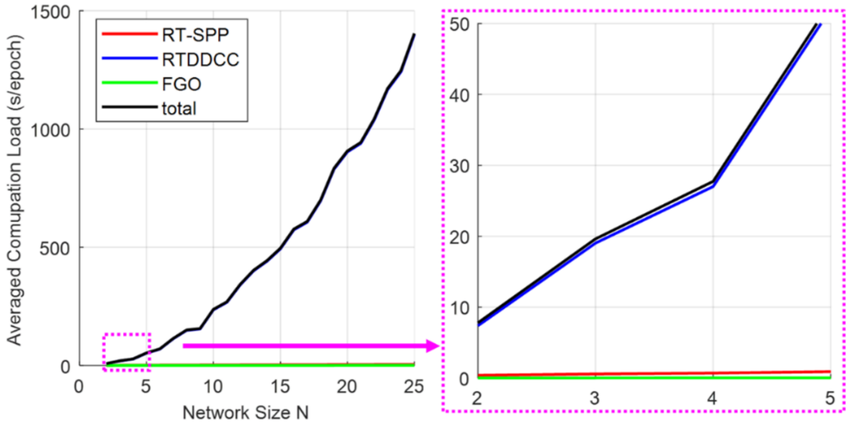

Fig. 6 The computation load during the centralized 3DMA GNSS CP. The total computation load is separated into three parts: RT-SPP) 3DMA GNSS ray-tracing positioning for the absolute positioning constraint of the factor graph; RTDDCC) the relative positioning constraint from the ray-tracing corrected DD positioning with consistency check; and FGO); factor graph construction and optimization.

5. LATENCY ANALYSIS FOR 3DMA GNSS CP

5.1 Analysis setup

During the real application of the centralized 3DMA GNSS CP, the communication latency between each road agent and the center server may introduce severe performance degradation. The delayed measurements from the road agent will make the constraints between that agent and other collaborators become unavailable, resulting in an out-of-date or degenerated factor graph with limited performance. To evaluate the latency degradation effect on the centralized 3DMA GNSS CP positioning, we employ the same GNSS measurement simulation from the scalability analysis (Section 4.1). The same target agent and the other four collaborators with effective relative positioning constraints are selected to form a 5-agent collaborative network for evaluation. Since the communication latency is always positive, a simulated delay with the half-normal distribution with the variance $\sigma_{latency}$ is randomly assigned to the measurement of each road agent on each epoch. The delayed measurements and all related inter-agent or inter-epoch constraints are not available for the factor graph, until the passing time from the delayed-measurement-sending time is larger than the latency. As an example demonstrated in Fig. 7, for the agent 2 on the epoch t with a simulated latency δ_t, all the related constraints are unavailable for the graph during the optimization process. When the operation epoch reaches $t + \Delta t$, where $\Delta t \geq \delta_t$, all the constraints related to the agent 2 are recovered for the factor graph optimization. To maintain the factor graph for optimization, the missed absolute positioning constraint for the delayed agent will be temporarily replaced by the constraints from the last available measurements. Since the GNSS measurements usually have a 1 Hz measurement rate for civil applications, the measurement with a delay below one second will still be treated as timely measurements. The latency-related degradation is evaluated by conducting the Monte Carlo method with 30 samples on the delay simulation model with different $\sigma_{latency}$ from 0 to 30 seconds.

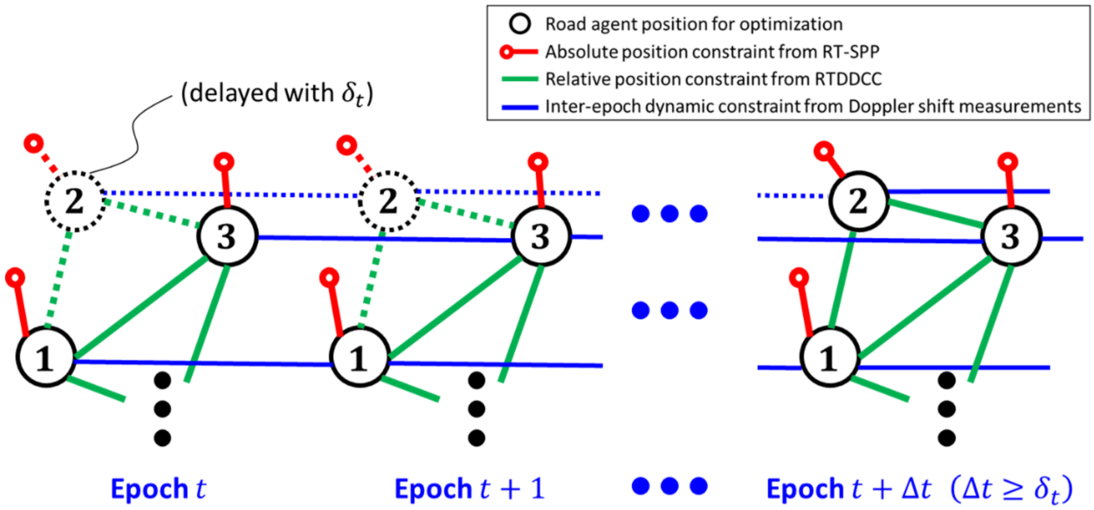

Fig. 7 Demonstration of the delayed-measurement-related constraints become unavailable for the factor graph, and their recovery when the passing time is over the delay time.

5.2 Analysis result

The positioning RMSEs of the centralized 3DMA GNSS CP from Monte Carlo method samples with different measurement latency setups are shown in Fig. 8. The averaged positioning performance for the target agent from the RT-FGO under the Monte Carlo method is compared with the stand-alone 3DMA GNSS ray-tracing positioning performance. For the case without any delay ($\sigma_{latency} = 0$), the RT-FGO achieves the positioning accuracy as 7.9 meters RMSE for the target agent of the selected network. When the simulated latency starts to be assigned, the RT-FGO positioning performance is severely degraded. With a larger $\sigma_{latency}$, more measurements are delayed with longer latency. Many of the inter-agent and inter-epoch constraints are not available in the factor graph, resulting in a poorly connect graph for optimization. As the $\sigma_{latency}$ is increased close to the simulated operation duration (30 seconds), only the out-of-date absolute constraints exist in the degenerated graph, whereas most of the inter-node constraints are not available in the whole process. Consequently, the performance of RT-FGO will be degraded close to the stand-alone ray-tracing positioning performance. Therefore, the impact of communication latency on the centralized 3DMA GNSS CP is significant.

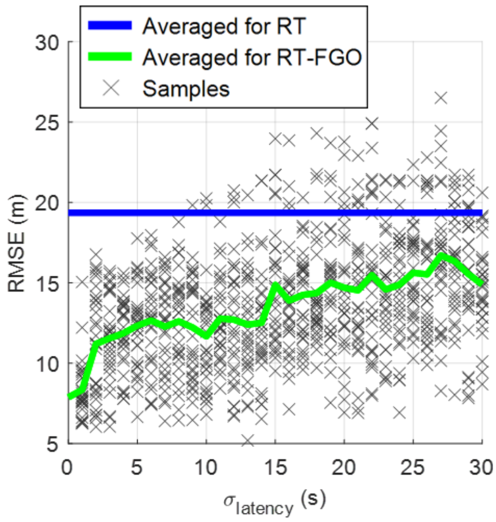

Fig. 8 The RMSEs of the centralized 3DMA GNSS CP and their averaged value (Averaged for RT-FGO) among 30 Monte Carlo method samples with different measurement latency setups, compared with the stand-alone ray-tracing positioning performance without collaboration (Averaged for RT).

6. CONCLUSIONS AND FUTURE WORKS

In this paper, the scalability and the latency degradation of the centralized 3DMA GNSS CP algorithm in the urban canyon are analyzed based on a realistic GNSS measurement simulator and the Monte Carlo method. From the scalability analysis result, the positioning performance is improved by enlarging the network size during CP, but starts to be degraded after the size is over five agents. Although the CP error due to the shared measurement noises is mitigated when the network size is increased, more unique errors from different agents are also involved in the meantime. As a result, the overall CP performance may be degraded after involving more agents with unreliable constraints in a certain size network. On the other hand, the computation load of the centralized 3DMA GNSS CP is dominated by the inter-agent constraint estimation approach, which follows a square increment to the network size. Finally, the latency analysis result indicates the degradation from communication delay for the centralized 3DMA GNSS CP is significant, which degenerates its performance to the stand-alone ray-tracing positioning performance under a sufficient large delaying circumstance.

The scalability analysis result shows a potential direction to further improve the performance of CP in a complicated environment. Although the CP is famous for mitigating the noise with similar behavior among the neighboring road agents, collaborating with other agents may also involve other error sources. Some of them are even uniquely related to individual road agents and unable to be mitigated by the CP, which degrades the overall performance. It is necessary to develop an intelligent collaborator selection strategy in the future, which employs the error correlations between agents to guarantee the effectiveness of the CP.

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