A Super-Resolution Algorithm with FRFT Towards GNSS TOA Estimation for Multipath Channel

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ABSTRACT

Multipath channels can highly reduce the performance of global navigation satellite system (GNSS) time-of-arrival (TOA) estimations, as a non-line-of-sight (NLOS) signal can distort the estimating accuracy for the line-of-sight (LOS) signal in a conventional GNSS receiver. This paper firstly investigates the dynamic difference of the LOS and NLOS signals for a moving GNSS antenna. Then, the fractional Fourier transform (FRFT) algorithm is presented to match the signals with different Doppler rate values in a multipath channel. A long coherent processing interval (CPI) is also adopted to enhance the sensitivity and accuracy of the FRFT match filter. Simulating results demonstrate the high performance for detections towards the LOS/NLOS signals in the fractional Fourier domain (FRFD) based on the proposed FRFT super-resolution algorithm (SRA).

INTRODUCTION

As the environments of the Earth are complicated and volatile, the global navigation satellite system (GNSS) signals are frequently confronted with severe reflection and refraction due to the blockages from different obstacles when the GNSS signals are received on the ground. This fact leads to the consequence that the multipath interferences and the non-line-of-sight (NLOS) receptions can highly degrades the localization performance behind the GNSS signal processing. Specifically speaking, the conventional tracking method may have troubles with precisely estimating the biased error in a multipath channel. More seriously, the occurrence of the multipath is always accompanied by the signal power attenuation. The tracking sensitivity would be evidently affected by this case. In order to overcome the drawback of the signal processing in a low carrier-to-noise ratio density (C/N_0) condition, the long coherent integration algorithm is acknowledged as an efficient method to improve the estimation accuracy [1], [2]. Also, a super-resolution algorithm (SRA) can be achieved based on the long coherent integration in signal processing for target detection [3]. And in the field of time-of-arrival (TOA) estimation, a recent research work has verified that the SRA has a superior performance in multipath mitigation [4]. However, this technique is also faced with massive issues in a practical situation. One of the most intractable issues

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is related to the dynamics including the ones caused by the clock and the user's motion. The dynamics induce in a frequency error behind the correlation of the incoming signal and the local replica. The integration gain could be severely reduced by this sort of frequency error if it is accumulated over a longer discrete time slot than the usual. The gain reduction with respect to the code cross-correlation would also become much more critical than the case where a conventional integration process is adopted [5], [6]. In other words, it can be said that the influence of the signal dynamics would be exaggerated in a long coherent integration process. Aiming to achieve a more reliable and more accurate GNSS TOA estimation, instead of the traditional algorithm stage, measures about dealing with the dynamics and the low- $\mathrm{C/N_0}$ condition, should be taken to improve the localization performance in the level of the signal channel.

The fractional Fourier transform (FRFT) has been proposed for decades of years and it has been verified with a promising performance on processing a linear frequency modulation (LFM) or a Chirp signal [3], [6], [7]. It has been commonly used in Radar Chirp signal processing and GNSS signal anti-jamming. As usual, the dynamic GNSS signal can be almost approximated as a Chirp signal model as well. Besides of the previously mentioned dynamic stress in the GNSS receivers, this paper will exploit the dynamic difference between the LOS and NLOS signals to improve the GNSS TOA estimation in a multipath channel as well. This idea will be investigated in the subsequent section.

By combining the concept of the FRFT with GNSS TOA estimation, the FRFT has the potential to solve the previously mentioned issue. In this paper, the FRFT in contrast to an original fast Fourier transform (FFT) will be utilized to achieve the SRA of TOA estimation for a multipath channel. The FRFT is a generalized form of the Fourier transform and the signal in the time domain can be mapped into a two-dimension (2D) space with respect to the Fourier domain and the fractional Fourier domain (FRFD), respectively. As it can be found that the mapping space becomes more abundant than the original FFT, the gains related to the carrier Doppler frequency, carrier Doppler rate and the $\mathrm{C/N_0}$ values can be taken into consideration to achieve the TOA estimation of the LOS signal in a multipath channel. Then, the performance of GNSS receivers can be accordingly improved in a harsh environment.

The main contributions of this paper can be summarized as follows:

1. A dynamic GNSS signal model for a multipath channel is proposed;
2. A FRFT-based super-resolution algorithm using a long coherent processing interval (CPI) is presented for the LOS/NLOS signal detection to improve the GNSS TOA estimation.

MULTIPATH SIGNAL MODEL

Assuming that no bit transition occurs, then, a channel with the incoming LOS and NLOS signals will be modeled and the complex signal in this channel after down-conversion is normally given by [8] [9]

$$x_l(t) = \sum_{l=0}^{L-1} c_l(t+\tau_l) \cdot s_l(t) e^{j2\pi f_I t} \tag{1}$$

with

$$s_l(t) = a_l \cdot e^{j\left(\phi_l + 2\pi f_l t + \pi \mu_l t^2\right)}$$

where

$x_l(t)$ is the real GNSS signal without noise;

L denotes the total paths of the incoming signals;

l is the index of the signal path where $l \in \mathbb{N}$ and it corresponds to the LOS signal when $l = 0$ while the other cases correspond to the NLOS signals;

f_I is the intermediate frequency;

f_l is the Doppler frequency linearized with the time variable;

μ_l is the Doppler rate reflecting a constant change of the Doppler frequency with respect to the time variable;

ϕ_l is the initial carrier phase of the received signal;

$c_l(t)$ is the code signal modulated on the radio frequency emitted by the satellite;

τ_l is the propagation delay of the l -th signal path;

a_l is the signal amplitude;

τ_l and ϕ_l are the functions of the user position vector $\mathbf{p}$, that is, $\tau_l \triangleq \tau_l(t, \mathbf{p})$ and $\phi_l \triangleq \phi_l(\mathbf{p})$.

Next, a diagram which demonstrates the evolution pattern of the NLOS and LOS signals over a certain period is provided for a kinematic user's receiver in Figure 1. In order to explain the Doppler rate term μ_l in (1), the double difference related to the LOS and NLOS distances between two adjacent epochs need to be assessed at the first time. According to the demonstration in Figure 1, based on the law of cosines, it is easy to know that $\Delta l_k \neq \Delta l_{k-1}$ when the azimuth angle and elevation angle in terms of reflecting points (e.g., caused by buildings and vehicles, etc., in an urban area) are being changed during the motion of the user. This case is easy to occur in practical cases. Under this circumstance, it can be concluded that there would be a Doppler rate offset between the LOS and NLOS signals when the user is not static. Especially, compared with the NLOS signal, the Doppler rate value of the LOS signal is closer to zero.

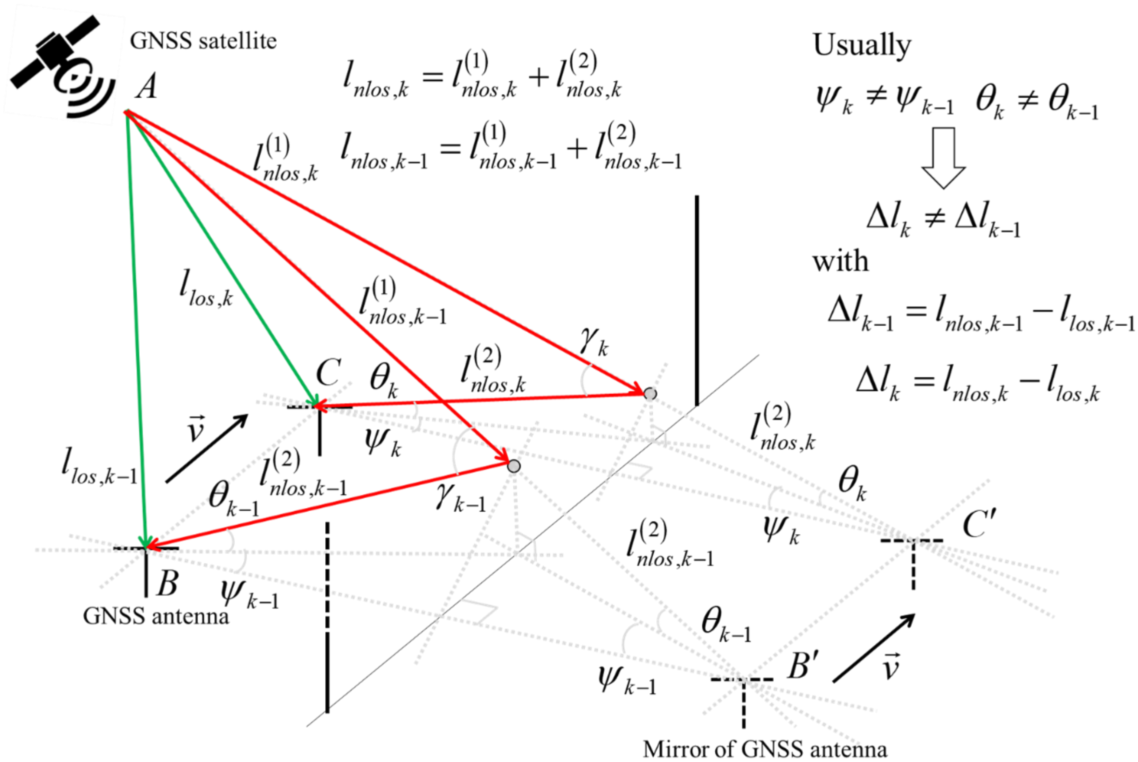

Figure 1 Diagram of the evolution pattern of the NLOS and LOS signals over a certain period for a kinematic user's receiver. (k denotes the index of updating epochs; θ_k and ψ_k correspond to the elevation and azimuth angles, respectively; γ_k is the angel between the reflected NLOS signal $l^{(2)}_{nlos,k}$ and the direct NLOS signal $l^{(1)}_{nlos,k}$; $\vec{v}$ denotes the velocity vector for the user; Δl_k and Δl_{k-1} denote the differences of the NLOS and LOS distances for k th and $k-1$th epochs, respectively; for practical cases, it can easily satisfy $\theta_k \neq \theta_{k-1}$ and $\psi_k \neq \psi_{k-1}$)

PROPOSED FRFT-BASED SRA

The FRFT match filter is used in this work to discriminate the difference of the Doppler rate term between the LOS and NLOS GNSS signal. The flow chart of the proposed algorithm for the TOA estimation in a single GNSS channel interfered by the multipath effect is displayed in Figure 2. In this algorithm, a type of the slow-time discrete signal is firstly formed with fixed locally replicated code and carrier frequencies over a long CPI. Each of the slow-time samples is obtained from the traditional correlating and integrating processes of GNSS receivers [1]. Thus, the original intermediate frequency samples are resampled to be at a lower sampling rate in this way. Next, the CPI samples are produced from the slow-time signal samples using the FRFT algorithm. By implementing power decision-making for the final CPI samples, the LOS and the NLOS signals can be identified and quantified by the different frequency and Doppler rate errors. Once the CPI sample power, Doppler frequency and Doppler rate values can be estimated for the respective LOS and NLOS signals, more accurate local replicas can be formed to improve the accuracy of the GNSS TOA estimation.

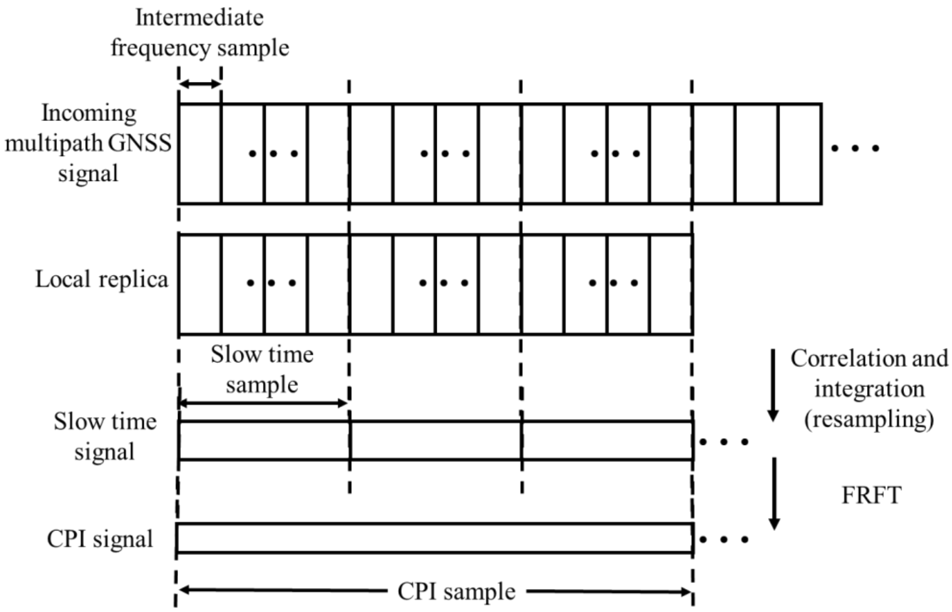

Figure 2 Proposed SRA with the FRFT match filter for GNSS TOA estimation in a multipath channel.

The model for the CPI sample based on the FRFT can be expressed by [5], [10]

$$X\left(m_p, m_u\right) \triangleq DF^p\left\{\chi\left(n_S\right)\right\} \tag{2}$$

with

$$\begin{aligned}\chi\left(n_S\right) &= \frac{1}{T_S}\int_{(n_S-1)T_S}^{n_S T_S}\sum_{l=0}^{L-1}\left(c_l\left(t+\tau_l\right)\cdot s_l\left(t\right)e^{j2\pi f_l t}\right)\cdot\left(\hat{c}_0\left(t+\hat{\tau}_0\right)\hat{s}_0\left(t\right)e^{j2\pi f_l t}\right)^* dt \\ &= \sum_{l=0}^{L-1} a_l \operatorname{sinc}\left(\Delta f_l T_S + \frac{\mu_l}{2} n_S T_S^2\right) R_{cc}\left[\Delta\tau_l\right] e^{j2\pi\left(\frac{\Delta\phi_l}{2\pi}+\Delta f_l n_S T_S + \frac{1}{2}\mu_l n_S^2 T_S^2\right)}\end{aligned}$$

$$\Delta\phi_l = \phi_l - \hat{\phi}_0,\quad \Delta f_l = f_l - \hat{f}_0,\quad \Delta\tau_l = \tau_l - \hat{\tau}_0$$

where

p is the FRFT order related to the Doppler rate estimation in the match filtering process, m_p is index of the FRFT order match filter in the FRFT domain, m_u is the index of the frequency match filter in the FRFT domain, $DF^p\{\cdot\}$ denotes the operator of the digital FRFT implementation [10];

$\chi\left(n_S\right)$ is denoted as a slow-time correlating sample formed with the incoming signal and the local replica, n_S is the index of the signal sample in the slow-time domain over a CPI processing, T_S is the slow-time interval;

$\hat{c}_0\left(t\right)$ and $\hat{s}_0\left(t\right)$ are locally replicated signals for the code and the carrier signals related to the LOS path, respectively;

$\hat{\phi}_0$, $\hat{f}_0$ and $\hat{\tau}_0$ are the locally replicated initial carrier phase, carrier Doppler frequency, and TOA estimation for the LOS signal, respectively;

$R_{cc}[\cdot]$ is the operator of the auto-correlation function for two spreading code sequences and vary with the time delay error $\Delta\tau_l$.

For $\mathcal{N}_{m_u} = \left\{x \middle| -\frac{N_S}{2\sin\alpha} \le x \le \frac{N_S}{2\sin\alpha}, x\in\mathbb{Z}\right\}$ and $\mathcal{N}_{m_p} = \left\{x \middle| -\frac{1}{\Delta p} \le x \le \frac{1}{\Delta p}, x\in\mathbb{Z}\right\}$, where N_S is the number of slow-time samples over a CPI processing, α is the rotation angle of the FRFT process and it is equal to $\frac{\pi}{2}p$, Δp is the FRFT order step. Then, the Doppler rate and the frequency of the LOS signal can be usually matched and estimated with respect to the amplitude peak formed with the FRFT CPI samples as follows

$$\left[\hat{m}_p,\hat{m}_u\right]=\underset{m_u\in\mathcal{N}_{m_u},m_p\in\mathcal{N}_{m_p}}{\arg\max}\left\{\sqrt{\Re^2\left\{X\left(m_p,m_u\right)\right\}+\Im^2\left\{X\left(m_p,m_u\right)\right\}}\right\} \tag{3}$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote the real and image part operators of a complex value, respectively; a proper FRFT order step Δp can be computed by the algorithms proposed by the previous paper of the authors [5]. Based on (3), the Doppler rate can be estimated with the basic FRFT match filter theory as [5]

$$\hat{\mu}_0\left(\hat{m}_p\right)=-\frac{\cot\left(\frac{\pi}{2}\left(1+\hat{m}_p\cdot\Delta p\right)\right)}{N_S T_S^2} \tag{4}$$

The averaging frequency error over the CPI can be estimated as

$$\hat{u}_0\left(\hat{m}_p,\hat{m}_u\right)=\frac{\hat{m}_u}{N_S T_S\sin\left(\frac{\pi}{2}\left(1+\hat{m}_p\cdot\Delta p\right)\right)}$$

Therefore, the initial frequency error can be estimated as

$$\Delta\hat{f}_0=\hat{u}_0\left(\hat{m}_p,\hat{m}_u\right)-\frac{\hat{\mu}_0\left(\hat{m}_p\right)}{2}N_S T_S \tag{5}$$

It is worth noting that the related NLOS signal parameters can be also estimated by the CPI samples with secondary power strengths.

SIMULATIONS AND DISCUSSIONS

The simulations are implemented in this work to verify the proposed FRFT SRA for the multipath channel estimation of the GNSS signal. Two signal paths (one path for the LOS signal and the other for the NLOS signal) are considered in the simulations and the simulating parameters are listed in Table 1 where λ is the wavelength of the radio-frequency signal and the TOA delay error corresponds to the TOA difference between the LOS and the NLOS signals. As to the signal parameters for the FRFT SRA implementation, the settings are listed in Table 2. The initial code phase error between the local replica and the LOS signal is set as zero in this simulation. The way to decide a proper FRFT order step Δp can refer to the algorithms proposed in [5].

Table 1 Simulation parameters of the multipath channel for the GNSS signal.

Parameter	**Value**
Signal Type	GPS L1 C/A
Radio frequency	1575.42 MHz
Code frequency	1.023 MHz
PRN number	1
Number of the signal channel	1
Number of the signal path in a channel	2 (one LOS signal and one NLOS signal)
TOA delay error	$0.1902(2\lambda)^{-1}$ cycles, $0.1902\lambda^{-1}$ cycles, 0.03 chips, 0.5 chips, 1 chip
C/N_0	45 dB-Hz
Amplitude ratio of the multipath signal $\frac{a_1}{a_0}$	0.5, 1
Input initial Doppler frequency for the LOS and NLOS signal	20 Hz
Input Doppler rate for the LOS signal	0 g
Input Doppler rate for the NLOS signal	0 g, 0.2 g, 0.4 g, 1 g

Table 2 Parameter settings of the FRFT processing for the SRA.

Parameter	**Value**
CPI $N_S T_S$	1.024 s
Slow time interval T_S	1 ms
FRFT order step Δp	0.001112

At first, considering that there is no Doppler rate difference between the LOS and the NLOS signal, and also the TOA delay error is considered to be very close to one and half of the carrier cycles, the match filtering results are illustrated in Figure 3 and Figure 4, respectively. It can be found that the signal power of the LOS and NLOS signals are overlapped in the FRFD if the Doppler rate difference is zero. Especially, the combined signal power will be enhanced once and be largely reduced for a full and a half of wavelength TOA delay difference, respectively. The weak power peak is still formed in the case of Figure 4, as the remained frequency error caused by the difference of the simulated TOA delay error $0.5\times0.1902\times c^{-1}$ for the NLOS signal and the delay error $0.5\lambda\cdot c^{-1}$ for the simulated LOS signal over a CPI time spanning dominates such power peak. But the power has been proved to be very weak and it is consistent with the theory related to the signal interference.

As explained in Figure 1, as usual, there is a dynamic error related to the Doppler rate between the LOS and the NLOS signal for a moving GNSS antenna. By exploiting this phenomenon in a multipath channel, the LOS and the NLOS signals can be separately detected in the FRFD. Then, the simulations for the cases with a very small TOA delay error are carried out to verify this idea. The TOA delay error is set to 0.03 chips (which can be equivalently considered as an around 10-meters multipath error). Also, the amplitude of the NLOS signal is considered as the same as the LOS signal in this extreme simulating scenario. As mentioned above, the LOS and the NLOS signals are considered with different input Doppler rate values. Thus, the simulating results in Figure 5 demonstrate the performances of the proposed FRFT SRA for the objective detection in a multipath channel. From the results, it can be found that a larger Doppler rate error accounts for an easier detecting process for the existing signal paths. However, when the real Doppler rate difference is not wide enough, it will become more difficult to accurately differentiate the two paths of the signals by the decision-making with the power of the FRFT match filter. For example, one extra ambiguous objective near the 0 g appears in the FRFD when the input Doppler rate error is 0.2 g. The power map for this ambiguous signal path is even higher than the one formed with the real NLOS signal. In order to solve this issue, one way is to improve the resolution of the FRFT SRA by increasing the CPI length. In summary, it has been proved that the proposed algorithm is theoretically efficient to differentiate the LOS and the NLOS signals. Then, the signal parameters related to the carrier frequency can be both estimated based on (4) and (5). For the code frequency, it can be estimated based on the measured $\mathrm{C/N_0}$ and the amplitude peaks of these two signals and the detailed process for this algorithm will be discussed in the future work.

When a Doppler rate difference exists between the LOS and the NLOS signals, the cases that the two paths of the signals have a smaller amplitude ratio and a larger TOA delay error are also simulated and processed with the proposed FRFT SRA. The related results are provided in Figure 6. From the results, it can be noticed that the proposed FRFT SRA is also sensitive to the power difference between the LOS signal and the NLOS. Both the amplitude attenuation and the TOA delay error can reduce the signal power of the NLOS path in the FRFD, but the FRFT match filter cannot identify these two differences by the power-based decision-making process. As we know a certain relationship between the carrier and the code Doppler frequencies, it can be exploited to estimate the TOA delay error with the matched frequency and the Doppler rate in the FRFT SRA. Details about this work will be followed in the future as well. In the simulation, it can be found that when the TOA delay error reaches one chip length, the NLOS signal disappears in the matched FRFD. This simulating result is reasonable due to the anti-jamming property for the code auto-correlation process from the design of the spreading code [1].

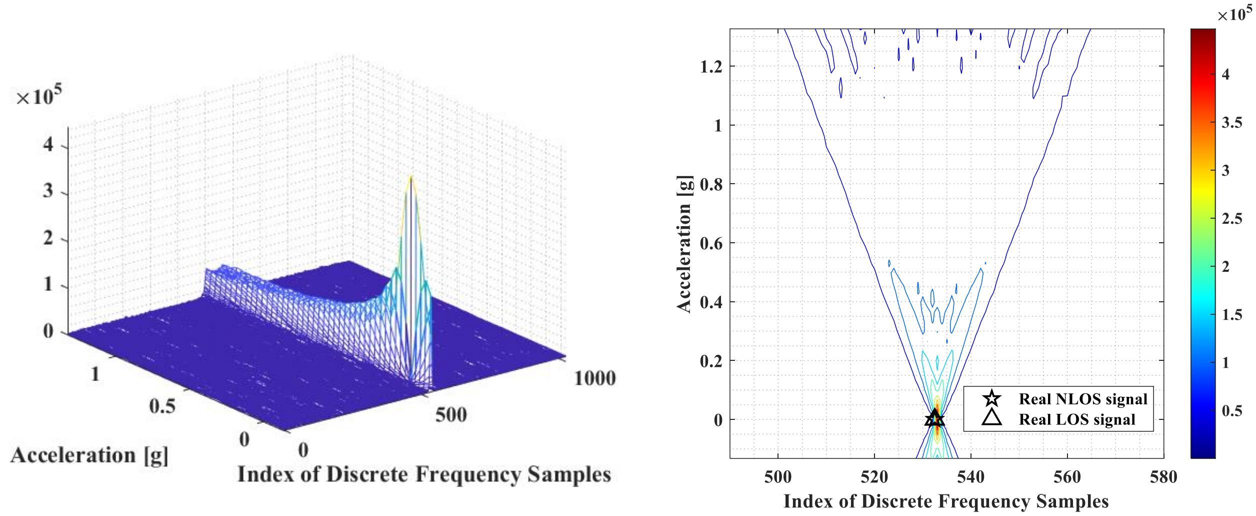

Figure 3 Amplitude match filter results and amplitude color map of the proposed FRFT-based SRA for the multipath channel. (Input TOA delay error, amplitude ratio $\frac{a_1}{a_0}$, and Doppler rate value for the NLOS signal are set to $0.1902\lambda^{-1}$ cycles, 1, and 0, respectively)

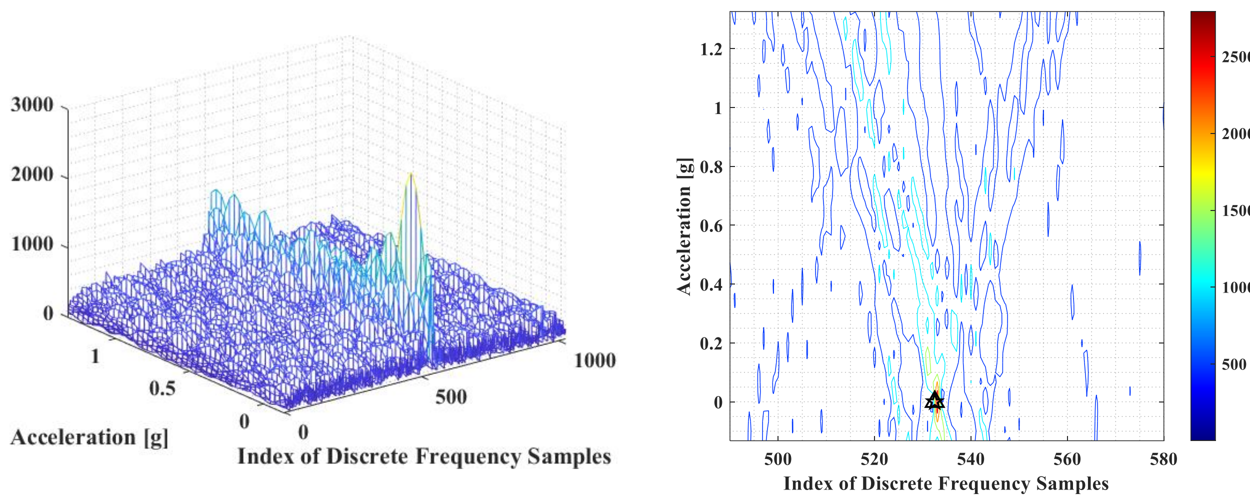

Figure 4 Amplitude match filter results and amplitude color map of the proposed FRFT-based SRA for the multipath channel. (Input TOA delay error, amplitude ratio $\frac{a_1}{a_0}$, and Doppler rate value for the NLOS signal are set to $0.1902(2\lambda)^{-1}$ cycles, 1, and 0, respectively)

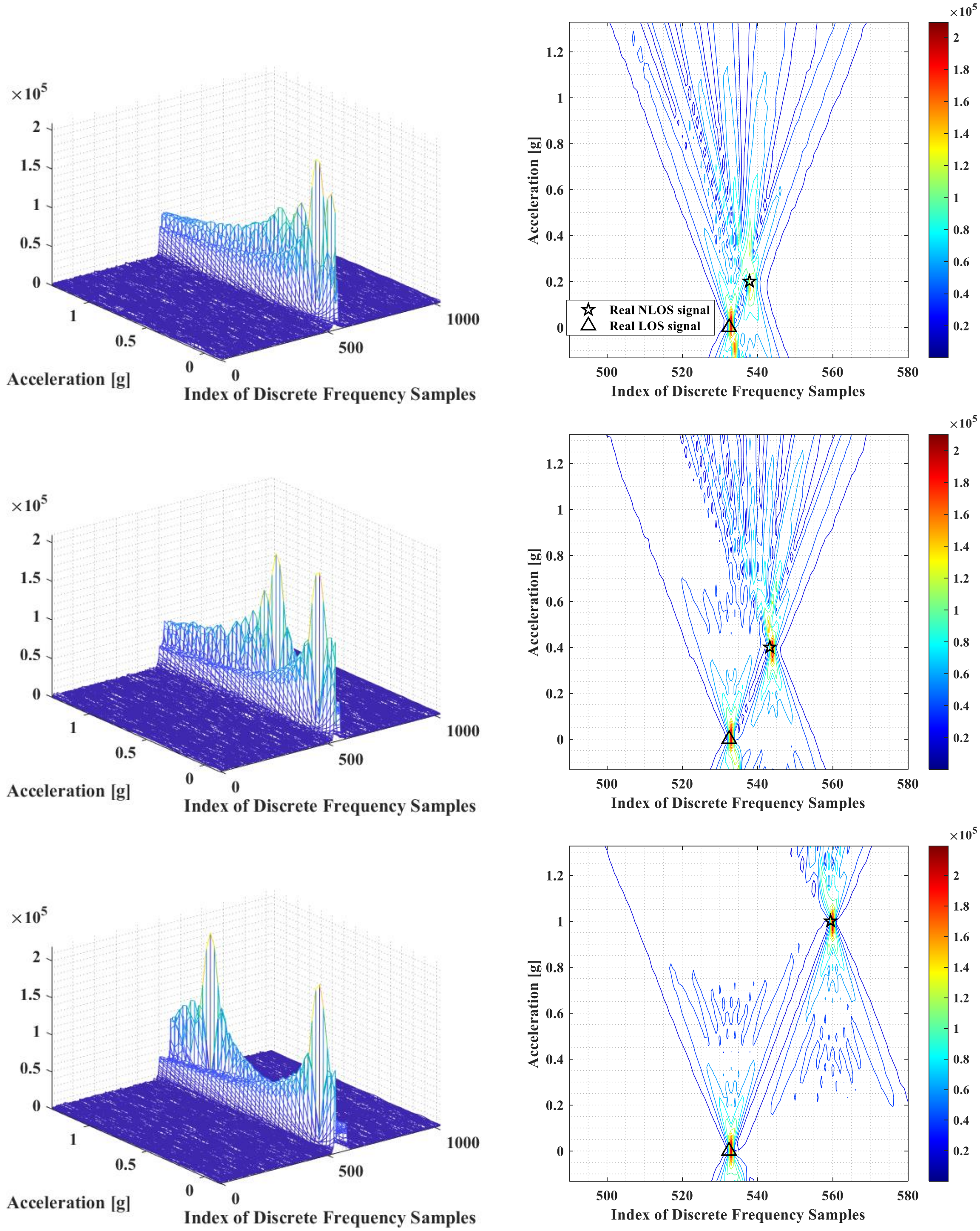

Figure 5 Amplitude match filter results and amplitude color map of the proposed FRFT-based SRA for the multipath channel. (Input TOA delay error and amplitude ratio $\frac{a_1}{a_0}$ for the NLOS signal are set to 0.03 chips and 1, respectively; input Doppler rate values for the NLOS signal are set to 0.2 g, 0.4 g, and 1 g.)

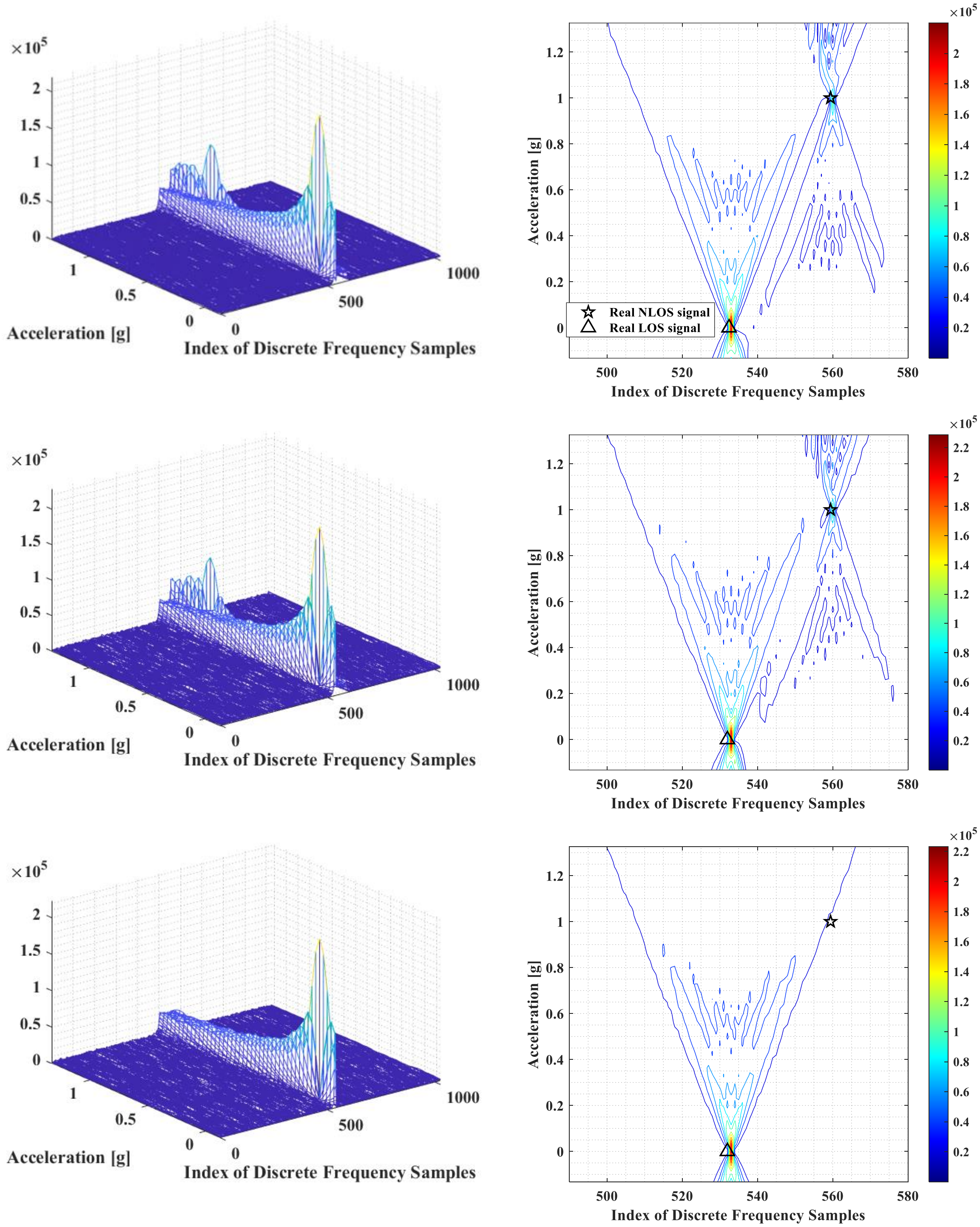

Figure 6 Amplitude match filter results and amplitude color map of the proposed FRFT-based SRA for the multipath channel. (Input Doppler rate is set to 1 g for all the three cases; input TOA delay error and amplitude ratio $\frac{a_1}{a_0}$ for the respective top, medium and bottom cases are set to 0.03 chips and 0.5, 0.5 chips and 1, 1 chip and 1)

CONCLUSIONS

By exploring the dynamic difference of the LOS and the NLOS signal, an FRFT-based super-resolution algorithm is proposed to estimate the GNSS TOA in a multipath channel in this paper. At first, a method that the different signal paths can be identified by the Doppler rate difference is proposed for the signal detection in a multipath channel. Then, a way to implement the FRFT with a long CPI is introduced to match the signals with different dynamics. Thus, the power produced by the FRFT-based super-resolution match filter can identify the different signals with a small dynamic difference. Simulating results verify that the proposed algorithm can effectively differentiate the LOS and NLOS signals on the fractional Fourier domain for a multipath channel. In the future works, the signal parameters of both LOS and NLOS signals will be computed and improved based on the proposed FRFT-based SRA. Finally, a more robust and accurate GNSS TOA estimation can be achieved in a harsh environment.

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