A Dispersion Study of Ground Penetrating Radar (GPR) Wave via Wide Angle Reflection and Refraction (WARR) for the characterization of cement-based materials

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Abstract

This paper studies the dispersion of a GPR wave's phase velocity at different wideband frequencies in plywood and concrete with varying moisture content. This study makes use of two GPR antennas with 2GHz centre frequency operating in wide angle reflection and refraction (WARR) mode and with computation of spectral analysis of the surface wave (SASW). Computation of phase velocities is based on the acquisition of the cross-power spectrum and phase unwrap of two distorted ground waves at positions closer to and farther away from the transmitting antenna. The velocities of the ground waves are found to experience greater dispersion in low frequency regimes within the effective frequency bandwidths determined by time-frequency analysis (TFA) and coherence plotting of the ground waves. This study validates not only the methodology, but also identifies the optimal distance between the first (R*x*1) and second (R*x*2) receivers as λ/2, which is based on a fixed transmitter (T*x*) minus the first receiver (R*x*1) distance. It serves as an indication of changeable separation distance when other lower frequency GPR is used because the distances of Tx-Rx1 and Rx1-Rx2 are wavelength dependent and thus also frequency dependent. This research also contributes to the building of the "GPR-WARR machine" (Annan & Jackson, 2017), within which the effects of wave dispersion on phase velocity can be inversely modelled to evaluate variations in the material properties of infrastructure as a means of detecting surface damage.

1. Introduction

The majority of ground penetrating radar (GPR) applications are aimed at "seeing the unseen" objects in engineering structures such as bridges, buildings, and buried pipes. These tasks are traditionally carried out using antennas with a fixed distance between one transmitter (Tx) and one receiver (Rx). In this research, these tasks are studied by using a variable Tx-Rx distance via wide angle reflection and refraction (WARR), which offers a means of measuring the effect of GPR surface wave dispersive behaviour on phase velocity. A failure to characterise surface damage is the root cause of the deterioration of roads and other types of infrastructure. The phenomenon of GPR wave dispersion in materials is seldom studied because the most frequently-used method of common offset profiling (COP) with a fixed distance between Tx and Rx does not yield such information. The adoption of WARR and methods of dispersion analysis, which is common in seismic imaging, can serve this purpose but is still in its infancy within the GPR community (Annan & Jackson, 2017). In this paper, algorithms used for seismic SASW were adopted for GPR use to obtain the dispersion curve of the GPR surface wave. The aim of which was to establish and validate procedures for measuring phase velocity dispersion through GPR-WARR (Annan & Jackson, 2017) and GPR-SASW dispersion analysis.

Most GPR surveys make use of GPR-COP setups consisting of one transmitting and one receiving antenna housed in the same shield. The antennas move together along the surveyed surface with a constant offset distance that is dependent on the antenna design and frequency. In this configuration, the two-way reflection ray-path from the antenna to the subsurface target and antenna orientation are both fixed (Lai et al., 2016; Sham & Lai, 2016). On the other hand, the multi-offset WARR configuration is not popular with the GPR community because of its relatively long data acquisition time, although the reflection and refraction ray-paths provide more information (Annan & Jackson, 2017) than the normal COP reflection ray-paths. Multi-offset WARR forms the basis of reflection/refraction seismic processing and experienced a growth in use soon after the enhancement of computer processing power that occurred around the turn of the millennium (Sheriff & Geldart, 1995; Yilmaz, 2001). Given the similarities between seismic and high-frequency electromagnetic wave propagation in the same subsurface media (Carcione, 1999; Carcione, 2007; Ursin, 1983), the data processing techniques are basically interchangeable. The major difference is the multiple longitudinal and transverse wave modes in seismic methods (e.g. body, shear, Rayleigh), whereas there is only a single transverse mode for GPR's electromagnetic wave. In addition, the physical parameters such as frequency, velocity, impedance and time base are also different in terms of their scales and dimensions. But with due care these data processing differences can be handled during the coding, for example by adopting different time bases and numbers of samples in A-scans.

The characterisation of GPR wave dispersion in materials using WARR offers two advantages. Firstly, the broadband GPR surface wave is dispersed within different frequency bandwidths. This results in velocity dispersion that can be measured by WARR's dispersion analysis, which gives a more comprehensive understanding of wave propagation than is possible in traditional COP with its fixed distance between transmitter and receiver. Secondly, such velocity dispersion is significantly affected by material defects such as cracks, delamination and water seepage. In this work, we begin the process of validating the measurement methods and algorithms by studying the effects of varying water content in plywood and concrete. Data collection was achieved using a high frequency (i.e. 2GHz) GPR antenna in order to obtain a wider bandwidth than is possible at lower frequencies. We also attempted to determine and generalize the wavelength-dependent distances between Tx–Rx1 and Rx1 and Rx2 in a typical GPR-SASW survey (Klysz et al., 2004).

2. Literature Review

Dielectric dispersion models have long been reported in small-scale investigations with coaxial cells, such as the pure materials studies of Debye's model (1929), composite materials by Cole and Cole (1941) and Von Hippel (1954), the 4-p Jonscher model (Jonscher, 1983; Jonscher, 1999), and the studies of geological materials by Davis and Annan (1989). A dielectric medium (or an insulator) possesses a number of dielectric mechanisms (i.e. ionic, dipolar, atomic, electronic) contributing to the complex dielectric permittivity over a large frequency range (Cole & Cole, 1941; De Waal et al., 1978; Debye, 1929; Von Hippel, 1954). According to these theories, electric charge carriers in the dielectric medium can be displaced by an incident electrical field through experiencing a torque on the dipoles. This displacement is known as polarisation $\varepsilon'(\varpi)$, which balances the electric field due to the positive and negative charges moving in opposite directions. The friction experienced by the dipolar orientation contributes to the losses in dielectric or permittivity values $\varepsilon''(\varpi)$. Both $\varepsilon'(\varpi)$ and $\varepsilon''(\varpi)$ are functions of angular frequency. At the microscopic level, several dielectric mechanisms have strong effects on the dielectric behaviour, in which dipole orientation governs the dielectric behaviour over the GPR frequency range from MHz to GHz. The orientation of these dipolar moments is random when an electric field is absent because no polarisation exists. The different polarization responses are frequency-dependent, which causes the mechanism underlying the dispersed travelling velocity of wide band GPR wave at different frequency ranges.

Velocity dispersion (i.e. velocity variation within the frequency spectrum) in the high frequency range of GPR (i.e. hundreds of MHz to GHz) is relatively small when compared with that occurring in the low frequency range (i.e. tens of MHz) in porous construction materials, such as concrete, soil, or asphalt (Davis and Annan (1989). So, the velocity within the hundreds of MHz to GHz frequency range is usually assumed to be constant and is therefore ignored, and this characteristic is known as the GPR plateau (Davis & Annan, 1989). On the one hand, this assumption of a plateau looks reasonably valid when the small velocity variation in the higher frequency range is compared with the large one in the lower frequency range in geophysical applications. On the other hand, this difference, which ranges in the order of 0.01m/ns when compared to the speed of light at 0.3m/ns, can be significant when mapping objects and in cases where either water (Topp et al., 1980) or clay content (Tosti et al., 2013), or material defects are significant. For the former case, objects of interest are usually embedded at shallow cover depths, for example involving tens of centimetres or less. For the latter situation, water tends to retard high frequency waves more than low frequency ones (Lai et al., 2011b). So, the variation of phase velocity at a particular frequency bandwidth can be used to study material defects and water seepage in concrete and pavement structures. To date, it is not clear which bandwidths of GPR waves would be affected by particular types of material defects and water content/seepage, although some groundwork has been carried out (Van der Kruk, 2006; Van Der Kruk et al., 2007; Van der Kruk et al., 2009). Thus, this paper makes use of GPR-WARR and dispersion analysis in order to study these unknown properties in composite construction materials, which is central to the diagnosis of material defects and water seepage.

3. Experiments

Four specimens were prepared. The first specimen was a 50 cm long x 40 cm wide x 18 mm thick piece of plywood suspended in air and tested in both bone-dry and completely saturated states (Figure 1). The setup ensured that in the GPR-WARR radargram, all recorded reflections arose only from the ground wave travelling from the transmitter to the receiver on the surface of the plywood. The experiment was therefore free from any reflected or refracted waves coming from below the plywood. For the plywood in the dry state, little dispersion was expected as the material should behave almost like the ground wave travelling in air. In the case of the saturated plywood, a much larger dispersion was expected according to the theories described in Section 2.

The second specimen was a 130 cm long x 49 cm wide x 15 cm thick plain concrete slab made of ordinary Portland cement (OPC), which again was tested in both wet and dry states, as shown in Figure 2 and Table 1. For the first measurement, the concrete was cured for 25 days after its fresh mixing, whereupon the initial hydration is expected to be stable (De Souza et al., 2004; Lai et al., 2009), which is also called a saturated surface dry (SSD) state. The specimen was then put in an oven drying chamber at 80^{o}C for 36 hours (Intermediate 1), 120 hours (Intermediate 2), and 204 hours (Intermediate 3). Three measurements were conducted for each state. The fifth measurement was made after the specimen was air-dried for 11 months. In between each oven-drying period, the specimens were weighed with a load cell and measured using GPR-WARR survey. The recorded weights from the four specimens are shown in Tables 1 and 2. Specimens 3 and 4 were respectively 2% and 4% chloride content mixed concretes with the same dimensions as specimen 2.

Two 2 GHz GSSI GPR antennas controlled with SIR-20 units were utilized in the WARR survey of the two specimens. The green survey wheel antenna (Antenna 1) in Figure 3 works with one transmitter and one receiver displayed in channel 1, and was the only moving antenna used during all the GPR-WARR surveys. Antenna 2's survey wheel and transmitting antenna were both disabled and it was then placed in a fixed position to receive the signals transmitted by antenna 1 during the WARR survey. So, the radargram shown in channel 2 was measured by the fixed antenna 2, while antenna 1 was moved to trigger the transmission of GPR waves into the specimen. Two typical radargrams collected in both plywood and concrete are shown in Figure 4.

4. Data Processing

4.1 Measuring velocity dispersion and associated real part of permittivity (ε') with GPR-SASW

SASW is widely used in seismic geophysics. It analyses the surface wave propagation between a transmitter and a receiver separated by variable distances. The two possible configurations used are: common middle point (CMP) and wide-angle reflection refraction (WARR). The most widely used method measures time shifts in order to estimate the wave speeds, while SASW emphasizes the frequency-dependent dispersion of velocity caused by dielectric dispersion ε'(ω). The latter can be measured using two steps (Klysz et al., 2004; Lai et al., 2010). The first step is to define signals by using the two ground wave signals in one full wavelength travelling from one transmitter to two receivers at a different distances (Figure 5) and then observing the two signals in the time-frequency spectrum (Lai et al., 2014) and magnitude-squared coherence plot in Figure 6. The second step is to compute the cross-power spectrum of these two signals (ground waves). The phases are then unwrapped to calculate and plot the phase velocities in the material v(ω), which is equal to the speed of light (c) divided by the square root of ε'(ω) in Figure 6.

Signals are recorded continuously for any two different antenna positions, as shown in the radargram in Figure 4. Two A-scans are extracted at three different distances in order to estimate the optimal X_1 - X_0 at position 1 to 2 in Figure 3, so that dispersion curves are obtained following the theories summarized in Section 2 (Klysz et al., 2004; Lai et al., 2010). These distances (X_1 - X_0) are assumed to be dependent on certain fractions of a wavelength, which are λ/4, λ/2 and ¾ λ of the wavelet at position 1 in Figure 3. The wavelength λ = v/f, where 'v' is the group velocity of the ground wave calculated based on the gradient of the ground wave reflections in Figure 4, and 'f' is the centre frequency of the first arrival wave's (i.e. position 1 in Figure 3) frequency spectrum after Fast Fourier Transform (FFT). Complete cycles of these two signals $A_1(t)$ and $A_2(t)$ were windowed and Fourier-transformed to $A_1(\omega)$ and $A_2(\omega)$. Then, the cross-power spectrum of these two signals was represented as $G_{X1X2}(\omega)$, which is a complex conjugate yielding real and imaginary parts. The phase spectrum $\Theta_{XY}(\omega)$ of the $G_{XY}(\omega)$ represents the number of cycles of a given frequency between the two antenna locations, and can be expressed as follows:

$$\theta_{X_1X_2}(\omega) = \theta_{X_1}(\omega) - \theta_{X_2}(\omega) = \tan^{-1}[\frac{\mathrm{Im}(G_{X_1X_2})}{\mathrm{Re}(G_{X_1X_2})}] \text{.....[1]}$$

where Im[G_{XY})] and Re[Im(G_{XY})] are the imaginary and real parts of G_{XY}.

As the arctangent function in Equation 1 provides phases only between –π/2 to +π/2, the discontinuous phase spectrum oscillates between –π/2 to +π/2, and is therefore what is known as 'wrapped'. The spectrum was therefore unwrapped to produce a continuous phase spectrum as a function of frequency. Based on the principle of a rotating vector, a phase shift of 2π is equivalent to the travel time of a period 'T', or the reciprocal of the frequency of the wave. Therefore, the travel time 't' and frequency-dependent phase velocity v(ω) between the two ground wave signals can be calculated by:

$$t(\omega) = \frac{\theta_{X_1X_2}(\omega)}{\omega} \text{.....[5]}$$

$$V(\omega) = \frac{D}{t(\omega)} \text{.....[6]}$$

where D is the distance between the two antennas, i.e. λ/4, λ/2 or ¾ λ of the ground wave at position 1 to 2 of Figure 3. The above signal processing steps are graphically explained in Figure 6.

GPR-WARR provides a velocity spectrum but does not constrain the effective upper and lower bounds of frequency bandwidth. These two bounds were estimated using a coarse and a fine method. The coarse method, or observation, was conducted using a wavelet transform in the GPR time-frequency domain (Lai et al., 2014; Lai et al., 2011a; Lai et al., 2012), as depicted in Figure 7. This method evaluates the spectral content of a particular reflector, or in this case, the ground wave in the WARR configuration. The fine method makes use of magnitude-squared coherence to study the relation between the two ground wave signals, as shown in the Figure 8. Coherence values always vary between 0 and 1. For an ideal constant parameter and linear system with a single input and a single output, this value will be equal to one. In this work, a coherence value of 0.5 was set as the threshold limiting the various upper and lower bound of the effective bandwidth. For example, the upper threshold of the frequency bandwidth for the specimen 0% Cl saturated surface dry (SDD) (X1-X0 = 0.75λ) is 0.33 GHz and 1.33 GHz, respectively, as indicated by the two arrows in Figure 8.

5. Findings and Data Analysis

5.1 Method Validation

The velocity dispersion plots from the plywood and concrete specimens are reported in Figures 9 to 11. The differences observable in these plots reflect the dispersion behaviour at varying distances X_1-X_0 (Figure 3) for λ/4 (Figure 9), λ/2 (Figure 10), and ¾ λ (Figure 11) of the wavelets at different positions remote from the stationary antenna in Figure 3. Velocities at all plots in dry plywood in Figure 9 (λ/4) and Figure 10 (λ/2) case produce a result close to that of a GPR wave travelling in air, demonstrating very little dispersion at a velocity of 2.8~2.9 x 10^9 m/s, which is close to the velocity of light (2.998 x 10^9 m/s) (Figure 11). The velocities in Figure 11 (¾ λ), i.e. about 2.6~2.7 x 10^9 m/s, are less satisfactory because they are much lower than the velocity of light. Then, when dry plywood was wetted to trigger dispersion, the wave velocity started to disperse at lower frequencies at 1.66 GHz as shown in Figure 9 and 10. For the concrete, the water content contained in the specimen was reduced gradually after drying, leading to velocity increase at all frequencies as presented in the five states in Figures 9 to 11. The above observations reported in the plywood also apply to the concrete. It is obvious that for both plywood and concrete, dispersion of the GPR wave velocity is clearly visible after changes in moisture content, and the velocity is in general reduced with the increase of moisture content at all frequencies.

There are two parameters to consider when a WARR machine (Annan and Jackson, 2017) or a GPR-SASW survey is setup. The first parameter is the optimal distance X_0 in Figure 3, where the first ground wave is captured. The design of this 'distance' should fulfil two criteria: (1) avoid near-field induction and maintain far-field wave propagation by selecting a distance larger than 2λ; and (2) minimize severe attenuation by selecting a distance X_0 that is not too large. In this case, if a velocity 0.1m/ns is assumed and the ground wave has a centre frequency of 1.6 GHz, then λ is 62.5 mm, as shown at Figure 7. An optimal 'distance' of 2λ is then equal to 125mm, which can be rounded up to 130mm, as shown in Figures 3 and 12.

The second parameter concerns the design of an optimal distance of X_1-X_0, as shown in Figure 12. Selection of the second ground wave in SASW is the major concern here because when X_2-X_0 is too large (i.e. they are too far apart), then the second ground wave would be seriously attenuated. But how far is too far? The effects of distance on the optimal X_1-X_0 in Figure 3 for λ/4, λ/2 and ¾ λ are studied and compared in Figures 9, 10 and 11 respectively. Plots with a distance larger than 1λ were not reported because the second ground wave was too weak to be identified. Of the three distances evaluated in Figures 9-11, a distance λ/2 between Tx and Rx is recommended. There are two reasons for this: (1) there is a small dispersion of velocity over the frequency bandwidth and the velocity is close to the speed of light as shown in Figure 10; and (2) the general trend of dispersion in wet plywood and concrete of various moisture contents follows nicely the theory set out in Section 2. When a low frequency GPR antenna is used, the X_0 = 2λ and X_1 - X_0= λ/2 can still be applied, but the physical distances between the transmitter and receivers must be increased due to longer wavelength.

5.2 Dispersion in wet/dry concrete and concrete with different levels of chloride content

Figures 13 to 15 illustrate the effects of (1) concrete wetness and (2) different levels of chloride content in the water during mixing on the frequency-dependent dispersion. We would focus on Figure 14 where the data at X_1-X_0 = 0.5 λ was validated in Section 5.1 as the most appropriate distance between the two transmitters. For effects of (1) concrete wetness, the velocity at all frequencies is increased significantly from about 0.040-0.120 m/ns to 0.085-0.155 m/ns because waves travel faster in concrete with lower moisture content (i.e. the air-dry concrete). For effects of (2) different levels of chloride content, both wet and dry concrete specimens containing nil or 2% chloride content show no significant differences in velocity across the frequency spectrum, presumably because such a small difference of chloride content in concrete has an insignificant effect on wave dispersion. But when the chloride content increases to 4%, there is a larger reduction in velocity as denoted by the arrows (blue and red) added to the plots for both wet and dry specimens. This implies that the radar wave is sensitive to an increase of chloride content at a level 4% but not lower.

5.3 Dispersion in different frequency bandwidths

A further observation is that in dry concrete, the magnitude of velocity reduction is fairly evenly distributed among all frequencies (NB: arrows indicating the differences are shown in Figures 14). But in wet concrete this reduction is small at lower frequencies but larger at higher frequencies (NB: no arrows were added at lower frequencies in the wet-concrete curves in Figures 14). This contrast implies that when in a wet state, the low frequency part is less affected by higher chloride content (i.e. 4%) because the effects on velocity are dominated by water content. However, in a dry state, the velocity in the low frequency part is affected as much as the high frequency part. An explanation for this contrast is given below.

As chloride in water distorts the pulse shape of the high-frequency GPR wave more significantly than its low-frequency counterpart (Lai et al., 2011b), this is also reflected in the phase velocity in these two plots. These phenomena are attributable to the electromagnetic wave propagation in materials with the presence of charge-carrying ions (Cassidy & Jol, 2009), which in this case are in the chloride. The leading and trailing edges of the reflection, which are affected by the chloride ions, yield a small displacement current radiating EM energy that is slightly out of phase with the incident pulse (Cassidy & Jol, 2009). This then slows down the main body of the propagating wave, or the ground wave in this study, mostly in the lower frequency part because that is where the leading and trailing edges of the reflection contribute most. The slightly out-of-phase localized energy interferes destructively with the ground wavelet, hence lengthening the pulse width and reducing wave velocity at lower frequencies.

6. Conclusions

Material defects typically evolve unseen in the shallow subsurface and usually remain unnoticed until serious failure occurs. Radargrams and 3D images obtained in common offset GPR imaging mostly give the location of objects rather than providing an evaluation of material properties. Dispersion used to be regarded as a drawback in GPR data analysis. But it is used in this study to differentiate between various phase velocities at different frequencies. It is potentially a useful tool, not only for understanding the underlying physics of wave travel in materials, but also for the evaluation of material properties in the spatial domain. The development of GPR-WARR methodologies will be attempted in order address a number of questions in the next stage of this research. For example, at which frequency bandwidths are GPR wave velocities most dramatically decelerated in damaged concrete and pavement structures exhibiting changes in material properties? Which frequency bandwidths remain unaffected or are least affected? How can the results be used to advance attempts to ‘fingerprint’ different material defects such as cracks and water seepage? Which physical parameters would most affect the computational process? Do the recommended 2λ and λ/2 rules in Section 5 work equally well in other GPR frequency bandwidths?

More studies are yet to be undertaken involving larger numbers of specimens and making use of other bandwidths in lower frequency GPR. This present work helps pave the way for studies of GPR wave dispersion phenomena in materials to be transformed into practical engineering applications.

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References

Annan, A. P., & Jackson, S. R. (2017). *The WARR machine.* Paper presented at the Advanced Ground Penetrating Radar (IWAGPR), 2017 9th International Workshop on.

Carcione, J. M. (1999). Staggered mesh for the anisotropic and viscoelastic wave equation. *Geophysics, 64*(6), 1863-1866.

Carcione, J. M. (2007). *Wave fields in real media: Wave propagation in anisotropic, anelastic, porous and electromagnetic media* (Vol. 38): Elsevier.

Cassidy, N. J. (2009). "Electrical and magnetic properties of rocks, soils and fluids." Chapter 2, Ground penetrating radar: Theory and applications, Harry M. Jol, ed., Elsevier, Amsterdam, The Netherlands.

Cole, K. S., & Cole, R. H. (1941). Dispersion and absorption in dielectrics I. Alternating current characteristics. *The Journal of chemical physics, 9*(4), 341-351.

Davis, J., & Annan, A. (1989). Ground‐penetrating radar for high resolution mapping of soil and rock stratigraphy: Geophysical Prospecting. *Geophysical prospecting, 37*(5), 531-551.

De Souza, T., Annan, A., Redman, J., & Hu, N. (2004). *Monitoring the GPR response of curing concrete.* Paper presented at the 16th World conference on NDT, Montreal. Canada.

De Waal, P., Nooteboom, J., & Poley, J. P. (1978). Use of VHF dielectric measurements for borehole formation analysis. *The Log Analyst, 19*(03).

Debye, P. J. W. (1929). *Polar molecules*: Chemical Catalog Company, Incorporated.

Jonscher, A. (1983). Dielectric relaxation in solids. 1983. *Chelsea Dielectric, London*.

Jonscher, A. K. (1999). Dielectric relaxation in solids. *Journal of Physics D: Applied Physics, 32*(14), R57.

Klysz, G., Balayssac, J. P., & Laurens, S. (2004). Spectral analysis of radar surface waves for non-destructive evaluation of cover concrete. *NDT & E International, 37*(3), 221-227. doi:10.1016/j.ndteint.2003.09.006

Lai, W.-L. W., Kind, T., Kruschwitz, S., Wöstmann, J., & Wiggenhauser, H. (2014). Spectral absorption of spatial and temporal ground penetrating radar signals by water in construction materials. *NDT & E International, 67*, 55-63. doi:10.1016/j.ndteint.2014.06.009

Lai, W. L., Kind, T., & Wiggenhauser, H. (2010). A Study of Concrete Hydration and Dielectric Relaxation Mechanism Using Ground Penetrating Radar and Short-Time Fourier Transform. *EURASIP Journal on Advances in Signal Processing, 2010*(1), 317216. doi:10.1155/2010/317216

Lai, W. L., Kind, T., & Wiggenhauser, H. (2011a). Using ground penetrating radar and time-frequency analysis to characterize construction materials. *NDT & E International, 44*(1), 111-120. doi:10.1016/j.ndteint.2010.10.002

Lai, W. L., Kou, S. C., & Poon, C. S. (2012). Unsaturated zone characterization in soil through transient wetting and drying using GPR joint time–frequency analysis and grayscale images. *Journal of Hydrology, 452-453*, 1-13. doi:10.1016/j.jhydrol.2012.03.044

Lai, W. L., Kou, S. C., Tsang, W. F., & Poon, C. S. (2009). Characterization of concrete properties from dielectric properties using ground penetrating radar. *Cement and Concrete Research, 39*(8), 687-695. doi:10.1016/j.cemconres.2009.05.004

Lai, W. W.-L., Kind, T., Sham, J. F.-C., & Wiggenhauser, H. (2016). Correction of GPR wave velocity at different oblique angles between traverses and alignment of line objects in a common offset antenna setting. *NDT and E International, 82*, 36-43. doi:10.1016/j.ndteint.2016.03.003

Lai, W. W. L., Kind, T., & Wiggenhauser, H. (2011b). Frequency-dependent dispersion of high-frequency ground penetrating radar wave in concrete. *NDT & E International, 44*(3), 267-273. doi:10.1016/j.ndteint.2010.12.004

Sham, J. F. C., & Lai, W. W. L. (2016). Development of a new algorithm for accurate estimation of GPR's wave propagation velocity by common-offset survey method. *NDT and E International, 83*, 104-113. doi:10.1016/j.ndteint.2016.05.002

Sheriff, R. E., & Geldart, L. P. (1995). *Exploration seismology*: Cambridge university press.

Topp, G. C., Davis, J., & Annan, A. P. (1980). Electromagnetic determination of soil water content: Measurements in coaxial transmission lines. *Water resources research, 16*(3), 574-582.

Tosti, F., Patriarca, C., Slob, E., Benedetto, A., & Lambot, S. (2013). Clay content evaluation in soils through GPR signal processing. *Journal of Applied Geophysics, 97*, 69-80. doi:10.1016/j.jappgeo.2013.04.006

Ursin, B. (1983). Review of elastic and electromagnetic wave propagation in horizontally layered media. *Geophysics, 48*(8), 1063-1081.

Van der Kruk, J. (2006). Properties of surface waveguides derived from inversion of fundamental and higher mode dispersive GPR data. *IEEE Transactions on Geoscience and Remote Sensing, 44*(10), 2908-2915.

Van Der Kruk, J., Arcone, S. A., & Liu, L. (2007). Fundamental and higher mode inversion of dispersed GPR waves propagating in an ice layer. *IEEE Transactions on Geoscience and Remote Sensing, 45*(8), 2483-2491.

Van der Kruk, J., Steelman, C., Endres, A., & Vereecken, H. (2009). Dispersion inversion of electromagnetic pulse propagation within freezing and thawing soil waveguides. *Geophysical Research Letters, 36*(18).

Von Hippel, A. (1954). Dielectric materials and applications. *London: Artech House*.

Yilmaz, Ö. (2001). *Seismic data analysis: Processing, inversion, and interpretation of seismic data*: Society of exploration geophysicists.