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A Generic Framework for Analytical Probabilistic Assessment of Frequency Stability in Modern Power System Operational Planning

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***Abstract*—In view of modern power system characterized by significant inertia reduction and booming uncertainty, this letter proposes an important operational planning tool to comprehensively analyze frequency stability including both steady-state frequency and rate of change of frequency (RoCoF) issues in a probabilistic manner for the first time. The proposed generic framework can tackle various frequency-related uncertainties and accommodate different system frequency response (SFR) models. The effectiveness and efficiency of the proposed analytical probabilistic assessment are demonstrated by comparing with numerical scenario-based simulation (SBS).**

***Index Terms*—Cumulant-based theory, frequency nadir/vertex (FN/FV), rate of change of frequency (RoCoF), renewable energy.**

I. INTRODUCTION

MODERN power system operation generally faces two inevitable and challenging facts brought by the gradual replacement of synchronous generators with power electronic converter-interfaced renewable energy sources: 1.a continuous reduction in system inertia and; 2.considerable growth in power supply uncertainty [1]. Those two facts have posed a critical threat to the system frequency stability. To tackle this challenge, current practice in many utilities is to contract on additional conventional generation (mostly more than actual needs) to accommodate the predicted arbitrary 'worst case scenario', which is neither an economical nor a robust solution. Hence, there is a pressing need to develop an operational tool to measure and identify how much renewable generation can be allowed to penetrate and how much conventional generation should be retained based on renewable energy and demand forecasting. Similar to Monte Carlo simulation (MCS), a numerical SBS approach is proposed to calculate the maximal renewable energy penetration limits to maintain the frequency performance by considering a large number of potential scenarios in [2], but it is actually quite time-consuming and unrealistic to exhaust all the possible scenarios and hence not used very often in industry especially in short-term or day-ahead planning.

This letter proposes a novel analytical probabilistic tool to meet the pressing need and provide a comprehensive but efficient assessment on the impact of above two facts on both FN/FV and RoCoF (FN/FV and RoCoF are the two major concerns and focus of system operator in frequency stability [3].) The proposed framework is generic as it can deal with multiple power uncertainties with different locations and incorporate various SFR model components, which are related to the specific condition of different networks.

II. METHODOLOGY FRAMEWORK

A. Modeling Power System Frequency-related Uncertainties

The major uncertainties related to power system frequency stability refer to the stochastic active power disturbance in the system, which could be caused by the fluctuation of renewable energy and system loads, or system faults such as generation loss. Since the former cause is a comparatively new issue and happens much more frequently with an increasing impact, it is the main focus of this letter. Therefore, the stochastic modeling of wind and solar power and demand are presented here. The impact of system faults can also be modeled and assessed by the framework according to the assessment need. It is reported that wind and solar power follow different distributions according to different assessment timescales and demand generally follows normal distribution. For instance, Beta distribution should be adopted for the wind and solar power during day-ahead planning, while for year-ahead planning the Weibull distribution is applied for wind power and normal distribution is suitable for solar power. The spatiotemporal correlations between different renewable energy sources and system loads can be properly modeled by the correlation coefficient matrix [4].

B. Deriving and Computing Sensitivity of Two Key Frequency Stability Margin Indices (FN/FV and RoCoF) w.r.t. System Active Power Disturbances

1) Analytical Sensitivity (AS)

The SFR model aggregated by the method in [5] is employed as an example in this letter for demonstration purpose and other SFR models associated with different power systems can also be accommodated by the designed framework. According to SFR model and derivations in [5], the worst RoCoF without action of AGC and FN/FV (represented by subscript n/v respectively) could be expressed in the following:

$$\frac{d\Delta f(0^+)}{dt} = \frac{1}{2H}\Delta P \tag{1}$$

$$\Delta f_{n/v} = \left[\frac{(1+a\sqrt{1-\varsigma^2}e^{-\varsigma\omega_n t_{n/v}})R}{DR+K_m}\right]\Delta P \tag{2}$$

where $t_{n/v} = \frac{1}{\omega_r}tan^{-1}\left(\frac{\omega_r T_R}{\varsigma\omega_n T_R-1}\right)$, $\omega_n^2 = \frac{DR+K_m}{2HRT_R}, \varsigma = \frac{2HR+DRT_R+K_m F_H T_R}{2(DR+K_m)}\omega_n, a = \sqrt{\frac{1-2T_R\varsigma\omega_n+T_R^2\omega_n^2}{1-\varsigma^2}}, \omega_r = \omega_n\sqrt{1-\varsigma^2}, \phi = tan^{-1}\left(\frac{\omega_r T_R}{1-\varsigma\omega_n T_R}\right) - tan^{-1}\left(\frac{\sqrt{1-\varsigma^2}}{-\varsigma}\right)$, $t_{n/v}$ is the time to reach the FN/FV, K_m is mechanical power gain, T_R is reheat time constant, F_H is high-pressure turbine fraction, R is governor speed droop constant, D is load damping constant and H is system inertia constant. ΔP is the active power disturbance ($\Delta P < 0$ will lead to FN and $\Delta P > 0$ will lead to FV).

With the sensitivities implied in (1) and (2), the relationship between the two key frequency stability margin indices (FN/FV and RoCoF) and active power disturbance can be established.

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2) Numerical Sensitivity (NS)

Obviously, the so-called AS (FN/FV or RoCoF) computed from the analytical SFR model is identical for all the disturbances with different locations. The uncertainties occurring in different locations of the network might have slightly different impacts on the system center frequency, which implies that the network could be considered in the sensitivity calculation to enhance the assessment accuracy in this case.

Therefore, a concept of numerical sensitivity (NS) is defined here to characterize and quantify the network impact of frequency-related uncertainties, which can be simply computed by the perturbation approach in the following.

$$NS = \frac{X_m}{\Delta P_t} \tag{3}$$

where X_m is RoCoF or FN/FV of the frequency response of system center (f_{coi}), which could be easily obtained by (4):

$$f_{coi} = \frac{\sum_{i=1}^{n} f_i H_i}{\sum_{i=1}^{n} H_i} \tag{4}$$

where f_i is the frequency response of the i^{th} generator, and n is the number of generators. Hence, there is no need of SFR model in the NS computation.

However, it should be noted that the SFR model-based AS is always beneficial to system planner, which can facilitate a deeper understanding of the impact mechanisms of uncertainties on frequency dynamics.

C. Identifying Probabilistic Distribution of Frequency Stability Margin Indices and Assessing Frequency Instability Risk

Compared with other analytical approaches and MCS, the cumulant-based analytical approach can easily accommodate arbitrary type of continuous or non-continuous distribution and correlation of stochastic variables [4], which is proved to be the most efficient and accurate way to deal with system uncertainties for probabilistic small-signal stability analysis in [6]. In this letter, the cumulant-based analytical approach is employed in the proposed assessment framework to construct the probabilistic distribution function (PDF) of frequency stability.

Based on the above Step A, B and C, the proposed framework of probabilistic assessment methodology for frequency stability is illustrated in Fig. 1. In Step A, according to the chosen assessment task, the wind/solar power and demand distributions, and their correlation coefficient matrices are determined. Then by selecting different calculation methods, the sensitivity of frequency stability margin indices is computed based on (1)(2) or (3) in Step B. The results of Step A and B above are sent into Step C as the inputs to obtain the PDF of frequency stability margin indices by adopting the cumulant-based approach, and the detailed procedure is described in [4]. The last step of Step C is to assess the frequency instability risk by using risk assessment matrix (RAM). The RAM can provide a two-dimensional assessment, i.e., occurrence probability and severity, and hence can provide a comprehensive and visible risk-based evaluation on stability, which has been applied to assess the risk of small disturbance security issues [7]. In most countries, the operational standards specify the operational/statutory limits for steady-state frequency and RoCoF (e.g., UK SQSS [3]). So based on these limits and obtained distributions of margin indices, RAM can be practically designed for the risk assessment of frequency instability.

III. NUMERICAL VALIDATION

The proposed assessment framework is verified by two test systems: 1) a modified IEEE 10-machine 39-bus test system with three wind farms connected to bus 6, 23 and 29 respectively as shown in Fig. 2, and 2) a modified 16-machine 68-bus test system with three wind farms connected to bus 29, 32 and 42 respectively [4]. Due to limited space, only the final probability for Case 2 are given (Table I), which are the most important and convincing results. The wind speed distribution in [8] is employed with the correlation coefficient matrix:

$$[\rho_{ij}]_{3\times3} = \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

The proposed analytical assessment is carried out according to the steps introduced in Section II and Fig. 1, and the numerical SBS is also conducted for 5000 times as the benchmark to test the accuracy and efficiency of the proposed assessment. The PDF results of both FN/FV and RoCoF produced by three methods are given in Fig. 3 (a) and (b) respectively, where both UK operational (green) and statutory (yellow) limit standard for frequency stability is applied as an example. It can be verified by Fig. 3 (a) and (b), the PDF curves

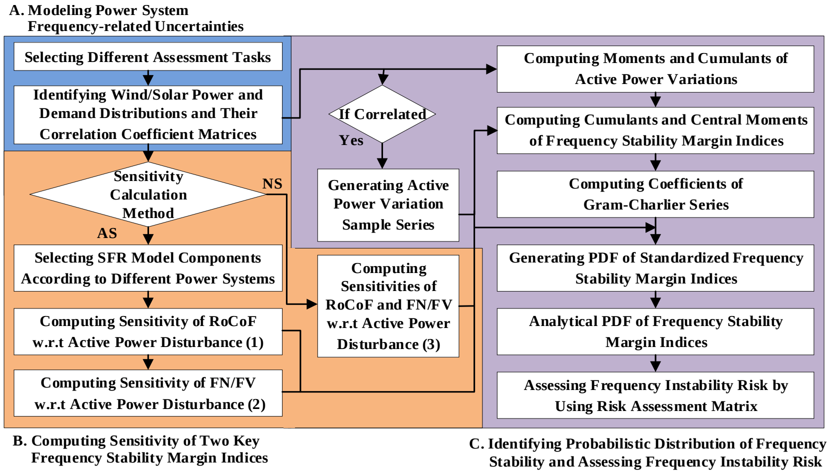


Fig. 1. Flowchart of proposed framework for frequency stability assessment.

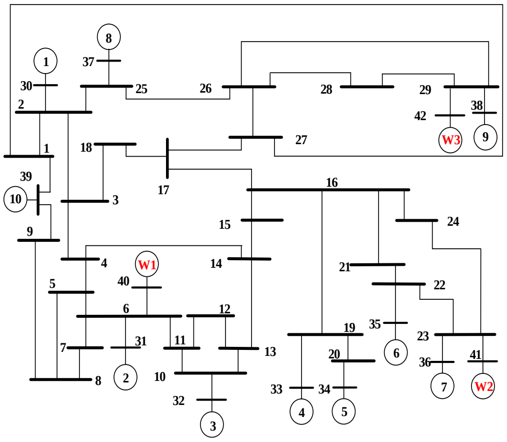
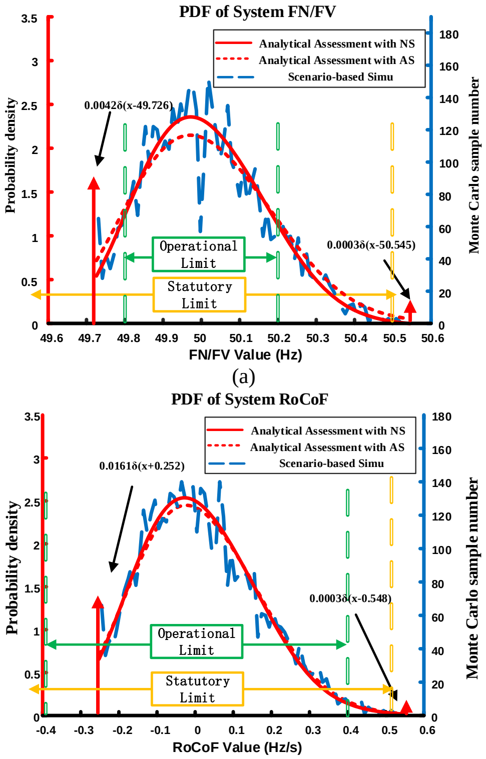


Fig. 2. Line diagram of modified IEEE 10-machine 39-bus test system with three wind farms.

by proposed analytical assessments with NS and AS are consistent with the ones by SBS for both FN/FV and RoCoF. Take the probability within the operational limits for further demonstration, the analytical assessment with NS has a marginal superiority over the one with AS when compared with SBS results shown in Table I. It is also proved by Table I that for a large-scale system (case 2), the proposed analytical assessments have satisfactory performances similar to case 1.



(a)

(b)

Fig. 3. PDFs by analytical assessment and SBS: (a) FN/FV, (b) RoCoF.

TABLE I

COMPARISON OF FREQUENCY STABILITY PROBABILITY WITHIN OPERATIONAL LIMITS

Margin Index Probability	FN/FV (Case 1)	RoCoF (Case 1)	FN/FV (Case 2)	RoCoF (Case 2)
Analytical Assessment (AS)	77.462%	98.231%	74.332%	99.072%
Analytical Assessment (NS)	80.808%	99.015%	74.411%	98.218%
Scenario-based Simu	79.640%	98.840%	76.081%	98.501%

TABLE II

COMPARISON OF COMPUTATIONAL TIME

Scenario-based Simu	Proposed Analytical Assessment
48569.54*s*	32.56*s*

Meanwhile, the computational time of two methods is also compared and displayed in Table II. It can be seen that the proposed assessment is around 1500 times faster than SBS.

Finally, the RAM of UK SQSS [3] (Table III) is applied as an example to evaluate whether the system is stable or not. The 'Green', 'Yellow' and 'Red' region in Table III indicates the state of 'Safe (Low Risk)', 'Alert (Medium Risk)' and 'Emergency (High Risk)' respectively. The RAM is filled with the probability results obtained by the proposed analytical assessment (with NS) in Table III. As revealed by the risk assessment in Table III, the system has around 81% probability to remain in the safe state in terms of steady-state frequency stability and around 99% for RoCoF. Although there are some circumstances for the frequency to breach the operational or even statutory limits as shown in Fig. 3, the chances are quite low as indicated in Table III. Therefore, the alert is on but no actual actions are needed, which demonstrates a typical case where the wind curtailments can be avoided.

TABLE III

PROPOSED ASSESSMENT RESULTS IN RAM

(a) **FN/FV RAM**

Probability \ Hz	<49.5	49.5~49.8	49.8~50.2	50.2~50.5	>50.5
0-1%	🟨 0%	🟩	🟩	🟩	🟨 0.197%
1%-30%	🟥	🟨 7.881%	🟩	🟨 11.113%	🟥
30%-100%	🟥	🟥	🟩 80.808%	🟥	🟥

(b) **RoCoF RAM**

Probability \ Hz/s	<-0.5	-0.5~-0.4	-0.4~0.4	0.4~0.5	>0.5
0-1%	🟨 0%	🟩 0%	🟩	🟩 0.774%	🟨 0.210%
1%-30%	🟥	🟨	🟩	🟨	🟥
30%-100%	🟥	🟥	🟩 99.015%	🟥	🟥

IV. CONCLUSIONS

This letter proposes a probabilistic assessment framework to comprehensively and efficiently evaluate system steady-state and dynamic frequency stability margins for the first time, which can significantly facilitate the system planner's decision-making process in the operational planning and effectively mitigate the renewable curtailments (Contribution 1). The RAM is practically applied to assess the risk of frequency stability (FN/FV and RoCoF) by incorporating a specific industrial standard for the first time (Contribution 2). The simulation results demonstrate that when comparing with the NS-based assessment considering the network impact and the existing scenario-based simulation, the performance of AS-based assessment is also quite satisfactory in the application of probabilistic stability analysis (Contribution 3).

V. ACKNOWLEDGMENT

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