

Security-Constrained Multi-Period Economic Dispatch with Renewable Energy Utilizing Distributionally Robust Optimization

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Abstract—This paper presents a security-constrained multi-period economic dispatch model (M-SCED) for systems with renewable energy sources (RES). A two-stage framework is adopted to model initial operation plans and recourse actions before and after the uncertainty realization of RES power. For ensuring superior system economic efficiency, distributionally robust optimization (DRO) is utilized to evaluate the expectations of operation costs affected by RES uncertainty. Practical issues, including boundedness of uncertainty and inaccurate information, are considered in modeling uncertainty in DRO. Within the framework of DRO, robust optimization is integrated to enhance system security. Besides, decision variables after the first period in M-SCED are approximated by segregated linear decision rules to achieve computational tractability without substantially degrading the model accuracy. A Constraint Generation algorithm is proposed to solve this problem with comprehensive case studies illustrating the effectiveness of the proposed M-SCED.

Index Terms—Economic dispatch, renewable energy, uncertainty, distributionally robust optimization, robust optimization, two-stage, multi-period.

I. NOMENCLATURE

In this paper, indices, decision variables and uncertainty are printed in italics while parameters are in non-italics.

A. Indices

i	Index of generators.
j	Index of RES
k	Index of loads
h	Index of transmission lines
t	Index of operation periods

B. Parameters

$a_i^{g,1}, a_i^{g,2}$	Generation cost coefficients of Generator i
a_i^{r+}, a_i^{r-}	Up, down reserve cost coefficient of Generator i

b_i^{p+}, b_i^{p-}	Up, down regulation cost coefficient of Generator i
b_k^{ls}	Load shedding penalty coefficient of Load k
b_j^w	Regulation cost coefficient of RES j
p_i^u, p_i^l	Maximal, minimal output limit of Generator i
Ra_i^+, Ra_i^-	Up, down ramping limit of Generator i
Re_i^+, Re_i^-	Up, down reserve capacity limit of Generator i
$\lambda_{h,i}, \lambda_{h,j}, \lambda_{h,k}$	Power transfer distribution factor of Line h from Generator i , RES j and Load k
Fl_h	Power flow limit of Line h
T	Number of periods in the operation horizon
N_G, N_W, N_L	Number of generators, RES and loads
$w_j^{f,t}$	Power forecast of RES j in Period t
d_k^t	Power demand of Load k in Period t
$d_k^{\lim,t}$	Allowed load shedding amount of Load k in Period t

C. First-stage decision variables

p_i^t	Scheduled output of Generator i in Period t
w_j^t	Scheduled output of RES j in Period t
re_i^{t+}, re_i^{t-}	Up, down reserve capacity offered by Generator i in Period t
\mathbf{x}^t	Vector of first-stage decision variables in Period t

D. Second-stage decision variables

$p_i^{r,t}$	Regulated output of Generator i in Period t
$d_k^{ls,t}$	Load shedding amount of Load k in Period t
$w_j^{r,t}$	Regulated output of RES j in Period t
\mathbf{y}^t	Vector of second-stage decision variables in Period t

E. Uncertainty

ξ^t, ξ	Uncertainty of power forecast of RES in Period t and the entire operation horizon, $\xi = [\xi^1; \dots; \xi^T]$
ξ_{seg}	Segregated uncertainty of ξ
μ_0	Statistical mean of ξ^1
Σ_0	Statistical covariance of ξ^1
S_0, S	Support of ξ^1, ξ
f_{ξ^1}, f_{ξ}	Distribution of ξ^1, ξ
D_1, D_2	Distributional set for ξ^1, ξ

F. Functions

$E(\cdot)$	Expectation
$\text{Tr}(\cdot)$	Trace of matrices
$\text{Pr}(\cdot)$	Probability of events

This work was supported by the Hong Kong Polytechnic University under Project G-UA3Z and Research Studentship RUH5, Beijing Natural Science Foundation (3174057), and Support Program for the Excellent Talents in Beijing City (2016000020124G079).

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$(x)^+, (x)^-$ Positive and negative operator,
 $(x)^+ = \max\{x, 0\}, (x)^- = \max\{-x, 0\}$

II. INTRODUCTION

RENEWABLE energies, such as wind and solar power, are taking a larger proportion in power supply of current power systems. Despite they have the advantage of being environment friendly, they can be greatly influenced by external factors, such as weather, and are therefore inherently unsteady and hard to predict [1-3]. The uncertainty in the forecasted RES power should therefore be carefully handled by system operators; otherwise, secure and economical system operation will be threatened, and power imbalance could be resulted in economic dispatch (ED) for example. ED is one of the fundamental decision problems in power systems and is often formulated as optimization problems [4], which are now contaminated by uncertainty because of vast allocation of RES.

In solving optimization problems involving uncertainty, robust optimization (RO) and stochastic optimization (SO) are the two major approaches to model and transform uncertainty. SO either uses samples to approximate uncertainty distribution, or assumes the distribution being known for analytical transformation [5-7]. However, the former is computationally demanding and the latter is often over-optimistic in assuming the uncertainty distribution. RO describes the possible region of uncertainty realizations through uncertainty sets and considers the worst case [8-10]. This approach has simple requirements on uncertainty information and is mathematically tractable. While strict constraints can be imposed by RO, the poor handling of expectations of uncertainty-related terms renders it insufficient in dealing with uncertainty-contaminated optimization problems.

Besides SO and RO, DRO is a new optimization approach being fast developed recently [11-15]. Different from SO, DRO is more practical in considering all possible uncertainty distributions according to available uncertainty information, such as statistical moments. It evaluates the worst expectation over all possible uncertainty distributions and is therefore worst-distribution oriented as compared to worst-case oriented RO.

Having practical consideration on uncertainty distributions, DRO is advantageous when expectations of uncertainty-related terms are concerned, and has been recently applied in power system operation, including reserve scheduling, optimal power flow and co-dispatch of energy and reserve [16-20]. Single-stage models are established in [16-19]. However, in modern power system operation, recourse actions are usually needed to eliminate the influence brought by uncertainty of forecast. In this regard, single-stage models in [16-19] roughly assume that recourse actions respond affinely to uncertainty realizations, which can be far from the optimal decision.

In contrast to single-stage models, two-stage formulations are more precise in modeling recourse actions. To achieve effective co-dispatch of energy and reserve, a two-stage model was built in [20] using DRO, and is referred as O-DRO here. In O-DRO, the first stage determines initial operation plans with respect to forecasts of RES power with consideration of

the second stage, and the second stage determines recourse actions with respect to uncertainty realizations. The second-stage problem minimizes the recourse cost, and the worst expectation of its optimal value is added into the objective of the first-stage problem through DRO techniques, forming the complete two-stage model. With this structure, proper current decisions can be made by accurately considering future recourse actions with respect to uncertainty realizations. [21] presented a similar two-stage model for co-dispatch of hydro, wind and thermal power sources.

Further to the O-DRO proposed in [20], the following three concerns are considered in this paper. Firstly, O-DRO is single-period, and thus coupled ramping constraints of generators between consecutive time periods cannot be incorporated. As a result, operational infeasibility or additional operation costs can be incurred in practice. Secondly, statistical moments (mean and covariance) used in O-DRO are assumed to be exact while they are in fact derived from historical samples and may deviate from actual values, especially when the available data is limited or with inferior quality, which can lead to sub-optimal solutions. Thirdly, in O-DRO, unlimited load shedding operation is allowed to avoid power imbalance, but could lead to undesirable load curtailment and thus degrade system security.

Therefore, a comprehensive multi-period security-constrained economic dispatch model, M-SCED, is proposed in this paper to address the above issues. The main contributions of this paper are as follows.

(1) Computationally-efficient accurate multi-period model: A two-stage formulation is adopted in M-SCED to accurately model recourse actions with respect to uncertainty realizations. To tackle the excessive computational difficulty of multi-period two-stage models, decision variables after the first period are approximated through segregated linear decision rules (SLDR), which assumes that optimal decisions are of piecewise-affine functions of earlier realized uncertainty. Such formulation ensures the computational tractability of M-SCED while the model accuracy can be mostly maintained.

(2) DRO performance: Instead of the one in O-DRO, a more realistic distributional set D_1 from [11] is applied in M-SCED to model possible distributions of the first-period RES uncertainty ξ^1 . In D_1 , uncertainty support is incorporated to embody the boundedness of RES uncertainty, and possible deviations of statistical moments are considered to guarantee DRO's performance under inaccurate uncertainty information. The explicit expression of the worst expectation over D_1 in M-SCED is derived through exploiting the dual second-stage problem of the first period and is simplified through a tailor-made Constraint Generation algorithm when M-SCED is solved.

(3) System security: Load shedding is limited in M-SCED to enhance system security. The solution difficulty caused by such limits is overcome with Farkas' Lemma. The proposed M-SCED is then transformed into a deterministic problem using RO and DRO techniques.

This paper is organized as follows. Section III introduces the theory of DRO. Section IV presents the formulation of M-

SCED. The solution algorithm for M-SCED is given in Section V. Then, case studies are conducted in Section VI to illustrate the effectiveness and improvements of M-SCED. Last but not least, conclusions are made in Section VII.

III. DRO TECHNIQUES

Because the exact uncertainty distribution cannot be obtained from limited data, DRO considers all possible distributions and defines sets containing them as distributional sets. Considering different requirements on modeling accuracy for ξ^1 and ξ , two distributional sets, D_1 and D_2 , are adopted in this paper.

For the first-period uncertainty ξ^1 , the more accurate set, D_1 , is used to model the possible distributions of ξ^1 as in (1) [11], where $(\cdot)'$ represents the transposition of (\cdot) .

$$D_1 = \left\{ \begin{array}{l} \Pr(\xi^1 \in S_0) = 1 \\ \left. \begin{array}{l} \mathbb{E}[\xi^1] - \mu_0 \\ \mathbb{E}[(\xi^1 - \mu_0) \cdot (\xi^1 - \mu_0)'] \end{array} \right\} \begin{array}{l} \leq r_1 \\ \preceq r_2 \Sigma_0 \end{array} \end{array} \right\} \quad \begin{array}{l} (1.1) \\ (1.2) \\ (1.3) \end{array}$$

(1.1) sets the uncertainty support, limiting the possible range of ξ^1 . (1.2) and (1.3) describe the possible range of uncertainty mean and covariance based on the statistical values, μ_0 and Σ_0 . (1.2) bounds the mean within an ellipsoid centered at μ_0 and (1.3) sets up a positive semidefinite cone for the covariance. r_1 and r_2 are parameters that limit the possible range of uncertainty mean and covariance, and decide the size of D_1 .

The core element of DRO is the worst expectation over the distributional set, which can be expressed as a semi-infinite optimization problem and is equal to Λ_1 under D_1 as in (2).

$$\Lambda_1(\Omega) = \sup_{f_{\xi^1} \in D_1} \mathbb{E}[\Omega] \\ = \sup_{f_{\xi^1} \in D_1} \int_{S_0} \Omega \cdot df_{\xi^1}(\xi^1) \quad (2.1)$$

$$s.t. \quad f_{\xi^1}(\xi^1) \geq 0, \quad \forall \xi^1 \in S_0 \quad (2.2)$$

$$\int_{S_0} df_{\xi^1}(\xi^1) = 1 \quad (2.3)$$

$$\int_{S_0} \begin{bmatrix} \Sigma_0 & \xi^1 - \mu_0 \\ (\xi^1 - \mu_0)' & r_1 \end{bmatrix} df_{\xi^1}(\xi^1) \succeq 0 \quad (2.4)$$

$$\int_{S_0} (\xi^1 - \mu_0) \cdot (\xi^1 - \mu_0)' df_{\xi^1}(\xi^1) \preceq r_2 \Sigma_0 \quad (2.5)$$

Probability densities are decision variables, so there are infinite variables and finite constraints in (2). (2.3)-(2.5) are transformed from (1.1)-(1.3), respectively. According to the duality theory [11], the optimal value of (2) is equal to that of the dual problem (3). (3) has finite variables and infinite constraints and is easier to solve. r^d, s^d, p^d, P^d, Q^d in (3) are dual variables.

$$\Lambda_1(\Omega) = \inf \quad r_2 \cdot \text{Tr}(\Sigma_0 \cdot Q^d) - \mu_0' Q^d \mu_0 + r^d + \text{Tr}(\Sigma_0 \cdot P^d) \\ - 2\mu_0' p^d + r_1 s^d \quad (3.1)$$

$$s.t. \quad \begin{bmatrix} P^d & p^d \\ (p^d)' & s^d \end{bmatrix} \succeq 0 \quad (3.2)$$

$$Q^d \succeq 0 \quad (3.3)$$

$$(\xi^1)' Q^d \xi^1 + 2(\xi^1)' (-p^d - Q^d \mu_0) + r^d \\ \geq \Omega, \quad \forall \xi^1 \in S_0 \quad (3.4)$$

Compared with the distributional set in [20-21], D_1 describes uncertainty moments by proper inequalities rather than equalities and thus attains the important condition (3.3), which facilitates the incorporation of uncertainty support information.

As decision variables after the first period are approximated by SLDR in M-SCED, the simpler distributional set D_2 can be adopted for the multi-period uncertainty ξ as in (4), where the mean of the segregated uncertainty under SLDR lies in a tractable conic representable set Ψ , and the worst expectation over D_2 is covered in Section V.A.

$$D_2 = \left\{ \begin{array}{l} \Pr(\xi \in S) = 1 \\ \mathbb{E}[\xi_{\text{seg}}] \in \Psi \end{array} \right\} \quad \begin{array}{l} (4.1) \\ (4.2) \end{array}$$

IV. MULTI-PERIOD ECONOMIC DISPATCH MODEL

In this section, separate models for first-stage and second-stage problems of the two-stage formulation are presented first. Then multi-period modeling and SLDR are discussed. At last, the multi period ED model M-SCED is established.

A. Separate models for first-stage and second-stage problems

As in (5), first-stage problems determine initial operation plans with respect to forecasts of RES power.

$$\inf \quad \sum_{i=1}^{N_G} \left[a_i^{g,1} (p_i^t)^2 + a_i^{g,2} p_i^t + a_i^{r,+} r e_i^{t,+} + a_i^{r,-} r e_i^{t,-} \right] \quad (5.1)$$

$$s.t. \quad \sum_{i=1}^{N_G} p_i^t + \sum_{j=1}^{N_W} w_j^t = \sum_{k=1}^{N_L} d_k^t \quad (5.2)$$

$$-Fl_h \leq \sum_{i=1}^{N_G} \lambda_{h,i} p_i^t + \sum_{j=1}^{N_W} \lambda_{h,j} w_j^t - \sum_{k=1}^{N_L} \lambda_{h,k} d_k^t \leq Fl_h, \quad \forall h \quad (5.3)$$

$$0 \leq w_j^t \leq w_j^{f,t}, \quad \forall j \quad (5.4)$$

$$p_i^t + r e_i^{t,+} \leq p_i^u, \quad \forall i \quad (5.5)$$

$$p_i^l \leq p_i^t - r e_i^{t,-}, \quad \forall i \quad (5.6)$$

$$0 \leq r e_i^{t,+} \leq Re_i^+, \quad \forall i \quad (5.7)$$

$$0 \leq r e_i^{t,-} \leq Re_i^-, \quad \forall i \quad (5.8)$$

The objective (5.1) minimizes the sum of generation costs and reserve costs. (5.2) ensures system power balance. (5.3) prevents overloading of transmission lines. (5.4) ensures scheduled RES power not exceeded the forecast values. (5.5)-(5.8) are power output and reserve capacity limits of generators. Quadratic generation costs in (5.1) can be approximated by piecewise-linear functions. Then, first-stage problems can be rewritten in compact forms as (6).

$$\inf_{\mathbf{x}^t} (\mathbf{a}^t)' \cdot \mathbf{x}^t \quad (6.1)$$

$$s.t. \quad \mathbf{A}^t \cdot \mathbf{x}^t \leq \mathbf{c}^t \quad (6.2)$$

Second-stage problems determine recourse actions according to actual realizations of RES uncertainty. The objective (7.1) minimizes recourse costs, including load shedding penalty and regulation costs of generators and RES. (7.2) and (7.3) avoid real-time power imbalance and overloading of transmission lines under recourse actions. (7.4) and (7.5) are constraints on availability of RES power and reserve capacity. (7.6) limits load shedding amounts, guaranteeing system security. The objective can be linearized through introducing slack variables, then the second-stage problems can be rewritten in compact forms as (8).

$$\inf \sum_{i=1}^{N_G} \mathbf{b}_i^{p+} (p_i^{r,t} - p_i^t)^+ + \sum_{i=1}^{N_G} \mathbf{b}_i^{p-} (p_i^{r,t} - p_i^t)^- + \sum_{j=1}^{N_W} \mathbf{b}_j^w |w_j^{r,t} - w_j^t| + \sum_{k=1}^{N_L} \mathbf{b}_k^{ls} d_k^{ls,t} \quad (7.1)$$

$$s.t. \quad \sum_{k=1}^{N_L} (d_k^t - d_k^{ls,t}) = \sum_{j=1}^{N_W} w_j^{r,t} + \sum_{i=1}^{N_G} p_i^{r,t} \quad (7.2)$$

$$-Fl_h \leq \sum_{i=1}^{N_G} \lambda_{h,i} p_i^{r,t} + \sum_{j=1}^{N_W} \lambda_{h,j} w_j^{r,t} - \sum_{k=1}^{N_L} \lambda_{h,k} (d_k^t - d_k^{ls,t}) \leq Fl_h, \forall h \quad (7.3)$$

$$0 \leq w_j^{r,t} \leq w_j^{f,t} + \zeta_j^t, \forall j \quad (7.4)$$

$$p_i^t - re_i^{t,-} \leq p_i^{r,t} \leq p_i^t + re_i^{t,+}, \forall i \quad (7.5)$$

$$0 \leq d_k^{ls,t} \leq d_k^{\lim,t}, \forall k \quad (7.6)$$

$$\Omega_t = \inf_{\mathbf{y}^t} (\mathbf{b}^t)' \cdot \mathbf{y}^t \quad (8.1)$$

$$s.t. \quad \mathbf{B}^t \cdot \mathbf{y}^t \leq \mathbf{g}^t - \mathbf{E}^t \cdot \mathbf{x}^t - \mathbf{F}^t \cdot \boldsymbol{\xi}^t \quad (8.2)$$

$$\mathbf{y}^t \geq \mathbf{0} \quad (8.3)$$

B. Multi-period modeling and segregated linear decision rules

The single-period two-stage O-DRO has the formulation as (9), in which $\Omega_1(\mathbf{x}^1, \boldsymbol{\xi}^1)$ represents the optimal value of the second-stage problem and implies satisfaction of relevant constraints. To solve this problem, the inexplicit term $\Omega_1(\mathbf{x}^1, \boldsymbol{\xi}^1)$ is eliminated through exploiting the dual second-stage problem. Although such formulation is accurate, it cannot be directly extended to multi-period as it would be too difficult to solve [22-23].

$$\inf_{\mathbf{x}^1} (\mathbf{a}^1)' \cdot \mathbf{x}^1 + \sup_{\boldsymbol{\xi}^1 \in \mathcal{D}_1} \mathbb{E}[\Omega_1(\mathbf{x}^1, \boldsymbol{\xi}^1)] \quad (9.1)$$

$$s.t. \quad \mathbf{A}^1 \cdot \mathbf{x}^1 \leq \mathbf{c}^1 \quad (9.2)$$

In multi-period ED problems, different periods in the operation horizon are not independent from each other, but are linked through ramping constraints of generators. As a result, for M-SCED, only \mathbf{x}^1 in the first period can be decided independently from actual realizations of uncertainty, while all

later decisions, $\mathbf{x}^2, \dots, \mathbf{x}^T$ and $\mathbf{y}^1, \dots, \mathbf{y}^T$, are influenced by relevant early uncertainty realizations. Because of this consistency requirement of time sequences, multi-period problems are difficult to solve and thus often approximated through linear decision rules (LDR) in literatures, which is shown to be effective and computationally efficient [19, 22-25].

The relationship between optimal decisions and realized uncertainty can be very complicated. Instead of considering the true relationship, LDR assumes that the optimal decision is of affine functions of relevant uncertainty realizations. For example, in the case of O-DRO, the second-stage optimal decision of \mathbf{y}^1 is influenced by the realization of $\boldsymbol{\xi}^1$ and is decided by the second-stage optimization problem (8). The relationship between optimal \mathbf{y}^1 and $\boldsymbol{\xi}^1$ generally cannot be expressed in closed forms, but will be assumed as (10) if LDR is adopted, where $\hat{\mathbf{y}}^1$ represents the approximation of \mathbf{y}^1 . In fact, two-stage models are equivalent to single-stage ones under LDR approximation. In multi-period problems, the relationship between optimal decisions in Period 2 to Period T and relevant uncertainty realizations is more complicated than that between optimal \mathbf{y}^1 and $\boldsymbol{\xi}^1$ in O-DRO, and will also be assumed to be affine under LDR.

SLDR extends LDR by segregating the primitive uncertainty and applying LDR on the segregated uncertainty to achieve better approximation, and is adopted here. Under SLDR, the primitive uncertainty $\boldsymbol{\xi}$ is segregated into $(\boldsymbol{\xi})^+$ and $(\boldsymbol{\xi})^-$, and the segregated uncertainty is $\boldsymbol{\xi}_{\text{seg}} = [(\boldsymbol{\xi})^+; (\boldsymbol{\xi})^-]$, where $(\boldsymbol{\xi})^+$ is the vector after element-wise operation $(\xi_i)^+ = \max\{\xi_i, 0\}$ to all elements in $\boldsymbol{\xi}$. Original decision variables are replaced by affine functions of the segregated uncertainty $\boldsymbol{\xi}_{\text{seg}}$, which are equivalent to piecewise-affine functions of the primitive uncertainty $\boldsymbol{\xi}$ as shown in (11). Coefficients of SLDR are optimized when M-SCED is solved. Details on LDR and SLDR can be found in [26-27].

$$\hat{\mathbf{y}}^1(\boldsymbol{\xi}) = \boldsymbol{\beta}_{\text{con}} + \boldsymbol{\beta} \cdot \boldsymbol{\xi} \quad (10)$$

$$\hat{\mathbf{x}}(\boldsymbol{\xi}) = \boldsymbol{\beta}_{\text{con}} + \boldsymbol{\beta}_{\text{seg}} \cdot \boldsymbol{\xi}_{\text{seg}} = \boldsymbol{\beta}_{\text{con}} + \boldsymbol{\beta}^+ \cdot (\boldsymbol{\xi})^+ + \boldsymbol{\beta}^- \cdot (\boldsymbol{\xi})^- \quad (11)$$

Under LDR or SLDR, time sequences can be regarded as being squeezed together. As a result, multi-period problems become mathematically equivalent to single-period ones as in [19], and the difficulty of solving them is greatly reduced.

C. Multi-period economic dispatch model M-SCED

In M-SCED, all later decision variables after \mathbf{x}^1 can be approximated by SLDR. But, to maintain modeling accuracy, \mathbf{y}^1 is remained intact and handled as in O-DRO. The multi-period model M-SCED is established as (12).

$$\inf \left\{ (\mathbf{a}^1)' \cdot \mathbf{x}^1 + \sup_{\boldsymbol{\xi}^1 \in \mathcal{D}_1} \mathbb{E}[\Omega_1(\mathbf{x}^1, \boldsymbol{\xi}^1)] + \sup_{\boldsymbol{\xi}^2 \in \mathcal{D}_2} \mathbb{E} \left[\sum_{t=2}^T \left[(\mathbf{a}^t)' \cdot \hat{\mathbf{x}}^t + (\mathbf{b}^t)' \cdot \hat{\mathbf{y}}^t \right] \right] \right\} \quad (12.1)$$

$$s.t. \quad \mathbf{A}^1 \cdot \mathbf{x}^1 \leq \mathbf{c}^1 \quad (12.2)$$

$$\mathbf{A}^t \cdot \hat{\mathbf{x}}^t \leq \mathbf{c}^t, \forall \xi^t, t=2, \dots, T \quad (12.3)$$

$$\mathbf{B}^t \cdot \hat{\mathbf{y}}^t \leq \mathbf{g}^t - \mathbf{E}^t \cdot \hat{\mathbf{x}}^t - \mathbf{F}^t \cdot \xi^t, \forall \xi^t, t=2, \dots, T \quad (12.4)$$

$$\hat{\mathbf{y}}^t \geq \mathbf{0}, \forall \xi^t, t=2, \dots, T \quad (12.5)$$

$$-\mathbf{Ra}_i^- \leq \hat{p}_i^{t,t} - \hat{p}_i^{t,t-1} \leq \mathbf{Ra}_i^+, \forall \xi^t, \forall i, t=2, \dots, T \quad (12.6)$$

In M-SCED, $\Omega_1(\mathbf{x}^1, \xi^1)$ is the optimal second-stage cost of the first period. Λ_1 and Λ_2 are the worst expectations of $\Omega_1(\mathbf{x}^1, \xi^1)$ and the total costs from Period 2 to Period T, respectively. They are minimized together with the deterministic first-stage cost of the first period. (12.2) is the deterministic first-stage constraints of the first period. (12.3)-(12.5) are constraints from Period 2 to Period T and (12.6) is ramping constraints. Constraints in M-SCED are required to be robust to all possible uncertainty realizations in uncertainty supports in order to ensure secure system operation, while the worst expected operation costs are minimized to pursue superior average economic performance with respect to the worst possible uncertainty distribution.

M-SCED only requires to solve for \mathbf{x}^1 and other variables help make the right decision for \mathbf{x}^1 by taking future circumstances into consideration. In rolling-plan operation, second-stage decisions \mathbf{y}^t are solved from (8) when they need to be carried out. First-stage decisions of Period 2 to Period T are solved later from updated M-SCED.

With this in mind, it is reasonable to keep \mathbf{y}^1 intact and approximate variables after \mathbf{y}^1 , because \mathbf{y}^1 has the most direct influence on the optimal decision of \mathbf{x}^1 . Load shedding is not modeled from Period 2 to Period T in M-SCED, because it cannot be properly approximated by LDR or SLDR. Relevant decision variables of load shedding are deleted from second-stage problems of Period 2 to Period T in M-SCED, and relevant constraints are accordingly modified. This means that the current decision \mathbf{x}^1 needs to be made to ensure feasible system operation even with no load shedding allowed in Period 2 to Period T. However, it should be noted that the current decision \mathbf{x}^1 has more direct influence on load shedding in the first period than in Period 2 to Period T, and load shedding in the first period is still modeled in \mathbf{y}^1 to help making a proper decision for \mathbf{x}^1 . Load shedding in Period 2 to Period T will be modeled later in \mathbf{y}^1 when M-SCED is updated in rolling-plan operation. With such formulation, M-SCED achieves tractable multi-period modeling while ensuring the quality of the current decision \mathbf{x}^1 .

While the consideration of RES uncertainty in the proposed model has just been presented, load uncertainty can also be incorporated into the model and handled similarly as the RES uncertainty. The incorporation of load uncertainty and its impacts will further be discussed in Section VI.D.

V. SOLUTION METHOD

In M-SCED, uncertainty supports of ξ^1 and ξ , S_0 and S , are taken as ellipsoids and polytopes respectively due to the computation considerations. Ellipsoidal S_0 makes it possible to derive the deterministic explicit expression of Λ_1 through S-lemma, and polyhedral S keeps the robust counterparts of (12.3)-(12.6) linear. Though ellipsoid and polytope may not be

the best uncertainty support shapes, both ellipsoidal and polyhedral supports are acceptable if they contain the clear majority of possible uncertainty realizations with reasonable sizes. In this section, M-SCED is transformed into an equivalent deterministic problem first and then a Constraint Generation algorithm is proposed to solve the problem efficiently.

A. Equivalent deterministic problem

Under SLDR, total costs from Period 2 to Period T are of affine functions of the segregated uncertainty ξ_{seg} . Therefore, Λ_2 can be rewritten as (13). If the mean of ξ_{seg} is expressed as $\bar{\xi}_{\text{seg}}$, then Λ_2 can be further transformed into (14) and thus can be replaced by its robust counterpart through standard RO techniques. Constraints (12.3)-(12.6) can be replaced by their robust counterparts as well. The explicit expression of Λ_1 is (3). Then, the only inexplicit term in M-SCED is $\Omega_1(\mathbf{x}^1, \xi^1)$ in (3.4).

$$\Lambda_2 = \sup_{f_{\xi} \in D_2} \mathbb{E} \left[\beta_{\text{con}} + (\beta_{\text{seg}})' \cdot \xi_{\text{seg}} \right] \quad (13)$$

$$\Lambda_2 = \beta_{\text{con}} + \sup_{\bar{\xi}_{\text{seg}} \in \Psi} (\beta_{\text{seg}})' \cdot \bar{\xi}_{\text{seg}} \quad (14)$$

According to the following proposition, $\Omega_1(\mathbf{x}^1, \xi^1)$ in (3.4) can be eliminated. Different from O-DRO, load shedding is limited in M-SCED. As a result, under improper first-stage decisions \mathbf{x}^1 , the second-stage problem of the first period could be infeasible because of power imbalance and $\Omega_1(\mathbf{x}^1, \xi^1)$ would become plus infinity. To overcome this, Farkas' Lemma is utilized to guarantee the second-stage feasibility in the proposition.

Proposition:

In M-SCED, (3.4) is equivalent to the combination of (15) and (16).

$$(\mathbf{g}^1 - \mathbf{E}^1 \cdot \mathbf{x}^1 - \mathbf{F}^1 \cdot \xi^1)' \cdot \mathbf{u}_r \leq 0, \forall \xi^1 \in S_0, \forall r \quad (15)$$

$$(\xi^1)' \mathbf{Q}^d \xi^1 + 2(\xi^1)' \cdot (-\mathbf{p}^d - \mathbf{Q}^d \mu_0) + r^d \geq$$

$$(\mathbf{g}^1 - \mathbf{E}^1 \cdot \mathbf{x}^1 - \mathbf{F}^1 \cdot \xi^1)' \cdot \mathbf{u}_e, \forall \xi^1 \in S_0, \forall e \quad (16)$$

$\mathbf{u}_r, r=1, \dots, N_R$ and $\mathbf{u}_e, e=1, \dots, N_E$ are all extreme rays and extreme points of the set $U = \{\mathbf{u} | (\mathbf{B}^1)' \cdot \mathbf{u} \leq \mathbf{b}^1, \mathbf{u} \leq \mathbf{0}\}$, which is the dual feasible region of the second-stage problem (8) of the first period.

Proof:

According to Farkas' Lemma [28], the feasibility of the second-stage problem (8) of the first period is equivalent to the infeasibility of (17) and can then be further transformed into (18).

$$\left\{ \begin{array}{l} \mathbf{u} \in U_c = \left\{ \mathbf{u} \mid (\mathbf{B}^1)' \cdot \mathbf{u} \leq \mathbf{0}, \mathbf{u} \leq \mathbf{0} \right\} \\ (\mathbf{g}^1 - \mathbf{E}^1 \cdot \mathbf{x}^1 - \mathbf{F}^1 \cdot \xi^1)' \cdot \mathbf{u} > 0 \end{array} \right\} \quad (17)$$

$$(\mathbf{g}^1 - \mathbf{E}^1 \cdot \mathbf{x}^1 - \mathbf{F}^1 \cdot \xi^1)' \cdot \mathbf{u} \leq 0, \forall \mathbf{u} \in U_c \quad (18)$$

Because U_c is a polyhedral cone, every element in it can be

expressed as a nonnegative linear combination of its extreme rays, which are the same as those of U . As a result, when \mathbf{x}^1 is fixed, (18) is equivalent to (15).

Under the feasibility guarantee (15), the optimal value $\Omega_1(\mathbf{x}^1, \xi^1)$ is attained at one of the extreme points of the dual feasible set U and then (19) holds [29-30].

$$\Omega_1(\mathbf{x}^1, \xi^1) = \max_{e \in \{1, 2, \dots, N_E\}} \left\{ (\mathbf{g}^1 - \mathbf{E}^1 \cdot \mathbf{x}^1 - \mathbf{F}^1 \cdot \xi^1)' \cdot \mathbf{u}_e \right\} \quad (19)$$

Therefore, (3.4) is equivalent to (16) under (15). \square

(15) guarantees the feasibility of the second-stage problem of the first period within the framework of DRO, and its effect is requiring enough reserve capacity for scheduled RES power. (16) ensures the optimality of \mathbf{x}^1 . (15) can be replaced by its deterministic robust counterpart through standard RO techniques and (16) can be transformed into semidefinite constraints through S-lemma [31]. Then, M-SCED becomes a deterministic semidefinite program, which can be solved by off-the-shelf solvers.

B. Constraint Generation algorithm

Although M-SCED is transformed into a deterministic problem as in Section V.A, constraints from (15) and (16) can be in vast numbers, relating with extreme rays and points of U , and only parts of them are active for the optimal solution. To solve the problem efficiently, a Constraint Generation algorithm is proposed here based on the structure of M-SCED. The core idea is to relax M-SCED by considering only parts of constraints from (15) and (16). Necessary constraints from (15) and (16) are found iteratively by solving the relaxed M-SCED and two sub-problems until the optimal solution is reached. The relaxed M-SCED is in the form of (20), where Φ_e and Φ_r are sets of necessary extreme points and rays. Sub-problem 1 and 2 are set as (21) and (22), respectively, where $\mathbf{x}^{1,*}$, $\mathbf{p}^{d,*}$, $\mathbf{Q}^{d,*}$ and $r^{d,*}$ are optimal values solved from the relaxed M-SCED.

$$\begin{aligned} & \inf (\mathbf{a}^1)' \cdot \mathbf{x}^1 + \Lambda_2 + (3.1) \\ & \text{s.t. } (12.2)-(12.6), (3.2)-(3.3) \\ & (15), \forall r \in \Phi_r \\ & (16), \forall e \in \Phi_e \end{aligned} \quad (20)$$

$$\begin{aligned} & \inf_{\xi^1, \mathbf{u}} (\xi^1)' \cdot \mathbf{Q}^{d,*} \cdot \xi^1 + 2(\xi^1)' \cdot (-\mathbf{p}^{d,*} - \mathbf{Q}^{d,*} \boldsymbol{\mu}_0) + r^{d,*} \\ & \quad - (\mathbf{g}^1 - \mathbf{E}^1 \cdot \mathbf{x}^{1,*} - \mathbf{F}^1 \cdot \xi^1)' \cdot \mathbf{u} \\ & \text{s.t. } \xi^1 \in S_0, \mathbf{u} \in U \end{aligned} \quad (21)$$

$$\begin{aligned} & \sup_{\xi^1, \mathbf{u}} (\mathbf{g}^1 - \mathbf{E}^1 \cdot \mathbf{x}^{1,*} - \mathbf{F}^1 \cdot \xi^1)' \cdot \mathbf{u} \\ & \text{s.t. } \xi^1 \in S_0, \mathbf{u} \in U_c \\ & (\mathbf{g}^1 - \mathbf{E}^1 \cdot \mathbf{x}^{1,*} - \mathbf{F}^1 \cdot \xi^1)' \cdot \mathbf{u} \leq 1 \end{aligned} \quad (22)$$

If there is any extreme ray \mathbf{u}_r violating (15), then Sub-problem 1 will be unbounded and the optimal \mathbf{u} of Sub-problem 2 corresponds to the most violating extreme ray for

(15), which should be added to Φ_r . If there is no violating extreme ray for (15) but are some violating extreme points for (16), then the optimal value of Sub-problem 1 will be a finite negative value and the optimal \mathbf{u} corresponds to the most violating extreme point for (16), which should be added to Φ_e . The complete algorithm of solving M-SCED is as follows.

- 1) Initialize Φ_e and Φ_r with proper extreme points and rays of U .
- 2) Solve the relaxed M-SCED (20) with parts of constraints from (15) and (16) corresponding to Φ_r and Φ_e , respectively.
- 3) Substitute the optimal $\mathbf{x}^{1,*}$, $\mathbf{p}^{d,*}$, $\mathbf{Q}^{d,*}$ and $r^{d,*}$ obtained in step 2 into Sub-problem 1 and 2, and solve Sub-problem 1.
- 4) If the optimal value of Sub-problem 1 is not less than zero, the optimal solution of M-SCED is attained and the algorithm ends. If the optimal value is less than zero but not minus infinity, add the corresponding extreme point into Φ_e . Otherwise, solve Sub-problem 2 and add the corresponding extreme ray into Φ_r .
- 5) Repeat Step 2-4 until the algorithm ends.

Sub-problem 1 and 2 are both biconvex problems. When \mathbf{u} is fixed, they are both convex in ξ^1 . When ξ^1 is fixed, they are both linear in \mathbf{u} . Biconvex problems are non-convex and difficult to solve. To solve the biconvex Sub-problem 1 and 2 efficiently, a sequentially alternating approach is adopted here, which solves the problems by fixing ξ^1 and \mathbf{u} as constants alternatively until a local optimum is attained [32]. Though this approach cannot guarantee to find the global optimal solutions of Sub-problem 1 and 2, its solution quality could be improved with the use of multiple initial values [20]. Similar heuristic methods have been widely adopted to effectively solve bilinear and biconvex problems in literatures adopting two-stage formulations [20-21, 33-35], and the suggested sequentially alternating approach has also been shown to be effective in the case studies.

It should be noted that if equalities were used instead of inequalities in the distributional set D_1 to describe uncertainty moments, condition (3.3) could not be attained unless the uncertainty support S_0 was set as the entire real space [20-21]; otherwise, the biconvexity of Sub-problem 1 could not be guaranteed and extra computational difficulty will be introduced as a result.

VI. RESULTS AND DISCUSSION

In this section, effects of DRO in improving the economy of ED operation are firstly demonstrated to show the necessity of utilizing DRO. Superiority of two-stage models over single-stage ones is then illustrated. Lastly, M-SCED is benchmarked against O-DRO [20] to show its improvements and the effects of incorporating load uncertainty are discussed.

A. Necessity of utilizing DRO in Economic Dispatch

In ED, RO has been commonly used to minimize the objective in the worst possible case. However, because the worst case rarely happens, the solution can often be over-conservative. In contrast, M-SCED can greatly improve the economic efficiency through evaluating the expected second-

stage cost by DRO. The traditional RO model in [20] is here referred as T-RO. Because both T-RO and O-DRO are single-period, only the first-period sub-model of M-SCED is used for comparison here by removing components after the first period in M-SCED, and it is referred as S-SCED.

Case studies in this part are conducted on a modified IEEE 118-bus system from [20], where all RES have power forecast of 100 MW. For simplicity, all RES uncertainty is assumed to have zero mean, the same variance and no correlation with each other. T-RO has a parameter called uncertainty budget Γ , which controls the size of uncertainty sets and decides the conservatism level of T-RO. To make a fair comparison, ED is solved by S-SCED, O-DRO and T-RO of different Γ under RES uncertainty of different variances. Their calculated total costs for two stages are shown in Fig.1. As explained above, costs of T-RO are much higher than those of S-SCED and O-DRO, which overlap each other in the figure.

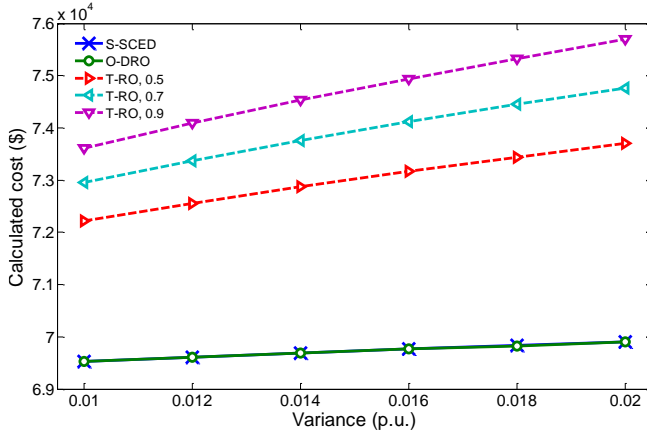


Fig. 1. Calculated costs from different models

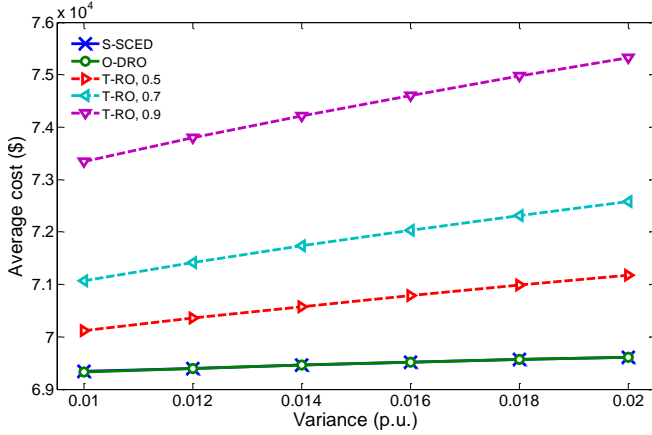


Fig. 2. Average costs from different models

To obtain the average performance of different models, for each set of variances, 3000 simulated uncertainty realizations are generated by normal distributions. Then based on the first-stage decisions of different models, recourse actions are solved from the second-stage problem (8) under simulated uncertainty realizations. The average total costs are shown in Fig. 2, which are the sum of deterministic first-stage costs and average second-stage costs with respect to all simulated uncertainty realizations. Average costs of T-RO are lower than its calculated costs, but still much higher than average costs of S-SCED and O-DRO. This is because T-RO is designed for

the worst case when actual available RES power is much lower than the forecast value. With this anticipation, T-RO schedules only part of forecasted RES power and thus plans more outputs from generators than S-SCED and O-DRO in the first stage, leading to unnecessary extra costs under most uncertainty realizations. Although T-RO will have lower total costs when the worst case does occur [20], the probability is small. Therefore, in terms of average economic performance, S-SCED and O-DRO are better than T-RO, proving the necessity of utilizing DRO in ED operation.

Computation times of different models are recorded in Table I which shows that S-SCED is comparable with O-DRO while the computation times of T-RO depend on Γ and could vary widely. Here, T-RO is solved by a column-and-constraint algorithm as in [20] with the bilinear sub-problem transformed into a mixed integer linear program (MILP). This bilinear sub-problem can also be solved by the sequentially alternating approach as adopted for S-SCED with the sacrifice of global optimality by solving a series of linear programs. While the method of transforming bilinear optimization problems into MILP is more computation demanding, it has been widely adopted in various applications [20, 36-39] because of its good solution quality, and hence is adopted here for benchmarking.

TABLE I
COMPUTATION TIMES OF DIFFERENT MODELS

Variance (p.u.)	S-SCED (s)	O-DRO (s)	T-RO (s)		
			$\Gamma=0.5$	$\Gamma=0.7$	$\Gamma=0.9$
0.01	119.50	89.88	149.61	119.59	30.99
0.014	116.52	107.24	132.86	122.42	29.86
0.018	114.35	107.66	166.70	121.89	34.42
0.02	113.51	113.02	150.10	121.86	30.28

B. Comparison between two-stage and single-stage models

Both S-SCED and O-DRO are two-stage models with recourse actions fully modeled in the second stage. In contrast, single-stage models simply assume that recourse actions respond affinely to uncertainty realizations. To illustrate the superiority of the two-stage S-SCED over single-stage models, comparison is made here between S-SCED and its single-stage counterpart, which is referred as S-S-SCED.

Case studies in this and following sections are conducted on a modified IEEE 30-bus system, where two RES are connected to Bus 22 and 25 with forecast power of 60 MW each. Variances of RES uncertainty are set to 0.02 p.u. and parameters of generators are given in Table II. Up and down reserve cost coefficients of all generators, a_i^{r+} and a_i^{r-} , are set to 1.2 \$/MW. 3000 simulated uncertainty realizations are generated by normal distributions to test S-SCED and S-S-SCED.

TABLE II
GENERATOR PARAMETERS IN IEEE 30-BUS SYSTEM

Bus No.	p_i^l (MW)	p_i^H (MW)	Re_i^+, Re_i^- (MW)	$a_i^{g,1}$ (\$/MW ²)	$a_i^{g,2}$ (\$/MW)	b_i^{p+}, b_i^{p-} (\$/MW)
1	50	100	20	3.75e-3	3	5
2	20	80	16	3.75e-3	3	5
5	15	50	10	3.75e-3	3	5
8	10	35	7	3.75e-3	3	5
11	12	60	10	3.75e-3	3	5
13	20	80	16	3.75e-3	3	5

Under this set of parameter settings, wind curtailment is cheaper than adjusting generators' outputs downwards. Therefore, in this and following case studies, only up reserve capacity is scheduled and presented in the results. The results of S-SCED and S-S-SCED are recorded in Table III. S-S-SCED schedules much less RES power than S-SCED and thus has much higher first-stage costs, leading to higher total costs than S-SCED. The reason is that the affine assumption on recourse actions of S-S-SCED is far from the optimal case. Taking load shedding as an example, if the first-stage plan of S-S-SCED is fixed to be the same as S-SCED, its decision rule for load shedding will be the dashed curve in Fig.3 when no transmission capacity constraint is binding. However, the optimal load shedding decision shall be the solid curve instead, as load shedding is the most expansive recourse action and should only be carried out when generators cannot provide enough regulation power. Besides load shedding, nonlinearity can also exist in other recourse actions, such as power regulation of generators. When the costs of generators are different, power regulation should be provided by lower cost generators first if possible; whereas under affine assumptions of single-stage models, regulation power is always provided by all generators.

The affine assumption on recourse actions leads to much more conservative anticipated costs of recourse actions for S-S-SCED. As a result, here S-S-SCED reduces the amount of scheduled RES power to avoid recourse actions as much as possible. The decision rule of S-SCED for recourse actions cannot be derived explicitly and thus cannot be directly compared with that of S-S-SCED. However, as shown in Table III, S-SCED achieves much better results than S-S-SCED, illustrating that it captures the nonlinearity of recourse actions better.

TABLE III
RESULTS OF S-SCED AND S-S-SCED

Model	S-SCED	S-S-SCED
Scheduled generator outputs (MW)	226.0	266.1
Scheduled RES outputs (MW)	120.0	80.0
Scheduled reserve capacity (MW)	20.7	0.0
First-stage costs (\$)	735.5	842.9
Average second-stage costs (\$)	29.9	2.0
Average total costs (\$)	765.4	844.8

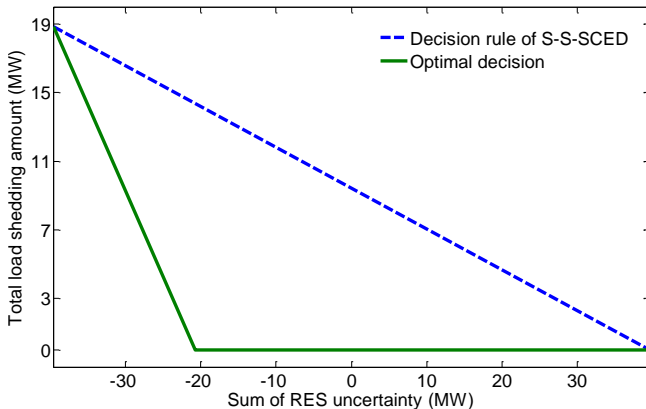


Fig. 3. Decision rule of S-S-SCED and the optimal decision on load shedding

Though the global optimal solutions of the biconvex sub-

problems in S-SCED cannot be guaranteed as explained in Section V.B, it is verified through case studies that the solution quality of S-SCED is better than that of single-stage models and is basically satisfactory.

C. Further comparison between S-SCED and O-DRO

In this section, the advantages of S-SCED over O-DRO will be shown from different aspects. Again, 3000 simulated uncertainty realizations are generated by normal distributions in each coming experiment and are used to obtain second-stage decisions from (8).

1) Effects of strict constraints on load shedding

Here, O-DRO is compared with S-SCED under different load shedding limits. In all considered circumstances, S-SCED schedules all forecasted RES power, and results of S-SCED are shown in Table IV. Reserve capacity scheduled in the first stage increases as the limit becomes stricter. As a result, load shedding is reduced in the second stage with corresponding reduced penalty. As the expense of ensuring system security, costs of the first stage are increased under stricter shedding limits because of increased reserve capacity, which also leads to higher average total costs of the two stages.

In contrast with S-SCED, O-DRO does not impose any load shedding limit. It schedules all forecasted RES power and 18.99 MW of reserve capacity. Because of the insufficient reserve capacity scheduled, load shedding limits will be violated in the second stage under O-DRO as shown in Table V. In this study, O-DRO has a probability up to 3.42% in having extra load shedding beyond the limit, which could be unacceptable. Although the load shedding penalty in O-DRO can be increased to reduce load shedding operation in the second stage, it is not clear how much the penalty should be set such that the load shedding limit will not be violated. Therefore, S-SCED is more capable to maintain system security compared with O-DRO.

TABLE IV
RESULTS OF S-SCED UNDER DIFFERENT LOAD SHEDDING LIMITS

Allowed total load shedding amounts (%)	Scheduled reserve capacity (MW)	First-stage costs (\$)	Average load shedding penalty (\$)	Average total costs (\$)
100	20.74	735.49	2.556	765.4
3	29.22	745.67	0.352	774.0
2.5	30.95	747.75	0.226	776.0
2	32.68	749.82	0.125	778.0
1.5	34.41	751.90	0.054	780.1
1	36.11	753.93	0.021	782.1
0.5	37.87	756.05	0.004	784.2

TABLE V
PROBABILITY OF VIOLATING LOAD SHEDDING LIMITS

Allowed total load shedding amounts (%)		3	2.5	2	1.5	1	0.5
Probability of violation (%)	S-SCED	0.00	0.00	0.00	0.00	0.00	0.00
	O-DRO	0.58	0.81	1.20	1.78	2.65	3.42

2) Effects of incorporating uncertainty support

RES uncertainty is always bounded with finite supports, because the power output of RES cannot be negative values nor exceed the installed capacity. As compared to O-DRO, S-SCED would incorporate uncertainty support information.

Here, both O-DRO and S-SCED are used to solve ED problems under different load shedding penalty and a summary of their dispatch plans is shown in Table VI. Under both models, scheduled reserve capacity increases as the shedding penalty grows, because in such circumstances, possibility of load shedding operation should be reduced accordingly to avoid excessive total costs. However, the reserve capacity gradually stops increasing under S-SCED when the penalty is high enough while it continues to increase under O-DRO. This is because uncertainty support information is absent in O-DRO and the solution becomes over-pessimistic in preventing cases that hardly happen. As a result, under high shedding penalty (75\$-135\$), the unnecessary reserve capacity scheduled by O-DRO makes its first-stage costs higher than those of S-SCED as shown in Table VII, which also leads to higher average total costs for O-DRO. Therefore, S-SCED can prevent over-conservative solutions compared with O-DRO by incorporating uncertainty support information.

Besides, under high shedding penalty (45\$-135\$), S-SCED schedules only part of forecasted RES power while O-DRO schedules all as shown in Table VI. This is not the major reason causing differences between the total costs of O-DRO and S-SCED. However, to avoid confusion, the reason of this phenomenon is briefly explained as follows. For S-SCED, its optimal dispatch plan of scheduling part of forecasted RES power can achieve a lower total cost compared with the plan of scheduling all forecasted RES power because of reduced reserve costs in the first stage and reduced recourse costs in the second stage. However, different from S-SCED, O-DRO has no support limitation for uncertainty distribution and thus the worst distribution it considers is more dispersed than that considered by S-SCED. As a result, the benefit of reducing recourse costs in the second stage for O-DRO by taking S-SCED's optimal plan is less than that for S-SCED and scheduling all forecasted RES power is still the optimal plan for O-DRO.

TABLE VI
SUMMARY OF DISPATCH PLANS OF S-SCED AND O-DRO UNDER DIFFERENT LOAD SHEDDING PENALTY

Load shedding penalty (\$)		15	45	75	105	135
Scheduled generator outputs (MW)	S-SCED	226.0	228.0	228.5	228.8	228.9
	O-DRO	226.0	226.0	226.0	226.0	226.0
Scheduled RES outputs (MW)	S-SCED	120.0	118.0	117.5	117.2	117.1
	O-DRO	120.0	120.0	120.0	120.0	120.0
Scheduled reserve capacity (MW)	S-SCED	20.74	35.46	36.03	36.17	36.24
	O-DRO	18.99	35.40	46.36	55.20	63.93

TABLE VII
OPERATION COSTS OF S-SCED AND O-DRO UNDER DIFFERENT LOAD SHEDDING PENALTY

Load shedding penalty (\$)		15	45	75	105	135
First-stage costs (\$)	S-SCED	735.5	759.6	762.1	763.0	763.5
	O-DRO	733.4	753.1	766.2	776.8	787.3
Average second-stage costs (\$)	S-SCED	29.34	22.58	21.29	20.88	20.70
	O-DRO	30.13	27.56	27.35	27.35	27.35
Average total costs (\$)	S-SCED	764.8	782.1	783.4	783.9	784.1
	O-DRO	763.5	780.7	793.6	804.2	814.7

3) Effects of considering deviations of uncertainty moments

Here, S-SCED is set with $r_1 = 0.2$ and $r_2 = 0$ to consider possible deviation of uncertainty mean. O-DRO and S-SCED are both used to solve ED problems and the summary of their dispatch plans is recorded in Table VIII. O-DRO schedules all forecasted RES power, 120.0MW, while S-SCED anticipates possible deviation of uncertainty mean and schedules only 116.0MW. Meanwhile, S-SCED schedules more power outputs from generators and thus has a higher first-stage cost than O-DRO.

TABLE VIII
SUMMARY OF DISPATCH PLANS OF S-SCED AND O-DRO WHEN MEAN DEVIATION CONSIDERED

Model	Scheduled generator outputs (MW)	Scheduled RES outputs (MW)	Scheduled reserve capacity (MW)	First-stage costs (\$)
S-SCED	230.0	116.0	17.46	744.6
O-DRO	226.0	120.0	18.99	733.4

An indicator, r_1^* , is used to measure the actual deviation of uncertainty mean. When $r_1^* = 0$, there is no deviation. When $r_1^* = r_1$, uncertainty mean deviates to the boundary of D_1 and has the lowest sum in D_1 . Simulated uncertainty realizations are generated by normal distributions according to the actual moments under different r_1^* . Operation costs of S-SCED and O-DRO are presented in Table IX. S-SCED has lower second-stage costs than O-DRO because it schedules less RES power and thus needs less recourse operation in the second stage. When $r_1^* = 0$, S-SCED has slightly higher average total cost than O-DRO. While in other cases recorded in Table IX, uncertainty mean deviates to be smaller than the statistical value, so there will be less available RES power on average than anticipated by O-DRO. As a result, O-DRO will have unexpected regulation costs on average in the second stage and thus has higher average total costs than S-SCED. Compared with O-DRO, cost savings of S-SCED, when uncertainty mean deviates to be smaller than the statistical value, are generally higher than its extra cost under no mean deviation. Besides, although the advantages of S-SCED over O-DRO shown here are not that significant, they will be amplified when RES penetration level increases or costs of recourse actions become more expansive. Therefore, it is more reasonable to adopt S-SCED rather than O-DRO. Besides, r_1 , r_2 of S-SCED can be adjusted based on historical uncertainty samples.

TABLE IX
OPERATION COSTS OF S-SCED AND O-DRO UNDER DIFFERENT MEAN DEVIATIONS

r_1^*		0	0.02	0.04	0.12	0.20
Deviation of average available RES power (MW)		0.00	-1.70	-2.40	-4.16	-5.37
Average second-stage costs (\$)	S-SCED	19.75	23.50	25.97	30.70	35.77
	O-DRO	30.06	36.05	39.21	44.99	51.52
Average total costs (\$)	S-SCED	764.3	768.1	770.5	775.3	780.3
	O-DRO	763.5	769.4	772.6	778.4	784.9

D. Incorporation of load uncertainty

As mentioned in Section IV.C, load uncertainty can also be incorporated into the model exactly like RES uncertainty. Here, the effects of incorporating load uncertainty are

discussed. Loads at Bus 2, 5, 7, 8, 21 and 30 contribute to 70% of the total load and they are assumed to be with uncertainty in this study. Results of comparison between models with and without load uncertainty are presented in Table X. Load uncertainty makes the ED problem more unpredictable, thus more reserve capacity is scheduled, and the operation costs increase as well.

TABLE X
EFFECTS OF INCORPORATING LOAD UNCERTAINTY

With load uncertainty	Scheduled reserve capacity (MW)	First-stage costs (\$)	Average second-stage costs (\$)	Average total costs (\$)
No	20.74	735.5	30.32	765.8
Yes	25.35	741.0	41.46	782.5

Correlation between uncertainty can be considered by the model through the covariance matrix. Effects of load uncertainty's correlation are also studied here. Load at Bus 5 is the largest and makes up of 31.5% of the total load. To simulate circumstances of different uncertainty correlation, a covariance matrix Δ is generated by assuming that uncertainties of loads at Bus 2, 7, 8, 21 and 30 are of 0.2, 0.2, 0.25, 0.15, and 0.2 times of the uncertainty of the load at Bus 5 respectively, and then Δ_α with different α are generated by multiplying the covariance terms in Δ between uncertainty of the load at Bus 5 and other load uncertainties by α . Δ_α is adopted as the covariance matrix of load uncertainty in the ED problem. Under positive and negative α , uncertainty of load at Bus 5 has positive and negative correlation respectively with other load uncertainties. The correlation is stronger with larger absolute value of α . Results of ED problems under Δ_α with different α are shown in Fig.4. Under stronger negative correlation, load uncertainties tend to offset each other better, thus less reserve capacity is scheduled and the operation cost decreases.

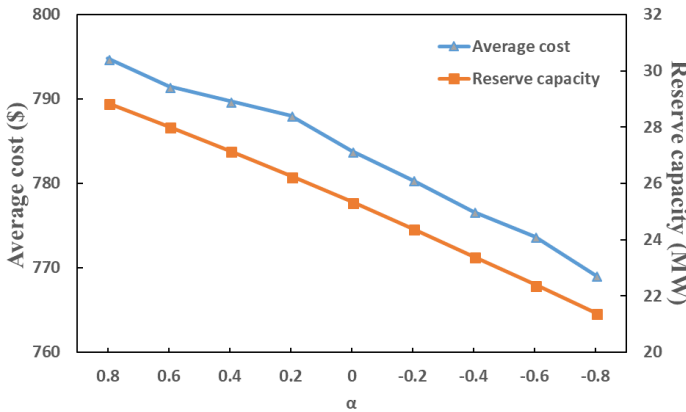


Fig. 4. Effects of load uncertainty's correlation

E. Necessity of multi-period modeling

Here, M-SCED is compared with O-DRO to illustrate the necessity of multi-period modeling for ED problems. M-SCED can incorporate any reasonable number of time periods. Here, for simplicity, T is set to three. Duration of each period is set to one hour. Generator 5 is set to be more expensive than Generator 1-4, and Generator 6 is set to be more expensive

than Generator 5. Cost difference between Generator 6 and Generator 5 is more significant than that between Generator 5 and Generator 1-4. Up and down ramping limits of all generators, Ra_i^+ and Ra_i^- , are set to two times of their up and down reserve capacity limits, respectively. Power forecasts of all RES in all periods are set as 30 MW. Total loads in the system are set as 318.4, 380.4, 380.4 MW from Period 1 to 3. In rolling-plan operation, with ramping constraints coupled with the previous period, M-SCED is updated in each period and O-DRO is adopted repeatedly for each period respectively to obtain ED operation plans. 1000 simulated uncertainty realizations are generated by normal distributions to evaluate the performance of M-SCED and O-DRO in multi-period ED operation.

Because Generator 6 is the most expensive, power generation should be preferentially scheduled to other generators if possible. As shown in Fig. 5, without anticipating future circumstances, O-DRO schedules Generator 5 and 6 at their minimum generation in Period 1. Then in Period 2 and 3, to meet the increased load, Generator 1-4 increase their power outputs to their maximum capacities, Generator 5 increases its power output and reserve capacity by reaching its up ramping limit, and the most expensive Generator 6 needs to increase its power output as well. In contrast, under M-SCED, because the load increase is anticipated, Generator 5 is scheduled with higher generation in Period 1 and thus it can provide enough power in Period 2 and 3, avoiding power output increase from expensive Generator 6.

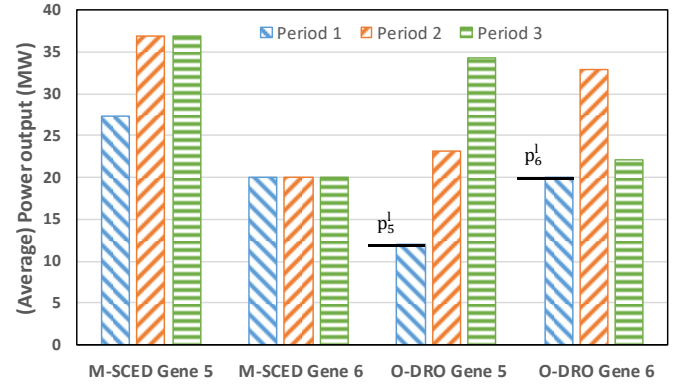


Fig. 5. Power outputs of Generator 5 and 6 under different models

As shown in Table XI, M-SCED has higher cost in Period 1 but lower costs in following periods than O-DRO, matching the results in Fig. 5. The total cost of M-SCED is lower as well. This is because M-SCED has an overall consideration over the entire operation horizon and sacrifices short-term benefit to pursue long-term profit. In contrast, O-DRO is shortsighted in considering only the current period. In more severe situations such as when the total load in Period 2 is 410 MW, O-DRO will even meet infeasible operation situations, leading to more serious consequences, while M-SCED will not. Besides, as stated earlier, O-DRO cannot be directly extended to be multi-period because of the complicated consistency requirement of time sequences. While according to Table XI, M-SCED achieves effective multi-period modeling without causing excessive computational burden.

TABLE XI
COMPUTATION TIMES OF M-SCED AND O-DRO AND THEIR OPERATION COSTS
OF MULTI-PERIOD ED

	Computation times (s)	Average costs in Period 1 (\$)	Average costs in Period 2 (\$)	Average costs in Period 3 (\$)	Average total costs (\$)
M-SCED	36.71	977.8	1204.9	1203.1	3385.8
O-DRO	18.72	967.9	1242.3	1208.7	3418.9

VII. CONCLUSIONS

In this paper, a multi-period economic dispatch model M-SCED is established to accommodate uncertainty from renewable energy, in which a two-stage formulation is adopted to effectively model recourse actions with respect to uncertainty realizations. Compared with traditional robust optimization based economic dispatch, M-SCED greatly reduces average operation costs through application of distributionally robust optimization. Compared with the previous work based on distributionally robust optimization, M-SCED avoids over-conservative solutions and prevents inferior performance under inaccurate uncertainty information through more realistic modeling of uncertainty as well as enhances system security by limiting load shedding in recourse actions. In addition, the efficient multi-period modeling guarantees economical and secure long-term economic dispatch operation without causing excessive computational burden.

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