The following publication H. Huang and A. V. Savkin, "Deployment of Charging Stations for Drone Delivery Assisted by Public Transportation Vehicles," in IEEE Transactions on Intelligent Transportation Systems, vol. 23, no. 9, pp. 15043-15054, Sept. 2022 is available at https://dx.doi.org/10.1109/TITS.2021.3136218.

Deployment of Charging Stations for Drone Delivery Assisted by Public Transportation Vehicles

Hailong Huang and Andrey V. Savkin

***Abstract*—To enable the drone delivery service in a remote area, this paper considers the approach of deploying charging stations and collaborating with public transportation vehicles. From the warehouse which is far from a customer, a drone takes some public transportation vehicles to reach some position close to the remote area. When the customer is unreachable from the position where the drone leaves the public transportation vehicle, the drone swaps the battery at a charging station. The focus of this paper is the deployment of charging stations. We propose a new model to characterize the delivery time for customers. We formulate the optimal deployment problem to minimize the average delivery time for the customers, which is a reflection of customer satisfaction. We then propose a sub-optimal algorithm that relocates the charging stations in sequence, which ensures that any movement of a charging station leads to a decrease in the average flight distance. The comparison with a baseline method confirms that the proposed model can more accurately estimate the flight distance of a customer than the commonly used model, and the proposed algorithm can relocate the charging stations achieving lower flight distance.**

***Index Terms*—Drones, unmanned aerial vehicle (UAV), parcel delivery, charging stations, public transportation vehicles.**

I. Introduction

MANY people may have seen the "Sorry, we missed you" cards in their letterboxes when they expect to receive their parcels. In many metropolises, such a card may be replaced by a text message like "Your parcel has been stored in a temporarily locked box, and you can collect it within 48 hours with a password". The main reason behind this is the fast growth of online shopping, resulting in the heavy working load on postmen. To "deliver" more parcels, many postmen choose the time-saving method by leaving a card or sending a text message.

To save the labour force, many logistics companies, such as Amazon [1], SF Express [2] and UPS [3], have started to develop unmanned aerial vehicles (UAVs), also known as drones, to conduct the last-mile delivery in recent years. There are two main types of drones: fixed-wing drones and multi-rotor drones. The former requires a launcher to get the drones into the air, and it is difficult for them to land. In contrast, multi-rotor drones do not require any facility for launching and landing, and they are preferred by most logistics companies. A bottleneck of this type of drones is the limited flying time. Most commercial drones can only fly for about half an hour. Consequently, they can only serve customers close to the warehouse.

In the literature, there are two main approaches to address the limited flight distance problem. The first one exploits ground vehicles to collaborate with drones [4]–[7]. Specifically, a ground vehicle such as a truck or van departs from the warehouse and takes some parcels and some drones. At some position, a drone can leave the vehicle and fly to a customer. After dropping off the parcel, the drone flies to a rendezvous position and docks with the vehicle. During this course, the vehicle can move to other customers and then go to the rendezvous position to meet the drone. Under such a design, the paper [4] targets scheduling the vehicle and the drone to minimize the latest return time of the vehicle and the drone to the warehouse. The reference [5] investigates a similar system. Differently, all the customers are served by the drone, and the vehicle only charges the drone. An optimal path planning problem on a road graph is formulated, and the goal is to find the shortest cooperative route to deliver parcels to all the target locations. The paper [6] considers minimizing the operational costs including the transportation cost and the one created by the waste time when one vehicle has to wait for the other. Some other variants have been discussed in [7], [8]. Compared with the traditional vehicle only method, the drone-vehicle collaboration approach has an advantage in reducing energy consumption and operation time, thanks to the participation of drones. A shortcoming is that it still requires a driver to participate in the delivery.

Another type of drone-vehicle collaboration approaches aims at removing human involvement. While the references [9], [10] consider the collaboration between drones and mobile robots, the papers [11], [12] have proposed the idea of using public transportation vehicles to transport drones. The paper [11] considers the path planning problem for a drone from the warehouse to a customer in the time-varying and stochastic public transportation network. A reliable path planning algorithm taking into account the worst-case uncertainty is proposed. The paper [12] further considers the round-trip planning problem. Though exploiting mobile robots is more flexible, the robots may meet various challenges especially in urban areas. Making use of public transportation vehicles avoids this issue because these vehicles are existing platforms. However, the uncertainty and the time variance need to be well considered in the planning of the drones' paths.

The second approach addressing the limited flight distance issue makes use of stationary facilities such as charging

This work was supported by the Australian Research Council and PolyU start-up fund (Project ID: P0038447).

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stations, similar to the charging stations for electric vehicles [13]. At a charging station, a drone can either recharge or replace the battery. Then, the drone can fly further to serve more customers. The paper [14] considers the scenario where customers locate far from the warehouse. So, it considers deploying a drone station near customers, which is assumed to furnish a sufficiently large number of drones. A vehicle leaves the warehouse with parcels and activates the drone station when it arrives at the drone station. Then, the drones deliver the parcels to customers. The paper [15] aims at selecting a number of charging stations from a given candidate set to maximize the coverage of customers in a certain area, while the goal of [16] is to minimize the total system cost including the charging stations, drone ownership, service congestion, etc. Different from [15], [16], the paper [17] considers the deployment of charging stations in a continuous space.

In this paper, we consider a scenario where a supplier wishes to extend its delivery service to a remote area. Such an area is far from the supplier's warehouse so that a drone cannot reach any customers in the area directly. Our strategy leverages the aforementioned two approaches. In particular, we assume that there are some public transportation vehicles that pass the warehouse and the area of interest. Then, a drone can take these vehicles to reach the area [11]. For a large area that cannot be covered by a drone without swapping its battery, we consider deploying some charging stations in the area. With the assistance of public transportation vehicles and charging stations, this strategy enables a drone to serve a remote customer.

We focus on the charging station deployment problem. Though it has been studied in existing publications, where the objectives of maximizing coverage [15] and minimizing system cost [16] have been considered, we investigate it from the point of the satisfaction of customers. Generally, a shorter delivery time means a higher satisfaction level. So, we consider minimizing the average delivery time, which can be approximated by flight distance. We propose a new model to characterize the flight distance for drones to serve customers, which is called the service model. Different from the commonly used coverage model which says a customer is covered by a charging station within a certain range [15], [17], the service model specifies the charging station via which a drone directly serves a customer in the shortest time. The main advantage of this model is that it can provide an accurate computation of the flight distance. With this model, we formulate the charging station deployment problem from a simple case with only one charging station to a complex case with multiple charging stations. While the former is easy to solve, the latter is challenging as the positions of charging stations are coupled in the objective function. Considering that this problem is NP-hard, to solve it within a reasonable time, we propose a sub-optimal solution. From the initial positions which are connected and cover all the customers, we construct the minimum spanning tree (MST). Then, we move the charging stations one by one in sequence to reduce the average flight distance of the impacted customers, while maintaining the topology of the MST and avoiding losing any customers.

The main contributions of the paper are as follows:

- We propose a service model to accurately compute the delivery time (approximated by flight distance) of customers.
- We formulate an optimization problem to minimize the average flight distance (which is a reflection of the satisfactory level of customers).
- We propose a sub-optimal algorithm to deploy the charging stations which ensures that any movement of a charging station leads to a decrease in the average flight distance (an improvement of the satisfactory level).

The rest of the paper is organized as follows. In Section II, we discuss other related work in a broad view. Section III provides an overview of the considered scenario. In Sections IV and V, we present the proposed service model and formulate the considered problems from simple to complex. Section VI shows the performance of the proposed method against a baseline method. Finally, Section VII concludes the paper with some future research directions.

II. Related Work

In this section, we briefly discuss the relevant work.

Under the battery capacity constraint, energy consumption models have been studied for drone delivery [18]. In [19], the authors present a battery-aware scheduling algorithm to accomplish more deliveries with a given battery capacity by taking into account the battery power transfers. The paper [20] develops the vehicle routing problem for drone delivery, where drones can make multiple returns to the warehouse to pick up parcels and swap their battery. This approach enables a drone to deliver multiple parcels in a single trip, while most logistic companies only deliver one parcel in a single travel. The reference [21] considers only delivering one parcel in a time-dependent graph. The edge cost in the graph represents the energy consumption, and the time variance can describe the wind effect. The authors present algorithms to construct offline and online routes and evaluate their feasibility. One common feature of these publications is that the drone can only serve a customer close to the warehouse.

The charging station deployment problem is the hub location problem or facility deployment problem [22]–[24], and the basic goal is to enable a long-term and large-scale operation. For example, in [23], the authors investigates the problem of optimally locating an automotive service centre to minimize the transportation cost of customers subject to stochastic customer demands, varying setup cost and regional constraints. The more general hub location problem concerns with locating hubs and allocating demands to hubs to route the traffic between origin–destination pairs [24]. Such a problem has rich applications including but not limited to postal operations, express shipment and cargo delivery, public transit, and computer and telecommunication networks. Regarding the facility deployment problem for electric vehicles, the commonly considered goal is to maximize the coverage of electric vehicle flows by deploying a number of charging stations on road segments. Various factors need to be taken into account including but not limited to budget constraints, charging time, and traffic flow [25], [26]. In addition to these general factors, the authors of [27] simultaneously consider the requirements of transportation infrastructures and electricity because charging EVs is a type of unconventional electric load, and they propose a graph-computing based integrated location planning model to maximize the charging convenience while ensuring the reliability of power grids. Following the observation of congestion at charging stations for electric vehicles, the reference [28] also investigates this issue in the charging station deployment for drone delivery. Different from the aforementioned context, the paper [29] considers positioning terror response facilities under the risk of disruption. The authors present a leader–follower game, which is translated into a minmaxmin problem, and propose a population-based heuristic algorithm to solve it.

The main difference between the deployment of charging stations for electric vehicles and that for drone delivery is the connectivity requirement. In general, a fully charged electric vehicle can travel for more than 100 km, which is much longer than a drone. The connectivity needs to be well considered in drone charging station deployment, but it is not necessary for electric vehicles. While the hub location problem cares more about routing the traffic between origin–destination pairs, our problem pays particular attention to the routing problem between one origin (depot) and multiple destinations (customers). In our problem, locating the charging stations needs to ensure that a drone is able to reach a customer and also safely return to the depot. However, such a constraint is not a necessity in the hub location problem. Moreover, in the hub location problem, a demand is allocated to a certain hub for the optimization of a particular metric. In other words, the demand is served by such a hub. However, in our problem, a customer can be connected with multiple charging stations. Since we aim at maximizing the customer satisfaction level, the customer will be served from a charging station that results in the shortest delivery time, and other charging stations are used for a safe return of the drone after the delivery.

The drone charging station deployment problem also shares similarities with the conventional set cover problem. Given a set of elements and a family of subsets of the elements, this problem aims at finding a minimum number of subsets to fully cover the elements [30]. Under the context of set cover, in papers [15], [17], a subset refers to the customers within a circle of a certain radius. Similar to the above group, connectivity is not explicitly required in the set cover problem.

Another group of related work studies the routing problem in wireless sensor networks. A hierarchical wireless sensor network consists of a sink, a number of cluster heads and a number of cluster members. The sensory data is propagated as follows. Whenever a sensor measures an event of interest, it sends the data to its cluster head. If the cluster head is close to the sink, the data is directly reported to the sink. Otherwise, the cluster head needs to send the data to other cluster heads with lower hop number for relay, until reaching the sink [31]. If the sink needs to send some command, it can do this in the reverse way, which is similar to the scenario where a drone travels from a charging station to a customer via other charging stations as relays. The main difference between routing in wireless sensor networks and flying among charging stations is that the wireless transmission distance can be easily increased by increasing the transmission power, while the flight distance of a drone is often strictly constrained.

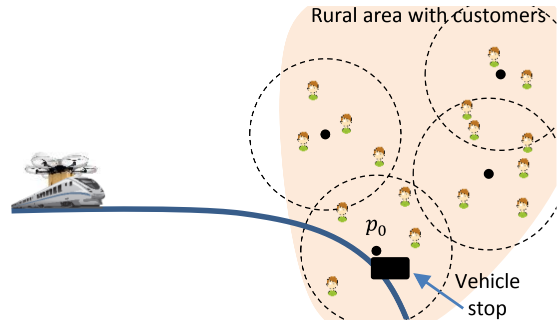


Fig. 1: Illustration of the considered scenario.

III. Scenario Description

In this section, we provide an overview of the considered scenario. We consider that a supplier wishes to provide drone delivery service to customers in a rural area that is far from the supplier's warehouse so that a drone cannot reach a customer in the area by direct flying. Following [11], a drone can take a public transportation vehicle, such as a train and a bus, to reach a vehicle stop that is close to the area of interest, see Fig. 1. However, when the area of interest is large, and many customers locate relatively far from the vehicle stop, due to the limited battery capacity, a drone may not be able to serve all the customers in the area.

We consider deploying a number of charging stations to extend the coverage area for drones. We assume that each charging station is equipped with a large number of batteries. Whenever a drone arrives, it replaces the used battery with a fresh one. This helps reduce the delivery time, which further helps increase the customer satisfaction level. The used battery can be recharged later. Because the time needed to replace a battery is relatively shorter than the UAV flight time, we ignore such a time in this paper. The first charging station is deployed near the vehicle stop, i.e., at p_0, see Fig. 1. With this charging station, we further deploy other charging stations so that all the customers in the area can be served by drones.

Our main objective is to serve the customers in the fastest way on average. The operation of a drone to serve a certain customer is as follows. Starting from the warehouse, it takes proper public transportation vehicles to reach the desired vehicle stop. It then travels to a customer by hopping from one charging station to another. After completing the task, it returns to p_0 and waits for a proper vehicle. Finally, it takes this vehicle to return to the warehouse. So, to serve one customer in the area of interest, a drone takes one or more public transportation vehicles to travel between the warehouse and p_0, and it travels between p_0 and the customer by hopping among the deployed charging stations. In this paper, we focus on the optimal deployment of charging stations. Regarding travelling with public transportation vehicles, readers are referred to [11]. In the next two sections, we mathematically formulate

TABLE I: Symbols and meanings

Symbol	Meaning
$p_j \in \mathbb{R}^2$	Location of node j
$O_j \subset \mathbb{R}^2$	Circle around node j
$C \subset \mathbb{R}^2$	The set of customers
$C_j \subset \mathbb{R}^2$	Subset of customers covered by node j
$R \in \mathbb{R}^2$	Coverage radius
$A_{ij} \subset \mathbb{R}^2$	Subset of customers covered by node j but served by node i
$B_{ij} \subset \mathbb{R}^2$	Subset of customers covered and served by node j but directly impacted by node i
$\mathcal{P}_j \subset \mathbb{R}^2$	The shortest path from node 0 to node j
$L(p_i, p_j) \in \mathbb{R}$	Length of the shortest path from node i to node j
$\phi(j) \in \mathbb{N}$	The parent of node j
$\psi(j) \subset \mathbb{N}$	The set of child nodes of node j
$F(p_{\phi(j)}, p_j) \in \mathbb{R}$	Average distance from $p_{\phi(j)}$ to the customers in C_j

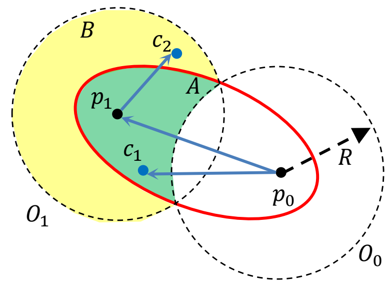


Fig. 2: Illustration of the proposed service model with two charging stations at p_0 and p_1. Without the charging station at p_1, the one at p_0 only serves the customers in an area inside the circle O_0. With p_1, the charging station at p_0 further serves the green region. Charging station p_1 serves the yellow region.

the problem from simple to complex and also present our solutions.

IV. Deployment of One Charging Station

Suppose that a fully charged drone can fly for a distance of $2R$. Then, for drone parcel delivery, a fixed charging station at the position $p_0 \in \mathbb{R}^2$ can serve the customers within a circular area centred at p_0 of radius $R \in \mathbb{R}$, see Fig. 2. It is worth pointing out that such a radius R depends on not only the onboard battery capacity but also the designed parcel weight. This paper focuses on the high level development problem. Readers that are interested in the low level control aspects are referred to some recent publications on aerial vehicles with uncertain payloads [32], [33]. We consider extending the coverage area by deploying a charging station $p_1 \in \mathbb{R}^2$. This charging station and the one at p_0 should be connected so that a drone starting from p_0 can reach the new charging station. At the new charging station, a drone can replace its battery with a fresh one. Let $C \subset \mathbb{R}^2$ denote a set of customers to be served. Let $O_0 \subset \mathbb{R}^2$ and $O_1 \subset \mathbb{R}^2$ be the circles centred at p_0 and p_1 of the radius R, respectively, see Fig. 2. We assume that there exists a position p_1 such that the customers in C can be enclosed by the circle O_1. The connectivity constraint requires that the distance between p_0 and p_1 is no greater than $2R$, i.e., $|p_0, p_1| \leq 2R$, where $|\cdot, \cdot|$ gives the standard Euclidean distance between two points. The mainly used symbols are listed in TABLE I.

A. Service model

We present a new model to characterize the service of customers and the delivery time, i.e., the service model:

Definition IV.1. *A customer is served by the charging station that can directly reach the customer and lead to the shortest delivery time. Here, the delivery time refers to the time duration between the instant when the drone leaves p_0 and the instant when the drone reaches the customer. Assuming that a drone flies at a constant speed and omitting other times such as replacing the battery, the delivery time is proportional to the flight distance.*

When $|p_0, p_1| < 2R$, we can construct an ellipse with p_0 and p_1 as the foci and passing the intersection points of O_0 and O_1, see the red ellipse in Fig. 2. An important property of such an ellipse is that for any point q on or inside the ellipse we have $|p_0, q| + |q, p_1| \leq 2R$. When $|p_0, p_1| = 2R$, such an ellipse reduces to a line segment between p_0 and p_1. Let $A \subset \mathbb{R}^2$ denote the subset of customers that are inside the ellipse but outside the circle O_0: $A = \{c| \ |p_1, c| + |p_0, c| \leq 2R \ \& \ |p_0, c| > R, c \in C\}$. In Fig. 2, the customers in A are in the green region. Let $B \subset \mathbb{R}^2$ denote the subset of customers that are inside the circle O_1 but outside the ellipse: $B = \{c| \ |p_1, c| \leq R \ \& \ |p_1, c| + |p_0, c| > 2R, c \in C\}$. In Fig. 2, the customers in B are in the yellow region. Moreover, we have $C = A \cup B$, and $A \cap B = \emptyset$. Under the proposed service model, the customers in the green region in Fig. 2 can be served with shorter flight distances (shorter time). Such a model is essentially based on a simple new concept, and to the best of our knowledge, we have not seen it in any existing publications. We formally state this benefit in Proposition IV.1.

Proposition IV.1. *For the given set of customers C and locations of p_0 and p_1, grouping the customers into the sets A and B and letting charging station p_0 serve the customers in A and charging station p_1 serve the customers in B ensure that every customer can be served and the flight distance for any customer is the lowest.*

Proof. If Proposition IV.1 is untrue, there must exist another partition so that all the customers can be served, the flight distance of any customer is no longer than the considered partition, and there must exist at least one customer such that the flight distance is lower than the considered partition. Consider there exists one customer c that falls into the set A but is grouped into the set B. Since the customers in set B are served by charging station p_1, the flight distance for c is $|p_0, c|+|c, p_1|$. If it is served by p_0, the flight distance is $|p_0, c|$. Clearly, $|p_0, c| + |c, p_1| \geq |p_0, c|$, and the equality holds only if p_1 is on the line segment connecting p_0 and c. So, grouping a customer falling into the set A to the set B does not lead to a lower flight distance for any customer. Moreover, suppose a customer falling into the set B is grouped into the set A. Since any customer c in B satisfies $|p_0, c| + |c, p_1| > 2R$, this customer cannot be served by p_0. Thus, grouping a customer falling into the set B to the set A results in that this customer cannot be served. Therefore, grouping the customers into the sets A and B and letting charging station p_0 serve the customers in A and charging station p_1 serve the customers in B ensure that every customer can be served and the flight distance for any customer is the lowest. □

B. Deploying a charging station in a continuous region

An interesting problem under this model is the optimal deployment of p_1. Formally, given p_0, R, and the positions of a set of customers C which are outside O_0, we look for the position p_1, such that p_1 is connected with p_0, all the customers are covered, and the average flight distance is minimized:

$$\min_{p_1} \frac{1}{|C|}\left(\sum_{c\in A}|p_0,c| + \sum_{c\in B}(|p_0,p_1|+|p_1,c|)\right) \tag{1}$$

s.t.

$$|p_0,p_1| \leq 2R, \tag{2}$$

$$|c,p_1| \leq R, \forall c \in C, \tag{3}$$

$$A = \{c|\ |p_1,c| + |p_0,c| \leq 2R \ \&\ |p_0,c| > R, c \in C\}, \tag{4}$$

$$B = \{c|\ |p_1,c| \leq R \ \&\ |p_1,c| + |p_0,c| > 2R, c \in C\}. \tag{5}$$

The objective function (1) minimizes the average flight distance. Constraint (2) requires that p_1 is connected with p_0, and constraint (3) requires that all the customers in C are within the distance R from p_1. Eqs. (4) and (5) define the sets A and B, which are both the functions of p_1. Note that the considered model can be extended to the case where customers are with weightings representing order frequency and the like. Some constants representing the battery swapping time can be added to the objective function. Also, the constant R can be adjusted to consider the energy consumption for take-off, landing, and other factors.

The problem (1) subject to (2), (3), (4) and (5) is non-convex. The variable p_1 determines the subsets A and B, and p_1, A and B all appear in the objective function (1). Additionally, p_1 is defined in a continuous space. A slight change of p_1 results in totally different A and B, which means that the objection function (1) is not continuous with respect to p_1. We can imagine that there is a customer c at the intersection of the circles O_0 and O_1. According to Definition IV.1, this customer belongs to the subset A, and the corresponding objective function value for this customer is given by $|p_0,c| = R$. However, a slight movement of p_1 may result in that this customer falls into the subset B. Then, the objective function value for c becomes $|p_0,p_1| + |p_1,c|$. Obviously, this is larger than $|p_0,c|$ [1], which means a jump in the objective function value due to a slight movement of p_1.

[1]Note that $|p_0,p_1|+|p_1,c| = |p_0,c|$ holds if and only if p_1 lies on the line segment connecting p_0 and c. For the considered c, $|p_0,p_1|+|p_1,c| > |p_0,c|$.

C. Deploying a charging station in a discrete set

In practice, a charging station cannot be deployed anywhere [15], [16]. So, we introduce a discrete set of candidate sites for deployment, denoted by $S = \{s_1,\ldots,s_m\}$, where $s_k \in \mathbb{R}^2$ is the location of site k and $m \in \mathbb{N}$ is the number of candidate sites. We also introduce a binary variable for each candidate site, i.e., x_k: $x_k = 1$ if the charging station is deployed at site k; $x_k = 0$ otherwise. Moreover, since the locations of the candidate sites and the customers are all known, we can pre-compute the subsets A and B for each candidate site. Similar to the above definition, for site k, let A_k denote the subset of customers falling into the ellipse but outside the circle O_0, and let B_k denote the subset of customers falling into the circle O_1 but outside the ellipse. Then, the aforementioned problem can be reformulated as follows:

$$\min_{x_1,\ldots,x_m} \sum_{k=1}^{m} \frac{x_k}{|C|}\left(\sum_{c\in A_k}|p_0,c| + \sum_{c\in B_k}(|p_0,p_1|+|p_1,c|)\right) \tag{6}$$

s.t.

$$x_k|p_0,s_k| \leq 2R, \exists k \in [1,m], \tag{7}$$

$$x_k|c,s_k| \leq R, \forall c \in C, \exists k \in [1,m], \tag{8}$$

$$\sum_{k=1}^{m} x_k = 1. \tag{9}$$

$$x_k \in \{0,1\}, \forall k \in [1,m]. \tag{10}$$

For any site k, the subsets A_k and B_k are fixed. Thus, the objective function (6) is a linear function of $x_1,\ldots,x_m$. Constraint (7) says that the charging station should be within the range of $2R$ from p_0. Constraint (8) requires that all the customers are covered by the deployed charging station. Constraint (9) specifies that there is only one charging station, and constraint (10) defines the binary variable of x_k. Clearly, the problem (6) subject to (7), (8), (9) and (10) is convex, which is easier to solve than the problem (1) subject to (2), (3), (4) and (5). In the case where the candidate set is not pre-defined, we can grid the field and take the grid points as the candidates. However, the converting the problem may take much pre-computation fro determining the sets A_k and B_k, which is only suitable to small-scale cases.

V. Deployment of Multiple Charging Stations

In this section, we discuss the complex scenario with multiple charging stations. We start from the model of coverage, formulate the problem of interest and then propose our solution.

A. Coverage model

To deploy multiple charging stations, an important tool is the grouping of customers. We introduce the concept of coverage in Definition V.1.

Definition V.1. *A customer is covered by the charging station that serves it if their distance is no larger than R; otherwise, this customer is covered by the closest charging station if their distance is no larger than R.*

Different from the concept of service, which pays attention to the calculation of flight distance, the concept of coverage defines how to group customers. In Fig. 2, the customer c_1 in A is served by p_0 according to Definition IV.1. Then, the flight distance is given by $|p_0, c_1|$. However, it is covered by p_1 according to Definition V.1, since the distance between c_1 and p_0 is larger than R. It is easy to understand that if we use the concept of coverage to compute the flight distance, we may obtain inaccurate results, as under this concept both c_1 and c_2 are covered by p_1. It is worth pointing out that Definition V.1 groups a customer to the charging station that serves it if the distance requirement holds. This differs from other existing coverage models. For example, in [17], a customer is grouped to the closest charging station. Consider a customer that is in the overlapping area of O_0 and O_1 and more closer to p_1 in Fig. 2. According to Definition V.1, this customer is covered by p_0. However, following the model in [17], it is covered by p_1. Since the concept of service generally leads to a shorter flight distance, determining the coverage of a customer by accounting for which charging station serving it can also result in a smaller flight distance.

B. Formulation of deploying multiple charging stations

Now, we consider a set of nodes labelled by $0, 1, \ldots, n$, where $n \in \mathbb{N}$ is the number of charging stations. Here, node 0 represents the fixed charging station near the vehicle stop, and nodes $1, \ldots, n$ represent charging stations to be deployed. Their locations are $p_0, p_1, \ldots, p_n$. Given $p_0, p_1, \ldots, p_n$, we construct the minimum spanning tree (MST) rooted at p_0. The basic requirement is that every charging station is linked with p_0. Then, any valid edge in the MST must be no longer than $2R$. For any node j, let $\mathcal{P}_j \subset \mathbb{R}^2$, a series of nodes, denote the shortest path from node 0 to node j. Let $\mathcal{P}_j[k]$ give the location of the kth node on the path $\mathcal{P}_j$, where $k = 1, \ldots, |\mathcal{P}_j|$. In particular, $\mathcal{P}_j[1] = p_0$, and $\mathcal{P}_j[|\mathcal{P}_j|] = p_j$. Then, the requirement of connectivity can be explicitly described by $|\mathcal{P}_j[k], \mathcal{P}_j[k+1]| \leq 2R$, $\forall k = 1, \ldots, |\mathcal{P}_j| - 1$. Moreover, let $\phi(j) \in \mathbb{N}$ denote the parent node of j and $\psi(j) \subset \mathbb{N}$ denote the set of child nodes of j in the MST. In the MST, a node (except the root) has a unique parent, but it may have more than one child node.

Similar to the single charging station case, we can construct an ellipse for a child-parent pair j and $\phi(j)$. Also, let O_j and $O_{\phi(j)}$ denote the circles centred at p_j and $p_{\phi(j)}$ of radius R, respectively. For node j, let C_j denote the subset of customers that are covered by node j (see Definition V.1). Let $A_{\phi(j)j}$ denote the subset of customers in C_j that are inside the ellipse but outside the circle $O_{\phi(j)}$, and let $B_{\phi(j)j}$ denote the subset of customers in C_j that are inside the circle O_j but outside the ellipse. For the customers in C_j, the flight distance can be broken down into the distance from p_0 to the parent of p_j, i.e., $p_{\phi(j)}$, and from $p_{\phi(j)}$ to the customers. Let $L(p_0, p_{\phi(j)})$ denote the length of the shortest path from p_0 to $p_{\phi(j)}$. Let $F(p_{\phi(j)}, p_j)$ denote the average distance from $p_{\phi(j)}$ to the customers in C_j. Similar to (1), $F(p_{\phi(j)}, p_j) = \frac{1}{|C_j|}\left(\sum_{c \in A_{\phi(j)j}} |p_{\phi(j)}, c| + \sum_{c \in B_{\phi(j)j}} (|p_{\phi(j)}, p_j| + |p_j, c|)\right)$. An illustration is shown in Fig. 3.

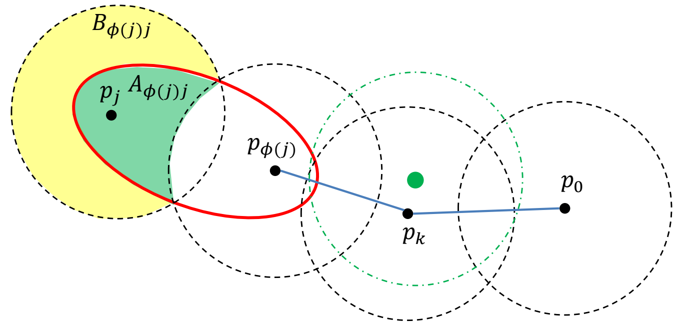


Fig. 3: The average flight distance for customers in C_j composes of the flight distance from p_0 to $p_{\phi(j)}$ and the average distance from $p_{\phi(j)}$ to these customers. The second part is the same as the model in Fig. 2.

Then, the objective function of minimizing the average flight distance of a set of customers C is formulated as follows:

$$\min_{p_1, \ldots, p_n} \frac{1}{|C|} \sum_{j=1}^{n} |C_j| \left(L(p_0, p_{\phi(j)}) + F(p_{\phi(j)}, p_j) \right). \quad (11)$$

Similar to the objective function of deploying a single charging station, i.e., (1), the term of $F(p_{\phi(j)}, p_j)$ couples the positions of the charging stations and the sets determined by the positions, i.e., $A_{\phi(j),j}$ and $B_{\phi(j),j}$. This problem is much more difficult than the single charging station case. It is also non-convex and NP-hard.

Now, we discretize the problem of deploying multiple charging stations. Let S denote the set of candidate sites to deploy charging stations. Suppose that the set S includes p_0. Given the locations of the set of customers C and the set of candidate sites S, we find the subset of customers covered by each candidate site, i.e., C_j. We can also construct the sets A_{ij} and B_{ij} for each pair of candidate sites i and j. Furthermore, let L_{ij} denote the distance between candidate sites i and j, and F_{ij} denote the average distance from candidate site i to the customers covered by candidate site j. Given the locations of the customers and the candidate sites, L_{ij} and F_{ij} can be pre-computed for each pair of charging stations i and j, and the complexity is $O(|C||S|^2)$.

Let x_i be a binary variable indicating if a charging station is deployed at candidate site i ($x_i = 1$, if yes; $x_i = 0$, otherwise). Let y_{ij} be another binary variable. If a charging station deployed at site j is connected with p_0 via the charging station deployed at site i, $y_{ij} = 1$; otherwise, $y_{ij} = 0$. For example, in Fig. 3, $y_{k\phi(j)} = 1$ because the charging station at $p_{\phi(j)}$ is connected with p_0 via the charging station at p_k, while $y_{\phi(j)k} = 0$. Moreover, let l_j be a non-negative real-value variable representing the path length from p_0 to site j.

With these variables, the problem of deploying multiple charging stations can be formulated in a discrete form:

$$\min_{x_i, y_{ij}, l_i} \frac{1}{|C|} \left(\sum_i |C_i| x_i l_i + \sum_i |C_i| \sum_j F_{ij} y_{ij} \right) \quad (12)$$

s.t.

$$\sum_i x_i = n, \quad (13)$$

$$\sum_{i \neq j} y_{ij} = n, \tag{14}$$

$$y_{ij} L_{ij} \leq 2R, \forall i \neq j, \tag{15}$$

$$\sum_{i \in S', j \in S', i \neq j} y_{ij} \leq |S'| - 1, \forall S' \subset S, \tag{16}$$

$$L_{0i} y_{0i} \leq l_i \leq l_j + L_{ij} y_{ij}, \forall i, j, \tag{17}$$

$$x_i, y_{ij} \in \{0, 1\}, \forall i, j \in S, i \neq j, \tag{18}$$

$$l_i > 0, \forall i. \tag{19}$$

The objective function (12) re-writes that of (11) in a discrete form. Constraint (13) specifies that exactly n sites are selected to deploy charging stations. Constraint (14) specifies that there are n edges in the MST. Constraint (15) says that if the charging stations deployed at sites i and j are connected by an edge, the length of the edge cannot be larger than $2R$. Constraint (16) eliminates sub-tours in the MST. Constraint (17) limits the length of the path from p_0 to a charging station. The left-hand side specifies the minimum length of a path, and the right-hand follows the triangulation law. Constraints (18) and (19) give the ranges of the variables.

Clearly, converting the problem to the discrete version, i.e., selecting positions from candidate sites, does not simplify the problem much. Many additional variables to formulate the connectivity requirement for each charging station are introduced, and the objective function is still non-linear. Although some existing solvers can be used to address the discrete version, they do not scale well. Below, we discuss a decentralized method to find sub-optimal positions for the charging stations.

C. Sub-optimal solution

Similar to deploying a single charging station, the objective function (11) more accurately characterizes the average flight distance than the existing models, thanks to the introduction of the ellipse. But, this new model makes the problem difficult to address, because moving one charging station in a decentralized manner impacts the coverage of not only the neighbour charging stations but also the non-neighbour charging stations. For a node j, its parent and children in the MST are called the neighbours of j, and the rest of the nodes are called the non-neighbours of j. Consider that we move p_k a bit in Fig. 3. Such a movement may change the subsets of $A_{k\phi(j)}$, $B_{k\phi(j)}$, A_{0k} and B_{0k} and the flight distance to customers in $C_{\phi(j)}$ and C_k. Moreover, the length of the shortest path from p_0 to $p_{\phi(j)}$ is likely to change because of the movement of p_k, so as the flight distance to the customers in C_j.

Fortunately, we notice that the movement of any node in the MST only impacts the flight distance of the customers covered by its downstream nodes, but not the upstream nodes. In Fig. 3, the movement of node k impacts the flight distance of the customers covered by nodes k, $\phi(j)$, and j, but not those covered by node 0. We call the node k and its downstream nodes a branch of nodes sub-rooted at node k. So, moving node k only affects the flight distance of customers covered by its branch. Moreover, we classify the nodes in a branch into two groups: the neighbours of the sub-root that are directly connected with the sub-root (the sub-root connects to itself); and non-neighbours that are not directly connected with the sub-root. We notice that for non-neighbours, the movement of the sub-root only affects the first part in (11), but not the second part. We call this the indirect impact. For instance, when we move node k in Fig. 3, the second part of the flight distance of the customers in C_j remains, because this part only depends on the positions of node j and its parent $\phi(j)$, rather than node k. For the neighbours of the sub-root, the movement of the sub-root affects both parts in (11), and we call this the direct impact. The relocation of a node depends on the evaluation of both the direct and indirect impacts. We relocate a node to a new position if the overall impact leads to a decrease in the average flight distance of the customers in its branch, and we move the node to the position corresponding to the maximum decrease of the average flight distance.

From the above discussion, we can see that the relocation of a leaf node (that does not have child nodes) can be addressed by the method discussed in Section IV. The relocation of a non-leaf node is more complex than that of a leaf node. We define $N_i \in \mathbb{N}$ as the number of the customers that are in the branch of node i and are indirectly impacted by node i. Moreover, let $G_i \in \mathbb{R}$ denote the average flight distance from node i to these customers. If i is a leaf node or the parent of a leaf node, $N_i = 0$ and $G_i = 0$ because the customers in the branch of node i are all directly impacted by node i. Let $H_1(p_j) \in \mathbb{R}$ denote the average flight distance from the parent of node j, i.e., $\phi(j)$, to all the indirectly impacted customers in the branch of node j. If $\sum_{i \in \psi(j)} N_i = 0$, $H_1(p_j) = 0$; otherwise, $H_1(p_j)$ is computed as follows:

$$H_1(p_j) = \frac{\sum_{i \in \psi(j)} N_i (G_i + |p_i, p_j| + |p_j, p_{\phi(j)}|)}{\sum_{i \in \psi(j)} N_i}. \tag{20}$$

In (20), the term $G_i + |p_i, p_j| + |p_j, p_{\phi(j)}|$ gives the average flight distance from node $\phi(j)$ to the customers in the branch of node i (where $i \in \psi(j)$) that are indirectly impacted by node j. An example is shown in Fig. 4. Clearly, $H_1(p_j)$ is a function of p_j, and other information in (20), including N_i, G_i and $p_{\phi(j)}$, is known if the downstream nodes and the parent node are fixed.

Let $H_2(p_j) \in \mathbb{R}$ denote the average flight distance from node $\phi(j)$ to the customers that are in the branch of node j and directly impacted by node j. Such customers include those covered by node j and the child nodes of node j. The number of these customers is $|C_j| + \sum_{i \in \psi(j)} |C_i|$. $H_2(p_j)$ is computed as follows:

$$H_2(p_j) = \frac{1}{|C_j| + \sum_{i \in \psi(j)} |C_i|} \Bigg[\sum_{c \in A_{\phi(j)j}} |p_{\phi(j)}, c| + \sum_{c \in B_{\phi(j)j}} (|p_{\phi(j)}, p_j| + |p_j, c|) + \sum_{i \in \psi(j)} \Bigg(\sum_{c \in A_{ji}} |p_j, c| + \sum_{c \in B_{ji}} (|p_j, p_i| + |p_i, c|) + \sum_{c \in C_i} |p_j, p_{\phi(j)}| \Bigg) \Bigg]. \tag{21}$$

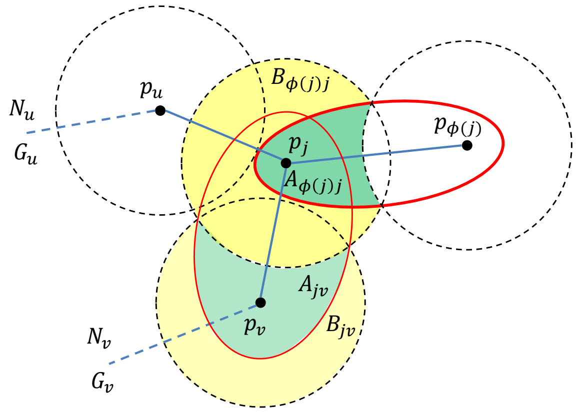


Fig. 4: Flight distance for indirectly and directly impacted customers. $G_v + |p_v, p_j| + |p_j, p_{\phi(j)}|$ gives the average flight distance from node $\phi(j)$ to customers in the branch of node v that are indirectly impacted by node j. $\sum_{c \in A_{\phi(j)j}} |p_{\phi(j)}, c| + \sum_{c \in B_{\phi(j)j}} (|p_{\phi(j)}, p_j| + |p_j, c|)$ gives the total flight distance from $p_{\phi(j)}$ to the customers in C_j. $\sum_{c \in A_{jv}} |p_j, c| + \sum_{c \in B_{jv}} (|p_j, p_v| + |p_v, c|) + \sum_{c \in C_v} |p_j, p_{\phi(j)}|$ gives the total flight distance from node $\phi(j)$ to the customers in C_v.

The term $\sum_{c \in A_{\phi(j)j}} |p_{\phi(j)}, c| + \sum_{c \in B_{\phi(j)j}} (|p_{\phi(j)}, p_j| + |p_j, c|)$ in (21) gives the total flight distance from node $\phi(j)$ to the customers in C_j. The term $\sum_{c \in A_{ji}} |p_j, c| + \sum_{c \in B_{ji}} (|p_j, p_i| + |p_i, c|) + \sum_{c \in C_i} |p_j, p_{\phi(j)}|$ in (21) gives the total flight distance from node $\phi(j)$ to the customers in C_i, where $i \in \psi(j)$. An example is available in Fig. 4. Clearly, $H_2(p_j)$ is a function of p_j, and other information in (21) is known if the downstream nodes and the parent node are fixed. Different from $H_1(j)$ which can be zero, $H_2(j) > 0$, because there exists a subset of customers that are directly impacted by node j. If this subset is empty, there is no need to have node j.

Let $H(p_j) \in \mathbb{R}$ denote the average flight distance from node $\phi(j)$ to all the indirectly and directly impacted customers by node j. Then,

$$H(p_j) = \frac{(|C_j| + \sum_{i \in \psi(j)} |C_i|) H_2(p_j) + \sum_{i \in \psi(j)} N_i H_1(p_j)}{|C_j| + \sum_{i \in \psi(j)} |C_i| + \sum_{i \in \psi(j)} N_i}. \tag{22}$$

Moreover, the average flight distance from node 0 to all the customers in the branch of node j is given by $L(p_0, p_{\phi(j)}) + H(p_j)$, where $L(p_0, p_{\phi(j)})$ denote the distance from node 0 to node $\phi(j)$ as defined at the beginning of this section. If at any time we only move one node in the MST, say node j, for the purpose of reducing the average flight distance, we only need to consider $H(p_j)$, because $L(p_0, p_{\phi(j)})$ is a constant. Therefore, we can relocate p_j to a new position that minimizes (22) subject to that the topology of the MST remains and node j does not lose any covered customers. Similar to (1), the function (22) is not continuous with respect to p_j. So, we solve it in a numerical manner, which is similar to what we do in Section IV.

Moreover, as the values of N and G are required in the computation of $H_1(p_j)$ in (20), we need a general formula for them to relocate each node in a decentralized manner. Given the N and G values of the child nodes of node j, we can compute these values for node j as follows:

$$N_j = \sum_{i \in \psi(j)} (N_i + \sum_{k \in \psi(i)} |C_k|), \tag{23}$$

$$G_j = \frac{1}{N_j} \sum_{i \in \psi(j)} \Bigg(N_i (G_i + |p_i, p_j|) + \sum_{c \in A_{ji}} |p_j, c| + \sum_{c \in B_{ji}} (|p_j, p_i| + |p_i, c|) \Bigg). \tag{24}$$

The values of N and G propagate in the upstream order. For a certain node, these values can be computed once those of its child nodes have been computed.

Now, we are in the position to present our method as a whole. Suppose that at the initial positions $p_1, \dots, p_n$, the n charging stations cover all the customers in the set C. We construct the MST for the nodes rooted at p_0. We assume that all the edges in the MST are no longer than $2R$. Additionally, with the initial positions, we can compute the subset of customers covered by each node. The main procedure of our method repeats relocating the nodes in sequence. In particular, whenever we are to relocate a node, we need to complete the relocation of all its child nodes. For two sibling nodes, either of them can come first in the sequence. With this rule, we start the relocation from the leaf nodes, then the parent of leaf nodes, and so on. For a leaf node, we find the position for this node by solving the problem (1) subject to (2), (3), (4) and (5), where p_1 in this problem is the leaf node, and p_0 is the parent of the leaf node. For a non-leaf node, we find the position for the node by minimizing (22) subject to that the topology of the MST remains when the node is relocated, and the node does not lose any customers. After the relocation, we use (23) and (24) to update the values of N and G for the node, respectively. After the relocation of all the nodes, we update the subset of customers covered by each node, including the update of the sets A and B. These procedures repeat until the nodes cannot be further moved. One termination condition can be that all the nodes stay at the previous positions in one round of relocation. This method is summarized in Algorithm 1.

Complexity. Algorithm 1 is an iterative algorithm, and the termination depends on the evaluation of the solutions in two consecutive rounds. We now analyze the complexity of each round. For any node in the MST, we need to compute the values of N and G. To this end, the child nodes of the considered node will be evaluated. The worst case is that this node has n child nodes. Then, the worst complexity is $O(n^2)$. It is worth pointing out that in practice, in an MST, the number of child nodes of a parent node is far less than n. Thus, the practical complexity of each round should be much smaller than $O(n^2)$.

Proposition V.1. *For a given set of customers, from the initial positions, Algorithm 1 ensures that any movement of a charging station leads to a lower average flight distance.*

Proof. Algorithm 1 consists of two main procedures. One is the relocation of charging stations in sequence, and the other is the update of customers covered by each charging station.

Algorithm 1 Relocating the nodes in MST

Input: $p_0, p_1, \ldots, p_n$

Output: $p_1, \ldots, p_n$

```
Construct the MST.
Compute the subset of customers covered by each node.
Construct a relocating sequence.
while Termination condition is unsatisfied do
    for Each node j in the sequence do
        if Node j is a leaf node then
            Find the new position by solving the problem (1) subject to (2), (3), (4) and (5).
            N_j ← 0, G_j ← 0.
        else
            Find the new position by minimizing (22) subject to that the topology of MST remains when node j relocates, and node j does not lose any customers.
            Update N_j and G_j by (23) and (24).
        end if
    end for
    Update the subset of customers covered by each node.
end while
```

For the former, as we only relocate one charging station at any time, and we relocate it only if the average flight distance of the downstream customers impacted by the charging station reduces. As analyzed above, such a relocation does not influence the upstream customers. Thus, the relocation procedure ensures the decrease of the average flight distance. Regarding the latter, according to Proposition IV.1, for the given locations of any pair of parent-child charging stations, the proposed customer grouping model ensures that any customer can be served and the flight distance for any customer is the lowest. Therefore, Algorithm 1 relocates the charging stations to positions with lower average flight distance. □

From Proposition V.1 we can see that starting from some initial positions, Algorithm 1 ensures that any movement of a charging station leads to a lower average flight distance. When there is no further decrease on the average flight distance, a local optimum is obtained. We can try different initial conditions to find different local optimums. Then, we can select the best among them.

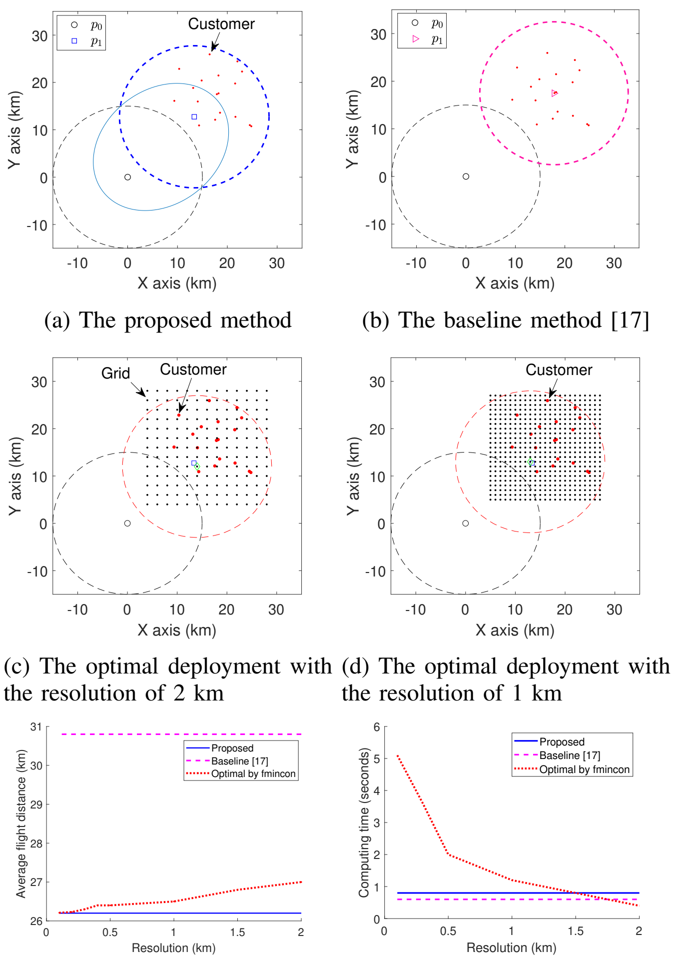
VI. Simulation Results

In this section, we present computer simulations to show the performance of the proposed method. As discussed in Section II, algorithms for the hub location problem do not pay attention to the connectivity constraint generally. The reference [15] considers the connectivity issue but aims at maximizing the coverage of customers. In contrast, the considered problem is to minimize the average delivery time. Suppose that we start from the same initial positions of charging stations that fully cover the customers, the method in [15] will terminate immediately, while the proposed method will exploit Algorithm 1 to reduce the flight distance. Thus, the comparison would be unfair. Minimizing the average distance between the customers and the charging stations is a part of the method in [17]. Therefore, the method in [17] is considered as a baseline method. Moreover, we use a MATLAB function 'fmincon' to address some small-scale instances to obtain the optimal solutions. Below, the coverage radius is $R = 15$ km.

We first consider a simple case with only one charging station to be deployed near a given charging station. Figs. 5a and 5b show the positions of the charging station by the the proposed method and the baseline method [17], respectively. We can see that the proposed method leads to an average flight distance of 26.2 km, which is shorter than the average flight distance of 30.8 km achieved by the baseline method. To apply the MATLAB function 'fmincon', we need to construct a set of candidate sites. For this case, we grid a square area around the customers with a certain resolution from 0.1 to 2 km. Figs. 5c and 5d demonstrate the results when the resolution takes 2 km and 1 km, respectively. Figs. 5e and 5f further show the performance of the function 'fmincon' in terms of the average flight distance and the computing time under different resolutions (counted on a normal computer with Intel Core i7-7500U CPU). It is as expected that when the resolution becomes large, the average flight distance increases and the computing time reduces. When the resolution is set as 0.2 km, the average flight distance achieved by the function 'fmincon' becomes the same as the proposed method.

In the second case, we consider 100 customers randomly located in a long but narrow area. Five charging stations are deployed to serve the customers. Fig. 6a shows the initial positions of the five charging stations together with p_0 at (0,0). Applying the proposed method and the baseline method [17], the final positions of the 5 charging stations are shown in Figs. 6b and 6c, respectively. The termination condition is that the average gap between the current positions and the previous positions of all charging stations is smaller than 0.01 km. In Fig. 6b, we also plot the ellipse for each parent-child pair. Fig. 6d compares the average flight distance to the customers in each round of relocation. We can observe from Fig. 6d that at the same positions (such as at the initial positions), the proposed model leads to about 6% shorter average flight distance than the baseline method. We can also see that the proposed Algorithm 1 moves the charging stations leading to a shorter average flight distance. In contrast, the baseline method does not guarantee the decrease of the average flight distance. The main reason is that during the relocation, the baseline method moves each charging station closer to the covered customer only without considering the flight distance from p_0 to this charging station. We also apply the function 'fmincon'. Similar to the above case, we discretize the area with a certain resolution, and take the grid points as the candidate sites. Figs. 5e and 5f show the average flight distance and the computing time, respectively. When the resolution is set lower than 2 km, the function 'fmincon' achieves better performance in terms of the average flight distance. However, the cost is that the computing time is more than 1 hour. In contrast, the proposed algorithm can complete in about 10 seconds.

We further consider a case in a real map as shown in Fig. 7. This is the greater Wollongong region, which is on the south side of Sydney and about 45 km along the coast. As we are not accessible to the population distribution, we assume that



(a) The proposed method

(b) The baseline method [17]

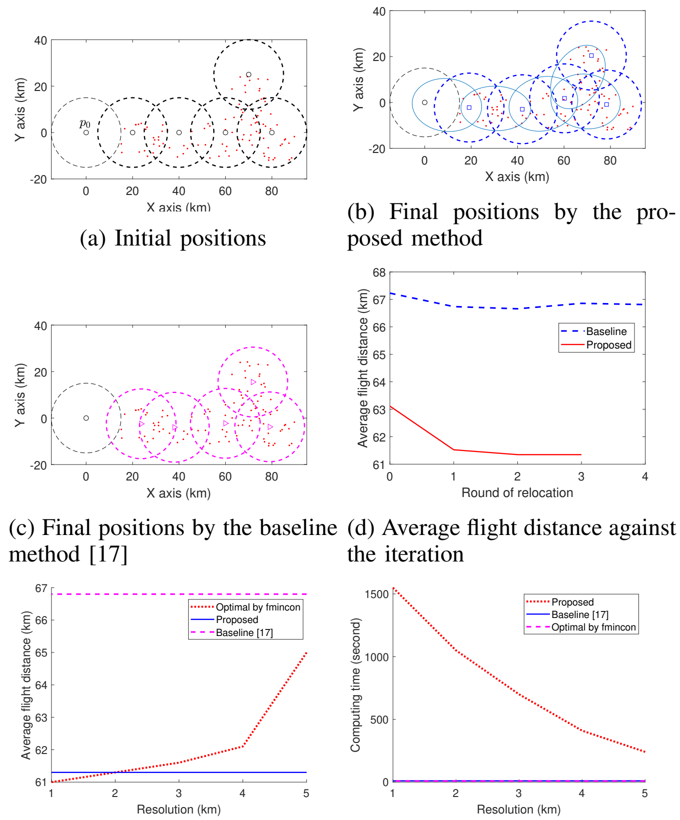
(c) The optimal deployment with the resolution of 2 km

(d) The optimal deployment with the resolution of 1 km

(e) Average flight distance VS resolution of grids

(f) Computing time VS resolution of grids

Fig. 5: Comparison of the proposed method, the baseline method [17], and the optimal solution achieved by 'fmincon' in a simple case.



(a) Initial positions

(b) Final positions by the proposed method

(c) Final positions by the baseline method [17]

(d) Average flight distance against the iteration

(e) Average flight distance VS resolution of grids

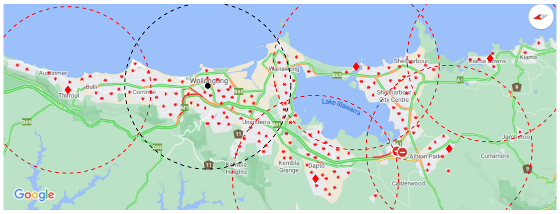
(f) Computing time VS resolution of grids

Fig. 6: Comparison of the proposed method, the baseline method [17], and the optimal solution achieved by 'fmincon' in a case with 5 charging stations.

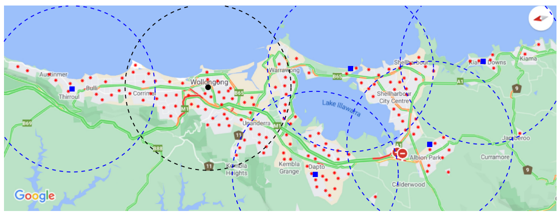
the customers are uniformly distributed in the resident area. p_0 is deployed at the position of the black square in Fig. 7. The coverage radius is $R = 15$ km, which is the same as above. To apply the optimal method, we randomly select 150 candidate sites in the map. Among the 150 candidate sites, the function 'fmincon' finds the optimal solution as shown in Fig. 7a. The computing time for this case is about 30 minutes, and the average flight distance is 33.1 km. Using this solution as the initial conditions, the proposed method is applied, and the corresponding deployment of the charging stations is shown in Fig. 7b. From the initial positions, the proposed method further moves the charging stations locally, and the final average distance drops to 32.3 km under the terminal condition of 0.1 km. The corresponding computing time is less than 1 minute. We also apply the baseline method to this case. Using the same initial condition, the deployment achieved by the baseline method [17] is shown in Fig. 7c. The average flight distance is 35.0 km, and the computing time of the baseline method.

We also demonstrate the scalability of the proposed method. We create some cases with more charging stations in larger areas with more customers randomly places in the areas. For each simulating case, we apply the proposed method to find the locations of charging stations. Some of the results are illustrated in Fig. 8. We repeat this 10 times for each case and record the computing time. As Fig. 9 shows, the proposed method can return solutions within a few minutes. However, the function 'fmincon' cannot address these large-scale cases in a reasonable time. Moreover, the computing time of the proposed method increases almost linearly with the number of charging stations. The main reason is that when we relocate a charging station, only the neighbour charging stations have an impact. This is consistent with our expectation as mentioned in Section V.

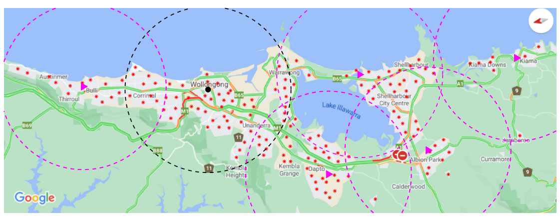
As a final remark, the function 'fmincon' to address the discretized version can find the optimal solution among the candidate sites. The computing time is very long if the number of candidate sites is large. So, we can first construct a set of candidate sites with low resolution and then use the function 'fmincon' to determine a 'rough' solution. Regarding this solution as an initial condition, we can apply the proposed method to further relocate the charging station locally to improve the solution.



(a) The deployment of the optimal solution. The red stars are the randomly selected candidate sites.

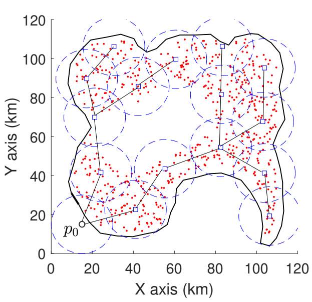


(b) The deployment by the proposed method

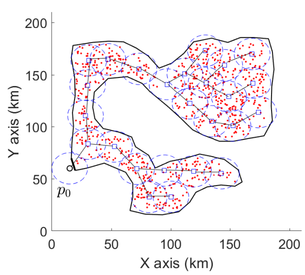


(c) The deployment by the baseline method [17]

Fig. 7: Comparison in a real map with uniformly distributed customers.



(a) 15 charging stations



(b) 25 charging stations

Fig. 8: Illustration of the proposed method in large-scale cases.

VII. Conclusion

To provide drone delivery service to customers in a remote area, this paper presented an approach using public transportation vehicles and charging stations. In particular, a drone can travel to some position near the remote area by taking public transportation vehicles, and then it may hop and swap the battery at charging stations in the area to reach a customer. For the deployment of charging stations, we proposed a new service model to characterize the delivery time, that provides an accurate estimation of the flight distance of a customer. The deployment problem was formulated with the objective of minimizing the average delivery time, which can be seen as a reflection of customer satisfaction level. To solve the problem, a sub-optimal algorithm that relocates the charging stations in the sequence was proposed. This algorithm ensures that any movement of a charging station leads to a decrease in the average flight distance. One shortcoming in the current approach is that we relocate a charging station by numerical evaluation. One future research work will look for an analytical solution to this issue. In addition, in the current work, the charging process is not considered as each charging station is assumed to have a large number of spare batteries. An interesting but also challenging problem is the routing planning problem for drones when the charging stations have limited battery resources. In this case, the status of the charging stations will be taken into account in the routing planning issue. To make the proposed scheme become a reality, accurate and timely information of the operations of PTVs should be accessible to drones because the drones need to collaborate with PTVs. Moreover, the current paper only addresses the facility deployment problem. The low-level air traffic control issue needs to be considered to achieve a safe and efficient schedule of the drones, especially when lots of drones are flying in the air.

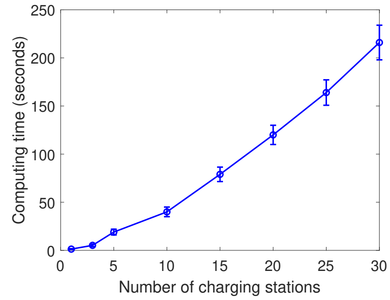


Fig. 9: Computing time of the proposed method under different number of charging stations.

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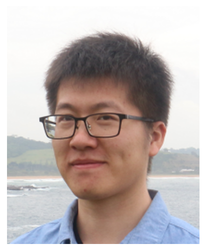

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