Case Study

Optimal Independent Baseline Searching for Global GNSS Networks

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Abstract: An n-station continuously operated global navigation satellite system (GNSS) network contains n-1 independent baselines. Baseline structure is critical to positioning accuracy, and the final result is dependent on the baseline selection strategies. The baseline length and amount of common observations are the primary principles for baseline selection. However, there are few discussions about the optimal strategy to determine the independent baseline of a huge GNSS network. To enhance the performance of the multibaseline solution, a comparison is drawn between the conventional method and a weighting strategy. Observations from continuous stations distributed globally within the International GNSS Service (IGS) are explored. At first, two conventional principles for baseline selection are tested. Subsequently, a weighting scheme is developed to exploit these two strategies. The enhanced method improves nearly 10% external accuracy compared with the classical methods, which can be verified from the experiment on January 1, 2012. Lastly, the network experiment is extended to the whole year of 2012 to increase statistical significance. It is therefore revealed that the novel weighting strategy (WEIGHT), with an equal chance of two conventional strategies, mitigates 0.4%–3.0% three-dimensional (3D) coordinate error of the whole year. Also, an analysis of the probability of gross errors indicates that WEIGHT exhibits better performance. Unlike the conventional view, it is shown that a proper weight of OBS-MAX and SHORTEST could form a better coordinate calculation result and a lower gross error rate. In conclusion, these experiments suggest a proposed method that synthetically considers the length of total stations and the total number of observations, and it is verified that WEIGHT is a better choice for searching independent baselines

Author keywords: Global navigation satellite system (GNSS) network; Independent baseline; Minimum spanning tree; Weight factor.

Introduction

The theories and methods of large network baseline solution are a persistent study topic. Multisite reference station differential GPS systems were designed by Johnston (1994a, b). Saalfeld (1999) studied the selection of receivers and satellite pairs. Wieser checked the reliability of the global navigation satellite system (GNSS) baseline (Wieser 2004; Wei et al. 2011) and investigated the posterior error of GPS control network. Yetkin et al. (2013) adopted the particle swarm optimization algorithm and Alizadeh-Khameneh et al. (2017) considered the baseline correlations.

For a large observatory network adjustment, the primary methods include multi-independent-baseline mode (Xu and Xu 2016) and network adjustment (Kowalczyk and Rapiński 2017). The characteristics of the entire network adjustment are to extract all the observation data simultaneously while estimating all the parameters participating in this network. Its essence is that it combines the methods of baseline calculating and network adjustment. The defect is that it is heavily determined by the calculating power and storage capacity of computers (Chen et al. 2013; Cui et al. 2017).

In terms of the multi-independent-baseline mode of n-stations, all the stations should be connected with sufficient lines without closed circles. If simultaneous observations are extracted from multiple stations at a certain period, $n(n+1)/2$ baseline vectors are an alternative. Subsequently, in all the mentioned synchronous observation baselines, n-1 independent baselines are taken from all the possibilities (Chen et al. 2013; Xu and Xu 2016), under the assumption that the data quality of all the stations is identical. Such algorithms used to choose n-1 baselines from $n(n+1)/2$ baseline

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are termed the minimum spanning tree (MST) (Kruskal 1964; Erciyes 2013).

When using the MST algorithm, a key value should exist between every two stations, while the critical problem is how to determine this key value. Once this value is determined, all the baselines are confirmed and the link from one station to any other station is unique (Xu and Xu 2016). In a multibaseline mode, this value causes different baseline structures and thus exerts a decisive impact on the final result.

Various strategies can be adopted to determine this key value. Of the original view, the overall length of the baselines should be as short as possible, which is known as the shortest baseline strategy (SHORTEST). Existing studies show the merits of adopting the shortest baselines. Omogunloye and Okorocha explored the positioning error of SHORTEST, medium, and long-baseline error; their results demonstrated that there is a significant difference when using the long-baseline (Omogunloye et al. 2017). Cui et al. (2015) gave an example to clarify how to select a baseline using the SHORTEST with high data utilization. Cai (Hua 2010) adopted the shortest path method to generate independent baselines and the precision of the coordinate was 0.007–0.010 m. To expedite the calculation, a Delaunay triangulation subdivision algorithm was conjoined before using MST to effectively simplify the computation of the minimum tree. There are two reasons to interpret why the shortest baseline could achieve a better result: first, the shorter the baseline, the more common-viewed satellites there would be. Second and more importantly, when the two base stations are close to each other, the signal propagation error can be eliminated more effectively (Loomis et al. 1991).

In theory, a shorter baseline scheme indicates more common observations, which is not always the case, especially when the stations are distributed worldwide. To use the maximal number of common satellite observations, the common observations of every two stations are taken as the key value mentioned previously. In other words, from all possible combinations, a set of baselines with maximal common observations is taken. Such a strategy is abbreviated as OBS-MAX. Many pieces of software dealing with the GNSS network are equipped with OBS-MAX as the default option. For instance, several studies (Wielgosz et al. 2011; Chen et al. 2013; Cui et al. 2015, 2017) adopted OBS-MAX with Bernese version 5.2 (Dach et al. 2015) and Gnsser software (Li et al. 2019a, b).

Thus far, the GNSS Double Difference (DD) network has been extensively used, including for producing precise satellite orbits (Alhamadani 2018; Ye et al. 2019), maintaining coordinate frame, and calculating earth rotation parameters (Zajdel et al. 2019). Most of these comply with the conventional OBS-MAX and SHORTEST experimental strategies, although both of these conventional methods exhibit defects. Theoretically, more observations indicate a better baseline, because there are more data involved in adjustment in the use of OBS-MAX. In practice, however, there exists a probable ionospheric and troposphere difference because the distance between stations on a baseline is relatively large, which would cause gross errors of the final coordinate. For the SHORTEST, it cannot fully exploit observations. To achieve high calculation precision of station coordinates, from a practical perspective, a weighting method is developed considering the advantages of the two strategies.

Theory of Weight Baseline Selection Strategy

Minimum Spanning Tree and Maximal Spanning Tree

It is assumed that there are z satellites for n receivers, and the DD model is formed as the difference between two single differences from two satellites (Parkinson and Spilker 1996; Kaplan and Hegarty 2005; Xu and Xu 2016; Teunissen and Montenbruck 2017), which can be expressed as a matrix form

$$\mathbf{DD}_{i,j}(\mathbf{O}) = (\mathbf{O}_{r_i,r_j}^{k_1,k_2}\mathbf{O}_{r_i,r_j}^{k_1,k_3} \cdots \mathbf{O}_{r_i,r_j}^{k_1,k_{z-1}})^{\mathrm{T}}, \quad i \neq j \tag{1}$$

where $\mathbf{O}$ = observation equation; $k_1, k_2, \cdots, k_{z-1}$ = satellite; and r_i, r_j = different receivers. The derivation process of the formulas can be found in the work of Xu and Xu (2016). For a network with n-stations, as mentioned previously, there exist n-1 independent baselines. The SHORTEST strategy takes MST to connect all the n-stations based on the length of baselines between every two stations. In contrast, if the OBS-MAX strategy is taken to form baselines, a maximal spanning tree should be exploited to maximize the number of observations of all the stations.

A weight matrix W is required to express the weight value between every two stations

$$\mathbf{W}^s = \begin{bmatrix} r_{11}^s & r_{12}^s & \cdots & r_{1n}^s \\ & r_{22}^s & & r_{2n}^s \\ & & \ddots & \vdots \\ & & & r_{nn}^s \end{bmatrix} \tag{2}$$

$$\mathbf{W}^o = \begin{bmatrix} r_{11}^o & r_{12}^o & \cdots & r_{1n}^o \\ & r_{22}^o & & r_{2n}^o \\ & & \ddots & \vdots \\ & & & r_{nn}^o \end{bmatrix} \tag{3}$$

where superscript s and o = SHORTEST and OBS-MAX strategy, respectively; and $r_{11}, r_{12}, \ldots, r_{nn}$ = weight index defined by the baseline forming strategy. For the SHORTEST strategy, r_{12}^s, $r_{13}^s, \ldots, r_{n\text{-}1,n}^s$ represent the Euclidean distance between every two stations ($r_{11}^s, r_{22}^s, \cdots, r_{nn}^s = 0$). For the OBS-MAX strategy, r_{12}^o, $r_{13}^o, \ldots, r_{n\text{-}1,n}^o$ represent the common satellites between every two stations, and $r_{11}^s, r_{22}^s, \ldots, r_{nn}^s$ are the observations for a single station in a certain period. The observation adopted in DD is different according to the different sampling rates. Subsequently, the MST algorithm is adopted to get a minimum total length or maximal common observations.

Principles of the WEIGHT Strategy

For most users, the strategy is often taken according to the software's default settings or by experience. To search more reliable baselines, which both have sufficient observations and a proper length, the SHORTEST and OBS-MAX strategy are combined by designing a weighting scheme.

First, the data should be normalized. A linear normalized model is employed here

$$r_{nn}^{s\prime} = \frac{r_{nn}^s - M_2^s}{M_1^s - M_2^s} \tag{4}$$

$$r_{nn}^{o\prime} = \frac{r_{nn}^o - M_2^o}{M_1^o - M_2^o} \tag{5}$$

where M_1 = maximal number of all the elements of W; M_2 = minimum number of all the elements of $\mathbf{W}$; and the new $\mathbf{W}$ for each strategy is expressed as $\mathbf{W}^{s\prime}$ and $\mathbf{W}^{o\prime}$.

Next, a novel weight matrix multiplied by a weight factor is

$$\mathbf{W}' = a \times \mathbf{W}^{s\prime} + b \times \mathbf{W}^{0\prime}(a + b = 1, a \subseteq [0, 1], b \subseteq [0, 1]) \tag{6}$$

where a and b = variable weight factors for matrix $\mathbf{W}^s$ and $\mathbf{W}^o$; and a and b = variables with days. It is assumed that there are x days. The matrix $\mathbf{W}'$ of each day is

$$\mathbf{W}'_x = \begin{bmatrix} a_x \times r^{s\prime}_{11} + b_x \times r^{o\prime}_{11} & a_x \times r^{s\prime}_{12} + b_x \times r^{o\prime}_{12} & \cdots & a_x \times r^{s\prime}_{1n} + b_x \times r^{o\prime}_{1n} \\ & a_x \times r^{s\prime}_{22} + b_x \times r^{o\prime}_{22} & & a_x \times r^{s\prime}_{2n} + b_x \times r^{o\prime}_{2n} \\ & & \ddots & \\ & & & a_x \times r^{s\prime}_{nn} + b_x \times r^{o\prime}_{nn} \end{bmatrix} \tag{7}$$

where $\mathbf{W}^s$, $\mathbf{W}^o$, and $\mathbf{W}'_x$ = symmetric matrices.

Deriving the Proper Weighting Factor

If the weight is determined as $0.5 + 0.5$, the weighted strategy can achieve better results for January 1, 2012 (see the next part). Further, a better proportion of this weight is analyzed. The normalized root mean square error (RMSE) of a network using OBS-MAX and SHORTEST is substituted as σ_o and σ_s, respectively. The proportion of the two strategies are substituted as w_o and w_s, respectively

$$w_o^{(1)} = \frac{\sigma_s}{\sigma_o + \sigma_s} \tag{8}$$

$$w_s^{(1)} = \frac{\sigma_o}{\sigma_o + \sigma_s} \tag{9}$$

Also, to amplify the effect, w_o and w_s can be defined as

$$w_o^{(2)} = \left(\frac{\sigma_s}{\sigma_o + \sigma_s}\right)^2 \tag{10}$$

$$w_s^{(2)} = \left(\frac{\sigma_o}{\sigma_o + \sigma_s}\right)^2 \tag{11}$$

Subsequently, the generally weighted matrix of all stations in one day is written as

$$\mathbf{W}' = w_s \times \mathbf{W}^s + w_o \times \mathbf{W}^o \; (w_s, w_o, a, b \in [0, 1]) \tag{12}$$

The matrix $\mathbf{W}'$ is written as

$$\mathbf{W}'_x = \begin{bmatrix} w_s \times r^s_{11} + w_o \times r^o_{11} & w_s \times r^s_{12} + w_o \times r^o_{12} & \cdots & w_s \times r^s_{1n} + w_o \times r^o_{1n} \\ & w_s \times r^s_{22} + w_o \times r^o_{22} & & w_s \times r^s_{2n} + w_o \times r^o_{2n} \\ & & \ddots & \vdots \\ & & & w_s \times r^s_{nn} + w_o \times r^o_{nn} \end{bmatrix} \tag{13}$$

where w_o covers $w_o^{(1)}$ and $w_o^{(2)}$; and w_s includes $w_s^{(1)}$ and $w_s^{(2)}$.

Proper Independent Baseline Searching by Experiment

At first, observations for 44 and 98 stations from IGS on January 1, 2012 are exploited to test the proposed algorithm. The selected stations are fairly globally distributed. The data range from 00:00 to 23:59:30 and the sampling and calculating frequency is 30 s. Only GPS satellite observation data are involved in the calculation. Subsequently, the 98 stations' network experiment is extended to the 366 days of 2012.

Network Solution Processing

This process is primarily based on Bernese. The flow chart is illustrated in Fig. 1. Such products or steps are involved in facilitating network processing: (1) a reference coordinate frame; (2) differential code bias corrections; (3) antenna file; (4) earth rotation parameter; (5) precise orbit files; (6) clock correction; and (7) tidal loading corrections.

This single-day process entails single-threaded computation on a computer. To process 44 stations, OBS-MAX takes 22 min 12 s and SHORTEST takes 21 min 1s. To process 98 stations, OBS-MAX takes 65 min 30 s, and SHORTEST takes 62 min 53 s. The calculation time is largely associated with the amount of observation. The WEIGHT calculation time is between OBS-MAX and SHORTEST.

Comparison with IGS Coordinate File

The RMSE of the multibaseline network coordinates in each direction is compared with IGS products in Table 1. The results with

44 stations presented in Fig. 2(a) suggest that almost all the results by WEIGHT are better than SHORTEST. The $0.5+0.5$ and $0.7+0.3$ weight schemes lead to optimal results. The RMSE of coordinate reaches 0.00557 m, which is 22.9% better than that of the SHORTEST scheme (0.00722 m) and 9.5% better than that of OBS-MAX (0.00615 m). Fig. 2(b) shows the experiment results with 98 stations. Better results can be achieved under the weighting scheme of $0.5+0.5$, which is 10.2% better than the OBS-MAX scheme (0.010498 m) and 10.9% better than the SHORTEST scheme (0.010588 m).

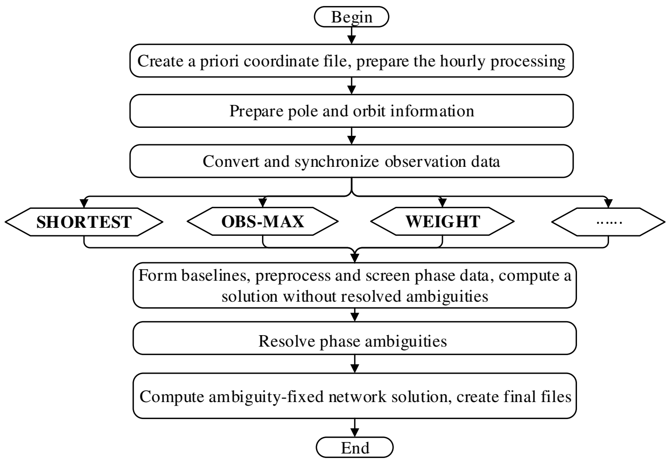

Fig. 1. Network solution processing for different baseline strategies.

The three-dimensional (3D) error of three strategies is listed in Table 2. WEIGHT represents the results considering OBS-MAX and SHORTEST equally. Given the calculation accuracy of station coordinate, OBS-MAX achieves a better result than SHORTEST, because more data participates in adjustment. It can be inferred that at the ratio of $0.5+0.5$ (namely, when the SHORTEST and OBS-MAX makes the identical contribution), a high-precision coordinate result is obtained.

This experiment extracts a whole day's observation to avoid the environmental factor that varies over time. All the selected reference stations in this study are continuously operated. Thus, the independent baseline calculation and station coordinate calculation are similar and repeatable. More experimentation across multiple days is developed hereinafter to illustrate that the single-day results are not by chance.

Shape of the Baseline Distribution

To verify whether the shape of baseline distribution impacts the final results, Fig. 3 presents three strategies. In Fig. 3, from the OBS-MAX map with 44 stations, the BRMU station in the Atlantic is linked with five stations, while it is only linked with three stations in the SHORTEST map. The most two obvious differences of the WEIGHT map are: (1) the GMAS station and the NKLG station in Africa are not connected each other, whereas they are linked to both

Table 1. RMSE of the single-day multibaseline network

	44 stations			98 stations		
Strategy\direction	x (m)	y (m)	z (m)	x (m)	y (m)	z (m)
SHORTEST	0.00754	0.00691	0.00720	0.00698	0.00765	0.01005
$0.1+0.9$	0.00664	0.00698	0.00800	0.00630	0.00829	0.01495
$0.2+0.8$	0.00550	**0.00710**	0.00712	0.00622	0.00806	0.01489
$0.3+0.7$	**0.00543**	**0.00721**	**0.00664**	**0.00605**	0.00747	0.01006
$0.4+0.6$	**0.00532**	0.00749	0.00781	0.00732	0.00742	**0.00784**
$0.5+0.5$	**0.00543**	**0.00721**	**0.00664**	0.00658	**0.00657**	**0.00857**
$0.6+0.4$	0.00551	0.00743	0.00750	**0.00549**	**0.00720**	0.00915
$0.7+0.3$	0.00717	0.00846	**0.00651**	**0.00564**	0.00733	**0.00887**
$0.8+0.2$	0.00692	0.00880	0.00865	0.00656	0.00749	0.00899
$0.9+0.1$	0.00609	0.00934	0.00989	0.00739	**0.00724**	0.00905
OBS-MAX	0.00681	0.00785	0.00989	0.00783	0.00942	0.00773

Note: The three smallest numbers in each column are indicated in bold. The smaller the number, the better the experimental results.

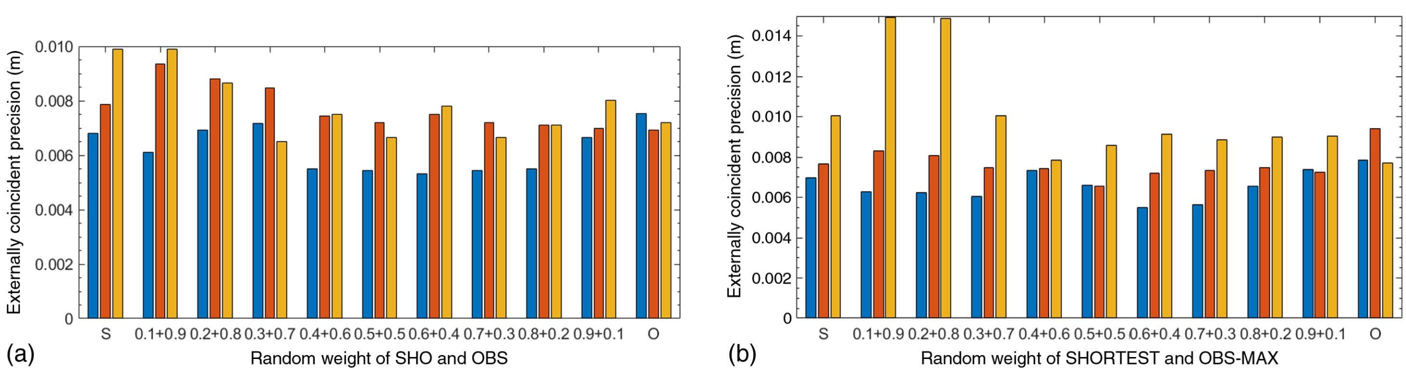

Fig. 2. Weighting scheme with SHORTEST and OBS-MAX of (a) 44 stations; and (b) calculation results for 98 stations on January 1, 2012. S at (a) of the x-axis represents SHORTEST; and O at (b) denotes OBS-MAX. The x-axis presents the weight varying from S to O, and the y-axis represents the difference between the station coordinate of this network solution with IGS products. The three bars in each weight block at the x-axis present the error in x, y, and z direction, respectively.

Table 2. Comparison of different strategies with 3D error

No. of stations	Strategy OBS-MAX (m)	SHORTEST (m)	WEIGHT (0.5 + 0.5) (m)
44 stations	0.01415	0.01725	0.01321
98 stations	0.01622	0.01664	0.01455

the OBS-MAX and SHORTEST strategies; and (2) REUN and COCO are linked in the WEIGHT map, but not linked when using other strategies.

In the OBS-MAX map with the 98 stations, the GOLD station in North America, the POVE station in South America, the GLSV station in Europe, and the KARR station in Australia are linked to a considerable number of stations, suggesting that these stations have numerous common observations. In the SHORTEST map, most of the stations are connected to no more than three other stations.

As shown in Fig. 3, the WEIGHT with 44 stations is similar to OBS-MAX, and the WEIGHT with 98 stations is closer to SHORTEST. In practice, the baseline of WEIGHT differs from neither SHORTEST nor OBS-MAX. When all stations are far apart, the number of observations per two stations is low, which highlights the role of OBS-MAX. When all stations are close together, the number of observations is stable and the distance becomes more important. Thus, the proportion of the SHORTEST can be considered more in a local network when using WEIGHT, while the OBS-MAX strategy should be considered more frequently than the SHORTEST when the common observation is insufficient, which confirms that the

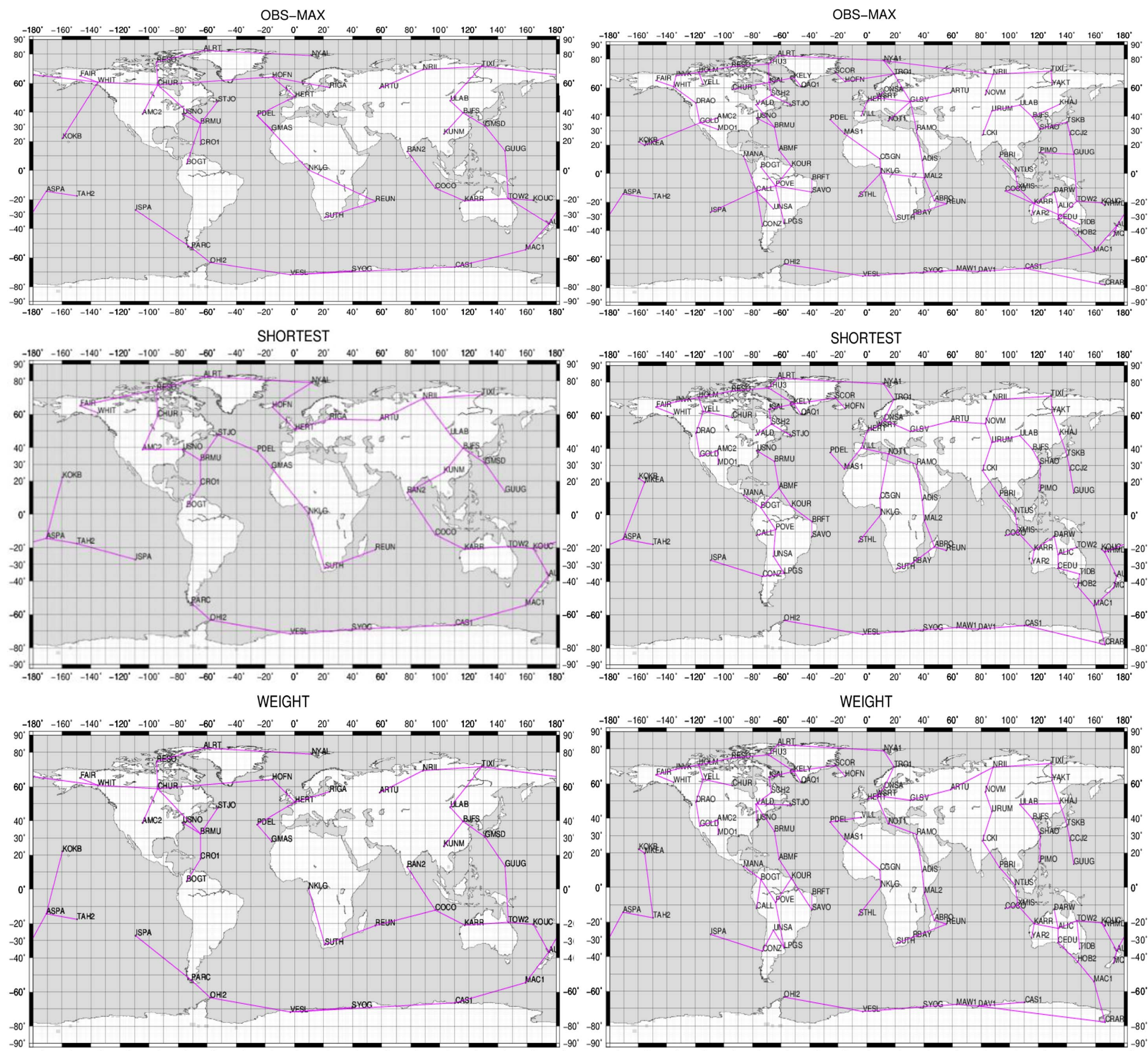

Fig. 3. Baseline structure with three strategies of 44- and 98-station networks. The three maps on the left display the shapes of the globally distributed baseline networks with 44 stations, and the three maps on the right have 98 stations.

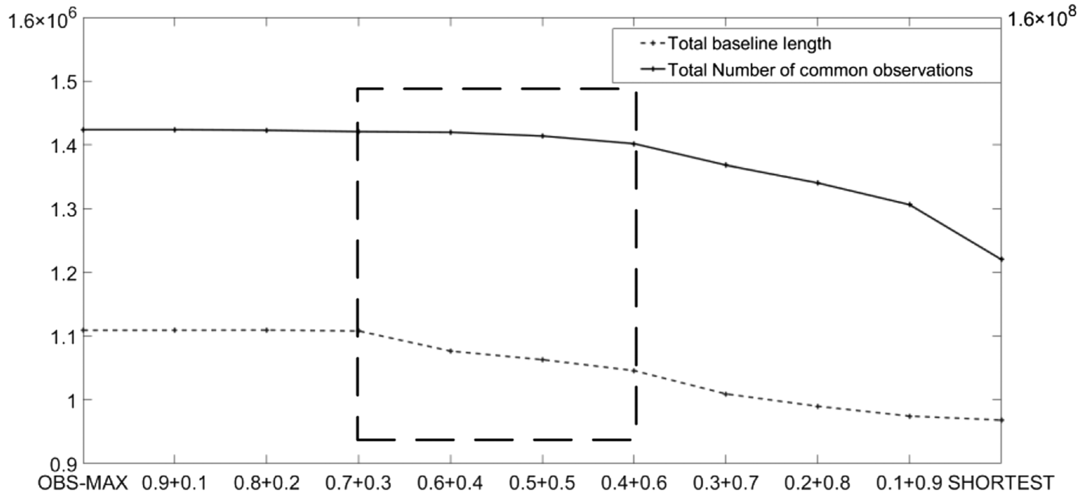

Fig. 4. With proportion changing between OBS-MAX and SHORTEST, the variation in total baseline length and the total number of common observations is presented. The *x* label shows the different weight between OBS-MAX and SHORTEST; the left *y* label shows the total number of common observations; and the right *y* label shows the total length of all the baselines (meters). Note that the numbers on the scale are the same, but they are of different orders of magnitude and unit.

WEIGHT strategy's advantage is that it is capable of adjusting the ratio of each strategy to adapt to different conditions.

Analysis of Variation of Total Common Observations and Baseline Length

A variation of the number of observations and total baseline lengths from the different ratios is shown in Fig. 4. There was a significant decline from $0.4 + 0.6$ to $0.1 + 0.9$ of total common observations, and $0.7 + 0.3$ to $0.3 + 0.7$ of total baseline length. In the overlap part ($0.7 + 0.3$ to $0.4 + 0.6$; notice the broken-line rectangles in Fig. 4), the loss of common observations is negligible and the reduction in baseline length is significant. That is to say, in this overlapping part, we can get both adequate observations and a relatively short baseline, so a better precision is obtained. This theoretical analysis is consistent with the calculation result.

Verification Through 1-Year GNSS Network

The experiment on 98 stations' multibaseline network with global continuously operating reference stations (CORS) stations is extended to one year (366 days) with OBS-MAX, SHORTEST, and WEIGHT strategies. Such an experiment is performed using parallel computation with multicores at the supercomputing center, and the solution time is about 50 min per day.

According to the statistics (Fig. 5) of 366 days, there are 119 days on which the weighted strategy achieves better station coordinate calculation results, as marked by *, while OBS-MAX has 169 days, and OBS-MAX has 73 days. It is therefore concluded that although OBS-MAX takes more data into adjustment, the proportion to achieve the optimal coordinate calculation result is only 47%. Moreover, the WEIGHT strategy is 33%, and the SHORTEST strategy is 20%. This is to say that more common observations do not always yield accurate results, and the final coordinate is also impacted by other factors, such as the atmosphere propagated error cancelled by short baselines.

Fig. 6 shows that almost every error of each direction of coordinate results is around 0.004 m, while there is a gross error at 0.016 appointed by the arrow in the application of OBS-MAX. Compared with SHORTEST, WEIGHT is closer to y label. The

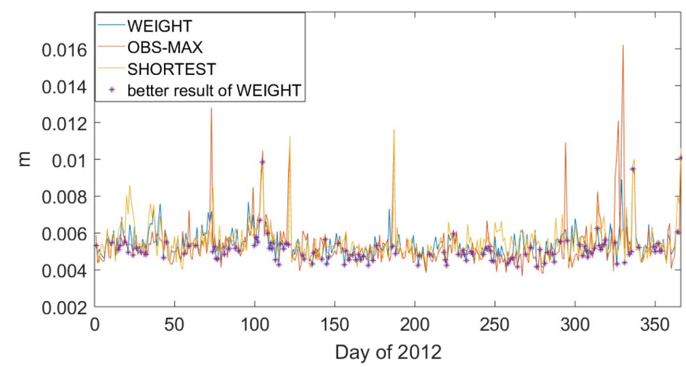

Fig. 5. Results of 1-year's network with 98 stations. The *x*-axis denotes the day of the year in 2012, and the *y*-axis denotes the final RMSE of the station coordinate every day. The different lines represent the WEIGHT, OBS-MAX, and SHORTEST, respectively. The asterisk marks the coordinate calculation results that WEIGHT exhibits the best performance.

final station coordinate result is listed in Table 3, and the calculation accuracy of OBS-MAX is 0.4% lower than WEIGHT, thereby causing error greater than about 0.008 m.

Quantified statistic results are listed in Table 4. The WEIGHT proportion larger than 0.006 m is 14.8%, slightly bigger than that of OBS-MAX, while the proportion of SHORTEST reaches 5.2% percent higher than OBS-MAX and 4.4% higher than WEIGHT. It can be verified that OBS-MAX tends to achieve more accuracy coordinate results. However, in the application of OBS-MAX, the average coordinate result at each direction larger than 0.008 and 0.010 m reaches 4.6% and 2.7%, whereas SHORTEST is 3.0% and 1.1%. Thus, it could be inferred that although OBS-MAX takes more observation data and tends to achieve an accuracy coordinate result, it considers low-quality observations as well (e.g., the common satellite when the baseline is overly long), which causes gross errors in the application of OBS-MAX. More importantly, the atmosphere propagated error is not well canceled. In contrast, the WEIGHT strategy synthetically considers common observations and baseline length, and the result reveals that the proportion that

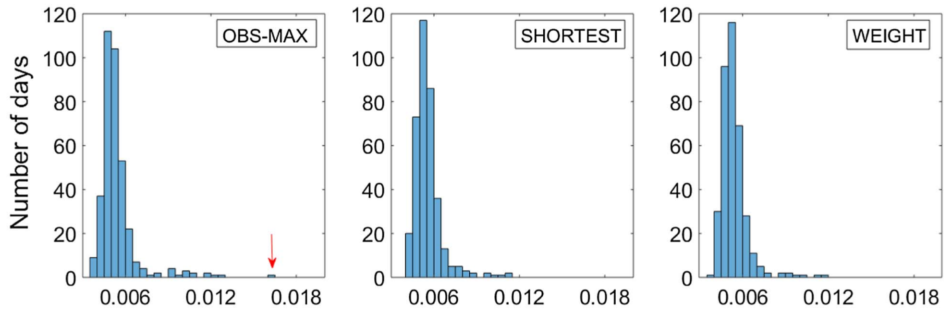

Fig. 6. Error distributions across the three strategies. The x-axis denotes the standard deviation, and the y-axis denotes the total number of days.

Table 3. Final accuracy of each strategy

Strategy	98 stations (m)
OBS-MAX	0.00541
SHORTEST	0.00555
WEIGHT	0.00539

Table 4. Probability statistics for the calculation error of each strategy

	Intervals of the coordinate calculation error (m)		
Strategy	>0.01	>0.008	>0.006
OBS-MAX	2.7%	4.6%	**13.9%**
SHORTEST	1.1%	3.0%	19.1%
WEIGHT	**0.8%**	**2.2%**	14.8%

Note: The smallest numbers in each column are indicated in bold. The smaller the number, the better the experimental results.

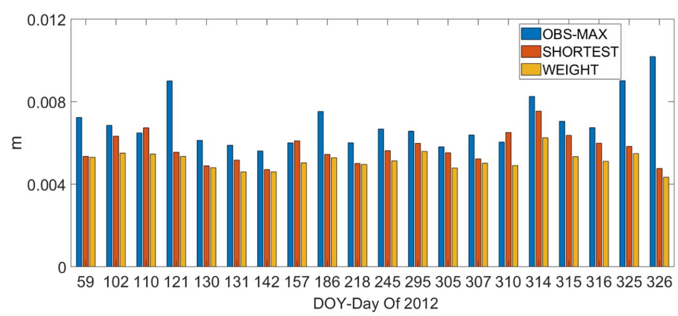

Fig. 7. RMSE of the station coordinates of 20 selected days. Value in x-axis is the number of the day in 2012 and the y-axis is the RMSE of the station coordinate. Different bars on the same day represent different strategies (OBS-MAX, SHORTEST, and WEIGHT, respectively).

is larger than 0.008 and 0.010 m is 2.2% and 0.8%. According to the statistic result, the ability of the WEIGHT strategy to resist gross errors is better than those of OBS-MAX and SHORTEST, while its calculation accuracy is identical to that of OBS-MAX.

More evidence is given in Fig. 7. There are 20 days on which WEIGHT performs significantly better. Note that on the 121, 186, 325, and 326 day of year (DOY)-day, WEIGHT performs better in the presence of a relatively big error in OBS-MAX. Another noteworthy finding is that the observations are distributed unevenly in all the stations. The OBS-MAX strategy attempts to maximize the observations of the globally distributed stations, whereas it can easily lead to local minimum observations; these station errors

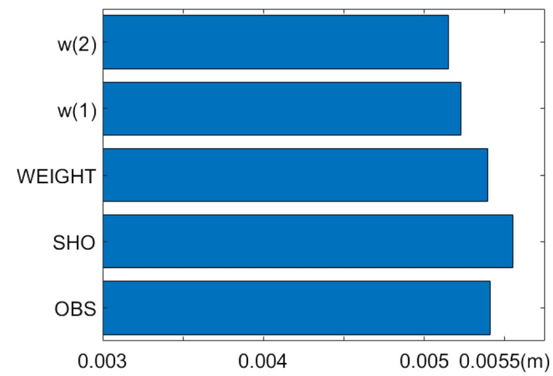

Fig. 8. Final results of the different strategies. The x-axis represents the standard deviation, and the y-axis contains different strategies.

adversely impact the final result. The WEIGHT strategy does not collect much data and keeps a good geometry structure to avoid a local minimum of length and observations.

Comparison of Proposed Weight Strategies

The multibaseline network of $w^{(1)}$ and $w^{(2)}$ of 366 days is calculated (Fig. 8). The RMSE of the SHORTEST is nearly 5% higher than OBS-MAX and WEIGHT. Because $w^{(1)}$ and $w^{(2)}$ consider the error conditions to determine the weight, the proposed method outperforms WEIGHT and OBS-MAX.

Fig. 9 provides the statistics of the optimal strategy searching across 366 days in two types of combination. The OBS-MAX exhibits higher accuracy than SHORTEST in most instances, whereas there are still certain days that the SHORTEST performs better. This figure reveals that $w^{(1)}$ and $w^{(2)}$ perform better in on more days within networks, whereas the result is not for all cases. It is therefore inferred that once the optimal proportion is determined, the calculation accuracy is enhanced by a range of about 5%–15%. Moreover, there is a preliminary conclusion that the proper strategy varies from day to day, and is dependent on the specific condition (e.g., satellite constellation, ionospheric disturbance, and observation quality). However, it is very time-consuming work to compute all the related parameters. If there are no prior statistics mentioned previously, it is feasible to exploit the proposed WEIGHT method.

Conclusion

In the present study, a WEIGHT strategy is proposed to select independent baselines for the GNSS global network. A year's data analysis reveals that the novel strategy is 0.4%–3.0% improved in the

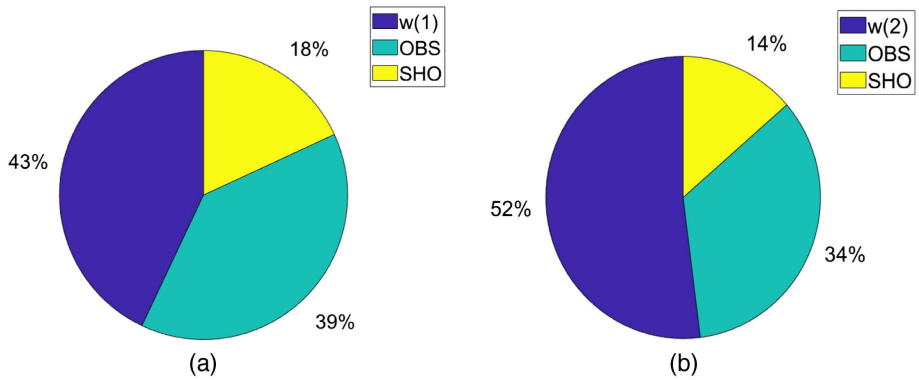

Fig. 9. Proportion of optimal result of some strategies. The (a) sector diagram shows the proportion of WEIGHT [w(1)], OBS-MAX, and SHORTEST; and the (b) sector diagram shows the proportion of WEIGHT [w(2)], OBS-MAX, and SHORTEST.

calculation of station coordinate error compared to the conventional strategies. Also, error distribution is analyzed. The proportion larger than 0.008 m is improved by 0.8% from 1.1% and 2.7%, and the proportion larger than 0.010 m is improved 2.2% from 3.0% to 4.6%.

OBS-MAX and SHORTEST both adopt the MST algorithm to form baselines. They may obtain local optimum based on only one method because a handful of low-quality baselines can be formed, which would significantly pollute the final coordinate result. The WEIGHT strategy can balance these two strategies and generate a more balanced structure; it exhibits better performance in achieving accurate coordinate results and avoiding gross errors. Furthermore, according to concrete conditions, the WEIGHT strategy can be adjusted flexibly by constructing an adaptive weight factor.

Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

The specific data that is available upon request:

1. All the source RINEX file and result files from January 1, 2012;
2. The 1-year coordinate result files of different strategies; and
3. The code for the minimum spanning tree.

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