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# A game theoretical analysis of metro-integrated city logistics systems

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## Abstract

The spare capacity of metro systems during non-peak hours can be utilized to transport parcels or freights, i.e., metro-integrated logistics systems (MILS). Existing studies regarding MILS mainly focused on operational level issues, e.g., parcel distribution problem and service scheduling problem. Little has been done to understand the strategic interactions between metro and logistics operators in the context of MILS and the resulting system-wide impacts. This study conducts a game theoretical analysis of MILS, where a metro company and a logistics company may work either independently or jointly (non-cooperative or cooperative games). In particular, the logistic company decides the number of parcels assigned to MILS, and the metro company controls the price of the MILS service. We examine the decisions of the metro company and the logistics company under different market power regimes, and quantify the system performance. Numerical studies are conducted to illustrate the analytical observations and provide further understanding. Our results show that introducing MILS has the potential to generate Pareto-improving outcomes for the metro company and the logistics company.

*Keywords:* Metro system, Parcel transportation, MILS, Cooperative game, Non-cooperative game

## 1. Introduction

In the era of E-commerce, there is a huge and continuously growing demand for parcel services in many large cities, which contributes to traffic congestion, fuel consumption, and emissions. 'Urban co-modality' has been proposed in this context to make use of existing urban passenger transportation systems to also carry freights/parcels. Urban co-modality advocates the shared use of public transportation systems, such as buses, metros, trams, and light rails, between passengers and freights/parcels. It

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is built upon the consideration that parcels may be transported by the under-utilized mass transit system during off-peak hours, where passenger flows typically can drop by more than 60% (compared to peak hours). The applications of co-modality are still evolving and different terminologies have been used, such as metro-integrated logistics systems or MILS (Liu et al., 2008; Kikuta et al., 2012), light rail freight (Arvidsson, 2010), integrated urban logistics service with bus transportation (Pimentel & Alvelos, 2018), and freight-on-tram systems (Pietrzak & Pietrzak, 2021).

Although co-modal systems are viable from the technological perspective, previous practices (e.g., CityCargo project in Amsterdam, Netherlands in 2007) indicate that financial challenges are the primary factors that hinder the realization of urban co-modality (Marinov et al., 2013). De Langhe et al. (2014) pointed out that the positive marketing is the key to the success of urban co-modality. To the best of our knowledge, existing studies mainly focused on addressing the tactical level and operational level problems of urban co-modality, including the location selection of distribution hubs (Zhao et al., 2018), transformation of transit vehicles and transit stations (Kelly & Marinov, 2017), dispatching and routing problems (Masson et al., 2017; Mourad et al., 2021), service scheduling problems (Behiri et al., 2018), and passenger-freight matching problems (Fatnassi et al., 2015). There is no analytically tractable model to generate strategic level understanding in relation to the interactions between operators in urban co-modality systems, the optimal operation decisions, and the economic feasibility of urban co-modality under different strategic alliances and market structures.[1]

This paper develops a tractable approach in order to provide insights into the complex interactions and optimal operation decisions of a metro-integrated logistics system (MILS). As a first step, we consider the MILS with one metro operator and one logistics service provider (referred to as metro company and logistics company, respectively). We model the strategic interactions between the two companies, where the metro company decides its pricing strategy for the MILS service (carrying freights/parcels) and the logistics company determines the numbers of parcels assigned to the MILS service and the conventional truck service (i.e., the freight/parcel modal-split decision). We consider that the metro company and the logistics company may work either independently or jointly and examine their optimal operation decisions under different market power regimes. In particular, we consider both the cooperative and non-cooperative markets, i.e., Nash bargaining model or Nash arbitration scheme in Nash (1950a), Nash equilibrium, and Stackelberg model. Regarding the Stackelberg model, we consider two scenarios where either the metro company or the logistics company leads (and the other

[1] Urban co-modal systems involve complex interactions among multi-operators and multi-type mixed flows. The mixed passenger and freight service in air transportation has been examined by many studies, which mainly focus on operations strategies of an airline or the competition among airlines (see e.g., Zhang & Zhang 2002; Zhang et al. 2004; Wong et al. 2009). For the co-modality in the city context (or urban co-modality), there is currently no analytically tractable model in the literature to uncover cross-modal interactions among multiple passenger and logistics service operators and the optimal operation decisions of service operators.

follows), i.e., Metro-company-Stackelberg or Logistics-company-Stackelberg model, respectively.

The studied MILS problem is particularly relevant to existing studies on the modal-split and pricing problem in freight transportation.[2] The coexistence of truck/road transportation and MILS indeed resembles a dual-channel distribution problem, where there exist one direct channel (i.e., truck/road transportation) and one indirect channel (i.e., truck-metro intermodal transportation or MILS). The direct/indirect channel selection problems have been examined in the literature of supply chain management, which mainly focused on competitions between channels, such as competitions between supplier and retailer(s) (Chiang et al., 2003; Cai, 2010) and competitions between direct and intermodal freight forwarders (Tamannaei et al., 2021). However, existing dual-channel problems often do not involve an indirect channel or mode based on passenger transit systems that accommodate both passenger and freight flows, where there can be both direct and indirect cross-type demand/flow interactions. For instance, in terms of direct interaction, passengers might be less likely to use the metro services if the metro also carries freights (e.g., due to negative perceptions of the mixed flows); and in terms of indirect interaction, the metro company and the logistics company may change their operation/pricing decisions (under different market regimes) that affect passenger and freight flow patterns and system performance. The current study extends the literature by considering the interactions between parcel/freight flow and passenger flow in the context of MILS with a dual-channel structure.

The main contributions of this paper are twofold. Firstly, this study formulates a novel modal-split and pricing problem in the context of urban co-modality with two modes: the conventional road/truck transportation and the truck-metro transportation (MILS), where both direct and indirect cross-type flow interactions between passengers and freights are considered. Secondly, in the context of co-modality, this study formulates tractable models to characterize the strategic interactions between a metro company and a logistics company under different market power regimes (non-cooperative and cooperative) and generate strategic level understanding in relation to business models and operation/pricing of co-modal systems.

The remainder of this paper is organized as follows. Section 2 introduces the problem and the model setting. Section 3 discusses the non-cooperative and cooperative game models for the MILS. Section 4 conducts numerical studies. Section 5 concludes the paper.

[2] Many studies developed mathematical models to characterize mode choice or modal split behavior for various transportation or logistics sectors, such as business coalition between freight operators (Saeed, 2013), competition between high-speed rail and air transportation (Yang & Zhang, 2012; Tsunoda, 2018), and cold chain shipping (Zhang et al., 2020).

## 2. Model formulation

This section begins with highlighting a few features of MILS (or similar transit-based co-modality applications) that this study aims to capture in the model formulation.

Firstly, MILS involves an intermodal mode (i.e., truck plus metro), and thus the operating cost of transporting parcels involves two modes. The truck mode is used for connections between service point(s) and the metro (transit) stations (Dampier & Marinov, 2015; Zhao et al., 2018), where the operating cost (of connection trips) is borne by the logistics company. The metro (transit) line is used to complete a part of the trip for parcels, where the metro company has to bear the operating cost due to carrying parcels on the metro (Arvidsson & Browne, 2013).

Secondly, while the metro company has to bear the operating cost due to carrying parcels on the metro, it also charges the logistics company service fares. The pricing of MILS service is a critical decision that yields system-wide impacts on the metro company, the logistics company, and the co-modality system (Hu et al., 2020). This indeed motivates the current study to examine the metro company's pricing decision under different market power regimes.

Thirdly, the introduction of MILS will result in mixed flow transit vehicles (i.e., passengers and parcels may have to share the same vehicle). This might cause direct negative impacts on passenger demand due to negative perceptions of the mixed flow (Cochrane et al., 2017). Such direct impact of parcels on the passenger demand will be explicitly considered.

With the above in mind, we are now ready to introduce the basic setting of the problem and the model formulation. In the following, we first describe the modal-split and pricing problem in the context of MILS with the consideration of the direct interaction between parcels and passengers. Then, the analytical conditions of Pareto-improving MILS (where both the metro company and logistics company are incentivized to adopt MILS) are derived. Table 1 summarizes the main notations in this paper. Those not included in Table 1 are specified in the text.

### *2.1. Problem setting*

Consider a generalized intra-city parcel transportation problem as shown in Fig. 1. Parcels are collected from senders and then prepared to be transported from a nearby service point (i.e., "origin service point", denoted as OSP in Fig. 1) to another service point near to the final destination of the parcel (i.e., "destination service point", denoted as DSP in Fig. 1). Recipients can collect their parcels at the destination service point. In the context of urban MILS, courier stores, post offices, parcel locker terminals or convenience stores might be used as OSP and/or DSP. There is a travel corridor connecting OSP and DSP which consists of a road and a parallel metro line (or underground transit line). The total number of parcels to be transported is $N > 0$, which is given. When the metro-integrated logistics system (MILS) is introduced, the logistics company will split the $N$ parcels into two groups: those transported by MILS and those transported by conventional road transportation, i.e., the modal-split for parcels. Let

**Table 1**
Notational glossary

| Symbol | Definition |
|---|---|
| $C_a$ | Cost of conventional road transportation |
| $C_t$ | Cost of connection trips |
| $g_{\mathrm{L}}$ | Fixed operating cost of the logistics company |
| $g_{\mathrm{M}}$ | Fixed operating cost of the metro company |
| $K$ | MILS-related operating cost of the metro company |
| $m$ | Unit price of the MILS service |
| $\tilde{m}$ | Upper bound of unit price of MILS service |
| $N$ | Total number of collected parcels |
| $n_a$ | Number of parcels transported by conventional road transportation |
| $n_b$ | Number of parcels transported by MILS |
| $p$ | Price per parcel unit |
| $Q$ | Metro passenger demand |
| $\tau$ | Metro fare |
| $v_b$ | Capacity of MILS |
| $w$ | Social benefit for transporting one passenger |
| $z_{\mathrm{M}}$ | Net benefit of the metro company |
| $\pi_{\mathrm{L}}$ | Profit of the logistics company |

$n_i$, $i = a, b$, denote the number of parcels transported by mode $i$, where '$a$' denotes conventional road transportation and '$b$' denotes MILS, then $N = n_a + n_b$. The number of parcels transported by MILS should be no greater than the available capacity for parcels in the metro system over the MILS operation duration. Let $v_b$ denote the MILS capacity, and we should have $n_b \leq \min\{v_b, N\}$.

For the $n_b$ parcels assigned to MILS, as shown in Fig. 1, the logistics company has to make connection trips between OSP and the departure metro station, and between the destination metro station and DSP. The connection between parcel senders and OSP, and the connection between DSP and the parcel recipients are not the focus of this study, and thus not considered. Readers interested in first-mile pickup and last-mile delivery problems in the context of the urban logistics network may refer to, e.g., Ghilas et al. (2016), Cattaruzza et al. (2017) and Masson et al. (2017).

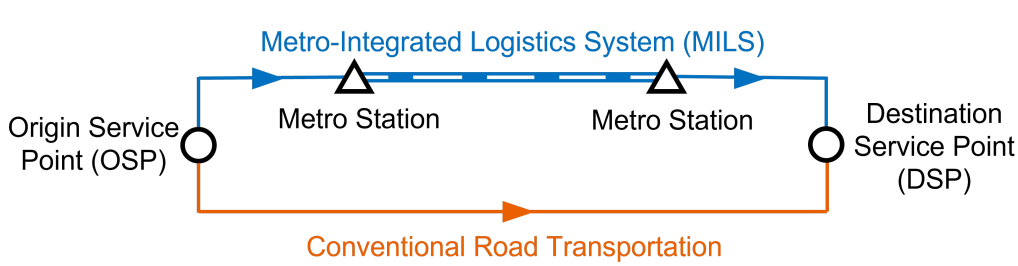



**Fig. 1.** The stylized bi-modal network

As the first step to examine the strategic interactions within MILS, for simplicity, we consider that the metro fare for passengers and metro service frequency are given, which are identical to those before introducing MILS. The MILS operation is during the hours with under-utilized metro capacity that can be potentially used for transporting parcels. Furthermore, a market with only one metro company and one logistics company (sometimes referred to as players later on) is considered.

Let $z_{\mathrm{M}}$ denote the net benefit received by the metro company; and let $\pi_{\mathrm{L}}$ denote the profit received by the logistics company. 'M' and 'L' stand for the metro company and the logistics company, respectively. The net benefit of the metro company can be expressed as:

$$z_{\mathrm{M}} = mn_b + \tau Q(n_b) + wQ(n_b) - K(n_b) - g_{\mathrm{M}} \tag{1}$$

On the right-hand side of Eq. (1), the first term $mn_b$ is the total parcel transportation fare collected from the logistics company, where $m$ is the unit price of the MILS service set by the metro company (AU\$/unit). The second term $\tau Q(n_b)$ represents total fare collected from metro passengers, where $\tau$ is the metro fare and $Q(n_b)$ is the total passenger demand as a decreasing function of $n_b$ (note that $Q$ should be non-negative). $Q(\cdot)$ captures the direct negative impacts of the mixed passenger-parcel flow on passenger demand, where $Q' = dQ/dn_b < 0$. We also assume that $Q(\cdot)$ is concave in $n_b$, i.e., $Q'' = d^2Q/dn_b^2 \leq 0$. Transporting passengers also brings social benefit, which is set as $w$ for each passenger served (AU\$/passenger).[3] The metro company also has a MILS-related operating cost $K(n_b)$ and other operating costs $g_{\mathrm{M}}$. The MILS-related operating cost increases with the number of parcels transported by MILS where $K' = dK/dn_b > 0$ and $K'' = d^2K/d{n_b}^2 > 0$. Other operating cost of the metro system, $g_{\mathrm{M}}$, is assumed to be constant regardless of whether there is MILS in place (e.g., equipment and station maintenance).

The profit of the logistics company can be expressed as:

$$\pi_{\mathrm{L}} = pN - mn_b - C_t(n_b) - C_a(N - n_b) - g_{\mathrm{L}} \tag{2}$$

which depends on the modal-split for parcel transportation. On the right-hand side of Eq. (2), the first term $pN$ is the total fare collected from customers (for the parcel service), where $p$ (AU\$/unit) is the price per parcel unit and $N$ is total number of parcels. $p$ and $N$ are taken as constants and this study focuses on $n_b$ as the decision variable.[4] The logistics company's expenditure on parcel transportation in-

[3]The term $wQ(n_b)$ also can be regarded as the total subsidy from the transportation authority or the government, where $w$ is the subsidy received by serving one passenger trip.

[4]This study focuses on the strategic interactions between the metro company and the logistics company under different market power regimes, but does not consider MILS system's potential positive/negative impacts on the parcel demand from customers. A further study may consider how MILS may further affect the parcel service quality for customers and thus incorporate the parcel demand endogenously in the modeling framework.

cludes the connection trip cost (between service points and metro stations), i.e., $C_t(n_b)$, and the conventional road transportation cost, i.e., $C_a(N-n_b)$. In particular, $C_t' = dC_t\left(n_b\right)/dn_b > 0$, and $C_t'' = d^2C_t\left(n_b\right)/dn_b^2 > 0$. Similarly, $C_a' = dC_a/d\left(N-n_b\right) > 0$ and $C_a'' = d^2C_a/d\left(N-n_b\right)^2 > 0$. Besides, the logistics company has to experience other fixed operating cost $g_{\mathrm{L}}$.

*2.2. Pareto-improving MILS*

While MILS is technologically viable, it does not necessarily mean that the metro company and the logistics company are financially incentivized to participate in such an integrated system. This subsection derives the conditions that both metro company and logistics company are better off or at least not worse off after introducing MILS, i.e., a Pareto-improving situation.

Prior to the introduction of MILS, the net benefit of the metro company is $\tilde{z}_{\mathrm{M}} = (\tau+w)Q(0) - g_{\mathrm{M}}$, where $(\tau+w)Q(0)$ is the sum of social benefit and total fare from serving passengers, $Q(0)$ is the passenger demand without MILS, and $g_{\mathrm{M}}$ is the metro operating cost. These terms are comparable to those in Eq. (1). The logistics company's profit is $\tilde{\pi}_{\mathrm{L}} = pN - C_a(N) - g_{\mathrm{L}}$, where $p$, $N$ and $g_{\mathrm{L}}$ are identical to those in Eq. (2). All parcels are transported through conventional road transportation (e.g., truck) and the cost is $C_a\left(N\right)$. The payoff pair $(\tilde{\pi}_{\mathrm{M}}, \tilde{\pi}_{\mathrm{L}})$ is taken as the status quo point without MILS.

The conditions to ensure that both companies are better off or at least not worse off after introducing MILS are:

$$\begin{aligned} &z_{\mathrm{M}} \geq \tilde{z}_{\mathrm{M}}, \text{ i.e., } mn_b + (\tau+w)Q(n_b) - g_{\mathrm{M}} - K(n_b) \geq (\tau+w)Q(0) - g_{\mathrm{M}} \\ &\pi_{\mathrm{L}} \geq \tilde{\pi}_{\mathrm{L}}, \text{ i.e., } pN - mn_b - C_t\left(n_b\right) - C_a(N-n_b) - g_{\mathrm{L}} \geq pN - C_a(N) - g_{\mathrm{L}} \end{aligned} \tag{3}$$

Multiple combinations of $n_b$ and $m$ may satisfy the above two conditions. From Eq. (3), we can further derive the following ($n_b \neq 0$):

$$m \geq m_{\mathrm{crit_M}}(n_b) = \frac{(\tau+w)\left[Q(0) - Q(n_b)\right] + K\left(n_b\right)}{n_b} \tag{4}$$

$$m \leq m_{\mathrm{crit_L}}(n_b) = \frac{C_a\left(N\right) - C_t\left(n_b\right) - C_a\left(N-n_b\right)}{n_b} \tag{5}$$

where $m = m_{\mathrm{crit_M}}(n_b)$ and $m = m_{\mathrm{crit_L}}(n_b)$ are the two critical unit prices where the metro company's net benefit and the logistics company's profit (later on might be referred to as payoffs of the two companies) remain the same before and after the introduction of MILS, respectively.

To ensure that Eq. (4) and Eq. (5) can simultaneously hold for at least one pair of $n_b$ and $m$, i.e., we can find a pair of $n_b$ and $m$ such that both companies are better off or at least not worse off, the following condition should hold for at least one given value of $n_b$ ($> 0$),

$$m_{\mathrm{crit_L}}(n_b) \geq m_{\mathrm{crit_M}}(n_b) \Leftrightarrow \; C_a(N) \geq F(n_b) \tag{6}$$

where

$$F(n_b) = (\tau + w)\left[Q(0) - Q(n_b)\right] + K(n_b) + C_t(n_b) + C_a(N - n_b) \tag{7}$$

**Remark 1**. Eqs. (6) and (7) indicate that if the sum of the social benefit loss (due to a decrease in passenger demand) and the total cost of freight transportation jointly shared by two companies (i.e., $F(n_b)$) is not more than the total road transportation cost solely covered by the logistics company before the introduction of MILS (i.e., $C_a(N)$), the MILS service can be Pareto-improving for the two companies, i.e., we can find a pair of $(m, n_b)$ where the two companies will be better off or at least not worse off after MILS is introduced.

## 3. Non-cooperative and cooperative games in MILS

This section considers non-cooperative and cooperative market structures for the metro company and the logistics company in the context of MILS, which can potentially yield Pareto-improving outcomes as those discussed in Section 2.2. In particular, we derive and analyze the optimal strategies taken by both companies in the context of the non-cooperative static game (Nash game), the Stackelberg leadership model, and the Nash arbitration scheme (or Nash bargaining model, Nash 1950a).

### *3.1. The non-cooperative static game or Nash equilibrium*

We first consider a non-cooperative market where the two companies choose strategies simultaneously, i.e., the non-cooperative static game or Nash equilibrium (NE) in Nash (1950b).

In the non-cooperative static game, the metro company decides the unit price $m$ for carrying parcels on metro in order to maximize its net benefit, i.e.,

$$\max z_{\mathrm{M}}(m) = mn_b + (\tau + w)Q(n_b) - K(n_b) - g_{\mathrm{M}} \tag{8}$$

subject to

$$0 \leq m \leq \tilde{m} \tag{9}$$

where $\tilde{m}$ is a price bound (e.g., subject to local policies or government regulations). The logistics company decides the modal-split strategy for parcel transportation in order to maximize its profit, i.e.,

$$\max \pi_{\mathrm{L}}(n_b) = pN - mn_b - C_t(n_b) - C_a(N - n_b) - g_{\mathrm{L}} \tag{10}$$

subject to

$$0 \leq n_b \leq \min\{v_b, N\} \tag{11}$$

where $v_b$ is the available capacity for parcels in the metro system over the MILS operation duration. Consider an interior solution $n_b^{\mathrm{NE}} < \min\{v_b, N\}$, we can derive the optimality condition for the above non-cooperative static game as follows:

$$m^{\mathrm{NE}} = \tilde{m} \tag{12a}$$

$$n_b^{\mathrm{NE}} : C_a' - m - C_t' = 0 \tag{12b}$$

As can be seen from Eq. (12), at NE the metro company sets its price $m$ to the allowed maximum (or upper bound), while logistics company has to balance the marginal saving and cost when deciding $n_b$. In particular, if there is a marginal increase in the number of parcels assigned to MILS, the marginal expenditure saving on conventional road transportation (i.e., $C_a'$) offsets the marginal expenditure increment on MILS services (the sum of MILS service fare and connection trip cost, i.e., $m + C_t'$).

**Proposition 1.** *There exists a unique Nash equilibrium (NE) solution under the non-cooperative static game for the metro company and the logistics company.*

**Proof.** The proof is given in Appendix A.1. □

Based on the optimality conditions presented in Eq. (12), the effect of a marginal change in the bound $\tilde{m}$ on the optimal strategies of both companies under NE can be derived, i.e.,

$$\frac{dm^{\mathrm{NE}}}{d\tilde{m}} = 1 > 0; \frac{dn_b^{\mathrm{NE}}}{d\tilde{m}} = -\frac{1}{C_t'' + C_a''} < 0 \tag{13}$$

Eq. (13) indicates that the optimal number of parcels transported by MILS decreases with the pricing bound.

By applying the point elasticity, we can rewrite Eq. (12) as follows

$$\begin{aligned} & m^{\mathrm{NE}} = \tilde{m} \\ & \sigma_{n_a}^{C_a} \frac{C_a}{(N - n_b)} - m - \sigma_{n_b}^{C_t} \frac{C_t}{n_b} = 0 \end{aligned} \tag{14}$$

where $\sigma_x^y$ is the point elasticity of $y$ with respect to $x$, i.e., $\sigma_x^y = \frac{\partial y}{\partial x}\frac{x}{y}$. One can then solve that

$$
\begin{aligned}
&m^{\mathrm{NE}} = \tilde{m} \\
&n_b^{\mathrm{NE}} = \frac{\delta}{2\tilde{m}} \\
&z_{\mathrm{M}}^{\mathrm{NE}} = \frac{1}{2}\delta + (\tau + w)Q - K - g_{\mathrm{M}} \\
&\pi_{\mathrm{L}}^{\mathrm{NE}} = -\frac{1}{2}\delta + pN - g_{\mathrm{L}} - C_t - C_a \\
&\text{with } \delta = \tilde{m}N - \sigma_{n_a}^{C_a}C_a - \sigma_{n_b}^{C_t}C_t + \sqrt{\left(\sigma_{n_a}^{C_a}C_a + \sigma_{n_b}^{C_t}C_t - \tilde{m}N\right)^2 + 4\tilde{m}\sigma_{n_b}^{C_t}C_tN} > 0
\end{aligned} \tag{15}
$$

By comparing $z_{\mathrm{M}}^{\mathrm{NE}}$ and $\pi_{\mathrm{L}}^{\mathrm{NE}}$ for NE with $\tilde{z}_{\mathrm{M}}$ and $\tilde{\pi}_{\mathrm{L}}$ for the status quo, we have

$$
z_{\mathrm{M}}^{\mathrm{NE}} - \tilde{z}_M = \frac{1}{2}\delta + (\tau + w)[Q - Q(0)] - K \tag{16a}
$$

$$
\pi_{\mathrm{L}}^{\mathrm{NE}} - \tilde{\pi}_{\mathrm{L}} = C_a(N) - \frac{1}{2}\delta - C_t - C_a \tag{16b}
$$

From Eq. (15) and Eq. (16), one can verify that a small price bound $\tilde{m}$ can yield a small $\delta$. It follows that the logistics company is more likely to have additional profit after introducing MILS. Note that $C_a(N) - C_t - C_a > 0$ is expected, which means that MILS will bring savings on the total road transportation cost (for transporting parcels) since less ground transportation efforts are needed; otherwise, the logistics company will never be incentivized to use MILS. Under a small $\tilde{m}$, when compared to the status quo, while the logistics company is more likely to benefit from MILS, the metro company is less likely to benefit from MILS. We will numerically examine different pricing bounds in Section 4.

## *3.2. The non-cooperative Stackelberg model*

This section considers two cases for the Stackelberg leadership model (Von Stackelberg, 1934), i.e., the metro company leads (Metro-company-Stackelberg model) and the logistics company leads (Logistics-company-Stackelberg model).

### *3.2.1. Logistics-company-Stackelberg model*

In the Logistics-company-Stackelberg model, the logistics company is the leader and the metro company is the follower. In the context of MILS, a part of the parcel transportation is outsourced to the metro company. The metro company can be regarded as a 'second-party logistics provider' (2LP), and is responsible for the parcel transportation between metro stations. The logistics company is similar to a 'manufacturer' in the traditional supply chain analysis who outsources the parcel transportation. The 'manufacturer-like' logistics company might have greater market power than the '2LP-like' metro company, which might occur when a large logistics company dominates the local market (e.g., SF Express and DHL) who partially outsources its parcel or freight transportation to the metro company.

The backward induction can be used to obtain the Logistics-company-Stackelberg equilibrium (LSE). In particular, the metro company (follower) solves the following optimization problem under any given parcel modal-split strategy of the logistics company (i.e., given $n_b \in [0, N]$):

$$\max z_{\mathrm{M}}(m|n_b) = mn_b + (\tau + w)Q(\tau, n_b) - K(n_b) - g_{\mathrm{M}} \tag{17}$$

subject to $0 \leq m \leq \tilde{m}$. The logistics company (leader) solves the following optimization problem with full information of the pricing set by the metro operator (i.e., $m^*$, the metro company's best response is known to the logistics company):

$$\max \pi_{\mathrm{L}}\left(n_b|m^*\right) = pN - m^* n_b - C_t\left(n_b\right) - C_a\left(N - n_b\right) - g_{\mathrm{L}} \tag{18}$$

subject to $0 \leq n_b \leq N$.

We then can derive the following regarding the solution to the Logistics-company-Stackelberg model (i.e., LSE):

$$m^*(n_b) = \tilde{m} \tag{19}$$

$$n_b^{\mathrm{LSE}} : C_a' - m^* - C_t' = 0 \tag{20}$$

where $m^*(n_b)$ is the best response function of the metro company given the logistics company's parcel modal-split strategy $n_b \in [0, N]$, and an interior solution for $n_b^{\mathrm{LSE}}$ is assumed. Eq. (19) says that the metro company's (follower's) best response is always setting the unit price of MILS service to the allowed maximum (upper bound) regardless of the logistics company's modal-split strategy for parcel transportation. This is identical to the solution in Eq. (12a) for the NE. Eq. (20) states that at the optimum, if there is a marginal increase in the number of parcels assigned to MILS, the marginal saving on the expenditure on the conventional road transportation (i.e., $C_a'$) offsets the marginal cost of using the MILS service (i.e., $m^* + C_t'$). This is similar to the solution in Eq. (12b) for UE. However, it is noteworthy that the optimality condition for $n_b^{\mathrm{LSE}}$ incorporates the metro company's best response $m^*$ while the optimality condition for $n_b^{\mathrm{NE}}$ only involves $m$. Such a difference indicates that unlike the non-cooperative static game, under the Stackelberg game, as a leader, the logistics company is able to utilize the information about the entire sequential game. However, since the optimal prices in Eq. (19) and Eq. (12a) are identical, the UE and LSE solutions are identical. It follows that properties and discussions of the UE solution also hold for the LSE solution, which are omitted (except the uniqueness result stated below).

**Proposition 2.** *There exists a unique solution (i.e., LSE) to the Logistics-company-*

*Stackelberg model.*

**Proof.** The proof is given in Appendix A.2. □

### 3.2.2. Metro-company-Stackelberg model

In the Metro-company-Stackelberg model, the metro company is the leader and the logistics company is the follower. For instance, a locally-operated small intra-city parcel delivery company may have no market power when compared to the metro company, while the monopolistic metro company of the whole city dominates the market.

Similarly, we can examine the Metro-company-Stackelberg equilibrium (MSE) via backward induction. In particular, given the metro company's MILS pricing scheme $m \in [0, \tilde{m}]$, the logistics company (follower) solves the following problem:

$$\max \pi_{\mathrm{L}} (n_b | m) = pN - mn_b - C_t (n_b) - C_a (N - n_b) - g_{\mathrm{L}}, \tag{21}$$

subject to $0 \leq n_b \leq \min\{v_b, N\}$. The metro company as the leader is able to incorporate the modal-split strategy of the logistics company and maximizes its net benefit, i.e., it solves:

$$\max z_{\mathrm{M}}(m|n_b^*(m)) = mn_b^*(m) + (\tau + w)Q(n_b^*(m)) - K(n_b^*(m)) - g_{\mathrm{M}}, \tag{22}$$

subject to $0 \leq m \leq \tilde{m}$, where $n_b^*(m)$ is the best response function of the logistics company given the leader's (metro company's) strategy $m \in [0, \tilde{m}]$.

Considering an interior solution for both $n_b$ and $m$, we can derive:

$$C_a' - m - C_t' = 0. \tag{23}$$

$$n_b + \frac{dn_b^*}{dm} \left[ (\tau + w)Q' - K' \right] + m\frac{dn_b^*}{dm} = 0. \tag{24}$$

By comparing Eqs. (12b) and (23), it can be seen that the first-order condition for $n_b$ in the Metro-company-Stackelberg model is identical to that in the non-cooperative static game model. This is because, under the Metro-company-Stackelberg game, as the follower, the logistics company is unable to predict leader's strategic move since information is not available. Eq. (24) indicates that at the interior optimum, the sum of the marginal profit from MILS services (due to an increased MILS price) and the marginal net benefit gain (due to an increase in passenger volume) together offset the profit loss caused by fewer parcels transported by MILS; and such a reduced number of parcels reflects the logistics company's response to an increased MILS price (as will be further discussed by Eq. (25)). Being a leader, the metro company takes into account the full information of this sequential game (note that if the pricing bound $\tilde{m}$ is too small, a corner solution may occur where $m^{\mathrm{MSE}} = \tilde{m}$, then MSE solution will be identical to those at NE and LSE).

Based on Eq. (23), by applying the implicit function theorem, we can further derive the following:

$$\frac{dn_b^*}{dm} = -\frac{1}{C_a'' + C_t''} < 0. \tag{25}$$

Eq. (25) means that the optimal number of parcels transported by MILS decreases with respect to the MILS service price set by the leader, i.e., the best response function $n_b^{\text{MSE}} = n_b^*(m)$ is decreasing with respect to $m$.

By combining Eq. (24) and Eq. (25), we can further derive the optimal MILS price $m^{\text{MSE}}$ as follows:

$$m^{\text{MSE}} = [K' - (\tau + w)Q'] + n_b^* \left(C_a'' + C_t''\right) \tag{26}$$

Since $C_a'' + C_t'' > 0$ and $n_b^* > 0$, we have $m^{\text{MSE}} > K' - (\tau + w)Q'$, or equivalently, $m^{\text{MSE}} + (\tau + w)Q' > K'$. This means that, at MSE, for the metro company the marginal revenue from a marginal increase in $n_b$ is greater than the marginal operating cost.

Moreover, since $m^{\text{MSE}} > K' - (\tau + w)Q'$, based on Eq. (24), we can derive that $\sigma_m^{n_b} < -1$, where $\sigma_m^{n_b} = \frac{\partial n_b}{\partial m}\frac{m}{n_b}$, i.e., the elasticity of $n_b$ with respect to $m$. This indicates that at the interior MSE solution, the system is operating at the elastic portion of the price-demand curve (MILS service demand).

**Proposition 3.** *There exists a unique solution (i.e., MSE) to the Metro-company-Stackelberg model.*

**Proof.** The proof is similar to that for Proposition 2, which is omitted. □

Similarly, we can rewrite Eq. (23) and Eq. (24) with relevant point elasticities, and further derive that

$$\begin{aligned}
&m^{\text{MSE}} = \theta \cdot \frac{\theta + \sigma_{n_b}^{C_t} C_t + \sigma_{n_a}^{C_a} C_a}{N\left(\theta + \sigma_{n_b}^{C_t} C_t\right)} \\
&n_b^{\text{MSE}} = N \cdot \frac{\theta + \sigma_{n_b}^{C_t} C_t}{\theta + \sigma_{n_b}^{C_t} C_t + \sigma_{n_a}^{C_a} C_a} \\
&z_{\text{M}}^{\text{MSE}} = \left(1 - \frac{\sigma_m^{n_b}}{\sigma_m^{n_b} + 1}\sigma_{n_b}^{Q}\right)(\tau + w)Q - \left(1 - \frac{\sigma_m^{n_b}}{\sigma_m^{n_b} + 1}\sigma_{n_b}^{K}\right)K - g_{\text{M}} \\
&\pi_{\text{L}}^{\text{MSE}} = -\theta + pN - g_{\text{L}} - C_t - C_a \\
&\text{with } \theta = \frac{\sigma_m^{n_b}}{\sigma_m^{n_b} + 1}\left[\sigma_{n_b}^{K} K - (\tau + w)\sigma_{n_b}^{Q} Q\right] > 0
\end{aligned} \tag{27}$$

where $\sigma_x^y$ is the point elasticity of $y$ with respect to $x$, i.e., $\sigma_x^y = \frac{\partial y}{\partial x}\frac{x}{y}$.

Eq. (27) further verifies that at the Stackelberg equilibrium, the metro company fully utilizes the information about the logistics company's on-road parcel transportation cost ($C_t$ and $C_a$) and its best response function when setting the optimal MILS price. The leadership with full information also enables the metro company to set an optimal MILS price such that the operating cost $K$ arising from providing MILS might be fully covered by the total parcel transportation fare collected from the logistics company.

We now compare $z_{\mathrm{M}}^{\mathrm{MSE}}$ and $\pi_{\mathrm{L}}^{\mathrm{MSE}}$ for MSE with $\tilde{z}_{\mathrm{M}}$ and $\tilde{\pi}_{\mathrm{L}}$ for the status quo, where we have

$$z_{\mathrm{M}}^{\mathrm{MSE}} - \tilde{z}_M = \left(1 - \frac{\sigma_m^{n_b}}{\sigma_m^{n_b}+1}\sigma_{n_b}^{Q}\right)(\tau+w)Q - \left(1 - \frac{\sigma_m^{n_b}}{\sigma_m^{n_b}+1}\sigma_{n_b}^{K}\right)K - (\tau+w)Q(0) \tag{28a}$$

$$\pi_{\mathrm{L}}^{\mathrm{MSE}} - \tilde{\pi}_{\mathrm{L}} = C_a(N) - \frac{\sigma_m^{n_b}}{\sigma_m^{n_b}+1}\left[\sigma_{n_b}^{K}K - (\tau+w)\sigma_{n_b}^{Q}Q\right] - C_t - C_a \tag{28b}$$

As can be seen from Eq. (28a), since $\sigma_m^{n_b} < -1$ (as discussed in the above), when $\sigma_{n_b}^{Q} < -\frac{\sigma_m^{n_b}+1}{\sigma_m^{n_b}}\left[\frac{Q(0)}{Q(n_b^{\mathrm{MSE}})} - 1\right]$ $(< 0)$ and $\sigma_{n_b}^{K} > \frac{\sigma_m^{n_b}+1}{\sigma_m^{n_b}}$ $(> 0)$, the metro company will benefit from introducing MILS (i.e., $z_{\mathrm{M}}^{\mathrm{MSE}} - \tilde{z}_M > 0$ will occur). Given that $\sigma_m^{n_b} < -1$, one can further verify if $\sigma_{n_b}^{Q} \leq -\left[\frac{Q(0)}{Q(n_b^{\mathrm{MSE}})} - 1\right]$ $(< 0)$ and $\sigma_{n_b}^{K} \geq 1$, MILS will bring the metro company additional benefit, i.e., $z_{\mathrm{M}}^{\mathrm{MSE}} - \tilde{z}_M > 0$.

Eq. (28b) indicates whether the logistics company will gain additional profit after introducing MILS (under MSE) are affected by how the metro company's MILS operating cost and the passenger demand vary against the number of parcels assigned to MILS (i.e., $\sigma_{n_b}^{K}$ and $\sigma_{n_b}^{Q}$), and the logistics company's response to the MILS price (i.e., $\sigma_m^{n_b}$). Again, $C_a(N) - C_t - C_a > 0$ since less ground transportation efforts are needed after introducing and utilizing the MILS. It can be verified that under a smaller $\sigma_{n_b}^{K}$ (carrying parcels is less costly), a larger $\sigma_{n_b}^{Q}$ (carrying parcels has less negative impact on passenger demand), a smaller $\sigma_m^{n_b}$ (the optimal MILS parcel volume $n_b$ is more sensitive to the optimal price $m$), the logistics company is more likely to benefit from introducing MILS (under the MSE). Also note that, when compared to NE (or LSE, refer to Eq. (16)), an interior optimal $m^{\mathrm{MSE}}$ means more flexibility for the metro company to benefit from introducing MILS.

### *3.3. Cooperative games in MILS*

This section investigates how well the metro company and the logistics company can be positioned against the non-cooperative scenarios if they choose to cooperate based on the Nash arbitration scheme (or Nash bargaining model) proposed by Nash (1950a).

In particular, the Nash arbitrated solution (NAS) can be obtained by solving the following optimization problem:

$$\begin{aligned}\max_{m,n_b} U &= (z_{\mathrm{M}} - \tilde{z}_{\mathrm{M}})(\pi_{\mathrm{L}} - \tilde{\pi}_{\mathrm{L}}) \\ &= \{mn_b + (\tau+w)[Q(n_b) - Q(0)] - K(n_b)\} \cdot [C_a(N) - mn_b - C_t(n_b) - C_a(n_b)]\end{aligned} \tag{29}$$

subject to

$$
\begin{aligned}
&0 \le m \le \tilde{m} \\
&0 \le n_b \le \min\{v_b, N\}
\end{aligned}
\tag{30}
$$

where $(\tilde{z}_{\text{M}}, \tilde{\pi}_{\text{L}})$ as the status quo before introducing MILS (refer to Section 2.2) is taken as the disagreement point in the Nash arbitration scheme. The disagreement point might be set as the Nash equilibrium, Metro- or Logistics-company-Stackelberg equilibrium, which will be examined in the numerical studies in Section 4.3.

The problem in Eqs. (29)-(30) jointly optimizes the MILS pricing strategy $m$ and modal-split strategy for parcel transportation $n_b$, which reflects the cooperation between the metro company and the logistics company. We can derive the optimality conditions for an interior solution as follows:

$$
n_b^{\text{NAS}} : C_a' - C_t' + (\tau + w)Q' - K' = 0 \tag{31}
$$

$$
\begin{aligned}
m^{\text{NAS}} &= \frac{C_a(N) - C_t(n_b^{\text{NAS}}) - C_a(N - n_b^{\text{NAS}}) + (\tau + w)[Q(0) - Q(n_b^{\text{NAS}})] + K(n_b^{\text{NAS}})}{2n_b^{\text{NAS}}} \\
&= \frac{m_{\text{crit}_\text{M}}(n_b^{\text{NAS}}) + m_{\text{crit}_\text{L}}(n_b^{\text{NAS}})}{2}
\end{aligned}
\tag{32}
$$

Eq. (31) indicates that at NAS, the marginal conventional road transportation cost has to offset the marginal MILS operating cost which is jointly shared by both metro company and logistics company (i.e., the sum of the connection trip cost, the MILS-related operating cost borne by the metro company and the total loss of net benefit due to the decrease in passenger demand) if there is a marginal increase in the number of parcels transported by MILS. At NAS, both companies' payoff formulations are considered in the decision-making, which results in the presence of not only the marginal impact of MILS-related underground operating cost $K'$ but also the marginal impact of passenger demand $Q'$ in the first-order condition with respect to $n_b^{\text{NAS}}$.

As can be seen from Eq. (32), the optimal MILS pricing strategy $m^{\text{NAS}}$ at NAS is a single-valued function of $n_b^{\text{NAS}}$ characterized by Eq. (31). One can see that the interior solution $m^{\text{NAS}}$ is exactly the average of the two critical unit prices at which the metro company's and logistics company's payoffs remain the same before and after the implementation of MILS if $n_b^{\text{NAS}}$ units of parcels are assigned to MILS, respectively, i.e., $m_{\text{crit}_\text{M}}(n_b^{\text{NAS}})$ and $m_{\text{crit}_\text{L}}(n_b^{\text{NAS}})$ (refer to Eqs. (4) and (5)). By comparing $m^{\text{NAS}}$ and the optimal pricing strategies under non-cooperative games, it can be seen that the pricing strategy at NAS yields a coordinated solution for both companies, leading to a Pareto optimal outcome ($m^{\text{NAS}}$ is between the two critical values, which is consistent with those conditions in Section 2.2 for a Pareto-improving MILS).

By applying the point elasticity in Eq. (31), one can further derive that

$$n_b^{\mathrm{NAS}} = N \cdot \frac{\sigma_{n_b}^{C_t} C_t - \sigma_{n_b}^{Q} Q(\tau + w) + \sigma_{n_b}^{K} K}{\sigma_{n_a}^{C_a} C_a + \sigma_{n_b}^{C_t} C_t - \sigma_{n_b}^{Q} Q(\tau + w) + \sigma_{n_b}^{K} K} \tag{33}$$

$$N - n_b^{\mathrm{NAS}} = N \cdot \frac{\sigma_{n_a}^{C_a} C_a}{\sigma_{n_a}^{C_a} C_a + \sigma_{n_b}^{C_t} C_t - \sigma_{n_b}^{Q} Q(\tau + w) + \sigma_{n_b}^{K} K} \tag{34}$$

Eq. (33) for NAS is different from those in NE, LSE and MSE, where here $n_b^{\mathrm{NAS}}$ incorporates the cost/demand functions in relation to both companies. This again highlights the cooperation between the metro company and logistics company. Eq. (33) and Eq. (34) together also reflect the consistency and relationship between the optimal freight modal-split and the costs/benefits related to the two modes.

**Remark 2.** By substituting the optimal strategy pair $(m^{\mathrm{NAS}}, n_b^{\mathrm{NAS}})$, where $m^{\mathrm{NAS}}$ is given by Eq. (32), into both companies' payoff functions, we have the following observation:

$$\begin{aligned} & z_{\mathrm{M}}^{\mathrm{NAS}} - \tilde{z}_{\mathrm{M}} = \pi_{\mathrm{L}}^{\mathrm{NAS}} - \tilde{\pi}_{\mathrm{L}} \\ & = \frac{1}{2} \left\{ C_a(N) - (\tau + w) \left[ Q(0) - Q(n_b^{\mathrm{NAS}}) \right] - K(n_b^{\mathrm{NAS}}) - C_t(n_b^{\mathrm{NAS}}) - C_a(N - n_b^{\mathrm{NAS}}) \right\} \end{aligned} \tag{35}$$

where Eq. (35) indicates that the additional gains of both companies after introducing MILS are equal under the Nash arbitration scheme.

## 4. Numerical studies

This section presents numerical studies to illustrate the proposed model and analysis. We firstly detail the numerical setting, and then illustrate and compare game outcomes under different market power regimes and pricing bounds. The Nash arbitrated solutions with respect to different disagreement points are also analyzed.

### *4.1. Numerical setting*

This subsection summarizes the numerical setting. Function specifications are given in Table 2 (which are assumed) and parameter values are presented in Table 3. The numerical studies are constructed in the context where the parcels are transported from a Sydney suburb, Hurstville, to Sydney central business district using the Sydney T4 Train Line. Please refer to the notes in Table 3 for the sources of numerical setting.

The metro passenger demand function that describes the direct interaction between passengers and parcels in the metro system, i.e., $Q(n_b)$ shown in Table 2, is derived according to the notion of supply-demand equilibrium, where the demand curve $D(\cdot)$

and the supply curve describing the generalized cost of a metro passenger $C_p(\cdot)$ are written as:

$$x = D(c_p) = x_m - e_1 c_p \tag{36}$$

$$c_p = C_p(x) = \tau + \rho\left[t_0 + \frac{1}{2f_m} + t_1\left(\frac{x}{v - \varphi n_b}\right)\right] \tag{37}$$

where $x$ is the metro passenger demand, $x_m$ is the potential passenger demand (constant), and $c_p$ is the generalized cost of the metro service. In particular, the demand function $D(\cdot)$ is linearly decreasing with $c_p$. Regarding the generalized cost function, it consists of the metro fare $\tau$ and the non-monetary cost term $\rho[t_0 + t_1(x/(v - \varphi n_b))]$, where $\rho$ is the value of time (assuming passengers are homogeneous), $t_0$ is the travel time between two metro stations, and $1/(2f_m)$ is the average service waiting time. To characterize the negative impact of the presence of the parcels on the transit service (i.e., the direct interaction between passengers and parcels), we include the term $t_1(x/(v - \varphi n_b))$ that captures congestion delay due to boarding and alighting which is dependent on the passenger demand $x$ and available capacity for passengers (i.e., $v - \varphi n_b$), where $v$ is the total metro capacity and $t_1$ is the coefficient of crowding. We assume that one passenger is equivalent to two parcel units, i.e., $\varphi = 0.5$. The term $(v - \varphi n_b)$ captures the direct interaction between parcels and passengers (e.g., crowding effect), where a larger number of parcels will yield fewer passengers. Based on Eqs. (36) and (37), one can readily derive the metro passenger demand $\bar{x}(n_b) = Q(n_b)$ as a function of $n_b$ based on $D^{-1}(x) = C_p(x)$ ($D^{-1}$ is the inverse demand function). Additionally, $Q(n_b)$ in Table 2 is strictly concave with $n_b$, which is consistent with the setting of $Q(n_b)$ in the analytical model (refer to Section 2).

**Table 2**
Function specifications

| Function | Specification |
|---|---|
| Metro passenger demand | $Q(n_b) = \left[\frac{1}{e_1}x_m - \rho\left(t_0 + \frac{1}{2f_m}\right) - \tau\right]\left(\frac{\rho t_1}{v - \varphi n_b} + \frac{1}{e_1}\right)^{-1}$ |
| MILS-related operating cost of the metro company | $K(n_b) = k_2 n_b^2 + k_1 n_b + k_0$ |
| Connection trip cost | $C_t(n_b) = y_2 l n_b^2 + \left(h_1 l + h_2 \frac{l}{s_a}\right)\frac{n_b}{v_a}$ |
| Conventional transportation cost | $C_a(N - n_b) = y_1 d(N - n_b)^2 + \left(h_1 d + h_2 \frac{d}{s_a}\right)\frac{N - n_b}{v_a}$ |

### *4.2. Comparison for different market equilibrium points*

We start from analyzing the game outcomes under a small pricing bound $\tilde{m} = 0.5$ AU\$/unit and a large pricing bound $\tilde{m} = 1.0$ AU\$/unit. Table 4 shows the non-cooperative and cooperative game outcomes with respect to the two bounds. Note that the solution to the cooperative game (i.e., the Nash arbitrated solution, NAS) shown in Table 4 is computed based on the status quo point (without MILS) in order to illustrate

**Table 3**
Summary of numerical settings

| Variable | Description | Value |
|---|---|---|
| $d$ | Distance between origin and destination service points | 20 km |
| $e_1$ | Coefficient in the metro company's passenger demand function | 2 |
| $f_m$ | Service frequency [i] | 4 services/h |
| $g_{\mathrm{L}}$ | Fixed operating cost of the logistics company | $5 \times 10^3$AU$/day |
| $g_{\mathrm{M}}$ | Fixed operating cost of the metro company | $2.5 \times 10^4$AU$/day |
| $h_1$ | Distance-based operating cost (fuels and maintenance) [ii] | 0.6 AU$/km |
| $h_2$ | Driver's hourly wage [iii] | 30 AU$/h |
| $k_0$ | Sink cost for MILS operation | 40 AU$/day |
| $k_1$ | Operating cost per each parcel unit | 0.1 AU$·unit$^{-1}$ · day$^{-1}$ |
| $k_2$ | Coefficient in MILS operating cost | $10^{-5}$ AU$·unit$^{-2}$ · day$^{-1}$ |
| $l$ | Total connection trip length | 3 km |
| $N$ | Total daily parcel demand | 20000 parcel units/day |
| $p$ | Price per parcel unit | 5 AU$/parcel unit |
| $\varphi$ | Passenger-parcel unit converting coefficient (homogeneous) | 0.5 passenger/parcel unit |
| $\rho$ | Metro user's value of time (homogeneous for all transit users) [iv] | 24.8 AU$/h |
| $s_a$ | Average speed of truck | 50 km/h |
| $t_0$ | Free flow travel time between two metro stations [i] | 0.375 h |
| $t_1$ | Coefficient of crowding | 0.1 h |
| $T_m$ | MILS operating duration [v] | 5 h |
| $\tau$ | Metro fare [vi] | 3.2 AU$/passenger |
| $v_a$ | Capacity of a truck | 30 parcel units/truck |
| $v_b$ | Capacity of MILS | 36000 parcel units |
| $v_m$ | Total metro capacity of each service [vii] | 900 passengers/service |
| $v$ | Total metro capacity over the entire MILS operating duration | 18000 passengers |
| $w$ | Social benefit for transporting one passenger | 0.1 AU$/passenger |
| $x_m$ | Potential metro passenger demand [i] | 6000 passengers |
| $y_1$ | Coefficient in conventional road transportation trip costs | $1 \times 10^{-6}$ AU$·unit$^{-2}$ · km$^{-1}$ |
| $y_2$ | Coefficient in connection trip costs | $6 \times 10^{-6}$ AU$·unit$^{-2}$ · km$^{-1}$ |

Notes: [i] The specifications regarding Sydney T4 transit services is based on smart transit card data and General Transit Feed Specification data (`https://opendata.transport.nsw.gov.au/dataset/timetables-complete-gtfs`). [ii] The distance-based operating cost for a is extracted from the Transport for NSW Technical Note on Calculating Road Vehicle Operating Costs (`https://www.transport.nsw.gov.au/news-and-events/reports-and-publications/transport-for-nsw-technical-note-on-calculating-road`). [iii] The driver's hourly wage is estimated by `https://au.indeed.com/career/driver/salaries`. [iv] The transit users' value of time is extracted from the Economic Parameter Values report published by Transport for NSW (`https://opendata.transport.nsw.gov.au/dataset/timetables-complete-gtfs`). [v] The MILS operation duration is set based on the length of the Sydney off-peak hour (10am - 3pm). [vi] The metro (transit) fare is calculated based on the adult fares from Hurstville station to Central station (`https://transportnsw.info/tickets-opal/opal/fares-payments/adult-fares`). [vii] The capacity of metro service is based on the seating specification of Waratah trains serving the Sydney T4 line (`https://www.railway-technology.com`).

the benchmark bargaining scenario (labelled as 'NAS-SQ'). Besides, as discussed in Section 3.2.1, the Logistics-company-Stackelberg equilibrium (LSE) coincides with the Nash equilibrium (NE).

We now summarize the main observations from Table 4. (i) When we have a small price bound ($\tilde{m} = 0.5$ AU$/unit), different non-cooperative games yield the identical outcome (and the optimal price equals the bound). A small bound encourages the logistics company to utilize MILS services more (i.e., $n_b^*$ is larger), and yield a larger total system revenue ($z_{\mathrm{M}} + \pi_{\mathrm{L}}$). (ii) When metro company leads (i.e., MSE) and the bound is large (here $\tilde{m} = 1.0$ AU\$/unit), we have an interior optimal $m$. This is consistent with the discussion in Section 3.2.2, where the leader is able to incorporate the logistics company's best response. A larger bound means more flexibility for metro company, and thus its net benefit is larger. (iii) When the bound is large, the Nash arbitrated solution follows that defined by Eqs. (31) and (32). When the value of $\tilde{m}$ is small, the corner solution occurs, i.e., $m^{\text{NAS}} = 0.5$ AU\$/unit. (iv) The total system payoff, i.e., the sum of the metro company's benefit and the logistics company's profit ($z_{\text{M}} + \pi_{\text{L}}$), can be further increased if both companies choose to cooperate. (v) The Pareto-improving MILS is achieved under all market power regimes. This highlights the potential of co-modality to generate Pareto-improving outcomes for the two companies.

**Table 4**
Game outcomes with respect to a large and a small pricing bound

| **Models** | **NE/LSE** | | **MSE** | | **NAS-SQ** | | **SQ** |
|---|---|---|---|---|---|---|---|
| Bound size | $\tilde{m} = 0.5$ | $\tilde{m} = 1.0$ | $\tilde{m} = 0.5$ | $\tilde{m} = 1.0$ | $\tilde{m} = 0.5$ | $\tilde{m} = 1.0$ | - |
| $m$ | 0.500 | 1.000 | 0.500 | 0.870 | 0.500 | 0.590 | - |
| $n_b$ | 12894.737 | 6315.789 | 12894.737 | 8021.806 | 15044.152 | 14370.653 | - |
| $z_{\text{M}}$ | -1856.261 | -64.222 | -1856.261 | 186.105 | -1597.884 | -370.813 | -5308.386 |
| $\pi_{\text{L}}$ | 77318.421 | 72515.789 | 77318.421 | 73445.276 | 77142.862 | 75937.573 | 71000.000 |
| $z_{\text{M}} + \pi_{\text{L}}$ | 75462.160 | 72451.568 | 75462.160 | 73631.381 | 75544.978 | 75566.759 | 65651.614 |

*4.3. Nash bargaining under different disagreement points*

We now compare the Nash arbitrated solutions with respect to different disagreement points. Table 5 shows the results of various Nash bargaining solutions under a small and a large bound. We label the bargaining case where the disagreement point is set at the status quo (SQ), the Nash equilibrium (NE) and Metro-company-Stackelberg equilibrium (MSE) as 'NAS-SQ', 'NAS-NE' and 'NAS-MSE', respectively. Again, since NE coincides with Logistics-company-Stackelberg equilibrium (LSE) given any pricing bound $\tilde{m} > 0$, the corresponding Nash arbitrated solutions with NE or LSE as disagreement points are also identical; thus, the results are reported together (refer to 'NAS-NE/LSE'). It is noteworthy that the Nash arbitrated solution and the corresponding disagreement point are both subject to the same pricing bound in concern.

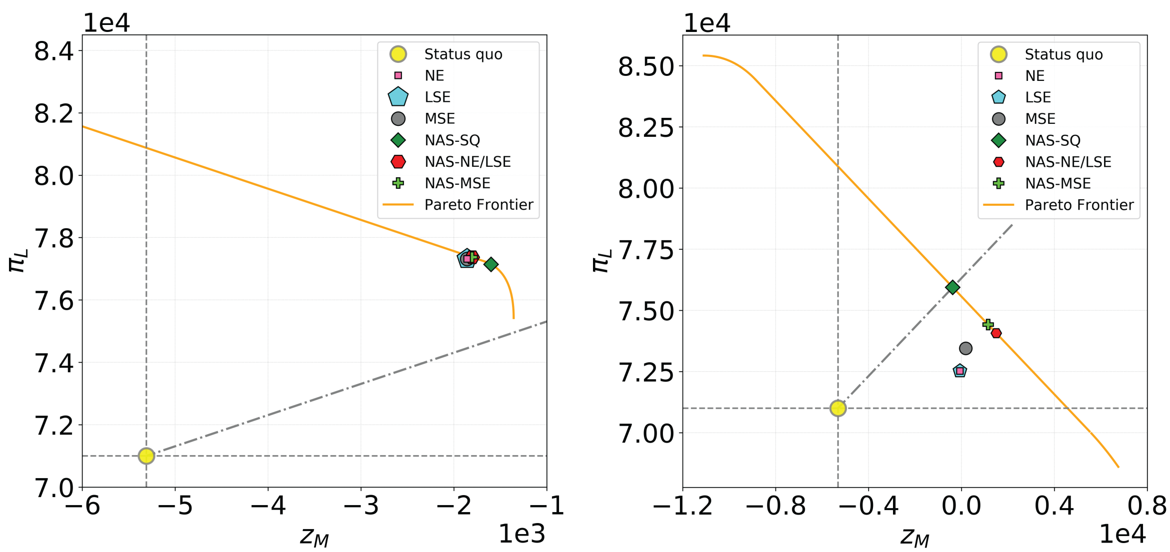
**Table 5**
Nash arbitrated solutions with respect to different disagreement points

| **Models** | **NAS-SQ** | | **NAS-NE/LSE** | | **NAS-MSE** | | **SQ** |
|---|---|---|---|---|---|---|---|
| Price Bound | $\tilde{m} = 0.5$ | $\tilde{m} = 1.0$ | $\tilde{m} = 0.5$ | $\tilde{m} = 1.0$ | $\tilde{m} = 0.5$ | $\tilde{m} = 1.0$ | - |
| $m$ | 0.500 | 0.590 | 0.491 | 0.720 | 0.491 | 0.696 | - |
| $n_b$ | 15044.152 | 14370.653 | 14370.653 | 14370.653 | 14370.653 | 14370.653 | - |
| $z_{\text{M}}$ | -1597.884 | -370.813 | -1803.961 | 1493.374 | -1803.961 | 1153.794 | -5308.386 |
| $\pi_{\text{L}}$ | 77142.862 | 75937.573 | 77370.721 | 74073.385 | 77370.721 | 74412.965 | 71000.000 |
| $z_{\text{M}} + \pi_{\text{L}}$ | 75544.978 | 75566.759 | 75566.759 | 75566.759 | 75566.759 | 75566.759 | 65651.614 |

We summarize the main observations from Table 5 as follows. (i) The Pareto-improving MILS is realized regardless of which disagreement point is chosen. (ii) With a large pricing bound, setting the disagreement point at NE or LSE allows the metro company to obtain the highest net benefit among all cases. Whereas, with a small pricing bound, setting the disagreement point at NE, LSE or MSE enables the logistics company to obtain the highest profit among all cases. (iii) Given a small pricing bound, setting the disagreement point at the status quo is more favorable to the metro company than it is to the logistics company.

### *4.4. Summary*

This subsection summarizes the game outcomes under different market power regimes and/or disagreement points (for the Nash arbitration scheme) in Table 4 and Table 5, and visualizes them in Fig. 2. Fig. 2 plots the game outcomes on the two-dimensional space of $(z_{\rm M}, \pi_{\rm L})$, where Fig. 2a and Fig. 2b correspond to the pricing bounds $\tilde{m} = 0.5$ and $\tilde{m} = 1.0$ AU\$/unit, respectively. In Fig. 2, we also mark the status quo point, and draw a dash-dotted black line, where points along this line indicate identical additional gains of both companies after implementing MILS (when compared to the status quo). It is evident that the points located above the dashed black line implying that logistics company receives more revenue than metro company and vice versa; and the Pareto-improving region is the entire first quadrant divided by the axes whose origin is the status quo point (grey dashed lines). To ease the illustration, we only present the portions of Pareto frontiers adjacent to the status quo point.




$(a)\ \tilde{m} = 0.5$AU\$/unit $(b)\ \tilde{m} = 1.0$AU\$/unit

**Fig. 2.** Pareto frontier and different game outcomes

We highlight a few observations from Table 4, Table 5 and Fig. 2. (i) Different game outcomes under the small pricing bounds $\tilde{m}$ are identical or very close (even for the NAS) since the flexibility of the whole system is very small when the MILS service price is over-regulated (and corner solutions occur). (ii) When the pricing bound is relatively large, in non-cooperative market cases, the profit or benefit of the company who leads is always larger than that when the company in concern is the follower. (iii) When the pricing bound is relatively large, the NAS-SQ lies at the dashed grey line, indicating identical additional gains of both companies after implementing MILS (when compared to the status quo). This is consist with the theoretical analysis for the NAS. When the pricing bound is relatively small, while NAS-SQ (a corner solution) does not lie in the dashed grey line, it is still closer to the dashed grey line when compared to other game outcomes. (iv) MILS will be more utilized to transport parcels if both companies choose to cooperate.

## 5. Conclusions

This paper provides an economic analysis of the emerging metro-integrated logistics system (MILS) under different market power regimes. The non-cooperative and cooperative game theoretical models are utilized to analytically characterize the strategical interactions between a metro company and a logistics company. The direct and indirect impacts of mixed passenger and parcel flows on passenger flow and freight flow patterns are also incorporated into the analysis.

In the non-cooperative market where both companies maximize their own payoffs, we find that, the metro company either sets the price at the allowed maximum or at an interior optimum. The game outcomes under a large pricing bound are more favorable to the metro company, as it has more pricing flexibility. Regarding the cooperative market where both companies work jointly to maximize the joint gains, we consider the Nash arbitration scheme (or Nash bargaining model) and compare different Nash arbitrated solutions under various disagreement points (i.e., status quo without MILS and Nash equilibrium). Essentially, when the pricing bound is sufficiently large and an interior Nash arbitrated optimal price exists, the additional benefit or profit of metro company and logistics company from introducing MILS (compared to the corresponding disagreement point) are equal. When a small pricing bound is applied, setting the disagreement point at the status quo would generate the most favorable outcome for the metro company; whereas, the logistics company could obtain a higher profit if the disagreement point is set as the Nash equilibrium (or Logistics-company Stackelberg equilibrium).

This study is the first in the literature to analytically examine the strategic interactions between the metro and logistics operators in MILS and illustrates the potential of MILS to yield a Pareto-improving outcome. As a first step, the model formulation and analysis presented in this paper mainly focused on the MILS pricing and modal-split strategies. Other factors or decisions that might affect the feasibility, efficiency and reliability of MILS are not fully examined. For instance, demand and/or supply heterogeneity and uncertainties, service coordination or synchronization, temporary storage and management of transfer facilities for parcels are not studied. This study can be further extended by incorporating these additional dimensions that more comprehensively capture features of MILS operation in practice, especially when we study tactical level planning problems and operational level optimization problems. In particular, one may examine the hyperconnected city logistics system, i.e., a complicated urban co-modal system with more than one modes of public transportation utilized to facilitate the parcel transportation or delivery (Crainic & Montreuil, 2016).

This paper considers that the total parcel demand is given (the passenger demand may vary depending on the parcel volume on the metro). This allows us to focus on the interactions between parcel modal-split and the pricing. The underlying assumption is that the MILS mode and the truck mode provide similar service quality. It is of our interest to incorporate endogenous passenger and parcel demands and endogenous levels of service for both the passengers and parcels. The major challenge is that the endogenous interaction between the demand and service quality further limits the analytical tractability of the models and more numerical analysis will be needed.

This study utilizes an abstract network with a single origin-destination (OD) pair, and only considers the parcel transportation between different service points (without considering first-mile and last-mile problems in the context of MILS). A future study may develop a service planning model that incorporates multiple OD pairs with multiple metro lines, and/or multiple logistics service providers, and/or the first-mile and last-mile problems.

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## Appendix A

**A.1. Proof for Proposition 1**. To prove the existence of the NE solution, we apply the theorem proposed by Debreu (1952), which states that there exists at least one NE solution in the game if (i) the strategy space for each player is compact and convex, and (ii) the payoff function is continuous and quasi-concave with respect to each player's own strategy. Considering the MILS non-cooperative game presented in Eqs. (8)-(11), it can be readily seen that the strategy space $\Omega = S_{\mathrm{M}} \times S_{\mathrm{L}}$ is compact and convex as the set $S_{\mathrm{M}} = \{m : 0 \leq m \leq \tilde{m}\}$ and $S_{\mathrm{L}} = \{n_b : 0 \leq m \leq N\}$ are both convex and compact, so is their Cartesian product $\Omega$. Next, one can verify that metro company's and logistics company's payoff functions (Eqs. (8) and (10)) are concave (implies the quasi-concavity) because $\partial^2 z_{\mathrm{M}}/\partial m^2 = 0$ and $\partial^2 \pi_{\mathrm{L}}/\partial n_b^2 < 0$. This completes the proof for the existence of the NE solution.

We now prove the uniqueness of the NE solution. As shown above, the strategy space $\Omega$ is compact and convex, and the $z_{\mathrm{M}}(m)$ and $\pi_{\mathrm{L}}(n_b)$ are convex in $m$ and $n_b$, respectively. Then, the problem in Eq. (8)-Eq. (11) can be reformulated into a variational inequality (VI) problem denoted as $\mathbf{VI}(\Omega, \mathbf{F})$ and every solution of the the variational inequality $\mathbf{VI}(\Omega, \mathbf{F})$ is a solution to the Nash equilibrium problem in Eq. (8)-Eq. (11), where

$$\mathbf{F}(m, n_b) = \begin{pmatrix} \nabla_m z_{\mathrm{M}} \\ \nabla_{n_b} \pi_{\mathrm{L}} \end{pmatrix} \tag{38}$$

To demonstrate the uniqueness of the NE solution, it is equivalent to show that $\mathbf{F}(m, n_b)$ is strictly monotone on $\Omega$, i.e., the Jacobian matrix of $\mathbf{F}$, $\mathbf{J}_{\mathbf{F}}$, is negative definite. Suppose that $\mathbf{J}_{\mathbf{F}}$ is negative definite, it has to fulfill the following condition: $\mathbf{v}^{\mathrm{T}}\mathbf{J}_{\mathbf{F}}\mathbf{v} < 0$ for all non-zero complex column vectors $\mathbf{v} \in \mathbb{C}^2$. Let $\mathbf{v} = \left(\begin{smallmatrix} a \\ b \end{smallmatrix}\right)$, where both $a$ and $b$ are complex numbers. Then,

$$\mathbf{v}^{\mathrm{T}}\mathbf{J}_{\mathbf{F}}\mathbf{v} = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & -C_t'' - C_a'' \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = -b^2(C_t'' + C_a'') < 0 \text{ for all } n_b \in [0, N]$$

Thus, $\mathbf{F}(m, n_b)$ is strictly monotone on $\Omega$ as $\mathbf{J}_{\mathbf{F}}$ is negative definite, and the NE solution is unique. This completes the proof. □

**A.2. Proof for Proposition 2**. Eq. (19) indicates that when the pricing bound is adopted, the metro company's optimal price is $m^*(n_b) = \tilde{m}$. Since the metro company's best response function $m^*(n_b) = \tilde{m}$ is single-valued (i.e., the best response set is a singleton), it can be directly substituted into the leader's (logistics company's) objective function.

To prove the existence and the uniqueness of the Stackelberg equilibrium, we demonstrate the concavity of logistics company's profit function $\pi_{\mathrm{L}}(n_b|m^*)$ as shown in Eq. (18). The first order derivative of the logistics company's profit function is:

$$\frac{d\pi_{\mathrm{L}}}{dn_b} = -\tilde{m} - C_t' + C_a'$$

The second derivative is:

$$\frac{d^2\pi_{\mathrm{L}}}{dn_b^2} = \frac{d}{dn_b}\left(-\tilde{m} - C_t' + C_a'\right) = -\tilde{m} - (C_t'' + C_a'') < 0$$

Thus, $\pi_{\mathrm{L}}(n_b|\tilde{m})$ is a strictly concave function. This means that the Logistics-company-Stackelberg model has a unique solution (i.e., a unique Stackelberg equilibrium). □

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