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Optimal Vehicle Trajectory Planning with Control Constraints and Recursive Implementation for Automated On-Ramp Merging

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***Abstract*— This paper proposes a vehicle trajectory planning method for automated on-ramp merging. Trajectory planning tasks of an on-ramp merging vehicle and a mainline facilitating vehicle are formulated as two related optimal control problems. Rather than specifying the merge point via external computational procedures, the location and time that the on-ramp vehicle merges into the mainline are determined endogenously by the optimal control problem of the facilitating vehicle. Bounds on vehicle acceleration are explicitly considered. The Pontryagin Maximum Principle is applied to find the solutions of the optimal control problems. In order to accommodate the constantly changing external environment, the proposed optimal control method is subsequently implemented in a recursive planning framework. Because of the nature of the problem, the length of the planning horizon is time-varying, unlike conventional model predictive control applications where the planning horizon is of fixed length. Numerical experiments are conducted to study performances of the proposed methodology under the influence of different leading vehicle trajectories and with different lengths of the planning updating interval. In particular, an experiment involving a real-world leading vehicle trajectory and considering different traffic demand levels are presented. The proposed methodology performs well in these experiments and has demonstrated a good potential in real-time applications.**

***Index Terms* — trajectory planning, on-ramp merging, optimal control, Maximum Principle, automated vehicles**

I. Introduction

MOTORWAY on-ramp merge sections have become stubborn bottlenecks with the expansion of motorcar ownerships. Such bottlenecks can contribute to travel delay [1], cause capacity drop [2] and be a source of traffic instabilities [3] and excessive vehicular emissions [4]. To alleviate the negative aspects, there are three main methodologies [5]: optimal infrastructure design, active traffic management strategies (e.g. ramp metering), and automated on-ramp merging strategies. Ramp metering methods regulate macroscopic traffic flow state variables (e.g. density or flow rate) by controlling on-ramp inflow rates. Ramp metering methods are unable to regulate movements of individual merging vehicles. More importantly, ramp metering strategies cannot regulate the movements of mainline vehicles, which are responsible for creating suitable gaps in the mainline in order to accommodate the on-ramp merging vehicles. With the development of connected and automated vehicle (CAV) technologies, regulation of individual vehicles' trajectories becomes possible; this has provided a new perspective for improving driving experience and traffic operations at on-ramp merge sections.

The focus of this paper is to present a method to plan vehicle trajectories for automated on-ramp merging and gap development under a CAV environment. It is assumed that the mainline traffic is in congestion mode, and thus a sufficiently large gap to accommodate an on-ramp merging vehicle is not readily available. Using the planned trajectories of the proposed method, a mainline vehicle will carry out a facilitating maneuver to develop a suitable gap and an on-ramp merging vehicle will maneuver into this gap. Thus, the scope of the paper is limited to trajectory planning in situations when a mainline facilitating vehicle has been selected and the merging process has been initiated (both of which may be determined by an existing upper-level control scheme). Also, the execution of the reference trajectory by lower-level controllers is considered beyond the scope of the paper, which is in line with previous studies in vehicle trajectory planning (e.g. [6-12]).

The core of the proposed method are two related optimal control problems, one for governing the gap development and the other for governing the merging. The Pontryagin Maximum Principle (PMP) is applied to solve the optimal control problems. While there are many computational approaches to solving constrained optimal control problems (e.g. Dynamic Programming [13], Nonlinear Programming [14]), PMP approaches seek to solve the optimal control problem analytically and thus has potential theoretical and potential computational advantages. In this paper, the PMP is used to develop algebraic conditions for the optimal control trajectory that are computationally simple to solve and yield insight into the characteristics of the optimal control trajectories. The solutions are implemented in a recursive planning framework so as to accommodate the constantly changing external disturbances. The proposed method is tested via numerical experiments, including an experiment which uses the trajectory of a real-world, human-driven, instrumented vehicle as the leading vehicle trajectory and which tests different levels of traffic demand of mainline and on-ramp.

The proposed method requires vehicle-to-vehicle (V2V) communications between three vehicles: the on-ramp

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merging vehicle, the mainline facilitating vehicle, and the leading vehicle of the mainline facilitating vehicle. Only the merging vehicle and the facilitating vehicle need to be automated vehicles. Therefore, the proposed strategy may be applied when the penetration rate of automated vehicles is low. However, low penetration rate may require an on-ramp vehicle to decelerate or even stop before carrying out the merging movement. Treatment of such situations is considered outside the scope of this paper and will be studied in future research.

Compared with existing studies, the contributions of this paper are as follows. First, the optimal control problems in this paper are formulated, solved and recursively implemented by explicitly considering acceleration bounds. Second, the time and location of the merge point are treated as decision variables rather than specified parameters, which offers more flexibilities when the proposed trajectory planning method is integrated with an upper-level traffic management program.

The remainder of this paper is organised as follows. Section II reviews and analyses existing studies of automated on-ramp merging strategies. Section III describes the overall mechanism of the proposed methodology. Section IV formulates the maneuvers of the mainline facilitating vehicle and the on-ramp merging vehicle as optimal control problems, applies the PMP to solve them, and then presents a recursive planning framework. Section V conducts numerical experiments based on both hypothetical and real-world leading vehicle trajectories. Finally, section VI summarises this study and lists future research directions.

II. Literature Review

A. A Summary of Existing Studies

Automated on-ramp merging strategies can be broadly classified into two categories: feedback control and optimization approaches. The core idea of feedback control strategies is the introduction of a "virtual vehicle", first proposed by [15]. The "virtual vehicle" method projects the image of an on-ramp vehicle onto the mainline so that a mainline vehicle will have a virtual leading vehicle to follow, and vice versa. By the introduction of "virtual vehicles", on-ramp merging tasks are converted to car-following tasks. Representative works include [16-20], and most recently, [21]. In these works, the feedback controller controls either the mainline facilitating vehicle only [18, 20] or both the mainline facilitating vehicle and the on-ramp merging vehicle [15-17, 19, 21].

The other major category of strategies applies optimization. In a series of studies, [14, 22-24] formulated an optimal control problem governing cooperative driving of all mainline vehicles and on-ramp merging vehicles in a merge section. Although the authors used model predictive control (MPC), they did not really take the advantage of MPC's "closing-the-loop" feature, because all the vehicles are controlled by a centralized controller and thus no vehicle is an external disturbance and no prediction is really needed. In [9], all vehicles that have entered a predefined control zone are to be coordinated in a centralized fashion in order to achieve a system optimum goal. The cost function minimizes the total quadratic of accelerations of all the vehicles, conserving energy consumption. Reference [25] formulated a centralized nonlinear program to optimize all the vehicles' second-by-second acceleration in a predefined merge section. The objective function was to maximize the total speed. Reference [8] reported a decentralized optimal control strategy for on-ramp merging trajectory planning. They formulated the vehicle trajectory planning task as linear-quadratic optimal control problems of fixed terminal time and specified terminal state. Two alternatives of quadratic cost functions were applied, one minimizing acceleration and the other minimizing jerk. All the vehicles were assumed to be connected and automated, and it was assumed that an upper level controller is in charge of determining the order by which each vehicle arrives at the predefined merge point. The strategy was implemented in a recursive planning framework.

It is worth noting that optimal control has also been widely applied in car-following studies, e.g. [11, 12, 26, 27]. In particular, the car-following task in [26] was formulated as a LQR of infinite time horizon. Moreover, [26] has considered control constraints. Notice that a major difference exists in the optimal control formulations for the task of car-following and for the task of merging and gap development: The former usually contain state variables, e.g. gap or velocity of the subject vehicle, in the running cost, e.g. [11, 12, 26, 27], while the latter usually do not, e.g. [8, 9]. This is because the objectives of the two tasks are different: car-following controllers seek to maintain a desired vehicle state (e.g. a desired gap or velocity) over the entire process, while the objective of a merging and gap development task is to achieve a desired terminal state at the time when the process is finished, with little regard to intermediate states of the process.

B. Feedback vs. Optimization Approaches

Both feedback and optimization approaches have their advantages and disadvantages. Feedback strategies are easy to implement and computationally cheap, but are not optimal in any sense. Another main shortcoming of feedback strategies is that they cannot compensate for the speed loss of the facilitating vehicle in gap development, e.g. [18, 20]. On the other hand, usually optimization-based strategies are not straightforward in implementation and are computationally expensive. But if handled properly, some optimization-based strategies can still be computationally cost-effective. For example, in [8, 9] closed-form solutions with only unknown constants are derived; these unknown constants can be solved efficiently, therefore the solutions can be utilized in an online fashion such as MPC. The main advantage of optimization-based strategies is that they can be optimal in a desired sense. Another advantage is that they allow respective specification of the terminal position and terminal speed, so the initial speed loss due to the gap development of a facilitating vehicle can be compensated, something not achievable by a feedback controller.

C. Limitations with Existing Optimization Approaches

Some limitations exist in reported optimization approaches. First, in existing optimal control approaches that achieved analytical solutions, e.g. [8, 9], acceleration bounds were not explicitly taken care of. Admittedly, under many circumstances acceleration bounds can be avoided by delicate pairing up a merging vehicle and a facilitating vehicle; however there are still many unfavourable situations under which the acceleration bounds can be a realistic restriction, such as when the mainline traffic is dense or when there are a limited number of automated vehicles in the mainline.

Second, the merge location was pre-specified and fixed, e.g. [8, 9]. However, a flexible merge point design may be desirable since it offers more flexibility to a supervisory traffic management scheme.

Third, the capabilities of recursive planning approaches (e.g. MPC) to compensate for imperfect estimation of trajectories have not been well studied. The approaches such as in [14, 22-25] have assumed that all vehicles are CAVs and are optimized by a centralized optimization program. A main merit of MPC is that it updates the trajectory plan recursively so as to compensate for errors associated with imperfect prediction of external disturbances. Thanks to this merit, application of an MPC framework renders a decentralized controller practical which is generally more computationally cost-effective than a centralized one and which better suits the long transitioning period of mixed conventional and connected and automated vehicles.

Based on these considerations, in this paper, we develop decentralized optimal control problems for the mainline facilitating vehicle and the on-ramp merging vehicle. Both the optimal control problems explicitly consider acceleration constraints, and the optimal control problem of the facilitating vehicle features free terminal time and terminal state being movable on a curve. We develop analytical solutions for the proposed optimal control problems. Finally, we apply a recursive planning framework to implement the optimal solutions and demonstrate effectiveness as well as computational efficiency of the recursive framework. Note that we chose in this paper not to use the term "MPC" but rather to use the term "recursive", because MPC approaches typically refer to fixed-length planning horizons [28, 29], but in our method the planning horizon is time-varying, as explained in section IV.

Consideration of control bounds greatly increases the complexity of the problem due to the following three factors: (1) It increases the number of admissible control sequences for each of the two optimal control problems from one to nine (section IV); (2) Solution of the only control sequence when control bounds are not considered happens to be the simplest – only control constants need to be solved, while solutions of the other admissible control sequences involve solving for both the control constants and the control switching times; (3) When control bounds are not considered, the recursive implementation of the solution is actually only to re-solve the system of algebraic equations with updated initial conditions at each recursive step. However, when control bounds are considered, at each recursive step, nine different systems of algebraic equations have to be considered.

III. Overview of The Methodology

In this paper, only longitudinal movements of vehicles are explicitly considered; lateral movements of merging vehicles are assumed to have no influence on longitudinal movements of mainline traffic and therefore can be treated separately, as in Ntousakis, et al. [8]. The on-ramp merging process considered here can generally be illustrated by Fig. 1. Fig. 1 (a) shows the condition at the time when the merging process begins. The shaded blue vehicle (with no flag) is the on-ramp merging vehicle whose front bumper is currently at the predefined call-for-assistance (CFA) point and who sends out a request for assistance to mainline vehicles. It is assumed for simplicity that communications are instantaneous. The flagged vehicle is the mainline vehicle who agrees to facilitate and who initiates the process of creating a suitable gap between itself and the leading vehicle, i.e. the grey shaded vehicle. Fig. 1 (b) is a snapshot at some time in the middle of the merging process. Note that the front gap and rear gap at this moment are not yet suitable for merging. Fig. 1 (c) shows the condition when the merging process terminates: The front gap and rear gap have satisfied certain conditions and the on-ramp vehicle merges into the mainline. As soon as the merging vehicle merges into the gap, it switches to car-following operation, so does the facilitating vehicle. Obviously, the time and location where the merging process terminates depends on the trajectory of the leading vehicle and the conditions specifying the successful development of a suitable gap. This point will be explained in more detail in section IV.

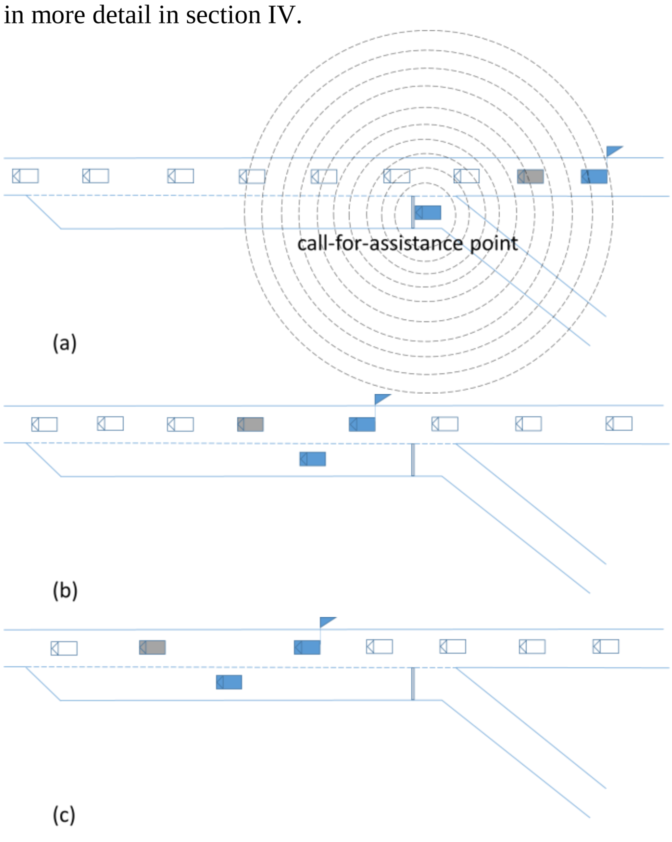


Fig. 1. The on-ramp merging process: (a) the time when the process is initiated; (b) a time in the middle of the process; (c) the time when the process is finished.

The leading vehicle is an external disturbance and its trajectory must be predicted. In practice, this prediction cannot be perfect. As a result, it is necessary to recursively update the prediction and recursively perform the trajectory planning task so as to compensate for errors associated with the imperfect prediction. In this study, the system dynamics are assumed to be perfectly captured by the governing kinematic equations.

IV. Optimal Control Problem Formulation and Solution, and a Recursive Planning Framework

A. Formulation of Optimal Control Problems

The optimal control problem of the mainline facilitating vehicle is given by (1) through (7).

$$\min_{u(t)} \int_0^{t_\mathrm{f}} \frac{1}{2}[u(t)^2+\lambda]\,dt \tag{1}$$

subject to:

$$\dot{\mathbf{x}}(t)=\begin{bmatrix}\dot{x}(t)\\ \dot{v}(t)\end{bmatrix}=\begin{bmatrix}v(t)\\ u(t)\end{bmatrix} \tag{2}$$

$$a_{\min}\le u(t)\le a_{\max} \tag{3}$$

with initial conditions:

$$x(0)=x_0^{\mathrm{fac}} \tag{4}$$

$$v(0)=v_0^{\mathrm{fac}} \tag{5}$$

with terminal state conditions:

$$x(t_\mathrm{f})=x_l(t_\mathrm{f})-(L_l+s_0)-(L_{mer}+s_0)-2\tau v(t_\mathrm{f}) \tag{6}$$

$$v(t_\mathrm{f})=v_l(t_\mathrm{f}) \tag{7}$$

In this formulation, t_f is the (free) terminal time; $x(t)$ and $v(t)$ are state variables, representing the location (measured with reference to the front bumper of the subject vehicle) and speed of the facilitating vehicle, respectively; $u(t)$ is the control variable, i.e. the acceleration of the facilitating vehicle; $a_{\min}<0$ and $a_{\max}>0$ are specified and represent the maximum allowable deceleration and acceleration, respectively; λ is a constant and serves as the weighting factor that penalizes the duration of the merging process; τ is the constant time-gap; L_l and L_{mer} are the lengths of the leading vehicle and merging vehicle, respectively; s_0 is the standstill clearance; $x_l(\cdot)$ and $v_l(\cdot)$ are the position and speed trajectories of the leading vehicle, respectively, and are assumed to be known.

We see that (1) through (7) formulate a linear-quadratic optimal control problem with free terminal time and a terminal state which moves along a curve as a known function of the final time. Although (2) may not be able to fully capture the real vehicle dynamics which can involve higher order dynamics, nonlinearity, and delay. However, for trajectory planning purposes, use of a simplified model for both planning and simulation has been common, e.g. [6-12]. Of course, these planned trajectories would then serve as input reference signals to a lower-level vehicle controllers which would more realistically consider the vehicle dynamics.

The physical implication of the terminal state conditions (6) and (7) is to render zero feedback errors in car-following strategy (a constant time-gap cooperative adaptive cruise control (CACC) or adaptive cruise control (ACC) strategy) when the merging process is completed and the facilitating and merging vehicles switch to car-following operations.

While the length of the acceleration lane can be a constraint, we have chosen not to include an additional inequality constraint on $x(t_f)$ in order to keep the analytical solution simple. Referring to the experiment results reported in section V, the length of a merging trajectory usually ranges from 150 to 300 meters. Most acceleration lanes can accommodate such a length. In situations when the acceleration lane is very short, the issue can be solved by a number of upper-level management strategies such as: 1) initiating the merging process earlier (i.e. advancing the call-for-assistance point upstream); 2) increase the terminal penalty weight to decrease the terminal time; 3) select a new facilitating vehicle that is further upstream.

The proposed optimal control problem that governs the on-ramp merging vehicle's maneuver is given as (8) through (14).

$$\min_{u(t)} \int_0^{t_\mathrm{f}} \frac{1}{2}u(t)^2\,dt+\frac{1}{2}\lambda_1[x(t_\mathrm{f})-x_M]^2 + \frac{1}{2}\lambda_2\left[v(t_\mathrm{f})-v_{t_\mathrm{f}}^{\mathrm{mer}}\right]^2 \tag{8}$$

subject to:

$$\dot{\mathbf{x}}(t)=\begin{bmatrix}\dot{x}(t)\\ \dot{v}(t)\end{bmatrix}=\begin{bmatrix}v(t)\\ u(t)\end{bmatrix} \tag{9}$$

$$a_{\min}\le u(t)\le a_{\max} \tag{10}$$

with initial conditions:

$$x(0)=x_0^{\mathrm{mer}} \tag{11}$$

$$v(0)=v_0^{\mathrm{mer}} \tag{12}$$

The desired terminal state is defined by:

$$x_M=x_l(t_\mathrm{f})-(L_l+s_0)-\tau v_{t_\mathrm{f}}^{\mathrm{mer}} \tag{13}$$

$$v_{t_\mathrm{f}}^{\mathrm{mer}}=v_l(t_\mathrm{f}) \tag{14}$$

In this formulation, $x(t)$, $v(t)$, $u(t)$ are the state and control variables, respectively; $x(t_\mathrm{f})$ and $v(t_\mathrm{f})$ are the terminal states of the merging vehicle; λ_1 and λ_2 are two constant weighting factors that penalize deviations of the terminal state from the desired values, i.e. x_M and $v_{t_\mathrm{f}}^{\mathrm{mer}}$, which are defined by (13) and (14), respectively. The physical implication of (13) and (14) are the same of (6) and (7). Note that in (8), t_f is specified, as determined by the optimal control problem of the facilitating vehicle.

B. Solution Using the Pontryagin Maximum Principle

The proposed optimal control problems can be solved analytically by applying the PMP. While solutions for linear-quadratic optimal control problems without control bounds and of fixed terminal times are well established [30, 31], the proposed problems of this paper contain control bounds and is of free terminal time, and therefore must be specifically derived.

Consider the optimal control problem of the facilitating vehicle as defined by (1) to (7). To apply the PMP, first covert (1) to be maximization of the cost function.

$$\max_{u(t)} \int_0^{t_\mathrm{f}} -\frac{1}{2}[u(t)^2+\lambda]\,dt \tag{15}$$

Then form the Hamiltonian:

$$\mathcal{H}=-\frac{1}{2}u(t)^2-\frac{1}{2}\lambda+p_1(t)v(t)+p_2(t)u(t) \tag{16}$$

In (16), $p_1(t)$ and $p_2(t)$ are the co-state variables associated with the state variables $x(t)$ and $v(t)$, respectively. To maximize the Hamiltonian, an optimal control must satisfy, $\forall t \in [0, t_{\rm f}]$:

$$u^*(t) = \begin{cases} p_2^*(t), & \text{for } a_{\min} < p_2^*(t) < a_{\max} \\ a_{\max}, & \text{for } p_2^*(t) \ge a_{\max} \\ a_{\min}, & \text{for } p_2^*(t) \le a_{\min} \end{cases} \quad (17)$$

The state equations are:

$$\dot{x}^*(t) = v^*(t) \quad (18)$$

$$\dot{v}^*(t) = u^*(t) \quad (19)$$

The co-state equations are:

$$\dot{p}_1^*(t) = -\frac{\partial \mathcal{H}^*}{\partial x} = 0 \quad (20)$$

$$\dot{p}_2^*(t) = -\frac{\partial \mathcal{H}^*}{\partial v} = -p_1^*(t) \quad (21)$$

Note that in (20) and (21), $\mathcal{H}^*$ is a shorthand for $\mathcal{H}$ evaluated along the extremal trajectory, i.e.

$$\mathcal{H}^* = -\frac{1}{2}u^*(t)^2 - \frac{1}{2}\lambda + p_1^*(t)v^*(t) + p_2^*(t)u^*(t) \quad (22)$$

The boundary conditions are:

$$x^*(0) = x_0^{\rm fac} \quad (23)$$

$$v^*(0) = v_0^{\rm fac} \quad (24)$$

$$x^*(t_{\rm f}) = x_l(t_{\rm f}) - 2(L + s_0) - 2\tau v(t_{\rm f}) \quad (25)$$

$$v^*(t_{\rm f}) = v_l(t_{\rm f}) \quad (26)$$

Plugging (26) into (25) yields

$$x^*(t_{\rm f}) = x_l(t_{\rm f}) - (L_l + s_0) - (L_{mer} + s_0) - 2\tau v_l(t_{\rm f}) \quad (27)$$

Combining (26) and (27) in the vector form gives

$$\boldsymbol{\theta}(t) := \begin{bmatrix} x_l(t) - (L_l + s_0) - (L_{mer} + s_0) - 2\tau v_l(t_{\rm f}) \\ v_l(t) \end{bmatrix} \quad (28)$$

Then one more boundary condition is

$$\mathcal{H}^* - \mathbf{p}^*(t_{\rm f})^T \frac{d\boldsymbol{\theta}}{dt}(t_{\rm f}) = 0 \quad (29)$$

which yields

$$-\frac{1}{2}u^*(t_{\rm f})^2 - \frac{1}{2}\lambda + p_1^*(t_{\rm f})v^*(t_{\rm f}) + p_2^*(t_{\rm f})u^*(t_{\rm f}) = p_1^*(t_{\rm f})[v_l(t_{\rm f}) - 2\tau \dot{v}_l(t_{\rm f})] + p_2^*(t_{\rm f})\dot{v}_l(t_{\rm f}) \quad (30)$$

Thus we have obtained a two-point boundary value problem (TBVP) defined by (18) to (21) with boundary conditions (23) to (26) and (30).

Next, we solve the TBVP. From (20),

$$p_1^*(t) \equiv c_1 \quad (31)$$

From (31) and (21),

$$p_2^*(t) = -c_1 t + c_2 \quad (32)$$

In (31) and (32), c_1 and c_2 are unknown constants. Note that (32) indicates that $p_2^*(t)$ is monotonic. Combining (32) and (17), it is clear that admissible extremal control sequences can be only $\{p_2^*(t)\}$, $\{a_{\min}, p_2^*(t)\}$, $\{a_{\min}, p_2^*(t), a_{\max}\}$, $\{p_2^*(t), a_{\max}\}$, $\{a_{\max}, p_2^*(t), a_{\min}\}$, $\{a_{\max}, p_2^*(t)\}$, $\{p_2^*(t), a_{\min}\}$, $\{a_{\min}\}$, and $\{a_{\max}\}$.

Having identified all the admissible extremal control sequences, the next step is to construct system of algebraic equations for each of them, from which the control constants and control switching times (if any) can be solved. The development of these algebraic equations for the most complex case, $\{a_{\min}, p_2^*(t), a_{\max}\}$, can be found in the Appendix, and the procedure for the other extremal sequences proceeds in a similar manner.

After these constants are determined from these conditions, they need to be checked for feasibility. For example, for the extremal control sequence $\{a_{\min}, p_2^*(t), a_{\max}\}$, the following conditions must be satisfied to be optimal:

$$0 < t_1 < t_2 < t_{\rm f} \quad (33)$$

$$c_2 < a_{\min} \quad (34)$$

$$-c_1 t_{\rm f} + c_2 > a_{\max} \quad (35)$$

$$-c_1 > 0 \quad (36)$$

In our problem, the Hessian of the Hamiltonian is negative-definite. Specifically, it can be calculated that $\frac{\partial^2 \mathcal{H}}{\partial u^2} = -1$. Moreover, the control constraints are linear. Thus, as long as a feasible solution is found, it is optimal. Note that we have verified the above analytical solution method by discretizing the optimal control problem and then solving using numerical method (the "fmincon" function of the MATLAB). Analytical solutions to the optimal control problem of the merging vehicle are similar and straightforward, and thus are not presented in this paper.

C. A Recursive Planning Framework

In practice, the leading vehicle's trajectories, i.e. $x_l(\cdot)$ and $v_l(\cdot)$, cannot be perfectly predicted. Therefore, to compensate for the errors it is necessary to recursively update the prediction of $x_l(\cdot)$ and $v_l(\cdot)$ and resolve the optimal control problems. In this paper, the future speed of the leading vehicle is assumed to be the same as the instant when the planning is updated, as in line with previous studies [8, 12]. At a given time t_0, the optimal control problems for the facilitating and merging vehicle are solved, respectively, yielding the optimal control histories $u^{\rm fac^*}(t)$ and $u^{\rm mer^*}(t)$ for $t_0 \le t \le t_{\rm f}$ and the first portion of the controls are applied over a pre-specified updating interval $\Delta t_{\rm up}$, e.g. 0.1 sec, 0.5 sec, etc. The process is repeated until the termination condition $t_{\rm f} < \overline{t_{\rm f}}$ is met, after which the entire optimal control histories are applied. Note that $\overline{t_{\rm f}}$ is needed because here the length of planning horizon is shrinking with the increase of the recursive step index. In this paper, the control implementation interval is 0.1 seconds, i.e. $\Delta t_{\rm im} = 0.1$ sec. Numerical simulation studies revealed that $\overline{t_{\rm f}} = 8\Delta t_{\rm im}$ is sufficient to generate a well-behaved solution.

To prevent a merging vehicle (or a facilitating vehicle) from dangerously approaching the preceding merging vehicle (or the leading vehicle), at each step, the more restrictive acceleration between the optimal control output and the output of a constant time-gap (CTG) car-following law which takes into account avoiding dangerous approaching, is implemented. Such a treatment was proposed by [8]. This paper adopts the CTG car-following law proposed by [32, 33] which can prevent the following car from dangerously approaching its predecessor.

V. Numerical Experiments

A. *Effects of Leading Vehicle Trajectories and Planning Updating Interval*

In this section we study the effects of leading vehicle trajectories and the lengths of the pre-specified updating interval (Δt_{up}) on the performance of the proposed methodology. To evaluate the effects, we present the resulting acceleration trajectories, as well as a series of quantitative performance measures, as summarized in Table I.

Two hypothetical sinusoidal speed trajectories, i.e. profile 1 and profile 2, are assumed for the leading vehicle, defined by (37) and (38), respectively, and are illustrated by Fig. 2.

$$v_l(t) = v_0^{\mathrm{lead}}\left[1 - \frac{1}{6}\sin\left(\frac{\pi}{15}t\right)\right] \quad (37)$$

$$v_l(t) = v_0^{\mathrm{lead}}\left[1 - \frac{1}{6}\sin\left(\frac{\pi}{10}t\right)\right] \quad (38)$$

We test three different lengths of Δt_{up}, i.e. 1 sec, 0.5 sec, and 0.1 sec. Although it is intuitive to only use $\Delta t_{\mathrm{up}} = 0.1$ sec, which is consistent with the sampling time interval length, however, in practice, computational time can be a restraint that must be considered, thus, we think it is desirable to test the performance of the proposed method with longer updating intervals as well, i.e. $\Delta t_{\mathrm{up}} = 0.5$ sec and $\Delta t_{\mathrm{up}} = 1$ sec. In MPC studies, it is not uncommon to have longer updating interval than sampling interval, e.g. [10, 34, 35].

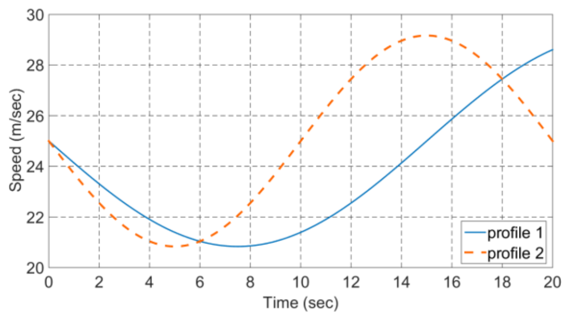


Fig. 2. Two sinusoidal speed profiles of the leading vehicle

Set the coordinate of the CFA point as 0 m. Suppose that the initial states of the leading vehicle, facilitating vehicle, and the merging vehicle are as follows: $v_0^{\mathrm{lead}} = 23$ m/sec, x_0^{lead}= -10 m;v_0^{fac}=25 m/sec, $x_0^{\mathrm{fac}} = -53$ m; $v_0^{\mathrm{mer}} =$ 10 m/sec, x_0^{mer} = 0 m. Set $\lambda = 10$, $\lambda_1 = 25$, $\lambda_2 = 25$. It is assumed that all the vehicles are of the same length and employ the same CTG car-following policy.

Fig. 3 illustrates the resulting acceleration trajectories of the facilitating and merging vehicles with the three different Δt_{up}, when the leading vehicle follows speed profile 1. Table II summaries the performance measures. Fig. 4 illustrates the resulting acceleration trajectories of the facilitating and merging vehicles with the three different Δt_{up}, when the leading vehicle follows speed profile 2. Table III summaries the performance measures.

TABLE I
Performance Measures of different MPC Updating Intervals

Name of the Performance Measure	**Symbol**
Time duration of the merging process	T
Merge location	X_M
Deviation of the merging vehicle's actual terminal spacing from the desired terminal spacing	$\mathrm{Dev\_S_T^{mer}}$
Deviation of the merging vehicle's actual terminal speed from the desired terminal speed	$\mathrm{Dev\_V_T^{mer}}$
Deviation of the facilitating vehicle's actual terminal spacing from the desired terminal spacing	$\mathrm{Dev\_S_T^{fac}}$
Deviation of the facilitating vehicle's actual terminal speed from the desired terminal speed	$\mathrm{Dev\_V_T^{fac}}$
CPU time of the merging vehicle	$\mathrm{CPU\_T^{mer}}$
CPU time of the facilitating vehicle	$\mathrm{CPU\_T^{fac}}$

TABLE II
Performance Measures of different Recursive planning Updating Intervals Under Leading Trajectory Profile 1

	1 sec	**0.5 sec**	**0.1 sec**
T(sec)	9.7	9.6	9.4
X_M(m)	149.70	147.79	143.98
$\mathrm{Dev\_S_T^{mer}}$	7%	5.7%	4.1%
$\mathrm{Dev\_V_T^{mer}}$	-14.5%	-13.9%	-13.6%
$\mathrm{Dev\_S_T^{fac}}$	0	0.8%	-0.1%
$\mathrm{Dev\_V_T^{fac}}$	0.2%	-0.7%	0.5%
$\mathrm{CPU\_T^{mer}}$(sec)	5.2	5.2	10.6
$\mathrm{CPU\_T^{fac}}$(sec)	16.9	22.5	92.8

TABLE III
Performance Measures of different Recursive planning Updating Intervals Under Leading Trajectory Profile 2

	1 sec	**0.5 sec**	**0.1 sec**
T(sec)	15.6	15.4	15.2
X_M(m)	290.46	285.10	279.75
$\mathrm{Dev\_S_T^{mer}}$	-0.2%	0.1%	0.3%
$\mathrm{Dev\_V_T^{mer}}$	0	0	-0.4%
$\mathrm{Dev\_S_T^{fac}}$	1.3%	1.1%	1.4%
$\mathrm{Dev\_V_T^{fac}}$	-0.9%	-0.8%	-0.1%
$\mathrm{CPU\_T^{mer}}$(sec)	7.3	8.2	17.1
$\mathrm{CPU\_T^{fac}}$(sec)	22.2	36	147.3

From these results, we see that for any of the two leading vehicle trajectories, and for any of the different lengths of Δt_{up}, a suitable gap can be developed between the facilitating vehicle and the leading vehicle, and the merging vehicle can maneuver to the gap accordingly, with acceptable terminal errors. When Δt_{up} = 1 sec and Δt_{up} = 0.5 sec, the planned acceleration profiles are chattering, especially when the leading vehicle is of speed profile 2. Such planned accelerations, if were to be directly applied to the vehicles, would cause discomfort of the passengers because of jerks. However, in line with previous studies, e.g. [36, 37], planned trajectories often serve as input references to a lower level vehicle controller, and it is the task of the lower-level controller to generate smooth control commands. When Δt_{up} = 0.1 sec, the planned accelerations are largely smooth, except for the ending portions when the merging processes transition to car-following.

The discontinuities at the ending portions is due to the characteristics of the analytical solutions of the optimal control problems, which often end with $a_{\max}$ for the facilitating vehicle when a large time penalty λ is imposed. We have purposely chosen a large time penalty to test performances of the proposed method under very unfavorable situations (e.g. when there are few CAVs in the mainline and meanwhile the on-ramp vehicles are given high priorities). When the value of λ is small, the ending portion discontinuity issue is minor, as will be shown in section V.B. Even so, the magnitudes of these discontinuities and/or chattering in our experiments are comparable to those in previous studies of MPC-based automated trajectory planning, e.g. [9-12, 27, 38]. One possible approach to mitigate the discontinuity issue in ending portions is to include additional terminal conditions to constrain the terminal accelerations. However, this will add extra complexity to the solutions of the optimal control problems, and thus we leave it to future work.

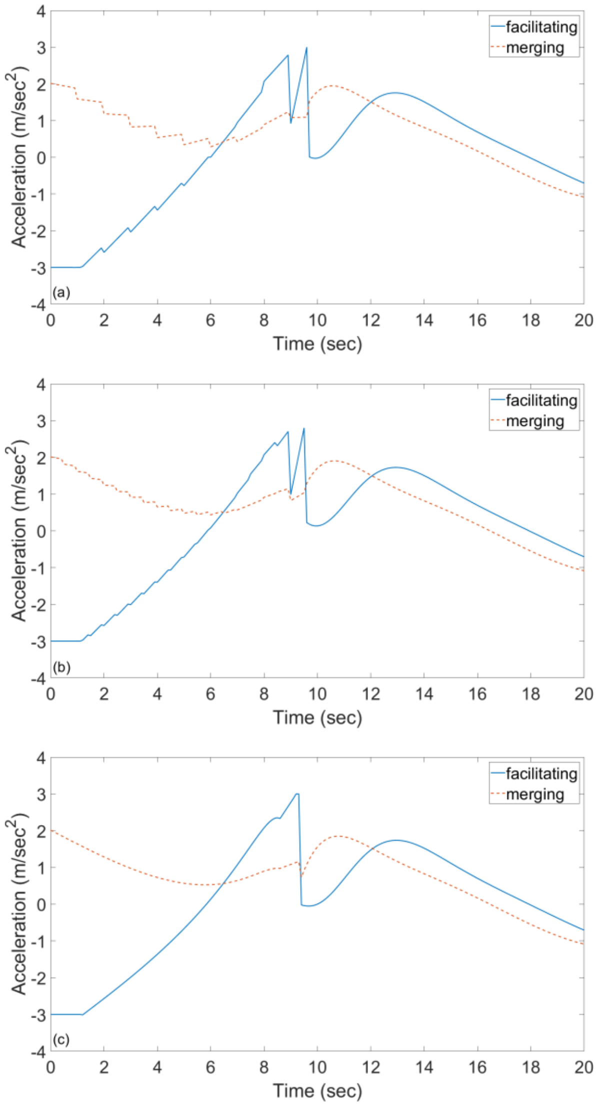


Fig. 3. Planned accelerations of the facilitating and merging vehicles under the influnece of the leading trajectory profile 1: (a) $\Delta t_{\text{up}} = 1$ sec; (b) $\Delta t_{\text{up}} = 0.5$ sec; (c) $\Delta t_{\text{up}} = 0.1$ sec.

Note that there will always be sufficiently high variation in the leading vehicle trajectory that will lead to non-convergence of the recursive planning scheme, but such trajectories are rare in practice (they require high accelerations from the leading vehicle) and can be mitigated with either a fallback control strategy (e.g. specifying conditions for human intervention), or directly by including prediction uncertainties in the optimization formulation (i.e. robust control). These important considerations are left to future work.

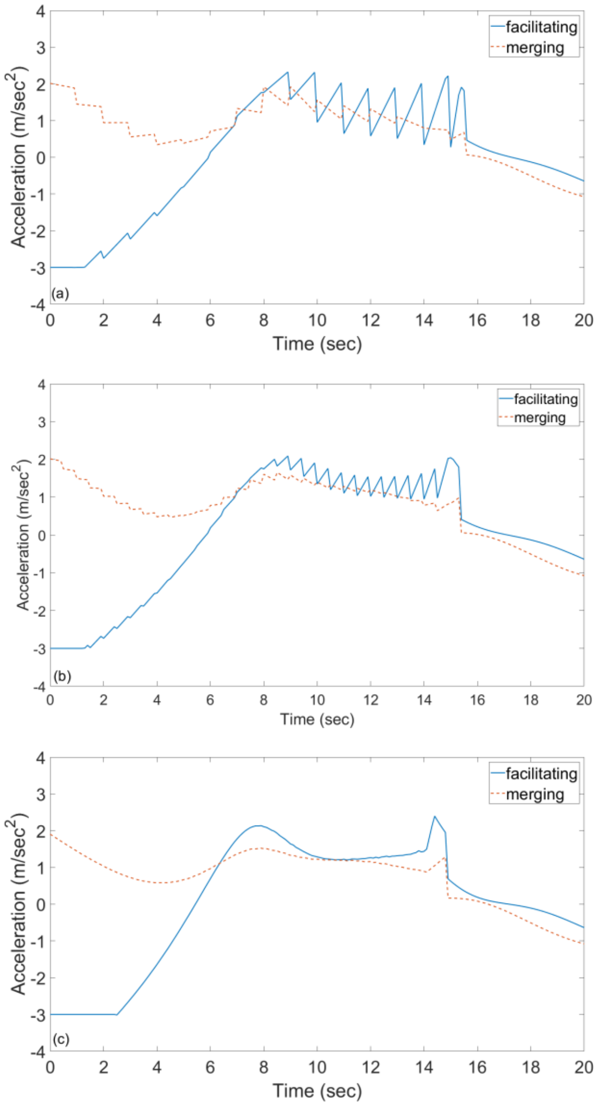


Fig. 4. Planned accelerations of the facilitating and merging vehicles under the influnece of the leading trajectory profile 2: (a) $\Delta t_{\text{up}} = 1$ sec; (b) $\Delta t_{\text{up}} = 0.5$ sec; (c) $\Delta t_{\text{up}} = 0.1$ sec.

With regard to computational efficiency, the reported CPU times were all obtained using a personal computer with an Intel® CORE™ i5-7300HQ CPU @ 2.5 GHz. Considering that the programs were implemented in MATLAB m-file, the proposed methodology has demonstrated a good potential in real-time applications considering the rapid progress of computational technology.

B. Tests Using a Real-World Vehicle Trajectory as The Leading Vehicle Trajectory

1) Description of the instrumented vehicle trajectory

To evaluate the proposed methodology's performance under the influence of a real-world leading vehicle, real vehicle trajectory data were collected by an instrumented vehicle operated by the Institute of Industrial Science, University of Tokyo, on March 31 2017. The instrumented vehicle was driven by a human driver. Note that the instrumented vehicle's trajectory only serves as a leading vehicle trajectory in this experiment. A most representative segment of the journey was identified when the instrumented vehicle drove northbound through the Asada on-ramp merge section on the Kanagawa Route No. 1, Yokohane Line, a major urban expressway in Tokyo-Yokohama Metropolitan Area. When the instrumented vehicle approached the on-ramp merge area, it briefly slowed down slightly and then soon recovered its speed, following which an on-ramp merging vehicle was present and the instrumented car again decelerated to yield for the merging vehicle, and after the merge, the instrumented car sped up and left the merge section. The speed profile (sampled at 10 HZ) of the above process as recorded by the instrumented vehicle is shown in Fig. 5. In the following, we will use the speed trajectory as the leading vehicle speed profile and conduct numerical experiment.

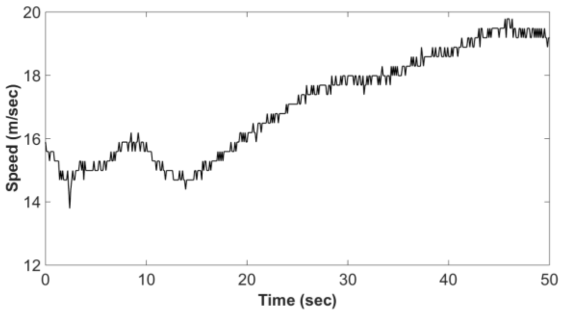


Fig. 5. Speed profile of the human-driven instrumented vehicle through an on-ramp merge area

2) Tests with multiple on-ramp merging vehicles

We study the proposed method's performances under multiple on-ramp merging vehicles. We assume all the vehicles are CAVs and adopt the same CTG car-following law. Situations with different penetration rates of CAVs in mixed CAV and conventional vehicle environment will be studied in future research. Since the proposed method is needed when the mainline traffic is congested, thus for simplicity but without loss of generality, it is assumed that the mainline traffic is consistently under car-following regime with the constant time-gap $\tau = 1.5$ sec. For on-ramp traffic, two scenarios will be tested: First, the on-ramp traffic demand is low, with the headway between arrivals of on-ramp vehicles $h = 20$ sec; second, the on-ramp traffic demand is high, with the headway between arrivals of on-ramp vehicles $h = 5$ sec.

As before, the coordinate of the CFA point is set to be 0. For simplicity but without loss of generality, it is assumed that the speeds of the on-ramp vehicles when they arrive at the CFA point are all 5 m/sec. Note that the arrival speeds of on-ramp vehicles can be conveniently specified because they are CAVs. It is also assumed that prior to first merging process, the mainline traffic is under steady car-following state, led by the instrumented leading vehicle. The initial state of the mainline leading vehicle (i.e. the Tokyo instrumented vehicle) is: $x_0^{\text{lead}} = -10.96$ m, $v_0^{\text{lead}} = 15.89$ m/s.

The optimal control parameters are set to $\lambda = 1$, $\lambda_1 = 25$, $\lambda_2 = 25$. The value of λ is set low to reflect the desire of not to generate high decelerations and accelerations. The values of λ_1 and λ_2 are set high in order to have small terminal errors. The tuning of these control parameters could be part of an upper-level program, e.g. as in [8]. We will briefly introduce the content of the upper-level program in section VI. In this experiment, we set the updating time interval length to be 0.1 sec, i.e. $\Delta t_{\text{up}} = 0.1$ sec.

Applying the recursive planning scheme with $\Delta t_{\text{up}} = 0.1$ sec, the resulting trajectories under the low and high on-ramp traffic demand scenarios are illustrated by Fig. 6 and Fig. 7, respectively.

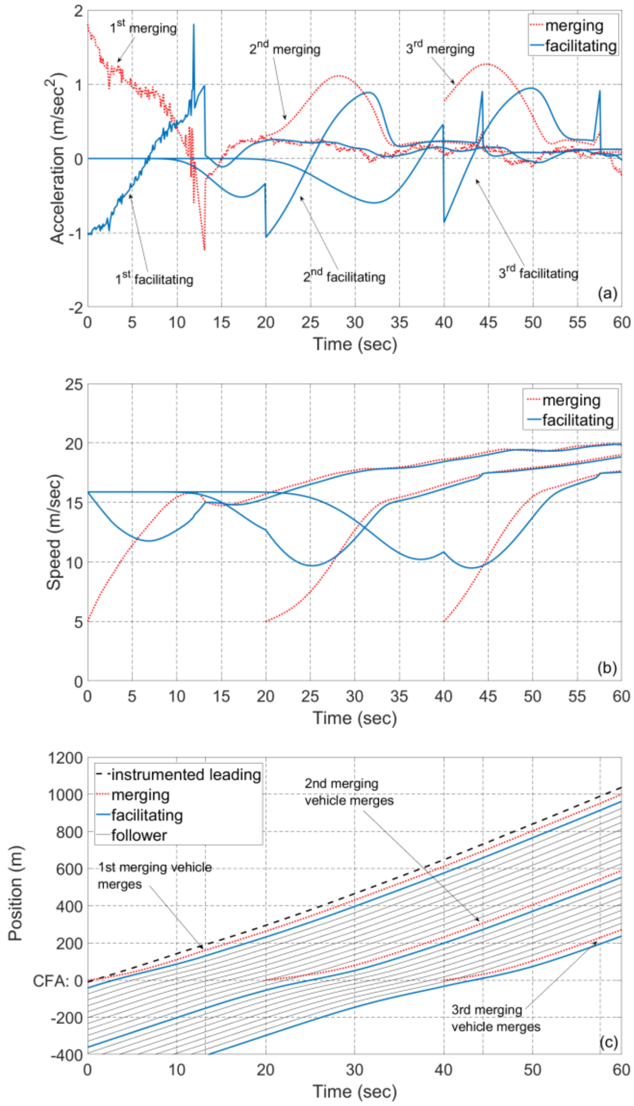
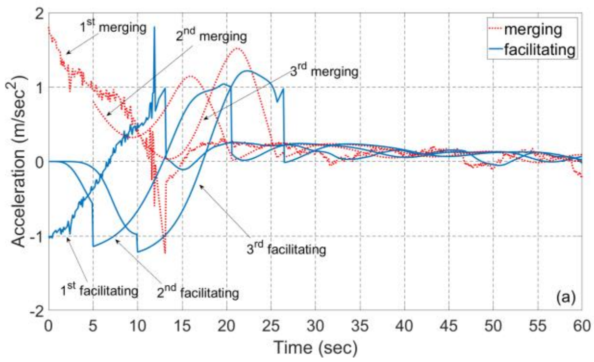


Fig. 6. Planned trajectories of the merging and facilitating vehicles for the low on-ramp demand scenario: (a) accelerations; (b) speeds; (c) positions.



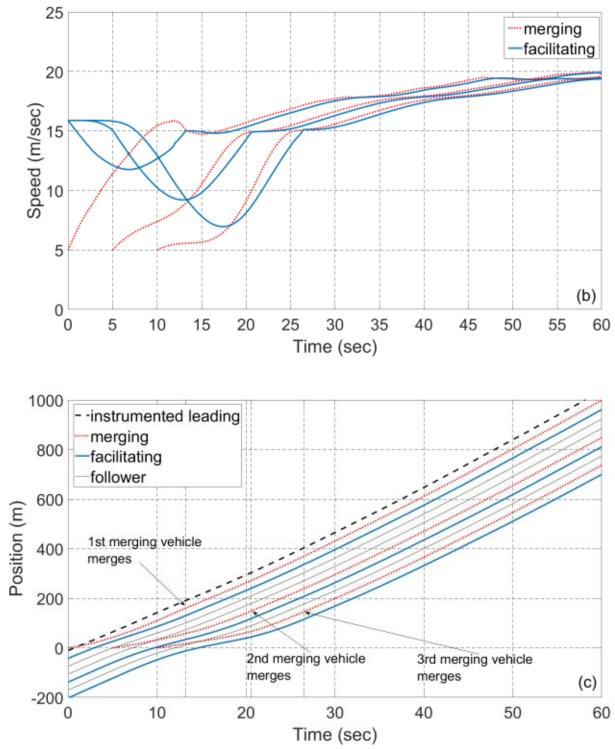


Fig. 7. Planned trajectories of the merging and facilitating vehicles for the high on-ramp demand scenario: (a) accelerations; (b) speeds; (c) positions.

3) Analysis of results

These results show that all the three facilitating vehicles are able to develop a suitable gap under the influence of their respective leading vehicles whose speeds are constantly changing, and all the three merging vehicles can maneuver to desired merge locations with desired merge-in speeds. It can be seen, for both traffic scenarios, the acceleration trajectories of the 2nd and 3rd pairs of facilitating and merging vehicles are largely smooth, while the 1st pair has demonstrated some chattering, but at a fairly low level. Note that the acceleration chattering associated with the 1st pair is due to the noisy speed data of the leading vehicle, rather than the recursive implementation, which is obvious from referring to the speed trajectory of the first leading vehicle as shown in Fig. 5.

From Fig. 7 (c), we see that the speed reduction of a facilitating vehicle due to the facilitating maneuver grows with the increase of the vehicle index. Similar phenomena can be observed from experiments in [9, 25]. This is as expected, because in this test, the headway between the arrivals of the on-ramp merging vehicles are as short as five seconds. Actually, as Fig. 7 (c) shows, when the third pair of merging and facilitating vehicles have started their merging process, the merging process of the first pair has not yet finished. Thus, the speed drop of the first facilitating vehicle is partly assimilated by the second facilitating vehicle, and the speed drop of the second facilitating vehicle is passed on to the third. In short, the speed reduction can accumulate. For the low on-ramp traffic demand scenario, the issue of speed reduction accumulation is minor, as shown by Fig. 6 (c). Obviously, when the on-ramp traffic demand is sufficiently low, this issue will become negligible. Comparison of the two tests has justified the necessity of a pre-merging on-ramp traffic management scheme when the on-ramp traffic demand is heavy in order to interfere with the on-ramp traffic. Actually, the first test (i.e. $h = 20$ sec) can be viewed as the second test (i.e. $h = 5$ sec) with an interrupted on-ramp traffic.

A simple on-ramp traffic management scheme for handling high on-ramp traffic demand can be as follows: Set a waiting zone at a proper location upstream of the CFA point at the on-ramp. An on-ramp vehicle will be held briefly at the waiting zone if the mainline speed has not yet recovered back to some threshold value. This method can be used alone or jointly applied with a changing λ value as in the facilitating vehicle's optimal control problem, to achieve a best overall performance. The development of such a scheme is part of the upper-level control program and will be studied in future.

VI. Conclusion and Prospects on Future Research

This paper presents a recursive optimal vehicle trajectory planning method for motorway on-ramp merging. The trajectory planning problem of the facilitating vehicle is formulated as a linear-quadratic optimal control problem with free terminal time and with terminal state moving on a given function of time. The trajectory planning problem of the merging vehicle is formulated as a linear-quadratic optimal control problem with fixed terminal time and with terminal state being penalized for deviating from the specified desired terminal state. In both problem, the control variable, i.e. acceleration, is constrained. The Pontryagin Maximum Principle is applied to solve the optimal control problems. Due to the existence of the control constraints, the PMP solution procedure is much more complicated than the unconstrained case. The solution procedure for a most representative and also most complicated case, which contains two control switching times, is presented. Solutions of the optimal control problems are implemented in a recursive planning framework in order to cope with uncertainties in prediction of the behavior of the leading vehicle.

Numerical experiments are conducted to test the effects of oscillation frequency of the leading vehicle's speed trajectory and the length of the planning updating interval. It is found that the recursive planning scheme can handle situations when the leading vehicle's speed oscillates. When the planning updating interval is consistent with the sampling interval length, i.e. 0.1 sec, the resulting planned acceleration trajectories are largely smooth; when the planning updating interval is longer, the planned acceleration profiles can be chattering.

A numerical experiment using a real-world, human-driven, instrumented vehicle's trajectory as the leading vehicle trajectory is also conducted. With such a leading vehicle, the proposed trajectory planning method is tested for two scenarios of on-ramp traffic demand, with the headways between consecutive merging vehicles being 5 sec and 20 sec, respectively. It is found that under both scenarios, the proposed method performs well in generating consistent acceleration trajectories, moving the merging and facilitating vehicles to desired positions and transitioning to car-following operations. It is also found that, as expected, for the scenario of short headway merging vehicles, speed reduction in facilitating vehicles can accumulate quickly if nothing would be done to manage the arrival rate of the on-ramp vehicles. An on-ramp traffic management measure is briefly introduced that can handle this issue.

Several prospects of future study have been planned. First, an upper-level controller will be developed which is responsible for handling pre-merging movements of on-ramp vehicles, selection of mainline facilitating vehicles, and determination of the weighting factors in the objective functions considering macroscopic traffic flow efficiency. Second, it may be desirable to filter noisy leading vehicle trajectories such as shown in Fig. 5 and investigate if this can result in planned trajectories of higher quality. Third, the proposed optimal control problems will be modified to include speed constraints, to mitigate the issue of speed reduction accumulation, among other measures. Fourth, it could be interesting to explore the effects of imperfect trajectory following using a more realistic vehicle model, and design a lower-level vehicle controller to execute planned reference trajectories. Fifth, it can be interesting to extend the method of this study to cover situation when the mainline traffic is uninterrupted.

APPENDIX

In this appendix, we construct the system of algebraic equations that define the control constants and control switching times for the most complicated candidate extremal control sequence, $\{a_{\min}, p_2^*(t), a_{\max}\}$. If the optimal control sequence is $\{a_{\min}, p_2^*(t), a_{\max}\}$, this implies that there exists two times t_1 and t_2, $0 < t_1 < t_2 < t_{\mathrm{f}}$, such that

$$u^*(t) = \begin{cases} a_{\min}, & \text{for } t \in [0, t_1) \\ p_2^*(t), & \text{for } t \in [t_1, t_2) \\ a_{\max}, & \text{for } t \in [t_2, t_{\mathrm{f}}] \end{cases} \tag{A1}$$

For $t \in [0, t_1)$,

$$u^*(t) = a_{\min} \tag{A2}$$

$$v^*(t) = v_0^{\mathrm{fac}} + a_{\min} t \tag{A3}$$

$$x^*(t) = x_0^{\mathrm{fac}} + v_0^{\mathrm{fac}} t + \frac{1}{2} a_{\min} t^2 \tag{A4}$$

For $t \in [t_1, t_2)$,

$$u^*(t) = -c_1 t + c_2 \tag{A5}$$

$$v^*(t) = -\frac{1}{2} c_1 t^2 + c_2 t + c_3 \tag{A6}$$

$$x^*(t) = -\frac{1}{6} c_1 t^3 + \frac{1}{2} c_2 t^2 + c_3 t + c_4 \tag{A7}$$

For $t \in [t_2, t_{\mathrm{f}}]$,

$$u^*(t) = a_{\max} \tag{A8}$$

$$v^*(t) = a_{\max} t + c_5 \tag{A9}$$

$$x^*(t) = \frac{1}{2} a_{\max} t^2 + c_5 t + c_6 \tag{A10}$$

Therefore, at $t = t_1$ we have:

$$a_{\min} = -c_1 t_1 + c_2 \tag{A11}$$

$$v_0^{\mathrm{fac}} + a_{\min} t_1 = -\frac{1}{2} c_1 t_1^2 + c_2 t_1 + c_3 \tag{A12}$$

$$x_0^{\mathrm{fac}} + v_0^{\mathrm{fac}} t_1 + \frac{1}{2} a_{\min} t_1^2 = -\frac{1}{6} c_1 t_1^3 + \frac{1}{2} c_2 t_1^2 + c_3 t_1 + c_4 \tag{A13}$$

At $t = t_2$ we have:

$$-c_1 t_2 + c_2 = a_{\max} \tag{A14}$$

$$-\frac{1}{2} c_1 t_2^2 + c_2 t_2 + c_3 = a_{\max} t_2 + c_5 \tag{A15}$$

$$-\frac{1}{6} c_1 t_2^3 + \frac{1}{2} c_2 t_2^2 + c_3 t_2 + c_4 = \frac{1}{2} a_{\max} t_2^2 + c_5 t_2 + c_6 \tag{A16}$$

At $t = t_{\mathrm{f}}$ we have:

$$a_{\max} t_{\mathrm{f}} + c_5 = v_l(t_{\mathrm{f}}) \tag{A17}$$

$$\frac{1}{2} a_{\max} t_{\mathrm{f}}^2 + c_5 t_{\mathrm{f}} + c_6 = x_l(t_{\mathrm{f}}) - 2(L + s_0) - 2\tau v_l(t_{\mathrm{f}}) \tag{A18}$$

$$-\frac{1}{2} a_{\max}^2 - \frac{1}{2}\lambda + (-c_1 t_{\mathrm{f}} + c_2) a_{\max} = (c_2 - 2\tau c_1 - c_1 t_{\mathrm{f}}) \dot{v}_l(t_{\mathrm{f}}) \tag{A19}$$

Note that (A19) is obtained from (30). Therefore, we have nine unknowns, c_1, c_2, c_3, c_4, c_5, c_6, t_1, t_2, and t_{f}, and nine nonlinear algebraic equations (A11) through (A19), which we solve using MATLAB's "vpasolve" function.

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