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Speed and Trading Behavior in an Order-Driven Market: An Analysis on a High Quality Dataset

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Abstract

This paper examines how the speed of order submission affects investor behavior when submitting orders in an order-driven market. We provide a theoretical model where the speed of investor and limit order placement is non-monotonic. This is rationalized by mid-speed traders submitting initial orders further away from the market price to avoid order pick-offs by faster traders when the underlying asset changes, at the same time, they can still benefit by revising quotes against slower traders. We also show that market orders and marketable limit orders are used differently depending on investor speed. Fast traders prefer marketable limit orders to market orders more than slow traders do, since slow traders face higher costs of marketable-intended limit orders when an order is not executed immediately.

JEL Classification: G10; G19

Keywords: Limit order market; Low-latency; Order submission; Marketable limit order; Market order

1 Introduction

The ability to trade quickly has become important in contemporary financial markets. With the growth of low-latency traders (LLTs) in the last decade, financial markets have changed radically. More orders and trades are coming from computerized algorithms, and exploiting speed in the limit order market allowed LLTs to trade more frequently, without holding large positions. Acknowledging the existence of LLTs caused non-LLTs to update their beliefs about their trading counterparts, since their orders can be picked off before they reflect new information and circumstances.

In this paper, we examine the investor behavior where investors differ in speed of order submission. The cost of submitting limit order closer to the market price is that it has higher pick off risk. When the fundamental value of the asset fluctuates, limit orders closer to the market price may be executed before the one can revise her quote whereas orders further away from the market price may not be executed and thus can be revised. However, the benefit of orders closer to the market price is that it increases the execution probability of a trade. Considering the possibility of pick offs, it seems natural that faster traders who has higher chances of revising her quote are more likely to submit orders closer to the market price than slower traders. If a trader who can beat slow traders is not fast enough to beat faster traders, she can submit orders further away to lower the chance of being picked off and revise her quote when she has to. When the original quote gets executed, she can still benefit from having a better price than submitting orders closer to the market price. However, slower traders who are not able to revise their quotes and looking for higher execution chances may submit orders closer to the market price. These incentives question the monotonicity of order submission strategy on speed.

Investor speed can be categorized in more than two groups. There exist traders who closely follow the market with high technical skills to execute their strategy on time, such as low latency traders. Some traders do follow the market but may not have access to quick order submissions as low-latency traders do. There are traders who do not have resources to trade fast and also those who are not observing the market every second. To examine the relationship of speed and order submission among more than two parties, we provide an extension of Hoffmann (2014)'s theoretical model where there exist three types of traders—slow, fast, and very fast. The extended model has an equilibrium in which very fast traders submit limit orders closer to the market price than slow traders do, but fast traders submit limit orders further away than slow traders do. This shows that speed and limit order placements need not be monotonically related.

In financial markets, faster trading speed (or low latency) is considered to be advantageous and also likely to be correlated with aggressiveness when submitting orders. Faster traders may observe some orders to be profitable while slower traders do not find it to be so, due to lack of execution ability. In this light, speed is likely to share similar characteristic on trading strategy as the private valuation of an asset. Hollifiield et al. (2004) test the monotonicity of the limit order strategy related to traders' valuation of the asset in a limit order market, finding that they cannot reject monotonicity for buy or sell orders separately, but can reject it when buy and sell orders are tested jointly. However, our results show that trader's speed is not monotonically aligned with their order strategies. Thus, we argue that individual speed and individual valuation of an asset or informational advantage should not be considered in a same way nor shall be projected in a single dimension. Extant literatures lack evidence on non-monotonic results, though monotonicity is an important condition in economic theory. Laffont and Vuong (1996) suggest that the in testing auction models, monotonicity condition can be used. and Guerre et al. (2000) show that for the existence of equilibrium in a first-price, sealed-bid auction with independent private values, a function of the observed bids must satisfy the monotonicity condition.

To support our theoretical findings, we provide empirical evidence that fast traders (e.g., LLTs or HFTs) and slow traders behave differently when submitting orders in the limit order book. Using novel data from the KOSPI 200 futures market, in which we can track orders and trades at the account level, we proxy for the ability to trade quickly by observing each account's minimum time difference between two orders. We find that slow traders submit non-marketable limit orders further from the market price. However, we do not find a monotonic relationship across entire speed groups. Our results are robust when controlling for market volatility and liquidity. Our empirical results are consistent with our theoretical model. We also find evidence that slow traders submit market orders with higher probability than fast traders do, and fast traders prefer using marketable limit orders than market orders conditioning on submitting orders with immediate execution.

Theories suggest that some factors may affect limit order behavior. Parlour (1998) proves that rational investors consider subsequent order arrivals and thickness of book when deciding what orders to submit. Hoffmann (2014) uses a variant of Foucault (1999) to show that slow traders use market orders more frequently, and provide conditions on a trader's optimal limit order strategy. Submitting limit orders further from the current price allows an order to earn more time due to the order of market order execution, though it comes at the cost of lowering the probability of being executed. Market orders do not experience pick-offs from fast traders, but might face less attractive trading prices. In the model, when the volatility of the underlying asset or the proportion of fast traders changes, slow traders might submit limit orders closer to the market value than fast traders do, or vice versa. However, since there are only two types of investors in Hoffmann (2014), the model is insufficient to show the (non-)monotonicity of trading behaviors.[1]

Existing literature such as Kirilenko et al. (2017), Malinova et al. (2018), Boehmer et al. (2018), and Biais et al. (2016) classify HFTs using number of order submissions or end-of-day inventory which are not direct measures of speed. Hautsch, Noé, Zhang (2017) additionally use time between two orders to classify HFTs. Our simple measure classifies investors into several groups only by looking at the time between two orders to proxy for speed. While previous literature focuses on the behavior and effects of HFT (firms), our analysis mainly focus on just how the speed of trading activity is reflected in order submissions. Also, our

[1] While Hoffmann (2014) also provides policy implications in his model, we only focus on the behavior of optimal order strategy and do not provide any welfare and policy implication from the extended model.

trading environment (KOSPI 200 futures) is also known for massive "boutique" or individual investors in the market. It is also known that these investors are actually using HFT strategies in the market.[2]

We also contribute to the literature by providing evidence of different uses of market orders and marketable limit orders by speed types of investors. Some literature distinguish market order with marketable limit orders (Boehmer, 2005; Boehmer et al., 2007; Harris and Hasbrouck, 1996; Peterson and Sirri, 2002) and while some neglect the differences between two types of orders (Anand et al., 2005; Bacidore et al., 2003; Bloomfield et al., 2005; Menkhoff et al., 2010). We show that traders in the fastest group submit marketable limit orders more often than market orders when they need immediate execution of an order. This suggests that the speed advantage of fast traders allows them to use marketable limit orders that secure prices if executed while forgoing the sure execution at a lower cost. If the order is not executed, fast traders can revise and cancel their quotes, and resubmit a new order at a lower cost that can be executed. Slow traders pay a higher cost if their marketable-intended limit orders are not executed. For example, suppose that new information increases the market price. An investor then submits a marketable limit-order at the best ask, but the order cannot be executed immediately due to quote cancellation at the best ask or another bid beating the queue. If the information is likely to drive the price up further, a trader can submit or revise an order at the new best ask or with a market order. Fast traders can do this in a short time so that execution can be made with minimal price changes, but slow traders take time when submitting a new order, that can lead to large disadvantage in terms of the price that they face.

Our findings of fast traders submitting more non-marketable limit orders and submitting limit orders closer to the execution price are consistent with the fact that fast traders act as liquidity suppliers. Hendershott and Riordan (2013) show that algorithmic traders consume liquidity when the spread is small and provide liquidity when the spread is large. Ranaldo

[2]See http://english.yonhapnews.co.kr/business/2015/04/26/0503000000AEN20150426001400320.html.

(2004) provides empirical evidence that order aggressiveness depends on the thickness of the limit order book. The market that we study is extremely liquid where the spread is normally one-tick, making it hard to directly test the findings in Hendershott and Riordan (2013) or Ranaldo (2004) in our sample.

Several papers study speed of trading in financial markets, but welfare consequences are mixed. Biais et al. (2015) and Budish et al. (2015) suggest that investing to trade faster causes negative welfare consequences, and Weller (2018) shows that stocks with greater algorithmic liquidity takers relative to liquidity providers suffer information losses. Brogaard et al. (2014) argue that HFTs facilitate price discovery, and Chordia et al. (2018) show that low-latency traders react quickly to macroeconomic announcements, and provide evidence of increasing competition among low-latency traders. Jovanovic and Menkveld (2016) show that HFTs reduce adverse-selection costs and Foucault et al. (2016) find that fast-informed traders account for higher trading volume. The current study finds that low-latency traders provide liquidity by using mostly limit orders, especially at the best bid or ask prices, which is consistent with our findings.

2 Three Speed Type Model

In this section, we develop a theory to show the relationship between trader's speed an order submission behavior. We of three different types of traders which only differs by speed. We follow the notation and the basic concept from Hoffmann (2014), which is a variant of Foucault (1999).

2.1 Limit Order Market with Three Types of Traders

There exists a single risky asset with fundamental value v_t. The value follows a random walk

$$v_t = v_{t-1} + \varepsilon_t \tag{1}$$

where $\varepsilon \in \{\sigma, -\sigma\}$ with equal probability. Risk neutral traders arrive sequentially at time $t = 1, 2, \ldots$. A trader arriving at $t \leq t'$ values the asset

$$R_{t'} = v_{t'} + y_t \tag{2}$$

where $y_t \in \{+L, -L\}$ is the time invariant private valuation and the realization of y_t occurs with equal probability.

Each trader can sell market orders execute at the currently best bid B_t^m, buy market orders execute at the currently best ask A_t^m, or set a limit order that might be executed during the next period. There are three types of traders: slow traders (STs), fast traders (FTs), and very fast Traders (VTs). The fraction of STs and FTs are denoted α and β, respectively, and the rest $(1 - \alpha - \beta)$ are VTs. FTs are able to cancel/revise their limit order and resubmit new ones after the realization of ε_{t+1}, conditional on $t+1$-th traders being an ST. VTs are able to cancel/revise their limit orders and resubmit new ones after the realization of ε_{t+1} conditional on $t+1$-th traders being an FT or ST.

2.2 Payoff and Strategies

Suppose $y_t = -L$. The seller's expected profit when choosing to post a limit order is equal to V_k^{LO}, $k \in \{ST, FT, VT\}$. She will "market sell" if

$$B^m - (v + \varepsilon - L) \geq V_k^{LO} \tag{3}$$

Let the cutoff price be

$$\hat{B}_k^{v+\varepsilon} \quad = \quad V_k^{LO} + (v + \varepsilon - L) \tag{4}$$

which makes the seller indifferent, if available. In the case of indifference, we assume that traders will choose market order to limit order if the expected payoffs are equal.

Now consider the buyer's quote-setting problem. Let $p(B)$ be the execution probability of ST. Since innovation (ε) and trader types are discrete, p is an increasing step function. Optimality implies that the price is set at a cutoff. The objective function of a slow buyer, who decides to submit the limit order, is:

$$V_{ST}^{LO} = \max_{B_{ST}} \{p(B_{ST})(v + E_{ex}[\varepsilon] + L - B_{ST})\} \tag{5}$$

where $E_{ex}[\cdot]$ is the expectation function conditional on execution. A fast buyer chooses a three-tuple bid price $(B_{FT}, B_{FT}^{+\sigma}, B_{FT}^{-\sigma})$. Let $q_{|k,\varepsilon}(B)$ denote execution probability of the FT's limit order with bid price B, given the next trader type and asset value innovation:[3]

$$\begin{aligned} V_{FT}^{LO} &= \max_{B_{FT}, B_{FT}^{+\sigma}, B_{FT}^{-\sigma}} \quad \{(1-\alpha) q_{|ST^-}(B_{FT})(v + E_{ex}[\varepsilon] + L - B_{FT}) \\ &\qquad + \frac{\alpha}{2} q_{|ST}(B_{FT}^{+\sigma})(v + \sigma + L - B_{FT}^{+\sigma}) \\ &\qquad + \frac{\alpha}{2} q_{|ST}(B_{FT}^{-\sigma})(v - \sigma + L - B_{FT}^{-\sigma})\} \end{aligned} \tag{6}$$

$B_{FT}^{+\sigma} = \hat{B}_{ST}^{v+\sigma}$ and $B_{FT}^{-\sigma} = \hat{B}_{ST}^{v-\sigma}$, and thus the maximization simplifies to:

$$\begin{aligned} & \max_{B_{FT}} \{(1-\alpha) q_{|ST^-}(B_{FT})(v + E_{ex}[\varepsilon] + L - B_{FT})\} \\ = & \max_{B_{FT}} \{\beta q_{|FT}(B_{FT})(v + E_{ex}[\varepsilon] + L - B_{FT}) + (1-\alpha-\beta) q_{|VT}(B_{FT})(v + E_{ex}[\varepsilon] + L - B_{FT})\} \end{aligned} \tag{7}$$

A very fast buyer chooses a three-tuple bid price $(B_{VT}, B_{VT}^{+\sigma}, B_{VT}^{-\sigma})$. Let $r_{|k,\varepsilon}(B)$ denote the execution probability of VT's limit order with bid price B, conditional on the next trader type and asset value innovation:

$$\begin{aligned} V_{VT}^{LO} &= \max_{B_{VT}, B_{VT}^{+\sigma}, B_{VT}^{-\sigma}} \quad \{(1-\alpha-\beta) r_{|VT}(B_{VT})(v + E_{ex}[\varepsilon] + L - B_{VT}) \\ &\qquad + \frac{(\alpha+\beta)}{2} r_{|VT^-}(B_{VT}^{+\sigma})(v + \sigma + L - B_{VT}^{+\sigma}) \\ &\qquad + \frac{(\alpha+\beta)}{2} r_{|VT^-}(B_{VT}^{-\sigma})(v - \sigma + L - B_{VT}^{-\sigma})\} \end{aligned} \tag{8}$$

[3] The negative superscript on ST^- refers to "all except" ST, which in this case refers to $FT \cup VT$.

For simplicity, we focus only on the positive innovation, where $\varepsilon = +\sigma$.[4] Choosing the optimal $B_{VT}^{+\sigma}$ may vary depending on the parameters. Since ability to trade faster cannot be worse, $V_{FT}^{LO} > V_{ST}^{LO}$, and thus have $\hat{B}_{FT}^{v+\sigma} > \hat{B}_{ST}^{v+\sigma}$. Since r is increasing and a step function, it is optimal to either choose $B_{VT}^{+\sigma} = \hat{B}_{ST}^{v+\sigma}$ or $B_{VT}^{+\sigma} = \hat{B}_{FT}^{v+\sigma}$. Note that if VT chooses $\hat{B}_{ST}^{+\sigma}$, FT seller will not trade since $\hat{B}_{FT}^{v+\sigma} > \hat{B}_{ST}^{v+\sigma}$ and only ST seller will trade. Thus:

$$\frac{\alpha+\beta}{2} r_{|VT^-}(\hat{B}_{ST}^{v+\sigma})(v+\sigma+L-\hat{B}_{ST}^{v+\sigma}) \gtreqless \frac{\alpha+\beta}{2} r_{|VT^-}(\hat{B}_{FT}^{v+\sigma})(v+\sigma+L-\hat{B}_{FT}^{v+\sigma}) \tag{9}$$

and since execution probability $r_{|VT^-}(\hat{B}_{ST}^{v+\sigma}) = \frac{\alpha}{2(\alpha+\beta)}$ and $r_{|VT^-}(\hat{B}_{FT}^{v+\sigma}) = \frac{1}{2}$,

$$\begin{aligned} \frac{\alpha}{2}(v+\sigma+L-\hat{B}_{ST}^{v+\sigma}) &\gtreqless \frac{(\alpha+\beta)}{2}(v+\sigma+L-\hat{B}_{FT}^{v+\sigma}) \\ \underbrace{\frac{\alpha}{\alpha+\beta}}_{LHS} &\gtreqless \underbrace{\frac{(v+\sigma+L-\hat{B}_{FT}^{v+\sigma})}{(v+\sigma+L-\hat{B}_{ST}^{v+\sigma})}}_{RHS} \end{aligned} \tag{10}$$

If the left-hand side of the equation (10) is greater, VTs target STs but not FTs. If the right-hand side is greater, VTs target both FTs and STs. Analogously from Hoffmann (2014), we have the following result which we elaborate the detailed proof in the appendix.

Lemma 1. *In equilibrium,* $\hat{B}_{ST}^{v-\sigma*} \leq \hat{B}_{FT}^{v-\sigma*} \leq \hat{B}_{VT}^{v-\sigma*} \leq \hat{B}_{ST}^{v+\sigma*} \leq \hat{B}_{FT}^{v+\sigma*} \leq \hat{B}_{VT}^{v+\sigma*}$

2.3 Equilibrium

For each equilibrium, note that each group of traders has following strategy set:

$$\{\hat{B}_{ST}^{v-\sigma*}, \hat{B}_{FT}^{v-\sigma*}, \hat{B}_{VT}^{v-\sigma*}, \hat{B}_{ST}^{v+\sigma*}, \hat{B}_{FT}^{v+\sigma*}, \hat{B}_{VT}^{v+\sigma*}\}$$

For each individuals, we must show that their strategy is incentive compatible. It is clear from the payoff structure (i.e., step function of execution probability) that investors will choose only one of the six strategies in $\{\hat{B}_{ST}^{v-\sigma*}, \hat{B}_{FT}^{v-\sigma*}, \hat{B}_{VT}^{v-\sigma*}, \hat{B}_{ST}^{v+\sigma*}, \hat{B}_{FT}^{v+\sigma*}, \hat{B}_{VT}^{v+\sigma*}\}$. We need to consider only the

[4] Negative innovation is analogous.

incentive compatibilities of VTs from $\{\hat{B}_{VT}^{v-\sigma*}, \hat{B}_{VT}^{v+\sigma*}\}$ and FTs from $\{\hat{B}_{FT}^{v-\sigma*}, \hat{B}_{VT}^{v-\sigma*}, \hat{B}_{FT}^{v+\sigma*}, \hat{B}_{VT}^{v+\sigma*}\}$, since other strategies are (weakly) dominated. Since we have two strategies for each $\hat{B}_{VT}^{v-\sigma*}$ and $\hat{B}_{VT}^{v+\sigma*}$, depending on the inequality in (10), we have maximum of $6 \times 4 \times 2 \times 2 = 96$ equilibrium strategy profiles. However, these strategies still may not all be feasible. We suggest the following result, which supports the findings from the empirical exercise in the previous section.

Proposition 1. *For some fixed parameters $(\alpha, \beta, \sigma, L)$, equilibrium exists where optimal strategy is non-monotonic in speed.*

This proposition can be examined by finding an example. Table 1 reports the number of equilibrium strategies for given $\sigma \in \{.1, .2, \ldots, .9\}$, and for each (α, β) pairs of $\alpha, \beta \in \{.01, .02, .03, \ldots, .99\}$ where $\alpha + \beta \leq 1$, and $L = 1$. Therefore, we examine 4,950 (α, β) pairs for each σ, with a total number of $44,550$ pairs of (α, β, σ). Table 1 shows, for example, that there are 35 (α, β) equilibrium pairs with $\hat{B}_{ST}^* = \hat{B}_{FT}^{v+\sigma*}$, $\hat{B}_{FT}^* = \hat{B}_{FT}^{v+\sigma*}$, and $\hat{B}_{VT}^* = \hat{B}_{VT}^{v-\sigma*}$ when $\sigma = .1$.[5] Graphical plots of equilibrium strategies are provided in Figure 2. With Lemma 1, we see that some type of equilibrium of speed on limit order behavior to be non-monotonic. It shows that fast traders submit orders further away from the market price than slow traders do.

Notice that submitting orders further away from the market price let i) stale orders that are picked off to have better prices and ii) decrease the chance of execution when the next (fast or very fast) investor arrives at a market, but can still revise the quotes to be executed by slow traders. Since slow traders cannot make any revisions of their stale orders due to lack of speed, their order submission strategy becomes simpler than the other two parties. However, as in our model, it represents a behavior that we would not be able to observe in a simple two-player setting. For example, when α is large, Figure 2 presents cases where STs submit limit orders closer to the market price than any other type of traders. More importantly, in our sample case of $\sigma = .8$, we find the non-monotonic results where FTs (black dots) submit orders further away than any others. Our model suggests incentives

[5] This is also an example that Proposition 1 holds. Clearly from Lemma 1, there can be other examples where Proposition 1 holds.

for middle player (FTs) to optimize by submitting the quotes further away than any other type's optimal strategy. In our model, FTs can still benefit by revising their quotes against the large number of slow traders but can minimize the loss from the very fast traders by submitting orders that very fast traders may not prefer to execute. Our theoretical result also aligns with our empirical findings that will come in the later section.

Similar to other games between players of more than two, strategies and equilibrium outcomes are complicated. While we do find equilibrium of non-monotonic behavior, our model has shortcomings due to complexity of the nature. Unlike the simple model of Hoffmann (2014), we find some cases where equilibrium may not exist for some profile of strategies. In the appendix, we show that not all the possible strategy profiles constitute an equilibrium.

3 Data

The underlying asset of the KOSPI 200 futures market is defined as the KOSPI 200 spot index, a value weighted stock price index that consists of the 200 largest common stocks listed on the Korea Exchange (KRX). The KOSPI 200 futures market is extremely liquid due to its low transaction costs and low entry barriers. No transaction tax is imposed on futures trading, and hence brokerage fees for KOSPI 200 futures trading are much lower than those for equity trading; about one-tenth compared to equity trading fees. Furthermore, there are no make-take fees in the market that may be crucial for strategic order selection.

With its rich liquidity, the futures index market is better suited to trading strategies based on macroeconomic, market-wide information. Since it consists of basket of assets, it is less likely to be affected by private information of a single firm. Another advantage of the futures market is that professional and informed traders can minimize exposure. Absence of designated market makers allows for anonymous trading when submitting orders and the abundant liquidity of the market helps them hide their informed trades.[6]

[6] Informed investors may split their orders and spread their trades more in illiquid markets. The order-splitting strategy can also be used when investors are more likely to be identified. This strategy is called

The tick size of the asset we study is 0.05 points, which equals to 25,000 KRW. The daily continuous trading session regularly opens at 9am and closes at 3:05pm. Batch auctions are used before and after the continuous trading session, which runs from 8am to 9am and 3:05pm to 3:15pm.[7] Expiration dates are second Thursdays of March, June, September, and December of each year.

Our sample period is from March 2010 to June 2014. Our data consist of both Trade and Quote (TAQ) for every trade and orders time-stamped at the millisecond level for all orders submitted to the market. The novelty of our dataset is that we can observe each trade and quote at the account level. We can also observe a trading account's group type (i.e., retail/individual investor, financial trading firm, institutional firm, etc.) and which country the account is from. The data also include types of orders for each order submitted (e.g., limit order, market order, stop order, order cancellations, order modifications, etc.).

Using the dataset, we are able to examine how different investors submit their orders by observing how frequently they use market orders and how far they place their limit orders from execution prices. We focus on trades and orders when each individual can observe the limit order book. Although the limit order book is available from 9am to 3:05pm, we restrict the sample to 9:10am through 2:50pm to eliminate microstructure bias that might occur during opening and near closing hours.[8] Analogously, we exclude trading days one week before maturities, and analyze only specific assets that have the nearest maturity date. Table 2 shows basic statistics by order type during the sample period. Less than two percent of all orders submitted during the period are market orders, but nearly 24.1 percent are marketable limit orders. We define marketable limit orders as buy (sell) limit orders that were submitted above the best ask (bid) price. Although marketable limit orders execute as

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stealth trading (see Anand and Chakravarty (2007)).

[7]There are exceptions to trading session times on the first trading day of the calendar year, and on the national College Scholastic Ability Test (CSAT) day. On these dates, opening of the continuous trading session is delayed one hour. For the CSAT day, the closing time of the continuous trading session is also delayed one hour.

[8]We make appropriate adjustments for the days that regular hours differ from these hours, which includes the first trading day of the calendar year and on the national College Scholastic Ability Test (CSAT) day.

market orders, we are able to distinguish them due to the rich dataset. Among the initial non-marketable limit orders submitted, 34.5 percent execute, on average, 188 seconds after submission, 22.8 percent 169 seconds after submission, and 42.3 percent 5 minutes. 883,340 non-marketable limit orders that do not belong to any of the aforementioned categories survive throughout the continuous trading hour (until 3:05pm), and are normally submitted further away from the market price. Although non-marketable limit orders are, on average, short-lived, as implied by the standard deviation of survival time, the distribution is wide.

To proxy account speed (i.e., the ability to trade fast), we use a simple measure where we track the smallest time difference between two messages from the same account.[9] The intuition is that traders able to trade quickly can submit multiple orders/revisions/cancellations in a very short period. This measure ignores the possibility of fast traders who submit only one order during a relatively long period, but we still pick up those able to submit multiple orders in a short period, and can identify them as fast traders. Thus, we use the following measure:

$$\ell = speed_{ij} = \min\{|time_{ijt} - time_{ijt+1}|\} \tag{11}$$

where i is the account number, j is the nearest futures maturity month, and $time_{ijt}$ is the t^{th} order that i made for j futures. $time$ is calculated in seconds clock time, not trading time. For further analysis we define the term "speed group" as simply as:

$$\hat{\ell} = \widehat{speed}_{ij} = \begin{cases} 7 & \text{if } speed_{ij} \leq 0.001 \\ 1 & \text{if } speed_{ij} > 100 \\ 4 - \lceil \log_{10} speed_{ij} \rceil & \text{otherwise} \end{cases} \tag{12}$$

where $\lceil \cdot \rceil$ is the ceiling function that rounds up to the next integer. For example, an account that submitted multiple orders within a millisecond will be in speed group $\hat{\ell} = 7$, and an

[9]Messages include submitting a new order, revising an existing limit order, or canceling a pre-submitted limit order.

account in which $\ell = -.017$ will be in speed group 5. In all, we have seven speed groups, ranging as integers from 1 to 7.

Our speed measure is not only to capture the physical ability (i.e. co-location) to submit orders. Rather, our measure incorporates the efficiency of trading algorithm, type of strategies, as well as (human) attention. Since we are not only assuming competition among LLTs, we believe our proxy which considers several reasons and measures actual timing of orders are suitable for our analysis.

Much high-frequency literature suggests proxies to estimate algorithmic trading. Hendershott et al. (2011) use number of messages, Hagströmer and Nordén (2013) use NASDAQ measure, Hasbrouck and Saar (2013) use strategic runs. Weller (2018) uses order-to-volume, cancel-to-trade, average trade size, and odd lot volume. In the current sample, most orders of futures contracts are relatively small, so average trade size does not explain what we observe. LLTs tend to have small average trade sizes, but individual investors also do not submit large orders in the derivatives market. The odd lot volume used by O'Hara et al. (2014) also does not apply in the current context since the smallest size tradable is one unit of contract, which is indivisible.

Table 3 Panel A shows basic statistics for market orders, marketable orders, limit spreads, order-to-trade ratios, and cancel-to-trade ratios for account-maturity pairs. $Dist$ is the difference between the limit buy (sell) order price and best ask (bid) price in number of ticks. To test whether a speed group relates to high-frequency proxies used in the literature, we use the order-to-trade and cancel-to-trade ratios as high-frequency proxies. Since market orders might distort each ratio, we use a limit orders to executed limit orders ratio ($\ln \frac{LimitOrder}{LimitTrade}$) and a canceled limit orders to executed limit orders ratio ($\ln(1 + \frac{Cancel}{LimitTrade})$). We also use the natural logarithm for each ratio to prevent statistical means from being affected heavily by right-skewed observations, and add one to the canceled to orders ratio to prevent accounts that do not use cancellations in their orders from giving output of $\ln 0$.[10] Panel A

[10] We find that approximately $\frac{1}{3}$ of accounts in the sample do not use cancellations in their orders. However, this does not imply that all of their orders were executed since the limit order book resets after the market

of Table 3 shows the basic statistics for these measures, and fraction of pure market orders and marketable orders (including marketable limit orders). Panel B of Table 3 shows simple correlations of the measures. We find that our measure of speed (ℓ) correlates positively with the high-frequency trading proxies, which is desired. All figures in the correlation tables are statistically significant at the 1% level.

Table 4 shows the number of active accounts by speed group. We find various accounts in terms of trading speed in the KOSPI 200 Index futures throughout the sample period. One interesting finding is that from the December 2011 contract (Column 12/8/11) onward, there was a greater number of extremely fast traders who could submit orders within one millisecond (row 7). The number of active accounts decreased for speed group 6 during the same period, suggesting a trading arms race among fast traders. We do not observe this phenomenon in other speed groups. Table 5 reports the total size of orders submitted for each speed group, and we find that most of the orders are dominated by very fast traders, primarily investors in speed groups 6 and 7.

4 Empirical Analysis

4.1 Limit order behavior

We now we test limit order behaviors among investors. As Hoffmann (2014) suggests, asset volatility might affect trading behavior, and thus we calculate a simple regression while controlling for volatility. Figure 1 shows volatility changes between January 2010 and June 2014. We calculate a regression for a subsample, that in which futures matured on September 8, 2011.[11] Since the difference between the market price and limit order must be non-negative,[12] we calculate a censored (tobit) regression as:

closes.

[11] We run the same regression by using different subsamples in our data. However, the results do not qualitatively differ from the results which are to shown in this section.

[12] Although we observe some marketable buy (sell) limit orders that are below the best ask (bid) price, we construct the variable to have a minimum value of zero.

$$Dist_{itd} = \alpha + \beta S_i + \gamma_{itd} Vol_{td} + BBQ_{td} + BAQ_{td} + \varepsilon_{itd} \quad (13)$$

where $Dist_{itd}$ is the tick difference (0.05) between a submitted limit order price and the mid-quote of the best bid and ask available. $Dist_{itd}$ has a minimum value of zero. Clearly, when $Dist_{itd}$ is zero, it represents a marketable limit order. We use the V-KOSPI index measure, derived from the KOSPI 200 options that an underlying asset is the same KOSPI 200.

Table 6 reports regression results. In the first four columns, where all intended limit orders are used (including marketable limit orders), we find that speed coefficients are non-monotonic. When we do not control for the top of the limit order book, all speed coefficients are positive, which implies that in comparison to the base group ($\hat{\ell} = 1$, slowest), other groups place orders further away from the market price. While this result may be very surprising, it may be coming from the investors using marketable limit orders as a strategy substituting market orders.

To take into account the use of marketable limit orders causing the problem, we exclude all marketable limit orders and re-run the regression. Columns (5) through (8) reports the results. We find that all groups submit orders closer to the market price than the slowest group (negative coefficient), implying that investors in the slowest group use limit orders but place marketable limit orders more frequently than others do, which is more consistent with our conjecture. Our results in Table 6 supports our theoretical findings of non-monotonic limit order behavior. This is mainly due to $\hat{\ell} = 5$ group, placing order further away from the market price than faster ($\hat{\ell} = 6, 7$) and slower ($\hat{\ell} = 2, 3, 4$) groups. The $\hat{\ell} = 5$ group is still fast trading group (FT in our theoretical model), since they are able to submit and revise orders within tenth of a second. However, they are still faced with the pick-off strategies from faster traders. Columns (2) to (4) and (6) to (8) in Table 6 show results including market volatility using KOSPI200 volatility index. Consistent with the literature, we find that with higher asset volatility, investors are more likely to submit limit orders further away from the market price.

While our results confirm the non-monotonic results in limit order behavior when using both all limit orders or non-marketable limit orders, it seems that traders are using limit orders to execute their orders similar to market order. Table 7 shows the number of immediate executable orders submitted per investor group. We find that fast traders prefer to submit marketable limit orders over market orders much more than slower traders do, which is consistent with our argument. Also, we find that a large number of immediate orders are actually marketable limit orders. Harris and Hasbrouck (1996) report that about 13 percent of immediate executable orders are marketable limit orders. In a later research, Peterson and Sirri (2002) report that 38 percent of immediate orders are marketable limit orders. Peterson and Sirri (2002) also observe that the usage of limit orders increase as the size or orders increase. In our sample, most of the orders in our sample are single unit orders, so we do not observe this phenomenon. However, we conjecture that marketable limit orders are more favored to investors when they compete on speed. This calls for an analysis on choice of market order and marketable orders.

4.2 Further analysis on market order and marketable order

Next, we investigate whether choosing market orders depends on the ability to trade quickly. Similar to (13), we run the regression, but with the dependent variable as a dummy variable as follows:

$$mktorder_{itd} = \alpha + \beta S_i + \gamma_{itd} Vol_{td} + BBQ_{td} + BAQ_{td} + \varepsilon_{itd} \tag{14}$$

$mktorder_{itd}$ is a dummy variable that is 1 if an order submitted at time t on day d by individual i is a market order and zero otherwise. Since the dependent variable is binary, we use logit regression for the equation above. S_i is a vector of dummies which represents each speed group ($\hat{\ell}$) from 2 to 7. Vol_{td} is the volatility index implied by KOSPI 200 options that is measured every 30 seconds. BBQ_{td} is the quantity of limit order placed at the best bid and BAQ_{td} is the quantity of limit order placed at the best ask. As mentioned in the earlier section, we do not observe much variation in terms of spread since we normally observe only one tick of a spread. Thus, we do not include spread in our regression and cannot induce arguments like in Bae et al. (2003). Columns (1) through (4) in Table 8 show regression results from equation (14). We find the coefficients from $Speed_2$ to $Speed_7$ decrease monotonically by going down the row, implying that faster traders submit fewer market orders. We also find positive results for volatility. Higher volatility implies greater uncertainty in the market, and thus risk-averse investors might prefer market orders to limit orders. We find a negative coefficient for best ask (bid) quantities for sell orders, and the opposite for buy orders, implying that when placing a sell (buy) limit order, placing a limit order in a longer queue is less likely to be executed, so market orders become more favorable in comparison to a short queue.

We also calculate the same regression using equation (14) but use marketable orders as the dependent variable, which includes marketable limit orders. Columns (5) to (8) report regression results, and show similar patterns for speed coefficients. However, the monotonicity is broken primarily by group six ($Speed_6$), implying that the second fastest group submits more marketable limit orders than the next (third) fastest group does. This also implies that when investors of the second fastest group try to absorb liquidity from the market, they might submit a marketable limit order rather than a market order to avoid the risk of price change between order submission and execution. The third fastest group uses more market orders and less marketable orders in comparison to the second fastest group, placing greater weight on execution risk than price change risk. We also find that the signs of non-speed related coefficients flip in comparison to results for market orders. These observations imply that market orders and marketable limit orders derive from disparate strategies.

Tables 8 and 6 imply that investors' option of market order, marketable limit order, and non-marketable limit order might have disparate costs and benefits by agent. Marketable-intended limit orders might not execute and become non-marketable limit orders when the top of the limit order book price changes between the time order requests are sent and when the market receives an order. This relates to human versus computer awareness, and market latency mentioned by Menkveld and Zoican (2017). Thus, when an investor is satisfied if she can make an order at the "current" price, but will not be satisfied when the order is executed at a different price and prefers to wait at the "current" price, she submits a limit order that might not be executed immediately. Marketable-intended limit orders also face risk, but the size of the risk differs by the ability to trade quickly. When limit orders are not executed and the price is moving away from the submitted quote, investors can resubmit an order for execution. However, if the executable price is moving at a rate proportional to time, slow traders incur higher costs than fast traders do. Market orders face only price risk since the execution of an order is certain as long as there exists orders outstanding in the limit order book. Thus, if risk of execution is similar across agents, risk of price is higher for slow traders such that slow traders will relatively prefer market orders more than fast traders do.

Table 9 shows orders that are placed one-tick away from the executable price. For the fastest group ($\hat{\ell} = 7$), we find that more than three-quarters of all canceled quotes initially on the top of the limit order book are canceled within three seconds. However, when an order is executed, the quote survival time is longer than a revised or canceled order. This statistic might be biased since revised or canceled quotes are pulled off before execution, but for slower traders, the survival time of executed orders is much lower (in median) than that for canceled or revised quotes. Thus, Table 9 provides evidence of marketable order use by fast traders.

Although many orders are canceled or revised by fast traders at the top of the limit order book, many orders are executed. This is not limited to orders submitted only at the top of the limit order book. In comparison to slow traders, fast traders are more likely to be liquidity providers in the market.

5 Conclusion

The ability to trade faster than others changes the behavior of not just the ones who are able, but also all potential counter-parties who trade with them. We evidence that slow investors fear the chances of their limit orders being picked off. Slow investors make market orders more often than fast investors do, and when they do make limit orders, they place them further from the market value, which lowers the possibility of execution. However, with multiple types of traders that differ by speed, we might not see monotonicity of limit order spread by speed, which is supported by data from the KOSPI 200 Futures market. Extant literature does not explain this phenomenon, but we provide a theoretical model that addresses this observation.

We also stress that when using marketable orders, we might need to distinguish market orders and marketable limit orders. The two type of orders can be used differently depending on the strategy. The set of strategies that individual can execute may differ by their abilities. We show that to submit and revise/cancel in a short period of time can be a factor that effects investors which orders to submit when they seek immediate order execution.

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A Proofs

A.1 Proof of Lemma 1

The proof of Lemma 1 is analogous to the Lemma 1 in Hoffmann (2014).

Since the ability to revise limit orders can never be inferior, $V_{VT}^{LO*} \geq V_{FT}^{LO*} \geq V_{ST}^{VO*}$. From (4), we have $\hat{B}_{ST}^{v-\sigma*} \leq \hat{B}_{FT}^{v-\sigma*} \leq \hat{B}_{VT}^{v-\sigma*}$ and $\hat{B}_{ST}^{v+\sigma*} \leq \hat{B}_{FT}^{v+\sigma*} \leq \hat{B}_{VT}^{v+\sigma*}$.

It remains to show $\hat{B}_{VT}^{v-\sigma*} \leq \hat{B}_{ST}^{v+\sigma*}$. First, L is the maximum expected profit from trade that period (when agents with different private values trade, they share a surplus of $2L$, which occurs at most with probability $\frac{1}{2}$), so $L \geq V_k^{LO*} \geq 0$ for all $k \in \{ST, FT, VT\}$.

Assume $\sigma \geq \frac{L}{2}$. Using (4), $v_t + \sigma \geq \hat{B}_{ST}^{v+\sigma*} \geq v_t + \sigma - L$ and $v_t - \sigma \geq \hat{B}_{VT}^{v-\sigma*} \geq v_t - \sigma - L$, which directly implies $\hat{B}_{VT}^{v-\sigma*} \leq \hat{B}_{ST}^{v+\sigma*}$.

Now Assume $\sigma < \frac{L}{2}$, and consider a very fast buyer submitting a buy limit order. It is easy to see that, in this case,

$$\frac{1-\alpha-\beta}{4}[v-\sigma+L-\hat{B}_{VT}^{v+\sigma*}]+\frac{1-\alpha-\beta}{4}[v+\sigma+L-\hat{B}_{VT}^{v+\sigma*}] \geq \frac{1-\alpha-\beta}{4}[v-\sigma+L-\hat{B}_{VT}^{v-\sigma*}]$$

such that his optimal choice is $B_{VT}^{*} = \hat{B}_{VT}^{v+\sigma*}$. A buyer arriving in the next period does not execute this order because $v - \sigma + L > \hat{B}_{VT}^{v+\sigma*}$.

Now consider a slow buyer where he submits a buy limit order with $B_{ST}^{*} = \hat{B}_{VT}^{v+\sigma*}$. Since this is not his equilibrium strategy, $V_{ST}^{LO*} \geq \frac{1}{2}[v + L - \hat{B}_{VT}^{v+\sigma*}]$. Now assume

$$\frac{\alpha}{\alpha+\beta} < \frac{(v+\sigma+L-\hat{B}_{FT}^{+\sigma})}{(v+\sigma+L-\hat{B}_{ST}^{+\sigma})}$$

Then,

$$\begin{aligned} V_{VT}^{LO} &= \{\frac{(1-\alpha-\beta)}{2}(v+L-\hat{B}_{VT}^{v+\sigma*}) \\ &\quad +\frac{(\alpha+\beta)}{4}(v+\sigma+L-\hat{B}_{FT}^{v+\sigma*}) \\ &\quad +\frac{(\alpha+\beta)}{4}(v-\sigma+L-\hat{B}_{FT}^{v-\sigma*})\} \end{aligned}$$

and therefore,

$$\begin{aligned} V_{VT}^{LO*}-V_{ST}^{LO*} &\leq \frac{\alpha+\beta}{4}(\hat{B}_{VT}^{v+\sigma*}-\hat{B}_{FT}^{v-\sigma*})+\frac{\alpha+\beta}{4}(\hat{B}_{VT}^{v+\sigma*}-\hat{B}_{FT}^{v+\sigma*}) \\ &\leq \frac{\alpha+\beta}{4}(\hat{B}_{VT}^{v+\sigma*}-\hat{B}_{ST}^{v-\sigma*})+\frac{\alpha+\beta}{4}(\hat{B}_{VT}^{v+\sigma*}-\hat{B}_{ST}^{v+\sigma*}) \\ &= \frac{\alpha+\beta}{2}(V_{VT}^{LO*}-V_{ST}^{LO*})+\frac{\alpha+\beta}{2}\sigma \end{aligned}$$

Using (4), $V_{VT}^{LO*}-V_{ST}^{LO*} \leq \frac{\alpha+\beta}{2-\alpha-\beta}\sigma$, so $\hat{B}_{VT}^{v-\sigma*} \leq \hat{B}_{ST}^{v+\sigma*}$.

Now assume

$$\frac{\alpha}{\alpha+\beta} > \frac{(v+\sigma+L-\hat{B}_{FT}^{+\sigma})}{(v+\sigma+L-\hat{B}_{ST}^{+\sigma})}$$

Then,

$$\begin{aligned} V_{VT}^{LO} &= \{\frac{(1-\alpha-\beta)}{2}(v+L-\hat{B}_{VT}^{v+\sigma*}) \\ &\quad +\frac{\alpha}{4}(v+\sigma+L-\hat{B}_{ST}^{v+\sigma*}) \\ &\quad +\frac{\alpha}{4}(v-\sigma+L-\hat{B}_{ST}^{v-\sigma*})\} \\ &\leq \{\frac{(1-\alpha-\beta)}{2}(v+L-\hat{B}_{VT}^{v+\sigma*}) \\ &\quad +\frac{\alpha+\beta}{4}(v+\sigma+L-\hat{B}_{ST}^{v+\sigma*}) \\ &\quad +\frac{\alpha+\beta}{4}(v-\sigma+L-\hat{B}_{ST}^{v-\sigma*})\} \end{aligned}$$

and therefore,

$$\begin{aligned} V_{VT}^{LO*} - V_{ST}^{LO*} &\leq \frac{\alpha+\beta}{4}(\hat{B}_{VT}^{v+\sigma*} - \hat{B}_{ST}^{v-\sigma*}) + \frac{\alpha+\beta}{4}(\hat{B}_{VT}^{v+\sigma*} - \hat{B}_{ST}^{v+\sigma*}) \\ &= \frac{\alpha+\beta}{2}(V_{VT}^{LO*} - V_{ST}^{LO*}) + \frac{\alpha+\beta}{2}\sigma \end{aligned}$$

so we again have $\hat{B}_{VT}^{v-\sigma*} \leq \hat{B}_{ST}^{v+\sigma*}$.

A.2 Not all strategy profiles are feasible

Proposition 2. *For some fixed parameters $(\alpha, \beta, \sigma, L)$, a pure strategy equilibrium may not exist. Furthermore, not all strategy profiles are feasible for equilibrium.*

Proof. For the first part of the proposition, it is enough to show an example where an equilibrium strategy does not exist for certain set of fixed parameter. When $\alpha = .5$, $\beta = .3$, $\sigma = .5$, and $L = 1$ we can easily find that none of the possible strategies satisfy an equilibrium strategy.

For the second part of the proposition, we can prove by showing that a certain strategy profile does not constitute an equilibrium.In this part, we only show the conditions one of the many possible cases for equilibrium. We focus on the case when $\hat{B}_{ST}^{*} = \hat{B}_{ST}^{v-\sigma*}$, $\hat{B}_{FT}^{*} = \hat{B}_{FT}^{v-\sigma*}$, and $\hat{B}_{VT}^{*} = \hat{B}_{VT}^{v-\sigma*}$.[13] First we check on the case where $\frac{\alpha}{\alpha+\beta} < \frac{(v+\sigma+L-\hat{B}_{FT}^{v+\sigma})}{(v+\sigma+L-\hat{B}_{ST}^{v+\sigma})}$. □

$V_{ST}^{LO*} = \frac{\alpha}{4}[v - \sigma + L - \hat{B}_{ST}^{v-\sigma*}]$

$V_{FT}^{LO*} = \frac{\beta}{4}[v - \sigma + L - \hat{B}_{FT}^{v-\sigma*}] + \frac{\alpha}{4}(v + \sigma + L - \hat{B}_{ST}^{v+\sigma*}) + \frac{\alpha}{4}(v - \sigma + L - \hat{B}_{ST}^{v-\sigma*})$

$V_{VT}^{LO*} = \frac{(1-\alpha-\beta)}{4}(v - \sigma + L - B_{VT}^{v+\sigma*}) + \frac{(1-\alpha-\beta)}{4}(v + \sigma + L - B_{VT}^{v+\sigma*}) + \frac{\alpha}{4}(v + \sigma + L - \hat{B}_{ST}^{v+\sigma*}) + \frac{\alpha}{4}(v - \sigma + L - \hat{B}_{ST}^{v-\sigma*})$

[13] There are other strategy profiles that are also non-feasible.

so we have

$$\begin{aligned}
V_{ST}^{LO*} = V_{ST}(\hat{B}_{ST}^{v-\sigma*}) &= \frac{\alpha}{4}[v - \sigma + L - \hat{B}_{ST}^{v-\sigma*}] \\
&= \frac{\alpha}{4}[2L - V_{ST}^{LO*}] \\
&= (\frac{\alpha}{4+\alpha})(2L). \qquad (15)
\end{aligned}$$

For FTs,

$$\begin{aligned}
V_{FT}^{LO*} &= \frac{\beta}{4}[2L - V_{FT}^{LO*}] + \frac{\alpha}{2}[2L - V_{ST}^{LO*}] \\
&= \frac{1}{4+\beta}[2L(2\alpha+\beta) - 2\alpha V_{ST}^{LO*}]. \qquad (16)
\end{aligned}$$

Plugging in (15),

$$V_{FT}^{LO*} = \frac{\beta(4+\alpha)+8\alpha}{(4+\beta)(4+\alpha)}2L. \qquad (17)$$

We also should have the optimal strategy for VTs,

$$\begin{aligned}
V_{VT}^{LO*} &= -\frac{1-\alpha-\beta}{2}\sigma + \frac{1-\alpha-\beta}{4}[2L - V_{VT}^{LO*}] + \frac{1-\alpha-\beta}{4}[2L - V_{VT}^{LO*}] \\
&\quad + \frac{\alpha}{4}[2L - V_{ST}^{LO*}] + \frac{\alpha}{4}[2L - V_{ST}^{LO*}] \\
&= -\frac{1-\alpha-\beta}{2}\sigma + \frac{1-\alpha-\beta}{2}[2L - V_{VT}^{LO*}] + \frac{\alpha}{2}[2L - V_{ST}^{LO*}] \\
&= -\frac{1-\alpha-\beta}{3-\alpha-\beta}\sigma + \frac{2}{3-\alpha-\beta}[(1-\beta)L - \frac{\alpha}{2}V_{ST}^{LO*}]. \qquad (18)
\end{aligned}$$

Applying (15) to (18) gives

$$V_{VT}^{LO*} = -\frac{1-\alpha-\beta}{3-\alpha-\beta}\sigma + \frac{(1-\beta)(4+\alpha)-\alpha^2}{(3-\alpha-\beta)(4+\alpha)}2L \qquad (19)$$

Since we assume $\frac{\alpha}{\alpha+\beta} < \frac{(v+\sigma+L-\hat{B}_{FT}^{v+\sigma})}{(v+\sigma+L-\hat{B}_{ST}^{v+\sigma})}$, (15) and (16) gives

$$\beta > \frac{\alpha^2}{4-2\alpha} \qquad (20)$$

V_{ST}^{LO*} should be greater than other available strategies for STs, i.e., slow traders using $\hat{B}_{FT}^{v-\sigma*}$ strategy gives

$$\begin{aligned} V_{ST}\left(\hat{B}_{FT}^{v-\sigma*}\right) &= \frac{\alpha+\beta}{4}[v-\sigma+L-\hat{B}_{FT}^{v-\sigma*}] \\ &= \frac{\alpha+\beta}{4}[2L-V_{FT}^{LO*}] \\ &= \frac{(\alpha+\beta)(4-\alpha)}{(4+\beta)(4+\alpha)}2L \end{aligned} \tag{21}$$

which should satisfy the incentive compatibility,

$$\begin{aligned} V_{ST}^{LO*} &\geq V_{ST}(\hat{B}_{FT}^{v-\sigma*}) \\ (\frac{\alpha}{4+\alpha})(2L) &\geq \frac{(\alpha+\beta)(4-\alpha)}{(4+\beta)(4+\alpha)}2L \end{aligned}$$

that gives

$$\beta \leq \frac{\alpha^2}{4-2\alpha}. \tag{22}$$

This contradicts with (20). Thus, $\hat{B}_{ST}^{*}=\hat{B}_{ST}^{v-\sigma*}$, $\hat{B}_{FT}^{*}=\hat{B}_{FT}^{v-\sigma*}$, and $\hat{B}_{VT}^{*}=\hat{B}_{VT}^{v+\sigma*}$, where $\frac{\alpha}{\alpha+\beta}<\frac{(v+\sigma+L-\hat{B}_{FT}^{v+\sigma})}{(v+\sigma+L-\hat{B}_{ST}^{v+\sigma})}$ cannot be an equilibrium.

Thus we should have $\frac{\alpha}{\alpha+\beta}>\frac{(v+\sigma+L-\hat{B}_{FT}^{v+\sigma})}{(v+\sigma+L-\hat{B}_{ST}^{v+\sigma})}$. Note that for the FTs, $\hat{B}_{VT}^{v-\sigma}$ strategy should not give higher value. So,

$$\begin{aligned} V_{FT}(\hat{B}_{VT}^{v-\sigma*}) &= \frac{1-\alpha}{4}(2L-V_{VT}^{LO*})+\frac{\alpha}{2}(2L-V_{ST}^{LO*}) \\ &= \frac{(1-\alpha)(1-\alpha-\beta)}{4(3-\alpha-\beta)}\sigma+\left(\frac{(1-\alpha)(8-2\alpha)}{4(3-\alpha-\beta)(4+\alpha)}+\frac{2\alpha}{4+\alpha}\right)2L \end{aligned}$$

Incentive compatible condition implies

$$
\begin{aligned}
V_{FT}^{LO*} &\geq V_{FT}(\hat{B}_{VT}^{v-\sigma*}) \\
\frac{\beta(4+\alpha)+8\alpha}{(4+\beta)(4+\alpha)}2L &\geq \frac{(1-\alpha)(1-\alpha-\beta)}{4(3-\alpha-\beta)}\sigma + \left(\frac{(1-\alpha)(8-2\alpha)}{4(3-\alpha-\beta)(4+\alpha)} + \frac{2\alpha}{4+\alpha}\right)2L \\
\frac{(4-\alpha)[(\beta-4)(1-\alpha-\beta)-\beta^2]}{(4+\alpha)(4+\beta)(1-\alpha)(1-\alpha-\beta)}4L &\geq \sigma \qquad (23)
\end{aligned}
$$

Since $\alpha, \beta \in [0,1]$ and $\alpha+\beta \in [0,1]$, the LHS of (23) is negative. Since $L > 0$ and $\sigma > 0$, it contradicts that, $\hat{B}_{ST}^{*} = \hat{B}_{ST}^{v-\sigma*}$, $\hat{B}_{FT}^{*} = \hat{B}_{FT}^{v-\sigma*}$, and $\hat{B}_{VT}^{*} = \hat{B}_{VT}^{v+\sigma*}$ is an equilibrium where $\frac{\alpha}{\alpha+\beta} > \frac{(v+\sigma+L-\hat{B}_{FT}^{v+\sigma})}{(v+\sigma+L-\hat{B}_{ST}^{v+\sigma})}$. Hence we conclude that strategy profile $\hat{B}_{ST}^{*} = \hat{B}_{ST}^{v-\sigma*}$, $\hat{B}_{FT}^{*} = \hat{B}_{FT}^{v-\sigma*}$, and $\hat{B}_{VT}^{*} = \hat{B}_{VT}^{v+\sigma*}$ cannot be an equilibrium strategy.

B Tables and Figures

Table 1: Equilibrium Frequency for Each Strategy Pairs
This table serves as numerical proof of Proposition 1. It reports the number of equilibrium strategies for given $\sigma \in \{.1, .2, \ldots, .9\}$, and for each α and β to have values of $\{.01, .02, .03, \ldots, .99\}$ where $\alpha + \beta \leq 1$ and $L = 1$. Therefore, we test 4,950 different (α, β) pairs for each σ given. For example, when $\sigma = .1$, it shows that there are 35 (α, β) pairs when $\hat{B}^*_{ST} = \hat{B}^{v+\sigma*}_{FT}$, $\hat{B}^*_{FT} = \hat{B}^{v+\sigma*}_{FT}$, and $\hat{B}^*_{VT} = \hat{B}^{v-\sigma*}_{VT}$. Number of non-monotonic order behavior equilibrium are in bold.

$\sigma = .1$	$\hat{B}^*_{VT} = \hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^*_{FT}$				$\hat{B}^*_{VT} = \hat{B}^{v+\sigma*}_{VT}$	$\hat{B}^*_{FT}$			
		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$
$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{ST}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{FT}$	0	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{FT}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	0	0
	$\hat{B}^{v+\sigma*}_{ST}$	**0**	0	**0**	**1**	$\hat{B}^{v+\sigma*}_{ST}$	**0**	**0**	0	17
	$\hat{B}^{v+\sigma*}_{FT}$	**0**	0	35	**0**	$\hat{B}^{v+\sigma*}_{FT}$	**0**	**0**	189	29
	$\hat{B}^{v+\sigma*}_{VT}$	**0**	0	31	12	$\hat{B}^{v+\sigma*}_{VT}$	**0**	**0**	**162**	2319

$\sigma = .2$	$\hat{B}^*_{VT} = \hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^*_{FT}$				$\hat{B}^*_{VT} = \hat{B}^{v+\sigma*}_{VT}$	$\hat{B}^*_{FT}$			
		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$
$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{ST}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{FT}$	0	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{FT}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	0	0
	$\hat{B}^{v+\sigma*}_{ST}$	**0**	0	**0**	**4**	$\hat{B}^{v+\sigma*}_{ST}$	**0**	**0**	0	53
	$\hat{B}^{v+\sigma*}_{FT}$	**0**	0	34	**0**	$\hat{B}^{v+\sigma*}_{FT}$	**0**	**0**	293	55
	$\hat{B}^{v+\sigma*}_{VT}$	**0**	0	26	5	$\hat{B}^{v+\sigma*}_{VT}$	**0**	**0**	**205**	2037

$\sigma = .3$	$\hat{B}^*_{VT} = \hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^*_{FT}$				$\hat{B}^*_{VT} = \hat{B}^{v+\sigma*}_{VT}$	$\hat{B}^*_{FT}$			
		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$
$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{ST}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{FT}$	0	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{FT}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	0	0
	$\hat{B}^{v+\sigma*}_{ST}$	**0**	0	**0**	**10**	$\hat{B}^{v+\sigma*}_{ST}$	**0**	**0**	0	120
	$\hat{B}^{v+\sigma*}_{FT}$	**0**	0	37	**0**	$\hat{B}^{v+\sigma*}_{FT}$	**0**	**0**	378	74
	$\hat{B}^{v+\sigma*}_{VT}$	**0**	0	28	3	$\hat{B}^{v+\sigma*}_{VT}$	**0**	**0**	**271**	1840

$\sigma = .4$	$\hat{B}^*_{VT} = \hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^*_{FT}$				$\hat{B}^*_{VT} = \hat{B}^{v+\sigma*}_{VT}$	$\hat{B}^*_{FT}$			
		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$
$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{ST}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{FT}$	0	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{FT}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	**0**	**0**	$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	0	0
	$\hat{B}^{v+\sigma*}_{ST}$	**0**	0	**0**	**9**	$\hat{B}^{v+\sigma*}_{ST}$	**0**	**0**	0	117
	$\hat{B}^{v+\sigma*}_{FT}$	**0**	0	33	**0**	$\hat{B}^{v+\sigma*}_{FT}$	**0**	**0**	498	96
	$\hat{B}^{v+\sigma*}_{VT}$	**0**	0	30	2	$\hat{B}^{v+\sigma*}_{VT}$	**0**	**0**	**426**	2025

(Table 1 continued)

$\sigma = .5$

$\hat{B}^*_{VT} = \hat{B}^{v-\sigma*}_{VT}$		$\hat{B}^*_{FT}$				$\hat{B}^*_{VT} = \hat{B}^{v+\sigma*}_{VT}$		$\hat{B}^*_{FT}$			
		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$			$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$
$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	**0**	**0**	$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{FT}$	0	0	**0**	**0**		$\hat{B}^{v-\sigma*}_{FT}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	**0**	**0**		$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	0	0
	$\hat{B}^{v+\sigma*}_{ST}$	**0**	0	**0**	**10**		$\hat{B}^{v+\sigma*}_{ST}$	**0**	**0**	0	170
	$\hat{B}^{v+\sigma*}_{FT}$	**0**	0	34	**0**		$\hat{B}^{v+\sigma*}_{FT}$	**0**	**0**	492	104
	$\hat{B}^{v+\sigma*}_{VT}$	**0**	0	18	1		$\hat{B}^{v+\sigma*}_{VT}$	**0**	**0**	**324**	1480

$\sigma = .6$

$\hat{B}^*_{VT} = \hat{B}^{v-\sigma*}_{VT}$		$\hat{B}^*_{FT}$				$\hat{B}^*_{VT} = \hat{B}^{v+\sigma*}_{VT}$		$\hat{B}^*_{FT}$			
		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$			$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$
$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	**0**	**0**	$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{FT}$	0	0	**23**	**0**		$\hat{B}^{v-\sigma*}_{FT}$	0	0	168	0
	$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	**0**	**0**		$\hat{B}^{v-\sigma*}_{VT}$	**0**	0	0	0
	$\hat{B}^{v+\sigma*}_{ST}$	**0**	0	**1**	**10**		$\hat{B}^{v+\sigma*}_{ST}$	**0**	**0**	2	277
	$\hat{B}^{v+\sigma*}_{FT}$	**0**	0	37	**0**		$\hat{B}^{v+\sigma*}_{FT}$	**0**	**0**	546	85
	$\hat{B}^{v+\sigma*}_{VT}$	**0**	0	2	0		$\hat{B}^{v+\sigma*}_{VT}$	**0**	**0**	**91**	516

$\sigma = .7$

$\hat{B}^*_{VT} = \hat{B}^{v-\sigma*}_{VT}$		$\hat{B}^*_{FT}$				$\hat{B}^*_{VT} = \hat{B}^{v+\sigma*}_{VT}$		$\hat{B}^*_{FT}$			
		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$			$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$
$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	**0**	**0**	$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	0	0
	$\hat{B}^{v-\sigma*}_{FT}$	15	22	**330**	**0**		$\hat{B}^{v-\sigma*}_{FT}$	7	2	34	0
	$\hat{B}^{v-\sigma*}_{VT}$	**0**	818	**102**	**0**		$\hat{B}^{v-\sigma*}_{VT}$	**0**	1	0	1
	$\hat{B}^{v+\sigma*}_{ST}$	**1**	343	**0**	**0**		$\hat{B}^{v+\sigma*}_{ST}$	**1**	**18**	0	0
	$\hat{B}^{v+\sigma*}_{FT}$	**0**	92	454	**0**		$\hat{B}^{v+\sigma*}_{FT}$	**0**	**0**	27	0
	$\hat{B}^{v+\sigma*}_{VT}$	**0**	0	0	0		$\hat{B}^{v+\sigma*}_{VT}$	**0**	**0**	**0**	824

$\sigma = .8$

$\hat{B}^*_{VT} = \hat{B}^{v-\sigma*}_{VT}$		$\hat{B}^*_{FT}$				$\hat{B}^*_{VT} = \hat{B}^{v+\sigma*}_{VT}$		$\hat{B}^*_{FT}$			
		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$			$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$
$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	**0**	**0**	$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	0	147	4
	$\hat{B}^{v-\sigma*}_{FT}$	341	117	**234**	**0**		$\hat{B}^{v-\sigma*}_{FT}$	39	11	27	0
	$\hat{B}^{v-\sigma*}_{VT}$	**0**	2102	**7**	**0**		$\hat{B}^{v-\sigma*}_{VT}$	**0**	6	0	0
	$\hat{B}^{v+\sigma*}_{ST}$	**0**	147	**0**	**0**		$\hat{B}^{v+\sigma*}_{ST}$	**0**	**4**	0	0
	$\hat{B}^{v+\sigma*}_{FT}$	**0**	8	104	**0**		$\hat{B}^{v+\sigma*}_{FT}$	**0**	**0**	**0**	0
	$\hat{B}^{v+\sigma*}_{VT}$	**0**	0	0	0		$\hat{B}^{v+\sigma*}_{VT}$	**0**	**0**	**0**	0

$\sigma = .9$

$\hat{B}^*_{VT} = \hat{B}^{v-\sigma*}_{VT}$		$\hat{B}^*_{FT}$				$\hat{B}^*_{VT} = \hat{B}^{v+\sigma*}_{VT}$		$\hat{B}^*_{FT}$			
		$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$			$\hat{B}^{v-\sigma*}_{FT}$	$\hat{B}^{v-\sigma*}_{VT}$	$\hat{B}^{v+\sigma*}_{FT}$	$\hat{B}^{v+\sigma*}_{VT}$
$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	137	**0**	**0**	$\hat{B}^*_{ST}$	$\hat{B}^{v-\sigma*}_{ST}$	0	7	0	0
	$\hat{B}^{v-\sigma*}_{FT}$	447	219	**133**	**0**		$\hat{B}^{v-\sigma*}_{FT}$	39	15	16	0
	$\hat{B}^{v-\sigma*}_{VT}$	**0**	2145	**0**	**0**		$\hat{B}^{v-\sigma*}_{VT}$	**0**	7	0	0
	$\hat{B}^{v+\sigma*}_{ST}$	**0**	0	**0**	**0**		$\hat{B}^{v+\sigma*}_{ST}$	**0**	**0**	0	0
	$\hat{B}^{v+\sigma*}_{FT}$	**0**	0	0	**0**		$\hat{B}^{v+\sigma*}_{FT}$	**0**	**0**	**0**	0
	$\hat{B}^{v+\sigma*}_{VT}$	**0**	0	0	0		$\hat{B}^{v+\sigma*}_{VT}$	**0**	**0**	**0**	0

Table 2: Summary Statistics

This table reports the basic statistics of orders submitted from 9:10 a.m. to 2:50 p.m. which are during continuous trading hours in the Korea Exchange for the KOSPI 200 Futures from March 12, 2010 to June 12, 2014 for nearest maturity futures, excluding those that matures within one week. *Qty* is order quantity per order submitted, *Surv* is survival time of non-marketable limit order, *Dist* is defined as the difference between limit buy (sell) order to best ask (bid) in ticks, but truncated at 10 ticks. One tick is 0.05 in index. Non-marketable limit orders that are classified as "else" are orders that are not executed, revised, nor canceled until the end of continuous trading hour (3:05 p.m.). All statistics for *Surv* and *Dist* are weighted by *Qty*.

		N	Mean(Qty)	Std(Qty)	Mean($Surv$)	Std($Surv$)	Mean($Dist$)	Std($Dist$)
Market Order		6,457,270	1.61	1.94				
Marketable Limit Order		90,355,609	2.13	3.65				
Non-Marketable	All	277,779,327	2.62	5.78	244.36	2171.62	2.81	4.54
Limit Order	Executed	95,926,104	2.10	3.57	188.51	1366.10	1.54	2.50
	Revise	63,426,742	2.11	3.85	169.37	1301.18	4.82	4.72
	Cancel	117,543,132	3.33	7.69	298.63	2941.29	2.75	4.83
	Else	883,349	2.89	8.67			8.59	4.51
Other		465,179	2.76	6.29				
Total		375,057,485	2.49	5.30				

Table 3: Speed, Trading Frequency, and Trading Behavior

This table presents descriptive statistics for every maturity-account pair with at least one trading volume during 9:10 a.m. to 2:50 p.m. of our sample period. Panel A shows the basic statistics for all account-maturity pair. Panel B shows the correlation of fraction of market orders submitted and limit order spread, as well as high-frequency trading proxies such as order-to-trade ratio and cancel-to-trade ratio. *Dist* is defined as the difference between the price of limit buy (sell) order to best ask (bid) price. $\ln \frac{LimitOrder}{LimitTrade}$ is the log of total number of limit orders submitted over total number of orders executed that were submitted via limit orders. $\ln(1 + \frac{Canel}{Trade})$ is the log of one plus the total number of cancellations over total number of orders executed that were submitted via limit orders. The two proxies are logged to prevent right skewed figures in raw numbers that affect the mean. Trading speed ℓ is constructed from (11).

Panel A: Basic Statistics

	N	Mean	Std.Dev.	10%	Median	90%
MktOrd%	306,049	1.11	396.09	0.00	0.00	1.73
MktbOrd%	306,049	21.77	1230.03	0.24	13.76	57.11
Dist	287,597	2.60	103.42	0.99	2.053	5.31
$\ln \frac{LimitOrder}{LimitTrade}$	268,359	1.89	108.57	0.44	1.16	4.17
$\ln(1 + \frac{Canel}{Trade})$	268,359	1.47	96.20	0.28	0.91	3.04

Panel B: Correlation Matrix

	MktOrd%	MktbOrd%	*Dist*	$\ln \frac{LimitOrder}{LimitTrade}$	$\ln(1 + \frac{Cancel}{LimitTrade})$
MktbOrd%	0.25322				
Dist	0.04547	-0.41519			
$\ln \frac{LimitOrder}{LimitTrade}$	-0.08839	-0.40842	0.28996		
$\ln(1 + \frac{Canel}{Trade})$	-0.08252	-0.2833	-0.01551	0.81947	
ℓ	-0.17265	-0.08917	-0.14706	0.17273	0.22469

Table 4: Number of Active Accounts by Trading Speed and Nearest Maturity Date

This table shows the number of active accounts during 9:10 a.m. to 2:50 p.m. by trading speed group ($\hat{\ell}$) and for each time periods between maturity dates in Korea Exchange trading KOSPI 200 Futures from March 12, 2010 to June 12, 2014 excluding those with less than one week of maturity. Active account refers to any existing account that has submitted one or more orders during the period. Trading speed is calculated by equation (12). For example, $\hat{\ell} = 7$ is the accounts observed to be able to submit multiple orders ore revise their orders within a millisecond. Larger number implies ability to trade faster.

$\hat{\ell}$		6/10/10	9/9/10	12/9/10	3/10/11	6/9/11	9/8/11	12/8/11	3/8/12	6/14/12	9/13/12	12/13/12	3/14/13	6/13/13	9/12/13	12/12/13	3/13/14	6/12/14
1	N	1735	1755	1480	1776	1871	1577	1558	1700	1526	1580	1490	1410	1495	1526	1334	1458	1603
	%	9.15	9.51	8.52	9.97	9.87	7.69	7.2	8.72	8.24	8.25	9.02	8.78	8.86	8.37	8.42	9.07	10.29
2	N	3806	3588	3480	3708	3728	3941	4059	3842	3430	3613	3150	3259	3255	3684	3301	3374	3236
	%	20.07	19.45	20.03	20.82	19.67	19.21	18.77	19.71	18.52	18.86	19.07	20.29	19.29	20.2	20.83	20.99	20.77
3	N	7180	6889	6407	6377	6917	7597	8261	7274	6670	7000	5905	5661	6026	6496	5632	5767	5337
	%	37.85	37.34	36.89	35.81	36.5	37.03	38.2	37.32	36.02	36.55	35.75	35.24	35.71	35.62	35.53	35.87	34.26
4	N	1433	1322	1317	1294	1434	1729	1859	1524	1601	1571	1233	1355	1371	1420	1250	1207	1108
	%	7.55	7.17	7.58	7.27	7.57	8.43	8.6	7.82	8.64	8.2	7.47	8.44	8.12	7.79	7.89	7.51	7.11
5	N	1896	1917	1723	1650	1839	2064	2112	1982	1994	2064	1791	1663	1746	1955	1638	1648	1492
	%	10	10.39	9.92	9.27	9.7	10.06	9.77	10.17	10.77	10.78	10.84	10.35	10.35	10.72	10.33	10.25	9.58
6	N	1988	1868	1853	1970	2067	2252	1876	1188	1155	1205	975	918	1024	1099	964	940	932
	%	10.48	10.13	10.67	11.06	10.91	10.98	8.67	6.1	6.24	6.29	5.9	5.72	6.07	6.03	6.08	5.85	5.98
7	N	930	1110	1110	1033	1094	1357	1902	1980	2144	2120	1972	1796	1957	2059	1731	1684	1870
	%	4.9	6.02	6.39	5.8	5.77	6.61	8.79	10.16	11.58	11.07	11.94	11.18	11.6	11.29	10.92	10.47	12
Total		18968	18449	17370	17808	18950	20517	21627	19490	18520	19153	16516	16062	16874	18239	15850	16078	15578

Table 5: Order Quantity by Trading Speed and Nearest Maturity Date

This table shows the number of orders submitted during 9:10 a.m. to 2:50 p.m. by trading speed group ($\hat{\ell}$) and for each time periods between maturity dates in Korea Exchange trading KOSPI 200 Futures from March 12, 2010 to June 12, 2014 excluding those with less than one week of maturity. Any order submitted count as a single order disregarding the size of order. Trading speed is calculated by equation (12). For example, $\hat{\ell} = 7$ is the accounts observed to be able to submit multiple orders ore revise their orders within a millisecond. Larger number implies ability to trade faster.

(in thousands)

$\hat{\ell}$		6/10/10	9/9/10	12/9/10	3/10/11	6/9/11	9/8/11	12/8/11	3/8/12	6/14/12	9/13/12	12/13/12	3/14/13	6/13/13	9/12/13	12/12/13	3/13/14	6/12/14
1	N	36	43	26	40	35	22	23	26	22	20	19	18	18	18	14	15	18
	%	0.05	0.07	0.04	0.07	0.06	0.03	0.03	0.05	0.04	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04
2	N	296	304	265	272	305	307	312	252	225	248	196	210	212	251	177	200	163
	%	0.45	0.48	0.43	0.48	0.50	0.41	0.45	0.50	0.43	0.40	0.37	0.46	0.48	0.48	0.46	0.51	0.40
3	N	2,812	2,784	2,480	2,410	2,920	3,331	3,530	2,054	2,083	2,054	1,557	1,523	1,623	1,825	1,331	1,221	1,029
	%	4.28	4.35	3.99	4.23	4.78	4.43	5.08	4.06	3.95	3.34	2.93	3.36	3.65	3.52	3.43	3.12	2.50
4	N	1,365	1,143	969	906	1,184	1,446	1,485	976	964	908	790	788	993	712	519	437	468
	%	2.08	1.79	1.56	1.59	1.94	1.92	2.14	1.93	1.83	1.47	1.49	1.74	2.23	1.37	1.34	1.12	1.14
5	N	2,736	2,797	2,241	1,743	2,293	3,572	3,761	2,771	2,826	2,558	2,100	1,722	1,641	1,791	1,436	1,403	1,188
	%	4.16	4.37	3.61	3.06	3.75	4.75	5.41	5.48	5.36	4.16	3.95	3.79	3.69	3.45	3.71	3.59	2.89
6	N	44,214	39,119	38,034	34,876	38,353	36,761	14,506	3,371	3,899	4,874	3,016	2,604	2,930	4,022	3,938	2,389	1,796
	%	67.24	61.14	61.19	61.18	62.77	48.85	20.87	6.67	7.40	7.92	5.68	5.74	6.59	7.75	10.16	6.11	4.37
7	N	14,292	17,790	18,140	16,757	16,012	29,810	45,892	41,080	42,682	50,892	45,470	38,528	37,063	43,278	31,335	33,449	36,461
	%	21.74	27.81	29.18	29.40	26.21	39.61	66.02	81.30	80.99	82.68	85.55	84.88	83.33	83.39	80.87	85.52	88.66
Total		65,752	63,979	62,156	57,003	61,101	75,248	69,509	50,530	52,702	61,554	53,149	45,393	44,479	51,897	38,749	39,114	41,123

Table 6: How Account Speed affects Investor's Limit Order Behavior

This table presents results from a following censored regression (Tobit) of limit order spread on on speed, volatility, and the top limit order book status variables for futures that matures at September 2011, excluding those with less than one week of maturity.

$$Dist_{itd} = \alpha + \beta S_i + \gamma_{itd} Vol_{dt} + BBQ_{td} + BAQ_{td} + \varepsilon_{itd}$$

$Dist_{itd}$ is the price difference between in terms of ticks (0.05) submitted limit buy (sell) order price and the best ask (bid) available, S_i is a vector of dummies representing each speed ($\hat{\ell}$) group from 2 to 7. Vol_d is the 30-second volatility from volatility index implied by KOSPI 200 options. BBQ_{td} is the quantity of limit order placed at the best bid and BAQ_{td} is the quantity of limit order placed at the best ask. $Speed_{\hat{\ell}}$ refers to the coefficient on the dummy variable of speed $\hat{\ell}$. Columns (1)–(4) report results including marketable limit orders. Columns (5)–(8) report results excluding marketable limit orders. Standard errors are reported in parenthesis.

	All Limit Orders				Non-marketable Limit Orders			
	All Orders	All Orders	Sell Orders	Buy Orders	All Orders	All Orders	Sell Orders	Buy Orders
Intercept	1.43991	0.97294	1.19610	2.06331	5.32804	4.80187	5.24970	4.82382
	(0.05539)	(0.05529)	(0.07517)	(0.08070)	(0.06206)	(0.06196)	(0.08506)	(0.09007)
$Speed_2$	1.74577	1.75581	1.70579	1.09678	-0.43861	-0.42445	-0.56171	-0.03020
	(0.05678)	(0.05664)	(0.07703)	(0.08250)	(0.06321)	(0.06308)	(0.08660)	(0.09157)
$Speed_3$	0.72958	0.74499	0.63426	0.15007	-1.41758	-1.40034	-1.55807	-0.98783
	(0.05550)	(0.05537)	(0.07520)	(0.08073)	(0.06216)	(0.06203)	(0.08510)	(0.09010)
$Speed_4$	0.62931	0.63213	0.51520	-0.03325	-1.84573	-1.83825	-1.94252	-1.46954
	(0.05565)	(0.05552)	(0.07542)	(0.08093)	(0.06227)	(0.06214)	(0.08526)	(0.09026)
$Speed_5$	2.14549	2.14160	2.04730	1.52019	-0.28192	-0.28814	-0.42987	0.10911
	(0.05549)	(0.05536)	(0.07518)	(0.08072)	(0.06214)	(0.06202)	(0.08507)	(0.09009)
$Speed_6$	0.13410	0.14711	0.06509	-0.55846	-2.31563	-2.28824	-2.40064	-1.87930
	(0.05540)	(0.05526)	(0.07505)	(0.08058)	(0.06207)	(0.06194)	(0.08496)	(0.08998)
$Speed_7$	0.16296	0.01706	0.18059	-0.62897	-2.72404	-2.74098	-2.75867	-2.43499
	(0.05540)	(0.05527)	(0.07505)	(0.08059)	(0.06207)	(0.06194)	(0.08497)	(0.08998)
Vol		0.01450	0.01162	0.01269		0.01918	0.01003	0.01342
		(0.00008)	(0.00013)	(0.00013)		(0.00007)	(0.00012)	(0.00012)
BAQ			-0.00520	0.00347			0.00006	-0.00235
			(0.00002)	(0.00002)			(0.00002)	(0.00002)
BBQ			0.00401	-0.00719			-0.00223	-0.00045
			(0.00002)	(0.00002)			(0.00002)	(0.00002)
Obs.	29,326,567	29,326,567	14,531,110	14,795,457	21,289,101	21,289,101	10,459,321	10,829,780

Table 7: Number of Immediate Executable Orders by Speed

This table reports the basic statistics of market and marketable limit orders by speed group $\hat{\ell}$. The sample consists all those orders submitted from 9:10 a.m. to 2:50 p.m. which are during continuous trading hours in the Korea Exchange for the KOSPI 200 Futures from March 12, 2010 to June 12, 2014 for nearest maturity futures, excluding those that matures within one week. *Qty* is order quantity submitted over the whole sample period per speed group.

$\hat{\ell}$	Limit Order	Number of orders
1	Marketable	77,397
	Market	115,866
2	Marketable	555,944
	Market	495,034
3	Marketable	4,296,477
	Market	1,456,527
4	Marketable	2,077,402
	Market	604,129
5	Marketable	4,485,589
	Market	1,536,473
6	Marketable	28,876,035
	Market	3,754,209
7	Marketable	49,986,765
	Market	2,438,381

Table 8: How speed affect investor's choice of order

This table presents results from a following logit regression of choosing to submit market order on speed, volatility, and the top limit order book status variables for futures that matures at September 2011, excluding those with less than one week of maturity.

$$mktorder_{idt} = \alpha + \beta S_i + \gamma_{idt} Vol_{dt} + BBQ_{dt} + BAQ_{dt} + \varepsilon_{idt}$$

S_i is a vector of dummies representing each speed ($\hat{\ell}$) group from 2 to 7. Vol_d is the 30-second volatility from volatility index implied by KOSPI 200 options. BBQ_{td} is the quantity of limit order placed at the best bid and BAQ_{td} is the quantity of limit order placed at the best ask. $Speed_{\hat{\ell}}$ refers to the coefficient on the dummy variable of speed $\hat{\ell}$. Standard errors are reported in parenthesis. For columns (1) to (4), $mktorder_{itd}$ is constructed as 1 if the order submitted by individual i at time t on day d is market order and 0 otherwise. For columns (5) to (8), market order also includes marketable orders, which are buy (sell) limit orders which is above (below) or at the best ask (bid) price.

	Market Orders				+Marketable Orders			
	All Orders	All Orders	Sell Orders	Buy Orders	All Orders	All Orders	Sell Orders	Buy Orders
Intercept	-0.73689	-1.38220	-1.21817	-1.19267	0.59483	0.58092	0.78827	0.64979
	(0.01438)	(0.01408)	(0.02116)	(0.02162)	(0.01406)	(0.01408)	(0.01992)	(0.02080)
$Speed_2$	-1.05676	-1.07419	-1.04071	-1.10900	-1.20735	-1.20751	-1.24879	-1.27408
	(0.01528)	(0.01456)	(0.02156)	(0.02205)	(0.01456)	(0.01456)	(0.02058)	(0.02143)
$Speed_3$	-2.19777	-2.20876	-2.16015	-2.25403	-1.39394	-1.39385	-1.40566	-1.50560
	(0.01460)	(0.01411)	(0.02057)	(0.02110)	(0.01411)	(0.01411)	(0.01992)	(0.02080)
$Speed_4$	-2.36482	-2.39593	-2.34110	-2.47283	-1.58792	-1.58823	-1.55716	-1.68578
	(0.01496)	(0.01418)	(0.02107)	(0.02161)	(0.01418)	(0.01418)	(0.02003)	(0.02090)
$Speed_5$	-2.58598	-2.62216	-2.58391	-2.66681	-1.77697	-1.77737	-1.80527	-1.88823
	(0.01467)	(0.01411)	(0.02067)	(0.02121)	(0.01411)	(0.01411)	(0.01993)	(0.02081)
$Speed_6$	-3.64648	-3.64042	-3.61806	-3.68611	-1.61710	-1.61662	-1.62659	-1.66120
	(0.01446)	(0.01406)	(0.02037)	(0.02091)	(0.01406)	(0.01406)	(0.01986)	(0.02073)
$Speed_7$	-4.34731	-4.40376	-4.38834	-4.42716	-1.98659	-1.98732	-2.05930	-2.07157
	(0.01457)	(0.01407)	(0.02054)	(0.02107)	(0.01407)	(0.01407)	(0.01986)	(0.02074)
Vol		0.02358	0.02049	0.02066		0.00052	-0.00188	-0.00028
		(0.00003)	(0.00016)	(0.00015)		(0.00003)	(0.00005)	(0.00005)
BAQ			-0.00253	0.00091			0.00445	-0.00582
			(0.00003)	(0.00002)			(0.00001)	(0.00001)
BBQ			0.00065	-0.00208			-0.00647	0.00560
			(0.00003)	(0.00003)			(0.00001)	(0.00001)
Obs.	30,048,272	30,048,272	14,892,524	15,155,748	30,048,272	30,048,272	14,892,524	15,155,748

Table 9: One-tick Limit Orders by Speed

This table reports the basic statistics of buy (sell) limit orders submitted one-tick away from the best ask (bid) price. The sample consists all those orders submitted from 9:10 a.m. to 2:50 p.m. which are during continuous trading hours in the Korea Exchange for the KOSPI 200 Futures from March 12, 2010 to June 12, 2014 for nearest maturity futures, excluding those that matures within one week. *Qty* is order quantity per order submitted, *Surv* is survival time of non-marketable limit order in seconds.

$\hat{\ell}$	Limit Order	Qty	Mean($Surv$)	1Q($Surv$)	Median($Surv$)	3Q($Surv$)
1	Revise	3,599	552.840	154.321	272.368	562.335
	Cancel	1,182	1004.670	162.931	299.663	916.157
	Execute	45,758	134.830	5.137	19.159	68.401
2	Revise	112,287	180.359	31.230	59.579	130.158
	Cancel	96,012	242.752	36.651	77.135	177.872
	Execute	743,828	68.841	5.088	17.000	50.037
3	Revise	1,383,558	94.948	11.696	26.342	65.362
	Cancel	1,537,492	113.552	12.077	27.638	71.441
	Execute	6,602,308	48.491	3.465	11.505	34.503
4	Revise	1,325,605	31.181	2.974	7.961	22.039
	Cancel	820,945	63.382	5.783	14.400	38.522
	Execute	2,832,675	34.766	2.373	8.590	25.645
5	Revise	1,506,509	46.391	4.585	11.987	32.458
	Cancel	1,840,326	74.190	4.607	15.010	42.872
	Execute	5,574,632	38.300	2.085	8.310	26.266
6	Revise	4,986,078	43.367	3.044	9.498	27.706
	Cancel	36,566,360	76.084	0.593	3.328	14.867
	Execute	49,294,680	47.274	0.776	4.768	18.268
7	Revise	11,051,272	26.469	0.351	2.889	13.409
	Cancel	141,158,917	38.241	0.000	0.318	2.941
	Execute	73,639,125	47.328	0.502	3.434	15.355

Figure 1: KOSPI 200 and Volatility Index

The figure shows the daily KOSPI 200 index and the daily V-KOSPI Index from Jan 1, 2010 to July 30, 2014. V-KOSPI is the volatility index implied by KOSPI 200 options.

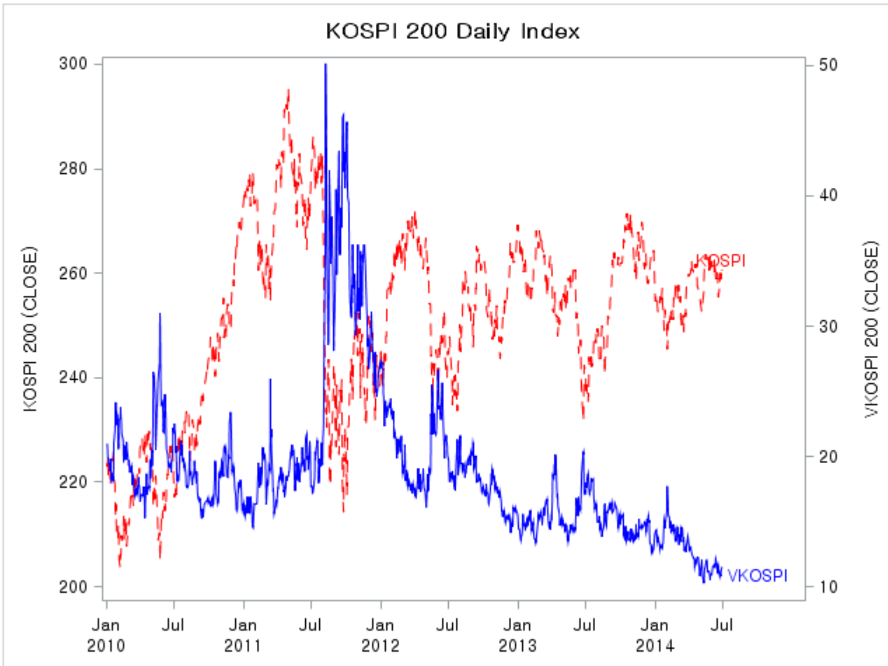
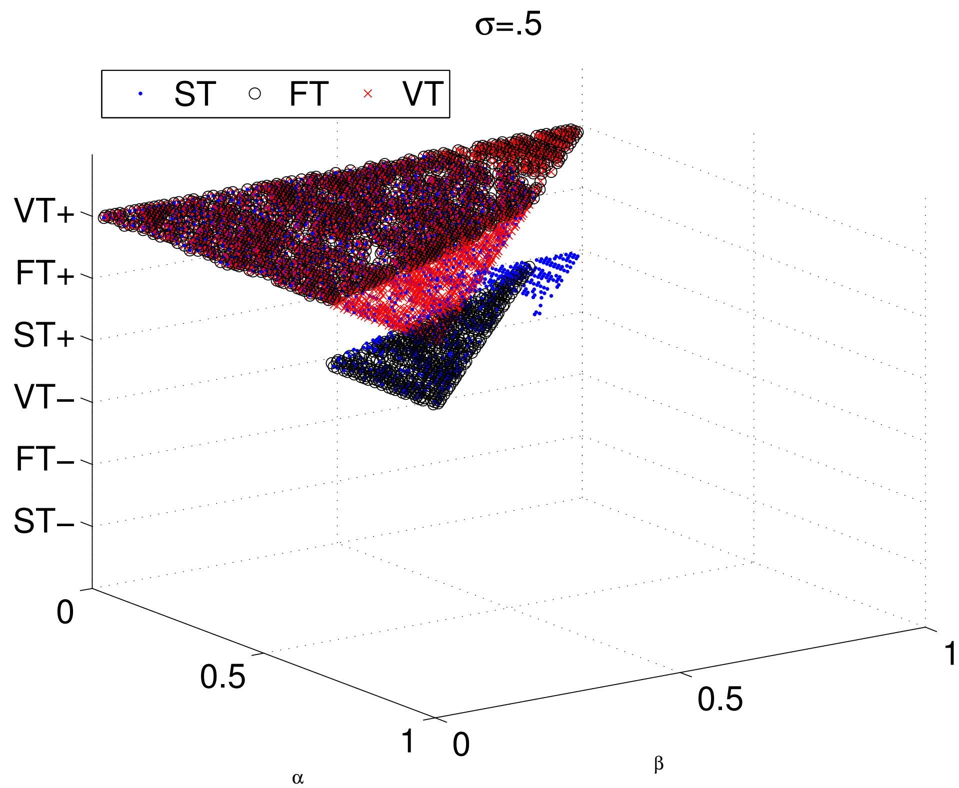
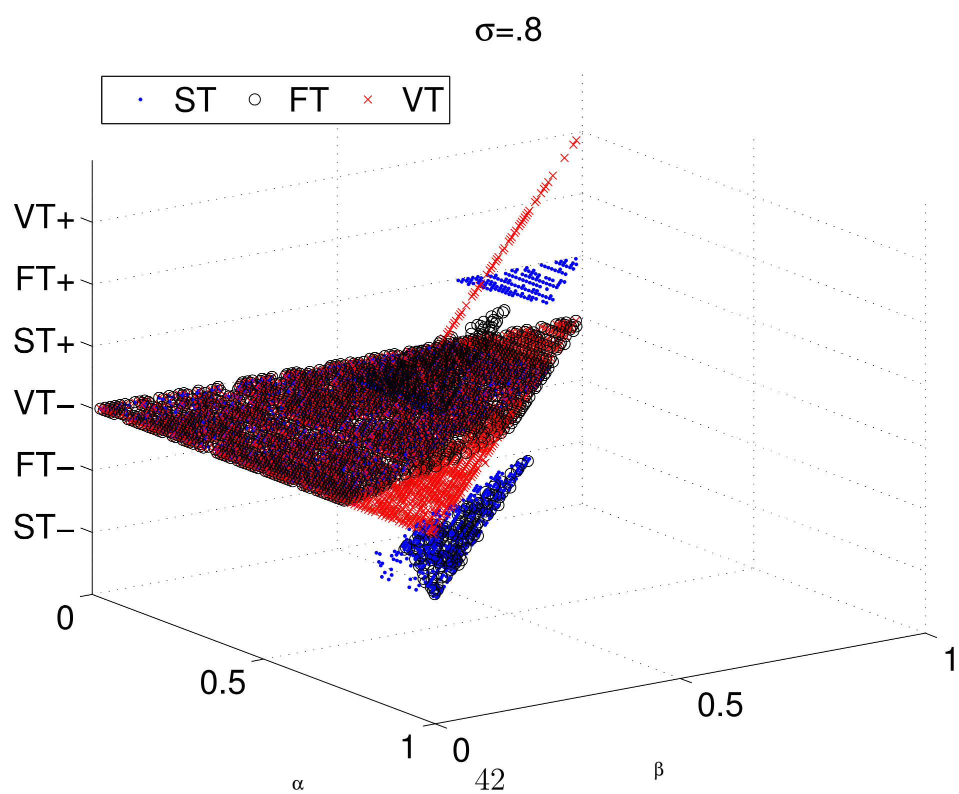


Figure 2: Equilibrium for Strategy Pairs by Asset Variance
This figure plots the equilibrium strategies for $\sigma \in \{.5, .8, .1.0\}$, and for each α and β to have values of $.01, .02, .03, \ldots, .99$, where $\alpha + \beta \leq 1$ and $L = 1$. The strategies are solved numerically. Therefore, each figure seen below can have, at most, 4,950 plots per investor type, namely, for slow traders (STs), fast traders (FTs), and very fast traders (VTs).





(Figure 2 continued)