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Remanufacturing strategies under product take-back regulation

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Abstract

We consider a profit-maximizing original equipment manufacturer (OEM) that produces a new product for the primary market. At the end of the first stage of product use, the OEM collects the used products (cores) and remanufactures them if they are eligible for remanufacturing; otherwise, the OEM sends them to material recycling. In the US and most European countries, the product take-back regulation mandated by the government holds the OEM accountable for the proper and safe treatment of end-of-life products in order to reduce their harmful environmental effects, and sets a collection rate target. In this paper, we take the collection rate of used products as an endogenous variable satisfying the set target and consider the uncertainty of the amount of collected cores qualified for remanufacturing. We formulate the problem and derive several optimal remanufacturing strategies for the OEM, depending on the inverse operating cost and new product manufacturing cost with and without the take-back regulation. Our major findings are: (i) The OEM should adopt the maximum (minimum) collection rate when the inverse operating cost is extremely low (high). (ii) A combination of the inverse operating cost and new product manufacturing cost leads to the optimal collection rate, thereby determining the new and remanufactured product prices, and remanufacturing quantity. (iii) A comparison between the cases with and without the take-back regulation indicates that, counter-intuitively, regulating the collection rate may not necessarily hurt the OEM.

Key words: take-back regulation; remanufacturing; uncertainty; cores

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1. Introduction

Have you ever wondered what happens to your electronic waste (e-waste) like the old laptop or cell phone when you throw it away? According to UN's Global E-waste Monitor 2020, 53.6 million metric tons (MT) of e-waste was generated around the world in 2019, only 17.4% of which was collected and recycled. The value of raw materials in the global e-waste generated in 2019 approaches 57 billion US dollars. Alarmingly, the report also predicts that the global generation of e-waste will grow to 74.7 MT by 2030, which indicates e-waste is the fastest-growing domestic waste stream in the world (The Global E-waste Monitor, 2020). This creates a great threat to human health and the environment. In response, different countries and regions have taken action to address the issue based on the extended producer responsibility (EPR), which advocates the enactment of the product take-back regulation that mandates manufacturers to provide environmentally safe management of their products at the end of their lives.

In recent years, the product take-back regulation has become very popular, particularly as a public policy to manage e-waste. The Waste Electrical and Electronic Equipment (WEEE) Directive in the European Union (EU) holds OEMs accountable for taking back and recycling end-of-life products and sets mandatory rate targets for the recycling and recovery of WEEE in order to reduce the harmful environmental effects of such products at the end of their lives (Esenduran et al., 2019). For instance, in Europe, 27 countries have enacted product take-back legislation for e-waste (Huang et al., 2019). In the US, there is no national e-waste law, but 25 states have passed statewide legislation on e-waste recycling (Esenduran et al., 2017). All the e-waste take-back regulations aim to reduce the environmental impact through increasing landfill diversion. One of the world's leading providers of technology, Dell recognizes the responsibility to ensure that technological equipment is disposed of properly at the end of its usable life, and announces a programme that takes any Dell product for free recycling. Similar activities can be seen at Apple Stores where customers can trade in their used products. Apple has found efficient ways to reuse and recycle its electronic products including iPhones, iPads, Macs, or PC computers. The used products collected by Apple will first be disassembled and the key components that can be reused are kept. The majority of the plastics can be pelletized into secondary raw materials. With materials reprocessing and component reuse, Apple has managed to achieve a 90% recovery rate by weight of the original product. Also focusing on environmental protection, Lenovo has announced its take-back programme offering free recycling of used Lenovo and some IBM products for customers (ETBC, 2016).

In this paper, we focus on electronic products, like cell phones and computers, and remanufacturing the used products (cores) collected if they are qualified for remanufacturing; otherwise, they are sent to material recycling. We consider an OEM that directly collects the used products from consumers, and determines if the cores are qualified for remanufacturing, in addition to selling new products. If qualified, the cores will be remanufactured. Otherwise, the cores will be salvaged through material recycling. Based on the EPR mandated by the government, our primary objectives are to understand the implications of imposing the collection rate target on the OEM, and to analyze how the inverse operating cost affects the OEM's performance when the uncertainty in the amount or quality of the cores collected is taken into account.

To this end, we formulate an economic model that consists of an OEM (we use the term manufacturer generically to refer to the OEM) facing the take-back regulation mandated by the government. There are four stages in the model: (i) In the first stage, the OEM sets the new product price. (ii) In the second stage, the OEM identifies the optimal collection rate for the used products sold in the previous stage. (iii) In the third stage, the OEM determines the optimal remanufacturing quantity. (iv) In the final stage, the OEM decides the price of the remanufactured product. We derive the optimal new product price, collection rate, remanufacturing ←R3.2 quantity, and remanufactured product price. Analyzing and comparing the optimal decisions under the cases with and without the take-back regulation, we find answers to the following main research questions:

(i) Which key operating factors influence the optimal collection rate?

(ii) Does imposing the collection rate target always harm the OEM?

Our results suggest the following: (i) A combination of the inverse operating cost and new product manufacturing cost determines the optimal collection rate. Intuitively, when the inverse operating cost is extremely high, the OEM evidently chooses the minimum collection rate under the take-back regulation, and the collection rate is zero without the take-back regulation regardless of the new product manufacturing cost. Similarly, when the inverse operating cost is extremely low, the OEM has the economic incentive to collect all the used products regardless of the new product manufacturing cost. That is, the real collection rate will be higher than the collection rate target. However, when the inverse operating cost is at a moderate level, the optimal collection rate is determined by a combination of inverse operating cost and new product manufacturing cost. (ii) The OEM's performance under the take-back regulation is always inferior or equal to that without the take-back regulation. Specifically, when the inverse operating cost is relatively high, imposing the collection rate target on the OEM decreases the landfill but hurts the ←R2.1 OEM's profit. In this case, the product take-back regulation has an impact on environmental efficiency. However, when the inverse operating cost is relatively low, the OEM's profits are the same for both cases. That is, imposing the collection rate target does not harm the OEM and the product take-back regulation is redundant. In summary, the answer to question (i) identifies the conditions under which the OEM's optimal solutions are obtained. The answer to question (ii) identifies the conditions under which imposing the target collection rate on the OEM does not hurt the OEM's profit.

We organize the rest of the paper as follows: In Section 2 we review the related literature. In Section 3 we introduce the basic model and discuss the model assumptions. In Section 4 we derive the OEM's optimal decisions for the new product price, collection rate, remanufacturing quantity, and remanufactured product price without the take-back regulation as the benchmark case. In Section 5 we analyze the model under the take-back regulation and compare the results with those of the benchmark case. Finally, in Section 6, we conclude the paper and suggest topics for future research. We provide all the proofs of the results in the Appendix.

2. Literature Review

Recently, operations management research considering the environmental issues has received increasing attention. Our work is closely associated with the research stream concerning product take-back legislation. An early study in the stream, Atasu et al. (2009) examined the impact of take-back legislation on the economy and discussed the efficiency of the WEEE Directive of the European Commission from the perspectives of competition, fairness to manufacturers, and consumer education (encouraging consumers to return their end-of-life products). They showed that take-back efficiency is closely related to environmental category of products, recovery cost structure, and consumer willingness to pay. Subsequently, various operations management problems with product take-back legislation have been studied. We discuss the representative studies in the following.

First, some researchers study the problems from the perspective of compliance decisions with regard to individual and collective producer responsibilities (IPR and CPR). Atasu and Subramanian (2012) modeled two competitive manufacturers producing high- and low-end products, respectively. Comparing the individual and collective systems, they concluded that preferences for IPR or CPR are highly parameter-dependent. Also examining compliance decisions, Esenduran and Kemahlioglu-Ziya (2015) considered n manufacturers and focused on cost minimization. They found that a collective scheme consisting of firms with large market shares achieves a higher collection rate than the individual scheme.

Gui et al. (2018) used biform game to study whether CPR dominates IPR by focusing on product design incentives. The authors ←R3.1 identified conditions under which improving product design is beneficial to the stability of CPR. In addition, cost allocation mechanisms are proposed to achieve superior product designs and stable collective system. After that, Gui (2020) studied the implementation of EPR in developing countries and examined how recycling infrastructure development is affected by cost sharing rules. The author derived the conditions under which joint investment in CPR may lead to a worse recycling infrastructure development outcome compared to independent investment in IPR. Also studying the stability of collective system, Tian et al. (2019) considered two models (asymmetric manufacturing and symmetric manufacturing) where manufacturers can switch between CPR and IPR, and studied the stability of recycling alliances. The authors suggested that IPR is preferred by manufacturers when market competition is intensive and market sizes are differentiated. However, CPR is the most common stable recycling alliance regardless of economies of scale when market competition is less intensive or market shares are more equitable. Different from Tian et al. (2019), Tian et al. (2020) focused on a single product type and believed there are two major obstacles to recycling: One is high cost of material separation which can be overcome by IPR, and the other is high fixed recycling cost which can be overcome by CPR. The authors analyzed the impact of these two determinants on the EPR implementation. Our paper does not consider compliance decisions, instead, we contribute to the literature by analyzing how the OEM's optimal collection rate changes with the inverse operating cost under the product take-back regulation. Contrary to intuition, we show that the OEM still chooses the upper bound of the collection rate even when the inverse operating cost is not extremely low. This is because the optimal collection rate depends not only on the inverse operating cost, but also on the new product manufacturing cost, as detailed in Section 5.

Second, some researchers consider the problems from the perspective of quality design decisions. Atasu and Souza (2013) investigated a firm's quality decision in the monopolistic setting, assuming that consumers perceive both the new and remanufactured products are the same. Considering three general forms of product recovery, they found that the impact of product recovery on quality choice depends on the form of product recovery, recovery cost structure, and legislative environment. Huang et al. (2019) studied the design implications of EPR from two dimensions, i.e., recyclable and durable, which could reduce the recycling cost and recycle volume, respectively. They found that stricter regulation targets do not necessarily yield improved product recyclability and durability. Zheng et al. (2019) studied the impacts of design for the environment (DfE) on firm's remanufacturing strategies in the monopolistic and competitive settings. Comparison of the two cases suggests that DfE encourages an independent remanufacturer to engage in remanufacturing in the presence of competition, while the result is quite opposite in the other case. Pazoki and Samarghandi (2020) considered a manufacturer facing two different choices, namely remanufacturing and eco-design, under the take-back regulation. They found that if eco-design is costly, the manufacturer is more willing to remanufacture. However, if eco-design can reduce the production cost, the take-back regulation is unnecessary. Our paper differs from the above works in that we consider the scenario where new and remanufactured products are sold in different markets, and the latter products are viewed as inferior.

Finally, some researchers investigate the problems with a focus on the remanufacturing strategies under take-back regulation. Esenduran et al. (2016) investigated the environmental implications of three levels of the take-back regulation, i.e., no regulation, regulation with collection rate targets, and regulation with collection and remanufacturing targets, in the monopolistic setting and characterized the manufacturer's optimal solutions under different regulations. By examining the implications for the environment, they demonstrated that increased remanufacturing does not necessarily yield better environmental benefits, and derived conditions under which increased remanufacturing under the take-back regulation leads to a reduction in the total environmental benefit. Finally, they examined the impacts of regulation from the perspectives of manufacturer's profit and consumer surplus, and characterized when total welfare will decrease under the take-back legislation.

Esenduran et al. (2017) extended the above model to competition between an OEM and an independent remanufacturer, and studied how legislation affects remanufacturing quantities, the OEM profit, consumer surplus and social welfare. They found, counter-intuitively, OEM facing take-back regulation may reduce remanufacturing level, which means that mandating more ←R3.1 stringent collect targets do not lead to more remanufacturing. However, the OEM and consumer may benefit from stringent targets. In addition, numerical study shows that social welfare increases as the collection target increases if the environmental benefit is high and vice versa. Moreover, Mazahir et al. (2019) modeled an OEM's new and remanufactured product quantity decision under six policy options from the perspective of environment, and examined which one is more suitable in improving environmental outcomes. Their findings show that there is no unified environmental policy suitable for all products, which means that policy options strongly depends on product category. Furthermore, comparisons of different policy options generate insights which can be used to guide the policy makers. In this paper, however, we only consider collection target and regard the collection rate as an endogenous variable, which has received little attention in the context of take-back regulation.

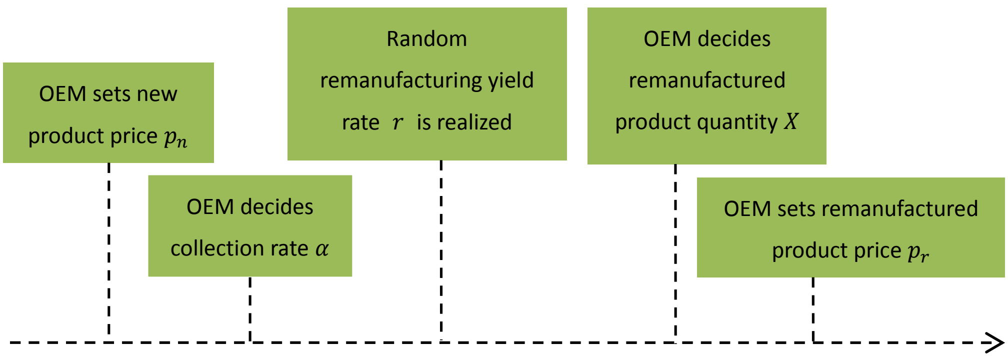
Among the works in the related literature, Raz et al. (2014) is the closest to ours. Studying a firm's DfE decisions under the product take-back and use stage regulations, they assessed the social cost in the primary and secondary markets. We do not consider the firm's DfE decisions; instead, we focus on the uncertainty in the amount or quality of the collected cores, i.e., we treat the remanufacturing yield rate as a random variable. Our work differs from Raz et al. (2014) in several other aspects, too: Raz et al. (2014) considered the firm's decision sequence as follows: (i) determining the DfE innovation investment, (ii) deciding the optimal prices to sell in the primary and secondary markets, and (iii) deciding the optimal collection rate. In contrast, in our paper, we take the yield rate as a random variable, and set the pricing decisions for the new and remanufactured products in different stages. Thus our model consists of four stages for the firm's decisions. First, the firm decides the new product price in the primary market for the product use stage; second, at the end of the product use stage, the firm determines the optimal collection rate, and collects the used products sold in the product use stage; and third, the firm sets the remanufacturing quantity. Note that in the third stage, the remanufacturing yield rate is realized and not a random variable, which is important to be aware of when we take expectation of the random yield rate in the second stage. Finally, the firm decides the price of the remanufactured product.

In addition, Raz et al. (2014) commented that a comparison between the cases with and without the take-back legislation did not yield many practical implications of the analytical results, so they resorted to numerical studies to generate managerial insights. In contrast, we analytically derive the conditions under which the OEM's optimal solutions change with its inverse operating cost in response to the take-back regulation and how the regulation affects the OEM's profit, so we provide more generic insights. To the best of our knowledge, research on product take-back legislation with random remanufacturing yield rate and endogenous collection rate considerations is limited in the literature. We conduct this study to fill this research gap.

3. Model setup

In this paper we consider the setting in which an OEM sells both new and remanufactured products, and confine our attention ←**R2.2** to the case where the new and remanufactured products are not substitutable. ←**R3.3** Specifically, the OEM first sells the new product at the price of p_n per unit in the primary market. The end-of-life products will be collected by the OEM at a cost of c_c, then the collected used products are remanufactured and sold in the secondary market at a price of p_r if they are eligible for remanufacturing. Those that are not remanufacturable are sent to material recycling at a salvage value of w. In addition, the leftover remanufactured products are salvaged at a price of k.

Figure 1. Decision sequence of the OEM.



We assume the market size is fixed and normalized to 1. There is a heterogeneous customer base, whose valuations of quality are uniformly distributed between 0 and 1 for the primary market and between 0 and u for the secondary market. Despite that both the new and remanufactured products have the same functionality, the latter is perceived to be inferior in quality by the consumers. We assume that $u < 1$, which means that the consumer's average utility of the remanufactured product in the secondary market is ←**R2.3** less than that of the new product in the primary market. We refer the reader to Raz et al. (2014) for more details of the model and assumptions. Given the above model setup, we derive the demand functions for the new product as $D_n = 1 - p_n$ and ←**R3.4** remanufactured product as $D_r = u - p_r$. The decision sequence of the OEM, illustrated in Figure 1, is as follows: First, the OEM determines the new product price p_n. Second, the OEM decides how many used products to collect from the primary market or, equivalently, the collection rate α. Third, the OEM sets the remanufacturing quantity X in response to the secondary market located in developing countries. Finally, the OEM sets the remanufactured product price p_r. We restrict our attention to the case where the OEM can collect enough quantity of the used products to satisfy the collection target α_0 mandated by the government (α_0 is the lower bound on the collection rate α, and α_1 is the upper bound on the collection rate α, representing the maximum rate of used products that can be collected). We summarize in Table 1 the notation used throughout the paper.

Table 1. Notation.

	Symbol	Definition
Decision variable	p_n	New product price
	p_r	Remanufactured product price
	α	Collection rate
	X	Remanufacturing quantity
Random variable	r	Random variable representing remanufacturing yield rate
Parameter	c_n	Cost to produce a new product
	c_r	Cost to produce a remanufactured product
	c_c	Collection cost
	w	Salvage value for cores that are not remanufacturable
	k	Leftover remanufactured product price
	u	Scaled size of the secondary market
Other notation	D_n	New product market demand
	D_r	Remanufactured product market demand

We formulate the OEM's profit function as follows:

$$max\ \pi(p_n, p_r, \alpha, X) = (p_n - c_n)\ D_n + (p_r - c_r)\ min(X, D_r) + (k - c_r)\ max(0, X - D_r) + w\ \alpha\ D_n\ (1 - r) - c_c\ \alpha\ D_n \quad (1)$$ ← R2.4

$$s.t.\ \alpha_0 \le \alpha \le \alpha_1, \quad (2)$$

$$0 \le X \le \alpha\ D_n\ r\ . \quad (3)$$

Objective function (1) seeks to maximize the profit of the OEM from the primary and the secondary markets. Specifically, the first term is the profit from the new product sold in the primary market. The second term is the profit from the remanufactured product sold in the secondary market. The third term is the profit from the leftover remanufactured products, and the fourth term ← R3.5 is the profit from the collected units that cannot be remanufactured and are thus sent to material recycling. The last term is the collection cost. Constraint (2) ensures that the collection rate set by the OEM is at least as much as required by the take-back legislation and less than the maximum collection rate that the OEM can attain. Constraint (3) imposes that the remanufacturing quantity is not greater than the quantity of collected products that are remanufacturable.

4. Remanufacturing without the take-back regulation

In this section we derive the optimal decisions of the OEM for the case without the take-back regulation. Since there are four stages in the model, we use the backward induction to solve the problem.

Fourth-stage Analysis. As stated in Section 3, we first consider the price of the remanufactured product, given p_n, X, and α. We consider the case where $min(X, D_r) = D_r$[1]. For this case, the OEM's profit function is ← R2.5

$$max\ \pi(p_n, p_r, \alpha, X) = (p_n - c_n + w\ \alpha\ (1 - r) - c_c\ \alpha)\ D_n + (p_r - k)\ \ D_r + (k - c_r\ )\ X. \quad (4)$$

[1] We show in the Appendix that when $min(X, D_r) = X$, the optimal remanufacturing quantity is D_r, i.e., $X = D_r$, which is a ← R2.5 particular case of $X \ge D_r$. Therefore, the case where $min(X, D_r) = X$ is dominated by the case where $min(X, D_r) = D_r$.

Re-arranging and simplifying the terms in (4), we convert the OEM's problem into

$$max\, \pi(p_r|p_n,\alpha,X) = -p_r^2 + (u+k)\,p_r + (p_n - c_n + (1-r)\,w\,\alpha - c_c\,\alpha)\,(1-p_n) + (k-c_r)\,X - u\,k \tag{5}$$

$$s.t. X \geq D_r. \tag{6}$$

Note that the OEM's problem is quadratic and concave in p_r ($\frac{\partial^2 \pi}{\partial p_r^2} = -2$). Let $A_1 = \frac{H}{2}$, $A_2 = \alpha\, D_n\, r$, and $H = u - k$. Proposition 1 gives the optimal solution for the OEM's fourth-stage problem.

Proposition 1. Given the new product price, collection rate, and remanufacturing quantity, OEM's optimal solution is as follows:

Case	p_r	D_r	$\pi(X\|p_n,\,\alpha)$
$X \leq A_2 \leq A_1$	$u - X$	X	$-X^2 + (u-c_r)\,X + (p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha)\,D_n$
$A_1 \leq X \leq A_2$	$\frac{u+k}{2}$	$\frac{u-k}{2}$	$(k-c_r)\,X + \frac{(u-k)^2}{4} + (p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha)\,D_n$

The proof of Proposition 1 is given in the Appendix. Proposition 1 confirms the intuition that the scaled size of the secondary market u has a positive effect on the optimal selling price of the remanufactured product, which is decreasing with the remanufacturing quantity. In addition, the selling price is increasing in the salvage value k. This is because a high salvage value indicates that the OEM can still obtain benefit even if the remanufactured product is not sold at a high selling price. ←R3.6

Third-stage Analysis. After characterizing the OEM's fourth-stage decisions, given p_n, α, and X, we can derive the OEM's third-stage quantity decision.

Proposition 2. Given the selling price of the remanufactured product obtained in the fourth stage, the OEM's optimal new product price, collection rate, remanufacturing quantity are as follows:

Case ($\boldsymbol{k \geq c_r}$)	p_r	D_r	X	$\pi(\alpha\|p_n)$
$X \leq A_2 \leq A_1$	$u - \alpha\,D_n\,r$	$\alpha\,D_n\,r$	$\alpha\,D_n\,r$	$-r^2 D_n^2\,\alpha^2 + (u-c_r)\,\alpha\,r\,D_n + (p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha)\,D_n$
$A_1 \leq X \leq A_2$	$\frac{u+k}{2}$	$\frac{u-k}{2}$	$\alpha\,D_n\,r$	$(k-c_r)\,\alpha\,D_n\,r + \frac{(u-k)^2}{4} + (p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha)\,D_n$

Case ($\boldsymbol{k \leq c_r}$)	p_r	D_r	X	$\pi(\alpha\|p_n)$
$X \leq A_2 \leq \frac{u-c_r}{2} \leq A_1$	$u - \alpha\,D_n\,r$	$\alpha\,D_n\,r$	$\alpha\,D_n\,r$	$-r^2 D_n^2\,\alpha^2 + (u-c_r)\,\alpha\,r\,D_n + (p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha)\,D_n$
or $X \leq \frac{u-c_r}{2} \leq A_2 \leq A_1$	$\frac{u+c_r}{2}$	$\frac{u-c_r}{2}$	$\frac{u-c_r}{2}$	$(p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha)\,D_n + \frac{(u-c_r)^2}{4}$
$A_1 \leq X \leq A_2$	$\frac{u+c_r}{2}$	$\frac{u-c_r}{2}$	$\frac{u-c_r}{2}$	$(p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha)\,D_n + \frac{(u-c_r)^2}{4}$

The proof of Proposition 2 is given in the Appendix. Proposition 2 shows that the remanufacturing quantity is higher than or equal to the market demand for the case where $k \geq c_r$. However, the optimal remanufacturing quantity is equal to the market demand for the case where $k \leq c_r$. Generally, the optimal quantity is always equal to the market demand when the demand is deterministic. Note that $k \geq c_r$ means that the salvage value of the remanufactured product is no less than the remanufacturing cost, which suggests that the OEM has an economic incentive to produce all the collected used products, i.e., $X = \alpha\,D_n\,r$. However, if $k \leq c_r$, the optimal remanufacturing quantity is equal to the market demand, which is intuitive. It is because any rational manufacturer will not set the quantity higher than the market demand if the salvage value is lower than the cost. Next, we only consider the case where $k \geq c_r$. In fact, we can derive the optimal solution for the case where $k \leq c_r$ in a similar way. ←R3.6

Second-stage Analysis. After characterizing the OEM's third and fourth-stage decisions, we can analyze the optimal second-stage collection rate decision. Recall that in the third and fourth stages, the yield rate r is realized and not a random variable. However, in this stage, the yield rate r is a random variable because the OEM has not yet started the collection activity when deciding the collection rate α.

Given the new product price, we make use of the selling price and remanufacturing quantity obtained in the third and fourth stages to formulate the OEM's problem as follows:

$$max\ \pi(\alpha|p_n) = \begin{cases} \frac{H^2}{4} + (p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha + (k - c_r)\,\alpha\,r)\,D_n, & r \geq m \\ -\alpha^2\,r^2\,D_n^2 + (p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha + (u - c_r)\,\alpha\,r)\,D_n, & r \leq m \end{cases}, \quad (7)$$

where $m = \frac{H}{2\ D_n\,\alpha}$ and $H = u - k$.

Recall that r is the yield rate of the collected used products that are remanufacturable. Following Galbreth and Blackburn (2010), we assume that the random variable r is uniformly distributed between 0 and 1. Let $f(r)$ be the probability density function of r. Recall from Equation (7) that the realized value of random variable r can be higher or lower than m, so we discuss the cases where $\boldsymbol{m} \geq \mathbf{1}$ and $\boldsymbol{m} \leq \mathbf{1}$ separately.

We call the case where $\boldsymbol{m} \geq \mathbf{1}$ **case *A***. Taking the expectation of r yields

$$E[\pi(\alpha)] = \int_0^1 [-\alpha^2\,r^2\,D_n^2 + (p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha + (u - c_r)\,\alpha\,r)\,D_n]\,f(r)\,dr$$
$$= -\tfrac{1}{3}\,D_n^2\,\alpha^2 + (p_n - c_n)\ D_n + \tfrac{1}{2}\,(w + u - 2\,c_c - c_r)\,D_n\,\alpha. \quad (8)$$

We call the case where $\boldsymbol{m} \leq \mathbf{1}$ **case *B***. Taking the expectation of r yields

$$E[\pi(\alpha)] = \int_0^m [-\alpha^2\,r^2\,D_n^2 + (p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha + (u - c_r)\,\alpha\,r)\,D_n]\,f(r)\,dr$$
$$+ \int_m^1 [\tfrac{(u-k)^2}{4} + (p_n - c_n + w\,\alpha\,(1-r) - c_c\,\alpha + (k - c_r)\,\alpha\,r)\,D_n]\,f(r)\,dr$$
$$= \left(p_n - c_n - \frac{1}{2}\,(2\,c_c + c_r - w - k)\,\alpha\right)\ D_n + \frac{H^2}{4} - \frac{H^3}{24\ D_n\,\alpha}. \quad (9)$$

In the next section we derive the optimal collection rate and introduce the case without the take-back regulation as the benchmark.

4.1. Benchmark: Without the take-back regulation

In this section we characterize the OEM's optimal decisions without the take-back regulation, i.e., the target collection rate is not binding. Note that when the collection target is not binding, it is equivalent to the case where the government does not hold the OEM accountable for collecting the used products, i.e., the lower bound on the collection rate is equal to zero, i.e., $\alpha_0 = 0$. In what follows, we let superscript N represent the case where α is not binding or, equivalently, there is no take-back regulation, and R represent the case where α is binding or, equivalently, there is the take-back regulation.

4.1.1. Without the take-back regulation: Case *NA*

First, we solve the case where $\boldsymbol{m} \geq \mathbf{1}$ and call case ***A*** without the take back regulation **case *NA*.** For **case *NA***, for simplicity, we let $G_0 = w + u - ccr$ and $ccr = 2\,c_c + c_r$, where ccr is the average inverse operating cost for dealing with a used product[2]. The inverse operating cost plays a key role in our analysis. When the inverse operating cost increases, the OEM finds it not profitable to be involved in the collection of the used products. However, when the inverse operating cost decreases, it is beneficial to the OEM

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[2] Since the random variable r is uniformly distributed between 0 and 1, its expected value is $\mu = \frac{1}{2}$, so the unit average inverse operating cost is $c_c/\mu + c_r = 2\,c_c + c_r$.

to collect the used products. When the OEM determines the optimal collection rate, one immediate observation is as follows:

Observation 1. If $G_0 \leq 0$ $(G_0 = w + u - ccr)$, then the OEM's optimal collection rate is $\alpha^{N*} = 0$.

Observation 1 confirms the intuition that when the unit inverse operating cost ccr is higher than the threshold $\overline{w} = w + u$, it is not optimal for the OEM to collect any used product. Therefore, the optimal collection rate is zero without the take-back regulation. Consequently, the OEM's problem is as follows:

$$\pi\,(p_n) = (p_n - c_n)\, D_n,$$

where $D_n = 1 - p_n$. Note that for this case, the OEM only decides the new product price. It is easy to find that the optimal price is $p_n^{NA*} = \frac{1+c_n}{2}$ under the condition that $ccr \geq \overline{w}$. Next, we consider the general case where $G_0 \geq 0$ and the collection rate α can be other values.

Note that $m = \frac{H}{2\,D_n\,\alpha} \geq 1$ is equivalent to $\alpha \leq \frac{H}{2\,D_n} = t$. Also, we assume that the maximum value of the collection rate is α_1, i.e., $\alpha \leq \alpha_1$[3]. Thus, this case includes two subcases, i.e., $\alpha \leq t \leq \alpha_1$ and $\alpha \leq \alpha_1 \leq t$. For case ***NA***, the OEM's problem is as follows:

$$\text{Case } NA-1\text{:}\;\; E[\pi(\alpha)] = -\frac{1}{3}\,D_n^2\,\alpha^2 + (p_n - c_n)\;D_n + \frac{1}{2}\,G_0\,D_n\,\alpha \tag{10}$$

$$s.t.\,\alpha \leq t \leq \alpha_1, \tag{11}$$

$$\text{Case } NA-2\text{:}\;\; E[\pi(\alpha)] = -\frac{1}{3}\,D_n^2\,\alpha^2 + (p_n - c_n)\;D_n + \frac{1}{2}\,G_0\,D_n\,\alpha$$

$$s.t.\;\alpha \leq \alpha_1 \leq t. \tag{12}$$

4.1.2. Without the take-back regulation: Case *NB*

Analogously, we call case ***B*** without the take-back regulation **case *NB*.** For case ***NB***, let $G = ccr - w - k$ and $ccr = 2\,c_c + c_r$. The meaning of ccr is the same as that in case ***NA***. When deciding the collection rate α, similar to case ***NA***, one immediate observation is as follows:

Observation 2. If ≤ 0 $(G = ccr - w - k)$, then α_1 is the optimal collection rate.

Observation 2 states that when the unit inverse operating cost ccr is low, remanufacturing is extremely attractive and the OEM has an economic incentive to collect all the used products. Note that k and w denote the unit salvage values of the collected used products that can and cannot be remanufactured, respectively. When $ccr < w + k$, collecting the used products is always profitable, so the OEM chooses the maximum collection rate. Next, we consider the case where $G \geq 0$ and the collection rate α can be other values in addition to α_1.

Note that for case ***NB***, $m = \frac{H}{2\,D_n\,\alpha} \leq 1$ is equivalent to $\alpha \geq \frac{H}{2\,D_n} = t$. Recall from equation (9) that the OEM's problem is

$$E[\pi(\alpha)] = \left(p_n - c_n - \frac{1}{2}\,G\,\alpha\right)D_n + \frac{H^2}{4} - \frac{H^3}{24\,D_n\,\alpha} \tag{13}$$

$$s.t.\,t \leq \alpha \leq \alpha_1. \tag{14}$$

In the proof of Proposition 3 in the Appendix, we show that the optimal collection rate and new product price depend on cases ***NA*** and ***NB***. Because we have to find the best solutions from all the feasible solutions for cases ***NA*** and ***NB***, we give the optimal solutions for the **First-** and **Second-Stage** problems simultaneously. In summary, based on the results for the case without the

[3] In accordance with reality, we assume that the maximum collection rate target is α_1, not 1, where $1 - \alpha_1$ is the fraction of the used products that leak out to the market. One critical reason is the leakage of the used products to developing countries, or some owners may keep the used products at home. Anyway, it is generally impossible for the OEM to realize 100% collection.

take-back regulation, we derive the next proposition.

Proposition 3. For the case without the take-back regulation, the OEM's optimal remanufacturing price, quantity, collection rate, and new product price are provided in Table 2.

Table 2. The OEM's optimal solutions without the take-back regulation.

Region	$E^{N*}(p_r)$	$E^{N*}(X)$	α^{N*}	p_n^{N*}
$NR1$	N/A	N/A	*0*	$\frac{1+c_n}{2}$
$NR2$	$u-\frac{3\,G_0}{8}$	$\frac{3\,G_0}{8}$	α_A	$\frac{1+c_n}{2}$
$NR3$	$\frac{\sqrt{3\,G\,H}+2\,u+2\,k}{4}$	$\frac{H}{4}\sqrt{\frac{H}{3\,G}}$	α_B	$\frac{1+c_n}{2}$
$NR4$	$u-\frac{3\,G_0\,\alpha_1^2+6\,\alpha_1(1-c_n)}{8\,(\alpha_1^2+3)}$	$\frac{3\,G_0\,\alpha_1^2+6\,\alpha_1(1-c_n)}{8\,(\alpha_1^2+3)}$	α_1	p_{n1}^{A*}
$NR5$	$\frac{H^2}{8\,(1-p_{n1}^{B*})\,\alpha_1}+\frac{u+k}{2}$	$\frac{1}{2}\,(1-p_{n1}^{B*})\,\alpha_1$	α_1	p_{n1}^{B*}

$N_1=1+c_n+\frac{1}{2}G\alpha_1,\ z_1=\alpha_1^2(\sqrt{8\alpha_1(2-N_1)^3+9H^3}-3\,H\sqrt{H})^2,\ G_0=\overline{w}-ccr, \overline{w}=w+u,\ G=ccr-w-k,\ H=u-k,$

$p_{n1}^{A*}=\frac{4\,\alpha_1^2-3\,G_0\,\alpha_1+6\,(1+c_n)}{4\,(\alpha_1^2+3)},\ p_{n1}^{B*}=\frac{2}{3}+\frac{N_1}{6}-\frac{\sqrt[3]{z_1}}{12\,\alpha_1}-\frac{\alpha_1\,(2-N_1)^2}{3\,\sqrt[3]{z_1}},\ \alpha_A=\frac{3\,G_0}{2\,(1-c_n)},\ \alpha_B=\frac{H}{(1-c_n)}\sqrt{\frac{H}{3\,G}},\ H_i=\alpha_i\,(1-c_n), i=0,1.$

Referring to Figure 2, Proposition 3 shows the following

(1) In region $NR1$ where $\overline{w}\le ccr\le\overline{w}-\frac{2(c_n-1)}{\alpha_1}$, the inverse operating cost is pretty high and the OEM has no economic motivation to collect any used product without the take-back regulation. Under this scenario, the OEM only decides the new product price in the monopolistic setting.

(2) In region $NR2$ where $\overline{w}-\frac{2\,H}{3}\le ccr\le\overline{w}$ and $c_n\le 1-\frac{H}{\alpha_1}$, or $\overline{w}-\frac{2\,H_1}{3}\le ccr\le\overline{w}\ and\ 1-\frac{H}{\alpha_1}\le c_n\le 1$, the inverse operating cost is relatively lower than that in region $NR1$ and the OEM chooses $\alpha_A=\frac{3\,G_0}{2\,(1-c_n)}$ as the optimal collection rate. Noting that α_A is closely associated with the salvage value w, scaled size of the secondary market u, and new product manufacturing cost c_n, we derive Corollary 1.

Corollary 1. In region $NR2$, the optimal collection rate α is monotonic in w, u, and c_n.

Corollary 1 confirms the intuition that the salvage value and secondary market size have a positive effect on the optimal collection rate. In addition, the optimal collection rate is increasing in the new product manufacturing cost, which indicates that a higher new product manufacturing cost leads to a higher collection rate. Evidently, managers always hope to recycle the used products when the cost of producing a new counterpart is pretty high and the life cycle is not much longer.

(3) In region $NR3$ where $w+k+\frac{H^3}{3\,H_1^2}\le ccr\le w+k+\frac{H}{3}\ and\ c_n\le 1-\frac{H}{\alpha_1}$, the inverse operating cost is lower than those in regions $NR1$ and $NR2$, and the OEM chooses $\alpha_B=\frac{(u-k)}{(1-c_n)}\sqrt{\frac{u-k}{3\,(ccr-w-k)}}$ as the optimal collection rate. Similar to region $NR2$, it is easy to see that the salvage value w and secondary market size u have a positive impact on the optimal collection rate. We derive the impact of the salvage value k (k is the salvage value of a remanufactured product, which is different from w) on the optimal collection rate in the following corollary.

Corollary 2. In region $NR3$, the optimal collection rate α is monotonic increasing in k.

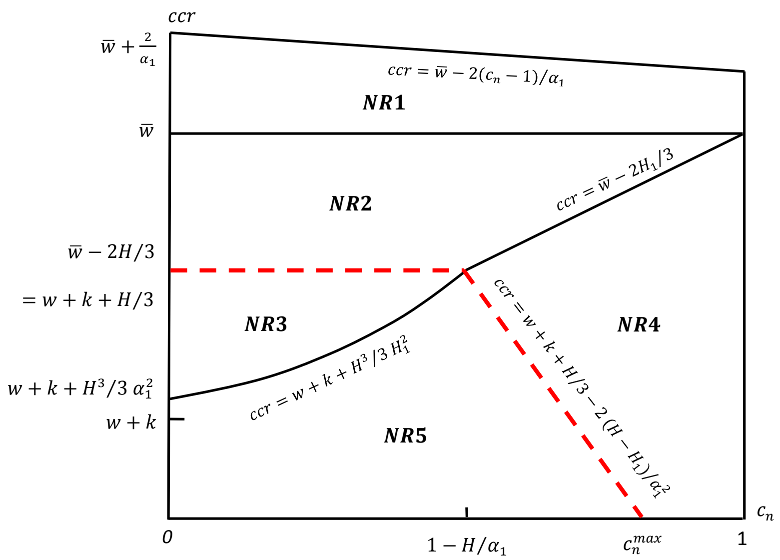
Corollary 2 states that the optimal collection rate is increasing in the salvage value k. By taking the first derivative of α_B with respect to k yields $\frac{\partial \alpha_B}{\partial k} = \frac{\sqrt{3}\,(u-k)}{6\,(1-c_n)} \frac{2\,k+u-3\,(ccr-w)}{\sqrt{(ccr-w-k)^3}}$. Note that $u-k$ and $1-c_n$ are evidently greater than zero. Also, $2\,k+u-3\,(ccr-w)$ is equivalent to $H-3\,(ccr-w-k)$, where $H = u-k$. Note that in region $NR3$ where $ccr \leq w+k+\frac{H}{3}$, we determine that $2\,k+u-3\,(ccr-w) \geq 0$.

(4) In region $NR4$ where $w+k+\frac{H}{3}-\frac{2\,(H-H_1)}{\alpha_1^2} \leq ccr \leq \overline{w}-\frac{2\,H_1}{3}$ and $1-\frac{H}{\alpha_1} \leq c_n \leq c_n^{max}$, or $0 \leq ccr \leq \overline{w}-\frac{2\,H_1}{3}$ and $c_n^{max} \leq c_n \leq 1$, whereas $c_n^{max} = \frac{(u+2k+3w)\alpha_1^2+6(k-u+\alpha_1)}{6\alpha_1}$. Since the inverse operating cost is not extremely low but the new product manufacturing cost is relatively higher, a combination of both leads to a higher collection rate α_1.

(5) In region $NR5$ where $0 \leq ccr \leq w+k+\frac{H^3}{3\,H_1^2}$ and $c_n \leq 1-\frac{H}{\alpha_1}$, or $0 \leq ccr \leq w+k+\frac{H}{3}-\frac{2\,(H-H_1)}{\alpha_1^2}$ and $1-\frac{H}{\alpha_1} \leq c_n \leq c_n^{max}$, the inverse operating cost is pretty small and the OEM has an economic incentive to collect all the used products.

Figure 2 graphically illustrates that different combinations of ccr and c_n lead to different optimal solutions in the proposition. Note that the optimal collection rate shifts from zero to α_1 as we move from region $NR1$ to region $NR5$. The red dashed line in Figure 2 illustrates the case where $m=1$, i.e., both cases ***NA*** and ***NB*** are the same. Furthermore, the regions above this line, i.e., $NR1$, $NR2$, and $NR4$, indicate the case where $m>1$, and the optimal collection rates are 0, α_A, and α_1, respectively. The regions below this line, i.e., $NR3$ and $NR5$, indicate the case where $m<1$, and the optimal collection rates are α_B and α_1, respectively. In other words, the dashed line is the boundary of cases ***NA*** and ***NB.***

Figure 2. Characterization of the OEM's optimal solutions in different regions without take-back regulation.



Proposition 3 states that when the inverse operating cost ccr is extremely high, the OEM has no motivation to collect any used product without the take-back regulation regardless of the value of c_n. Similarly, when the inverse operating cost ccr is pretty low and the OEM has an economic incentive to collect all the used products to remanufacture and sell them in the secondary market regardless of the value of c_n. This is intuitive and consistent with what we know from reality. However, when the inverse operating cost is at a moderate level, a combination of ccr and c_n determines the optimal collection rate α. When the new product manufacturing cost is high, α_1 can still be the optimal solution at a moderate inverse operating cost, which is not straightforward for the OEM to find the optimal solution for this case. In summary, the optimal collection rate depends not only on the inverse operating cost, but also on the new manufacturing cost. Furthermore, it affects the new and remanufactured product pricing decisions, and the remanufacturing quantity. These findings are helpful managerial insights for the managers to make the optimal decisions when the inverse operating cost is not extremely high or extremely low. In what follows, we discuss the case under the take-back legislation.

5. Remanufacturing under the take-back regulation

In this section we analyze the case where there is a take-back regulation imposed on the OEM and the government sets a minimum collection rate target, which we assume α_0 in this paper. Afterwards, we compare the results between the cases with and without the take-back regulation to generate some meaningful managerial insights.

5.1. With the take-back regulation: Case *RA*

Similar to the case without the take-back regulation, we use backward induction to solve the OEM's problem. Note that the **Third- and Fourth-Stage Analyses** under the take-back regulation are the same as those for the case without the take-back regulation, so we omit them here. Recall that we let ***R*** represent the case under the take-back regulation. Analogous to Section 4, we call case ***A*** under the take-back regulation **case *RA.***

Now we solve the OEM's problem in the second stage. For **case *RA***, when the OEM determines the optimal collection rate, one immediate observation is as follows:

Observation 3. If $G_0 \le 0$ $(G_0 = w + u - ccr)$, then the OEM's optimal collection rate is $\alpha = \alpha_0$.

Observation 3 indicates that when the inverse operating cost ccr is higher than the threshold $\bar{w} = w + u$, the OEM sets the collection rate target mandated by the government. Recall from Observation 1 that when the inverse operating cost is extremely high, the OEM is not involved in the collection of the used products at all without the take-back regulation.

Next, we consider the general case where $G_0 \ge 0$ and the optimal collection rate can be other values in addition to α_0. Similar to Section 4.1, we consider the value of m. Note that $m = \frac{H}{2\, D_n\, \alpha} \ge 1$ is equivalent to $\alpha \le \frac{H}{2\, D_n} = t$. Different from the case without the take-back regulation, under the take-back regulation, the government sets a minimum collection rate target α_0. Thus, this case includes two subcases, i.e., $\alpha_0 \le \alpha \le t \le \alpha_1$ and $\alpha_0 \le \alpha \le \alpha_1 \le t$. Note that the OEM's problem remains the same as that for case ***NA*** (equation (8)) with the exception that the collection rate target constraint has to be added. Consequently, the OEM's problem is as follows:

$$\text{Case}\quad RA - 1: E[\pi(\alpha)] = -\frac{1}{3}\, D_n^2\, \alpha^2 + (p_n - c_n)\; D_n + \frac{1}{2}\, G_0\, D_n\, \alpha$$

$$s.t.\ \alpha_0 \le \alpha \le \alpha_1 \le t, \tag{15}$$

$$\text{Case}\quad RA - 2: E[\pi(\alpha)] = -\frac{1}{3}\, D_n^2\, \alpha^2 + (p_n - c_n)\; D_n + \frac{1}{2}\, G_0\, D_n\, \alpha$$

$$s.t.\ \alpha_0 \le \alpha \le t \le \alpha_1. \tag{16}$$

5.2. With the take-back regulation: Case *RB*

Analogously, we call case ***B*** under the take-back regulation **case *RB.*** For case ***RB***, let $G = ccr - w - k$ and $ccr = 2\, c_c + c_r$. The meaning of ccr is the same as that for case ***NB***. When deciding the collection rate α, similar to case ***NB***, we can determine that α_1 is the optimal collection rate when $G \le 0$, i.e., $ccr \le w + k$. In what follows, we consider the case where $G \ge 0$ and the optimal collection rate α can be other values in addition to α_1.

Note that $m = \frac{H}{2\, D_n\, \alpha} \le 1$ is equivalent to $\alpha \ge \frac{H}{2\, D_n} = t$. Recall from equation (9) that the OEM's problem is

$$\text{Case } RB - 1: E[\pi(\alpha)] = \left(p_n - c_n - \frac{1}{2}\, G\, \alpha\right) D_n + \frac{H^2}{4} - \frac{H^3}{24\, D_n\, \alpha}$$

$$s.t.\ t \le \alpha_0 \le \alpha \le \alpha_1, \tag{17}$$

$$\text{Case } RB - 2: E[\pi(\alpha)] = \left(p_n - c_n - \frac{1}{2}\, G\, \alpha\right) D_n + \frac{H^2}{4} - \frac{H^3}{24\, D_n\, \alpha}$$

$$s.t.\ \alpha_0 \le t \le \alpha \le \alpha_1 . \tag{18}$$

Analogous to the case without the take-back regulation, we give the optimal solutions for the **First-** and **Second-Stage** problems simultaneously, and derive the next proposition.

Proposition 4. For the case under the take-back regulation, the OEM's optimal remanufacturing price, quantity, collection rate, and new product price are provided in Table 3.

Table 3. The OEM's optimal solutions under the take-back regulation.

Region	$E^{R*}(p_r)$	$E^{R*}(X)$	α^{R*}	p_n^{R*}
$R1$	$u - \dfrac{3\,G_0\,\alpha_0^2 + 6\,\alpha_0(1-c_n)}{8\,(\alpha_0^2+3)}$	$\dfrac{3\,G_0\,\alpha_0^2 + 6\,\alpha_0(1-c_n)}{8\,(\alpha_0^2+3)}$	α_0	p_{n0}^{A*}
$R2$	$\dfrac{H^2}{8\,(1-p_{n0}^{B*})\,\alpha_0} + \dfrac{u+k}{2}$	$\dfrac{1}{2}\,(1-p_{n0}^{B*})\,\alpha_0$	α_0	p_{n0}^{B*}
$R3$	$u - \dfrac{3\,G_0}{8}$	$\dfrac{3\,G_0}{8}$	α_A	$\dfrac{1+c_n}{2}$
$R4$	$\dfrac{\sqrt{3\,G\,H} + 2\,u + 2\,k}{4}$	$\dfrac{H}{4}\sqrt{\dfrac{H}{3\,G}}$	α_B	$\dfrac{1+c_n}{2}$
$R5$	$u - \dfrac{3\,G_0\,\alpha_1^2 + 6\,\alpha_1(1-c_n)}{8\,(\alpha_1^2+3)}$	$\dfrac{3\,G_0\,\alpha_1^2 + 6\,\alpha_1(1-c_n)}{8\,(\alpha_1^2+3)}$	α_1	p_{n1}^{A*}
$R6$	$\dfrac{H^2}{8\,(1-p_{n1}^{B*})\,\alpha_1} + \dfrac{u+k}{2}$	$\dfrac{1}{2}\,(1-p_{n1}^{B*})\,\alpha_1$	α_1	p_{n1}^{B*}

$N_i = 1 + c_n + \frac{1}{2}G\alpha_i,\ z_i = \alpha_i^2(\sqrt{8\alpha_i(2-N_i)^3 + 9H^3} - 3H\sqrt{H})^2,\ G_0 = \overline{w} - ccr, \overline{w} = w + u,\ G = ccr - w - k,\ H = u - k,$

$p_{ni}^{A*} = \frac{4\,\alpha_i^2 - 3\,G_0\,\alpha_i + 6\,(1+c_n)}{4\,(\alpha_i^2+3)},\ p_{ni}^{B*} = \frac{2}{3} + \frac{N_i}{6} - \frac{\sqrt[3]{z_i}}{12\,\alpha_i} - \frac{\alpha_i\,(2-N_i)^2}{3\,\sqrt[3]{z_i}}, H_i = \alpha_i\,(1-c_n), i = 0,1,\ \alpha_A = \frac{3\,G_0}{2\,(1-c_n)},\ \alpha_B = \frac{H}{(1-c_n)}\sqrt{\frac{H}{3\,G}}.$

Referring to Figure 3, Proposition 4 shows the following

(1) In region $R1$ where $\overline{w} - \frac{2\,H}{3} - \frac{2\,(H-H_0)}{\alpha_0^2} \le ccr \le \overline{w} - \frac{2(c_n-1)}{\alpha_1}$ and $c_n^{min} \le c_n \le 1 - \frac{H}{\alpha_0}$, or $\overline{w} - \frac{2\,H_0}{3} \le ccr \le \overline{w} - \frac{2(c_n-1)}{\alpha_1}$ and $1 - \frac{H}{\alpha_0} \le c_n \le 1$, whereas $c_n^{min} = max\left(0,\ \frac{H\alpha_1(3+\alpha_0^2)+3\alpha_0^2-3\alpha_0\alpha_1}{3\alpha_0^2-3\alpha_0\alpha_1}\right)$. A higher inverse operating cost means that remanufacturing is not profitable, so the OEM sets the collection rate target mandated by the government.

(2) In region $R2$ where $+k + \frac{H^3}{3\,H_0^2} \le ccr \le \overline{w} - \frac{2\,H}{3} - \frac{2\,(H-H_0)}{\alpha_0^2}$ and $c_n^{min} \le c_n \le 1 - \frac{H}{\alpha_0}$, the optimal collection rate target is the same as that in region $R1$.

(3) In region $R3$ where $\overline{w} - \frac{2\,H}{3} \le ccr \le \overline{w} - \frac{2\,H_0}{3}$ and $1 - \frac{H}{\alpha_0} \le c_n \le 1 - \frac{H}{\alpha_1}$, or $\overline{w} - \frac{2\,H_1}{3} \le \mathrm{ccr} \le \overline{w} - \frac{2\,H_0}{3}$ and $1 - \frac{H}{\alpha_1} \le c_n \le 1$, α_A is the optimal collection rate. Similar to $NR2$, α_A is monotonic in $w, u,$ and c_n.

(4) In region $R4$ where $w + k + \frac{H^3}{3\,H_1^2} \le ccr \le w + k + \frac{H^3}{3\,H_0^2}$ and $c_n^{min} \le c_n \le 1 - \frac{H}{\alpha_0}$, or $w + k + \frac{H^3}{3\,H_1^2} \le ccr \le \overline{w} - \frac{2\,H}{3}$ and $1 - \frac{H}{\alpha_0} \le c_n \le 1 - \frac{H}{\alpha_1}$, α_B is the optimal collection rate. Furthermore, we find that $\alpha_B = \frac{H}{(1-c_n)}\sqrt{\frac{H}{3\,G}}$ is increasing in $w, u,$ and k.

(5) In region $R5$ where $w + k + \frac{H}{3} - \frac{2\,(H-H_1)}{\alpha_1^2} \le ccr \le \overline{w} - \frac{2\,H_1}{3}$ and $1 - \frac{H}{\alpha_1} \le c_n \le c_n^{max}$, or $0 \le ccr \le \overline{w} - \frac{2\,H_1}{3}$ $and\ c_n^{max} \le c_n \le 1$, the inverse operating cost is not extremely low, but the new product manufacturing cost is relatively higher, so a

combination of both leads to a higher collection rate α_1.

(6) In region $R6$ where $0 \le ccr \le w + k + \frac{H^3}{3\,H_1^2}$ and $c_n^{min} \le c_n \le 1 - \frac{H}{\alpha_1}$, or $0 \le ccr \le w + k + \frac{H}{3} - \frac{2\,(H - H_1)}{\alpha_1^2}$ and $1 - \frac{H}{\alpha_1} \le c_n \le c_n^{max}$, the inverse operating cost is pretty small and it is beneficial to the OEM to collect all the used products.

Based on the above results, we make two observations as follows:

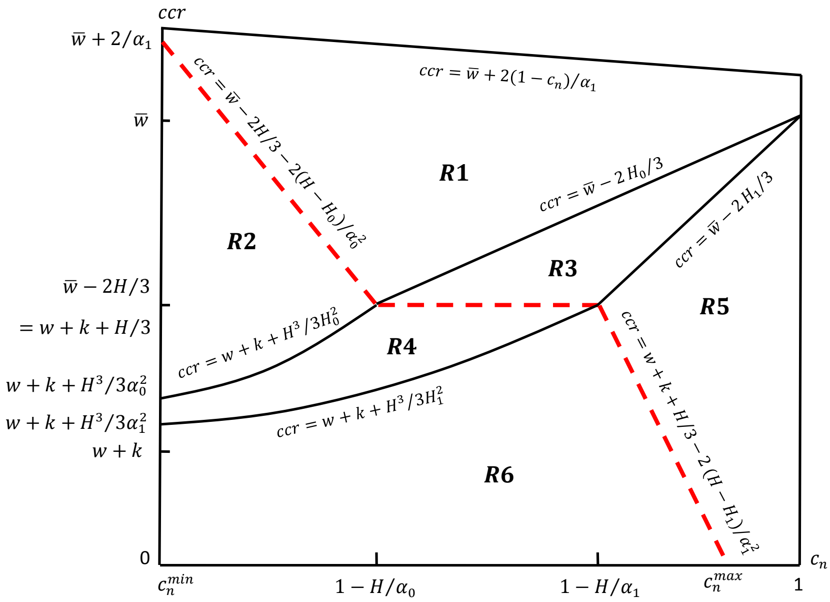
Observation 4. In regions $R3$ and $R4$, the optimal new product price p_n is the same as that for the case where $\alpha^{N*} = 0$. In addition, it is independent of the inverse operating cost.

Observation 4 states that the optimal new product price is the same in regions $NR1$, $R3$, and $R4$. Region $NR1$ is the case where the OEM is not involved in the collection of the used products at all due to the extremely high inverse operating cost, so the OEM only considers the new product pricing decision, i.e., $p_n^{N*} = \frac{1+c_n}{2}$, which suggests that it is only associated with the manufacturing cost and independent of the inverse operating cost. In region $R3$, substituting the optimal collection rate α_A into the profit function yields $(p_n - c_n)\,(1 - p_n) + \frac{(u - c_c - c_r)^2}{4}$, where the first and second terms are the new and remanufactured product profits, respectively. It is easy to see that the remanufacturing profit part has nothing to do with the new product price. The optimal new product price $\frac{1+c_n}{2}$ only depends on the first one. In region $R4$, substituting the optimal collection rate α_B into the profit produces $(p_n - c_n)\,(1 - p_n) + \frac{H^2}{4} - \frac{\sqrt{3}\,H}{6}\sqrt{G\,H}$. Similar to region $R3$, the optimal new product price $\frac{1+c_n}{2}$ is completely independent of the inverse operating cost. In summary, in regions $R3$ and $R4$, the inverse operating cost only affects the remanufacturing pricing and quantity decisions, not the new product pricing decision. The new product pricing decision is only related to the new product profit, which is the same as the case where there is no remanufacturing activity.

Observation 5. Regions $R1$ and $R2$ have the same optimal collection rate, but their new and remanufactured product prices, and remanufacturing quantities are distinct.

Table 3 shows that the optimal remanufacturing quantity depends not only on the collection rate α, but also on other parameters like the new manufacturing cost c_n, scaled size of the secondary market u, salvage value of the leftover remanufactured products w, and inverse operating cost ccr. Note that the inverse operating cost in region $R1$ is completely different from that in region $R2$ despite that the other parameters are the same, so they have different quantities. For the same reason, we demonstrate that their new and remanufactured product prices are distinct. The discussion above can also be used to explain the scenarios in regions $R5$ and $R6$.

Figure 3. Characterization of the OEM's optimal solutions in different regions under take-back regulation.



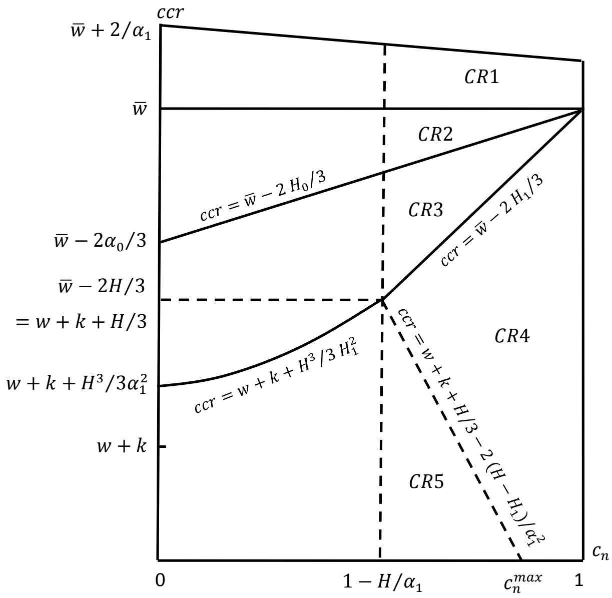
Similar to Figure 2, Figure 3 graphically illustrates how the OEM's optimal solutions change in different regions of the inverse operating cost ccr and new product manufacturing cost c_n. The optimal collection rate shifts from zero to α_1 as we move from region $R1$ to region $R6$ and the red dashed line is the boundary of cases ***RA*** and ***RB.***

Proposition 4 suggests that there are six possibilities, each of which arises under certain conditions. In the two extreme regions $R1$ and $R6$, the optimal collection rates are α_0 and α_1, respectively. However, in regions $R2$, $R3$, $R4$, and $R5$, obtaining the corresponding optimal collection rates is not straightforward. Therefore, our contribution is that we identify the six regions, and the optimal new product price, collection rate, remanufacturing quantity, and remanufactured product price in each region.

5.3. Comparison of the two cases with and without the take-back regulation

In this section we compare the optimal profits between the cases with and without the take-back regulation. Figure 4 illustrates the common regions between the cases with and without the take-back regulation. We derive the results in the following proposition.

Figure 4. Characterization of the common regions between the cases with and without take-back regulation.



Proposition 5. A threshold inverse operating cost value exists, i.e., $t_{ccr}=\bar{w}-\frac{2H_0}{3}$, for the remanufactured product such that, if

$t_{ccr} \leq ccr \leq \bar{w} + \frac{2(1-c_n)}{\alpha_1}$, then $E\ (\pi^{N*}) \geq E(\pi^{R*})$; if $0 \leq ccr \leq t_{ccr}$, then $E\ (\pi^{N*}) = E(\pi^{R*})$.

Proposition 5 states that the optimal expected profit without the take-back regulation is always higher than or equal to that under the take-back regulation. For example, for the case where $1 - \frac{H}{\alpha_1} \leq c_n \leq 1$, in the common region $CR1$, $\bar{w} \leq ccr \leq \bar{w} + 2/\alpha_1$. The inverse operating cost is extremely high, the profit under the take-back regulation is lower than that without the take-back regulation. Under this scenario, the take-back regulation greatly helps increase landfill diversion because the OEM has no economic incentive to collect any used product at a high cost. In the common region $CR2$, $\bar{w} - \frac{2H_0}{3} \leq ccr \leq \bar{w}$. Likewise, the profit without the take-back regulation is higher than that under the take-back regulation. For the remaining regions, i.e., $CR3$, $CR4$, and $CR5$, the profits are the same for both cases with and without the take-back regulation, which indicates that the collection rate target does not have an impact on the OEM's decisions. For the case where $0 \leq c_n \leq 1 - \frac{H}{\alpha_1}$, we can make similar observations and we omit them here.

In summary, Figure 4 suggests that line $L1$ is the boundary of the cases between $E\ (\pi^{N*}) \geq E(\pi^{R*})$ and $E\ (\pi^{N*}) = E(\pi^{R*})$. Specifically, when the inverse operating cost is relatively small, i.e., in the regions below line $L1$, the OEM makes the same decisions regardless of the take-back regulation. However, when the inverse operating cost is relatively large, remanufacturing may not be profitable under regulation. Therefore, the collection rate target imposed on the OEM plays an important role in increasing landfill diversion, while harming the OEM's profit. In this case, the product take-back regulation has an impact on environmental efficiency. In practice, our intuition is that imposing a collection rate target on the OEM is always harmful to the OEM. However, we find that regulating the collection rate may not necessarily hurt the OEM when the inverse operating cost is relatively low, which is quite opposite to what is expected by intuition.

6. Conclusions and future research

In this paper we consider an OEM selling new and remanufactured products in the primary and secondary markets (developed and developing countries), respectively, i.e., there is no competition between the two products. Most of the existing research on product take-back legislation regards the collection rate as an exogenous variable. One of our contributions is that the collection rate is endogenous in our study. In addition, we consider uncertainty in the amount of collected cores qualified for remanufacturing, which has received limited attention in the literature. Following Galbreth and Blackburn (2010), assuming that the remanufacturing yield rate is uniformly distributed between 0 and 1, we derive closed-form solutions in this setting.

We first consider the case without the take-back regulation as the benchmark. In fact, this is a special case of the case under the take-back regulation where $\alpha_0 = 0$. Using backward induction, we derive the optimal new product price, collection rate, remanufacturing quantity, and remanufactured product price in different regions demarcated by combinations of the model parameters. We identify six remanufacturing strategies for the OEM, depending on the inverse operating cost ccr and the new product manufacturing cost c_n, and the results demonstrate that: (i) When the inverse operating cost is extremely high, the OEM collects nothing from the primary market without the take-back regulation and sets the collection rate target under the take-back regulation. For this case, product take-back legislation helps reduce the adversary environmental impact of manufacturing, while harming the OEM's profit. (ii) When the inverse operating cost is extremely low, the OEM has an economic incentive to collect all the used products even without the take-back regulation. (iii) However, when the inverse operating cost is neither extremely high nor extremely low, a combination of the inverse operating cost and new manufacturing cost determines the optimal collection rate, thereby generating the OEM's optimal solution.

In addition, we find that both regions $R1$ and $R2$ have the same optimal collection rate under the take-back regulation, but their remanufacturing quantities are quite different. The intuition behind this is that the optimal remanufacturing quantity depends not only on the collection rate, but also on other parameters like the new manufacturing cost, scaled size of the secondary market, salvage value of the leftover remanufactured product, and inverse operating cost. Note that the inverse operating cost in region $R1$ is completely different from that in region $R2$, despite that other parameters are the same, so they have different quantities. For the same reason, we demonstrate that their new and remanufactured product prices are distinct. Regions $R5$ and $R6$ can be explained in the same way. Furthermore, we show that both regions $R3$ and $R4$ have the same new product price despite the different collection rates. It is because in these two regions the inverse operating cost only affects the remanufacturing pricing and quantity decisions, not the new products pricing decision, which is the same as the case where there is no remanufacturing activity (see Observation 4). Finally, a comparison between the cases with and without the take-back regulation confirms the intuition that the optimal profit without the regulation is always higher than or equal to that under the regulation. More importantly, we identify a threshold inverse operating cost value, below which the OEM's profits are the same for both cases. The collection rate target is realized even without the take-back regulation. In other words, the product take-back regulation is redundant under this scenario.

We conclude with a brief note of caution. For the ease of exposition, we assume the collection cost is linear. In fact, nonlinear cost is also common. We refer the reader to Atasu et al. (2013), who discussed the impacts of different cost structures on the manufacturer's reverse channel choice. Also, in some cases, there may be competition from third-party remanufacturers, especially when the inverse operating cost is small. In addition, we restrict our model to the case where the market demands for both the new and remanufactured products are deterministic. Our results may not apply when demands are uncertain. Finally, we confine our attention to the context where the remanufacturing yield rate r is uniformly distributed between 0 and 1. The managerial insights may be more general when r follows a general distribution. All these assumptions must be relaxed in future research to generate comprehensive implications for the take-back regulation.

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Appendix

For convenience, we let $\overline{w} = w + u,\ m = \frac{H}{2\, D_n\, \alpha},\ t = \frac{H}{2\, D_n},\ H = u - k,\ A_1 = \frac{u-k}{2},\ A_2 = \alpha\, D_n\, r,\ \eta = \frac{3}{3+\alpha_0^2},\ \theta = \frac{3\, \alpha_1}{3\, \alpha_1 + \alpha_0^2\, \alpha_1 - \alpha_0^3},$

$\gamma_0 = \frac{6\, \alpha_1}{6\, \alpha_1 - \alpha_0^3},\ \gamma = \frac{6}{6 - \alpha_1^2 + \alpha_0^2},\ \gamma_1 = \frac{6}{6 - \alpha_1^2},\ H_i = \alpha_i\, (1 - c_n),\ p_{Hi} = 1 - \frac{H}{2\, \alpha_i},\ p_{ni}^{A} = 1 - \frac{3\, G_0}{4\, \alpha_i},\ p_{ni}^{B} = 1 - \frac{H}{6\, \alpha_i}\sqrt{\frac{3\, H}{G}},\ p_{nt}^{A*} = p_{nt}^{B*} = \frac{1+c_n}{2}$

$N_i = 1 + c_n + \frac{G\, \alpha_i}{2},\ z_i = \alpha_i^2\, (\sqrt{8\, \alpha_i\, (2 - N_i)^3 + 9\, H^3} - 3\, H\, \sqrt{H})^2,\ i = 0, 1.$

All these notations will be used throughout this paper. We use the backward induction to solve the OEM's problem, so we first consider the price of the remanufactured product. For the case where $X \le D_r$, we convert Equation (1) into

$$max\, \pi\, (p_r | p_n\, , \alpha, X) = \big(p_n - c_n + w\, \alpha\, (1 - r)\big)\, D_n + (p_r - c_r)\, X - c_c\, \alpha\, D_n \tag{A1}$$

$$s.t. X \le D_r \tag{A2}$$

Taking the first derivative of $(A1)$ with respect to p_r yields $\frac{\partial\, \pi}{\partial\, p_r} = X \ge 0$, which means that the OEM's profit function is increasing in the price of the remanufactured product. Also note that $X \le D_r$ is equivalent to $p_r \le u - X$. Therefore, we can find that the OEM's profit is optimal when $p_r = u - X$, i.e., $X = D_r$. This is a particular case of $X \ge D_r$.

Proof of Propositions 1 and 2. As discussed above, the case where $X \le D_r$ is dominated by the case where $X \ge D_r$, so in the following we only analyze the case where $X \ge D_r$. The OEM's problem is

$$max\, \pi(p_r | p_n, \alpha, X) = -p_r^2 + (u + k)\, p_r + (p_n - c_n + (1 - r)\, w\, \alpha - c_c\, \alpha)\, (1 - p_n) + (k - c_r)\, X - u\, k \tag{A3}$$

$$s.t.\ D_r \le\ X \le \alpha\, D_n\, r \tag{A4}$$

It is easy to see that $(A3)$ is quadratic in p_r. Taking the first derivative of $(A3)$ with respect to p_r and letting it be zero yield $p_{r1} = \frac{u+k}{2}$. Also, the constraint $X \ge D_r$ is equivalent to $p_r \ge u - X = p_{r0}$. We can find that the optimal price is p_{r1}, if $p_{r1} \ge p_{r0}$, which is equivalent to $X \ge \frac{u-k}{2} = A_1$. Otherwise, the optimal price is p_{r0}. We let $A_2 = \alpha\, D_n\, r$. In summary, for the case where $A_1 \ge A_2$, the optimal price of the remanufactured product is as follows:

Case	p_r	D_r	$\pi(X,\ \vert p_n,\ \alpha)$
$X \le A_2 \le A_1$	$u - X$	X	$-X^2 + (u - c_r)\, X + (p_n - c_n + w\, \alpha\, (1 - r) - c_c\, \alpha)\, D_n$

Note that the case where $X \ge A_1 \ge A_2$ conflicts with condition A4, and thus we omit it here.

For the case where $A_1 \le A_2$, the optimal prices of the remanufactured product are as follows:

Case	p_r	D_r	$\pi(X,\ \vert p_n,\ \alpha)$
$X \le A_1$	$u - X$	X	$-X^2 + (u - c_r)\, X + (p_n - c_n + w\, \alpha\, (1 - r) - c_c\, \alpha)\, D_n$
$A_1 \le X \le A_2$	$\frac{u + k}{2}$	$\frac{u - k}{2}$	$(k - c_r)\, X + \frac{(u - k)^2}{4} + (p_n - c_n + w\, \alpha\, (1 - r) - c_c\, \alpha)\, D_n$

We are now ready to solve for the OEM's third-stage problem. We first consider the case where $k \ge c_r$. It is easy to see that the optimal remanufacturing quantity is $X = A_1$ when $X \le A_1$, which is a particular case of $A_1 \le X \le A_2$. So we omit the case where $X \le A_1$. Correspondingly, the optimal price and quantity of the remanufactured product are provided in Propositions 1 and 2 for the case where $k \ge c_r$. Next, we study the case where $k \le c_r$.

When $A_1 \le A_2$, we can easily find that the OEM's optimal remanufacturing quantity is $X = \frac{u-c_r}{2}$, if $X \le A_1$. We have $\frac{u-c_r}{2} \le$

$\frac{u-k}{2} = A_1$. Note that the OEM's profit function is decreasing in the remanufacturing quantity if $A_1 \le X \le A_2$, so the optimal solution to the quantity is $X = A_1$, which is dominated by the case where $X \le A_1$. For the case where $A_1 \ge A_2$, there are two subcases, i.e., $A_2 \le \frac{u-c_r}{2} \le A_1$ and $\frac{u-c_r}{2} \le A_2 \le A_1$. The optimal remanufacturing quantities are A_2 and $\frac{u-c_r}{2}$, respectively, which is provided in Proposition 2.

Proof of Observation 1. For the **case *NA***, the OEM's problem is

$$E[\pi(\alpha)] = -\frac{1}{3} D_n^2 \alpha^2 + \frac{1}{2} G_0 D_n \alpha + (p_n - c_n) D_n \qquad (A5)$$

Taking the first derivative of $(A5)$ with respect to α yields $\frac{\partial E[\pi(\alpha)]}{\partial \alpha} = -\frac{2}{3} D_n^2 \alpha + \frac{1}{2} G_0 D_n \le 0$, if $G_0 \le 0$. Therefore, we can find that $(A5)$ is decreasing with the collection rate α and the OEM chooses to collect nothing without the take-back regulation. This suggests that it is not beneficial to the OEM to collect cores when the inverse operating cost is extremely high.

Proof of Observation 2. For the **case *NB***, the OEM's problem is

$$E[\pi(\alpha)] = \left(p_n - c_n - \frac{1}{2} G \alpha\right) D_n + \frac{H^2}{4} - \frac{H^3}{24 D_n \alpha} \qquad (A6)$$

Solving the first order condition produces $\frac{\partial E[\pi(\alpha)]}{\partial \alpha} = -\frac{1}{2} G D_n + \frac{H^3}{24 D_n \alpha^2} \ge 0$, if $G \le 0$. We can find that $(A6)$ decreases with the collection rate α and the OEM chooses to collect all the used products he can attain, i.e., $\alpha = \alpha_1$. Similar to Observation 1, this indicates that the OEM has an economic incentive to collect all the cores when the inverse operating cost is extremely low.

Proof of Observation 3. Similar to the proof of Observation 1, we can find that the OEM's expected profit function is decreasing in the collection rate, if $G_0 \le 0$. So the optimal collection rate is α_0 under the take-back regulation.

Proof of Proposition 3. Note that the case without the take-back legislation is a particular case of that with the take-back legislation where $\alpha_0 = 0$. We refer the readers to the proof of Proposition 4.

Proof of Proposition 4. Recall that there are two possibilities for the case where $m \ge 1$, i.e., $\alpha_0 \le \alpha \le \alpha_1 \le t$ and $\alpha_0 \le \alpha \le t \le \alpha_1$. We first research the case where $\alpha_0 \le \alpha \le \alpha_1 \le t$, and the OEM's problem is

$$\text{Case } RA-1: E[\pi(\alpha)] = -\tfrac{1}{3} D_n^2 \alpha^2 + \tfrac{1}{2} G_0 D_n \alpha + (p_n - c_n) D_n \qquad (A7)$$

$$s.t.\ \alpha_0 \le \alpha \le \alpha_1 \le t \qquad (A8)$$

Note that the expected profit function $(A7)$ is quadratic and concave in the collection rate α, because $\frac{\partial^2 E}{\partial \alpha^2} = -\frac{2 D_n^2}{3} \le 0$. Taking the first derivative of the expected profit function with respect to α and letting it be zero yield $\alpha_A = \frac{3 G_0}{4 D_n}$. The optimal collection rate α^{A*} can have three possibilities, i.e., $\alpha^{A*} = \alpha_0$, if $\alpha_A \le \alpha_0$; $\alpha^{A*} = \alpha_A$, if $\alpha_0 \le \alpha_A \le \alpha_1$; and $\alpha^{A*} = \alpha_1$, if $\alpha_A \ge \alpha_1$. In what follows, we analyze new product prices regarding the three cases.

i) $\alpha_A \le \alpha_0,\ \alpha^{A*} = \alpha_0$.

$$E[\pi(p_n)] = -\frac{1}{3}\,(1-p_n)^2\,\alpha_0^2 + \frac{1}{2}\,G_0\,(1-p_n)\alpha_0 + (p_n - c_n)\,(1-p_n) \tag{A9}$$

$$s.t.\begin{cases} m \geq 1 \\ \alpha_1 \leq t \\ \alpha_A \leq \alpha_0 \\ G_0 \geq 0 \end{cases} \tag{A10}$$

Rearranging and simplifying the terms, we have the OEM's problem for the case where $\alpha^{A*} = \alpha_0$ as follows:

$$E[\pi(p_n)] = -\left(1 + \frac{\alpha_0^2}{3}\right) p_n^2 + \left(1 + c_n + \frac{2}{3}\,\alpha_0^2 - \frac{1}{2}\,G_0\,\alpha_0\right) p_n - \frac{\alpha_0^2}{3} + \frac{1}{2}\,G_0\,\alpha_0 - c_n \tag{A11}$$

$$s.t.\begin{cases} p_{H1} \leq p_n \leq p_{n0}^A \\ \overline{w} - \dfrac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w} \end{cases} \tag{A12}$$

We can see that the expected profit function $(A11)$ is quadratic and concave in the new product price p_n, since $\frac{\partial^2 E}{\partial\, p_n^2} = -2\,\left(1 + \frac{\alpha_0^2}{3}\right) \leq 0$. Solving the first order condition produces $p_{n0}^{A*} = \frac{6\,(1+c_n) - 3\,G_0\,\alpha_0 + 4\,\alpha_0^2}{4\,(\alpha_0^2+3)}$. Analogous to the optimal solution to the collection rate α, there are also three possibilities for the new product price p_n^{A*}, i.e., $p_n^{A*} = p_{H1}$, if $p_{n0}^{A*} \leq p_{H1}$; $p_n^{A*} = p_{n0}^{A*}$, if $p_{H1} \leq p_{n0}^{A*} \leq p_{n0}^{A}$; and $p_n^{A*} = p_{n0}^{A}$, if $p_{n0}^{A*} \geq p_{n0}^{A}$. Next, we derive the conditions under which the optimal solutions are obtained.

i)-1 $p_n^{A*} = p_{H1}$, conditions $p_{n0}^{A*} \leq p_{H1}$ and $\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w}$ have to be satisfied simultaneously. Note that $p_{n0}^{A*} \leq p_{H1}$ is equivalent to $ccr \leq \overline{w} - \frac{6\,(H-H_1)+2\,H\,\alpha_0^2}{3\,\alpha_0\,\alpha_1}$. In summary, the optimal conditions are: $\left(\frac{H}{H_1} \leq \frac{3}{3+\,\alpha_0^2}\;\&\;\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w}\right)$and $\left(\frac{3}{3+\,\alpha_0^2} \leq \frac{H}{H_1} \leq 1\;\&\;\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w} - \frac{6\,(H-H_1)+2\,H\,\alpha_0^2}{3\,\alpha_0\,\alpha_1}\right)$.

i)-2 $p_n^{A*} = p_{n0}^{A*}$, conditions $p_{H1} \leq p_{n0}^{A*} \leq p_{n0}^{A}$ and $\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w}$ are to be held simultaneously. Note that $p_{H1} \leq p_{n0}^{A*} \leq p_{n0}^{A}$ is equivalent to $ccr \geq \overline{w} - \frac{6\,(H-H_1)+2\,H\,\alpha_0^2}{3\,\alpha_0\,\alpha_1}$ & $ccr \geq \overline{w} - \frac{2\,H_0}{3}$. Therefore, the optimal conditions are $\left(\frac{3}{3+\,\alpha_0^2} \leq \frac{H}{H_1} \leq 1\;\&\;\overline{w} - \frac{6\,(H-H_1)+2\,H\,\alpha_0^2}{3\,\alpha_0\,\alpha_1} \leq ccr \leq \overline{w}\right)$ and $\left(\frac{H}{H_1} \geq 1\;\&\;\overline{w} - \frac{2\,H_0}{3} \leq ccr \leq \overline{w}\right)$.

i)-3 $p_n^{A*} = p_{n0}^{A}$, conditions $p_{n0}^{A*} \geq p_{n0}^{A}$ and $\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w}$ are to be satisfied simultaneously. Note that $p_{n0}^{A*} \geq p_{n0}^{A}$ is equivalent to $ccr \leq \overline{w} - \frac{2\,H_0}{3}$. Thus, the optimal conditions are $\frac{H}{H_1} \geq 1\;\&\;\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w} - \frac{2\,H_0}{3}$.

ii) $\alpha_0 \leq \alpha_A \leq \alpha_1,\; \alpha^{A*} = \alpha_A$.

$$E[\pi(p_n)] = -\,p_n^2 + (1 + c_n)\,p_n + \frac{3\,G_0^2}{16} - c_n \tag{A13}$$

$$s.t.\begin{cases} p_{n0}^A \leq p_n \leq p_{n1}^A \\ \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w} \end{cases} \tag{A14}$$

or $E[\pi(p_n)] = -p_n^2 + (1+c_n)\, p_n + \frac{3\, G_0^2}{16} - c_n$

$$s.t.\begin{cases} p_{H1} \le p_n \le p_{n1}^A \\ \overline{w} - \frac{2\, H}{3} \le ccr \le \overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1} \end{cases} \qquad (A15)$$

Note that there are two possibilities on constraints for this case. We first examine constraint $(A14)$. we can find that the expected profit function is quadratic and concave in new product price. Taking the first derivative of profit function with respect to p_n and letting it be zero yield $p_{nA}^{A*} = \frac{1+c_n}{2}$. The optimal new product prices have three possibilities, i.e., $p_n^{A*} = p_{n0}^A$, if $p_{nA}^{A*} \le p_{n0}^A$; $p_n^{A*} = p_{nA}^{A*}$, if $p_{n0}^A \le p_{nA}^{A*} \le p_{n1}^A$; and $p_n^{A*} = p_{n1}^A$, if $p_{nA}^{A*} \ge p_{n1}^A$. Next, we derive the conditions under which the optimal solutions are satisfied.

ii)-11 $p_n^{A*} = p_{n0}^A$, conditions $p_{nA}^{A*} \le p_{n0}^A$ and $\overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1} \le ccr \le \overline{w}$ have to be satisfied simultaneously. Note that $p_{nA}^{A*} \le p_{n0}^A$ is equivalent to $ccr \ge \overline{w} - \frac{2\, H_0}{3}$, so the optimal conditions are: $\left(\frac{H}{H_1} \le 1 \,\&\, \overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1} \le ccr \le \overline{w}\right)$ and $\left(\frac{H}{H_1} \ge 1 \,\&\, \overline{w} - \frac{2H_0}{3} \le ccr \le \overline{w}\right)$.

ii)-12 $p_n^{A*} = p_{nA}^{A*}$, conditions $p_{n0}^A \le p_{nA}^{A*} \le p_{n1}^A$ and $\overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1} \le ccr \le \overline{w}$ are equivalent to $\left(1 \le \frac{H}{H_1} \le \frac{\alpha_1}{\alpha_0} \,\&\, \overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1} \le ccr \le \overline{w} - \frac{2H_0}{3}\right)$ and $\left(\frac{H}{H_1} \ge \frac{\alpha_1}{\alpha_0} \,\&\, \overline{w} - \frac{2H_1}{3} \le ccr \le \overline{w} - \frac{2H_0}{3}\right)$.

ii)-13 $p_n^{A*} = p_{n1}^A$, conditions $p_{nA}^{A*} \ge p_{n1}^A$ and $\overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1} \le ccr \le \overline{w}$ are equivalent to $\frac{H}{H_1} \ge \frac{\alpha_1}{\alpha_0}$ and $\overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1} \le ccr \le \overline{w} - \frac{2H_1}{3}$.

For constraint $(A15)$, we can get the optimal conditions using similar methods, which are as follows.

ii)-21 $p_n^{A*} = p_{H1}$, conditions $p_{nA}^{A*} \le p_{H1}$ and $\overline{w} - \frac{2\, H}{3} \le ccr \le \overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1}$ are equivalent to $\frac{H}{H_1} \le 1$ and $\overline{w} - \frac{2\, H}{3} \le ccr \le \overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1}$.

ii)-22 $p_n^{A*} = p_{nA}^{A*}$, conditions $p_{H1} \le p_{nA}^{A*} \le p_{n1}^A$ and $\overline{w} - \frac{2\, H}{3} \le ccr \le \overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1}$ are equivalent to $1 \le \frac{H}{H_1} \le \frac{\alpha_1}{\alpha_0} \,\&\, \overline{w} - \frac{2H_1}{3} \le ccr \le \overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1}$.

ii)-23 $p_n^{A*} = p_{n1}^A$, conditions $p_{nA}^{A*} \ge p_{n1}^A$ and $\overline{w} - \frac{2\, H}{3} \le ccr \le \overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1}$ are equivalent to $\left(1 \le \frac{H}{H_1} \le \frac{\alpha_1}{\alpha_0} \,\&\, \overline{w} - \frac{2H}{3} \le ccr \le \overline{w} - \frac{2H_1}{3}\right)$ and $(\frac{H}{H_1} \ge \frac{\alpha_1}{\alpha_0} \,\&\, \overline{w} - \frac{2H}{3} \le ccr \le \overline{w} - \frac{2\, H\, \alpha_0}{3\, \alpha_1})$.

iii) $\alpha_A \ge \alpha_1,\ \alpha^{A*} = \alpha_1$.

$$E[\pi(p_n)] = -\left(1+\frac{\alpha_1^2}{3}\right) p_n^2 + \left(1+c_n+\frac{2}{3}\,\alpha_1^2 - \frac{1}{2}\,G_0\,\alpha_1\right) p_n - \frac{\alpha_1^2}{3} + \frac{1}{2}\,G_0\,\alpha_1 - c_n \qquad (A16)$$

$$s.t.\begin{cases} p_n \geq p_{H1} \\ ccr \leq \overline{w} - \dfrac{2\,H}{3} \end{cases} \qquad (A17)$$

or $E[\pi(p_n)] = -\left(1+\frac{\alpha_1^2}{3}\right) p_n^2 + \left(1+c_n+\frac{2}{3}\,\alpha_1^2 - \frac{1}{2}\,G_0\,\alpha_1\right) p_n - \frac{\alpha_1^2}{3} + \frac{1}{2}\,G_0\,\alpha_1 - c_n$

$$s.t.\begin{cases} p_n \geq p_{n1}^A \\ \overline{w} - \dfrac{2\,H}{3} \leq ccr \leq \overline{w} \end{cases} \qquad (A18)$$

Likewise, the expected profit function $(A16)$ is concave in the new product price. Solving the first order condition yields $p_{n1}^{A*} = \frac{6\,(1+c_n)-3\,G_0\,\alpha_1+4\,\alpha_1^2}{4\,(\alpha_1^2+3)}$, and the optimal conditions under constraints $(A17)$ and $(A18)$ are as follows.

iii)-11 $p_n^{A*} = p_{n1}^{A*}$, conditions $p_{n1}^{A*} \geq p_{H1}$ and $ccr \leq \overline{w} - \frac{2\,H}{3}$ are equivalent to $\frac{H}{H_1} \geq 1\,\&\,\overline{w} - \frac{6\,(H-H_1)+2\,H\,\alpha_1^2}{3\,\alpha_1^2} \leq ccr \leq \overline{w} - \frac{2\,H}{3}$.

iii)-12 $p_n^{A*} = p_{H1}$, conditions $p_{n1}^{A*} \leq p_{H1}$ and $ccr \leq \overline{w} - \frac{2\,H}{3}$ are equivalent to $\left(\frac{H}{H_1} \geq 1\,\&\,ccr \leq \overline{w} - \frac{6\,(H-H_1)+2\,H\,\alpha_1^2}{3\,\alpha_1^2}\right)$ and $(\frac{H}{H_1} \leq 1\,\&\,ccr \leq \overline{w} - \frac{2\,H}{3})$.

iii)-21 $p_n^{A*} = p_{n1}^{A*}$, conditions $p_{n1}^{A*} \geq p_{n1}^A$ and $\overline{w} - \frac{2\,H}{3} \leq ccr \leq \overline{w}$ are equivalent to $\frac{H}{H_1} \geq 1\,\&\,\overline{w} - \frac{2\,H}{3} \leq ccr \leq \overline{w} - \frac{2\,H_1}{3}$.

iii)-22 $p_n^{A*} = p_{n1}^A$, conditions $p_{n1}^{A*} \leq p_{n1}^A$ and $\overline{w} - \frac{2\,H}{3} \leq ccr \leq \overline{w}$ are equivalent to $\left(\frac{H}{H_1} \geq 1\,\&\,\overline{w} - \frac{2\,H_1}{3} \leq ccr \leq \overline{w}\right)$ and $\left(\frac{H}{H_1} \leq 1\,\&\,\overline{w} - \frac{2\,H}{3} \leq ccr \leq \overline{w}\right)$.

We summarize all the possible solutions discussed above under the case where $m \geq 1\,\&\,\alpha_0 \leq \alpha \leq \alpha_1 \leq t$ in **Table RA** for the ease of reading. Next, we examine the case where $m \geq 1\,\&\,\alpha_0 \leq \alpha \leq t \leq \alpha_1$, and the OEM's problem is

$$\text{Case } RA-2\text{: } E[\pi(\alpha)] = -\frac{1}{3}\,D_n^2\,\alpha^2 + \frac{1}{2}\,G_0\,D_n\,\alpha + (p_n - c_n)\,D_n \qquad (A19)$$

$$s.t.\ \alpha_0 \leq \alpha \leq t \leq \alpha_1 \qquad (A20)$$

Analogous to the case where $m \geq 1\,\&\,\alpha_0 \leq \alpha \leq \alpha_1 \leq t$, all the possible solutions to the case where $m \geq 1\,\&\,\alpha_0 \leq \alpha \leq t \leq \alpha_1$ can be derived using similar methods, which are summarized in **Table RB**.

Now, we study the case where $m \leq 1$. Recall that there are two possibilities for the case where $m \leq 1$, i.e., $t \leq \alpha_0 \leq \alpha \leq \alpha_1$ or $\alpha_0 \leq t \leq \alpha \leq \alpha_1$. We first research the case where $t \leq \alpha_0 \leq \alpha \leq \alpha_1$, and the OEM's problem is

$$\text{Case } RB-1\text{: } E[\pi(\alpha)] = \left(p_n - c_n - \frac{1}{2}\,G\,\alpha\right) D_n + \frac{H^2}{4} - \frac{H^3}{24\,D_n\,\alpha} \qquad (A21)$$

$$s.t.\ t \le \alpha_0 \le \alpha \le \alpha_1 \tag{A22}$$

Note that the expected profit function $(A21)$ is concave in the collection rate α, because $\frac{\partial^2 E}{\partial \alpha^2} = -\frac{H^3}{12\, D_n\, \alpha^3} \le 0$. Taking the first derivative of the expected profit function with respect to α and letting it be zero yield $\alpha_B = \frac{H}{6\ D_n}\sqrt{\frac{3\,H}{G}}$. The optimal collection rate α^{B*} can have three possibilities, i.e., $\alpha^{B*} = \alpha_0$, if $\alpha_B \le \alpha_0$; $\alpha^{B*} = \alpha_B$, if $\alpha_0 \le \alpha_B \le \alpha_1$; and $\alpha^{B*} = \alpha_1$, if $\alpha_B \ge \alpha_1$. In what follows, we analyze the new product price regarding the three cases.

i) $\alpha_B \le \alpha_0,\ \alpha^{B*} = \alpha_0$.

$$E[\pi(p_n)] = \left(p_n - c_n - \frac{1}{2}\, G\, \alpha_0\right) D_n + \frac{H^2}{4} - \frac{H^3}{24\, D_n\, \alpha_0} \tag{A23}$$

$$s.t. \begin{cases} m \le 1 \\ \alpha_0 \ge t \\ \alpha_B \le \alpha_0 \\ G \ge 0 \end{cases} \tag{A24}$$

Rearranging and simplifying the terms, we obtain the OEM's problem for the case where $\alpha^{B*} = \alpha_0$ as follows:

$$E[\pi(p_n)] = \left(p_n - c_n - \frac{1}{2}\, G\, \alpha_0\right)(1 - p_n) + \frac{H^2}{4} - \frac{H^3}{24\,(1 - p_n)\alpha_0} \tag{A25}$$

$$s.t. \begin{cases} p_n \le p_{n0}^B \\ w + k \le ccr \le w + k + \frac{H}{3} \end{cases} \tag{A26}$$

or $E[\pi(p_n)] = \left(p_n - c_n - \frac{1}{2}\, G\, \alpha_0\right)(1 - p_n) + \frac{H^2}{4} - \frac{H^3}{24\,(1-p_n)\alpha_0}$

$$s.t. \begin{cases} p_n \le p_{H0} \\ ccr \ge w + k + \frac{H}{3} \end{cases} \tag{A27}$$

Note that there are two possibilities on constraints for this case. We first examine constraint $(A26)$. It is easy to see that the expected profit function $(A25)$ is concave in new product price p_n, since $\frac{\partial^2 E}{\partial p_n^2} = -2 - \frac{H^3}{12\, \alpha_0\, (1-p_n)^3} \le 0$. Solving the first order condition produces $p_{n0}^{B*} = \frac{2}{3} + \frac{N_0}{6} - \frac{\sqrt[3]{z_0}}{12\ \alpha_0} - \frac{\alpha_0\ (2-N_0)^2}{3\ \sqrt[3]{z_0}}$. Analogous to the optimal solution to the collection rate α, there are two possibilities for the new product price p_n^{B*}, i.e., $p_n^{B*} = p_{n0}^{B*}$, if $p_{n0}^{B*} \le p_{n0}^{B}$; and $p_n^{B*} = p_{n0}^{B}$, if $p_{n0}^{B*} \ge p_{n0}^{B}$. Next, we derive the conditions under which the optimal solutions are obtained.

i)-11 $p_n^{B*} = p_{n0}^{B*}$, conditions $p_{n0}^{B*} \le p_{n0}^{B}$ and $w + k \le ccr \le w + k + \frac{H}{3}$ have to be held simultaneously. Note that $p_{n0}^{B*} \le p_{n0}^{B}$ is equivalent to $ccr \ge w + k + \frac{H^3}{3\, H_0^2}$. In summary, the optimal conditions are: $\frac{H}{H_0} \le 1\ \&\ w + k + \frac{H^3}{3\, H_0^2} \le ccr \le w + k + \frac{H}{3}$.

i)-12 $p_n^{B*} = p_{n0}^{B}$, conditions $p_{n0}^{B*} \ge p_{n0}^{B}$ and $w + k \le ccr \le w + k + \frac{H}{3}$ are to be held simultaneously. Note that $p_{n0}^{B*} \ge p_{n0}^{B}$ is equivalent to $ccr \le w + k + \frac{H^3}{3\,H_0^2}$. Therefore, the optimal conditions are $\left(\frac{H}{H_0} \le 1 \;\&\; w + k \le ccr \le w + k + \frac{H^3}{3\,H_0^2}\right)$ and $\left(\frac{H}{H_0} \ge 1 \;\&\; w + k \le ccr \le w + k + \frac{H}{3}\right)$.

For the case under constraint $(A27)$, we can get the optimal conditions using similar methods, which are as follows.

i)-21 $p_n^{B*} = p_{n0}^{B*}$, conditions $p_{n0}^{B*} \le p_{H0}$ and $ccr \ge w + k + \frac{H}{3}$ are to be satisfied simultaneously. Note that $p_{n0}^{B*} \le p_{H0}$ is equivalent to $ccr \le w + k + \frac{H}{3} - \frac{2\,(H - H_0)}{\alpha_0^2}$. In summary, the optimal conditions are: $\frac{H}{H_0} \le 1 \;\&\; w + k + \frac{H}{3} \le ccr \le w + k + \frac{H}{3} - \frac{2\,(H - H_0)}{\alpha_0^2}$.

i)-22 $p_n^{B*} = p_{H0}$, conditions $p_{n0}^{B*} \ge p_{H0}$ and $ccr \ge w + k + \frac{H}{3}$ are to be held simultaneously. Note that $p_{n0}^{B*} \ge p_{H0}$ is equivalent to $ccr \ge w + k + \frac{H}{3} - \frac{2\,(H - H_0)}{\alpha_0^2}$. Therefore, the optimal conditions are $\left(\frac{H}{H_0} \le 1 \;\&\; ccr \ge w + k + \frac{H}{3} - \frac{2\,(H - H_0)}{\alpha_0^2}\right)$ and $\left(\frac{H}{H_0} \ge 1 \;\&\; ccr \ge w + k + \frac{H}{3}\right)$.

ii) $\alpha_0 \le \alpha_B \le \alpha_1,\ \alpha^{B*} = \alpha_B$.

$$E[\pi(p_n)] = -\,p_n^2 + (1 + c_n)\,p_n + \frac{H^2}{4} - c_n - \frac{H}{6}\,\sqrt{3\,G\,H} \qquad (A28)$$

$$s.t. \begin{cases} p_{n0}^B \le p_n \le p_{n1}^B \\ w + k \le ccr \le w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \end{cases} \qquad (A29)$$

or $E[\pi(p_n)] = -\,p_n^2 + (1 + c_n)\,p_n + \frac{H^2}{4} - c_n - \frac{H}{6}\,\sqrt{3\,G\,H}$

$$s.t. \begin{cases} p_{n0}^B \le p_n \le p_{H0} \\ w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H}{3} \end{cases} \qquad (A30)$$

We can easily find that the expected profit function $(A28)$ is quadratic and concave in the new product price. Taking the first derivative of the expected profit function with respect to p_n and letting it be zero yield $p_{nB}^{B*} = \frac{1+c_n}{2}$. Similarly, all possible solutions to the new product price under the constraint $(A29)$ can be derived as follows:

ii)-11 $p_n^{B*} = p_{n0}^B$, the optimal conditions are: $\frac{H}{H_0} \le \frac{\alpha_0}{\alpha_1} \;\&\; w + k + \frac{H^3}{3\,H_0^2} \le ccr \le w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2}$.

ii)-12 $p_n^{B*} = p_{nB}^{B*}$, the optimal conditions are: $(\frac{H}{H_0} \le \frac{\alpha_0}{\alpha_1} \;\&\; w + k + \frac{H^3}{3\,H_1^2} \le ccr \le w + k + \frac{H^3}{3\,H_0^2})$ and $(\frac{\alpha_0}{\alpha_1} \le \frac{H}{H_0} \le 1 \;\&\; w + k + \frac{H^3}{3\,H_1^2} \le ccr \le w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2})$.

ii)-13 $p_n^{B*} = p_{n1}^B$, the optimal conditions are: $(\frac{H}{H_0} \le 1 \;\&\; w + k \le ccr \le w + k + \frac{H^3}{3\,H_1^2})$ and $(\frac{H}{H_0} \ge 1 \;\&\; w + k \le ccr \le w +$

$k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$).

All the possible solutions to the new product price under the constraint $(A30)$ are as follows:

ii)-21 $p_n^{B*}=p_{n0}^{B}$, the optimal conditions are: $(\frac{H}{H_0}\le\frac{\alpha_0}{\alpha_1}$ & $w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}\le ccr\le w+k+\frac{H}{3})$ and $(\frac{\alpha_0}{\alpha_1}\le\frac{H}{H_0}\le 1$ & $w+k+\frac{H^3}{3\,H_0^2}\le ccr\le w+k+\frac{H}{3})$.

ii)-22 $p_n^{B*}=p_{nB}^{B*}$, the optimal conditions are: $\frac{\alpha_0}{\alpha_1}\le\frac{H}{H_0}\le 1$ & $w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}\le ccr\le w+k+\frac{H^3}{3\,H_0^2}$.

ii)-23 $p_n^{B*}=p_{H0}$, the optimal conditions are: $\frac{H}{H_0}\ge 1$ & $w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}\le ccr\le w+k+\frac{H}{3}$.

iii) $\alpha_B\ge\alpha_1,\ \alpha^{B*}=\alpha_1$.

$$E[\pi(p_n)]=\left(p_n-c_n-\frac{1}{2}\,G\,\alpha_1\right)(1-p_n)+\frac{H^2}{4}-\frac{H^3}{24\,(1-p_n)\alpha_1}\qquad(A31)$$

$$s.t.\begin{cases}p_{n1}^{B}\le p_n\le p_{H0}\\ w+k\le ccr\le w+k+\dfrac{H\,\alpha_0^2}{3\,\alpha_1^2}\end{cases}\qquad(A32)$$

iii)-1 $p_n^{B*}=p_{n1}^{B}$, the optimal conditions are: $\frac{H}{H_0}\le 1$ & $w+k+\frac{H^3}{3\,H_1^2}\le ccr\le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$.

iii)-2 $p_n^{B*}=p_{n1}^{B*}$, the optimal conditions are: $(\frac{H}{H_0}\le 1$ & $w+k\le ccr\le w+k+\frac{H^3}{3\,H_1^2})$ and $(1\le\frac{H}{H_0}\le\frac{6\,\alpha_1}{6\,\alpha_1-\alpha_0^3}$ & $w+k\le ccr\le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}-\frac{2\,(H-H_0)}{\alpha_0\,\alpha_1})$.

iii)-3 $p_n^{B*}=p_{H0}$, the optimal conditions are: $(1\le\frac{H}{H_0}\le\frac{6\,\alpha_1}{6\,\alpha_1-\alpha_0^3}$ & $w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}-\frac{2\,(H-H_0)}{\alpha_0\,\alpha_1}\le ccr\le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2})$ and $(\frac{H}{H_0}\ge\frac{6\,\alpha_1}{6\,\alpha_1-\alpha_0^3}$ & $w+k\le ccr\le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2})$.

We summarize all the possible solutions discussed above for the case where $m\le 1$ & $t\le\alpha_0\le\alpha\le\alpha_1$ in Table **RC** for the ease of reading. Next, we study the case where $m\le 1$ & $\alpha_0\le t\le\alpha\le\alpha_1$, and the OEM's problem is

$$\text{Case } RB-2: E[\pi(\alpha)]=\left(p_n-c_n-\frac{1}{2}\,G\,\alpha\right)D_n+\frac{H^2}{4}-\frac{H^3}{24\,D_n\,\alpha}\qquad(A33)$$

$$s.t.\ \alpha_0\le t\le\alpha\le\alpha_1\qquad(A34)$$

Analogous to the case where $m\le 1$ & $t\le\alpha_0\le\alpha\le\alpha_1$, all the possible solutions to the case where $m\le 1$ & $\alpha_0\le t\le\alpha\le\alpha_1$ are summarized in Table **RD**.

Particular case: Note that the above proofs focus on the general ***case RA*** where $G_0 \geq 0$ and ***case RB*** where $G \geq 0$. Next, we discuss the particular case where $G_0 \leq 0$ and $G \leq 0$.

Particular case *PRA:* We call case ***RA*** under the condition $m \geq 1 \,\&\, G_0 \leq 0$ case ***PRA***. For this case, α_0 is the optimal collection rate, and the OEM's problem is

$$E[\pi(p_n)] = -\left(1 + \frac{\alpha_0^2}{3}\right) p_n^2 + \left(1 + c_n + \frac{2}{3}\,\alpha_0^2 - \frac{1}{2}\,G_0\,\alpha_0\right) p_n - \frac{\alpha_0^2}{3} + \frac{1}{2}\,G_0\,\alpha_0 - c_n \qquad (A35)$$

$$s.t. \begin{cases} p_n \geq p_{H0} \\ ccr \geq \overline{w} \end{cases} \qquad (A36)$$

Similar to the proof of the general case, we summarize all the possible solutions to the case where $m \geq 1 \,\&\, G_0 \leq 0$ in Table **RE**.

Particular case *PRB:* We call case ***RB*** under the condition $m \leq 1 \,\&\, G \leq 0$ case ***PRB*.** For this case, α_1 is the optimal collection rate, and the OEM's problem is

$$E[\pi(p_n)] = \left(p_n - c_n - \frac{1}{2}\,G\,\alpha_1\right)(1 - p_n) + \frac{H^2}{4} - \frac{H^3}{24\,(1 - p_n)\alpha_1} \qquad (A37)$$

$$s.t. \begin{cases} p_n \leq p_{H1} \\ ccr \leq w + k \end{cases} \qquad (A38)$$

Analogously, we summarize all the possible solutions to the case where $m \leq 1 \,\&\, G \leq 0$ in Table **RF**.

So far, we have discussed all the possible cases. Specifically, there are four general cases ($m \geq 1 \,\&\, \alpha_0 \leq \alpha \leq \alpha_1 \leq t;\ m \geq 1 \,\&\, \alpha_0 \leq \alpha \leq t \leq \alpha_1;\ m \leq 1 \,\&\, t \leq \alpha_0 \leq \alpha \leq \alpha_1$; and $m \leq 1 \,\&\, \alpha_0 \leq t \leq \alpha \leq \alpha_1$). For each case, we derive the conditions under which optimal solutions are satisfied. In addition, we have two particular cases where $m \geq 1 \,\&\, G_0 \leq 0$ and $m \leq 1 \,\&\, G \leq 0$. For $G_0 \leq 0$, which represents the extreme case where the inverse operating cost is pretty high and the OEM has no economic incentive to collect any used product under the product take-back legislation. For the case where $G \leq 0$, which represents the extreme case when the inverse operating cost is very low and the OEM has an economic incentive to collect all the used products, even without the product take-back regulation. Based on the analysis above, we can use Figure S to illustrate all the solutions in Tables RA, RB, RC, RD, RE and RF, and find the only optimal solution in each region among all the possible solutions. Note that there are 11 columns in Figure **S**. We denote columns 1 to 11 by B, C, D, E, F, G, H, I, J, K, L, and let B_i $(i = 1, 2, 3, 4, 5, 6, 7, 8, 9)$ represent the regions of column B from top to bottom. In what follows, we only derive the optimal solutions in column B, optimal solutions in other columns can be obtained in a similar way.

Proof of region B_1**.** In region B_1, $\overline{w} - \frac{6\,(H - H_0) + 2\,H\,\alpha_0^2}{3\,\alpha_0^2} \leq ccr \leq \overline{ccr}$, which is equivalent to $G_0 \leq \frac{6\,(H - H_0) + 2\,H\,\alpha_0^2}{3\,\alpha_0\,\alpha_1}$. Note that there are three possible feasible solutions, $(\alpha_0,\ p_{H0})$, $(t,\ p_{H0})$, and $(\alpha_0,\ p_{n0}^{A*})$, which are satisfied under cases where $m \leq 1 \,\&\, t \leq \alpha_0 \leq \alpha \leq \alpha_1;\ m \leq 1 \,\&\, \alpha_0 \leq t \leq \alpha \leq \alpha_1;$ and $m \geq 1 \,\&\, G_0 \leq 0$, respectively. For convenience, we only give optimal collection rate and new product price. The optimal price and quantity of remanufactured product vary according to both of them and we omit here. For feasible solution $(\alpha_0,\ p_{H0})$, we can find $m = \frac{H}{2\,D_n\,\alpha} = \frac{H}{2\,(1 - p_{H0})\,\alpha_0} = 1$, which is a particular case where $m \geq 1$. Similarly, for feasible solution $(t,\ p_{H0})$, we can find $m = \frac{H}{2\,D_n\,\alpha} = \frac{H}{2\,D_n\,t} = 1$, which is also a particular case where $m \geq 1$. Consequently, we determine that $(\alpha_0,\ p_{n0}^{A*})$ is the first best among all the three feasible solutions.

Proof of region B_2. In region B_2, $\bar{w} \le ccr \le \bar{w} - \frac{6\,(H-H_0)+2\,H\,\alpha_0^2}{3\,\alpha_0^2}$, which is equivalent to $\frac{6\,(H-H_0)+2\,H\,\alpha_0^2}{3\,\alpha_0^2} \le G_0 \le 0$. We have three feasible solutions, $(\alpha_0,\ p_{n0}^{B*})$, $(t,\ p_{H0})$, and $(\alpha_0,\ p_{H0})$, which are satisfied under cases where $m \le 1\ \&\ t \le \alpha_0 \le \alpha \le \alpha_1$; $m \le 1\ \&\ \alpha_0 \le t \le \alpha \le \alpha_1$; and $m \ge 1\ \&\ G_0 \le 0$, respectively. Similar to the proof discussed in region B_1, solutions $(t,\ p_{H0})$ and $(\alpha_0,\ p_{H0})$ are dominated by solution $(\alpha_0,\ p_{n0}^{B*})$, so $(\alpha_0,\ p_{n0}^{B*})$ is the best.

Proof of region B_3. In region B_3, $\bar{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \le ccr \le \bar{w}$, which is equivalent to $0 \le G_0 \le \frac{2\,H\,\alpha_0}{3\,\alpha_1}$. Feasible solutions $(\alpha_0,\ p_{H1})$, $(\alpha_A,\ p_{n0}^{A})$ and $(\alpha_1,\ p_{n1}^{A})$ are satisfied under case where $m \ge 1\ \&\ \alpha_0 \le \alpha \le \alpha_1 \le t$. Solutions $(\alpha_0,\ p_{H0})$, $(\alpha_0,\ p_{n0}^{B*})$ and $(t,\ p_{H0})$ are satisfied under the cases where $m \ge 1\ \&\ \alpha_0 \le \alpha \le t \le \alpha_1$; $m \le 1\ \&\ t \le \alpha_0 \le \alpha \le \alpha_1$; and $m \le 1\ \&\ \alpha_0 \le t \le \alpha \le \alpha_1$, respectively.

$$
\begin{aligned}
&E\left[\pi\,(\alpha_0,\ p_{H1})\right] - E\left[\pi\,(\alpha_A,\ p_{n0}^{A})\right] \\
&\quad = \frac{1}{48\,\alpha_0^2\,\alpha_1^2}\,[(27\,\alpha_1^2 - 9\,\alpha_0^2\,\alpha_1^2)\,G_0^2 + \left(12H\,\alpha_0^3\ \alpha_1 - 36\,\alpha_0\,\alpha_1^2\,(1-c_n)\right)\,G_0 - 4\,H^2\,\alpha_0^4 + 24\,H\,\alpha_0^2\,\alpha_1\,(1-c_n) \\
&\quad -12\,H^2\,\alpha_0^2] \qquad (A39)
\end{aligned}
$$

Note that $(A39)$ is quadratic and convex in G_0, and has two real roots: $G_{00} = \frac{2\,H\,\alpha_0}{3\,\alpha_1}$, $G_{01} = \frac{2\,\left(H\,\alpha_0^2 - 6\,\alpha_1\,(1-c_n) + 3\,H\right)\,\alpha_0}{3\,(\alpha_0^2-3)\,\alpha_1}$. Also, $G_{00} \le G_{01}$. Since $\frac{2\,H\,\alpha_0}{3\,\alpha_1} \le \frac{2\,\left(H\,\alpha_0^2 - 6\,\alpha_1\,(1-c_n) + 3\,H\right)\,\alpha_0}{3\,(\alpha_0^2-3)\,\alpha_1}$ is equivalent to $H \le H_1$. Note from Figure S that in column B, $H \le \frac{\alpha_0}{\alpha_1} H_0 \le H_1$. Otherwise, there is a contradiction. In summary, we conclude that $E\left[\pi\,(\alpha_0,\ p_{H1})\right] \ge E\left[\pi\,(\alpha_A,\ p_{n0}^{A})\right]$.

$$
E\left[\pi\,(\alpha_A,\ p_{n0}^{A})\right] - E\left[\pi\,(\alpha_1,\ p_{n1}^{A})\right] = \frac{3}{16\,\alpha_0^2\,\alpha_1^2}\,[(3\,\alpha_0^2 - 3\,\alpha_1^2)\,G_0^2 + 4\,\alpha_0\,\alpha_1\,(1-c_n)\,(\alpha_1 - \alpha_0)\ G_0] \qquad (A40)
$$

We can see that $(A40)$ is quadratic and concave in G_0, and has two real roots: $G_{00} = 0$, $G_{01} = \frac{4\,\alpha_0\,\alpha_1\,(1-c_n)}{3\,(\alpha_0 + \alpha_1)}$. Also, $G_{01} \ge \frac{2\,H\,\alpha_0}{3\,\alpha_1}$. If $G_{01} \le \frac{2\,H\,\alpha_0}{3\,\alpha_1}$, then $\frac{H}{H_1} \ge \frac{2\,\alpha_1}{\alpha_0 + \alpha_1} \ge 1$, which contradicts to the condition $H \le \frac{\alpha_0}{\alpha_1}\,H_0 \le H_1$ in column B. In summary, we can determine $E\left[\pi\,(\alpha_A,\ p_{n0}^{A})\right] \ge E\left[\pi\,(\alpha_1,\ p_{n1}^{A})\right]$.

Solution $(\alpha_0,\ p_{H1})$ is satisfied under the case where $m \ge 1\ \&\ \alpha_0 \le \alpha \le \alpha_1 \le t$. Note that $t = \frac{H}{2\,D_n} = \alpha_1$, which means that the case where $m \ge 1\ \&\ \alpha_0 \le \alpha \le \alpha_1 \le t$ is dominated by the case where $m \ge 1\ \&\ \alpha_0 \le \alpha \le t \le \alpha_1$. Also note that solution $(t,\ p_{H0})$ to the case where $m \le 1\ \&\ \alpha_0 \le t \le \alpha \le \alpha_1$ is dominated by that of the case where $m \ge 1$. To sum, we conclude that $(\alpha_0,\ p_{n0}^{B*})$ is the best.

Proof of region B_4. In region B_4, $\bar{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \le ccr \le \bar{w} - \frac{2\,H}{3}$, which is equivalent to $\frac{2\,H\,\alpha_0}{3\,\alpha_1} \le G_0 \le \frac{2\,H}{3}$. Feasible solutions $(\alpha_A,\ p_{H1})$ and $(\alpha_1,\ p_{n1}^{A})$ are satisfied under the case where $m \ge 1\ \&\ \alpha_0 \le \alpha \le \alpha_1 \le t$. Solutions $(\alpha_0,\ p_{H0})$ and $(\alpha_A,\ p_{n0}^{A})$ are satisfied under the case where $m \ge 1\ \&\ \alpha_0 \le \alpha \le t \le \alpha_1$. Solutions $(\alpha_0,\ p_{n0}^{B*})$ and $(t,\ p_{H0})$ are satisfied under the cases where $(m \le 1\ \&\ t \le \alpha_0 \le \alpha \le \alpha_1)$ and $(m \le 1\ \&\ \alpha_0 \le t \le \alpha \le \alpha_1)$, respectively.

$E\left[\pi\,(\alpha_A,\ p_{n0}^{A})\right] - E\left[\pi\,(\alpha_1,\ p_{n1}^{A})\right] = \frac{3}{16\,\alpha_0^2\,\alpha_1^2}\,[(3\,\alpha_0^2 - 3\,\alpha_1^2)\,G_0^2 + 4\,\alpha_0\,\alpha_1\,(1-c_n)\,(\alpha_1 - \alpha_0)\ G_0] \ge 0$, please refer to the proof of B_3.

$$
E\left[\pi\,(\alpha_0,\ p_{H0})\right] - E\left[\pi\,(\alpha_A,\ p_{n0}^{A})\right]
$$

$$= \frac{1}{48\,\alpha_0^2}[(27 - 9\,\alpha_0^2)\,G_0^2 + \left(12\,H\,\alpha_0^2 - 36\,\alpha_0\,(1 - c_n)\right)G_0 - 4\,H^2\,\alpha_0^2 + 24\,H\,\alpha_0\,(1 - c_n) - 12\,H^2)] \qquad (A41)$$

$(A41)$ is quadratic and convex in G_0, and has two real roots: $G_{00} = \frac{2\,H}{3}$, $G_{01} = \frac{2\,(H\,\alpha_0^2 - 6\,\alpha_0\,(1-c_n) + 3\,H)}{3\,(\alpha_0^2 - 3)}$, and $G_{00} \le G_{01}$. We can determine that $E[\pi\,(\alpha_0,\,p_{H0})] \ge E[\pi\,(\alpha_A,\,p_{n0}^A)]$.

$$E[\pi\,(\alpha_A,\,p_{n0}^A)] - E[\pi\,(\alpha_A,\,p_{H1})] = \frac{1}{16\alpha_0^2\alpha_1^2}[-9\,\alpha_1^2\,\,G_0^2 + 12\,\,\alpha_0\,\alpha_1^2\,(1 - c_n)\,\,G_0 - 8\,H\,\alpha_0^2\,\,\alpha_1\,(1 - c_n) + 4\,H^2\,\alpha_0^2\,] \quad (A42)$$

Note that (A42) is quadratic and concave in G_0, and has two real roots: $G_{00} = \frac{2\,H\,\alpha_0}{3\,\alpha_1}$, $G_{01} = \frac{2\,\alpha_0(2\alpha_1(1-c_n) - H)}{3\,\alpha_1}$. We determine that $E[\pi\,(\alpha_A,\,p_{n0}^A)] \ge E[\pi\,(\alpha_A,\,p_{H1})]$, since $G_{00} \le \frac{2\,H}{3} \le \frac{2\,H_0}{3} \le G_{01}$. Again, solutions $(\alpha_0,\,p_{H0})$ and $(t,\,p_{H0})$ are dominated by $(\alpha_0,\,p_{n0}^{B*})$, refer to the proof in region B_1. So we conclude $(\alpha_0,\,p_{n0}^{B*})$ is the best solution.

Proof of region B_5. In region B_5, $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H}{3}$ (because $\overline{w} - \frac{2\,H}{3} = w + k + \frac{H}{3}$), which is equivalent to $\frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le G \le \frac{H}{3}$. Feasible solutions $(\alpha_1,\,p_{H1})$ and $(t,\,p_{H0})$ are satisfied under the cases where $(m \ge 1\,\&\,\alpha_0 \le \alpha \le \alpha_1 \le t)$ and $(m \ge 1\,\&\,\alpha_0 \le \alpha \le t \le \alpha_1)$, resepctively. Solutions $(\alpha_B,\,p_{n0}^B)$ and $(\alpha_0,\,p_{n0}^{B*})$ are satisfied under the case where $m \le 1\,\&\,t \le \alpha_0 \le \alpha \le \alpha_1$. Solutions $(\alpha_1,\,p_{n1}^B)$ and $(\alpha_B,\,p_{H0})$ are satisfied under the case where $m \le 1\,\&\,\alpha_0 \le t \le \alpha \le \alpha_1$.

$$E[\pi\,(\alpha_B,\,p_{n0}^B)] - E[\pi\,(\alpha_B,\,p_{H0})] = \frac{H}{12\,\alpha_0^2}\,(-6\,\alpha_0\,(1 - c_n) + 2\,\sqrt{3\,H}\,\alpha_0\,(1 - c_n)\frac{1}{\sqrt{G}} + 3\,H - \frac{H^2}{G})$$

$$= \frac{H}{12\,\alpha_0^2}\,(-H^2\,\varphi^2 + 2\,\sqrt{3\,H}\,H_0\,\varphi - 6\,H_0 + 3\,H) \qquad (\text{where } \varphi = \tfrac{1}{\sqrt{G}}) \qquad (A43)$$

$$s.t.\,\sqrt{\frac{3}{H}} \le \varphi \le \sqrt{\frac{3}{H}}\,\frac{\alpha_1}{\alpha_0} \qquad (A44)$$

Note that equation $(A43)$ is quadratic and concave in φ, and has two real roots: $\varphi_0 = \sqrt{\frac{3}{H}}$, $\varphi_1 = \frac{\sqrt{3\,H}\,(2\,H_0 - H)}{H^2}$ and $\varphi_0 \le \varphi_1$. $\sqrt{\frac{3}{H}}\,\frac{\alpha_1}{\alpha_0} \le \varphi_1$, because $\sqrt{\frac{3}{H}}\,\frac{\alpha_1}{\alpha_0} \ge \varphi_1$ is equivalent to $\frac{H}{H_0} \ge \frac{2\,\alpha_0}{\alpha_0 + \alpha_1} \ge \frac{\alpha_0}{\alpha_1}$, which contradicts to $\frac{H}{H_0} \le \frac{\alpha_0}{\alpha_1}$. In summary, we conclude that $E[\pi\,(\alpha_B,\,p_{n0}^B)] \ge E[\pi\,(\alpha_B,\,p_{H0})]$.

$$E[\pi\,(\alpha_1,\,p_{n1}^B)] - E[\pi\,(\alpha_B,\,p_{H0})]$$
$$= \frac{H}{12\,\alpha_0^2\,\alpha_1^2}\,(-H^2\alpha_0^2\,\varphi^2 + 2\,\sqrt{3\,H}\,\alpha_0^2\,\alpha_1\,(1 - c_n)\,\varphi + 3\,H\,\alpha_1^2 - 6\,\alpha_0\,\alpha_1^2\,(1 - c_n) \quad (\text{where } \varphi = \tfrac{1}{\sqrt{G}}) \qquad (A45)$$

$$s.t.\,\sqrt{\frac{3}{H}} \le \varphi \le \sqrt{\frac{3}{H}}\,\frac{\alpha_1}{\alpha_0} \qquad (A46)$$

Equation $(A45)$ is quadratic and concave in φ and has two roots: $\varphi_0 = \sqrt{\frac{3}{H}}\,\frac{\alpha_1}{\alpha_0}$, $\varphi_1 = \frac{\sqrt{3\,H}\,(2\,H_0 - H)}{H^2}\,\frac{\alpha_1}{\alpha_0}$ and $\varphi_0 \le \varphi_1$. It is easy to see that $E[\pi\,(\alpha_1,\,p_{n1}^B)] \le E[\pi\,(\alpha_B,\,p_{H0})]$. Note from Table **RC** that solution $(\alpha_B,\,p_{n0}^B)$ is dominated by $(\alpha_0,\,p_{n0}^{B*})$ under the case where $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H}{3}$. In summary, we conclude that $(\alpha_0,\,p_{n0}^{B*})$ is the best strategy.

Proof of region B_6. In region B_6, $w+k+\frac{H^3}{3\,H_0^2} \le ccr \le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$, which is equivalent to $\frac{H^3}{3\,H_0^2} \le G \le \frac{H\,\alpha_0^2}{3\,\alpha_1^2}$. Feasible solutions $(\alpha_1,\ p_{H1})$ and $(t,\ p_{H0})$ are satisfied under the cases where $(m \ge 1\ \&\ \alpha_0 \le \alpha \le \alpha_1 \le t)$ and $(m \ge 1\ \&\ \alpha_0 \le \alpha \le t \le \alpha_1)$, resepctively. Solutions $(\alpha_B,\ p_{n0}^{B})$, $(\alpha_0,\ p_{n0}^{B*})$ and $(\alpha_1,\ p_{n1}^{B})$ are satisfied under the case where $m \le 1\ \&\ t \le \alpha_0 \le \alpha \le \alpha_1$. Solution $(\alpha_1,\ p_{H0})$ is satisfied under the case where $m \le 1\ \&\ \alpha_0 \le t \le \alpha \le \alpha_1$.

$$E\left[\pi\left(\alpha_B,\ p_{n0}^{B}\right)\right]-E\left[\pi\left(\alpha_1,\ p_{n1}^{B}\right)\right]$$

$$=\frac{H^2}{12\,\alpha_0^2\,\alpha_1^2}\left[\left(H\,\alpha_0^2-H\,\alpha_1^2\right)\varphi^2+\left(2\sqrt{3\,H}\,\alpha_1^2\,\alpha_0\,(1-c_n)-2\sqrt{3H}\,\alpha_0^2\,\alpha_1\,(1-c_n)\right)\varphi\right] \quad (A47)$$

$$s.t.\ \sqrt{\frac{3}{H}}\,\frac{\alpha_1}{\alpha_0} \le \varphi \le \sqrt{\frac{3\,H_0^2}{H^3}} \quad \text{(where } \varphi=\tfrac{1}{\sqrt{G}}\text{)} \quad (A48)$$

Equation $(A47)$ is quadratic and concave in φ, and has two roots: $\varphi_0=0$, $\varphi_1=\frac{2\sqrt{3}\ \alpha_0\alpha_1\,(1-c_n)}{(\alpha_0+\alpha_1)\,\sqrt{H^3}}$. Note that $\sqrt{\frac{3\,H_0^2}{H^3}} \le \varphi_1$, so we can determine that $E\left[\pi\left(\alpha_B,\ p_{n0}^{B}\right)\right] \ge E\left[\pi\left(\alpha_1,\ p_{n1}^{B}\right)\right]$. Similar to the proof of B_5, Table **RC** shows that solution $\left(\alpha_B,\ p_{n0}^{B}\right)$ is dominated by $(\alpha_0,\ p_{n0}^{B*})$. In summary, we determine that $(\alpha_0,\ p_{n0}^{B*})$ is the optimal solution in this region.

Proof of region B_7. In region B_7, $w+k+\frac{H^3}{3\,H_1^2} \le ccr \le w+k+\frac{H^3}{3\,H_0^2}$, which is equivalent to $\frac{H^3}{3\,H_1^2} \le G \le \frac{H^3}{3\,H_0^2}$. Feasible solutions $(\alpha_1,\ p_{H1})$ and $(t,\ p_{H0})$ are satisfied under the cases where $(m \ge 1\ \&\ \alpha_0 \le \alpha \le \alpha_1 \le t)$ and $(m \ge 1\ \&\ \alpha_0 \le \alpha \le t \le \alpha_1)$, resepctively. Solutions $(\alpha_B,\ p_{nB}^{B*})$, $(\alpha_0,\ p_{n0}^{B})$ and $(\alpha_1,\ p_{n1}^{B})$ are satisfied under the case where $m \le 1\ \&\ t \le \alpha_0 \le \alpha \le \alpha_1$. Solution $(\alpha_1,\ p_{H0})$ is satisfied under the case where $m \le 1\ \&\ \alpha_0 \le t \le \alpha \le \alpha_1$.

$$E\left[\pi\left(\alpha_B,\ p_{nB}^{B*}\right)\right]-E\left[\pi\left(\alpha_0,\ p_{n0}^{B}\right)\right]=\frac{H^3}{12\,\alpha_0^2}\left(\varphi-\frac{\sqrt{3\,H^3}\,H_0}{H^3}\right)^2 \ge 0.$$

$$E\left[\pi\left(\alpha_B,\ p_{nB}^{B*}\right)\right]-E\left[\pi\left(\alpha_1,\ p_{n1}^{B}\right)\right]=\frac{H^3}{12\,\alpha_1^2}\left(\varphi-\frac{\sqrt{3\,H^3}\,H_1}{H^3}\right)^2 \ge 0 \quad \text{(where } \varphi=\tfrac{1}{\sqrt{G}}\text{)}.$$

Analogously, we can find that solutions $(\alpha_1,\ p_{H1})$ and $(t,\ p_{H0})$ to the case where $(m \ge 1\ \&\ \alpha_0 \le \alpha \le \alpha_1 \le t)$ are dominated by the case where $m \le 1$, solution $(\alpha_1,\ p_{H0})$ to the case where $m \le 1\ \&\ \alpha_0 \le t \le \alpha \le \alpha_1$ is dominated by the case where $m \le 1\ \&\ t \le \alpha_0 \le \alpha \le \alpha_1$. In summary, $(\alpha_B,\ p_{nB}^{B*})$ is the best solution.

Proof of region B_8. In region B_8, $w+k \le ccr \le w+k+\frac{H^3}{3\,H_1^2}$, which is equivalent to $0 \le G \le \frac{H^3}{3\,H_1^2}$. Feasible solutions $(\alpha_1,\ p_{H1})$ and $(t,\ p_{H0})$ are satisfied under the cases where $(m \ge 1\ \&\ \alpha_0 \le \alpha \le \alpha_1 \le t)$ and $(m \ge 1\ \&\ \alpha_0 \le \alpha \le t \le \alpha_1)$, resepctively. Solutions $(\alpha_B,\ p_{n1}^{B})$, $(\alpha_0,\ p_{n0}^{B})$ and $(\alpha_1,\ p_{n1}^{B*})$ are satisfied under the case where $m \le 1\ \&\ t \le \alpha_0 \le \alpha \le \alpha_1$. Solution $(\alpha_1,\ p_{H0})$ is satisfied under the case where $m \le 1\ \&\ \alpha_0 \le t \le \alpha \le \alpha_1$.

$$E\left[\pi\left(\alpha_B,\ p_{n1}^{B}\right)\right]-E\left[\pi\left(\alpha_0,\ p_{n0}^{B}\right)\right]=\frac{H}{12\alpha_0^2\alpha_1^2}\left(H\,\alpha_1^2-H\,\alpha_0^2\right)\varphi^2+2\sqrt{3\,H}\,\alpha_0\,\alpha_1(1-c_n)(\alpha_0-\alpha_1)\,\varphi \quad (A49)$$

$$s.t.\ \varphi \ge \sqrt{\frac{3\,H_1^2}{H^3}} \quad (A50)$$

Equation $(A49)$ is quadratic and convex in φ, and has two real roots: $\varphi_0=0$, $\varphi_1=\frac{2\sqrt{3}\,\alpha_0\,\alpha_1(1-c_n)}{(\alpha_0+\alpha_1)\sqrt{H^3}}$. Since $\sqrt{\frac{3\,H_1^2}{H^3}} \ge \varphi_1$, we conclude that $E\left[\pi\left(\alpha_B,\ p_{n1}^{B}\right)\right] \ge E\left[\pi\left(\alpha_0,\ p_{n0}^{B}\right)\right]$. Solution $(\alpha_1,\ p_{H0})$ to the case where $m \le 1\ \&\ \alpha_0 \le t \le \alpha \le \alpha_1$ is dominated by the case where $m \leq 1$ & $t \leq \alpha_0 \leq \alpha \leq \alpha_1$. Similarly, solution $(\alpha_B,\ p_{n1}^{B})$ is dominated by $(\alpha_1,\ p_{n1}^{B*})$ for the case where $w + k \leq ccr \leq w + k + \frac{H^3}{3\,H_1^2}$ $(\frac{H^3}{3\,H_1^2} \leq \frac{H\,\alpha_0^2}{3\,\alpha_1^2}$ in column B). In summary, $(\alpha_1,\ p_{n1}^{B*})$ is the best strategy.

Proof of region B_9. In region B_9, $ccr \leq w + k$, which is equivalent to $G \leq 0$. Feasible solutions $(\alpha_1,\ p_{H1})$ and $(t,\ p_{H0})$ are satisfied under the cases where $(m \geq 1$ & $\alpha_0 \leq \alpha \leq \alpha_1 \leq t)$ and $(m \geq 1$ & $\alpha_0 \leq \alpha \leq t \leq \alpha_1)$, respectively. Solution $(\alpha_1,\ p_{n1}^{B*})$ is satisfied under the case where $m \leq 1$ & $G \leq 0$. Note that solutions $(\alpha_1,\ p_{H1})$ and $(t,\ p_{H0})$ to the cases where $(m \geq 1$ & $\alpha_0 \leq \alpha \leq \alpha_1 \leq t)$ and $(m \geq 1$ & $\alpha_0 \leq \alpha \leq t \leq \alpha_1)$ are dominated by the case where $m \leq 1$, and we determine that $(\alpha_1,\ p_{n1}^{B*})$ is the optimal solution in region B_9.

For the rest columns ranging from C to G, proofs can be derived using similar methods. Note that we need condition $D_n \geq 0$ to hold, i.e., $G_0 \geq 2(c_n - 1)/\alpha_1$, which can be transformed as $ccr \leq \overline{ccr} = \overline{w} - 2(c_n - 1)/\alpha_1$. In addition, the condition $0 \leq \overline{w} - \frac{2H}{3} - \frac{2(H-H_1)}{\alpha_1^2} \leq \overline{w} - \frac{2(c_n-1)}{\alpha_1}$ also needs to hold, which can be transformed as $\frac{H\alpha_1(3+\alpha_0^2)+3\alpha_0^2-3\alpha_0\alpha_1}{3\alpha_0^2-3\alpha_0\alpha_1} \leq c_n \leq \frac{(u+2k+3w)\alpha_1^2+6(k-u+\alpha_1)}{6\alpha_1}$. The lower bound of new production cost is $c_n^{min} = max\left(0,\ \frac{H\alpha_1(3+\alpha_0^2)+3\alpha_0^2-3\alpha_0\alpha_1}{3\alpha_0^2-3\alpha_0\alpha_1}\right)$, and we have the upper bound of new production cost, $c_n^{max} = \frac{(u+2k+3w)\alpha_1^2+6(k-u+\alpha_1)}{6\alpha_1}$. After summarizing the analysis discussed above, we derive six regions ranging from $R1$ to $R6$, which are shown in Figure 3. Meanwhile, we can derive the optimal quantities and prices of the remanufactured product in each region.

In region $R1$, for the case where $m \geq 1$, the OEM's optimal remanufacturing quantity is

$$E(X) = \int_0^1 \alpha^{R*}\,D_n\,r\,dr = \frac{1}{2}\,\alpha^{R*}\,D_n = \frac{1}{2}\,\alpha^{R*}\,(1 - p_n^{R*}) = \frac{1}{2}\,\alpha_0\,(1 - p_{n0}^{A*}) = \frac{1}{2}\,\alpha_0\,\left(1 - \frac{4\,\alpha_0^2 - 3\,G_0\,\alpha_0 + 6\,(1+c_n)}{4\,(\alpha_0^2+3)}\right) = \frac{3\,G_0\,\alpha_0^2 + 6\,\alpha_0\,(1-c_n)}{8\,(\alpha_0^2+3)}.$$

Correspondingly, the OEM's optimal remanufacturing price is

$$E(p_r) = \int_0^1 (u - \alpha^{R*}\,D_n\,r\,)\,dr = u - E(X) = u - \frac{3\,G_0\,\alpha_0^2 + 6\,\alpha_0\,(1-c_n)}{8\,(\alpha_0^2+3)}.$$

In region $R2$, for the case where $m \leq 1$, the OEM's optimal remanufacturing quantity is

$$E(X) = \int_0^m \alpha^{R*}\,D_n\,r\,dr + \int_m^1 \alpha^{R*}\,D_n\,r\,dr = \int_0^1 \alpha^{R*}\,D_n\,r\,dr = \frac{1}{2}\,\alpha^{R*}\,D_n = \frac{1}{2}\,\alpha^{R*}\,(1 - p_n^{R*}) = \frac{1}{2}\,\alpha_0\,(1 - p_{n0}^{B*}).$$

Correspondingly, the OEM's optimal remanufacturing price is

$$E(p_r) = \int_0^m (u - \alpha^{R*}\,D_n\,r\,)dr + \int_m^1 \frac{u+k}{2}\,dr = \frac{(u-k)^2}{8\,\alpha_0\,(1-p_{n0}^{B*})} + \frac{u+k}{2}.$$

In region $R3$, for the case where $m \geq 1$,the OEM's optimal remanufacturing quantity is

$$E(X) = \int_0^1 \alpha^{R*}\,D_n\,r\,dr = \frac{1}{2}\,\frac{3\,G_0}{4\,D_n}\,D_n = \frac{3\,G_0}{8}.$$

Correspondingly, the OEM's optimal remanufacturing price is

$$E(p_r) = \int_0^1 (u - \alpha^{R*}\,D_n\,r\,)\,dr = u - E(X) = u - \frac{3\,G_0}{8}.$$

In region $R4$, for the case where $m \leq 1$, the OEM's optimal remanufacturing quantity is

$$E(X)=\int_0^m \alpha^{R*}\,D_n\,r\,dr+\int_m^1 \alpha^{R*}\,D_n\,r\,dr=\int_0^1 \alpha^{R*}\,D_n\,r\,dr=\frac{1}{2}\,\alpha^{R*}\,D_n=\frac{1}{2}\,\frac{H}{2\,D_n}\,\sqrt{\frac{H}{3\,G}}\,D_n=\frac{H}{4}\,\sqrt{\frac{H}{3\,G}}.$$

Correspondingly, the OEM's optimal remanufacturing price is

$$E(p_r)=\int_0^m (u-\alpha^{R*}\,D_n\,r\,)dr+\int_m^1 \frac{u+k}{2}\,dr=\frac{\sqrt{3\,G\,(u-k)}+2u+2k}{4}.$$

In region $R5$, for the case where $m\geq 1$, the OEM's optimal remanufacturing quantity is

$$E(X)=\int_0^1 \,\alpha^{R*}\,D_n\,r\,dr=\frac{1}{2}\,\alpha^{R*}\,D_n=\frac{1}{2}\,\alpha^{R*}\,(1-p_n^{R*})=\frac{1}{2}\,\alpha_1\,(1-p_{n1}^{A*})=\frac{3\,G_0\,\alpha_1^2+6\,\alpha_1\,(1-c_n)}{8\,(\alpha_1^2+3)}.$$

Correspondingly, the OEM's optimal remanufacturing price is

$$E(p_r)=\int_0^1 (u-\alpha^{R*}\,D_n\,r\,)\,dr=u-E(X)=u-\,\frac{3\,G_0\,\alpha_1^2+6\,\alpha_1\,(1-c_n)}{8\,(\alpha_1^2+3)}.$$

In region $R6$, for the case where $m\leq 1$, the OEM's optimal remanufacturing quantity is

$$E(X)=\int_0^m \alpha^{R*}\,D_n\,r\,dr+\int_m^1 \alpha^{R*}\,D_n\,r\,dr=\int_0^1 \alpha^{R*}\,D_n\,r\,dr=\frac{1}{2}\,\alpha^{R*}\,D_n=\frac{1}{2}\,\alpha^{R*}\,(1-p_n^{R*})=\frac{1}{2}\,\alpha_1\,(1-p_{n1}^{B*}).$$

Correspondingly, the OEM's optimal remanufacturing price is

$$E(p_r)=\int_0^m (u-\alpha^{R*}\,D_n\,r\,)dr+\int_m^1 \frac{u+k}{2}\,dr=\frac{(u-k)^2}{8\,\alpha_1\,(1-p_{n1}^{B*})}+\frac{u+k}{2}.$$

Proof of Proposition 5. We only give the case where $1-\frac{H}{\alpha_1}\leq c_n\leq 1$. For the case where $0\leq c_n\leq 1-\frac{H}{\alpha_1}$, which can be proved using similar methods. The common regions between with and without the take-back regulation are illustrated in Figure 4 in our main manuscript. Note that we need the condition $D_n\geq 0$ to hold, i.e., $G_0\geq 2(c_n-1)/\alpha_1$, which can be transformed as $ccr\leq\overline{ccr}=\overline{w}-2(c_n-1)/\alpha_1$.

(i) In common region $\mathbf{CR1}$, $\mathbf{\overline{w}\leq ccr\leq\overline{ccr}}$, i.e., $\mathbf{2(c_n-1)/\alpha_1\leq G_0\leq 0}$, the profit difference between the cases with and without the regulation is

Proof of Proposition 5. We only give the case where $1-\frac{H}{\alpha_1}\leq c_n\leq 1$. For the case where $0\leq c_n\leq 1-\frac{H}{\alpha_1}$, which can be proved using similar methods. The common regions between with and without the take-back regulation are illustrated in Figure 4 in our main manuscript. Note that we need the condition $D_n\geq 0$ to hold, i.e., $G_0\geq 2(c_n-1)/\alpha_1$, which can be transformed as $ccr\leq\overline{ccr}=\overline{w}-2(c_n-1)/\alpha_1$.

(i) In common region $CR1$, $\overline{w}\leq ccr\leq\overline{ccr}$, i.e., $2(c_n-1)/\alpha_1\leq G_0\leq 0$, the profit difference between the cases with and without the regulation is

$$E(\pi^{N*})-E(\pi^{R*})=\frac{\alpha_0}{16(\alpha_0^2+3)}(-3\,\alpha_0\,G_0^2-12\,(1-c_n)\,G_0+4\,\alpha_0\,(1-c_n)^2)\qquad\text{(A51)}$$

(A51) is quadratic and concave in G_0, and has two real roots: $G_{00}=\frac{2\left(3+\sqrt{3\,\alpha_0^2+9}\right)(-1+c_n)}{3\,\alpha_0}$, $G_{01}=-\frac{2\left(-3+\sqrt{3\,\alpha_0^2+9}\right)(-1+c_n)}{3\,\alpha_0}$. Also, $G_{00}\leq 2(c_n-1)/\alpha_1\leq 0\leq G_{01}$. Therefore, we can determine that $E\,(\pi^{N*})>E(\pi^{R*})$.

(ii) In common region $CR2$, $\overline{w}-\frac{2\,H_0}{3}\leq ccr\leq\overline{w}$, i.e., $0\leq G_0\leq\frac{2\,H_0}{3}$. The profit difference between the cases with and without the regulation is

$$E(\pi^{N*})-E(\pi^{R*})=\frac{1}{16(\alpha_0^2+3)}(9G_0^2-12H_0G_0+4H_0^2)=\frac{1}{16(\alpha_0^2+3)}(3G_0-2H_0)^2\geq 0$$

The profit without the regulation is no less than that with regulation.

(iii) In common regions $CR3$, $CR4$ and $CR5$, profits with and without the regulation are the same. i.e., $E(\pi^{N*})=E(\pi^{R*})$.

In summary, we conclude that the profit without the regulation is higher than or equal to that with the regulation. For the case where $0 \le c_n \le 1 - \frac{H}{\alpha_1}$, it can be proved in a similar way.

Table *RA* Solutions to the case where $m \ge 1 \ \& \ \alpha_0 \le \alpha \le \alpha_1 \le t \ \& \ G_0 \ge 0$

α^{A*}	p_n^{A*}	*Conditions*
$\alpha_0 \ (\alpha_A \le \alpha_0)$ $p_{H1} \le p_n \le p_{n0}^A$ $\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \le ccr \le \overline{w}$	p_{H1} $(p_{n0}^{A*} \le p_{H1})$	$\frac{H}{H_1} \le \eta \ \& \ \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \le ccr \le \overline{w}$ $\eta \le \frac{H}{H_1} \le 1 \ \& \ \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \le ccr \le \overline{w} - \frac{6\,(H - H_1) + 2\,H\,\alpha_0^2}{3\ \alpha_0\,\alpha_1}$
	p_{n0}^{A*} $(p_{H1} \le p_{n0}^{A*} \le p_{n0}^A)$	$\eta \le \frac{H}{H_1} \le 1 \ \& \ \overline{w} - \frac{6\,(H - H_1) + 2\,H\,\alpha_0^2}{3\ \alpha_0\,\alpha_1} \le ccr \le \overline{w}$ $\frac{H}{H_1} \ge 1 \ \& \ \overline{w} - \frac{2\,H_0}{3} \le ccr \le \overline{w}$
	p_{n0}^A $(p_{n0}^{A*} \ge p_{n0}^A)$	$\frac{H}{H_1} \ge 1 \ \& \ \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \le ccr \le \overline{w} - \frac{2\,H_0}{3}$
$\alpha_A \ (\alpha_0 \le \alpha_A \le \alpha_1)$ $p_{n0}^A \le p_n \le p_{n1}^A$ $\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \le ccr \le \overline{w}$	p_{n0}^A $(p_{nA}^{A*} \le p_{n0}^A)$	$\frac{H}{H_1} \le 1 \ \& \ \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \le ccr \le \overline{w}$ $\frac{H}{H_1} \ge 1 \ \& \ \overline{w} - \frac{2H_0}{3} \le ccr \le \overline{w}$
	p_{nA}^{A*} $(p_{n0}^A \le p_{nA}^{A*} \le p_{n1}^A)$	$1 \le \frac{H}{H_1} \le \frac{\alpha_1}{\alpha_0} \ \& \ \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \le ccr \le \overline{w} - \frac{2H_0}{3}$ $\frac{H}{H_1} \ge \frac{\alpha_1}{\alpha_0} \ \& \ \overline{w} - \frac{2H_1}{3} \le ccr \le \overline{w} - \frac{2H_0}{3}$
	p_{n1}^A $(p_{nA}^{A*} \ge p_{n1}^A)$	$\frac{H}{H_1} \ge \frac{\alpha_1}{\alpha_0} \ \& \ \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \le ccr \le \overline{w} - \frac{2H_1}{3}$
$\alpha_A \ (\alpha_0 \le \alpha_A \le \alpha_1)$ $p_{H1} \le p_n \le p_{n1}^A$ $\overline{w} - \frac{2H}{3} \le ccr \le \overline{w} - \frac{2H\alpha_0}{3\alpha_1}$	p_{H1} $(p_{nA}^{A*} \le p_{H1})$	$\frac{H}{H_1} \le 1 \ \& \ \overline{w} - \frac{2\,H}{3} \le ccr \le \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1}$
	p_{nA}^{A*} $(p_{H1} \le p_{nA}^{A*} \le p_{n1}^A)$	$1 \le \frac{H}{H_1} \le \frac{\alpha_1}{\alpha_0} \ \& \ \overline{w} - \frac{2H_1}{3} \le ccr \le \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1}$
	p_{n1}^A $(p_{nA}^{A*} \ge p_{n1}^A)$	$1 \le \frac{H}{H_1} \le \frac{\alpha_1}{\alpha_0} \ \& \ \overline{w} - \frac{2H}{3} \le ccr \le \overline{w} - \frac{2H_1}{3}$ $\frac{H}{H_1} \ge \frac{\alpha_1}{\alpha_0} \ \& \ \overline{w} - \frac{2H}{3} \le ccr \le \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1}$
$\alpha_1 \ (\alpha_A \ge \alpha_1)$ $p_n \ge p_{H1}$ $ccr \le \overline{w} - \frac{2H}{3}$	p_{n1}^{A*} $(p_{n1}^{A*} \ge p_{H1})$	$\frac{H}{H_1} \ge 1 \ \& \ \overline{w} - \frac{6\,(H - H_1) + 2\,H\,\alpha_1^2}{3\ \alpha_1^2} \le ccr \le \ \overline{w} - \frac{2\,H}{3}$
	p_{H1} $(p_{n1}^{A*} \le p_{H1})$	$\frac{H}{H_1} \ge 1 \ \& \ ccr \le \overline{w} - \frac{6\,(H - H_1) + 2\,H\,\alpha_1^2}{3\ \alpha_1^2}$ $\frac{H}{H_1} \le 1 \ \& \ ccr \le \overline{w} - \frac{2\,H}{3}$
$\alpha_1 \ (\alpha_A \ge \alpha_1)$ $p_n \ge p_{n1}^A$ $\overline{w} - \frac{2H}{3} \le ccr \le \overline{w}$	p_{n1}^{A*} $(p_{n1}^{A*} \ge p_{n1}^A)$	$\frac{H}{H_1} \ge 1 \ \& \ \overline{w} - \frac{2\,H}{3} \le ccr \le \overline{w} - \frac{2\,H_1}{3}$
	p_{n1}^A $(p_{n1}^{A*} \le p_{n1}^A)$	$\frac{H}{H_1} \ge 1 \ \& \ \overline{w} - \frac{2\,H_1}{3} \le ccr \le \overline{w}$ $\frac{H}{H_1} \le 1 \ \& \ \overline{w} - \frac{2\,H}{3} \le ccr \le \overline{w}$

Table RB Solutions to the case where $m \geq 1$ & $\alpha_0 \leq \alpha \leq t \leq \alpha_1$ & $G_0 \geq 0$

α^{A*}	p_n^{A*}	*Conditions*
$\alpha_0\ (\alpha_A \leq \alpha_0)$ $p_{H0} \leq p_n \leq p_{n0}^A$ $\overline{w} - \frac{2H}{3} \leq ccr \leq \overline{w} - \frac{2H\alpha_0}{3\alpha_1}$	p_{H0} $(p_{n0}^{A*} \leq p_{H0})$	$\frac{H}{H_0} \leq \theta$ & $\overline{w} - \frac{2\,H}{3} \leq ccr \leq \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1}$ $\theta \leq \frac{H}{H_0} \leq 1$ & $\overline{w} - \frac{2\,H}{3} \leq ccr \leq \overline{w} - \frac{6\,(H - H_0) + 2\,H\,\alpha_0^2}{3\,\alpha_0^2}$
	p_{n0}^{A*} $(p_{H0} \leq p_{n0}^{A*} \leq p_{n0}^A)$	$\theta \leq \frac{H}{H_0} \leq 1$ & $\overline{w} - \frac{6\,(H - H_0) + 2\,H\,\alpha_0^2}{3\,\alpha_0^2} \leq ccr \leq \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1}$ $H_0 \leq H \leq H_1$ & $\overline{w} - \frac{2\,H_0}{3} \leq ccr \leq \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1}$
	p_{n0}^A $(p_{n0}^{A*} \geq p_{n0}^A)$	$\frac{H}{H_1} \geq 1$ & $\overline{w} - \frac{2\,H}{3} \leq ccr \leq \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1}$ $H_0 \leq H \leq H_1$ & $\overline{w} - \frac{2\,H}{3} \leq ccr \leq \overline{w} - \frac{2\,H_0}{3}$
$\alpha_0\ (\alpha_A \leq \alpha_0)$ $p_{H0} \leq p_n \leq p_{H1}$ $\overline{w} - \frac{2H\alpha_0}{3\alpha_1} \leq ccr \leq \overline{w}$	p_{H0} $(p_{n0}^{A*} \leq p_{H0})$	$\frac{H}{H_0} \leq \eta$ & $\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w}$ $\eta \leq \frac{H}{H_0} \leq \theta$ & $\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w} - \frac{6\,(H - H_0) + 2\,H\,\alpha_0^2}{3\,\alpha_0^2}$
	p_{n0}^{A*} $(p_{H0} \leq p_{n0}^{A*} \leq p_{H1})$	$\eta \leq \frac{H}{H_0} \leq \theta$ & $\overline{w} - \frac{6\,(H - H_0) + 2\,H\,\alpha_0^2}{3\,\alpha_0^2} \leq ccr \leq \overline{w}$ $\theta \leq \frac{H}{H_0} \leq \frac{\eta\,H_1}{H_0}$ & $\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w}$ $\frac{\eta\,H_1}{H_0} \leq \frac{H}{H_0} \leq \frac{H_1}{H_0}$ & $\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w} - \frac{6\,(H - H_1) + 2\,H\,\alpha_0^2}{3\,\alpha_0\,\alpha_1}$
	p_{H1} $(p_{n0}^{A*} \geq p_{H1})$	$\eta \leq \frac{H}{H_1} \leq 1$ & $\overline{w} - \frac{6\,(H - H_1) + 2\,H\,\alpha_0^2}{3\,\alpha_0\,\alpha_1} \leq ccr \leq \overline{w}$ $\frac{H}{H_1} \geq 1$ & $\overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1} \leq ccr \leq \overline{w}$
$\alpha_A\ (\alpha_0 \leq \alpha_A \leq t)$ $p_{n0}^A \leq p_n \leq p_{H1}$ $\overline{w} - \frac{2H}{3} \leq ccr \leq \overline{w} - \frac{2H\alpha_0}{3\alpha_1}$	p_{n0}^A $(p_{nA}^{A*} \leq p_{n0}^A)$	$H_0 \leq H \leq H_1$ & $\overline{w} - \frac{2\,H_0}{3} \leq ccr \leq \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1}$ $H \leq H_0$ & $\overline{w} - \frac{2H}{3} \leq ccr \leq \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1}$
	p_{nA}^{A*} $(p_{n0}^A \leq p_{nA}^{A*} \leq p_{H1})$	$H_0 \leq H \leq H_1$ & $\overline{w} - \frac{2H}{3} \leq ccr \leq \overline{w} - \frac{2\,H_0}{3}$
	p_{H1} $(p_{nA}^{A*} \geq p_{H1})$	$H \geq H_1$ & $\overline{w} - \frac{2H}{3} \leq ccr \leq \overline{w} - \frac{2\,H\,\alpha_0}{3\,\alpha_1}$
$t\ (\alpha_A \geq t)$ $p_{H0} \leq p_n \leq p_{H1}$ $ccr \leq \overline{w} - \frac{2H}{3}$	p_{H0} $(p_{nt}^{A*} \leq p_{H0})$	$H \leq H_0$ & $ccr \leq \overline{w} - \frac{2\,H}{3}$
	p_{nt}^{A*} $(p_{H0} \leq p_{nt}^{A*} \leq p_{H1})$	$H_0 \leq H \leq H_1$ & $ccr \leq \overline{w} - \frac{2\,H}{3}$
	p_{H1} $(p_{nt}^{A*} \geq p_{H1})$	$H \geq H_1$ & $ccr \leq \overline{w} - \frac{2\,H}{3}$

Table RC Solutions to the case where $m \le 1 \,\&\, t \le \alpha_0 \le \alpha \le \alpha_1 \,\&\, G \ge 0$

α^{B*}	p_n^{B*}	*Conditions*
$\alpha_0\ (\alpha_B \le \alpha_0)$ $p_n \le p_{n0}^B$ $w+k \le ccr \le w+k+\frac{H}{3}$	p_{n0}^{B*} $(p_{n0}^{B*} \le p_{n0}^B)$	$\frac{H}{H_0} \le 1 \,\&\, w+k+\frac{H^3}{3\,H_0^2} \le ccr \le w+k+\frac{H}{3}$
	p_{n0}^B $(p_{n0}^{B*} \ge p_{n0}^B)$	$\frac{H}{H_0} \le 1 \,\&\, w+k \le ccr \le w+k+\frac{H^3}{3\,H_0^2}$ $\frac{H}{H_0} \ge 1 \,\&\, w+k \le ccr \le w+k+\frac{H}{3}$
$\alpha_0\ (\alpha_B \le \alpha_0)$ $p_n \le p_{H0}$ $ccr \ge w+k+\frac{H}{3}$	p_{n0}^{B*} $(p_{n0}^{B*} \le p_{H0})$	$\frac{H}{H_0} \le 1 \,\&\, w+k+\frac{H}{3} \le ccr \le w+k+\frac{H}{3}-\frac{2\,(H-H_0)}{\alpha_0^2}$
	p_{H0} $(p_{n0}^{B*} \ge p_{H0})$	$\frac{H}{H_0} \le 1 \,\&\, ccr \ge w+k+\frac{H}{3}-\frac{2\,(H-H_0)}{\alpha_0^2}$ $\frac{H}{H_0} \ge 1 \,\&\, ccr \ge w+k+\frac{H}{3}$
$\alpha_B\ (\alpha_0 \le \alpha_B \le \alpha_1)$ $p_{n0}^B \le p_n \le p_{n1}^B$ $w+k \le ccr \le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$	p_{n0}^B $(p_{nB}^{B*} \le p_{n0}^B)$	$\frac{H}{H_0} \le \frac{\alpha_0}{\alpha_1} \,\&\, w+k+\frac{H^3}{3\,H_0^2} \le ccr \le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$
	p_{nB}^{B*} $(p_{n0}^B \le p_{nB}^{B*} \le p_{n1}^B)$	$\frac{H}{H_0} \le \frac{\alpha_0}{\alpha_1} \,\&\, w+k+\frac{H^3}{3\,H_1^2} \le ccr \le w+k+\frac{H^3}{3\,H_0^2}$ $\frac{\alpha_0}{\alpha_1} \le \frac{H}{H_0} \le 1 \,\&\, w+k+\frac{H^3}{3\,H_1^2} \le ccr \le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$
	p_{n1}^B $(p_{nB}^{B*} \ge p_{n1}^B)$	$\frac{H}{H_0} \le 1 \,\&\, w+k \le ccr \le w+k+\frac{H^3}{3\,H_1^2}$ $\frac{H}{H_0} \ge 1 \,\&\, w+k \le ccr \le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$
$\alpha_B\ (\alpha_0 \le \alpha_B \le \alpha_1)$ $p_{n0}^B \le p_n \le p_{H0}$ $w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w+k+\frac{H}{3}$	p_{n0}^B $(p_{nB}^{B*} \le p_{n0}^B)$	$\frac{H}{H_0} \le \frac{\alpha_0}{\alpha_1} \,\&\, w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w+k+\frac{H}{3}$ $\frac{\alpha_0}{\alpha_1} \le \frac{H}{H_0} \le 1 \,\&\, w+k+\frac{H^3}{3\,H_0^2} \le ccr \le w+k+\frac{H}{3}$
	p_{nB}^{B*} $(p_{n0}^B \le p_{nB}^{B*} \le p_{H0})$	$\frac{\alpha_0}{\alpha_1} \le \frac{H}{H_0} \le 1 \,\&\, w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w+k+\frac{H^3}{3\,H_0^2}$
	p_{H0} $(p_{nB}^{B*} \ge p_{H0})$	$\frac{H}{H_0} \ge 1 \,\&\, w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w+k+\frac{H}{3}$
$\alpha_1\ (\alpha_B \ge \alpha_1)$ $p_{n1}^B \le p_n \le p_{H0}$ $w+k \le ccr \le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$	p_{n1}^B $(p_{n1}^{B*} \le p_{n1}^B)$	$\frac{H}{H_0} \le 1 \,\&\, w+k+\frac{H^3}{3\,H_1^2} \le ccr \le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$
	p_{n1}^{B*} $(p_{n1}^B \le p_{n1}^{B*} \le p_{H0})$	$\frac{H}{H_0} \le 1 \,\&\, w+k \le ccr \le w+k+\frac{H^3}{3\,H_1^2}$ $1 \le \frac{H}{H_0} \le \gamma_0 \,\&\, w+k \le ccr \le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}-\frac{2\,(H-H_0)}{\alpha_0\,\alpha_1}$
	p_{H0} $(p_{n1}^{B*} \ge p_{H0})$	$1 \le \frac{H}{H_0} \le \gamma_0 \,\&\, w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}-\frac{2\,(H-H_0)}{\alpha_0\,\alpha_1} \le ccr \le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$ $\frac{H}{H_0} \ge \gamma_0 \,\&\, w+k \le ccr \le w+k+\frac{H\,\alpha_0^2}{3\,\alpha_1^2}$

Table RD Solutions to the case where $m \le 1$ & $\alpha_0 \le t \le \alpha \le \alpha_1$ & $G \ge 0$

α^{B*}	p_n^{B*}	*Conditions*
$t\ (\alpha_B \le t)$ $p_{H0} \le p_n \le p_{H1}$ $ccr \ge w + k + \frac{H}{3}$	p_{H0} $(p_{nt}^{B*} \le p_{H0})$	$H \le H_0$ & $ccr \ge w + k + \frac{H}{3}$
	p_{nt}^{B*} $(p_{H0} \le p_{nt}^{B*} \le p_{H1})$	$H_0 \le H \le H_1$ & $ccr \ge w + k + \frac{H}{3}$
	p_{H1} $(p_{nt}^{B*} \ge p_{H1})$	$H \ge H_1$ & $ccr \ge w + k + \frac{H}{3}$
$\alpha_B\ (t \le \alpha_B \le \alpha_1)$ $p_{H0} \le p_n \le p_{n1}^B$ $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H}{3}$	p_{H0} $(p_{nB}^{B*} \le p_{H0})$	$H \le H_0$ & $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H}{3}$
	p_{nB}^{B*} $(p_{H0} \le p_{nB}^{B*} \le p_{n1}^B)$	$H_0 \le H \le H_1$ & $w + k + \frac{H^3}{3\,H_1^2} \le ccr \le w + k + \frac{H}{3}$
	p_{n1}^B $(p_{nB}^{B*} \ge p_{n1}^B)$	$H_0 \le H \le H_1$ & $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H^3}{3\,H_1^2}$ $H \ge H_1$ & $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H}{3}$
$\alpha_1\ (\alpha_B \ge \alpha_1)$ $p_{H0} \le p_n \le p_{H1}$ $w + k \le ccr \le w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2}$	p_{H0} $(p_{n1}^{B*} \le p_{H0})$	$H \le H_0$ & $w + k \le ccr \le w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2}$ $1 \le \frac{H}{H_0} \le \gamma_0$ & $w + k \le ccr \le w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} - \frac{2\,(H - H_0)}{\alpha_0\,\alpha_1}$
	p_{n1}^{B*} $(p_{H0} \le p_{n1}^{B*} \le p_{H1})$	$1 \le \frac{H}{H_0} \le \gamma_0$ & $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} - \frac{2\,(H - H_0)}{\alpha_0\,\alpha_1} \le ccr \le w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2}$ $\gamma_0 H_0 \le H \le \gamma\,H_1$ & $w + k \le ccr \le w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2}$ $\gamma \le \frac{H}{H_1} \le \gamma_1$ & $w + k \le ccr \le w + k + \frac{H}{3} - \frac{2\,(H - H_1)}{\alpha_1^2}$
	p_{H1} $(p_{n1}^{B*} \ge p_{H1})$	$\gamma \le \frac{H}{H_1} \le \gamma_1$ & $w + k + \frac{H}{3} - \frac{2\,(H - H_1)}{\alpha_1^2} \le ccr \le w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2}$ $\frac{H}{H_1} \ge \gamma_1$ & $w + k \le ccr \le w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2}$
$\alpha_1\ (\alpha_B \ge \alpha_1)$ $p_{n1}^B \le p_n \le p_{H1}$ $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H}{3}$	p_{n1}^B $(p_{n1}^{B*} \le p_{n1}^B)$	$\frac{H}{H_0} \le 1$ & $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H}{3}$ $H_0 \le H \le H_1$ & $w + k + \frac{H^3}{3\,H_1^2} \le ccr \le w + k + \frac{H}{3}$
	p_{n1}^{B*} $(p_{n1}^B \le p_{n1}^{B*} \le p_{H1})$	$H_0 \le H \le H_1$ & $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H^3}{3\,H_1^2}$ $H_1 \le H \le \gamma\,H_1$ & $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H}{3} - \frac{2\,(H - H_1)}{\alpha_1^2}$
	p_{H1} $(p_{n1}^{B*} \ge p_{H1})$	$H_1 \le H \le \gamma\,H_1$ & $w + k + \frac{H}{3} - \frac{2\,(H - H_1)}{\alpha_1^2} \le ccr \le w + k + \frac{H}{3}$ $H \ge \gamma\,H_1$ & $w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} \le ccr \le w + k + \frac{H}{3}$

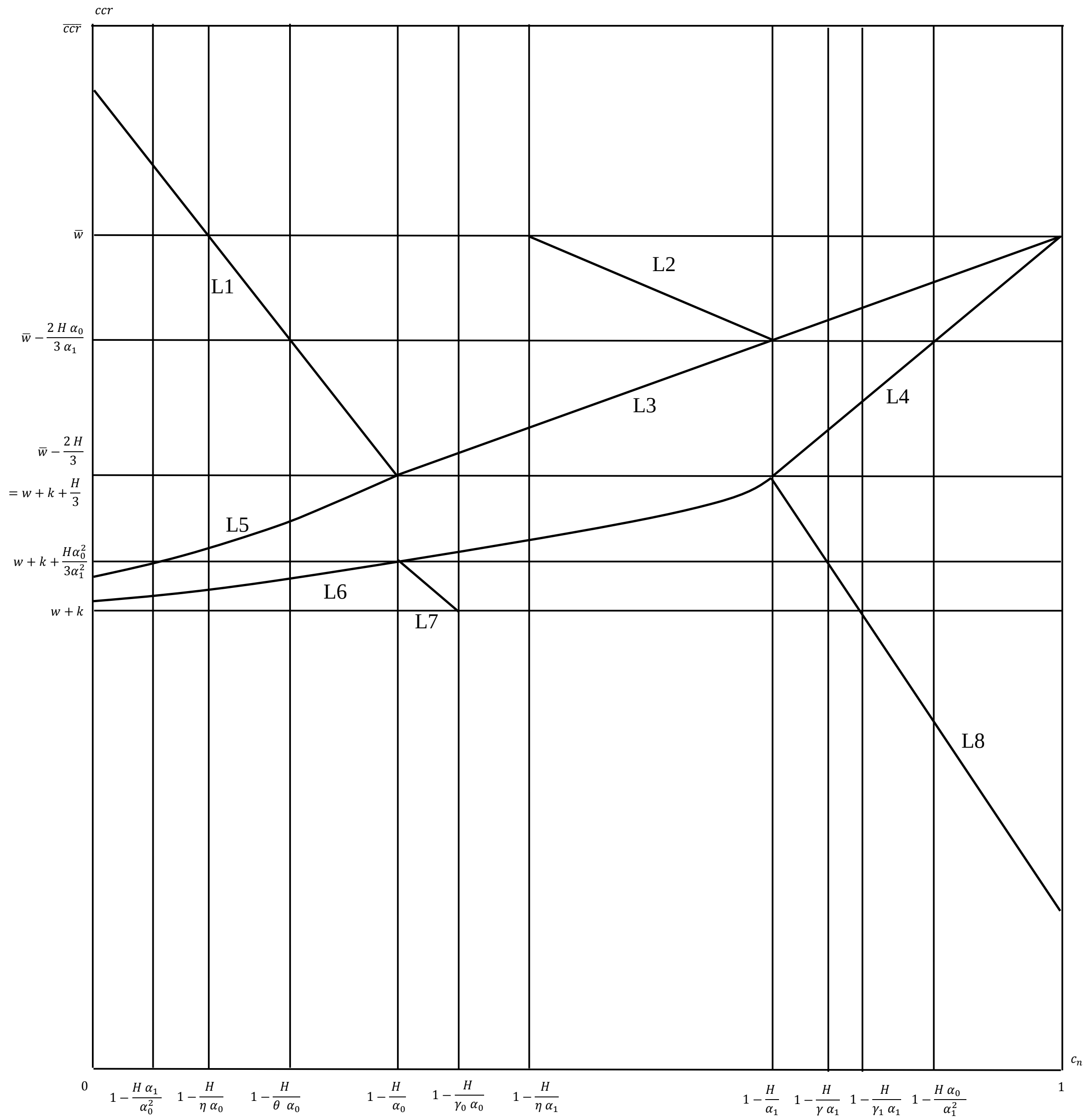
Table RE Solutions to the particular case where $m \geq 1 \,\&\, G_0 \leq 0$

α^{A*}	p_n^{A*}	*Conditions*
$\alpha_0 (G_0 \leq 0)$ $p_n \geq p_{H0}$ $ccr \geq \overline{w}$	p_{n0}^{A*} $(p_{n0}^{A*} \geq p_{H0})$	$\frac{H}{H_0} \leq \eta \,\&\, ccr \geq \overline{w} - \frac{6\,(H - H_0) + 2\,H\,\alpha_0^2}{3\,\alpha_0^2}$ $\frac{H}{H_0} \geq \eta \,\&\, ccr \geq \overline{w}$
	p_{H0} $(p_{n0}^{A*} \leq p_{H0})$	$\frac{H}{H_0} \leq \eta \,\&\, \overline{w} \leq ccr \leq \overline{w} - \frac{6\,(H - H_0) + 2\,H\,\alpha_0^2}{3\,\alpha_0^2}$

Table RF Solutions to the particular case where $m \leq 1 \,\&\, G \leq 0$

α^{B*}	p_n^{B*}	*Conditions*
$\alpha_1 (G \leq 0)$ $p_n \leq p_{H1}$ $ccr \leq w + k$	p_{n1}^{B*} $(p_{n1}^{B*} \leq p_{H1})$	$\frac{H}{H_1} \leq \gamma_1 \,\&\, ccr \leq w + k$ $\frac{H}{H_1} \geq \gamma_1 \,\&\, ccr \leq w + k + \frac{H}{3} - \frac{2\,(H - H_1)}{\alpha_1^2}$
	p_{H1} $(p_{n1}^{B*} \geq p_{H1})$	$\frac{H}{H_1} \geq \gamma_1 \,\&\, w + k + \frac{H}{3} - \frac{2\,(H - H_1)}{\alpha_1^2} \leq ccr \leq w + k$

Figure S. Characterization of all the possible solutions in different regions.



$$\text{L1: } ccr = w + k + \frac{H}{3} - \frac{2\,(H - H_0)}{\alpha_0^2},\ \text{L2: } ccr = \overline{w} - \frac{6\,(H - H_1) + 2\,H\,\alpha_0^2}{3\ \alpha_0\,\alpha_1},\ \text{L3: } ccr = \overline{w} - \frac{2\,H_0}{3}$$

$$\text{L4: } ccr = \overline{w} - \frac{2\,H_1}{3},\ \text{L5: } ccr = w + k + \frac{H^3}{3\,H_0^2},\ \text{L6: } ccr = w + k + \frac{H^3}{3\,H_1^2}$$

$$\text{L7: } ccr = w + k + \frac{H\,\alpha_0^2}{3\,\alpha_1^2} - \frac{2\,(H - H_0)}{\alpha_0\,\alpha_1},\ \text{L8: } ccr = w + k + \frac{H}{3} - \frac{2\,(H - H_1)}{\alpha_1^2}$$