Supplier Encroachment with Multiple Retailers

Baoshan Liu

School of Economics and Management, Center for Energy and Environmental Management and Decision-making (CE2MD), China University of Geosciences, Wuhan 430074, China
baoshanl@163.com

Xu Guan

School of Management, Huazhong University of Science and Technology, Wuhan, 430074 China,
guanxu@hust.edu.cn

Yulan Wang*

Faculty of Business, The Hong Kong Polytechnic University, Kowloon, Hong Kong,
yulan.wang@polyu.edu.hk

In this paper, we investigate the supplier's encroachment incentive when it distributes the product through multiple retailers. We show that the number of enrolled downstream retailers plays a pivotal role in determining the supplier's encroachment incentive and the channel members' performances. There exists a threshold value with respect to the number of downstream retailers, below which the bright side of supplier encroachment documented in the existing literature exists; that is, encroachment can benefit not only the encroaching supplier itself but also the retailers. However, when the number of downstream retailers exceeds this threshold value, the further intensified downstream competition dampens the effect of wholesale price reduction arising from supplier encroachment. Supplier encroachment becomes always detrimental to the retailer. Moreover, with the increasing number of retailers, the supplier may become worse off when being endowed with the option of downstream encroachment, even when the supplier does not actually execute this option. We further investigate the supplier's optimal market penetration strategy when it can enroll a new retailer or open a direct channel, or it is costly to establish the indirect channel. We show that the main results remain qualitatively unchanged when the two selling channels are imperfect substitutes or retailers are asymmetric.

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*Corresponding author.

1 Introduction

Nowadays, many manufacturers set up direct selling channels to complement their incumbent retail networks. Such channel encroachment has been widely acknowledged as an important means to improve a manufacturer's profitability as it endows the manufacturer with more pricing and production flexibility. Furthermore, the advance of information technology and the popularity of e-commerce have made the setup cost of encroachment almost negligible, allowing the manufacturer to encroach into the retail market much more easily than ever before. Nonetheless, it is interesting to see that still many companies insist on the traditional indirect selling (i.e., wholesaling) mode and achieve great success. Procter & Gamble (P&G), the world second largest fast-moving consumer goods company[1], normally sells its products via retail outlets such as supermarkets and convenience stores rather than establish its own direct selling channel. For instance, in Hong Kong, P&G does not own any direct selling channel but rely on retail stores to achieve market penetration. One potential reason that hinders P&G's direct selling is the extremely highly competitive retailing sector in Hong Kong, where retail stores can be find on every street and consumer can buy products easily within a short walking distance.[2] Other examples like Scopely, a game developer founded in 2011, sells its products through indirect sales channels made up of Internet application outlets and has launched six No. 1 ranked games in the app stores. Nerium, a cosmetic company established in 2011, develops anti-aging skin products and relies on the indirect sales channel partners exclusively, which are called independent brand partners, to distribute its products. It has achieved a three-year growth of 16,617% in the industry.[3]

In the above examples, all the upstream manufacturers do not adopt the encroachment strategy even when it is costless to do so. Although a number of existing studies have shown that encroaching into the retail market may make the manufacturer worse off, they are mainly based on a specific information asymmetry structure: either the manufacturer lacks the demand information while the retailer has it (Li et al., 2014, 2015) or the manufacturer holds the private information about its product quality which needs

[1]See https://www.consultancy.uk/news/26874/the-worlds-40-largest-fast-moving-consumer-goods-companies.

[2]For more details, please see "https://www.pghongkong.com/en-us/contact-us " and "https://www.hongkongtripguide.com/hong-kong-convenience-stores.html."

[3]For more details, please see "https://www.logicbay.com/blog/10-companies-with-indirect-sales-channels-that-made-the-inc-500-list-this-year."

to be disclosed to the public (Guan et al., 2020). When there is no information asymmetry, it is well documented that a manufacturer should encroach into the retail market as long as its benefit can cover the entry cost (Chiang et al., 2003; Arya et al., 2007), which, however, is based on a premise that there is only one retailer in its indirect distribution channel. Here, to fill the gap between practice and research, we seek to provide another justification for the manufacturer's abandonment of encroachment without considering information asymmetry: market competition environment, that is, the number of retailers in the manufacturer's indirect distribution channel shall greatly affect the manufacturer's profitability from encroachment and thus its incentive to encroach.

Specifically, we consider that a supplier who sells indirectly via distributing its product through multiple retailers now decides whether or not to encroach into the retail market. In particular, we are interested in the following research questions:

1. How does retailer competition affect the effects of supplier encroachment on the encroaching supplier and its retailers?
2. How can retailers competition change the impact of encroachment on both the supplier's and the retailer's profitability?
3. Regarding the two market penetration strategies, enrolling a new retailer into the incumbent indirect distribution channel and opening a direct channel, which strategy shall the supplier adopt and under what conditions?
4. How the endowment of encroachment option affects the number of retailers that the supplier would sell through?

We show that internal competition among retailers plays a pivotal role in determining the supplier's equilibrium encroachment strategy. When more retailers are enrolled in the supplier's indirect distribution channel, each retailer becomes more aggressive with product ordering, thereby mitigating the double marginalization and leading to a more efficient indirect channel. Consequently, the supplier is less likely to encroach unless its unit direct-selling cost is sufficiently low. However, under such a situation, once the supplier encroaches, it results in a prominent demand reduction from retailers as each retailer's order is unduly diminished given the supplier encroachment. This in turn propels the supplier into reducing its wholesale price to uphold the retailers' ordering incentive.

Differently, when the supplier's unit direct-selling cost falls into an intermediate range, retailers would jointly deter the supplier's encroachment by over-ordering the product. As the number of retailers increases, each retailer just needs to over-order fewer units and then the successful deterrence of encroachment can be achievable. Under such a circumstance, each retailer becomes more tolerable with the wholesale price increment. Accordingly, in equilibrium, the supplier is able to charge a higher wholesale price when more retailers are present in its indirect distribution channel, a result in sharp contrast to that when the unit direct-selling cost is sufficiently low and the supplier indeed encroaches.

Interestingly, we show that when a supplier sells through multiple retailers, it may become worse off when being endowed with the option to encroach into the retail market even when it does not necessarily build up the direct distribution channel. This result, to our best knowledge, has been seldom identified by the existing literature. When a supplier encroaches into the retail market, it faces a tradeoff between the additional gain generated from selling directly to consumers and the potential loss in wholesaling induced by the retailers' diminished demands. However, an increase in the number of downstream retailers undermines the supplier's profits in both direct-selling and wholesaling. As the number of retailers increases, the competition in the retail market is further exacerbated, leading to a profit margin squeeze. This hurts the supplier's direct-selling profit. Regarding the supplier's wholesaling profit, although the encroaching supplier would reduce its wholesale price to uphold the retailers' ordering incentives, the effect of such wholesale price reduction is significantly weakened by the increasing number of retailers. Note that if only one retailer exists in the indirect channel, the retailer's reduced ordering incentive due to supplier encroachment can be effectively mitigated by the wholesale price reduction. In this situation, the supplier's profit gain brought by direct selling surpasses its loss incurred in wholesaling induced by encroachment, and thus the supplier benefits from being endowed with the option of encroachment. Nonetheless, when there are more than three retailers engaged in reselling and the direct selling cost is relatively high, both the profit gain in direct selling and the effect of wholesale price reduction on boosting the retailers' ordering incentives diminish. Having the option of encroachment is thus no longer beneficial. Under such a circumstance, the supplier should forgo the option of encroachment and commit to wholesaling only.

Moreover, we show that the bright side of encroachment for the retailer well documented in the existing literature (Arya et al., 2007) no longer exists in our game context when the number of reselling retailers exceeds a threshold value. In particular, the likelihood that a retailer can benefit from supplier encroachment decreases with the increasing number of retailers. When there are more than four retailers in the supplier's indirect distribution channel, a retailer can never benefit from supplier encroachment even though it can still enjoy a reduced wholesale price offered by the supplier. This result complements Arya et al. (2007) by identifying a boundary of the bright side of encroachment, whose underlying reason can be explained as follows. When there is only one retailer in the supplier's indirect distribution channel, the benefit of wholesale price reduction can be fully absorbed by this retailer. However, when more retailers are enrolled in reselling the supplier's product, intensified internal competition among retailers induces them to order more from the supplier, leading to a higher total order quantity but a lower profit margin. In this sense, the internal competition among the retailers actually mitigates the positive effect from wholesale price reduction. Consequently, when the number of reselling retailers is large enough, the benefit that a retailer can enjoy from wholesale price reduction is significantly reduced and can no loner compensate the downside brought to the retailer from supplier encroachment. The bright side of supplier encroachment then vanishes.

We further identify the supplier's optimal market penetration strategy with channel expansion, wherein the supplier can either enroll a new retailer or open a direct channel in addition to the incumbent indirect distribution channel. We show that the supplier prefers setting up a direct selling channel under one of the following two conditions. One, the cost of direct selling is sufficiently low and thus the supplier is highly capable of direct selling. Two, the cost of direct selling is medium-high such that the supplier can use encroachment to induce the retailers' deterrence behavior. Under this circumstance, encroachment does not actually materialize but the supplier can utilize this threat to induce the retailers to order more products and thus achieve market penetration. We also consider the scenario in which it is costly for the supplier to establish the indirect distribution channel. We show that when the supplier can determine upfront how many retailers to enroll, in equilibrium a retailer's profit remains unchanged regardless of whether the supplier actually encroaches or not. Moreover, our main results remain qualitatively unchanged if the substitution between the two selling channels is imperfect, retailers hold different market sizes, or retailers make their ordering decisions sequentially.

The rest of this paper is organized as follows. Section 2 reviews the relevant literature.

In Section 3, we lay out the model setup. The firms' equilibrium strategies and payoffs are analyzed in Section 4. We discuss some extensions in Section 5. Concluding remarks are presented in Section 6. All of the proofs are relegated to the online Appendix A.

2 Literature Review

Supplier encroachment has been investigated in a vast literature. A key result in this stream of research is that supplier encroachment can hurt the retailers' profit as the additional channel can lower retailers' sales effort (Fein and Anderson, 1997), attract away a part of consumers (Alba et al., 1997), affect the brand image (Frazier and Lassar, 1996), transfer more inventory risk (Chen et al., 2008) and more quality improvement cost (Ha et al., 2016) to retailers and so forth. Some studies have shown that the supplier can adopt some cooperation strategies along with the encroachment option to mitigate the channel conflict and improve the retailers' profits. Based on the horizontal differentiation of consumers' heterogeneous channel preferences, Cattani et al. (2006) find that the supplier's commitment on offering the same retail price as the retailer can benefit all the supply chain members. Liu and Zhang (2006) study the firms' personalized pricing strategy under the encroachment context, and find that the retailer becomes worse off and the manufacturer becomes better off when the manufacturer has the encroachment option. Wu et al. (2015) show that the online manufacturer can mitigate channel conflict by referring online consumers to its indirect channel. Other studies that incorporate different market conditions or operations strategies find that supplier encroachment can conditionally improve the retailers' profits (Tsay and Agrawal, 2004; Cai, 2010; Li et al., 2014; Huang et al., 2018; Arya and Mittendorf, 2013; Gao et al., 2021). For example, Tsay and Agrawal (2004) demonstrate that the retailer can benefit from the positive externality of the supplier's exerting sales effort in its direct channel. By formulating the direct channel as a threat to the retailer, Chiang et al. (2003) have shown that the option of encroachment itself (with no sales occurring in the direct channel) can induce the supplier to set a lower wholesale price and the retailer can conditionally benefit from such proactive wholesale price reduction. Gao et al. (2021) show that the upstream manufacturer holding the private direct-selling cost information will cut the wholesale price to signal her cost, which benefits the retailer. Our work is most closely related to Arya et al. (2007) who use a quantity competition model to investigate the supplier's encroachment decision when facing a downstream retailer. They show that the retailer can be better off from supplier encroachment when the supplier's direct channel exhibits an intermediate efficiency. Based on their work, we incorporate multi-retailer competition into supplier encroachment. We show that when the number of downstream retailers surpasses four, the retailers can no longer benefit from supplier encroachment due to the intensified downstream competition.

The aforementioned studies all show that the endowment with the voluntary option of encroachment can always benefit the supplier in a weak sense. We note that the following studies find that when there exists information asymmetry, the supplier is not necessarily better off with the option of encroachment. Li et al. (2014) and Li et al. (2015) consider supplier encroachment when the retailer holds the private information. They demonstrate that the supplier can be worse off by encroaching into the retail market. Their studies reveal that the threat of supplier encroachment induces the retailer to distort the demand information to deter encroachment by the supplier, which in turn impairs the supply chain efficiency and hurts the profitability of channel members. Guan et al. (2020) show that when the upstream supplier has private information, its encroachment can hurt itself as well. Here, in this study, by assuming information symmetry among the channel members, we uncover a new driver for the *dark side* of supplier encroachment in terms of encroachment worsening the supplier performance. That is, we show that the magnitude of downstream retailer competition can hugely affect the encroaching supplier's profit. When the number of retailers in its indirect distribution channel is large enough, a supplier may become worse off when being endowed with a voluntary option to encroach into the retail market.

Our article also contributes to the literature on multi-retailer competition, which has studied the manufacturer's channel coordination strategies (Ingene and Parry, 1995; Padmanabhan and Png, 1997), the effect of exogenous product differentiation (Choi, 1996; Harutyunyan and Jiang, 2019), endogenous quality decision (Banker et al., 1998) and so forth. Besides, Padmanabhan and Png (1997) shows that more intense competition between retailers can decrease *double marginalization*, benefiting the manufacturer and hurting the retailers. However,in general situation, Tyagi (1999) indicates the important role of *elasticity of slope of inverse consumer demand function* which determines whether the intense competition reduces *double marginalization* or not. Differently, we study the retailers' competition incorporating with the supplier encroachment. More importantly, we show that the supplier's encroachment option can reduce the alleviated double marginalization derived from the downstream retailers' competition. David and Adida (2015) also investigate the mutual effect of encroachment and multi-retailer competition. In their study, the supplier can commit the total product supply. It ensures that the supplier can credibly abandon the direct channel if it occurs damage, i.e., the free encroachment option will never hurt the supplier. In contrast, we assume the supplier can easily input product to the market that the supplier has no way to credibly commit to refrain from revising his own order quantity after receiving the retailers' orders (Arya et al., 2007; Li et al., 2014). And we derive that the supplier can be conditionally worse off by holding the encroachment option.

3 Model Setting

Consider a supply chain that consists of one supplier (he, denoted by s) and multiple independent retailers (she, denoted by $r_i, i = 1, 2, \cdots, n$). The supplier is currently selling his product to the end market through n retailers at a wholesale price w. Meanwhile, the supplier also has an option to encroach into the retail market and sell directly to consumers. For example, the supplier can sell the product via the self-managed online store or the flag shop, which serves as a substitute for the traditional indirect sales channel. Without loss of generality, the supplier's marginal production cost is normalized to be zero.

The retailers are symmetric and the inverse demand function is given by $p = a - Q$, where $a > 0$ is the market potential, Q is the total quantity of the product in the market and p is the market clearing price. When the supplier sells exclusively via the retailers, $Q = \sum_{i=1}^{n} q_{r_i}$, in which q_{r_i} is the order/selling quantity from retailer r_i, $i = 1, 2, \cdots, n$. Otherwise, if the supplier sets up the direct sales channel, then $Q = \sum_{i=1}^{n} q_{r_i} + q_s$, in which q_s is the supplier's direct selling quantity. The supplier incurs a unit selling cost c when selling directly to consumers via his own direct channel, where $c \in (0, a)$. We normalize the retailer's unit selling cost to be zero to reflect her relative cost effectiveness and higher selling ability compared to the supplier, which is also consistent with the business practice. For example, retailers might take advantage of their direct contact with customers and economies of scope from other retailing activities. Moreover, one can easily show that in a setting where the supplier and retailers incur the same selling cost, the supplier should sell only through his own direct distribution channel, which is trivial and uninteresting. Such assumptions of quantity-setting Cournot competition, linear inverse demand, and nonzero direct selling cost have been commonly used in the existing literature on encroachment; see, e.g., Arya et al. (2007), Li et al. (2015) and Guan et al. (2020).

The decision sequences are defined as follows. First, the supplier decides his unit wholesale price w. Next, the retailers determine their respective profit-maximizing order quantity q_{r_i}, $i = 1, 2, \cdots, n$ simultaneously. After observing the retailers' orders, the supplier decides his direct-selling quantity q_s if the supplier encroaches into the retail market. Last, the retail market is cleared according to the inverse demand function, and firms collect their respective profits. Here, we assume that the supplier determines his direct-selling quantity after the retailers' order decisions. The underlying reason is that the supplier cannot credibly commit not to revise his own selling quantity after observing the retailers' orders (Arya et al., 2007; Li et al., 2014).

We assume that the supplier and retailers are all risk neutral and aim to maximize their respective profits. Since the game contains multiple rounds of strategic interactions, backward induction is applied to ensure subgame perfection.

4 Analysis

In this section, we consider two scenarios based on whether or not the supplier has the capability to encroach into the retail market. We first derive the equilibrium outcome associated with no encroachment option. We then derive the equilibrium outcome associated with the existence of encroachment option. We then compare these equilibrium outcomes to derive the supplier's encroachment preference and the impact of supplier encroachment on the channel members' performances.

4.1 No Supplier Encroachment

Here, we first investigate a benchmark scenario wherein the supplier is unable to set up a direct-selling channel and does not encroach. For ease of exposition, we use the superscript "n" to denote the equilibrium outcome associated with this scenario. Under this scenario, the profit functions of the supplier and each retailer r_i, $i = 1, 2, \cdots, n$ can be easily written as

$$\pi_s = w\sum_{j=1}^{n} q_{r_j} \text{ and } \pi_{r_i} = (a - \sum_{j=1}^{n} q_{r_j}^n - w)q_{r_i}^n.$$

With backward induction, we first derive the retailer r_i's best-response order decision given the wholesale price w. It can be easily shown that

$$q_{r_i}(w) = \frac{a-w}{(n+1)}, i = 1, 2, \cdots, n.$$

Anticipating the retailers' optimal order decision, the supplier decides the wholesale price w to maximize his own profit. We can show that the optimal wholesale price

$$w^n = \frac{a}{2}.$$

The corresponding retailer order quantity and the resulting equilibrium profits of the supplier and retailers are respectively,

$$q_{r_i}^n = \frac{a}{2(n+1)}, \ \pi_{r_i}^n = \frac{a^2}{4(n+1)^2} \text{ and } \pi_s^n = \frac{na^2}{4(n+1)}.$$

We then examine how the number of retailers in the indirect channel n affects the system performance and have the following result.

Proposition 1. *When the supplier cannot encroach into the retail market, in equilibrium,*

(1). the optimal wholesale price w^n is independent of n, the number of downstream retailers;

(2). each retailer's order quantity $q_{r_i}^n$. $i = 1, 2, \cdots, n$, decreases in n while the total demand of all retailers $\sum_{j=1}^{n} q_{r_j}^n$ increases in n;

(3). the supplier's profit π_s^n increases in n while that of a retailer $\pi_{r_i}^n$ decreases in n. The total supply chain profit $\pi_c^n = \pi_s^n + \sum_{j=1}^{n} \pi_{r_j}^n$ increases in n.

Proposition 1 shows that when the supplier delegates the product selling solely to the retailers, his optimal wholesale price remains constant no matter how many retailers are enrolled in his indirect distribution channel. As the number of retailers increases, each retailer's order quantity becomes smaller due to intensified downstream competition and limited market potential a. This hurts each retailer's profitability. However, the total demand from all the retailers becomes larger. That is, the increasing number of downstream retailers generates a strategic *competition effect* among them, which helps mitigate the double marginalization effect in the decentralized indirect channel and results in a higher total order quantity. This makes both the supplier and the whole supply chain better off. Note that when the number of downstream retailers becomes extremely large (i.e., $n \to \infty$), the whole supply chain can be fully coordinated from the competition effect. In such a situation, the supplier extracts all the surplus ($\pi_s^n = \frac{a^2}{4}$) while each retailer's profit is squeezed to zero ($\pi_{r_i}^n = 0$). Similar results have also been observed in the literature (Ingene and Parry, 1995; Padmanabhan and Png, 1997). Proposition 1 implies that when direct selling is difficult to establish or impossible, a supplier should enroll more retailers into his indirect distribution channel.

4.2 Potential Supplier Encroachment

We now consider the scenario in which the supplier is capable of encroaching into the retail market and can voluntarily decide whether or not to encroach (if doing so is profitable). For ease of exposition, we use the superscript "en" to denote the equilibrium outcome associated with this scenario.

Under such a circumstance, given the retailers' order decisions, the supplier decides his direct selling quantity q_s to maximize his profit

$$\pi_s = w \sum_{j=1}^{n} q_{r_j} + (a - \sum_{j=1}^{n} q_{r_j} - q_s - c) q_s. \tag{1}$$

Solving the first-order condition yields the following optimal quantity decision:

$$q_s(q_{r_i}) = \left(\frac{a - \sum_{j=1}^{n} q_{r_j} - c}{2} \right)^{+}. \tag{2}$$

Given the wholesale price w and anticipating the supplier's encroachment quantity $q_s(q_{r_i})$, each retailer r_i, $i = 1, 2, \cdots, n$ decides the order quantity q_{r_i} to maximize her profit

$$\pi_{r_i} = \left(a - \sum_{j=1}^{n} q_{r_j} - q_s(q_{r_i}) - w \right) q_{r_i},$$

which is concave. We can then derive the retailers' optimal order decisions based on the first-order conditions and the subsequent direct selling quantity of the supplier. It can be easily derived that

$$(q_{r_i}(w), q_s(w)) = \begin{cases} \left(\frac{a+c-2w}{(n+1)}, \frac{a-(2n+1)c+2nw}{2(n+1)} \right) & \text{if } w > \frac{(2n+1)c-a}{2n}; \\ \left(\frac{a-c}{n}, 0 \right) & \text{if } \frac{(n+1)c-a}{n} < w \leq \frac{(2n+1)c-a}{2n}; \\ \left(\frac{a-w}{(n+1)}, 0 \right) & \text{if } w \leq \frac{(n+1)c-a}{n}. \end{cases} \tag{3}$$

As the retailers are symmetric, their order quantities are the same. Equation (3) implies that when the wholesale price w is sufficiently low ($w \leq \frac{(n+1)c-a}{n}$), the retailers' total order quantity will be sufficiently high, thereby subsequently squeezing the supplier out of the retail market. Under such a situation, the supplier has no incentive to encroach and the retailer's optimal order quantity equals exactly that stated in §4.1 when there is no encroachment. When the wholesale price falls into an intermediate range $\left(\frac{(n+1)c-a}{n}, \frac{(2n+1)c-a}{2n}\right]$, it is optimal for the retailers to jointly order a total quantity that leads to zero profit margin for the supplier if he wants to encroach and sell directly. In other words, under this situation, the market clearing price equals the supplier's direct selling cost c, i.e., $p = a - nq_{r_i} = c$. Given the same wholesale price w, each retailer now orders more than that stated in §4.1 when there is no threat of supplier encroachment to deter the supplier's possible encroachment. Last, when the wholesale price is sufficiently high, it impairs the retailers' order incentives and the total demand from all the retailers is limited. Thus, the supplier also sells directly.

Anticipating the retailer's ordering behavior stated in (3), the supplier then decides the wholesale price w to maximize his profit. Plugging (3) into (1) yields

$$\pi_s(w) = \begin{cases} nw\frac{a+c-2w}{(n+1)} + \left(\frac{a-(2n+1)c+2nw}{2(n+1)}\right)^2, & \text{if } w > \frac{(2n+1)c-a}{2n}; \\ w(a-c), & \text{if } \frac{(n+1)c-a}{n} < w \leq \frac{(2n+1)c-a}{2n}; \\ nw\frac{a-w}{(n+1)}, & \text{if } w \leq \frac{(n+1)c-a}{n}. \end{cases}$$

The following proposition summarizes the equilibrium wholesale price, ordering quantities and the profits of the supplier and retailers.

Proposition 2. *When the supplier has the option to encroach into the retail market, the supplier encroaches only when his unit direct-selling cost $c \leq \underline{c}$ and never encroaches when $c > \bar{c}$, where*

$$\bar{c} = \frac{2(n+1)^2 + n\sqrt{2(n+1)}}{2(n+1)\,(2n+1)}a \text{ and } \underline{c} = \frac{n+2}{2+3n}a.$$

When $c \in (\underline{c}, \bar{c}]$, the retailers face the threat of encroachment from the supplier and order so that the market clearing price equals c. The corresponding equilibrium wholesale price, order quantities and profits are listed in Table 1.

For ease of reference, we name the three encroachment outcomes stated in Proposition 2 as "Encroachment Occurrence (Denoted as EO)", "No Encroachment (Denoted as NE)" and "Threat of Encroachment (Denoted as TE)", respectively. Proposition 2 indicates that

Table 1: Equilibrium Wholesale Price, Quantities and Profits: Potential Encroachment

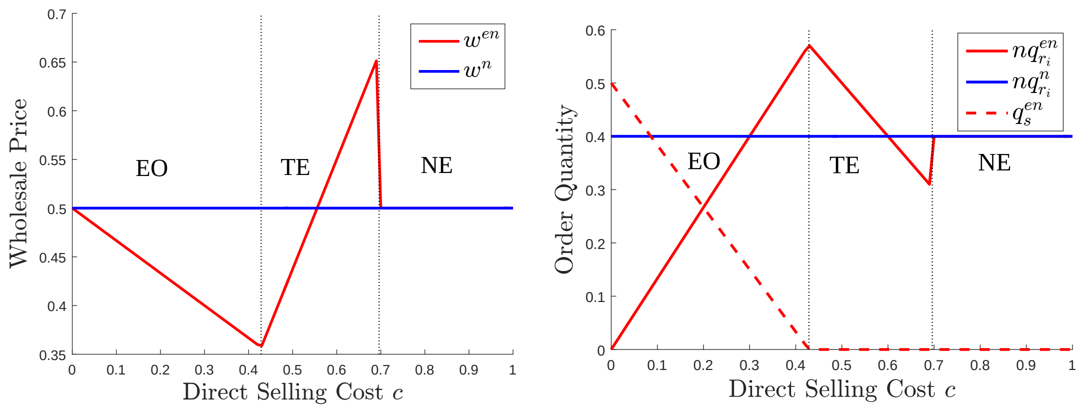
	Encroachment Occurrence (EO) $c \leq \underline{c}$	Threat of Encroachment (TE) $\underline{c} < c \leq \bar{c}$	No Encroachment (NE) $c > \bar{c}$
w^{en}	$\frac{(n+2)a-nc}{2(n+2)}$	$\frac{(2n+1)c-a}{2n}$	$\frac{a}{2}$
$q_{r_i}^{en}$	$\frac{2c}{(n+2)}$	$\frac{a-c}{n}$	$\frac{a}{2(n+1)}$
q_s^{en}	$\frac{(n+2)a-(2+3n)c}{2(n+2)}$	0	0
$\pi_{r_i}^{en}$	$\frac{2c^2}{(n+2)^2}$	$\frac{(a-c)^2}{2n^2}$	$\frac{a^2}{4(n+1)^2}$
π_s^{en}	$\frac{nc((n+2)a-nc)}{(n+2)^2} + \frac{((n+2)a-(2+3n)c)^2}{4(n+2)^2}$	$\frac{((2n+1)c-a)(a-c)}{2n}$	$\frac{na^2}{4(n+1)}$

the supplier's equilibrium encroachment decisions greatly hinges upon the magnitude of his unit direct selling cost c, that is, his direct selling efficiency. Notably, as c increase, the option of encroaching into the retail market to direct sell becomes less efficient and attractive to the supplier. There exists an upper threshold direct selling cost $\bar{c}$, above which the supplier never executes the encroachment option as direct channel is too costly. The threat of supplier encroachment thus becomes incredible. The retailers would order as if the encroachment were not an option for the supplier, and thus the equilibrium outcomes remain the same as that stated in §4.1. There also exists a lower threshold direct selling cost $\underline{c}$, below which the supplier always encroaches into the retail market and direct sells. Under such a circumstance, one can verify that the optimal wholesale price w^{en} monotonically decreases with the direct selling cost c and is always lower that w^n, the one under no supplier encroachment; see Figure 1(a) for the illustration. This implies that to ensure his encroachment would not unduly reduce the retailers' order incentives, the encroaching supplier has to cut down his wholesale price. When the direct selling cost is medium-low ($c \in \left(\frac{(n+2)a}{4(n+1)}, \underline{c}\right)$), such wholesale price reduction can actually stimulate the retailers to order more in comparison to that with no supplier encroachment as the competition between the direct and indirect distribution channels is not very intensified. This phenomenon is called the "bright side of encroachment" and was first identified by Arya et al. (2007).

When the unit direct selling cost falls between the two thresholds, the supplier has the intermediate direct selling efficiency and can potentially benefit from selling directly. However, it is now in the best interest of retailers to deter the supplier's encroachment by over-ordering so that it is no longer profitable for the supplier to enter the retail market. That is, there exists the "threat of encroachment" but such threat does not materialize.

Under this circumstance, the retailers' order quantities satisfy $\sum_{j=1}^{n} q_{r_j} = a - c$, leading to a market clearance price equal the supplier's unit direct-selling cost c. It can be verified that the retailer now orders more than what she should under no supplier encroachment when the wholesale price $w^{en} = \frac{(2n+1)c-a}{2n}$ as depicted in Figure 1(b), that is,

$$q_{r_i}^{en} = \frac{a-c}{n} > q_{r_i}(w^{en}) = \frac{a-w}{(n+1)}\Big|_{w=\frac{(2n+1)c-a}{2n}}.$$



(a) (b)

Figure 1: Equilibrium Wholesale Prices and Quantities: $n = 4, a = 1$

In other words, the deterrence of downstream encroachment is actually achieved via the retailer's over-ordering. Interestingly, the supplier's wholesale price w^{en} now increases in his direct selling cost c. The intuition behind is that when direct selling becomes more costly and less efficient for the supplier, it is easier for the retailers to deter the supplier's encroachment. The supplier then can increase the wholesale price without dampening the retailers' deterrence incentive. Moreover, the supplier can even charge a wholesale price w^{en} that is higher than the one when no encroachment occurs when c becomes large enough; see Figure 1(a) for the illustration.

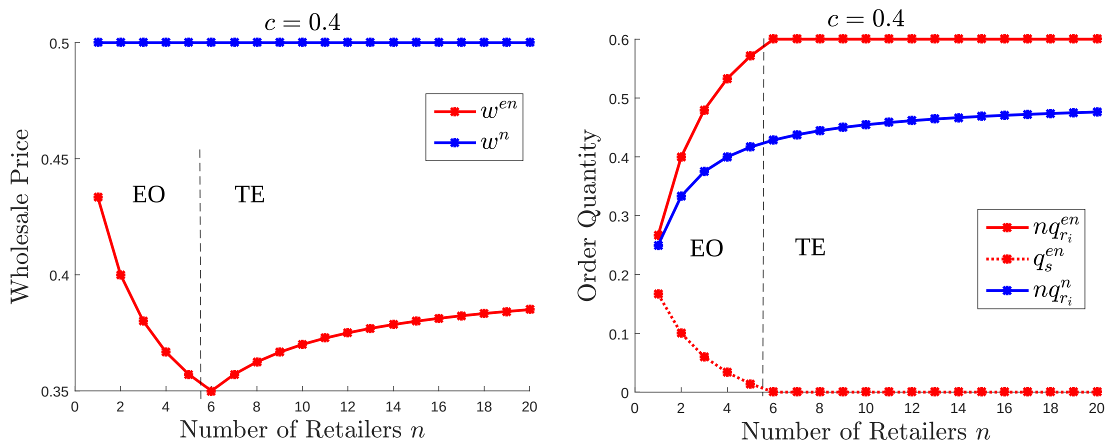
We now examine how the competition intensity among the retailers, i.e., the number of enrolled retailers n, affects the system performance and obtain the following result.

Corollary 1. *When the supplier has the option to encroach into the retail market, in equilibrium*

(1). the two direct-selling cost thresholds $\underline{c}$ and $\bar{c}$ both decrease in n;

(2). the optimal wholesale price w^{en} decreases in n under encroachment occurrence while increases in n under the threat of encroachment.

(3). Each retailer's order quantity $q_{r_i}^{en}$ decreases in n. Nonetheless, the total order quantity $\Sigma_{j=1}^{n} q_{r_j}^{en}$ remains constant (i.e., $\Sigma_{j=1}^{n} q_{r_j}^{en} = a - c$) under the threat of encroachment, and increases in n under encroachment occurrence.



(a) (b)

Figure 2: Impact of n on equilibrium wholesale price and order quantities: $a = 1$

Corollary 1 shows that the two threshold direct selling costs both decrease in the number of retailers. This implies that with more retailers in the indirect channel, it becomes less likely for the supplier to encroach into the retail market to direct sell due to the fiercer downstream market competition. As discussed in §4.1, the increasing number of retailers generates a strategic *competition effect* that mitigates the double marginalization and increases the system efficiency of the indirect distribution channel. Accordingly, direct selling becomes less attractive unless the supplier is very cost-effective with this selling option, that is, with a sufficiently low c.

Corollary 1 further shows that the wholesale price exhibits distinctive relationship with the number of retailers, depending on which encroachment outcome shows up in equilibrium. Specifically, under encroachment occurrence, the supplier decreases his wholesale price as more retailers are engaged in the indirect channel; see Figure 2(a). The reason is that when the number of retailers increases, the competition among retailers becomes fiercer and makes them more sensitive to the wholesale price change. Supplier encroachment will further intensify the downstream competition. As the total demand from retailers decreases with the wholesale price at a rate $2n/(n+1)$, which is higher than the corresponding rate $n/(n+1)$ under no encroachment, as shown in (3). Moreover, the difference between these two rates $[2n/(n+1) - n/(n+1)]$ increases in n. This implies that the total order quantity in the indirect channel is much more sensitive to the wholesale price under encroachment occurrence. To ensure that the orders from retailers are not unduly diminished, the supplier has to further reduce his wholesale price with the increasing number of retailers.

In contrast, under the threat of encroachment, the supplier is able to charge a higher wholesale price in face of more retailers in the indirect channel. Note that here the supplier has the potential to encroach but this option does not materialize because the retailers jointly deter the supplier's potential encroachment via over-ordering. Under this situation, the retailers' total order quantity remains a constant that makes the market clearing price equal c. Thus, when more retailers are enrolled, each retailer can over-order less. Anticipating the retailer's order behavior, the supplier can raise the wholesale price but meanwhile maintain the retailers' deterrence incentive as their number increases.

4.3 Impact of Supplier Encroachment

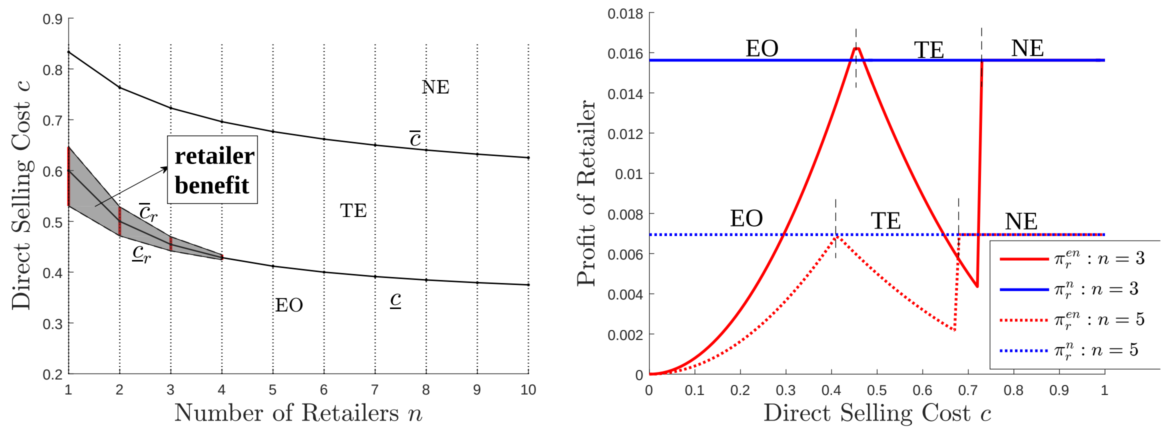
After deriving the equilibrium outcomes associated with no supplier encroachment and those associated with the potential supplier encroachment, we now examine how the option of encroachment affects the profits of the supplier and retailers. It is widely acknowledged (Chiang et al., 2003; Arya et al., 2007) that a supplier should benefit from having the option of encroachment and a retailer can conditionally benefit from the supplier's encroachment when there is no information asymmetry. Below, we will demonstrate that those results may no longer hold, depending on the number of retailers.

Proposition 3. *When the supplier can encroach into the retail market, in equilibrium,*

(1). if the number of retailers $n > 4$, a retailer is always worse off with supplier encroachment.

(2). if $n \leq 4$, a retailer can benefit from supplier encroachment when the direct-selling cost $c \in [\underline{c}_r, \overline{c}_r]$, where $\underline{c}_r = \frac{\sqrt{2}(n+2)a}{4(n+1)}$ and $\overline{c}_r = \frac{2(n+1)-\sqrt{2n}}{2(n+1)}a$.

The bright side of encroachment was first introduced in the classic paper of Arya et al. (2007), in which the supplier has to cut down his wholesale price when he encroaches, and the positive effect of wholesale price reduction can surpass the downside of encroachment on intensifying the downstream competition when direct selling is relatively costly, leading to a higher profit for the retailer. Nonetheless, here, in Proposition 3, we demonstrate that such a bright side still exists only when the number of retailers in the indirect



(a) (b)

Figure 3: Impact of potential supplier encroachment on the retailer's profit: $a = 1$

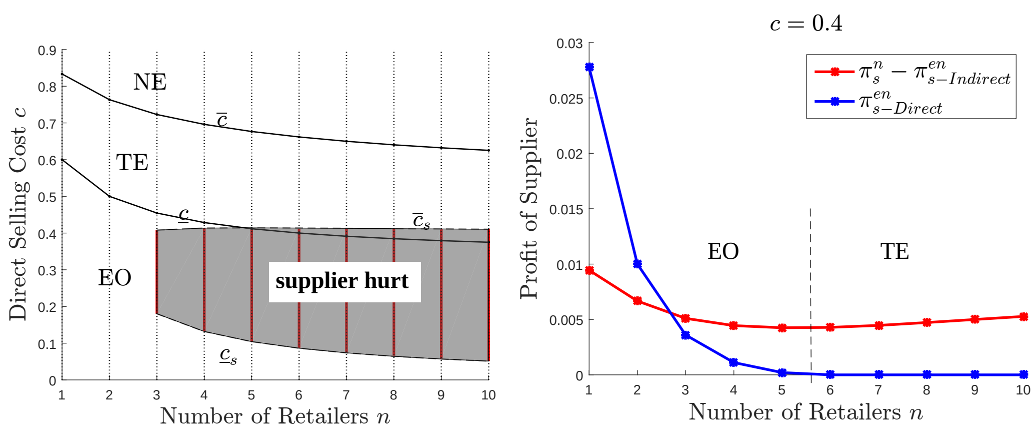
distribution channel n is no more than four under which the inter-competition among retailers is not too fierce. Otherwise, encroachment always hurts the retailer. As depicted in Figure 3 (a), when the cost of direct selling falls into an intermediate region, the bright side of encroachment for the retailer(s) appears. However, such a region shrinks with the increasing number of retailers and no longer exists when $n \geq 4$.

The intuition is that when there is only one retailer in the indirect channel (Arya et al., 2007), the benefit from wholesale price reduction can be fully absorbed by this retailer. However, when there are more retailers in the indirect channel, internal competition induces the retailers to order more, leading to a higher total order quantity but a lower profit margin; that is, $\partial(p^{en} - w^{en})/\partial n < 0$. In this sense, the positive impact of wholesale price reduction on the retailers' side is weakened by their internal competition and diluted by the number of retailers. This is confirmed by Figure 3(b), which depicts the retailer's profits with and without the potential supplier encroachment. It shows that the potential supplier encroachment makes the retailer worse off when the number of retailers is large enough. Under such a circumstance, the benefit from wholesale price reduction cannot surpass the potential loss brought by the retailer over-ordering induced by the supplier encroachment. This result complements Arya et al. (2007) by identifying a boundary of the bright side of encroachment, i.e., the number of existing retailers in the indirect channel. Note that the threshold retailer number $n = 4$ is obtained under the assumption that retailers are symmetric and make their ordering decisions simultaneously. This result can be applied to more general settings such as imperfect substitution between direct and indirect channels, asymmetric retailers and sequential ordering, under which the bright side of encroachment vanishes when the number of retailers exceeds a certain threshold; see the related discussion in §5.

Proposition 4. *When the supplier can encroach into the retail market, in equilibrium,*

(1). if the number of retailers $n < 3$, the supplier always benefits from having the option to encroach into the retail market.

(2). if $n \geq 3$, the supplier becomes worse off when being endowed with the option of encroachment if his direct-selling cost $\underline{c}_s \leq c \leq \bar{c}_s$, where $\underline{c}_s = \frac{(n+1)(n+2)-\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a$ and $\bar{c}_s = \min\left\{\frac{(n+1)(n+2)+\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a, \frac{2(n+1)^2-n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a\right\}$.



(a) (b)

Figure 4: Impact of potential supplier encroachment on the supplier's profit: $a = 1$

Interestingly, Proposition 4 shows that with retailers' competition, a supplier may become worse off from the fact that he has the option of encroachment, even when he does not actually execute such option. This result, to our best knowledge, has been seldom observed by the prior literature.[4] Recall that the supplier can extract all the supply chain surplus when the number of retailers n is sufficiently high (see Proposition 1). That is, encroachment may not bring a higher profit to the supplier when n is very large. Proposition 4 demonstrates that the supplier could even be hurt by the endowment of the option to

[4]In a recent paper, Guan et al. (2020) show that the supplier can be worse off by having the option of encroachment due to the change of information transparency from the supplier's disclosure behavior.

encroach when the number of retailers exceeds a certain threshold. In this sense, a monopolistic supplier can be worse off by the more intensified retailer competition, which is contrary to the conventional wisdom that 'an upstream monopolist should benefit from the downstream competition'. Specifically, when the number of retailers is limited ($n < 3$), having the option of encroachment is beneficial to the supplier; otherwise, the supplier could be hurt by being endowed with the option to voluntarily encroach into the retail market. As depicted in Figure 4(a), the supplier becomes worse off with downstream encroachment and direct selling when his direct selling cost is relatively high.[5]

Intuitively, encroachment generates two conflicting effects with regard to the supplier's profits in direct and indirect channels respectively. On one hand, encroachment allows the supplier to extract more surplus from the retail market by selling directly. On the other hand, encroachment intensifies the channel competition and reduces the retailers' ordering incentive in the indirect channel so that it undermines the supplier's profit in the indirect channel. Nonetheless, in anticipation of the potential loss in the indirect channel, the supplier would undercut the wholesale price to incentivize the retailers to order more so that the demand from retailers is not severely reduced. When there are a fewer retailers in the indirect channel, wholesale price reduction can be quite effective in maintaining the retailers' ordering incentive. As a result, the benefit from direct selling can fully cover the loss from indirect selling, and the supplier is better off with encroachment.

However, the increasing number of retailers further exerts two negative effects on the supplier's profits in the respective channels. One, because the retail market becomes more crowded, the benefit from selling directly shrinks. Two, as aforementioned, because the effect of wholesale price reduction is greatly diluted by the intensified competition among retailers, the supplier has no choice but to offer an even lower wholesale price in the indirect channel. This consequently reduces the supplier's profit in the indirect channel when it encroaches. Combining them together, when $n \geq 3$ and the direct selling cost falls into an intermediate range, the loss from indirect selling $\pi_s^n - \pi_{s-Indirect}^{en}$ surpasses the benefit from direct selling $\pi_{s-Direct}^{en}$, as shown in Figure 4(b). Consequently, the supplier becomes worse off with the endowment of the option to encroach. However,

[5] The threshold $n = 3$ for the arising of the darkside of encroachment is obtained under the assumption that retailers are symmetric and make their ordering decisions simultaneously. Similar results can be derived under more general settings when the number of retailers exceeds a certain threshold; see the related discussion in §5.

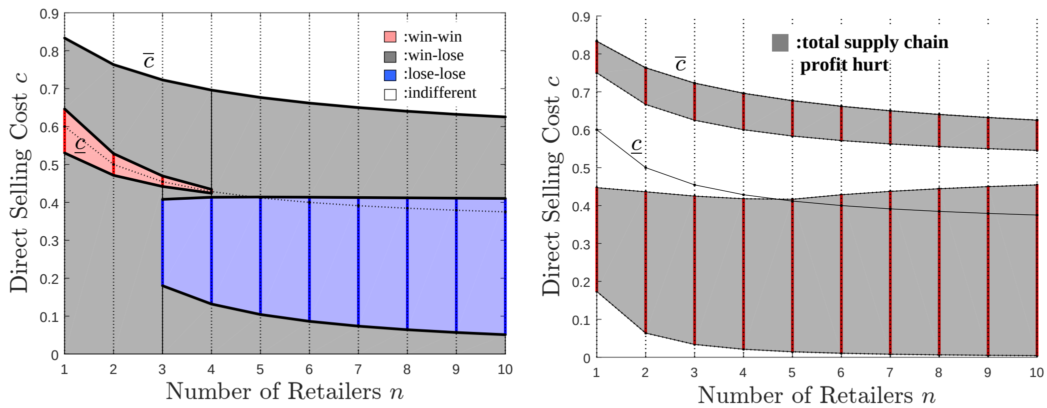
when the direct selling cost is either low or sufficiently high, the supplier benefits from the endowment of encroachment option. The underlying reasons are as follows: with a medium-high direct selling cost, encroachment becomes a credible threat to the retailers (under the threat of encroachment), who must over-order to counteract, whereas an extremely high direct selling cost makes the direct channel so much less competitive that retailers become tolerant towards a high wholesale price. These make the supplier better off.

By Propositions 3 and 4, we can conclude that the potential supplier encroachment can result in a "win-win" outcome for the supplier and the retailer only when the number of retailers is small while the direct selling cost is intermediate and it can even lead to a "lose-lose" outcome when the number of retailers is large enough; see Figure 5(a) for the illustration. Next, we examine how the potential supplier encroachment affects the supply chain performance. Interestingly, we can show that in most situations, the total supply chain profit $\Pi_{sc} = \pi_s + n\pi_{r_i}$ is actually hurt by the supplier's endowment with an option to encroach, as illustrated in Figure 5(b).[6] Again, this is driven by the intriguing interaction between the supplier's potential encroachment and the downstream retailer competition: encroachment might bring a sales volume to the direct channel but it also reduces the sales volume in the indirect channel, whose downside is further amplified by the increase of the number of retailers. When the loss induced by the latter dominates the benefit brought by the former, the total supply chain profit hurts.

5 Discussion

Below, we extend our baseline model to further consider the following two questions: one, when the supplier decides to further penetrate the retail market, which option shall he adopt, enrolling one more retailer into his indirect channel or establishing a direct channel? Two, in the baseline model, we assume that the products sold via the direct and indirect channel are perfect substitutes, the establishment of the indirect channel is

[6] It can be shown that the total supply chain profit can be hurt by the potential supplier encroachment when either $\frac{(n+2)\left((n+2)(n+1)-\sqrt{n(n^3+6n^2+8n-8)}\right)}{(n+1)(5n^2+20n+4)}a < c < \max\left\{\frac{(n+2)\left((n+2)(n+1)+\sqrt{n(n^3+6n^2+8n-8)}\right)}{(n+1)(5n^2+20n+4)}a, \frac{n}{2(n+1)}a\right\}$ or $\frac{n+2}{2(n+1)}a < c < \bar{c}$.



(a) (b)

Figure 5: Impact of potential supplier encroachment on the supply chain performance: $a = 1$

costless, and retailers are symmetric and make their ordering decisions simultaneously. Here, we relax these assumptions to check whether or not our main results still hold.

5.1 Market Penetration via Channel Expansion

Here, we consider a scenario in which the supplier distributes his product via n retailers and is considering further penetrating the retail market by opening another selling channel. Should he set up a direct selling channel or enroll a new retailer into the incumbent indirect distribution channel? In line with the baseline model, if the supplier chooses direct selling, he incurs a direct-selling cost c for each product sold directly. On the contrary, if he chooses to enroll a new retailer, the new retailer does not incur any additional selling cost due to her expertise in retailing operation. As the analysis is quite tedious and standard, we omit the details for the sake of space saving. The following proposition characterizes the supplier's optimal market penetration strategy.

Proposition 5. *When there are n incumbent retailers in his indirect distribution channel, the supplier should encroach into the retail market and direct sell when his direct-selling cost*

$$c \in \left[0, \frac{n+2-\sqrt{n^2-n+2}}{5n+2}a\right] \cup \left[\frac{2(n+1)(n+2)-\sqrt{2n(n-1)(n+2)}}{2(2n+1)(n+2)}a, \frac{2(n+1)(n+2)+\sqrt{2n(n-1)(n+2)}}{2(2n+1)(n+2)}a\right].$$

Otherwise, the supplier should enroll a new retailer into his indirect channel.

Proposition 5 shows that the supplier has the incentive to encroach only when his direct-selling cost is either sufficiently low or medium-high, under which the option of encroachment makes the supplier better off. Otherwise, he would prefer enrolling one

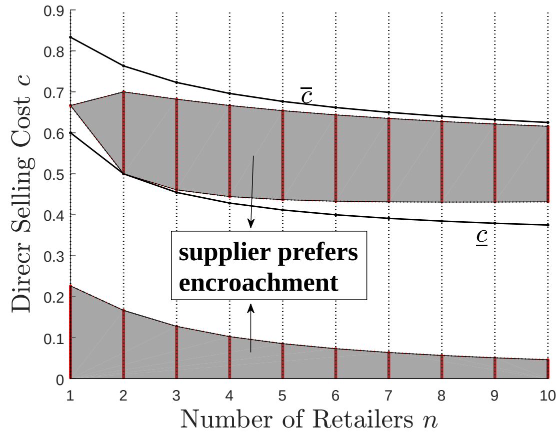


Figure 6: Supplier's preference over penetrating via encroachment

more retailer to achieve market penetration. Moreover, with the increasing numbers of retailers, the supplier is less likely adopting encroachment as the means of market penetration, as depicted in Figure 6. This is consistent with our observations under the baseline model stated in §4.3. Recall from Proposition 1 that increasing the number of retailers can alleviate the double marginalization between the supplier and retailers, thereby benefiting the supplier. However, such a surplus decreases in the number of incumbent retailers. Nonetheless, we have shown in Proposition 4 that downstream encroachment can be a double-edged sword on the supplier, and encroaching into the retail market may be detrimental to the supplier when his direct-selling cost falls into an intermediate range.

Taking together the aforementioned consequences of either option, the supplier prefers adding a direct channel only if it can improve his profit and the surplus surpasses that from enrolling one more retailer. As illustrated in Figures 4(a) and 6, this happens when the direct-selling cost is either sufficiently low or medium-high. Note that in the second condition, encroachment actually does not materialize but the supplier can utilize this threat to induce the incumbent retailers to over-order. By doing so, the supplier also achieves market penetration by taking advantage of the incumbent retailers' strategic deterrence of encroachment.

5.2 Imperfect Product Substitution

In the baseline model, we assume that products sold via different channels are perfect substitutes. Here, we relax that assumption and assume that products sold in direct and indirect distribution channels are imperfect substitutes for the consumers. Accordingly, the inverse demand functions for the respective channels are

$$p_{r_i} = a - \sum_{j=1}^{n} q_{r_j} - \theta q_s, i = 1, 2, \cdots, n \text{ and } p_s = a - q_s - \theta \sum_{j=1}^{n} q_{r_j},$$

where $0 < \theta \leq 1$ is the substitution rate of one channel's product over the other and measures the competition intensity between the direct and indirect channels. That is, a higher θ indicates a higher degree of competition intensity between the two channels. Under this scenario, the channel members' profits are respectively

$$\begin{aligned} \pi_s &= w \sum_{j=1}^{n} q_{r_j} + (a - \theta \sum_{j=1}^{n} q_{r_j} - q_s - c) q_s, \\ \pi_{r_i} &= (a - \sum_{j=1}^{n} q_{r_j} - \theta q_s - w) q_{r_i}, i = 1, 2, \cdots, n. \end{aligned}$$

Solving this problem via backward induction, we then can obtain the following results.

Proposition 6. *When the channels are imperfect substitutes, the equilibrium wholesale price, quantities and profits are as shown in Table 2. Moreover,*

(1). if $n \leq \left[\frac{2(2-\theta^2)+2\sqrt{2(2-\theta^2)}}{\theta^2}\right]$*, the retailer can benefit from supplier encroachment when* $c \in [\underline{c}_{r-\theta}, \overline{c}_{r-\theta}]$*, where* $\overline{c}_{r-\theta} = \frac{\sqrt{2(2-\theta^2)}(n+1)-n\theta}{\sqrt{2(2-\theta^2)}(n+1)} a$ *and* $\underline{c}_{r-\theta} = \frac{4(n+1)-(3n+2)\theta^2-2(1-\theta)\sqrt{2(2-\theta^2)}(n+1)}{2\theta\sqrt{2(2-\theta^2)}(n+1)} a.$

(2). if $n \geq 3$*, the supplier becomes worse off when being endowed with the option of encroachment when* $c \in [\underline{c}_{s-\theta}, \overline{c}_{s-\theta}]$*, where* $\underline{c}_{s-\theta} = \frac{4n^2\theta^2-8n^2\theta+4n^2+n\theta^2-8n\theta+8n-2\theta^2+4}{(2-\theta)^2 n^2+(8-4\theta-\theta^2)n+4-2\theta^2+\theta\sqrt{n(-n^2+n+2)(2\theta^2-4n+3n\theta^2-4)}} a,$ *and* $\overline{c}_{s-\theta} = \min\left(\frac{\left((4-2\theta)n+4-2\theta^2\right)(n+1)-\theta^2 n\sqrt{2(n+1)}}{4n^2-2n\theta^2+8n-2\theta^2+4} a, \frac{4n^2\theta^2-8n^2\theta+4n^2+n\theta^2-8n\theta+8n-2\theta^2+4}{(2-\theta)^2 n^2+(8-4\theta-\theta^2)n+4-2\theta^2-\theta\sqrt{n(-n^2+n+2)(2\theta^2-4n+3n\theta^2-4)}} a\right).$

Table 2: Equilibrium Outcomes under Imperfect Substitution with Rate θ

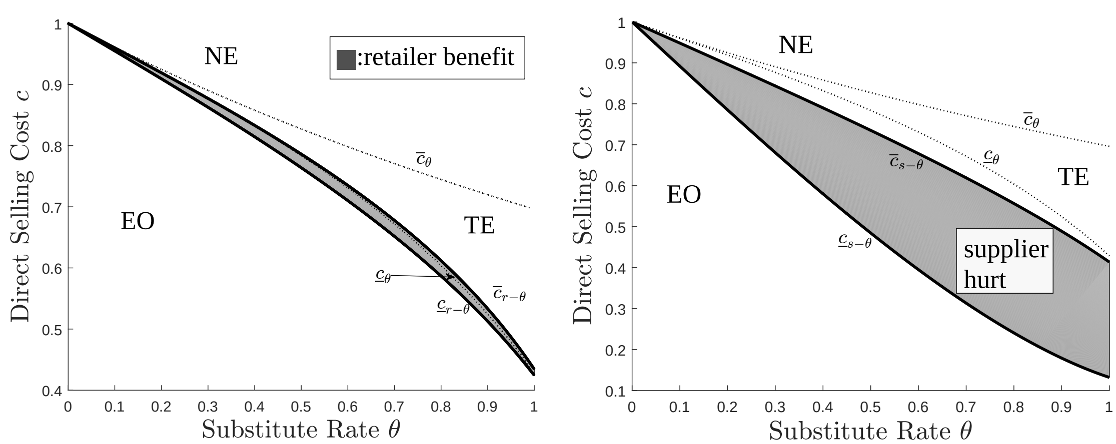
	Encroachment Occurrence $c < \underline{c}_\theta$	Threat of Encroachment $\underline{c}_\theta < c \leq \overline{c}_\theta$	No Encroachment $c \geq \overline{c}_\theta$
w^{en}	$\frac{(4-2\theta^2+n\theta^3+4n(1-\theta^2))a-n\theta^3 c}{8(1+n)-2(3n+2)\theta^2}$	$\frac{(2n+2-\theta^2)c-(2n(1-\theta)+(2-\theta^2))a}{2n\theta}$	$\frac{a}{2}$
$q_{r_i}^{en}$	$\frac{2(1-\theta)a+2c\theta}{4(1+n)-(3n+2)\theta^2}$	$\frac{a-c}{n\theta}$	$\frac{a}{2(n+1)}$
q_s^{en}	$\frac{(4-2\theta^2+(4-2\theta-\theta^2)n)a-(4-2\theta^2+(4-\theta^2)n)c}{8(1+n)-2(3n+2)\theta^2}$	0	0
$\pi_{r_i}^{en}$	$\frac{2(2-\theta^2)(a(1-\theta)+c\theta)^2}{(4(n+1)-(3n+2)\theta^2)^2}$	$\frac{(2-\theta^2)(a-c)^2}{2n^2\theta^2}$	$\frac{a^2}{4(n+1)^2}$
π_s^{en}	$\frac{(4-2\theta^2+n\theta^2)(a-c)^2+8n(1-\theta)(a-c)a+4nc^2}{16(n+1)-4(3n+2)\theta^2}$	$\frac{((2n+2-\theta^2)c-(2n(1-\theta)+2-\theta^2)a)(a-c)}{2n\theta^2}$	$\frac{na^2}{4(n+1)}$

where $\overline{c}_\theta = \frac{\left((4-2\theta)n+4-2\theta^2\right)(n+1)+\theta^2 n\sqrt{2(n+1)}}{(4n+4-2\theta^2)(n+1)} a$ and $\underline{c}_\theta = \frac{4-2\theta^2+(4-2\theta-\theta^2)n}{4-2\theta^2+(4-\theta^2)n} a.$

A comparison of the equilibrium outcome stated in Proposition 6 and that stated in Propositions 2–4 reveals that how the endowment of the option to encroach affects the performances of the supplier and retailers remain qualitatively the same no matter whether the two channels are perfect or imperfect substitutes. That is, the results under the baseline model are quite robust. Furthermore, the threshold retailer number for the vanishing of the bright side of encroachment $\left\lceil \frac{2(2-\theta^2)+2\sqrt{2(2-\theta^2)}}{\theta^2} \right\rceil > 4$ for all $\theta < 1$ and decreases with θ. This implies that compared to that under perfect substitution (see Proposition 3), imperfect substitution between the two channels makes the retailer more likely better off from the supplier encroachment. This is because the weakened competition mitigates the negative effect of encroachment on the retailer: a retailer can sell more when the two channels are imperfect substitutes than when they are perfect substitutes.

Next, we examine how the channel substitution impacts the firms' profits with and without the potential supplier encroachment, which is illustrated in Corollary 2 and depicted in Figure 7.

Corollary 2. *When the direct and indirect channels are imperfect substitutes, the thresholds stated in Proposition 6,* $\bar{c}_{r-\theta}, \underline{c}_{r-\theta}, \bar{c}_{s-\theta}, \underline{c}_{s-\theta}$ *all decrease in* θ.



(a) (b)

Figure 7: Impact of substitution rate θ on the retailer and supplier: $n = 4, a = 1$

Recall that a large substitution rate θ indicates a higher competition intensity between the two channels. The channel substitutability has the following two conflicting effects on the encroachment option. One, a higher substitution rate enhances the credibility of the threat from the direct channel encroachment. It can be easily verified that the optimal wholesale price w^{en} increases in θ under the threat of encroachment (TE). Thus, as θ increases, the retailers can benefit from supplier encroachment only when the supplier's direct selling cost is further reduced to mitigate the effect of wholesale price increase. Thus, $\bar{c}_{r-\theta}$ decreases in the substitute rate (Figure 7(a)). Moreover, the supplier is more likely benefit from the endowment of the encroachment option when the competition between two channels is relative low. As illustrated in Figure 7(b), $\bar{c}_{s-\theta}$ decreases in the substitution rate θ. Second, a higher competition level can decrease both the direct channel's demand and the retailers' order under the encroachment occurrence (EO). To maintain the retailers' order incentive, the supplier needs to reduce the wholesale price ($\partial w^{en}/\partial\theta < 0$). This means that with a higher channel competition, the supplier suffers more profit loss in the indirect channel. The supplier can profit from his encroachment option only when his gain from direct selling can surpass his loss from indirect selling. This can be achievable when the direct selling cost is sufficiently low. Subsequently, $\underline{c}_{s-\theta}$ decreases in the substitution rate.

5.3 Costly Channel Expansion

In this subsection, we consider that it is also costly for the supplier to establish his indirect channel. Specifically, the supplier incurs a fixed cost f to enroll a retailer. Note that $f < \frac{c^2}{2}$ is required to ensure that the supplier has the incentive to sell indirectly via the retailer.[7] Under such a circumstance, how many retailers shall the supplier enroll into his indirect channel?

When the supplier cannot encroach into the retail market, his profit when selling indirectly through n retailers can be expressed as $\pi_s^{nf}(n) = \frac{na^2}{4(n+1)} - nf$. Then, we can show that the optimal number of retailers that shall be enrolled in the indirect channel, n^{nf*} is one of the integers around $n^{nf} = \frac{a}{2\sqrt{f}} - 1$, i.e., $n^{nf*} = \arg\max_{n\in\{\lfloor n^{nf}\rfloor,\lfloor n^{nf}\rfloor+1\}} \pi_s^{nf}(n)$, where $\lfloor x \rfloor$ rounds x down to an integer.

When the supplier is capable of encroaching into the retail market, by applying the results stated in Table 1, we know that the first order differential equation of $\pi_s^{enf}(n) = \pi_s^{en} - nf$ with respect to n is continuous piecewise concave. Solving this problem via backward induction, we can obtain the following result.

[7] It can be verified that if $f \geq \frac{a^2}{4}$, the supplier will abandon the indirect channel.

Proposition 7. *When the supplier can proactively choose the number of retailers to enroll into his indirect channel, the equilibrium encroachment strategy, the optimal number of enrolled retailers and equilibrium profits are presented in Table 3, where n^{enf} is a real number, n^{enf*} is an integer,*

$$\bar{c}_f = \frac{a + \sqrt{2f} + \sqrt{2\left(2-\sqrt{2}\right) a\sqrt{f} - 2f}}{2} \text{ and } \underline{c}_f = \frac{2\sqrt{2f} + a}{3}.$$

Table 3: Equilibrium Strategies and Outcomes under Costly Channel Expansion

	Encroachment Occurrence (EO) $c \leq \underline{c}_f$	Threat of Encroachment (TE) $\underline{c}_f < c \leq \bar{c}_f$	No Encroachment (NE) $c > \bar{c}_f$
n^{enf}	$c\sqrt{\frac{2}{f}} - 2$	$\frac{a-c}{\sqrt{2f}}$	$\frac{a}{2\sqrt{f}} - 1$
n^{enf*}	$\arg\max_{n \in \{\lfloor n^{enf} \rfloor, \lfloor n^{enf} \rfloor + 1, \lfloor n^{nf} \rfloor, \lfloor n^{nf} \rfloor + 1\}} \pi_s^{enf}(n)$		
$\pi_{r_i}^{enf}(n^{enf})$	f	f	f
$\pi_s^{enf}(n^{enf})$	$(c - \sqrt{2f})^2 + \frac{(a-c)^2}{4}$	$(c - \sqrt{2f})(a - c)$	$\frac{(a-2\sqrt{f})^2}{4}$

Proposition 7 shows that when it is costly for the supplier to enroll the retailer without considering the integer constraint on n (the number of retailers), in equilibrium, the retailer's profit equals exactly the enrollment cost that the supplier incurs to expand his indirect channel no matter whether the supplier indeed encroaches or not. That is, under this situation, the option of supplier encroachment has no impact on the retailer: the retailer's profit remains unchanged no matter whether the supplier encroaches into the retail market or not. As to the supplier, the existence of encroachment option now always makes him better off. The underlying reason is that he now can choose the proper number of retailers for his indirect channel while taking into consideration the possibility of downstream encroachment. This helps control the level of downstream competition. However, for the retailer, the existence of potential supplier encroachment may make her worse off. For example, when the supplier's unit direct selling cost $c < \underline{c}_f$ and $c\sqrt{\frac{2}{f}} - 2$ is an integer, in equilibrium, each enrolled retailer's profit is f when the supplier is endowed with the encroachment option whereas it is $\left(\frac{a}{2(n^{nf*}+1)}\right)^2$ when the supplier never encroaches. It can be found that supplier encroachment can benefit the enrolled retailers when the optimal number of enrolled retailer $n^{nf*} = \lfloor n^{nf} \rfloor + 1$ but hurt then if $n^{nf*} = \lfloor n^{nf} \rfloor$.

5.4 Asymmetric Retailers

The retailers in the baseline model are assumed to be symmetric: they have the same market potential and also make the ordering decisions simultaneously. Here, we shall relax these assumptions and demonstrate that all the key results still hold. We first consider an alternative scenario in which retailers make their ordering decisions sequentially. That is, retailer r_{i+1} decides her order quantity after the retailer r_i, $i = 1, 2, \cdots, n-1$. Other settings remain the same as that in the baseline model. For the sake of brevity and space saving, we move the related analysis to the online Appendix B and provide the results in the following proposition.

Proposition 8. *When retailers make their ordering decisions sequentially,*

(1.) if $n \leq 6$, retailer r_1 can benefit from supplier encroachment when $c \in [\underline{c}_{r-S}, \overline{c}_{r-S}]$, where
$$\underline{c}_{r-S} = \min\left\{\frac{2^n+1}{2^{n+3/2}}a, \underline{c}_S\right\}, \overline{c}_{r-S} = \left(1 - \frac{2^{2n}-2^{n+1}+2}{\sqrt{2^{n+2}\left(2^{3n-1}+2^{2n}-2-2^n\sqrt{2^{n-1}(2^n-2)}\right)}}\right) a \text{ and}$$
$$\underline{c}_S = \frac{(2^n+1)\left(3\times 2^{2n}-2^{n+1}+2-\sqrt{2^{n+3}(2^n-2)}-2\sqrt{2}\sqrt{(2^n+1)^{-1}\left(2^{2n}-2^{n+1}+2\right)\left((2^n-1)\sqrt{2^{n+1}(2^n-2)}-2^n(2^n-2)\right)}\right)}{9\times 2^{3n}-13\times 2^{2n}+2^{n+4}-2-\left(2^{n+1}+2\right)\sqrt{2^{n+1}(2^n-2)}}a.$$

(2.) Retailer r_i, $i = 2, 3, \cdots, n$, is always worse off with supplier encroachment.

(3.) if $n \geq 2$, the supplier becomes worse off when being endowed with the option to encroach if $c \in [\underline{c}_{s-S}, \overline{c}_{s-S}]$, where $\underline{c}_{s-S} = \frac{2^n(2^n+1)-\sqrt{2^n(2^n-1)(2^n-3)(2^n+1)}}{5\times 2^{2n}-3\times 2^n}a$ and $\overline{c}_{s-S} =$
$$\frac{2^{3n}-2^n\sqrt{2^{n+1}(2^n-2)}-\sqrt{2^n(2^{2n-1}-2^n+1)\left((2^n-1)\sqrt{2^{n+1}(2^n-2)}+2\right)}}{2^n(2^{2n+1}-2^{n+1}+2-\sqrt{2^{n+1}(2^n-2)})}a.$$

When retailers make their decisions sequentially, the one who moves first enjoys the first-mover advantage and can seize a larger market size than those who move afterwards. Also, as shown in the online Appendix B, compared to that in the baseline model with the simultaneous move, the total order quantity from the indirect channel becomes larger under the sequential move. Consequently, the potential supplier encroachment would result in a more pronounced negative impact on his indirect selling. Proposition 8 shows that under the sequential move, only the first-moving retailer could benefit from the supplier encroachment. The sequential move among retailers further diminishes the bright side of encroachment. Moreover, it requires the supplier to charge an even lower wholesale price to uphold the retailers' ordering incentives compared to that under simultaneous move. As such, the downside of encroachment is further exacerbated by the

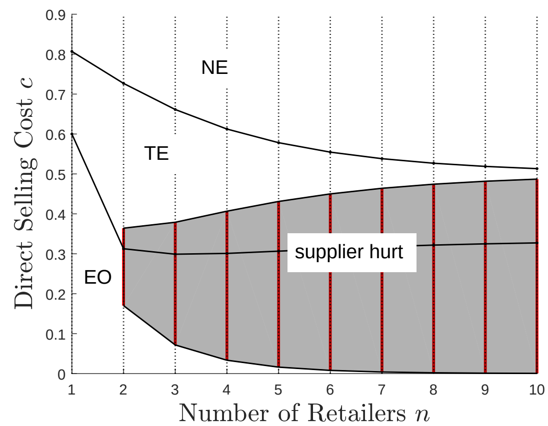


Figure 8: Impact of potential encroachment on the supplier's profit when retailers order sequentially: a=1

retailers' sequential move. As shown in Figure 8, the shadowed region within which the supplier is hurt by having the option to encroach is larger than that depicted in Figure 4 with simultaneous move. Proposition 8 indicates that when there are only two retailers, the supplier can still be hurt by having the option to encroach if the direct selling cost is intermediate. This result complements our findings in the baseline model by revealing another driving force for the dark side of encroachment– the *ordering timing* of retailers.

We now consider that retailers have different market potentials, which differ from the market potential of the direct channel as well. Denote a_i as the market potential of retailer r_i, $i = 1, 2 \cdots, n$ and a_s as that of the supplier in the direct channel, and assume that $a_1 \geq a_2 \geq \cdots \geq a_n \geq a_s$. Again, for the sake of brevity and space saving, we relegate the related analysis and discussion to the online Appendix C. We present our main results under this setting below. First, the retailers' optimal ordering decisions now hinge not only on the number of retailers but also on their market potentials. For those retailers with sufficiently low market potentials, they may actually order nothing (i.e., $q_i = 0$) and exit the market. The potential supplier encroachment would further squeeze those retailers with small market potentials out. Second, the retailers with larger market potentials are more active in deterring the supplier's potential encroachment via over-ordering. This subsequently further shrinks the market share of those retailers with small market potentials and reduces their profit margins. Third, the endowment with the option to encroach still can make the supplier worse off and the bright side of encroachment again disappears when the number of retailer n is large enough. In particular, we can show that when

$n = 2$, the supplier is always weakly better off with the option to encroach regardless of the market potentials of two retailers. However, the supplier can be worse off with the endowment of the encroachment option when $n \geq 3$.

6 Conclusion

In this study, we revisit the classic encroachment setting by taking into account the downstream retailer competition. The supplier sells his product through multiple retailers and meanwhile decides whether or not to open a direct channel. We show that the number of existing retailers plays a significant role in determining the supplier's encroachment action, pricing and selling strategy, leading to some unintended results. First, we show that retailer competition would mitigate the benefit a retailer can enjoy from the reduction of wholesale price by the encroaching supplier. Consequently, the classic bright side of encroachment may no longer exist when the number of retailers exceeds a threshold value. Second, we show that a supplier may become worse off after being endowed with the option to encroach into the retail market even when in equilibrium he does not encroach at all. The underlying reason is that with the increase of retailer number, the effect of wholesale price reduction weakens. As a result, the supplier is unable to maintain the retailers' ordering incentives via undercutting his wholesale price. Under such a circumstance, the profit gain from selling directly cannot compensate his wholesaling loss in the indirect channel, especially when his direct selling is not that cost-effective. In this sense, our study complements the existing literature by uncovering a new driver besides information asymmetry for the dark side of encroachment on the supplier, that is, the retailer competition.

Admittedly, our study has the following limitations. One, in our study we have considered heterogeneous retailer in terms of their market potentials and ordering sequence. In practice, retailers can be heterogeneous in many other dimensions. Second, we consider consumers are indifferent to the selling channels and retailers. However, some consumers may be loyal to certain channel(s) or retailer(s). Third, under our setting, all the retailers are competing for the same market. However, for some supplier, its retailers may be located geographically in different regions, and thus the competition among retailers is softened or even vanishes. Taking the above factors into consideration would enrich our understanding of the impact of potential supplier encroachment. We would like to leave them for future research. Besides, it is also worthwhile to study the optimal encroachment strategy with multiple retailers under other competition models such as pricing competition or Hotelling model.

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Online Appendix

"Supplier Encroachment With Multiple Retailers"

Appendix A: Proofs

Proof of Proposition 1: The results can be directly obtained based on the first order conditions.

Proof of Proposition 2: First, consider that the optimal wholesale price satisfies $w \geq \frac{(2n+1)c-a}{2n}$, his profit is then $\pi_s = nw\frac{a+c-2w}{(n+1)} + \left(\frac{a-(2n+1)c+2nw}{2(n+1)}\right)^2$. It leads to the optimal $w = \frac{(n+2)a-nc}{2(n+2)}$ if and only if $\frac{(n+2)a-nc}{2(n+2)} \geq \frac{(2n+1)c-a}{2n} \to a \geq \frac{2+3n}{n+2}c$; otherwise, when $a < \frac{2+3n}{n+2}c$, the optimal wholesale price should be $w = \frac{(2n+1)c-a}{2n}$. Second, if the optimal wholesale price satisfies $\frac{(n+1)c-a}{n} < w \leq \frac{(2n+1)c-a}{2n}$, the supplier will choose $w = \frac{(2n+1)c-a}{2n}$. Third, if the optimal wholesale price satisfies $w \leq \frac{(n+1)c-a}{n}$, the supplier will choose $w = \frac{a}{2}$ if and only if $\frac{a}{2} \leq \frac{(n+1)c-a}{n} \to a \leq \frac{2(n+1)c}{n+2}$. Taking the above three situations into consideration, we can find that the global optimal pricing should be $w = \frac{(n+2)a-nc}{2(n+2)}$ when $a > \frac{2+3n}{n+2}c$. While if the global optimal wholesale price is $w = \frac{a}{2}$, the constraint should be $a \leq \frac{2(n+1)c}{n+2}$ and $\frac{na^2}{4(n+1)b} \geq \frac{(2n+1)c-a}{2n}\frac{a-c}{b} \to a \leq \frac{2(n+1)^2-n\sqrt{2(n+1)}}{2n+n^2+2}c$ or $a \geq \frac{2(n+1)^2+n\sqrt{2(n+1)}}{2n+n^2+2}c$. That is, $a \in [0, \frac{2(n+1)c}{n+2}] \cap \{[0, \frac{2(n+1)^2-n\sqrt{2(n+1)}}{2n+n^2+2}c] \cup [\frac{2(n+1)^2+n\sqrt{2(n+1)}}{2n+n^2+2}c, 1]\} = [0, \frac{2(n+1)^2-n\sqrt{2(n+1)}}{2n+n^2+2}c]$. In the remaining interval $\frac{2(n+1)^2-n\sqrt{2(n+1)}}{2n+n^2+2} < a \leq \frac{2+3n}{n+2}c$, the global optimal pricing decision is $w = \frac{(2n+1)c-a}{2n}$. This ends the proof.

Proof of Corollary 1: The results can be easily obtained based on Table 1.

Proof of Proposition 3: For the retailer r_i, when $c \leq \frac{2(n+1)^2+n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a$, her profit is the same under the two scenario. When $\frac{n+2}{3n+2}a < c \leq \frac{2(n+1)^2+n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a$, the retailer can benefit from the supplier's encroachment option if and only if $\frac{(a-c)^2}{2n^2} \geq \frac{a^2}{4(n+1)^2} \to c \leq \left(1 - \frac{\sqrt{2}n}{2(n+1)}\right)a$. Moreover, $\left(1 - \frac{\sqrt{2}n}{2(n+1)}\right)a < \frac{2(n+1)^2+n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a$ always holds, and $\left(1 - \frac{\sqrt{2}n}{2(n+1)}\right)a > \frac{n+2}{3n+2}a$ if and only if $n < 2\left(\sqrt{2}+1\right) \approx 4.8$. When $c \leq \frac{n+2}{3n+2}a$, the retailer can benefit from the supplier's encroachment option if and only if $\frac{2c^2}{(n+2)^2} \geq \frac{a^2}{4(n+1)^2} \to c \geq \frac{\sqrt{2}(n+2)a}{4(n+1)}$. Moreover, $\frac{\sqrt{2}(n+2)a}{4(n+1)} < \frac{n+2}{3n+2}a$ if and only if $n < 2\left(\sqrt{2}+1\right) \approx 4.8$. Therefore, a retailer can benefit from the supplier's encroachment option only when the number of the symmetric retailers is less than 4 and the market potential satisfies $\frac{\sqrt{2}(n+2)a}{4(n+1)} \leq c \leq \left(1 - \frac{\sqrt{2}n}{2(n+1)}\right)a$. This ends the proof.

Proof of Proposition 4: For the supplier, when $\frac{n+2}{3n+2}a < c \leq \frac{2(n+1)^2+n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a$, the encroachment option can hurt the supplier's profit if and only if $\frac{((2n+1)c-a)(a-c)}{2n} \leq \frac{na^2}{4(n+1)} \rightarrow c < \frac{2(n+1)^2-n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a$. Moreover, $\frac{n+2}{3n+2}a < \frac{2(n+1)^2-n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a$ if and only if $n \geq 5$. When $c \leq \frac{n+2}{3n+2}a$, the encroachment option can hurt the supplier'a profit if and only if $\frac{(n+2)a^2-2(n+2)ac+(5n+2)c^2}{4(n+2)} \leq \frac{na^2}{4(n+1)} \rightarrow n \geq 2$ and $\frac{(n+1)(n+2)-\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a \leq c \leq \frac{(n+1)(n+2)+\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a$. Moreover, when $2 \leq n \leq 4$, there are $\frac{n+2}{3n+2}a > \frac{(n+1)(n+2)+\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a > \frac{(n+1)(n+2)-\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a$ and when $n \geq 5$, there are $\frac{(n+1)(n+2)-\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a < \frac{n+2}{3n+2}a < \frac{(n+1)(n+2)+\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a$. Therefore, the supplier's encroachment option can hurt himself if and only $n \geq 3$ and $\frac{(n+1)(n+2)-\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a \leq c \leq \min\left\{\frac{(n+1)(n+2)+\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a, \frac{2(n+1)^2-n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a\right\}$. This ends the proof.

Proof of Proposition 5: This result is the direct comparison between the supplier's profit under encroachment with n retailers with that under no-encroachment with $n+1$ retailers.

When $c < \frac{n+2}{3n+2}a$, the inequation is $\frac{(n+2)a^2-2(n+2)ac+(5n+2)c^2}{4(n+2)} \geq \frac{(n+1)a^2}{4(n+2)} \rightarrow c \leq \frac{n+2-\sqrt{n^2-n+2}}{5n+2}a$.

When $\frac{2(n+1)^2-n\sqrt{2(n+1)}}{2n+n^2+2}\frac{n+2}{3n+2}a < c \leq \frac{2(n+1)^2+n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a$, the inequation is $\frac{((2n+1)c-a)(a-c)}{2n} > \frac{(n+1)a^2}{4(n+2)} \rightarrow \frac{2(n+1)(n+2)-\sqrt{2n(n-1)(n+2)}}{2(2n+1)(n+2)}a \leq c \leq \frac{2(n+1)(n+2)+\sqrt{2n(n-1)(n+2)}}{2(2n+1)(n+2)}a$.

Proof of Proposition 6: The proof follows the same logic as that of proposition 2. It is quite straightforward and routine. We thus omit the details here.

Proof of Corollary 2: The results can be easily obtained based on the first order derivatives of $\bar{c}_{r-\theta}, \underline{c}_{r-\theta}, \bar{c}_{s-\theta}$ and $\underline{c}_{s-\theta}$ with respect to θ.

Proof of Proposition 7:. From Table 1, we can get

$$\pi_s^{enf} = \begin{cases} \frac{nc((n+2)a-nc)}{(n+2)^2} + \frac{((n+2)a-(2+3n)c)^2}{4(n+2)^2} - nf, & c < \underline{c}, \\ \frac{((2n+1)c-a)(a-c)}{2n} - nf, & \underline{c} \leq c < \bar{c} \\ \frac{na^2}{4(n+1)} - nf, & c \geq \bar{c}. \end{cases}$$

The first order differential with respect to n is

$$\frac{\partial \pi_s^{enf}}{\partial n} = \begin{cases} \frac{2c^2}{(n+2)^2} - f, & c < \underline{c}, \\ \frac{(a-c)^2}{2n^2} - f, & \underline{c} \leq c < \bar{c} \\ \frac{a^2}{4(n+1)^2} - f, & c \geq \bar{c}. \end{cases}$$

We can find that $\frac{\partial \pi_s^{enf}}{\partial n}$ is piecewise concave in n and has a jump at n that satisfies $c = \bar{c}$. If $c < \underline{c} = \frac{n+2}{2+3n}a$, the first order differential leads to $n = \frac{c\sqrt{2}}{\sqrt{f}} - 2$ and correspondingly, $\pi_s^{enf} = (c - \sqrt{2f})^2 + \frac{(a-c)^2}{4}, \pi_r^{enf} = f$. Plugging the solution into the constraint $c < \frac{n+2}{2+3n}a \rightarrow c < \frac{2\sqrt{2f}+a}{3}$. When $\frac{n+2}{2+3n}a < c < \frac{2(n+1)^2+n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a$, the first order differential leads to $n = \frac{a-c}{\sqrt{2f}}$ and correspondingly, $\pi_s^{enf} = (c - \sqrt{2f})(a-c), \pi_r^{enf} = f$. Plugging the solution into the constraint leads to $\frac{2\sqrt{2f}+a}{3} < c < \frac{a+3\sqrt{2f}+\sqrt{10f+2a\sqrt{2f}}}{2}$. When $c > \frac{2(n+1)^2+n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a$, the first order differential leads to $n = \frac{a}{2\sqrt{f}} - 1$ and correspondingly, $\pi_s^{enf} = \frac{(a-2\sqrt{f})^2}{4}, \pi_r^{enf} = f$. Plugging the solution into the constraint leads to $c > \frac{a^2+\left(a-2\sqrt{f}\right)\sqrt{a\sqrt{f}}}{2\left(a-\sqrt{f}\right)}$.

As $\frac{2\sqrt{2f}+a}{3} < \frac{a^2+\left(a-2\sqrt{f}\right)\sqrt{a\sqrt{f}}}{2\left(a-\sqrt{f}\right)} < \frac{a+3\sqrt{2f}+\sqrt{10f+2a\sqrt{2f}}}{2}$, we should compare $\left(\frac{((2n+1)c-a)(a-c)}{2n} - nf\right)_{n=\frac{a-c}{\sqrt{2f}}}$ with $\left(\frac{na^2}{4(n+1)} - nf\right)_{n=\frac{a}{2\sqrt{f}}-1}$ when $\frac{a^2+\left(a-2\sqrt{f}\right)\sqrt{a\sqrt{f}}}{2\left(a-\sqrt{f}\right)} < c < \frac{a+3\sqrt{2f}+\sqrt{10f+2a\sqrt{2f}}}{2}$. Further, $\left(\frac{((2n+1)c-a)(a-c)}{2n} - nf\right)_{n=\frac{a-c}{\sqrt{2f}}} < \left(\frac{na^2}{4(n+1)} - nf\right)_{n=\frac{a}{2\sqrt{f}}-1} \rightarrow c > \frac{a+\sqrt{2f}+\sqrt{2(2-\sqrt{2})a\sqrt{f}-2f}}{2}$.

Based on the above analysis, we can conclude the proof. We denote the optimal solution under supplier encroachment as n^{enf} and that under no supplier encroachment as n^{nf}. When we restrict n to be an integer, we should find the optimal n around n^{enf} and n^{nf} which is due to that $\frac{\partial \pi_s^{enf}(n)}{\partial n}$ is discontinuous at n that satisfies $c = \bar{c}$. Define $\Psi = \{\lfloor n^{enf} \rfloor, \lfloor n^{enf} \rfloor + 1, \lfloor n^{nf} \rfloor, \lfloor n^{nf} \rfloor + 1\}$, where $\lfloor x \rfloor$ rounds x down to an integer. Then, the optimal number of retailers shall be $n^{enf*} = \arg\max_{n\in\Psi} \pi_s^{enf}(n)$.

Appendix B: When Retailers Make Sequential Ordering Decisions

Here, we consider that retailers make their order decision sequentially, and retailer r_{i+1} decides her order quantity after retailer r_i, $i = 1, 2, ..., n$. Below, we focus on the analysis of the potential encroachment scenario as the one without encroachment is a degenerated case.

If the supplier encroaches into the retail market, he decides his direct selling quantity

q_s in the last stage; that is,

$$\pi_s(q_s) = w\sum_{j=1}^{n} q_{r_j} + (a - \sum_{j=1}^{n} q_{r_j} - q_s - c)q_s,$$

which leads to $q_s(q_{r_j}) = \left(\frac{a-\sum_{j=1}^{n} q_{r_j} - c}{2}\right)^+$.

For the retailers, given the wholesale price w and anticipating the supplier's encroachment quantity $q_s(q_{r_j})$, they decides their order quantities one by one. We can obtain the following lemma.

Lemma 1. *Given the wholesale price w and the ordering quantities of those retailers before her, retailer r_i's optimal order quantity is as follows:*

$$q_{r_i} = \begin{cases} \left(\frac{a-\sum_{j=1}^{i-1} q_{r_j} - 2w + c}{2}\right)^+, & \text{if } \sum_{j=1}^{i-1} q_{r_j} \leq a - 2w + c - \left(2^{n-i+2} + \sqrt{2^{n-i+3}\left(2^{n-i}-1\right)}\right)(c-w), \\ a - w - 2^{n-i}(c-w) - \sum_{j=1}^{i-1} q_{r_j}, & \text{if } a - 2w + c - \left(2^{n-i+2} + \sqrt{2^{n-i+3}\left(2^{n-i}-1\right)}\right)(c-w) \\ & \quad < \sum_{j=1}^{i-1} q_{r_j} \leq a - w - 2^{n-i+1}(c-w), \\ \frac{a-\sum_{j=1}^{i-1} q_{r_j} - w}{2} & \text{if } a - w - 2^{n-i+1}(c-w) < \sum_{j=1}^{i-1} q_{r_j} \leq a - w; \end{cases} \tag{4}$$

where $i = 1, 2, \cdots, n$ and $\sum_{j=1}^{0} q_{r_j} = 0$.

Based on Lemma 1, we can obtain the following result.

Lemma 2. *Given the wholesale price w, retailers' optimal order quantities are as follows:*

$$q_{r_i} = \begin{cases} \frac{a-c+2(c-w)}{2^i}, & \text{if } c - w < \frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}}; \\ \left(a - w - 2^{n-1}(c-w)\right) I_{i=1} + \left(2^{n-i}(c-w)\right) I_{i>1}, & \text{if } \frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}} < c - w \leq \frac{a-c}{2^n-1}; \\ \frac{a-w}{2^i}, & \text{if } c - w > \frac{a-c}{2^n-1}, \end{cases} \tag{5}$$

where I_S is the indicator function and $I_S = 1$ when the event S happens; otherwise, $I_S = 0$.

From lemma 2, we can further obtain the supplier's direct-selling quantity for the given wholesale price. In particular, when $c - w \leq \frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}}$, $q_s = \frac{a-(2-\frac{1}{2^{n-1}})\frac{a-2w+c}{2}-c}{2} = \frac{2w-2c+\frac{a-2w+c}{2^n}}{2} = \frac{a-(2^{n+1}-1)c+2(2^n-1)w}{2^{n+1}}$; otherwise, $q_s = 0$.

We now derive the supplier's optimal wholesale price decision. Anticipating the quantity decisions of retailers and himself, the supplier's profit can be written as

$$\pi_s = \begin{cases} w(2 - \frac{1}{2^{n-1}})\frac{a-2w+c}{2} + \frac{\left(a-(2^{n+1}-1)c+2(2^n-1)w\right)^2}{2^{2n+2}}, & \text{if } c - w \leq \frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}}; \\ w(a-c), & \text{if } \frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}} < c - w \leq \frac{a-c}{2^n-1}; \\ w(2 - \frac{1}{2^{n-1}})\frac{a-w}{2}, & \text{if } c - w > \frac{a-c}{2^n-1}. \end{cases}$$

Let $\underline{c}_S := \frac{(2^n+1)\left(3\times 2^{2n}-2^{n+1}+2-\sqrt{2^{n+3}(2^n-2)}-2\sqrt{2}\sqrt{(2^n+1)^{-1}\left(2^{2n}-2^{n+1}+2\right)\left((2^n-1)\sqrt{2^{n+1}(2^n-2)}-2^n(2^n-2)\right)}\right)}{9\times 2^{3n}-13\times 2^{2n}+2^{n+4}-2-\left(2^{n+1}+2\right)\sqrt{2^{n+1}(2^n-2)}}a$, and $\bar{c}_S := \frac{2^{3n}-2^n\sqrt{2^{n+1}(2^n-2)}+\sqrt{2^n\left(2^{2n-1}-2^n+1\right)\left((2^n-1)\sqrt{2^{n+1}(2^n-2)}+2\right)}}{2^n\left(2^{2n+1}-2^{n+1}+2-\sqrt{2^{n+1}(2^n-2)}\right)}a$. Then, we can show that the supplier's optimal wholesale pricing decision as follows:

$$w = \begin{cases} \frac{a(2^n+1)-c(2^n-1)}{2\times 2^n+2}, & \text{if } c \le \underline{c}_S, \\ c - \frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}}, & \text{if } \underline{c}_S < c \le \bar{c}_S, \\ \frac{a}{2}, & \text{if } c > \bar{c}_S. \end{cases}$$

Table 4 summarize the equilibrium outcomes.

Table 4: Equilibrium Strategies and Outcomes under Sequential Move

	Encroachment Occurrence (EO) $c \le \underline{c}_S$	Threat of Encroachment (TE) $\underline{c}_S < c \le \bar{c}_S$	No Encroachment (NE) $c > \bar{c}_S$
w^{e-S}	$\frac{a(2^n+1)-c(2^n-1)}{2\times 2^n+2}$	$c - \frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}}$	$\frac{a}{2}$
$q_{r_i}^{e-S}$	$\frac{2^{n+1-i}}{(2^n+1)}c$	$i=1: \frac{\left((2^n-2)\sqrt{2^{n-1}(2^n-2)}+\left(2^{2n}-2^n+2\right)\right)(a-c)}{2(2-2\times 2^n+2^{2n})}$ $i>1: \frac{2^{n-i}\left(2^{n+1}-\sqrt{2^{n+1}(2^n-2)}-2\right)(a-c)}{2(2-2\times 2^n+2^{2n})}$	$\frac{a}{2^{i+1}}$
q_s^{e-S}	$\frac{(2^n+1)a-(3\times 2^n-1)c}{2(2^n+1)}$	0	0
$\pi_{r_i}^{e-S}$	$\frac{2^{n+1-i}}{(2^n+1)^2}c^2$	$i=1: \frac{\left(2^{3n-1}+2^n-2-2^n\sqrt{2^{n-1}(2^n-2)}\right)(a-c)^2}{2(2-2\times 2^n+2^{2n})^2}$ $i>1: \frac{\left(2^n-\sqrt{2^{n-1}(2^n-2)}-1\right)^2(a-c)^2}{2^{i-n}(2-2\times 2^n+2^{2n})^2}$	$\frac{a^2}{2^{n+i+2}}$
π_s^{e-S}	$\frac{(2^n+1)a-2(2^n+1)ac+(5\times 2^n-3)c^2}{4(2^n+1)}$	$\left(c - \frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}}\right)(a-c)$	$\frac{(2^n-1)a^2}{2^{n+2}}$

Proof of Proposition 8: Note that the equilibrium outcome without the potential supplier encroachment is the same as that when 'no encroachment' strategy is adopted as listed in Table 4. Then, a comparison of the equilibrium outcomes listed in Table 4 can lead to the following results.

For retailer r_1, when $c \le \underline{c}_S$, she can benefit from supplier encroachment only if $\frac{2^n}{(2^n+1)^2}c^2 \ge \frac{a^2}{2^{n+3}}$, which requires $c \ge \frac{2^n+1}{2^{n+3/2}}a$. When $\underline{c}_S \le c \le \bar{c}_S$, she can benefit from supplier encroachment only if $\frac{\left(2^{3n-1}+2^n-2-2^n\sqrt{2^{n-1}(2^n-2)}\right)(a-c)^2}{2(2-2\times 2^n+2^{2n})^2} \ge \frac{a^2}{2^{n+3}}$. This requires $c < \left(1 - \frac{\left(2-2^{n+1}+2^{2n}\right)}{\sqrt{2^{n+2}\left(2^{3n-1}+2^{2n}-2-2^n\sqrt{2^{n-1}(2^n-2)}\right)}}\right)a$, which is higher than $\underline{c}_S$ if and only if $n \le 6$.

For retailer r_i, $i = 2, 3, ...n$, when $c \le \underline{c}_S$, she can benefit from the supplier encroachment only if $\frac{2^{n+1-i}}{(2^n+1)^2}c^2 \ge \frac{a^2}{2^{n+2+i}}$. This requires $c \ge \frac{2^n+1}{2^{n+3/2}}a$, which, however, cannot hold when $n \ge 2$. Also, $\frac{\left(2^n - \sqrt{2^{n-1}(2^n-2)} - 1\right)^2 (a-c)^2}{2^{i-n}(2-2\times 2^n + 2^{2n})^2} < \frac{a^2}{2^{n+2+i}}$ always holds for $n \ge i \ge 2$. Thus, retailer r_i, $i = 2, 3, ...n$, cannot benefit from supplier encroachment.

For the supplier, he is hurt by the option of encroachment either if $\frac{(2^n+1)a^2 - 2(2^n+1)ac + (5\times 2^n - 3)c^2}{4(2^n+1)} < \left(1 - \frac{1}{2^n}\right)\frac{a^2}{4}$ when $c \le \underline{c}_S$, or if $\left(c - \frac{2(2^n-1) - \sqrt{2^{n+2}(2^{n-1}-1)}}{2(2^{2n} - 2\times 2^n + 2)}(a-c)\right)(a-c) < \left(1 - \frac{1}{2^n}\right)\frac{a^2}{4}$ when $\underline{c}_S \le c \le \bar{c}_S$. Based on which, we can derive the interval $\underline{c}_{s-S} \le c \le \bar{c}_{s-S}$, which, however, exists if and only if $n \ge 2$.

Proof of Lemma 1: We prove the result by the mathematical induction. We first assume that given the first i retailers' order decisions and in anticipation of the order decisions of the following retailers and the supplier, the market price is

$$p_{r_i} = \begin{cases} \frac{a - \sum_{j=1}^{i} q_{r_j} - 2w + c}{2^{n-i+1}} + w, & \text{if } \sum_{j=1}^{i} q_{r_j} \le a - 2w + c - \left(2^{n-i+1} + \sqrt{2^{n-i+2}\left(2^{n-i-1} - 1\right)}\right)(c-w); \\ c, & \text{if } \begin{array}{l} a - 2w + c - \left(2^{n-i+1} + \sqrt{2^{n-i+2}\left(2^{n-i-1}-1\right)}\right)(c-w) \\ < \sum_{j=1}^{i} q_{r_j} \le a - w - 2^{n-i}(c-w); \end{array} \\ \frac{a - \sum_{j=1}^{i} q_{r_j} - w}{2^{n-i}} + w, & \text{if } a - w - 2^{n-i}(c-w) < \sum_{j=1}^{i} q_{r_j} \le a - w. \end{cases}$$

For retailer r_n, her profit function then can be written as

$$\pi_{r_n} = (a - \sum_{j=1}^{n} q_{r_j} - q_s(q_{r_j}) - w)q_{r_n} = \begin{cases} \frac{a - \sum_{j=1}^{n} q_{r_j} - 2w + c}{2} q_{r_n}, & \text{if } \sum_{j=1}^{n} q_{r_j} < a - c; \\ (a - \sum_{j=1}^{n} q_{r_j} - w)q_{r_n}, & \text{if } \sum_{j=1}^{n} q_{r_j} \ge a - c. \end{cases}$$

Then, retailer r_n's optimal ordering decision can be derived as

$$q_{r_n} = \begin{cases} \left(\frac{a - \sum_{j=1}^{n-1} q_{r_j} - 2w + c}{2}\right)^+, & \text{if } \sum_{j=1}^{n-1} q_{r_j} \le a - 3c + 2w; \\ a - c - \sum_{j=1}^{n-1} q_{r_j}, & \text{if } a - 3c + 2w < \sum_{j=1}^{n-1} q_{r_j} \le a + w - 2c; \\ \frac{a - \sum_{j=1}^{n-1} q_{r_j} - w}{2}, & \text{if } a + w - 2c < \sum_{j=1}^{n-1} q_{r_j} \le a - w. \end{cases}$$

This proves that when $i = n$, the lemma holds.

Now, suppose that when $i = k+1$, the lemma holds. We shall prove that when $i = k$, the lemma holds as well. When $i = k$, retailer r_k decides q_{r_k} to maximize her profit as follows:

$$\pi_{r_k} = \begin{cases} \text{Case 1: } \frac{a - \sum_{j=1}^{k} q_{r_j} - 2w + c}{2^{n-k+1}} q_{r_k}, & \text{if } \sum_{j=1}^{k} q_{r_j} \le \left(a - 2w + c - 2^{n-k+1}(c-w)\right) - (c-w)\sqrt{2^{n-k+2}\left(2^{n-k-1} - 1\right)}; \\ \text{Case 2: } (c-w)\, q_{r_k}, & \text{if } \begin{array}{l} \left(a - 2w + c - 2^{n-k+1}(c-w)\right) - (c-w)\sqrt{2^{n-k+2}\left(2^{n-k-1}-1\right)} \\ < \sum_{j=1}^{k} q_{r_j} \le a - w - 2^{n-k}(c-w); \end{array} \\ \text{Case 3: } \frac{a - \sum_{j=1}^{k} q_{r_j} - w}{2^{n-k}} q_{r_k}, & \text{if } a - w - 2^{n-k}(c-w) < \sum_{j=1}^{k} q_{r_j} \le a - w. \end{cases}$$

Based on the above equation, under the three cases, the retailer's global optimal order decision shall be: (1) $q_{r_k} = \frac{a-\sum_{j=1}^{k-1} q_{r_j} - 2w + c}{2}$ if and only if $\sum_{j=1}^{k} q_{r_j} \le \left(a - 2w + c - 2^{n-k+1}\right) - (c-w)\sqrt{2^{n-k+2}\left(2^{n-k-1}-1\right)}$ and $\frac{\left(a-\sum_{j=1}^{k-1} q_{r_j}-2w+c\right)^2}{2^{n-k+3}} \ge (c-w)\left(a - w - 2^{n-k}(c-w) - \sum_{j=1}^{k-1} q_{r_j}\right)$, which is equal to $\sum_{j=1}^{k-1} q_{r_j} \le \left(a - 2w + c - 2^{n-k+2}(c-w)\right) - (c-w)\sqrt{2^{n-k+3}\left(2^{n-k}-1\right)}$; (2) $q_{r_k} = \frac{a-\sum_{j=1}^{k-1} q_{r_j} - w}{2}$ if and only if $\sum_{j=1}^{k} q_{r_j} > a - w - 2^{n-k}(c-w)$ and $\frac{\left(a-\sum_{j=1}^{k-1} q_{r_j}-w\right)^2}{8} > (c-w)\left(a - w - 2^{n-k}(c-w) - \sum_{j=1}^{k-1} q_{r_j}\right)$, which is equal to $\sum_{j=1}^{k-1} q_{r_j} > a - w - 2^{n-k+1}(c-w)$; (3) $q_{r_k} = a - w - 2^{n-k}(c-w) - \sum_{j=1}^{k-1} q_{r_j}$ if and only if $\left(a - 2w + c - 2^{n-k+2}(c-w)\right) - (c-w)\sqrt{2^{n-k+3}\left(2^{n-k}-1\right)} \le \sum_{j=1}^{k-1} q_{r_j} < a - w - 2^{n-k+1}(c-w)$. As such, when $i = k$, the lemma also holds. This ends our proof.

Proof of Lemma 2: Based on the equation (4) stated in Lemma 1, we can obtain that

$$q_{r_1} = \begin{cases} \frac{a-c+2(c-w)}{2}, & \text{if } 0 \le a - 2w + c - 2^{n+1}(c-w) - (c-w)\sqrt{2^{n+2}\left(2^{n-1}-1\right)}; \\ a - w - 2^{n-1}(c-w), & \text{if } a - 2w + c - (c-w)\left(2^{n+1} + \sqrt{2^{n+2}\left(2^{n-1}-1\right)}\right) < 0 \le a - w - 2^{n}(c-w); \\ \frac{a-w}{2}, & \text{if } 0 > a - w - 2^{n}(c-w), \end{cases}$$

which can be rewritten as

$$q_{r_1} = \begin{cases} \text{Case 1: } \frac{a-c+2(c-w)}{2}, & \text{if } c - w \le \frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}}; \\ \text{Case 2: } a - w - 2^{n-1}(c-w), & \text{if } \frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}} < c - w \le \frac{a-c}{2^n-1}; \\ \text{Case 3: } \frac{a-w}{2}, & \text{if } c - w > \frac{a-c}{2^n-1}. \end{cases}$$

Again, based on (4), we have the following three cases:

Case 1: when $i \ge 1$ and $\sum_{j=1}^{i-1} q_{r_j} < a - 2w + c - \left(2^{n-i+2} + \sqrt{2^{n-i+3}\left(2^{n-i}-1\right)}\right)(c-w)$, we have $q_{r_i} = \frac{a-\sum_{j=1}^{i-1} q_{r_j} - 2w + c}{2}$ and $\sum_{j=1}^{i} q_{r_j} < a - 2w + c - \left(2^{n-i+1} + \sqrt{2^{n-i+2}\left(2^{n-i-1}-1\right)}\right)(c-w)$. Then, according to (4), we have $q_{r_{i+1}} = \frac{a-\sum_{j=1}^{i} q_{r_j} - 2w + c}{2}$. Thus, $q_{r_{i+1}} - q_{r_i} = -\frac{q_{r_i}}{2}$. Consequently, we can show that $q_{r_{i+1}} = \frac{q_{r_i}}{2} = \ldots = \frac{q_{r_1}}{2^i}$ and $\sum_{j=1}^{i-1} q_{r_j} = \left(2 - \frac{1}{2^{i-2}}\right) q_{r_1}$. Then, the corresponding constraint of retailer r_i under this case can be rewritten as $\left(2 - \frac{1}{2^{i-2}}\right) q_{r_1} < a - 2w + c - 2^{n-i+2}(c-w) - (c-w)\sqrt{2^{n-i+3}\left(2^{n-i}-1\right)}$. This can be further rewritten as $c - w < \frac{a-c}{2^{n+1}-2+\sqrt{2^{n+2}\left(2^{n-1}-2^{i-1}\right)}}$, whose right-hand side is increasing in i. That is, for any $i > 1$, $\frac{a-c}{2^{n+1}-2+\sqrt{2^{n+2}\left(2^{n-1}-2^{i-1}\right)}} > \frac{a-c}{2^{n+1}-2+\sqrt{2^{n+2}\left(2^{n-1}-1\right)}}$. This implies that once retailer r_1's quantity decision falls into this case (i..e, to allow the supplier to enter the retail market), the following retailers' quantity decisions also fall into this case

Case 2: When $i \ge 1$ and $\sum_{j=1}^{i-1} q_{r_j} > a - w - 2^{n-i+1}(c-w)$, we have $q_{r_i} = \frac{a-b\sum_{j=1}^{i-1} q_{r_j} - w}{2}$ and $\sum_{j=1}^{i} q_{r_j} > a - w - 2^{n-i}(c-w)$. Then, according to (4), we have $q_{r_{i+1}} = \frac{a-\sum_{j=1}^{i} q_{r_j} - w}{2}$.

Thus, $q_{r_{i+1}} - q_{r_i} = -\frac{q_{r_i}}{2}$. Consequently, we can show that $q_{r_{i+1}} = \frac{q_{r_i}}{2} = ... = \frac{q_{r_1}}{2^i}$ and $\sum_{j=1}^{i-1} q_{r_j} = \left(2 - \frac{1}{2^{i-2}}\right) q_{r_1}$. Then, the corresponding constraint of retailer r_i under this case can be rewritten as $\left(2 - \frac{1}{2^{i-2}}\right) q_{r_1} > a - w - 2^{n-i+1}(c - w)$. This can be further rewritten as $c - w > \frac{a-c}{2^n-1}$. This implies that once retailer r_1's quantity decision falls into this case (i..e, without considering the supplier encroachment), the following retailers' quantity decisions also fall into this case.

Case 3: When $i \geq 1$ and $a - 2w + c - \left(2^{n-i+2} + \sqrt{2^{n-i+3}(2^{n-i} - 1)}\right)(c - w) < \sum_{j=1}^{i-1} q_{r_j} \leq a - w - 2^{n-i+1}(c - w)$, we have $q_{r_i} = a - w - 2^{n-i}(c - w) - \sum_{j=1}^{i-1} q_{r_j}$ and $\sum_{j=1}^{i} q_{r_j} = a - w - 2^{n-i}(c - w)$. Then, according to (4), we have $q_{r_{i+1}} = a - w - 2^{n-i-1}(c - w) - \sum_{j=1}^{i} q_{r_j}$. Thus, $q_{r_{i+1}} - q_{r_i} = 2^{n-i-1}(c - w) - q_{r_i}$. Consequently, we can show that $q_{r_{i+1}} = 2^{n-i-1}(c - w)$ and $\sum_{j=1}^{i-1} q_{r_j} = a - w - 2^{n-i+1}(c - w)$. This implies that once retailer r_1's quantity decision falls into this case (i..e, to over-order to deter the supplier's encroachment), the following retailers' quantity decisions also fall into this case. And the corresponding constraint under this case is $\frac{a-c}{2(2^n-1)+\sqrt{2^{n+2}(2^{n-1}-1)}} < c - w \leq \frac{a-c}{2^n-1}$ (the remaining interval excluding the two intervals in the above two cases). This ends the proof.

Appendix C: When Retailers Have Asymmetric Market Potentials

We now consider that the market potentials of retailers and the direct channel are different. We also assume that $a_1 \geq a_2 \geq \cdots \geq a_n$ and $a_s > c$, and moreover, $(k+1)a_k > \sum_{j=1}^{k} a_j$ for all $k \leq n$.[8] Let

$$F(t) = (t+1)a_t - \sum_{j=1}^{t} a_j = ta_t - \sum_{j=1}^{t-1} a_j,$$

and we can prove that $F(t)$ decreases in t.

Scenario 1: No Supplier Encroachment. We first consider the scenario without supplier encroachment. For retailer r_i, her profit function is $\pi_{r_i} = (a_i - \sum_{j=1}^{n} q_{r_j} - w)q_{r_i}$. It is easy to show that $q_{r_i}^* = 0$ for any $a_i \leq w$ and her best response is $q_{r_i} = \left(\frac{a_i - \sum_{j \neq i} q_{r_j} - w}{2}\right)^+$. Suppose that there exists a $k \leq n$ such that $q_{r_i}^* > 0$ for $i \leq k$ and $q_{r_i}^* = 0$ for $i \geq k+1$. Then, k shall satisfy $a_{k+1} - w \leq \frac{\sum_{j=1}^{k}(a_j - w)}{k+1}$ and $a_k - w > \frac{\sum_{j=1}^{k}(a_j - w)}{k+1}$, which is equivalent to

[8] Subsequent analysis will reveal that retailer r_k will never enter the market if $(k+1)a_i \leq \sum_{j=1}^{k} a_j$. We consider $a_k = 0$ for $k > n$ when needed.

$F(k+1) \leq w < F(k)$. Based on the above discussion, we can easily obtain the following lemma.

Lemma 3. *Given the wholesale price w, there exists a unique integer $k_w = K(w)$ satisfying*

$$K(w) = \sum_{k=1}^{n} k \cdot I_{F(k+1) \leq w < F(k)}{}^{9},$$

and retailers' optimal order decisions are as follows:

$$q_{r_i} = a_i - w - \frac{\sum_{j=1}^{k_w}(a_j - w)}{k_w + 1}, i \leq k_w; q_{r_i} = 0, i \geq k_w + 1.$$

Anticipating the retailers' order decisions, the supplier makes the wholesale price decision to maximize his profit $\pi_s(w) = w\frac{\sum_{j=1}^{k_w}(a_j - w)}{k_w+1}$. We then have the following result.

Proposition 9. *The supplier's optimal wholesale price is $w^*_{no} = \frac{\sum_{j=1}^{j=k^*_{no}} a_j}{2k^*_{no}}$, where*

$$k^*_{no} = \arg\max_k \left\{ \frac{k}{k+1} \left(\frac{\sum_{j=1}^{k} a_j}{2k} \right)^2 : k = 1, 2, \cdots, n \right\}.$$

The above proposition indicates that the supplier takes all the retailers' market potentials into account to decide the wholesale price even though some retailer may not sell in equilibrium.

Scenario 2: Potential Supplier Encroachment. Now consider that the supplier has the option to encroach into the retail market with a market potential a_s. The supplier decides q_s to maximize his profit $\pi_s(q_s) = w\sum_{j=1}^{n} q_{r_j} + (a_s - c - \sum_{j=1}^{n} q_{r_j})q_s$. We can show that $q_s(q_{r_j}) = \left(\frac{a_s - c - \sum_{j=1}^{n} q_{r_j}}{2}\right)^+$. Anticipating $q_s(q_{r_j})$, retailer r_i decides the order quantity to maximize the profit

$$\pi_{r_i} = \begin{cases} (a_i - \sum_{j=1}^{n} q_{r_j} - q_s - w)q_{r_i}, & \text{if } \sum_{j=1}^{n} q_{r_j} < a_s - c, \\ (a_i - \sum_{j=1}^{n} q_{r_j} - w)q_{r_i}, & \text{if } \sum_{j=1}^{n} q_{r_j} \geq a_s - c. \end{cases}$$

Retailer r_i's best response function can be derived as

$$q_{r_i} = \begin{cases} \frac{2a_i - 2w - (a_s - c) - \sum_{j\neq i} q_{r_j}}{2}, & \text{if } \sum_{j\neq i} q_{r_j} < \min\{3(a_s - c) - 2(a_i - w), 2(a_i - w) - (a_s - c)\}; \\ a_s - c - \sum_{j\neq i} q_{r_j}, & \text{if } 3(a_s - c) - 2(a_i - w) < \sum_{j\neq i} q_{r_j} < 2(a_s - c) - (a_i - w); \\ \frac{a_i - w - \sum_{j\neq i} q_{r_j}}{2}, & \text{if } 2(a_s - c) - (a_i - w) < \sum_{j\neq i} q_{r_j} < a_i - w; \\ 0, & \text{if } \sum_{j\neq i} q_{r_j} \geq a_i - w \text{ or } \sum_{j\neq i} q_{r_j} > 2(a_i - w) - (a_s - c). \end{cases}$$

[9] I_S is an indicator function: $I_S = 1$ if the event S occurs and $I_S = 0$ otherwise.

Let

$$G(k) := \sum_{j=1}^{k} a_j - k a_k.$$

Then, it can be easily shown that $G(k)$ is increasing in k and we can find a unique $k_s \leq k_t \leq n$ such that

$$G(k_t) \leq a_s - c < G(k_t + 1) \;\; and \;\; 2G(k_s) < a_s - c \leq 2G(k_s + 1).$$

The following proposition summarizes the optimal selling/ordering quantities of retailers and the supplier.

Proposition 10. *Given the wholesale price $w \geq 0$, the optimal selling/ordering quantities of retailers and the supplier are as follows:*

1. *if $w < \frac{\sum_{j=1}^{k_t} a_j - (k_t+1)(a_s-c)}{k_t}$, there exists a unique integer $k_w \geq k_t$ satisfying $k_w = K(w)$ so that*

$$q^*_{r_i} = a_i - w - \frac{\sum_{j=1}^{k_w}(a_j - w)}{k+1} \; for \; i = 1,2,...k_w, q^*_{r_i} = 0 \; for \; i = k_w+1,...n, \; and \; q^*_s = 0. \quad (6)$$

2. *if $\frac{\sum_{j=1}^{k_t} a_j - (k_t+1)(a_s-c)}{k_t} \leq w \leq \frac{\sum_{j=1}^{k_s} a_j - (k_s+1)(a_s-c)}{k_s} + \frac{a_s-c}{2k_s}$, there exists a unique integer $k_w \in [k_s, k_t]$ satisfying $a_{k_w+1} \leq a_s - c + w < a_{k_w}$ so that $\sum_{j=1}^{k_w} q^*_{r_j} = a_s - c$ and* [10]

$$q^*_{r_i} = \frac{a_i - w - (a_s - c)}{\sum_{j=1}^{k_w}(a_i - w) - k_w(a_s - c)}(a_s - c) \; for \; i = 1,2,...k_w, q^*_{r_i} = 0 \; for \; i = k_w+1,...n, \; and \; q^*_s = 0. \quad (7)$$

3. *if $w > \frac{\sum_{j=1}^{k_s} a_j - (k_s+1)(a_s-c)}{k_s} + \frac{a_s-c}{2k_s}$, there exists a unique $k_w \leq k_s$ satisfying $k_w = K\left(w + \frac{a_s-c}{2}\right)$ so that*

$$\begin{aligned} q^*_{r_i} &= \frac{2(k_w+1)(a_i-w) - (a_s-c) - 2\sum_{j=1}^{k_w}(a_j-w)}{k_w+1} \; for \; i = 1,2,...k_w, q^*_{r_i} = 0 \; for \; i = k_w+1,...n; \\ and \; q^*_s &= \frac{(2k_w+1)(a_s-c) - 2\sum_{j=1}^{k_w}(a_j-w)}{2(k_w+1)}. \end{aligned} \quad (8)$$

Proposition 10 shows that given the wholesale price w, we can always derive a unique integer k_w (which depends on the sequence $\{a_k : k = 1,2,...n\}$ and w) so that all the retailers with market potentials larger than a_{k_w} will place a positive order while the remaining retailers will order nothing. We now derive the supplier's optimal wholesale price by maximizing his profit as follows:

[10] Here, we assume that retailers proportionally choose their orders according to their own market potentials.

$$\pi_s(w) = \begin{cases} w\left(\frac{\sum_{j=1}^{k_w}(a_j-w)}{k_w+1}\right), & \text{if } k_w \geq k_t; \\ w(a_s-c), & \text{if } k_s \leq k_w < k_t, w \leq \frac{2\sum_{j=1}^{k_s}a_j-(2k_s+1)(a_s-c)}{2k_s}; \\ \frac{\left(\sum_{j=1}^{k_w}a_j-k_w(a_s-c)\right)^2}{k_w(k_w+2)} + \frac{(a_s-c)^2}{4} \\ -\frac{k_w(k_w+2)}{(k_w+1)^2}\left(w-\frac{2\sum_{j=1}^{k_w}a_j+k_w^2(a_s-c)}{2k_w(k_w+2)}\right)^2, & \text{if } k_w \leq k_s, w > \frac{2\sum_{j=1}^{k_s}a_j-(2k_s+1)(a_s-c)}{2k_s}. \end{cases} \tag{9}$$

It is easy to show that $\pi_s(w)$ is continuous and piecewise concave with respect to w.

Proposition 11. *When the supplier is endowed with the option to encroach, he sets the optimal wholesale price as follows:*

$$w_{en}^* = \begin{cases} \frac{\sum_{j=1}^{k_{en}^*}a_j}{2k_{en}^*}, & \text{if } k_{en}^* \geq k_t \\ w_{k_s}, & \text{if } k_{en}^* = k_s \\ \frac{2\sum_{j=1}^{k_{en}^*}a_j+k_{en}^{*2}(a_s-c)}{2k_{en}^*(k_{en}^*+2)}, & \text{if } k_{en}^* < k_s. \end{cases}$$

where $k_{en}^* = \arg\max_k \left\{\frac{k}{k+1}\left(\frac{\sum_{j=1}^{k}a_j}{2k}\right)^2, k \geq k_t; \Pi(k_s); \frac{\left(\sum_{j=1}^{k}a_j-k(a_s-c)\right)^2}{k(k+2)} + \frac{(a_s-c)^2}{4}, k < k_s\right\}$ *and*

$$\begin{cases} w_{k_s} = \frac{2\sum_{j=1}^{k_s}a_j-(2k_s+1)(a_s-c)}{2k_s} \text{ and } \Pi(k_s) = w_{k_s}(a_s-c), & \text{if } a_s - c \leq \frac{2\sum_{j=1}^{k_s}a_j}{3k_s+2}, \\ w_{k_s} = \frac{2\sum_{j=1}^{k_s}a_j+k_s^2(a_s-c)}{2k_s(k_s+2)} \text{ and } \Pi(k_s) = \frac{\left(\sum_{j=1}^{k_s}a_j-k_s(a_s-c)\right)^2}{k_s(k_s+2)} + \frac{(a_s-c)^2}{4}, & \text{if } a_s - c > \frac{2\sum_{j=1}^{k_s}a_j}{3k_s+2}. \end{cases}$$

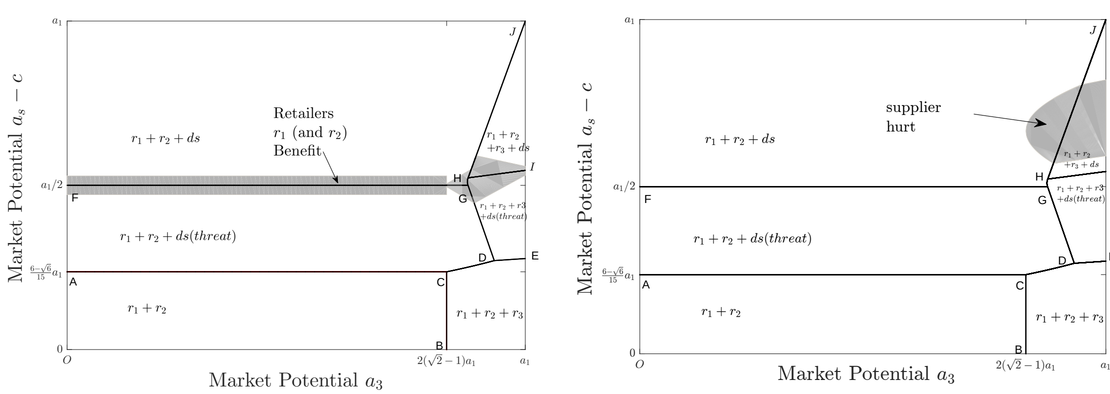

Figure 9: Impact of potential supplier encroachment on the firm profits when market potentials are asymmetric: $n = 3$ and $a_1 = a_2 > a_3$

Figure 9 illustrates the equilibrium regions and comparison results when $n = 3$ and $a_1 = a_2 > a_3$. In the regions denoted as $r_1 + r_2$, $r_1 + r_2 + ds(threat)$ and $r_1 + r_2 + ds$, only retailers r_1 and r_2 have positive sales while retailer r_3 sells nothing. Besides, in the region r_1+r_2, the supplier has no incentive to encroach; in the region $r_1+r_2+ds(threat)$, although the encroachment does not materialize, the potential supplier encroachment is a threat and induces retailers r_1 and r_2 to over-order so as to deter the supplier's encroachment; and in the region r_1+r_2+ds, the supplier indeed encroaches into the retail market and the direct channel incurs sales. The region denoted as $r_1+r_2+r_3/r_1+r_2+r_3+ds(threat)/\ r_1+r_2+r_3+ds$ are similar to the region $r_1+r_2/r_1+r_2+ds(threat)/r_1+r_2+ds$, except that now, in the former region retailer r_3 also has positive sales. Under this situation, the potential supplier encroachment may make retailers r_1 and r_2 better off while always hurts retailer r_3. As to the supplier, he can also become worse off with the endowment of the encroachment option.

Proof of Proposition 9: Let $w_k := \frac{\sum_{j=1}^{k} a_j}{2k}$, which decreases in k. If there exists a w_0 satisfying

$$w_0 = \arg\max_w \pi_s(w) = w\frac{\sum_{j=1}^{k_w}(a_j - w)}{k_w + 1},$$

then we must have $w_0 = w_{k_0} = \frac{\sum_{j=1}^{k_0} a_j}{2k_0}$, where $k_0 := K(w_0)$. Otherwise, we can set

$$w^\# = \begin{cases} \frac{\min\{F(k_0), w_{k_0}\}+w_0}{2}, & \text{if } w_0 < w_{k_0}, \\ w_{k_0}, & \text{if } w_0 > w_{k_0} \geq F(k_0+1), \\ \frac{F(k_0+1)+\max\{F(k_0+2), w_{k_0+1}\}}{2}, & \text{if } w_0 \geq F(k_0+1) > w_{k_0}. \end{cases}$$

Thus,

$$\pi_s(w^\#) = \begin{cases} w^\# \frac{\sum_{j=1}^{k_0}(a_j - w^\#)}{k_0+1}, & \text{if } w_0 < w_{k_0}, \\ w_{k_0}\frac{\sum_{j=1}^{k_0}(a_j - w_{k_0})}{k_0+1}, & \text{if } w_0 > w_{k_0} \geq F(k_0+1), \\ w^\# \frac{\sum_{j=1}^{k_0+1}(a_j - w^\#)}{(k_0+1)+1}, & \text{if } w_0 \geq F(k_0+1) > w_{k_0}. \end{cases}$$

There is $\pi_s(w^\#) > \pi_s(w_0)$, which contradicts the definition of w_0.

We now prove that if $\frac{k^*}{k^*+1}\left(\frac{\sum_{j=1}^{k^*} a_j}{2k^*}\right)^2$ is the maximal value of the sequence: $\left\{\frac{k}{k+1}\left(\frac{\sum_{j=1}^{k} a_j}{2k}\right)^2 : k = 1, 2, \cdots, n\right\}$, it must satisfy $F(k^*+1) \leq w_{k^*} < F(k^*)$. As $\frac{k^*}{k^*+1}\left(\frac{\sum_{j=1}^{k^*} a_j}{2k^*}\right)^2 \geq \frac{k^*-1}{(k^*-1)+1}\left(\frac{\sum_{j=1}^{k^*-1} a_j}{2(k^*-1)}\right)^2$, we have $a_{k^*} \geq \left(\sqrt{\frac{k^*+1}{k^*-1}} - 1\right)\sum_{j=1}^{k^*-1} a_j$. Since $\sqrt{\frac{k^*+1}{k^*-1}} - 1 > \frac{2k^*+1}{2k^{*2}-1}$, this implies $a_{k^*} > \frac{2k^*+1}{2k^{*2}-1}\sum_{j=1}^{k^*-1} a_j$. Thus, $\frac{\sum_{j=1}^{k^*} a_j}{2k^*} < (k^*+1)a_{k^*} - \sum_{j=1}^{k^*} a_j$. That is, $w_{k^*} < F(k^*)$. Similarly, as $\frac{k^*}{k^*+1}\left(\frac{\sum_{j=1}^{k^*} a_j}{2k^*}\right)^2 \geq \frac{k^*+1}{(k^*+1)+1}\left(\frac{\sum_{j=1}^{k^*+1} a_j}{2(k^*+1)}\right)^2$, we have $a_{k^*+1} \leq \left(\sqrt{\frac{k^*+2}{k^*}} - 1\right)\sum_{j=1}^{k^*} a_j$.

Since $\sqrt{\frac{k^*+2}{k^*}} - 1 \leq \frac{2k^*+1}{2k^*(k^*+1)}$, we can get $a_{k^*+1} \leq \frac{2k^*+1}{2k^*(k^*+1)} \sum_{j=1}^{k^*} a_j$. This implies $w_{k^*} \geq F(k^*+1)$. This ends the proof.

Proof of Proposition 10: If the retailers' ordering decisions with respect to w are consistent with that under the 'no supplier encroachment' scenario, there should exist $k_w = K(w)$ and $\frac{\sum_{j=1}^{k_w}(a_j - w)}{k_w+1} > a_s - c$, which requires $w < \frac{\sum_{j=1}^{k_w} a_j - (k_w+1)(a_s-c)}{k_w}$. As $G(k_t) \leq a_s - c < G(k_t+1)$ is equal to $F(k_t+1) < \frac{\sum_{j=1}^{k_t} a_j - (k_t+1)(a_s-c)}{k_t} \leq F(k_t)$, and for any $k < k_t$, we have $\frac{\sum_{j=1}^{k} a_j - (k+1)(a_s-c)}{k} < \frac{\sum_{j=1}^{k_t} a_j - (k_t+1)(a_s-c)}{k_t}$, we then can show that when $w < \frac{\sum_{j=1}^{k_t} a_j - (k_t+1)(a_s-c)}{k_t}$, there exists a unique integer $k_w \geq k_t$ satisfying $F(k_w+1) \leq w < F(k_w)$ and the direct channel will not enter the market. The retailers will make their quantity decisions as those stated in (6).

If under the retailers' best responses to the given wholesale price w the supplier encroaches, i.e, $q_s > 0$, and only the first k retailers order positively, then by $q_{r_i} = \left(\frac{2a_i - 2w - (a_s-c) - \sum_{j \neq i} q_{r_j}}{2}\right)^+$, the necessary and sufficient condition for this scenario is

$$\sum_{j=1}^{k} q_{r_j}^* < a_s - c \text{ and } 2(a_{k+1} - w) - (a_s - c) \leq \sum_{j=1}^{k} q_{r_j}^* < 2(a_k - w) - (a_s - c). \tag{10}$$

Then, we can get $q_{r_i}^* = \frac{2(k+1)(a_i-w) - (a_s-c) - 2\sum_{j=1}^{k}(a_j-w)}{k+1}$ for $i = 1, 2...k$ and $\sum_{j=1}^{k} q_{r_j}^* = \frac{2\sum_{j=1}^{k}(a_j-w) - k(a_s-c)}{k+1}$. Thus, (10) can be further written as $w > \frac{\sum_{j=1}^{k} a_j - (k+1)(a_s-c)}{k} + \frac{a_s-c}{2k}$ and $F(k+1) - \frac{a_s-c}{2} \leq w < F(k) - \frac{a_s-c}{2}$. Let $H(k) := \frac{\sum_{j=1}^{k} a_j - (k+1)(a_s-c)}{k} + \frac{a_s-c}{2k}$. As $2G(k_s) < a_s - c \leq 2G(k_s+1)$, we can show that the sequence $\{H(k) : k = 1, 2, \cdots, k_t\}$ is first increasing until $k = k_s$ and then decreasing afterwards. Then, we can have the following inequations:

$$H(k) < F(k+1) - \frac{a_s - c}{2} \text{ for } k = 1, 2, \cdots, k_s - 1;$$
$$H(k) > F(k) - \frac{a_s - c}{2} \text{ for } k = k_s + 1, k_s + 2, \cdots, k_t;$$
$$\text{and } F(k_s+1) - \frac{a_s - c}{2} \leq H(k_s) < F(k_s) - \frac{a_s - c}{2}.$$

Therefore, when $w > H(k_s)$, there always exists a unique $k_w \leq k_s$ satisfying $F(k_w+1) - \frac{a_s-c}{2} \leq w < F(k_w) - \frac{a_s-c}{2}$, i.e., $k_w = K(w + \frac{a_s-c}{2})$, and the quantity decisions of retailers and the supplier are the same as those stated in (8).

The aforemention analysis indicates that when $\frac{\sum_{j=1}^{k_t} a_j-(k_t+1)(a_s-c)}{k_t} = H(k_t) - \frac{a_s-c}{2k_t} \leq w \leq H(k_s)$, the retailers will order the total quantities up to $a_s - c$ to deter the supplier encroachment. Moreover, if $a_k - w \leq a_s - c$, retailer r_k will order nothing. That is, we can find a unique integer k_w satisfying $a_{k_w+1} \leq a_s - c - w < a_{k_w}$ so that only the first k_w retailers will place the positive order and their quantity decisions are the same as those stated in (7).

Proof of Proposition 11: Here, we have the following three cases:

Case 1: if the wholesale price is set such that $w < \frac{\sum_{j=1}^{k_t} a_j-(k_t+1)(a_s-c)}{k_t}$, then the suboptimal wholesale price must be w_{k^*}, where $k^* = \arg\max_k \left\{ \frac{k}{k+1} \left(\frac{\sum_{j=1}^{k} a_j}{2k} \right)^2, k \geq k_t \right\}$; otherwise this case will be dominated by one of the following cases.

Case 2: if the wholesale price is set such that $\frac{\sum_{j=1}^{k_t} a_j-(k_t+1)(a_s-c)}{k_t} \leq w \leq \frac{2\sum_{j=1}^{k_s} a_j-(2k_s+1)(a_s-c)}{2k_s}$, the suboptimal wholesale price w^* must be $\frac{2\sum_{j=1}^{k_s} a_j-(2k_s+1)(a_s-c)}{2k_s}$. As $a_{k_s} - w^* = \frac{(2k_s+1)(a_s-c)-2G(k_s)}{2k_s} > a_s - c > 0$, and $a_{k_s+1} - w^* = \frac{(2k_s+1)(a_s-c)-2G(k_s+1)}{2k_s} \leq a_s - c$, the first k_s retailers place a positive order and the others order nothing.

Case 3: if the wholesale price is set such that $w > \frac{2\sum_{j=1}^{k_s} a_j-(2k_s+1)(a_s-c)}{2k_s}$, the supplier's profit function is the same as that in (9). Let $w_k := \frac{2\sum_{j=1}^{k} a_j+k^2(a_s-c)}{2k(k+2)}$ and $\pi_s(w_k) := \frac{\left(\sum_{j=1}^{k} a_j-k(a_s-c)\right)^2}{k(k+2)} \frac{(a_s-c)^2}{4}$. Similar to the proof of Proposition 9, we can prove that (1) the optimal wholesale price must be one of the sequence $\{w_k : k \leq k_s\}$, and (2) if $\pi_s(w_{k^*})$ is the maximum of $\{\pi_s(w_k) : k \leq k_s\}$, it must satisfy $F(k^*+1) - \frac{a_s-c}{2} \leq w_{k^*} < F(k^*) - \frac{a_s-c}{2}$.

All in all, the supplier should choose the wholesale price to maximize his payoff from one of the above three cases as summarized in the proposition. Note that if the global optimal $k^*_{en} = k_s$, w^*_{en} may be either $\frac{2\sum_{j=1}^{k_s} a_j-(2k_s+1)(a_s-c)}{2k_s}$ (as that of the above Case 2) or $\frac{2\sum_{j=1}^{k_s} a_j+k_s^2(a_s-c)}{2k_s(k_s+2)}$ (as that of the above Case 3), with the corresponding respective profit.