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Train Timetabling with Stop-Skipping, Passenger Flow, and Platform Choice Considerations

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Abstract

In conventional railway planning processes, stop-skipping decisions are often made at the line planning stage, which is executed prior to train timetabling and platform assignment. However, stop-skipping can shorten passenger journey time and also save on train operating costs. Hence, integrating train timetabling, stop-skipping, and platform choice decisions can help generate train timetables with improved passenger convenience and higher train operating efficiency. Integrating these decisions is a challenging task, as these decisions affect passenger train transfer behavior, which in turn affects the entire passenger flow. This study is a first attempt at integrating these decisions while simultaneously taking into account the passenger flow. We consider a train timetabling problem on a single, one-way track with stop-skipping, platform choice, and passenger flow considerations, and we formulate it as a constrained minimum-cost multi-commodity network flow problem on a time-space network. We analyze the problem's complexity and develop a Lagrangian relaxation heuristic to solve the problem. We conduct a computational study with randomly generated data that captures the characteristics of the Beijing-Shanghai high-speed railway line. The computational results report the effectiveness of our Lagrangian relaxation heuristic and how the railway's service capacity and passenger traffic intensity affect the solution.

Keywords: Train timetabling; dynamic passenger demand; stop-skipping; platform assignment; Lagrangian relaxation

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1 Introduction

Due to the complexity of railway systems, the railway planning process is often divided into sequential planning phases. A typical sequence comprises the strategic phase, tactical phase, and operational phase (Lusby et al. 2011). The strategic phase includes network planning and line planning. The tactical phase includes train timetabling, rolling stock scheduling, and crew scheduling. The operational phase involves real-time railway management, such as disruption management. One obvious drawback of such a sequential planning process is that the decisions generally result in suboptimal overall solutions. To overcome this drawback, one approach is to integrate two or more decisions that traditionally belong to different phases.

In conventional railway planning processes, stop-skipping decisions are often made at the line planning stage, which is executed prior to train timetabling (see, e.g., Goossens et al. 2006). Stop-skipping enables one train to overtake another train. This can shorten passenger journey time and also save on train operating costs. However, this not only complicates the train timetable, but also increases the number of passenger transfers at stations. For example, in order to minimize the journey time, a passenger may get off a regular train at a particular station, transfer to an express train which skips stops, and then get off the express train and transfer back to another regular train to get to his/her destination. Hence, integrating train timetabling and stop-skipping decisions while simultaneously taking into consideration the passenger flow can potentially reduce both passenger inconvenience and train operating costs.

In this paper, we consider a train-timetabling model with dynamic (i.e., time-dependent) passenger demand and stop-skipping decisions on a single, one-way rail track. Since stop-skipping has a direct impact on passenger journey time, we consider the passenger flow among all the origin-destination (OD) pairs of the rail line. In addition, since stop-skipping will result in additional passenger transfers, our model considers the assignment of platform tracks (i.e., sidings) to trains at transfer stations, as well as the time and effort required by passengers to walk between platforms when changing trains. Our model aims to simultaneously minimize train operating costs and maximize passenger satisfaction, where passenger satisfaction is measured by how many passenger demands are satisfied, how long the passengers' journey times are, and how much walking the passengers need to do to transfer trains.

Given the railway infrastructure and passenger demand data, the line planning problem aims to determine a line plan, which specifies the paths operated between origin and destination stations, the hourly frequency of the lines, and the stop-skipping patterns (see, e.g., Goossens et al. 2004, 2006). Given a set of lines and their frequencies of use, the train timetabling problem determines the train arrival and departure times at each station along the operated path such that certain safety requirements, such as headway constraints and overtaking constraints, are satisfied. A great deal of research on different variants of the train timetabling problem is available in the literature. Various solution approaches, such as meta-heuristics, simulation-based heuristics, and mathematical programming-based methods, have been proposed. See Caprara et al. (2007), Lusby et al. (2011), Cacchiani et al. (2014), and Kroon et al. (2014) for comprehensive reviews on train timetabling research.

Some research has considered integrated optimization of line planning and train timetabling; see, for example, Kaspi and Raviv (2013) and Burggraeve et al. (2017). In particular, some studies have incorporated stop-skipping decisions into train timetabling so as to improve passenger service and reduce operating cost. These include studies on various train timetabling problems that consider the saving in acceleration and deceleration time when a train bypasses a station; see Zhou and Zhong (2005), Ren et al. (2009), and Shafia et al. (2012). Jiang et al. (2017) consider a train timetabling problem with stop-skipping decisions for a highly congested double-track railway line. Their model aims to add as many new trains as possible to the existing schedule, while taking into account the changes needed to the existing schedule. Wang et al. (2015a) study a train scheduling problem with stop-skipping decisions on urban rail lines and consider the passenger flow on such rail lines. No overtaking of trains at stations are allowed in their model. Yang et al. (2016) analyze a train timetabling problem with stop-skipping decisions for high-speed trains. Their model considers passenger demand at each station without tracking the detailed number of passengers getting on/off each train at each station. Altazin et al. (2017) study a train rescheduling problem with flexible stopping, where the number of passengers boarding and alighting at each station is considered, and passenger waiting is part of the objective. Zhu and Goverde (2019) study a train rescheduling problem with flexible stopping and short-turning, where the objective is to minimize the impacts of train delay, train cancellation, and stop-skipping on passengers.

Some studies have incorporated stop-skipping decisions into train timetabling, where the passenger demand of each OD pair is given explicitly as input. Wang et al. (2014) study a train timetabling problem for urban rail lines with stop-skipping decisions, where the passenger arrival rate at the origin station of each OD pair is given, and passengers do not change trains. Yue et al. (2016) study a timetabling problem for high-speed trains with a given minimum number of trains required for serving passengers of each OD pair. Their model includes stop-skipping decisions and considers multiple sidetracks at each station. However, unlike our problem, their model does not consider the start and end time of passenger journeys, how passengers change trains, or the cost of passengers' journeys. Gao et al. (2016) consider a rescheduling problem of a double-track metro line in an overcrowded situation after disruptions. Their model includes stop-skipping decisions in the recovery period and explicitly considers the number of passengers with different current and destination stations. However, their model does not consider stations with multiple tracks, overtaking of trains at stations, and passenger choice over changing trains. Shang et al. (2018) analyze a timetabling problem in an over-saturated urban rail transit network. Their model includes stop-skipping decisions and aims to maximize the systemwide equity performance, where an equity index is used for measuring the number of missed trains that a passenger encounters. Qi et al. (2018a, 2018b) and Cacchiani et al. (2020) study different train timetabling problems with stop-skipping decisions. Unlike our model, their models consider time-independent passenger demands and assume that each passenger takes only one train from his/her origin to destination without changing trains. Yan and Goverde (2019) study a combined line planning and train timetabling problem, where the line planning problem aims to obtain a line plan with different train types, stop patterns, and frequencies that satisfies the given day-based passenger demand, and the train timetabling problem aims to obtain a periodic train timetable. They present an iterative framework that solves the two problems iteratively. Dong et al. (2020) study a train timetabling problem with stop-skipping decisions and time-dependent passenger demands for a congested commuter railway line. Their model assumes that the number of trains in operation is a decision variable and that there are no predefined skip-stop candidate sets. These studies do not consider platform assignments or passenger transfers between platforms at transfer stations.

Our research is related to train platforming, as our problem includes the decision of assigning platforms to trains at each station. Train platforming problems, which are typically solved after train timetabling is done, aim to determine an assignment of trains to platforms at a railway station, while taking into account the train arrival and departure times at the station and the routing of the trains within the station (Caprara et al. 2011). Some works consider train timetabling problems with platform assignment decisions at those stations that have multiple parallel platforms; see, for example, Carey (1994), Ghoseiri et al. (2004), and Petering et al. (2016). Some works consider integrated train timetabling and platforming problems that involve stations with more complicated track layouts; see, for example, Carey and Crawford (2007) and Lee and Chen (2009).

Some studies have integrated the decisions of timetabling and passenger route choice. In contrast to traditional vehicle timetabling problems in which passenger routes are considered as input, timetabling with passenger routing aims to determining a vehicle timetable and a passenger routing simultaneously (see Schmidt 2014). Various models with different objectives and various solution approaches using event-activity networks for periodic vehicle timetabling with passenger routing have been developed by Siebert and Goerigk (2013), Schmidt and Schöbel (2015), Gattermann et al. (2016), and Borndörfer et al. (2017), and Schiewe and Schöbel (2020). These models' objectives focus on optimizing passengers' waiting time or travel time and ignore the trains' operating costs. Except for Gattermann et al. (2016) who consider distributing the passengers temporally using "time-slices," these models assume that passenger demands are time-independent. Martin-Iradi and Ropke (2019) also study a periodic train timetabling problem with passenger routing decisions. Their solution method bears some similarities as ours in that their solution procedure iteratively modifies the solution by taking passenger routing into account. Their model allows passenger transfers, where a minimum transfer time is imposed, and the transfer time is independent of the station platforms. Unlike our model, their model considers time-independent passenger demand OD pairs and assumes that all passengers belonging to the same OD pair traverse the same path from the origin to the destination, and its objective is to minimize total passenger travel time.

Our research falls into the category of demand-responsive train timetabling, where the train timetable is optimized according to the passenger demand pattern. Many of the abovementioned articles also involve demand-responsive train timetabling. Some representative works on demand-responsiveness of train timetabling include Niu and Zhou (2013), Barrena et al. (2014a; 2014b), Sun et al. (2014), Niu et al. (2015), Zhu et al. (2017), and Shi et al. (2018) who present various solution methods for different train timetabling models, so as to minimize passengers' time and cost. Demand-responsive train timetabling problems that consider

Table 1: Comparison of demand-responsive train timetabling research.

Reference	Infrastructure*	Train line selection or stop-skipping decision	Platform assign-ments	Objective(s)	Passenger demand	Passenger transfers at stations
Barrena et al. (2014a)	Unidirectional	No	No	Average passenger waiting time	Time-dependent	No transfers
Barrena et al. (2014b)	Unidirectional	No	No	Average passenger waiting time	Time-dependent	No transfers
Borndörfer et al. (2017)	General network	No	No	Total and maximum weighted passenger travel time	Time-independent	Transfers allowed
Burggraeve et al. (2017)	Bidirectional	Yes	No	Weighted sum of operating cost and passenger travel time	Time-independent	Transfers allowed
Cacchiani et al. (2020)	Unidirectional	Yes	No	Total unsatisfied passenger demand	Uncertain and time-independent	No transfers
Dong et al. (2020)	Unidirectional	Yes	No	Weighted sum of waiting/delay time of passengers and running time of trains	Time-dependent	No transfers
Gao et al. (2016)	Bidirectional	Yes	No	Total travel time of services; number of passenger waiting at stations	Time-dependent	Transfers take place after service disruption
Gattermann et al. (2016)	General network	No	No	Total passenger travel time plus penalty on change of journey start time	Time-dependent	Transfers allowed
Kaspi and Raviv (2013)	A given set of possible routes	Yes	No	Weighted sum of operating cost and passenger travel time	Time-dependent	Transfers allowed
Martin-Iradi and Ropke (2019)	General network	Yes	No	Total passenger travel time	Time-independent	Transfers allowed
Niu and Zhou (2013)	Bidirectional	No	No	Total passenger waiting time	Time-dependent	No transfers
Niu et al. (2015)	Unidirectional	No	No	Total passenger waiting time	Time-dependent	No transfers
Qi et al. (2018a)	Unidirectional	Yes	No	Total unsatisfied passenger demand	Uncertain and time-independent	No transfers
Qi et al. (2018b)	Unidirectional	Yes	No	Total train travel time; total passenger travel time	Time-independent	No transfers
Schiewe and Schöbel (2020)	General network	No	No	Total passenger travel time	Time-independent	Transfers allowed
Schmidt (2014)	General network	No	No	Total passenger travel time	Time-independent	Transfers allowed
Schmidt and Schöbel (2015)	General network	No	No	Total passenger travel time	Time-independent	Transfers allowed
Shang et al. (2018)	General network	Yes	No	Systemwide equity performance	Time-dependent	No transfers
Shi et al. (2018)	Bidirectional	No	No	Total passenger waiting time	Time-dependent	No transfers
Siebert and Goerigk (2013)	General network	No	No	Total passenger travel time	Time-independent	Transfers allowed
Sun et al. (2014)	Unidirectional	No	No	Total passenger waiting time	Time-dependent	No transfers
Wang et al. (2014)	Unidirectional with single terminus	Yes	No	Weighted sum of total passenger travel time and energy consumption of trains	Time-independent	No transfers

Table 1: (cont'd).

Reference	Infrastructure*	Train line selection or stop-skipping decision	Platform assignments	Objective(s)	Passenger demand	Passenger transfers at stations
Wang et al. (2015b)	General network	No	No	Weighted sum of total passenger travel time and energy consumption of trains	Time-independent	Transfers with platform-dependent transfer time
Yan and Goverde (2019)	Unidirectional	Yes	No	Multiple objectives	Time-independent	No transfers
Yang et al. (2020)	Bidirectional	No	No	Total passenger travel time; energy consumption	Time-dependent	No transfers
Yin et al. (2016)	Bidirectional	No	No	Expected passenger delay & travel time; energy consumption	Uncertain and time-dependent	No transfers
Yin et al. (2017)	Bidirectional	No	No	Total passenger waiting time; energy consumption	Time-dependent	No transfers
Yue et al. (2016)	Unidirectional	Yes	No	Total train profit	Time-independent	No transfers
Zhu et al. (2017)	Bidirectional	No	No	Total cost of passengers' traveling and waiting	Time-independent	No transfers
This study	Unidirectional	Yes	Yes	Total train and passenger cost	Time-dependent	Transfers with platform-dependent transfer time

* "Bidirectional" infrastructure includes railway systems that contain two parallel tracks, with each track used exclusively in one direction.

energy consumption of trains have also been studied; see, for example, Yin et al. (2016, 2017), and Yang et al. (2020). Wang et al. (2015b) study a train timetabling problem that take into consideration passengers' walking time between platforms when changing trains. However, unlike our problem, their model does not assign platforms to trains and does not consider overtaking of trains. Table 1 summarizes the above-mentioned demand-responsive train timetabling research. As can be seen in Table 1, demand-responsive train timetabling models with a unidirectional rail line generally ignore passenger transfers. Our work, however, considers the fact that even within a unidirectional line, passengers may take advantage of stop-skipping via transferring trains. Our model specifically takes in account passengers' time saving obtained from stop-skipping as well as the inconvenience caused by transfers, and we model this characteristic by imposing a unit cost of traveling and a unit cost of walking between platforms. Hence, our model has a unique feature that the train timetabling decision, stop-skipping decision, platform assignment decision, and passengers' reactions are integrated.

Our main contributions are twofold. First, this study is a first attempt at integrating train timetabling, stop-skipping, and platform assignment decisions. Integrating these decisions is a challenging task, as these decisions affect passenger train transfer behavior, which affects the entire passenger flow. The passenger flow in turn affects the optimal train timetabling, stop-skipping, and platform assignment decisions. We develop a mathematical model for making integrated decisions while simultaneously taking into account the passenger flow. Our model considers passenger demand for different OD pairs within different time intervals. It takes into account the time and effort that passengers need to spend on train transfers when the trains skip stops. Second, we develop a time-space network formulation and a Lagrangian relaxation heuristic for our model. Our time-space network uses multiple vertices to represent different states of a train at each combination of time and station, so that our train timetabling problem can be represented by a constrained minimum-cost multi-commodity network flow (MCMCNF) problem on the time-space network. This constrained MCMCNF problem possesses a nice property, in that after some constraints are relaxed, the problem is decomposed into a number of independent shortest path subproblems. This design enables us to determine a lower bound efficiently.

The rest of this paper is organized as follows. Section 2 provides a detailed mathematical description of our problem. Section 3 presents the time-space network, as well as a mixed integer linear programming formulation of the corresponding constrained MCMCNF problem in the network. Section 4 describes the Lagrangian relaxation heuristic. Section 5 reports the results of a computational study that tests the performance of the proposed heuristic. Some concluding remarks are made in Section 6.

2 Problem Description

The problem that we study aims to develop a train timetable for a single-track railway line with the objective of minimizing both the total train operating cost and the total passenger cost. One important characteristic of our problem is that we consider stop skipping at stations, as well as the acceleration and deceleration time incurred when a train stops at a station. Another important characteristic is that we consider the forecast demand of passengers on each possible OD pair and attempt to satisfy this demand, while taking into account the journey durations of the passengers and the operating costs of the trains. To determine the passengers' journeys, we treat the flows of passengers along each track segment as decision variables. A third characteristic is that we consider the utilization of multiple platforms at train stations, allow passengers to transfer trains at stations, and take into consideration the time required for passengers to walk from one platform to another.

2.1 Basic assumptions

The following assumptions are made throughout the paper.

- Assumption 1: This study focuses on trains and passengers traveling along a single-track, unidirectional railway line. However, since trains have different speed and may skip stops, passengers may transfer from one train to another to obtain a shorter travel time.
- Assumption 2: The time for a train to travel between the main track and the platform is ignored.
- Assumption 3: The operating cost of a train includes a cost per unit time that the train spends on running, plus a cost per unit time that the train spends of dwelling. Different trains may have different unit operating costs. Trains services cannot be canceled. The unit cost of a passenger includes a cost per unit time that the passenger spends on waiting and train-riding, plus a cost per unit time that the passenger spends on walking between station platforms. All passengers have the same unit cost of waiting and train-riding, and have the same unit cost of walking.
- Assumption 4: Passenger demand is known with certainty. Passengers are divided into passenger groups.

Each passenger group is characterized by the number of passengers in the group, the origin station, the destination station, and the start time of the passengers' journey. Passengers belonging to the same group do not need to travel together. There is a penalty if a passenger's demand is not satisfied.

- Assumption 5: Passengers with the same OD pair may travel separately. The passenger distribution on different trains is part of the decision of the model.

Assumption 1 captures the key features of our model, which specifically considers passengers' time saving obtained from stop-skipping as well as the inconvenience caused by transfers when passengers are traveling along a single line. Assumptions 2 and 3 are made for the sake of simplicity. Our model and solution method can be generalized to handle more general passenger travel and waiting costs, platform-dependent train travel times, and train cancellations. Assumption 4 holds when passenger demands can be forecast based on historical data. Assumption 5 implies that in the optimal solution of our model, the passengers are optimally distributed among all train services. In other words, for any given train timetable, the passenger flow that we are considering reflects the best-case scenario of passenger distribution.

2.2 Input data

We first describe the input parameters of our model. Table 2 summarizes these parameters, where all time-related parameters are integer-valued, and the planning horizon is given as $[0, T]$.

2.2.1 Railway network data

The considered railway network is a single-track, unidirectional railway line $s_1 \to s_2 \to \cdots \to s_n$ with stations $s_1, s_2, \ldots, s_n$ and mono-directional track segments $s_1 \to s_2$, $s_2 \to s_3$, …, $s_{n-1} \to s_n$. Stations s_1 and s_n are terminal stations, while stations $s_2, s_3, \ldots, s_{n-1}$ are intermediate stations. Each station has one or more parallel *platform tracks*, and has either zero or one *passing track*. Platform tracks enable trains to dwell at the station for passenger boarding and alighting. Each platform track can accommodate one train at a time. A passing track only allows a train to pass through the station without stopping. On a typical railway line, each terminal station has multiple platform tracks and no passing tracks, while each intermediate station has one passing track, which is part of the main line of the railway (see Figure 1). For $i = 1, 2, \ldots, n$, we let m_i and m'_i denote the number of platform tracks and passing tracks, respectively, at station s_i, where $m_i \in \{1, 2, \ldots\}$ and $m'_i \in \{0, 1\}$. Thus, at most m_i trains may stop at station s_i at the same time. For example, in the railway network corresponding to the system depicted in Figure 1, we have $n = 5$, $(m_1, m_2, m_3, m_4, m_5) = (3, 2, 3, 2, 3)$, and $(m'_1, m'_2, m'_3, m'_4, m'_5) = (0, 1, 1, 1, 0)$. Each platform track has a corresponding platform. When a train arrives at a platform track of a station, passengers may stay on the train, get off the train and wait at the current platform for another train, or get off the train and walk to another platform for changing trains.

To ensure safety, there is a minimum time interval g_i (i.e., arrival headway) required between two consecutive arrivals at station s_i (for $i = 2, 3, \ldots, n$), and there is a minimum time interval h_i (i.e., departure headway)

Table 2: Summary of input data.

Type of data	Notation	Description
Railway network data	S	set of all stations ($S = \{s_1, s_2, \dots, s_n\}$)
	g_i	minimum headway between arrivals at station s_i
	h_i	minimum headway between departures from station s_i
	m_i	number of platform tracks at station s_i ($m_i \geq 1$)
	m'_i	number of passing tracks at station s_i ($m'_i = 0$ or 1)
	$\tau_{ill'}$	time for a passenger to walk from the lth platform to the l'th platform of station s_i
Train data	K	set of all trains
	o_k	station index of train k's origin station
	d_k	station index of train k's destination station
	S_k	set of stations that train k may skip ($S_k \subseteq S$)
	$\bar{S}_k$	set of stations that train k must skip ($\bar{S}_k \subseteq S_k$)
	p_k	earliest possible start time of operation of train k at station s_{o_k}
	q_k	latest allowed completion time of operation of train k at station s_{d_k}
	α_{ki}	time for train k to traverse $s_i \to s_{i+1}$ if train k does not stop at stations s_i and s_{i+1}
	α'_k	additional amount of time for train k to traverse a track segment due to acceleration
	α''_k	additional amount of time for train k to traverse a track segment due to deceleration
	β_{ki}	minimum required dwell time of train k at station s_i if train k stops at s_i
	Γ_k	capacity of train k
	c_k	operating cost per unit time for train k to run on a track
	c'_k	operating cost per unit time for train k to dwell at a station
Passenger flow data	R	set of all passenger groups
	z_r	number of passengers in group r
	$\hat{o}_r$	station index of passenger group r's origin station
	$\hat{d}_r$	station index of passenger group r's destination station
	$\hat{p}_r$	arrival time of passenger group r at station $s_{\hat{o}_r}$
	$\hat{c}$	unit cost of a passenger's time spent on riding the trains and waiting for the trains
	$\hat{c}'$	unit cost of a passenger's time spent on walking between station platforms ($\hat{c}' \geq \hat{c}$)
	π_r	penalty for each unsatisfied passenger in group r

required between two consecutive departures at station s_i (for $i = 1, 2, \dots, n-1$). The passengers are allowed to change trains at any intermediate station, provided that the trains stop at that station. We let $\tau_{ill'}$ denote the amount of time that a passenger needs to walk from the lth platform to the l'th platform of station s_i, for $i \in \{2, 3, \dots, n-1\}$ and $l, l' \in \{1, 2, \dots, m_i\}$. We let $\tau_{ill'} = 0$ if $l = l'$.

2.2.2 Train data

Let K be the set of trains considered. For each $k \in K$, the input data of train k include: (i) the origin station s_{o_k}, where $o_k \in \{1, 2, \dots, n-1\}$; (ii) the destination station s_{d_k}, where $d_k \in \{2, 3, \dots, n\}$; (iii) a set S_k of stations

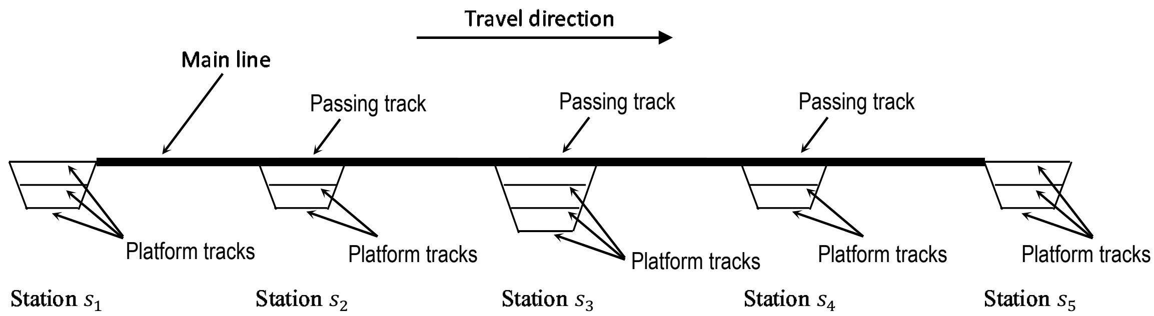

Figure 1: A railway line.

at which train k is allowed to skip, where $S_k \subseteq \{s_{o_k+1}, s_{o_k+2}, \ldots, s_{d_k-1}\}$; (iv) a set $\bar{S}_k$ of stations at which train k is required to skip, where $\bar{S}_k \subseteq S_k$; (v) the earliest time p_k that train k can start its operation at station s_{o_k}, and the latest time q_k that train k can complete its operation at station s_{d_k}, where $0 \leq p_k \leq q_k \leq T$; (vi) the amount of time α_{ki} that train k takes to traverse track segment $s_i \to s_{i+1}$ when the train skips stopping at both stations s_i and s_{i+1}; (vii) the additional amount of time α'_k that train k takes to traverse any track segment $s_i \to s_{i+1}$ incurred by the train's acceleration if the train stops at station s_i; (viii) the additional amount of time α''_k that train k takes to traverse any track segment $s_i \to s_{i+1}$ incurred by the train's deceleration if the train stops at station s_{i+1}; (ix) the minimum dwell time β_{ki} that train k needs to spend at station s_i if train k stops at that station; (x) the maximum number of passengers Γ_k that train k can carry at any time point; (xi) the operating cost c_k incurred per time unit when the train is traversing a track; and (xii) the operating cost c'_k incurred per time unit when the train is dwelling at a station while the train is in operation.

2.2.3 Passenger flow data

Let R be the set of all passenger groups. Each passenger group $r \in R$ is characterized by: (i) the number of passengers z_r in the group; (ii) an origin station $s_{\hat{o}_r}$, where $\hat{o}_r \in \{1, 2, \ldots, n-1\}$; (iii) a destination station $s_{\hat{d}_r}$, where $\hat{d}_r \in \{2, 3, \ldots, n\}$; (iv) the start time $\hat{p}_r$ of the passengers' journey.

We assume that passengers belonging to the same passenger group do not need to travel together, and that not every passenger's demand needs to be satisfied. If the demand of a passenger is satisfied, then a cost of $\hat{c}$ is imposed on each time unit that the passenger spends on riding the trains and waiting for the trains, and a cost of $\hat{c}'$ is imposed on each time unit that the passenger spends on walking between station platforms for transferring trains. Due to limited capacity of the railway system, some passengers' demand may not be satisfied, and the unsatisfied passengers need to seek other means of transportation. We model this by allowing unsatisfied passenger demand and imposing a penalty of π_r on each unsatisfied passenger in group r.

Remark 1 *In many train timetabling models with time-dependent passenger demand, a parameter is used to denote the number of passengers arriving at their origin station during a certain time interval and traveling to a particular destination; for example, a parameter D^t_{ij} is used to denote the number of passengers who arrive* *at station s_i during time interval $(t-1, t]$ and have a destination station s_j. If the input demand data is given as D_{ij}^t, then in our model, a passenger group r represents the triple (i, j, t), and we have $R = \{(i, j, t) \mid i < j;\ i, j = 1, 2, \ldots, n;\ t = 0, 1, \ldots, T\}$, $z_{(i,j,t)} = D_{ij}^t$, $\hat{o}_{(i,j,t)} = i$, $\hat{d}_{(i,j,t)} = j$, and $\hat{p}_{(i,j,t)} = t$. Note that in our model a time unit can be one minute, one hour, or any duration, and we do not consider the conversion between hour-dependent demand data and minute-dependent demand data; see Niu et al. (2015) for a discussion of formulations for high- and low-resolution demand input data.*

2.3 Objective and constraints

The objective is to determine a timetable for the trains, assign platform tracks to trains, and assign passengers to the train services, so that the total cost is minimized. The train timetabling decisions include determining each train's start time of operation, stop-skipping stations, and dwell time at every station. The passenger assignment decisions include determining the subset of passengers whose demands are to be satisfied. For those passengers to be served, the decisions also include determining the passengers' journey start time, the trains that they will take, and their transfer stations. The total cost includes the following cost components: (i) total operating cost of all trains; (ii) total penalty of unsatisfied passengers; and (iii) total cost of passengers' journey time, including the time spent by passengers on the trains or waiting at platforms, and the time spent on walking from one platform to another.

Note that in our model we measure passenger satisfaction by penalizing unsatisfied passenger demand and passenger journey time. Other commonly used passenger satisfaction measurements in train timetabling include penalties imposed on the shift and stretch of the timetable compared to an "ideal timetable" (see, e.g., Caprara et al. 2002), as well as penalties imposed on stop-skipping (see, e.g., Jiang et al. 2017). It is not difficult to include shift, stretch, and stop-skipping penalties into our time-space network formulation. However, for ease of presentation, we do not consider these passenger satisfaction measurements in this study. In some applications, it is reasonable to assume that passengers who cannot be served by the most desirable train service can always be served by another train at a later time. When our model is applied to such applications, the unsatisfied passenger penalty π_r should be set equal to infinity, and the length of the planning horizon may need to be increased in order to ensure that all the passengers can be served.

The following constraints need to be satisfied:

- Train operation constraints: Each train k is initially located at station s_{o_k} and is available for operation at time p_k. It needs to complete its operation at station s_{d_k} no later than time q_k. In addition, the travel time of train k on each track segment $s_i \to s_{i+1}$ must be equal to α_{ki} plus the additional time incurred by acceleration and deceleration, and the dwell time of train k at each station s_i must be no less than β_{ki} if train k stops at that station.
- Train capacity constraints: For each $k \in K$, at most Γ_k passengers can be carried by train k at any time point.

- Passenger journey time constraints: For each passenger belonging to passenger group r, if we choose to satisfy this passenger's demand, then the passenger must board a train at station $s_{\hat{o}_r}$ no earlier than $\hat{p}_k$. In addition, a passenger's transfer time between the lth platform and the l'th platform of a transfer station s_i must be no less than $\tau_{ill'}$.
- Headway constraints at stations: For each $i = 2, 3, \ldots, n$, the arrivals of any two trains to station s_i, regardless of which two platform/passing tracks of s_i these trains occupy, must be at least g_i time units apart. Similarly, the departures of any two trains from station s_i, regardless of which two platform/passing tracks of s_i these trains occupy, must be at least h_i time units apart.
- Overtaking constraints: For each $i = 1, 2, \ldots, n-1$, a train is forbidden to overtake another train on track segment $s_i \to s_{i+1}$; that is, overtaking can only take place at stations.
- Platform track constraints: Each platform track of a station can be occupied by at most one train at any time point.

Since the integrality of the number of passengers riding a train is not crucial in practice, in our model we do not require the number of passengers assigned to each part of the train service to be integer-valued.

3 The Time-Space Network Formulation

In this section we formulate our problem as a constrained MCMCNF problem. The underlying network is an acyclic directed time-space network $G = (V, A)$ which includes a dummy origin $\bar{o} \in V$ and a dummy destination $\bar{d} \in V$. There are $|K| + |R|$ commodities representing the flows of $|K|$ trains and the flows of $|R|$ passenger groups. For each $k \in K$, there is 1 unit of inflow (respectively outflow) of train k at origin $\bar{o}$ (respectively destination $\bar{d}$). For each $r \in R$, there are z_r units of inflow (respectively outflow) of passenger group r at origin $\bar{o}$ (respectively destination $\bar{d}$).

3.1 Network construction

The "time" dimension of network G covers the time instants $0, 1, \ldots, T$. Corresponding to each station s_i and each time instant t are $2m_i + m'_i + 1$ vertices. They include: (i) vertices ρ^t_{il} and $\bar{\rho}^t_{il}$ for $l = 1, 2, \ldots, m_i$, (ii) vertex φ^t_i if $m'_i = 1$, and (iii) vertex σ^t_i. Vertex ρ^t_{il} represents the arrival of a train at the lth platform track of station s_i at time t. Vertex $\bar{\rho}^t_{il}$ represents the departure of a train from the lth platform track of station s_i at time t. Vertex φ^t_i represents the passing track of station s_i at time t. Vertex σ^t_i represents the departure of a train from station s_i at time t after the train has dwelled at one of the platform tracks of s_i. Hence, the "space" dimension covers $\sum_{i=1}^n (2m_i + m'_i + 1)$ possible states at different stations, and

$$\begin{aligned} V = \{\bar{o}, \bar{d}\} &\cup \left\{\rho^t_{il}, \bar{\rho}^t_{il} \;\middle|\; i = 1, 2, \ldots, n;\ l = 1, 2, \ldots, m_i;\ t = 0, 1, \ldots, T\right\} \\ &\cup \left\{\varphi^t_i \;\middle|\; i = 1, 2, \ldots, n \text{ s.t. } m'_i = 1;\ t = 0, 1, \ldots, T\right\} \cup \left\{\sigma^t_i \;\middle|\; i = 1, 2, \ldots, n;\ t = 0, 1, \ldots, T\right\} \end{aligned}$$

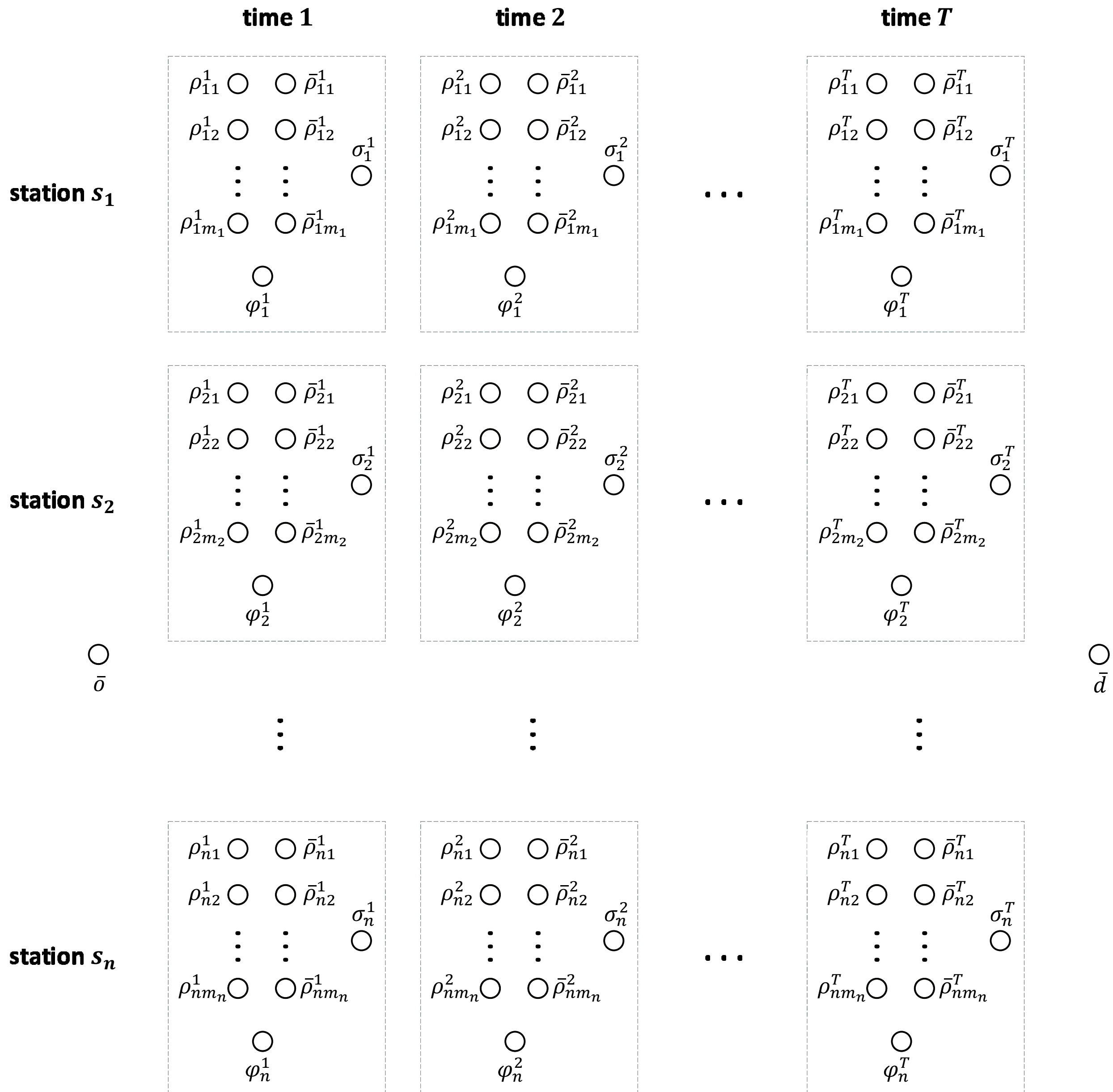

Figure 2: Vertices in time-space network G.

(see Figure 2). Arc set A of network G contains several types of arcs, with a vector of cost coefficients $\big(\xi^{k_1}(u,v),\ldots,\xi^{k_{|K|}}(u,v);\zeta^{r_1}(u,v),\ldots,\zeta^{r_{|R|}}(u,v)\big)$ associated with each arc $u \to v \in A$. The cost coefficient $\xi^k(u,v)$ represents the cost for train k to traverse arc $u \to v$, and the cost coefficient $\zeta^r(u,v)$ represents the cost for each passenger of group r to traverse arc $u \to v$. Each arc $u \to v \in A$ has an infinite capacity for each train (respectively passenger) when the cost coefficient for that train (respectively passenger) is finite. Descriptions of different types of arcs are given as follows.

3.1.1 Starting, ending, and unsatisfied-demand arcs

There are starting and ending arcs to ensure that the train operation constraints are satisfied. There are also starting and ending arcs to ensure that the passenger journey time constraints are satisfied. These arcs are given as follows.

For each $i \in \{1, 2, \ldots, n-1\}$, $l \in \{1, 2, \ldots, m_i\}$, and $t \in \{0, 1, \ldots, T\}$, there is a *starting arc* $\bar{o} \to \rho_{il}^t$. For each $k \in K$, if $i = o_k$ and $t \geq p_k$, then $\xi^k(\bar{o}, \rho_{il}^t) = 0$; otherwise, $\xi^k(\bar{o}, \rho_{il}^t) = +\infty$. This arc allows train k to start its operation at its origin station s_{o_k} at time $t \geq p_k$, and train k starts incurring its operating cost at time t. Passengers are not allowed to traverse this arc. Hence, $\zeta^r(\bar{o}, \rho_{il}^t) = +\infty$ for all $r \in R$.

For each $i \in \{1, 2, \ldots, n-1\}$, $l \in \{1, 2, \ldots, m_i\}$, and $t \in \{0, 1, \ldots, T\}$, there is also a starting arc $\bar{o} \to \bar{\rho}_{il}^t$. For each $r \in R$, if $i = \hat{o}_r$ and $t = \hat{p}_r$, then $\zeta^r(\bar{o}, \bar{\rho}_{il}^t) = 0$; otherwise, $\zeta^r(\bar{o}, \bar{\rho}_{il}^t) = +\infty$. This arc allows passengers of group r to start their trips from their origin station $s_{\hat{o}_r}$ at time $t = \hat{p}_r$. Note that $\hat{p}_r$ is the arrival time of passenger group r at station $s_{\hat{o}_r}$, and thus each passenger of group r starts incurring his/her unit cost $\hat{c}$ at time $\hat{p}_r$. Trains are not allowed to traverse this arc. Hence, $\xi^k(\bar{o}, \bar{\rho}_{il}^t) = +\infty$ for all $k \in K$.

For each $i \in \{2, 3, \ldots, n\}$ and $t \in \{0, 1, \ldots, T\}$, there is an *ending arc* $\sigma_i^t \to \bar{d}$. For each $k \in K$, if $i = d_k$ and $t \leq q_k$, then $\xi^k(\sigma_i^t, \bar{d}) = 0$; otherwise, $\xi^k(\sigma_i^t, \bar{d}) = +\infty$. This arc allows train k to complete its operation at its destination station s_{d_k} at time $t \leq q_k$. Passengers are not allowed to traverse this arc. Hence, $\zeta^r(\sigma_i^t, \bar{d}) = +\infty$ for all $r \in R$.

For each $i \in \{2, 3, \ldots, n\}$, $l \in \{1, 2, \ldots, m_i\}$, and $t \in \{0, 1, \ldots, T\}$, there is also an ending arc $\rho_{il}^t \to \bar{d}$. For each $r \in R$, if $i = \hat{d}_r$, then $\zeta^r(\rho_{il}^t, \bar{d}) = 0$; otherwise, $\zeta^r(\rho_{il}^t, \bar{d}) = +\infty$. This arc allows passengers of group r to complete their trips at their destination station $s_{\hat{d}_r}$ at time t. Trains are not allowed to traverse this arc. Hence, $\xi^k(\rho_{il}^t, \bar{d}) = +\infty$ for all $k \in K$.

There is an *unsatisfied-demand arc* $\bar{o} \to \bar{d}$. A passenger traversing this arc represents the situation that this passenger's demand is not satisfied. The unit cost for passenger group r to traverse this arc is $\zeta^r(\bar{o}, \bar{d}) = \pi_r$, for any $r \in R$. Trains are not allowed to traverse this arc. Hence, $\xi^k(\bar{o}, \bar{d}) = +\infty$ for all $k \in K$.

3.1.2 Dwelling, waiting, and transfer arcs

For each $i \in \{1, 2, \ldots, n\}$, $l \in \{1, 2, \ldots, m_i\}$, and $t, t' \in \{0, 1, \ldots, T\}$, there is a *dwelling arc* $\rho_{il}^t \to \bar{\rho}_{il}^{t'}$. For each $k \in K$, if $o_k \leq i \leq d_k$, $s_i \in S \setminus \bar{S}_k$, $t \geq p_k$, and $t' = t + \beta_{ki} \leq q_k$, then $\xi^k(\rho_{il}^t, \bar{\rho}_{il}^{t'}) = c'_k(t' - t)$; otherwise, $\xi^k(\rho_{il}^t, \bar{\rho}_{il}^{t'}) = +\infty$. This arc allows train k to stop at the lth platform track of station s_i during the time interval $[t, t']$ to satisfy its minimum dwell time requirement. During this time interval, passengers can alight and board the train. A passenger traversing arc $\rho_{il}^t \to \bar{\rho}_{il}^{t'}$ represents the situation where the passenger is staying in the train when the train is dwelling at the lth platform track of station s_i during time interval $[t, t']$. For each $r \in R$, the cost for a passenger of group r to traverse this arc is $\zeta^r(\rho_{il}^t, \bar{\rho}_{il}^{t'}) = \hat{c}(t' - t)$.

For each $i \in \{1, 2, \ldots, n\}$, $l \in \{1, 2, \ldots, m_i\}$, and $t \in \{0, 1, \ldots, T-1\}$, there is a *waiting arc* $\bar{\rho}_{il}^t \to \bar{\rho}_{il}^{t+1}$. This arc allows a train to continue to dwell at the lth platform track of station s_i during the time interval $[t, t+1]$

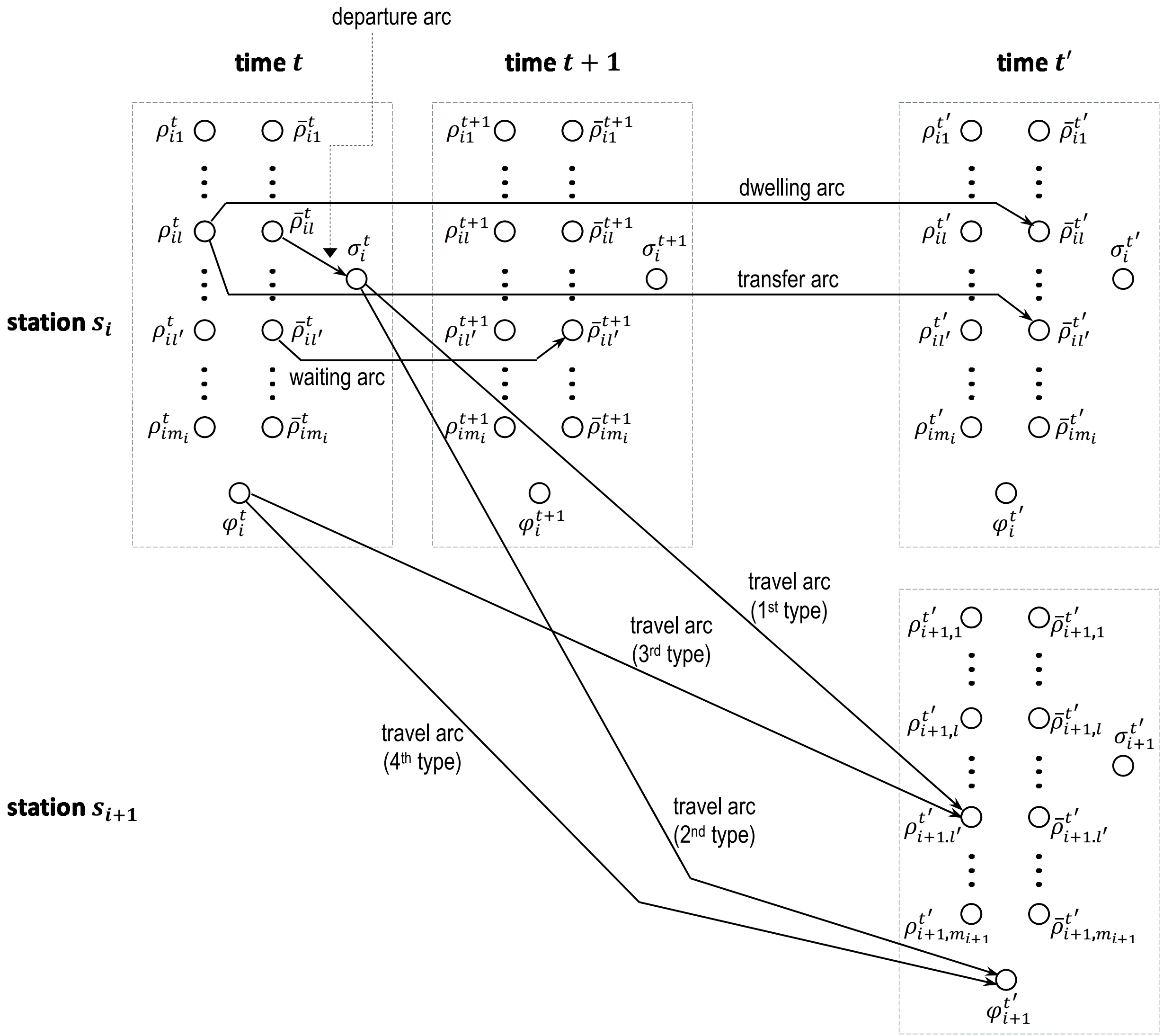

Figure 3: Dwelling, waiting, transfer, departure, and travel arcs in time-space network G.

after that train has satisfied its minimum dwell time requirement at s_i. It also allows passengers to wait at the lth platform of station s_i during the time interval $[t, t+1]$ if they arrive at the platform too early. For each $k \in K$, if $o_k \le i \le d_k$, $s_i \in S \setminus \bar{S}_k$, and $p_k \le t \le q_k - 1$, then $\xi^k(\bar{\rho}^t_{il}, \bar{\rho}^{t+1}_{il}) = c'_k$; otherwise, $\xi^k(\bar{\rho}^t_{il}, \bar{\rho}^{t+1}_{il}) = +\infty$. For each $r \in R$, if $\hat{o}_r \le i < \hat{d}_r$ and $t \ge \hat{p}_r$, then $\zeta^r(\bar{\rho}^t_{il}, \bar{\rho}^{t+1}_{il}) = \hat{c}$; otherwise, $\zeta^r(\bar{\rho}^t_{il}, \bar{\rho}^{t+1}_{il}) = +\infty$.

For each $i \in \{2, 3, \ldots, n-1\}$, $l, l' \in \{1, 2, \ldots, m_i\}$, and $t, t' \in \{0, 1, \ldots, T\}$, there is a *transfer arc* $\rho^t_{il} \to \bar{\rho}^{t'}_{il'}$. For each passenger group $r \in R$, if $\hat{o}_r \le i \le \hat{d}_r$, $t \ge \hat{p}_r$, then $\zeta^r(\rho^t_{il}, \bar{\rho}^{t'}_{il'}) = \hat{c}'(t'-t)$, otherwise, $\zeta^r(\rho^t_{il}, \bar{\rho}^{t'}_{il'}) = +\infty$. This arc allows passengers to walk from the lth platform to the l'th platform of station s_i during the time interval $[t, t']$ for changing trains. If $l = l'$, then this arc represents the situation where the passengers simply get off the train so that they can wait for another train at the same platform. Trains are not allowed to traverse this arc. Hence, $\xi^k(\rho^t_{il}, \bar{\rho}^{t'}_{il'}) = +\infty$ for all $k \in K$. Figure 3 depicts the dwelling, waiting, and transfer arcs for station s_i.

3.1.3 Departure and travel arcs

For each $i \in \{1, 2, \ldots, n\}$, $l \in \{1, 2, \ldots, m_i\}$, and $t \in \{0, 1, \ldots, T\}$, there is a *departure arc* $\bar{\rho}_{il}^t \to \sigma_i^t$ (see Figure 3). This arc represents the situation where a train has finished dwelling at the lth platform track of station s_i at time t and is about to leave the station. For each train $k \in K$, if $o_k \leq i \leq d_k$, $s_i \in S \setminus \bar{S}_k$, and $t \leq q_k$, then $\xi^k(\bar{\rho}_{il}^t, \sigma_i^t) = 0$; otherwise, $\xi^k(\bar{\rho}_{il}^t, \sigma_i^t) = +\infty$. For each passenger group $r \in R$, if $\hat{o}_r \leq i \leq \hat{d}_r - 1$, then $\zeta^r(\bar{\rho}_{il}^t, \sigma_i^t) = 0$; otherwise, $\zeta^r(\bar{\rho}_{il}^t, \sigma_i^t) = +\infty$.

There are *travel arcs* in network G to allow trains to travel from one station s_i to the next station s_{i+1}. There are four types of travel arcs that cover four different scenarios, depending on whether or not the train stops at station s_i, and whether or not the train stops at station s_{i+1} (see Figure 3).

The first type of travel arcs is for the scenario where the train stops at both stations s_i and s_{i+1}. For each $i \in \{1, 2, \ldots, n-1\}$, $l \in \{1, 2, \ldots, m_{i+1}\}$, and $t, t' \in \{0, 1, \ldots, T\}$, there is a travel arc $\sigma_i^t \to \rho_{i+1,l}^{t'}$ of this type. A train traversing this arc represents the situation where (i) the train has finished stopping at station s_i, (ii) it is traveling from station s_i to station s_{i+1}, (iii) it will stop at the lth platform track of station s_{i+1}, and (iv) the traveling takes place during the time interval $[t, t']$. Note that if train k traverses this arc, then train k stops at both s_i and s_{i+1}. This implies that train k's travel time on track segment $s_i \to s_{i+1}$ is $\alpha_{ki} + \alpha'_k + \alpha''_k$, and thus $t' = t + \alpha_{ki} + \alpha'_k + \alpha''_k$. For each $k \in K$, if $o_k \leq i \leq d_k - 1$, $s_i \in S \setminus \bar{S}_k$, $s_{i+1} \in S \setminus \bar{S}_k$, $t \geq p_k$, and $t' = t + \alpha_{ki} + \alpha'_k + \alpha''_k \leq q_k$, then $\xi^k(\sigma_i^t, \rho_{i+1,l}^{t'}) = c_k(t' - t)$; otherwise, $\xi^k(\sigma_i^t, \rho_{i+1,l}^{t'}) = +\infty$.

The second type of travel arcs is for the scenario where the train stops at stations s_i but skips station s_{i+1}. For each $i \in \{1, 2, \ldots, n-1\}$ and $t, t' \in \{0, 1, \ldots, T\}$ such that $m'_{i+1} = 1$, there is a travel arc $\sigma_i^t \to \varphi_{i+1}^{t'}$ of this type. A train traversing this arc represents the situation where (i) the train has finished stopping at station s_i, (ii) it is traveling from station s_i to station s_{i+1}, (iii) it will skip stopping at station s_{i+1}, and (iv) the traveling takes place during the time interval $[t, t']$. Note that if train k traverses this arc, then train k stops at s_i but not at s_{i+1}. This implies that train k's travel time on track segment $s_i \to s_{i+1}$ is $\alpha_{ki} + \alpha'_k$, and thus $t' = t + \alpha_{ki} + \alpha'_k$. For each $k \in K$, if $o_k \leq i \leq d_k - 1$, $s_i \in S \setminus \bar{S}_k$, $s_{i+1} \in S_k$, $t \geq p_k$, and $t' = t + \alpha_{ki} + \alpha'_k \leq q_k$, then $\xi^k(\sigma_i^t, \varphi_{i+1}^{t'}) = c_k(t' - t)$; otherwise, $\xi^k(\sigma_i^t, \varphi_{i+1}^{t'}) = +\infty$.

The third type of travel arcs is for the scenario where the train skips stations s_i but stops at station s_{i+1}. For each $i \in \{1, 2, \ldots, n-1\}$, $l \in \{1, 2, \ldots, m_{i+1}\}$, and $t, t' \in \{0, 1, \ldots, T\}$ such that $m'_i = 1$, there is a travel arc $\varphi_i^t \to \rho_{i+1,l}^{t'}$ of this type. A train traversing this arc represents the situation where (i) the train has skipped stopping at station s_i, (ii) it is traveling from station s_i to station s_{i+1}, (iii) it will stop at the lth platform track of station s_{i+1}, and (iv) the traveling takes place during the time interval $[t, t']$. Note that if train k traverses this arc, then train k stops at s_{i+1} but not at s_i. This implies that train k's travel time on track segment $s_i \to s_{i+1}$ is $\alpha_{ki} + \alpha''_k$, and thus $t' = t + \alpha_{ki} + \alpha''_k$. For each $k \in K$, if $o_k \leq i \leq d_k - 1$, $s_i \in S_k$, $s_{i+1} \in S \setminus \bar{S}_k$, $t \geq p_k$, and $t' = t + \alpha_{ki} + \alpha''_k \leq q_k$, then $\xi^k(\varphi_i^t, \rho_{i+1,l}^{t'}) = c_k(t' - t)$; otherwise, $\xi^k(\varphi_i^t, \rho_{i+1,l}^{t'}) = +\infty$.

The fourth type of travel arcs is for the scenario where the train skips both stations s_i and s_{i+1}. For each $i \in \{1, 2, \ldots, n-1\}$ and $t, t' \in \{0, 1, \ldots, T\}$ such that $m'_i = m'_{i+1} = 1$, there is a travel arc $\varphi_i^t \to \varphi_{i+1}^{t'}$ of this type. A train traversing this arc represents the situation where (i) the train has skipped stopping at station s_i, (ii) it is traveling from station s_i to station s_{i+1}, (iii) it will skip stopping at station s_{i+1}, and (iv) the traveling takes place during the time interval $[t, t']$. For each $k \in K$, if $o_k \leq i \leq d_k - 1$, $s_i \in S_k$, $s_{i+1} \in S_k$, $t \geq p_k$, and $t' = t + \alpha_{ki} \leq q_k$, then $\xi^k(\varphi_i^t, \varphi_{i+1}^{t'}) = c_k(t' - t)$; otherwise, $\xi^k(\varphi_i^t, \varphi_{i+1}^{t'}) = +\infty$.

Passengers are allowed to traverse travel arcs $\sigma_i^t \to \rho_{i+1,l}^{t'}$, $\sigma_i^t \to \varphi_{i+1}^{t'}$, $\varphi_i^t \to \rho_{i+1,l}^{t'}$, and $\varphi_i^t \to \varphi_{i+1}^{t'}$. A passenger traversing one of these arcs represents the situation where the passenger is riding a train from station s_i to station s_{i+1} during the time interval $[t, t']$. For each $r \in R$, if $\hat{o}_r \leq i \leq \hat{d} - 1$ and $t \geq \hat{p}_r$, then $\zeta^r(\sigma_i^t, \rho_{i+1,l}^{t'}) = \zeta^r(\sigma_i^t, \varphi_{i+1}^{t'}) = \zeta^r(\varphi_i^t, \rho_{i+1,l}^{t'}) = \zeta^r(\varphi_i^t, \varphi_{i+1}^{t'}) = \hat{c}(t' - t)$; otherwise, $\zeta^r(\sigma_i^t, \rho_{i+1,l}^{t'}) = \zeta^r(\sigma_i^t, \varphi_{i+1}^{t'}) = \zeta^r(\varphi_i^t, \rho_{i+1,l}^{t'}) = \zeta^r(\varphi_i^t, \varphi_{i+1}^{t'}) = +\infty$.

Remark 2 *The size of network $G = (V, A)$ can be reduced substantially by not including some of the arcs that cannot traversed by any train or passenger. This can be done by not creating the following arcs during the construction of the network: (i) any arc $u \to v$ where $\xi^k(u, v) = +\infty$ for all $k \in K$ and $\zeta^r(u, v) = +\infty$ for all $r \in R$; and (ii) any dwelling, departure, or travel arc $u \to v$ where $\xi^k(u, v) = +\infty$ for all $k \in K$. In addition, for any $v \in V \setminus \{\bar{o}\}$, if v does not have any incoming arc, then none of the trains and passengers can reach vertex v, and thus vertex v and all of its outgoing arcs do not need to be included in the network.*

3.2 Constraints on the multi-commodity flow

We assume that all α_{ki} and β_{ki} values are strictly positive. Note that each arc $u \to v \in A$ is of one of the following forms: (i) $\bar{o} \to v$ for some $v \in V \setminus \{\bar{o}\}$; (ii) $u \to \bar{d}$ for some $u \in V \setminus \{\bar{d}\}$; (iii) $\rho_{il}^t \to \bar{\rho}_{il}^t$ for some i, l, and t; (iv) $\bar{\rho}_{il}^t \to \sigma_i^t$ for some i, l, and t; or (v) $u \to v$ where the time index of v is larger than the time index of u. Hence, network G is acyclic.

A path connecting $\bar{o}$ and $\bar{d}$ in this acyclic network can potentially be a train's schedule (i.e., a sequence of changes in location of a train over time) or a passenger's schedule (i.e., a sequence of changes in location of a passenger over time). For any $k \in K$, if there exists a path P connecting $\bar{o}$ and $\bar{d}$ in which the cost coefficient $\xi^k(u, v)$ is finite for all $u \to v \in P$, then path P corresponds to a feasible schedule for train k. For any $r \in R$, if there exists a path P connecting $\bar{o}$ and $\bar{d}$ in which the cost coefficient $\zeta^r(u, v)$ is finite for all $u \to v \in P$, then either path P is the unsatisfied-demand arc $\bar{o} \to \bar{d}$, or path P corresponds to a feasible schedule for a passenger that belongs to passenger group r. However, a solution of this MCMCNF problem yields a feasible solution to our train timetabling problem only when the multi-commodity flow satisfies the following constraints:

- Integer flow constraints: The train flow along each $u \to v \in A$ is required to be binary-valued.
- Train capacity constraints: The total flow of passengers along a departure (respectively travel) arc must be zero if no train traverses this arc, and the total flow of passengers along a departure (respectively travel) arc must be no greater than Γ_k if train k also traverses this arc.
- Arrival headway constraints: For each $i = 2, 3, \ldots, n$, the arrivals of any two trains to station s_i must be least g_i time units apart (see Section 2.3). Thus, for each $i = 2, 3, \ldots, n$ and each $t_1 = 0, 1, \ldots, T - g_i + 1$,

we allow no more than one train to finish traversing the track segment $s_{i-1} \to s_i$ during the time interval $[t_1, t_1 + g_i - 1]$. Hence, for $i = 2, 3, \ldots, n$ and $t_1 = 0, 1, \ldots, T - g_i + 1$, we impose a constraint that the total train flow along the travel arcs in the arc subset

$$\begin{aligned} C^1_{it_1} = A \cap \Big[& \{\sigma^t_{i-1} \to \rho^{t'}_{il} \mid l = 1, 2, \ldots, m_i;\ t, t' = 0, 1, \ldots, T;\ t_1 \le t' \le t_1 + g_i - 1\} \\ & \cup \{\varphi^t_{i-1} \to \rho^{t'}_{il} \mid m'_{i-1} = 1;\ l = 1, 2, \ldots, m_i;\ t, t' = 0, 1, \ldots, T;\ t_1 \le t' \le t_1 + g_i - 1\} \\ & \cup \{\sigma^t_{i-1} \to \varphi^{t'}_{i} \mid m'_i = 1;\ t, t' = 0, 1, \ldots, T;\ t_1 \le t' \le t_1 + g_i - 1\} \\ & \cup \{\varphi^t_{i-1} \to \varphi^{t'}_{i} \mid m'_{i-1} = m'_i = 1;\ t, t' = 0, 1, \ldots, T;\ t_1 \le t' \le t_1 + g_i - 1\} \Big] \end{aligned}$$

is at most one. Note that a train that finishes traversing the track segment $s_{i-1} \to s_i$ during the time interval $[t_1, t_1 + g_i - 1]$ must traverse one of the travel arcs $\sigma^t_{i-1} \to \rho^{t'}_{il}$, $\varphi^t_{i-1} \to \rho^{t'}_{il}$, $\sigma^t_{i-1} \to \varphi^{t'}_{i}$, and $\varphi^t_{i-1} \to \varphi^{t'}_{i}$ for some l, t, and t' such that $t' \in [t_1, t_1 + g_i - 1]$. Thus, the arc subset $C^1_{it_1}$ collects all such travel arcs.

- Departure headway constraints: For each $i = 1, 2, \ldots, n-1$, the departures of any two trains from station s_i must be at least h_i time units apart (see Section 2.3). Thus, for each $i = 1, 2, \ldots, n-1$ and each $t_1 = 0, 1, \ldots, T - h_i + 1$, we allow no more than one train to start traversing the track segment $s_i \to s_{i+1}$ during the time interval $[t_1, t_1 + h_i - 1]$. Hence, for $i = 1, 2, \ldots, n-1$ and $t_1 = 0, 1, \ldots, T - h_i + 1$, we impose a constraint that the total train flow along the travel arcs in the arc subset

$$\begin{aligned} C^2_{it_1} = A \cap \Big[& \{\sigma^t_{i} \to \rho^{t'}_{i+1,l} \mid l = 1, 2, \ldots, m_{i+1};\ t, t' = 0, 1, \ldots, T;\ t_1 \le t \le t_1 + h_i - 1\} \\ & \cup \{\sigma^t_{i} \to \varphi^{t'}_{i+1} \mid m'_{i+1} = 1;\ t, t' = 0, 1, \ldots, T;\ t_1 \le t \le t_1 + h_i - 1\} \\ & \cup \{\varphi^t_{i} \to \rho^{t'}_{i+1,l} \mid m'_i = 1;\ l = 1, 2, \ldots, m_{i+1};\ t, t' = 0, 1, \ldots, T;\ t_1 \le t \le t_1 + h_i - 1\} \\ & \cup \{\varphi^t_{i} \to \varphi^{t'}_{i+1} \mid m'_i = m'_{i+1} = 1;\ t, t' = 0, 1, \ldots, T;\ t_1 \le t \le t_1 + h_i - 1\} \Big] \end{aligned}$$

is at most one. Note that a train that starts traversing the track segment $s_i \to s_{i+1}$ during the time interval $[t_1, t_1 + h_i - 1]$ must traverse one of the travel arcs $\sigma^t_i \to \rho^{t'}_{i+1,l}$, $\sigma^t_i \to \varphi^{t'}_{i+1}$, $\varphi^t_i \to \rho^{t'}_{i+1,l}$, and $\varphi^t_i \to \varphi^{t'}_{i+1}$ for some l, t, and t' such that $t \in [t_1, t_1 + h_i - 1]$. Thus, the arc subset $C^2_{it_1}$ collects all such travel arcs.

- Overtaking constraints: For each $i = 1, 2, \ldots, n-1$, a train is not allowed to overtake another train when traveling on the track segment $s_i \to s_{i+1}$. Thus, for each $i = 1, 2, \ldots, n-1$ and each pair of distinct time points t_1 and t_2 such that $t_1 < t_2$, whenever there are a train k arriving at station s_{i+1} at time t_2 and another train k' departing from station s_i at time t_1, either train k departs from s_i earlier than t_1, or train k' arrives at s_{i+1} earlier than t_2. In other words, we disallow the situation where train k departs from s_i later than t_1 and train k' arrives at s_{i+1} later than t_2. Denote

$$\begin{aligned} B_{it_1t_2} = A \cap \Big[& \{\sigma^t_{i} \to \rho^{t_2}_{i+1,l} \mid l = 1, 2, \ldots, m_{i+1};\ t_1 < t < t_2\} \\ & \cup \{\sigma^t_{i} \to \varphi^{t_2}_{i+1} \mid m'_{i+1} = 1;\ t_1 < t < t_2\} \\ & \cup \{\varphi^t_{i} \to \rho^{t_2}_{i+1,l} \mid m'_i = 1;\ l = 1, 2, \ldots, m_{i+1};\ t_1 < t < t_2\} \\ & \cup \{\varphi^t_{i} \to \varphi^{t_2}_{i+1} \mid m'_i = m'_{i+1} = 1;\ t_1 < t < t_2\} \Big] \end{aligned}$$

and

$$
\begin{aligned}
B'_{it_1t_2} = A \cap \Big[& \{\sigma_i^{t_1} \to \rho_{i+1,l}^{t'} \mid l = 1, 2, \ldots, m_{i+1};\ t_2 < t' \leq T\} \\
& \cup \{\sigma_i^{t_1} \to \varphi_{i+1}^{t'} \mid m'_{i+1} = 1;\ t_2 < t' \leq T\} \\
& \cup \{\varphi_i^{t_1} \to \rho_{i+1,l}^{t'} \mid m'_i = 1;\ l = 1, 2, \ldots, m_{i+1};\ t_2 < t' \leq T\} \\
& \cup \{\varphi_i^{t_1} \to \varphi_{i+1}^{t'} \mid m'_i = m'_{i+1} = 1;\ t_2 < t' \leq T\} \Big].
\end{aligned}
$$

Then, for $i = 1, 2, \ldots, n-1$ and $0 \leq t_1 < t_2 < T$ such that $B_{it_1t_2}, B'_{it_1t_2} \neq \emptyset$, we impose a constraint that the total train flow along the travel arcs in the arc subset

$$C^3_{it_1t_2} = B_{it_1t_2} \cup B'_{it_1t_2}$$

is at most one.

- Platform track constraints: For each $i = 1, 2, \ldots, n$ and each $l = 1, 2, \ldots, m_i$, the lth platform track of station s_i can be occupied by at most one train at each time instant. Thus, for each $i = 1, 2, \ldots, n$, each $l = 1, 2, \ldots, m_i$, and each $t_1 = 0, 1, \ldots, T$, we impose a constraint that the total train flow along the dwelling and waiting arcs in the arc subset

$$C^4_{ilt_1} = A \cap \Big[\{\rho_{il}^{t} \to \bar{\rho}_{il}^{t'} \mid t \leq t_1 \leq t'\} \cup \{\bar{\rho}_{il}^{t_1-1} \to \bar{\rho}_{il}^{t_1}\} \Big]$$

is at most one.

The arrival headway constraints, departure headway constraints, overtaking constraints, and platform track constraints share a common feature that they can be expressed as a collection of *incompatible arcs*. Denote

$$
\begin{aligned}
\mathcal{C} = & \{C^1_{it} \mid i = 2, 3, \ldots, n;\ t = 0, 1, \ldots, T - g_i + 1\} \cup \{C^2_{it} \mid i = 1, 2, \ldots, n-1;\ t = 0, 1, \ldots, T - h_i + 1\} \\
& \cup \{C^3_{it_1t_2} \mid i = 1, \ldots, n-1;\ 0 \leq t_1 < t_2 < T \text{ s.t. } B_{it_1t_2}, B'_{it_1t_2} \neq \emptyset\} \\
& \cup \{C^4_{ilt} \mid i = 1, \ldots, n;\ l = 1, 2, \ldots, m_i;\ t = 0, 1, \ldots, T\}.
\end{aligned}
$$

Then, for any $C \in \mathcal{C}$, the total flow along the arcs in C cannot exceed one.

Note that there are other ways to represent the overtaking constraints in a time-space network (see, e.g., Caprara et al. 2002; Xu et al. 2018). Because these constraints will be relaxed in our Lagrangian relaxation algorithm (see Section 4.1), a large number of overtaking constraints will result in a large number of Lagrangian multipliers, which will affect the convergence of the algorithm. Thus, a smaller number of overtaking constraints is more desirable. Here, we have defined $C^3_{it_1t_2}$ in such a way that there are only $O(nT^2)$ overtaking constraints in total.

3.3 Mixed integer programming formulation and computational complexity

For any $k \in K$, define $x^k_{uv} = 1$ if train k traverses arc $u \to v$ in the time-space network G, and $x^k_{uv} = 0$ otherwise. For any $r \in R$, define y^r_{uv} as the number of passengers in group r that traverse arc $u \to v$. Let A_{dep}

and A_{trv} denote the set of all departure arcs and the set of all travel arcs, respectively, in network G. The above constrained MCMCNF problem can be formulated as the following mixed integer linear program:

$$
\begin{aligned}
\mathbf{P}: \text{ Minimize } \quad & \textstyle\sum_{k\in K}\sum_{u\to v\in A}\xi^k(u,v)x_{uv}^k + \sum_{r\in R}\sum_{u\to v\in A}\zeta^r(u,v)y_{uv}^r && && (1)\\
\text{subject to } \quad & \textstyle\sum_{\{v:\bar{o}\to v\in A\}} x_{\bar{o}v}^k = 1, && \text{for all } k\in K && (2)\\
& \textstyle\sum_{\{u:u\to \bar{d}\in A\}} x_{u\bar{d}}^k = 1, && \text{for all } k\in K && (3)\\
& \textstyle\sum_{\{u:u\to v\in A\}} x_{uv}^k = \sum_{\{w:v\to w\in A\}} x_{vw}^k, && \text{for all } k\in K;\ v\in V\setminus\{\bar{o},\bar{d}\} && (4)\\
& \textstyle\sum_{\{v:\bar{o}\to v\in A\}} y_{\bar{o}v}^r = z_r, && \text{for all } r\in R && (5)\\
& \textstyle\sum_{\{u:u\to \bar{d}\in A\}} y_{u\bar{d}}^r = z_r, && \text{for all } r\in R && (6)\\
& \textstyle\sum_{\{u:u\to v\in A\}} y_{uv}^r = \sum_{\{w:v\to w\in A\}} y_{vw}^r, && \text{for all } r\in R;\ v\in V\setminus\{\bar{o},\bar{d}\} && (7)\\
& \textstyle\sum_{k\in K}\sum_{u\to v\in C} x_{uv}^k \le 1, && \text{for all } C\in\mathcal{C} && (8)\\
& \textstyle\sum_{r\in R} y_{uv}^r \le \sum_{k\in K}\Gamma_k x_{uv}^k, && \text{for all } u\to v\in A_{dep}\cup A_{trv} && (9)\\
& x_{uv}^k\in\{0,1\}, && \text{for all } k\in K;\ u\to v\in A && (10)\\
& y_{uv}^r\ge 0, && \text{for all } r\in R;\ u\to v\in A && (11)
\end{aligned}
$$

In objective function (1), the first part is the total operating cost of all trains, while the second part includes the total penalty of unsatisfied passengers and the total cost of time spent by passengers on traveling, waiting, and walking. Constraints (2) require the inflow of each train at vertex $\bar{o}$ to be 1, while constraints (3) require the outflow of each train at vertex $\bar{d}$ to be 1. Constraints (4) are the flow balance constraints for trains. Constraints (5) require the inflow of each passenger group r at $\bar{o}$ to be z_r, while constraints (6) require the outflow of each passenger group r at $\bar{d}$ to be z_r. Constraints (7) are the flow balance constraints for passengers. Constraints (8) cover all the arrival headway constraints, departure headway constraints, overtaking constraints, and platform track constraints described in Section 3.2. Constraints (9) are the train capacity constraints. Under these constraints, passengers can traverse a departure (respectively travel) arc only when a train is also traversing that arc, and the number of passengers traversing a departure (respectively travel) arc cannot exceed the capacity of the train traversing that arc. Constraints (10) specify the integer flow requirements for trains, while constraints (11) specify the nonnegativity requirements for passenger flows.

Similar to that of many other train timetabling models, the computational complexity of problem **P** is very high, as stated in the following theorem.

Theorem 1 *Problem* **P** *is strongly NP-hard.*

Proof. See Appendix A.

4 Lagrangian Relaxation Heuristic

In this section, we present a Lagrangian relaxation heuristic for solving the proposed model.

4.1 The Lagrangian relaxation

We use Lagrangian relaxation to relax constraints (8) and (9) of problem $\mathbf{P}$. Let $\lambda_C \geq 0$ and $\mu_{uv} \geq 0$ be the Lagrangian multipliers associated with constraints (8) and (9), respectively. Let λ and μ denote the vector of λ_C values and vector of μ_{uv} values, respectively. The relaxed problem is:

$$
\begin{array}{lll}
\tilde{\mathbf{P}}(\lambda,\mu): \text{ Minimize} & \sum_{k\in K}\sum_{u\to v\in A}\xi^k(u,v)x^k_{uv} + \sum_{r\in R}\sum_{u\to v\in A}\zeta^r(u,v)y^r_{uv} & \\
& + \sum_{C\in\mathcal{C}}\lambda_C\big(\sum_{k\in K}\sum_{u\to v\in C}x^k_{uv} - 1\big) & \\
& + \sum_{u\to v\in A_{dep}\cup A_{trv}}\mu_{uv}\big(\sum_{r\in R}y^r_{uv} - \sum_{k\in K}\Gamma_k x^k_{uv}\big) & \\
\text{subject to} & \sum_{\{v:\bar{o}\to v\in A\}}x^k_{\bar{o}v} = 1, & \text{for all } k\in K\\
& \sum_{\{u:u\to\bar{d}\in A\}}x^k_{u\bar{d}} = 1, & \text{for all } k\in K\\
& \sum_{\{u:u\to v\in A\}}x^k_{uv} = \sum_{\{w:v\to w\in A\}}x^k_{vw}, & \text{for all } k\in K;\ v\in V\setminus\{\bar{o},\bar{d}\}\\
& \sum_{\{v:\bar{o}\to v\in A\}}y^r_{\bar{o}v} = z_r, & \text{for all } r\in R\\
& \sum_{\{u:u\to\bar{d}\in A\}}y^r_{u\bar{d}} = z_r, & \text{for all } r\in R\\
& \sum_{\{u:u\to v\in A\}}y^r_{uv} = \sum_{\{w:v\to w\in A\}}y^r_{vw}, & \text{for all } r\in R;\ v\in V\setminus\{\bar{o},\bar{d}\}\\
& x^k_{uv}\in\{0,1\}, & \text{for all } k\in K;\ u\to v\in A\\
& y^r_{uv}\geq 0, & \text{for all } r\in R;\ u\to v\in A
\end{array}
$$

After removing the constant $-\sum_{C\in\mathcal{C}}\lambda_C$ from the objective function, this relaxed problem can be decomposed into $|K|+|R|$ independent subproblems.

The subproblem corresponding to each $k \in K$ is:

$$
\begin{array}{lll}
\tilde{\mathbf{P}}'_k(\lambda,\mu): \text{ Minimize} & \sum_{u\to v\in A}\xi^k(u,v)x^k_{uv} + \sum_{C\in\mathcal{C}}\lambda_C\sum_{u\to v\in C}x^k_{uv} - \sum_{u\to v\in A_{dep}\cup A_{trv}}\mu_{uv}\Gamma_k x^k_{uv} & \\
\text{subject to} & \sum_{\{v:\bar{o}\to v\in A\}}x^k_{\bar{o}v} = 1, & \\
& \sum_{\{u:u\to\bar{d}\in A\}}x^k_{u\bar{d}} = 1, & \\
& \sum_{\{u:u\to v\in A\}}x^k_{uv} = \sum_{\{w:v\to w\in A\}}x^k_{vw}, & \text{for all } v\in V\setminus\{\bar{o},\bar{d}\}\\
& x^k_{uv}\in\{0,1\}, & \text{for all } u\to v\in A
\end{array}
$$

Note that $A_{dep}\cap C = \emptyset$ for all $C\in\mathcal{C}$. Thus, $\sum_{\{C\in\mathcal{C}:u\to v\in C\}}\lambda_C = 0$ when $u\to v\in A_{dep}$. Let

$$
\delta^k_{uv} = \begin{cases}
\xi^k(u,v) - \mu_{uv}\Gamma_k, & \text{if } u\to v\in A_{dep};\\
\xi^k(u,v) + \sum_{\{C\in\mathcal{C}:u\to v\in C\}}\lambda_C - \mu_{uv}\Gamma_k, & \text{if } u\to v\in A_{trv};\\
\xi^k(u,v) + \sum_{\{C\in\mathcal{C}:u\to v\in C\}}\lambda_C, & \text{otherwise.}
\end{cases}
$$

Then, each subproblem $\tilde{\mathbf{P}}'_k(\lambda,\mu)$ is a shortest path problem with arc lengths δ^k_{uv}.

The subproblem corresponding to each $r \in R$ is:

$$\begin{aligned}
\text{Minimize} \quad & \textstyle\sum_{u\to v\in A} \zeta^r(u,v) y^r_{uv} + \sum_{u\to v\in A_{dep}\cup A_{trv}} \mu_{uv} y^r_{uv} \\
\text{subject to} \quad & \textstyle\sum_{\{v:\bar{o}\to v\in A\}} y^r_{\bar{o}v} = z_r, \\
& \textstyle\sum_{\{u:u\to \bar{d}\in A\}} y^r_{u\bar{d}} = z_r, \\
& \textstyle\sum_{\{u:u\to v\in A\}} y^r_{uv} = \sum_{\{w:v\to w\in A\}} y^r_{vw}, && \text{for all } v \in V\setminus\{\bar{o},\bar{d}\} \\
& y^r_{uv} \geq 0, && \text{for all } u\to v\in A
\end{aligned}$$

Letting $\bar{y}^r_{uv} = y^r_{uv}/z_r$, this subproblem can be rewritten as:

$$\begin{aligned}
\tilde{\mathbf{P}}''_r(\mu): \quad \text{Minimize} \quad & \textstyle\sum_{u\to v\in A} \zeta^r(u,v) z_r \bar{y}^r_{uv} + \sum_{u\to v\in A_{dep}\cup A_{trv}} \mu_{uv} z_r \bar{y}^r_{uv} \\
\text{subject to} \quad & \textstyle\sum_{\{v:\bar{o}\to v\in A\}} \bar{y}^r_{\bar{o}v} = 1, \\
& \textstyle\sum_{\{u:u\to \bar{d}\in A\}} \bar{y}^r_{u\bar{d}} = 1, \\
& \textstyle\sum_{\{u:u\to v\in A\}} \bar{y}^r_{uv} = \sum_{\{w:v\to w\in A\}} \bar{y}^r_{vw}, && \text{for all } v \in V\setminus\{\bar{o},\bar{d}\} \\
& \bar{y}^r_{uv} \geq 0, && \text{for all } u\to v\in A
\end{aligned}$$

Let

$$\gamma^r_{uv} = \begin{cases} \zeta^r(u,v) z_r + \mu_{uv} z_r, & \text{if } u\to v\in A_{dep}\cup A_{trv}; \\ \zeta^r(u,v) z_r, & \text{otherwise.} \end{cases}$$

Then, each subproblem $\tilde{\mathbf{P}}''_r(\mu)$ is a minimum cost network flow problem with unit flow from vertex $\bar{o}$ to vertex $\bar{d}$. By the integrality property of minimum cost network flows (Ahuja et al. 1993, p. 318), subproblem $\tilde{\mathbf{P}}''_r(\mu)$ has an integer minimum cost flow. Hence, each subproblem $\tilde{\mathbf{P}}''_r(\mu)$ is a shortest path problem with arc lengths γ^r_{uv}. Since G is acyclic, problems $\tilde{\mathbf{P}}'_k(\lambda,\mu)$ and $\tilde{\mathbf{P}}''_r(\mu)$ can be solved efficiently via a standard dynamic programming algorithm.

Let Z^* denote the optimal objective value of problem $\mathbf{P}$. Let $L(\lambda,\mu)$ denote the optimal objective value of $\tilde{\mathbf{P}}(\lambda,\mu)$. For any nonnegative vectors λ and μ, $L(\lambda,\mu)$ is a lower bound on Z^*.

4.2 Upper bound heuristic

Solving the lower bound problem yields a relaxed solution of problem $\mathbf{P}$. We now propose an upper bound heuristic for generating a feasible solution of $\mathbf{P}$ based on this relaxed solution. This heuristic first determines a train timetable and then determines the passenger schedule.

Recall that problem $\tilde{\mathbf{P}}(\lambda,\mu)$ is decomposable into $|K|+|R|$ independent shortest path subproblems in network G. Among these shortest path subproblems, $|K|$ of them correspond to the $|K|$ trains. To determine a train timetable, we apply a basic constructive heuristic (Caprara et al. 2002), which first ranks k by increasing optimal objective values of the $|K|$ shortest path subproblems and then schedules the trains one by one according to the ranked order. For each train k, we consider the paths in network G that do not contain any arc incompatible with those arcs traversed by the scheduled trains, and select one that has the minimum objective function value of the shortest path subproblem. To do so, we solve a shortest path problem in network G by excluding those arcs that violate arrival headway constraints, departure headway constraints, overtaking constraints, or platform track constraints caused by the scheduled trains.

The output of the above basic constructive heuristic is a set of x_{uv}^k values for all $k \in K$ and $u \to v \in A$. To determine the passenger schedule, we treat the x_{uv}^k values of the train timetable as input data and solve problem $\mathbf{P}$ optimally with only y_{uv}^r variables. After treating the x_{uv}^k values as input data, the problem becomes a standard MCMCNF problem with $|R|$ commodities. Note that the passenger schedule can be determined more efficiently by creating a smaller size version of network G before solving this MCMCNF problem. This smaller size version of G can be created as follows. First, any dwelling arc, departure arc, or travel arc $u \to v$ can be removed from G if the arc is not traversed by any train (i.e., $x_{uv}^k = 0$ for all $k \in K$). Second, starting arcs of the form $\bar{o} \to \rho_{il}^t$ and ending arcs of the form $\sigma_i^t \to \bar{d}$ can be removed from network G. Third, if no trains arrive at platform l of station s_i at time t, then all transfer arcs emanating from vertex ρ_{il}^t can be removed from G. Hence, a large proportion of dwelling arcs, departure arcs, travel arcs, starting arcs, ending arcs, and transfer arcs can be removed from G. Finally, we merge the consecutive waiting arcs along the train paths. Specifically, if a train traverses waiting arcs $\bar{\rho}_{il}^t \to \bar{\rho}_{il}^{t+1}, \bar{\rho}_{il}^{t+1} \to \bar{\rho}_{il}^{t+2}, \ldots, \bar{\rho}_{il}^{t'-1} \to \bar{\rho}_{il}^{t'}$, then we replace these waiting arcs by a new waiting arc $\bar{\rho}_{il}^t \to \bar{\rho}_{il}^{t'}$, where the unit cost for each passenger group to traverse the new waiting arc is equal to the sum of the corresponding unit costs of the original waiting arcs. For those starting and transfer arcs that terminate at vertices $\bar{\rho}_{il}^{t+1}, \bar{\rho}_{il}^{t+2}, \ldots, \bar{\rho}_{il}^{t'-1}$, we change the ends of the arcs to vertex $\bar{\rho}_{il}^{t'}$ and adjust the unit costs of the arcs accordingly. For example, if there is a starting or transfer arc $u \to \bar{\rho}_{il}^j$ with $t+1 \le j \le t'-1$, then we replace this arc by $u \to \bar{\rho}_{il}^{t'}$, where the unit cost for each passenger group to traverse this new arc is equal to the sum of the corresponding unit costs of arcs $u \to \bar{\rho}_{il}^j, \bar{\rho}_{il}^j \to \bar{\rho}_{il}^{j+1}, \ldots, \bar{\rho}_{il}^{t'-1} \to \bar{\rho}_{il}^{t'}$, and the capacity of this new arc is infinity.

Remark 3 *A simpler version of this upper bound heuristic is as follows: After constructing the train schedule, we construct the passenger schedules of the passenger groups one by one via solving a series of single-commodity minimum cost network flow problems. However, determining the passenger schedules of the passenger groups one by one requires almost the same amount of computational time required by solving an MCMCNF, while solving a standard MCMCNF is guaranteed no worse (and often better) than the passenger flow obtained by constructing the passenger schedules one by one. See Appendix B for a computational comparison of these two approaches.*

4.3 The overall solution procedure

For any λ and μ, $L(\lambda, \mu)$ is a lower bound on the optimal solution value of problem $\mathbf{P}$. We need to obtain near-optimal values of vectors λ and μ. We search for near-optimal λ and μ values via a subgradient optimization procedure.

Note that the lower bound $L(\lambda, \mu)$ equals the optimal objective value of $\tilde{\mathbf{P}}(\lambda, \mu)$. Thus, given an optimal solution $(\mathbf{x}, \mathbf{y})$ of $\tilde{\mathbf{P}}(\lambda, \mu)$, the value "$\sum_{k \in K} \sum_{u \to v \in C} x_{uv}^k - 1$" for each $C \in \mathcal{C}$ and the value "$\sum_{r \in R} y_{uv}^r - \sum_{k \in K} \Gamma_k x_{uv}^k$" for each $u \to v \in A_{dep} \cup A_{trv}$ form a subgradient vector η of the solution. Let η^m denote the mth component of η, for $m = 1, 2, \ldots, |\mathcal{C}| + |A_{dep} \cup A_{trv}|$. Let λ^m denote the mth component of the λ vector, for $m = 1, 2, \ldots, |\mathcal{C}|$. Let μ^m denote the mth component of the μ vector, for $m = 1, 2, \ldots, |A_{dep} \cup A_{trv}|$. The values of λ and μ are updated as follows:

$$\lambda^m \leftarrow \max\left\{\lambda^m + \theta \cdot \frac{UB - L(\lambda, \mu)}{\|\eta\|^2} \cdot \eta^m, \, 0\right\} \quad (m = 1, 2, \ldots, |\mathcal{C}|)$$

and

$$\mu^m \leftarrow \max\left\{\mu^m + \theta \cdot \frac{UB - L(\lambda, \mu)}{\|\eta\|^2} \cdot \eta^{m+|\mathcal{C}|}, \, 0\right\} \quad (m = 1, 2, \ldots, |A_{dep} \cup A_{trv}|),$$

where $\theta > 0$ is a prespecified step size parameter, and UB is the best feasible solution of problem $\mathbf{P}$ identified so far.

To improve the convergence of the subgradient optimization procedure, we further apply the *modified subgradient technique* proposed in Camerini et al. (1975). Let $\eta^{(i)}$ denote the η vector in the ith iteration of the procedure. We use a modified subgradient vector $\tilde{\eta}$ instead of η to update the Lagrangian multipliers. In the ith iteration, the modified subgradient vector $\tilde{\eta}^{(i)}$ is updated by

$$\tilde{\eta}^{(i)} \leftarrow \eta^{(i)} + b\tilde{\eta}^{(i-1)},$$

where b is a scalar defined as

$$b = \begin{cases} -a \cdot \frac{\tilde{\eta}^{(i-1)} \cdot \eta^{(i)}}{\|\tilde{\eta}^{(i-1)}\|^2}, & \text{if } \tilde{\eta}^{(i-1)} \cdot \eta^{(i)} < 0; \\ 0, & \text{otherwise;} \end{cases}$$

and a is a prespecified value such that $0 \leq a \leq 2$ (note: in the first iteration, $\tilde{\eta}^{(0)}$ is the vector with all components equal to 0). Moreover, because the number of relaxed constraints is very large, during implementation we use a *dynamic constraint-generation scheme* similar to that in Caprara et al. (2002) and Xu et al. (2018) to handle the relaxed constraints and determine the corresponding multipliers. Under this scheme, we dynamically identify constraints that the relaxed solution violates and store them in a constraint pool, and we update the multipliers corresponding to these violated constraints in the pool using the method described above.

Each iteration of the subgradient optimization procedure includes the following steps: (i) obtain a lower bound on problem $\mathbf{P}$; (ii) obtain a feasible solution of problem $\mathbf{P}$ using the upper bound heuristic presented in Section 4.2; (iii) identify constraints that the current lower bound solution has violated; (iv) update the modified subgradient vector; and (v) update the Lagrangian multipliers. Step (ii) involves an MCMCNF problem and is time-consuming to execute. We may skip this step in some iterations (see Section 5.1 for more details). This procedure is terminated when the computational time hits a prespecified limit.

5 Computational Study

We conduct a computational study to evaluate the performance of our Lagrangian relaxation heuristic. The heuristic is implemented in C# using a personal computer with a 3.70 GHz 10-core processor (Intel Core i9-10900 Processor) and 32 GB RAM. The MCMCNF problems in the upper bound procedure are solved by the commercial solver IBM CPLEX 12.6.0 (64-bit edition). The test data generation process is described in Section 5.1, and the computational results are reported in Section 5.2.

5.1 Generation of test instances

In our computational study, the test instances are generated randomly with the parameter settings selected based on the characteristics of the Beijing-Shanghai high-speed railway line (or JingHu line for short). Specifically, we use one direction of the JingHu line, including its major and minor stations, number of platform tracks at the stations, and distances between stations, as our railway network. We estimate the other train-related parameters such as train capacities, minimum arrival and departure headways, and unit operating costs by considering the characteristics of the trains operated by the Beijing-Shanghai High Speed Railway Co., Ltd. We estimate the passenger-related parameters in such a way that the resulting passenger traffic intensity and percentage of unsatisfied passenger closely match the reality.

The JingHu line has 23 stations, including 7 major stations and 16 minor stations. Major stations can be used as origin and destination stations for trains and also be visited by trains, while minor stations can only be visited by trains. Table 3 lists the detailed information of each station s_i from Beijing South to Shanghai Hongqiao along the JingHu line, including station name, station index i, number of platform tracks m_i, and number of passing tracks m_i'. The "Major/Minor" rows indicate whether station s_i is a major station or minor station. The "Distance" rows provide the distance between station s_i and the Beijing South station. The minimum headway between arrivals at each station is set equal to 4 minutes, while the minimum headway between departures at each station is set equal to 2 minutes.

To generate the train data, we consider those trains that are operated by the Beijing-Shanghai High Speed Railway Co., Ltd., and generate random data that capture their characteristics. According to the data in 2015, there are about 70 trains traveling in the direction "Beijing South to Shanghai Hongqiao" per day, where different OD pairs have different service frequencies. To capture these characteristics, we randomly select each train k's OD pair (o_k, d_k) from

$$\{(0,5),(0,10),(0,15),(0,22),(2,22),(5,22),(10,15),(10,22),(12,22),(15,22)\}.$$

To test the performance of our heuristic for different problem sizes, we divide the computational study into two parts. In the first part, we set $|K| = 30, 50, 70$, where OD pair $(0,22)$ is selected with probability 40/70, each of OD pairs $(0,5)$ and $(10,22)$ is selected with probability 6/70, each of OD pairs $(0,15)$, $(2,22)$, $(12,22)$, and $(15,22)$ is selected with probability 3/70, and each of OD pairs $(0,10)$, $(5,22)$, and $(10,15)$ is selected with

Table 3: Input data of JingHu line.

Station name	Beijing South	Langfang	Tianjin South	Cangzhou West	Dezhou East	Jinan West
Station index i	0	1	2	3	4	5
m_i	6	1	2	1	2	7
m'_i	0	1	1	1	1	1
Major/Minor	Major	Minor	Major	Minor	Minor	Major
Distance (km)	0	59	131	219	327	419
Station name	**Tai'an**	**Qufu West**	**Tengzhou East**	**Zaozhuang**	**Xuzhou East**	**Suzhou East**
Station index i	6	7	8	9	10	11
m_i	1	1	1	1	6	1
m'_i	1	1	1	1	1	1
Major/Minor	Minor	Minor	Minor	Minor	Major	Minor
Distance (km)	462	533	589	625	688	767
Station name	**Bengbu South**	**Dingyuan**	**Chuzhou**	**Nanjing South**	**Zhenjiang South**	**Danyang North**
Station index i	12	13	14	15	16	17
m_i	4	1	1	5	1	1
m'_i	1	1	1	1	1	1
Major/Minor	Major	Minor	Minor	Major	Minor	Minor
Distance (km)	844	897	959	1018	1087	1112
Station name	**Changzhou North**	**Wuxi East**	**Suzhou North**	**Kunshan South**	**Shanghai Hongqiao**	
Station index i	18	19	20	21	22	
m_i	1	1	1	1	10	
m'_i	1	1	1	1	0	
Major/Minor	Minor	Minor	Minor	Minor	Major	
Distance (km)	1144	1201	1227	1259	1302	

probability 2/70; see Table 4. In the second part, we test the performance of our heuristic with a larger number of trains but shorter travel distances. In this part, we set $|K| = 70, 110, 150$, where each of OD pairs $(0, 5)$, $(0, 10)$, $(0, 22)$, $(10, 15)$, $(12, 22)$, and $(15, 22)$ is selected with probability 1/6.

For each $k \in K$, the station sets S_k and $\bar{S}_k$ are generated as follows. We let S'_k denote the set of stations that train k may choose to skip or stop at (i.e., $S'_k = S_k \setminus \bar{S}_k$). Each station s_i along train k's route such that $s_i \notin \{s_{o_k}, s_{d_k}\}$ is selected and inserted into set S'_k with probability 1/10. Each station s_i along train k's route such that $s_i \notin \{s_{o_k}, s_{d_k}\} \cup S'_k$ is selected and inserted into set $\bar{S}_k$ with probability 1/10 (respectively 3/10) if s_i is a major (respectively minor) station. Then, we set $S_k = S'_k \cup \bar{S}_k$.

We consider two types of trains, including "G trains" with average speed of 300 km per hour (km/h) and "D trains" with average speed of 250 km per hour (see, e.g., Jiang et al. 2017). Based on the data in 2015,

Table 4: Parameter setting of computational study.

Train OD pair	$(0,5)$	$(0,10)$	$(0,15)$	$(0,22)$	$(2,22)$	$(5,22)$	$(10,15)$	$(10,22)$	$(12,22)$	$(15,22)$
Select probability (part 1)	6/70	2/70	3/70	40/70	3/70	2/70	2/70	6/70	3/70	3/70
Select probability (part 2)	1/6	1/6	0	1/6	0	0	1/6	0	1/6	1/6

Train speed type	G		D	
Train capacity	1200	600	1200	600
c_k	$(1.00)c$	$(0.50)c$	$(0.80)c$	$(0.40)c$
c'_k	$(0.90)c$	$(0.45)c$	$(0.72)c$	$(0.36)c$

(origin station, destination station)	(major, major)	(major, minor)	(minor, major)	(minor, minor)
ϑ's range	$[0.8, 1.0]$	$[0.5, 0.7]$	$[0.5, 0.7]$	$[0.2, 0.4]$

about one-seventh of the trains on the JingHu line were type D trains. Thus, in our computational study, for each $k \in K$, we randomly select train k's type from $\{\mathrm{G}, \mathrm{D}\}$, where type D is selected with probability 1/7, and type G is selected with probability 6/7. After determining train k's type, the time for train k to traverse each track segment along its route can be determined and is rounded to the nearest integer. For example, if train k is of type D, then its average speed is 250 km/h, and the amount of time for this train to traverse track segment $s_0 \to s_1$ with a length of 59 km is $(59/250) \cdot 60 \approx 14$ minutes. If train k is of type D, then both α'_k and α''_k are set to be 2 minutes. If train k is of type G, then α'_k and α''_k are set to be 2 minutes and 3 minutes, respectively. The minimum required dwell time β_{ki} of train k at a minor station s_i, $s_i \notin \bar{S}_k$, is randomly selected from $\{3, 4, 5\}$, where the probability of each value being selected is 1/3. The minimum required dwell time β_{ki} of train k at a major station s_i, $s_i \notin \bar{S}_k$, is randomly selected from $\{6, 8, 10\}$, where the probability of each value being selected is 1/3.

We set $T = 1080$ (minutes), which represents an 18-hour daily operation. Train k's latest allowed operation completion time q_k is set equal to T. Train k's earliest possible operation start time p_k is randomly generated from a discrete uniform distribution between 0 and $(0.8)T - T_k$, where T_k is the required minimum time for train k to finish its trip. The constant factor "0.8" is used to ensure that train k starts early enough so that it can finish its operation no later than T. Train k's capacity Γ_k is randomly selected from $\{600, 1200\}$, where 600 is selected with probability 1/4, and 1200 is selected with probability 3/4. Using the train capacities and travel distances, we can compute the total available seat-kilometers ($TASK$) defined as

$$TASK = \sum_{k \in K} \Gamma_k \cdot (\text{distance between train } k\text{'s origin and destination stations}),$$

which is the maximum total distance that passengers can travel by taking the trains. Let c denote the operating cost for a type G train with a 1200-passenger capacity to run on a track; that is, $c_k = c$ if train k is of type G and $\Gamma_k = 1200$. We set $c_k = (0.5)c$ if train k is of type G and $\Gamma_k = 600$; $c_k = (0.8)c$ if train k is of type D and $\Gamma_k = 1200$; and $c_k = (0.4)c$ if train k is of type D and $\Gamma_k = 600$. We set $c'_k = (0.9)c_k$ for all train k; see Table 4. For simplicity, the monetary unit is scaled in such a way that $c = 1$.

In order to simplify our computational analysis, we focus on the passenger OD pairs with significant passenger volume and ignore those passenger OD pairs that have small volume. Thus, we only consider passenger groups traveling among 11 large-passenger-volume stations, including the seven major stations and minor stations s_4, s_7, s_{19}, and s_{20}. Hence, there are 55 passenger OD pairs in total. For each passenger OD pair, there are 18 passenger groups whose arrival times at their origin stations are $0, 60, 120, \ldots, 1020$. This implies that there are $55 \times 18 = 990$ potential passenger groups. However, in some of these passenger groups, the passenger arrival times at their origin stations are quite late and the travel distances are long, so that it is impossible for the passengers to reach their destination stations by time T if they take a type D train. Let

$$t_r = \frac{\text{distance between passenger group } r\text{'s origin and destination stations}}{\text{speed of type D train}}.$$

Then, we exclude those passenger groups with $\hat{p}_r + t_r > T$. As a result, we consider only 896 passenger groups.

We generate the number of passengers z_r in passenger group r by setting it equal to $\vartheta_r \cdot \hat{z}$, where $\hat{z}$ is a parameter that represents the maximum possible number of passengers in a passenger group, and ϑ_r is a random value such that $\vartheta_r \leq 1$. This allows different passenger groups to have different sizes, while the maximum size is capped by $\hat{z}$. The value of ϑ_r is generated from a uniform distribution on $[\bar{\vartheta}_r - 0.1, \bar{\vartheta}_r + 0.1]$, where parameter $\bar{\vartheta}_r$ is the mean of ϑ_r and is dependent on the passenger group's origin and destination stations. Specifically, $\bar{\vartheta}_r = 0.9$ if the passenger group's origin and destination are both major stations, $\bar{\vartheta}_r = 0.6$ if the passenger group's origin is a major station and destination is a minor station, $\bar{\vartheta}_r = 0.6$ if the passenger group's origin is a minor station and destination is a major station, and $\bar{\vartheta}_r = 0.3$ if the passenger group's origin and destination are both minor stations; see Table 4.

The value of parameter $\hat{z}$ is determined as follows. Given $\hat{z}$ and $\bar{\vartheta}_r$, the expected value of z_r is equal to $\bar{\vartheta}_r \cdot \hat{z}$. We define the total expected passenger-kilometers ($TEPK$) as

$$TEPK = \sum_{r \in R} \bar{\vartheta}_r \cdot \hat{z} \cdot (\text{distance between passenger group } r\text{'s origin and destination stations}),$$

which is the expected total distance that all the passengers wish to travel. We then introduce a parameter κ defined as

$$\kappa = \frac{TEPK}{TASK},$$

which represents the passenger traffic intensity of the railway system. We consider test instances with different values of κ, representing different passenger traffic conditions. Given κ, $\bar{\vartheta}_r$, Γ_k, and the distance parameters, we have

$$\hat{z} = \frac{\kappa \sum_{k \in K} \Gamma_k \cdot (\text{distance between train } k\text{'s origin and destination stations})}{\sum_{r \in R} \bar{\vartheta}_r \cdot (\text{distance between passenger group } r\text{'s origin and destination stations})}.$$

The time $\tau_{ill'}$ for a passenger to walk from the lth platform to the l'th platform of station s_i is set equal to $\tau \cdot |l - l'|$, where parameter τ is set to be 2 minutes. The unit cost of a passenger's time spent on riding the trains and waiting for the trains, $\hat{c}$, is set equal to $(0.0010)c$, while the unit cost of a passenger's time spent on walking between station platforms, $\hat{c}'$, is set equal to $(0.0012)c$. The penalty π_r of each unsatisfied passenger in group r is set equal to $(0.2)c + \hat{c}t_r$, where the term $(0.2)c$ represents a fixed cost incurred by having an unsatisfied passenger, and the term $\hat{c}t_r$ is an additional penalty that is proportional to the travel distance t_r of the passenger's journey.

In the first part of the computational study, we test the performance of our Lagrangian relaxation heuristic by setting $|K| = 30, 50, 70$ and setting the OD pairs of the trains as shown in Table 4. For each value of $|K|$, we first set $\kappa = 0.8$ and generate 5 random test instances. Thus, there are 15 such test instances in total. Next, we investigate how traffic intensity affects the solution. To do so, we generate a train schedule with $|K| = 30$, and then generate random test instances with different passenger requirements. We set $\kappa = 0.2, 0.4, 0.6, 0.8, 1.0$. For each value of κ, we generate 3 random test instances. Thus, there are 15 such test instances. In the second part of the computational study, we set $|K| = 70, 110, 150$ and set the OD pairs of the trains as shown in Table 4.

For each value of $|K|$, we set $\kappa = 0.8$ and generate 5 random test instances. Hence, there are 15 test instances in this part of the study.

In our implementation of the subgradient optimization procedure, parameter a is set to 0.1. The initial value of the step size θ is set to 2.0. During the solution process, θ is reduced by 20% if the best lower bound identified has no improvement for 20 consecutive iterations. The computation is terminated if the running time limit has been reached. For each parameter setting, we set the running time limit to 5 hours per test instance. For the first 10 iterations of the subgradient optimization procedure, we execute the upper bound heuristic and update the upper bound at each iteration. After 10 iterations, since the upper bound heuristic becomes less likely to be able to identify a better upper bound, we execute the upper bound heuristic with only probability 20% at each iteration.

5.2 Computational results

Tables 5 and 6 summarize the results of the first part of the computational study. The "No. of iterations" column reports the number of iterations that the subgradient optimization process has gone through during the 5-hour computation. The "No. of executions of UB heuristic" column reports the number of times that the subgradient optimization process has executed the upper bound heuristic. The "No. of improving solutions" column reports the number of new upper bound solutions obtained during this process. The "*Gap*" column reports the optimality gap, where

$$Gap = \frac{UB^* - LB^*}{LB^*} \times 100\%,$$

UB^* is the objective function value of the heuristic solution, and LB^* is the best lower bound value identified in the subgradient optimization process. The "service level" of each solution is also reported, where

$$\text{service level} = \left(1 - \frac{\text{number of unsatisfied passengers}}{\text{total number of passengers}}\right) \times 100\%.$$

As can be seen from Table 5, the heuristic solution value increases as $|K|$ increases, because the cost of operating the trains is higher as the number of trains increases. The number of iterations and number of executions of the upper bound heuristic decrease as $|K|$ increases. This is because the upper bound heuristic, which needs to solve an MCMCNF problem on a large time-space network with a large number of commodities, is time-consuming to execute, and the computational burden of each execution increases as the problem size increases. The number of improving solutions tends to increase as $|K|$ increases. This indicates that for small-size instances, the upper bound heuristic can identify a near-optimal solution in the first iteration of the subgradient optimization process, and relatively few improvements need to be made in later iterations. The performance gaps of these test instances range between 18.8% and 29.7%. We observe that the performance gaps have a similar scale for instances with $|K| = 30$ and instances with $|K| = 50$, and tend to be larger for instances with $|K| = 70$. This indicates that as the computational burden reaches a certain level, the performance of the heuristic begins to drop. We also observe that the service level tends to increase as $|K|$ increases. This

Table 5: Computational results: Part 1.

Instance	$\lvert K \rvert$	κ	No. of iterations	No. of executions of UB heuristic	No. of improving solutions	UB^*	LB^*	*Gap*	Service level
1	30	0.8	2714	532	5	21041.7	17273.8	21.8%	72.8%
2	30	0.8	3673	714	1	17609.3	14611.7	20.5%	57.4%
3	30	0.8	2583	551	4	19596.7	16262.6	20.5%	74.1%
4	30	0.8	2939	600	3	19212.8	15745.4	22.0%	73.3%
5	30	0.8	2728	529	8	20518.7	17274.1	18.8%	71.2%
Average:			2927.4	585.2	4.2	19595.8	16233.5	20.7%	69.8%
6	50	0.8	1299	282	8	36022.1	29946.2	20.3%	76.8%
7	50	0.8	1305	279	12	32643.2	27145.3	20.3%	74.1%
8	50	0.8	1416	277	5	33766.8	27998.8	20.6%	76.4%
9	50	0.8	1248	281	2	31712.3	26697.4	18.8%	77.2%
10	50	0.8	1202	242	9	36373.8	30487.8	19.3%	75.9%
Average:			1294.0	272.2	7.2	34103.6	28455.1	19.9%	76.1%
11	70	0.8	534	142	15	46716.9	37473.9	24.7%	81.2%
12	70	0.8	603	121	7	48629.0	38767.9	25.4%	78.2%
13	70	0.8	713	151	7	45661.9	36938.3	23.6%	77.5%
14	70	0.8	522	124	4	49833.6	38421.4	29.7%	74.5%
15	70	0.8	837	160	9	45495.4	38029.4	19.6%	79.3%
Average:			641.8	139.6	8.4	47267.4	37926.2	24.6%	78.1%

is because as the number of trains increases, the service capacity of the system increases, and thus a higher percentage of passengers can be served.

As can be seen from Table 6, the heuristic solution value increases as κ increases. This is because the cost of handling more passengers is higher as the passenger traffic intensity increases. The performance gap is

Table 6: Computational results: Part 1 (cont'd).

Instance	$\lvert K \rvert$	κ	No. of iterations	No. of executions of UB heuristic	No. of improving solutions	UB^*	LB^*	*Gap*	Service level
16	30	0.2	3295	656	2	11994.4	10518.1	14.0%	64.8%
17	30	0.2	3227	661	4	12013.0	10508.9	14.3%	65.4%
18	30	0.2	3229	672	4	12009.5	10515.8	14.2%	67.1%
Average:			3250.3	663.0	3.3	12005.6	10514.3	14.2%	65.8%
19	30	0.4	3032	637	4	15108.7	12851.4	17.6%	66.3%
20	30	0.4	3091	610	11	15083.7	12812.2	17.7%	72.6%
21	30	0.4	3044	634	6	15143.1	12875.1	17.6%	73.9%
Average:			3055.7	627.0	7.0	15111.8	12846.2	17.6%	70.9%
22	30	0.6	2879	594	9	18258.5	15361.1	18.9%	72.6%
23	30	0.6	2720	617	9	18238.1	15302.7	19.2%	74.1%
24	30	0.6	2854	604	5	18206.0	15305.6	18.9%	75.8%
Average:			2817.7	605.0	7.7	18234.2	15323.1	19.0%	74.2%
25	30	0.8	2566	525	4	21399.0	17893.5	19.6%	73.1%
26	30	0.8	2727	533	8	21355.7	17935.9	19.1%	72.5%
27	30	0.8	2691	529	10	21370.2	17817.5	19.9%	70.3%
Average:			2661.3	529.0	7.3	21375.0	17882.3	19.5%	72.0%
28	30	1.0	2370	480	9	24457.4	20621.1	18.6%	69.4%
29	30	1.0	2238	473	13	24620.0	20568.5	19.7%	67.9%
30	30	1.0	2375	476	9	24596.1	20612.6	19.3%	70.4%
Average:			2327.7	476.3	10.3	24557.8	20600.7	19.2%	69.2%

Table 7: Computational results: Part 2.

Instance	$\lvert K\rvert$	κ	No. of iterations	No. of executions of UB heuristic	No. of improving solutions	UB^*	LB^*	Gap	Service level
31	70	0.8	1831	375	3	28577.2	24029.2	18.9%	69.5%
32	70	0.8	2341	464	1	24291.8	20610.8	17.9%	59.3%
33	70	0.8	2015	384	1	24304.0	20363.9	19.3%	65.7%
34	70	0.8	2267	453	2	21724.4	18574.8	17.0%	63.5%
35	70	0.8	1821	367	4	30320.8	25325.2	19.7%	71.0%
Average:			2055.0	408.6	2.2	25843.6	21780.8	18.6%	65.8%
36	110	0.8	803	149	4	45180.8	37089.4	21.8%	74.9%
37	110	0.8	900	178	5	45039.6	37347.5	20.6%	75.3%
38	110	0.8	1030	202	2	41101.7	34289.4	19.9%	74.2%
39	110	0.8	897	213	1	38397.2	31706.9	21.1%	64.2%
40	110	0.8	1113	230	3	38764.9	32723.1	18.5%	67.6%
Average:			948.6	194.4	3.0	41696.8	34631.3	20.4%	71.2%
41	150	0.8	684	147	3	56293.6	45318.7	24.2%	70.1%
42	150	0.8	547	111	6	54200.2	42945.0	26.2%	74.6%
43	150	0.8	429	99	1	61352.3	47613.0	28.9%	68.7%
44	150	0.8	780	174	2	61246.7	49598.9	23.5%	73.7%
45	150	0.8	479	97	5	55183.8	43089.1	28.1%	76.4%
Average:			583.8	125.6	3.4	57655.3	45712.9	26.2%	72.7%

significantly smaller when $\kappa = 0.2$. This indicates that the performance of the heuristic is significantly better when the passenger traffic is low. We also observe that the service level is the highest when $\kappa = 0.6$. When κ is greater than 0.6, the service level decreases as κ increases. This is because when the passenger traffic intensity is high, more passengers fail to get satisfied as the demand increases. However, when κ is less than 0.6, the service level tends to be lower when κ is smaller. This is because when the traffic intensity drops below a certain level, minimizing train operating costs becomes more important than minimizing passenger costs. Therefore, the trains are assigned more skipped stops and less dwell time at stations. As a result, less passenger demand can be satisfied by the train service.

Table 7 summarizes the results of the second part of the computational study. These results bear a lot of similarities as the results in Table 5. For example, as $|K|$ increases, the heuristic solution value increases, the number of iterations decreases, the number of executions of the upper bound heuristic decreases, the number of improving solutions tends to increase, and the service level tends to increase. Note that both instances 11–15 and instances 31–35 have $|K| = 70$. However, the number of iterations and the number of executions of the upper bound heuristic for instances 31–35 are larger than those for instances 11–15. This is because the trains in instances 31–35 have shorter travel distances than those in instances 11–15, and thus instances 31–35 require less computational work per execution of the upper bound heuristic. The lower computational burden of instances 31–35 also results in smaller performance gaps than instances 11–15.

6 Conclusions

We have studied a train timetabling problem with stop-skipping and platform assignment decisions while taking into account the passenger flow. We formulated this problem as a constrained MCMCNF problem on a time-space network, and proved that the problem is NP-hard in the strong sense. We developed a Lagrangian relaxation heuristic to solve the problem. Our computational results demonstrate the effectiveness of our Lagrangian relaxation heuristic and report how the solution is influenced by passenger traffic intensity and railway service capacity.

One limitation of this study is that our Lagrangian relaxation heuristic requires a large amount of computational time for the subgradient optimization procedure to converge, particularly when solving large-size instances. Thus, an interesting future research topic is to develop mathematical techniques to improve the tightness of the lower bound and speed up the convergence of the solution process. Developing other solution methods to tackle this difficult problem is also an interesting research topic. One reason for the heavy computational burden required by our Lagrangian relaxation heuristic is that every time the upper bound heuristic is executed, it needs to solve an MCMCNF problem on a large time-space network with a large number of commodities to determine the passengers' schedule. Hence, exploring other approaches to modeling passenger flows in train-timetabling problems with stop-skipping and platform choice considerations so as to improve the complexity of the model is another possible research direction.

Rail operations are vulnerable to unexpected disruptions. When a disruption occurs, efficient rescheduling methods that can generate passenger-friendly timetables are desirable. Since stop-skipping patterns, platform choices, and passenger flows are also key factors in the rescheduling process, an important research direction is to develop efficient methods to re-optimize the solution in our model when facing a disruption, so as to minimize the impact on the passenger flow. Another interesting future research direction is to consider other variants of our problem. This includes the extension of our model to bidirectional rail networks, models with energy consumption considerations, etc.

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Appendix A. Proof of Theorem 1

We transform Exact Cover by 3-Sets (X3C) to the decision version of problem **P**. Given a set X of $3b$ elements and a collection $\mathcal{K}$ of 3-element subsets of X, the X3C problem asks whether there exists $\mathcal{K}' \subseteq \mathcal{K}$ such that every element of X occurs in exactly one member of $\mathcal{K}'$. X3C is known to be strongly NP-hard (Garey and Johnson 1979). Let $u = |\mathcal{K}|$. For notational convenience, we denote $X = \{\phi_1, \phi_2, \ldots, \phi_{3b}\}$ and $\mathcal{K} = \{\{\phi_{\nu_{k1}}, \phi_{\nu_{k2}}, \phi_{\nu_{k3}}\} \mid k = 1, \ldots, u\}$.

Given an arbitrary instance of X3C, we construct a corresponding instance of problem **P** as follows. There are $6b+3$ stations, u trains, and $3b+1$ passenger groups. Each station has u platform tracks. Each terminal station has zero passing tracks, and each intermediate station has one passing track. The origin station of each train is s_1, and the destination station of each train is s_{6b+3}. For each $k = 1, 2, \ldots, u$, train k must stop at stations s_1, $s_{2\nu_{k1}}$, $s_{2\nu_{k1}+1}$, $s_{2\nu_{k2}}$, $s_{2\nu_{k2}+1}$, $s_{2\nu_{k3}}$, $s_{2\nu_{k3}+1}$, s_{6b+2}, and s_{6b+3}, and it must skip the other stations (i.e., $S_k = \bar{S}_k = S \setminus \{s_1, s_{2\nu_{k1}}, s_{2\nu_{k1}+1}, s_{2\nu_{k2}}, s_{2\nu_{k2}+1}, s_{2\nu_{k3}}, s_{2\nu_{k3}+1}, s_{6b+2}, s_{6b+3}\}$). The time for each train k to traverse each track segment $s_i \to s_{i+1}$ is 1 time unit, regardless of whether train k skips stations s_i and s_{i+1} or not (i.e., $\alpha_{ki} = 1$ and $\alpha'_{ki} = \alpha''_{ki} = 0$). The minimum required dwell time of each train at a station that it stops at is 1 time unit. The minimum headway between arrivals and the minimum headway between departures at each station are both 0. The earliest start time of operation of each train is 0, and the latest completion time of operation of each train is $12b + 22$. The capacity of each train is 1. The number of passengers in each of passenger groups $1, 2, \ldots, 3b$ is 1, and the number of passengers in group $3b+1$ is $u - b$. For $r = 1, 2, \ldots, 3b$, the origin station and destination station of passenger group r are s_{2r} and s_{2r+1}, respectively. The origin station and destination station of passenger group $3b+1$ are s_{6b+2} and s_{6b+3}, respectively. For $r = 1, 2, \ldots, 3b$, the earliest possible start time of the trip of a passenger in group r is 0, and the latest allowed completion time of the trip of a passenger in group r is $6b + 11$. The earliest possible start time of the trip of a passenger in group $3b+1$ is $12b + 20$, and the latest allowed completion time of the trip of a passenger in group $3b+1$ is $12b + 21$. The operating cost of each train is 1 per time unit regardless of whether the train is running or dwelling (i.e., $c_k = c'_k = 1$ for each k). The unit cost of each passenger's time is 0 regardless of whether the passenger is riding a train, waiting for a train, or walking between station platforms (i.e., $\hat{c} = \hat{c}' = 0$). The penalty of each unsatisfied passenger in any passenger group is $u(6b + 11) + 1$. Clearly, this construction can be done in polynomial time. Let $Y = u(6b + 11)$. We will show that there exists a feasible solution to this constructed instance of problem **P** with a total cost no greater than Y if and only if the answer to the given X3C problem is "yes."

Suppose the answer to the given X3C problem is "yes." Then, there exists $\mathcal{K}' \subseteq \mathcal{K}$ such that every element of X occurs in exactly one member of $\mathcal{K}'$. Let $K' = \{k \mid \{\phi_{\nu_{k1}}, \phi_{\nu_{k2}}, \phi_{\nu_{k3}}\} \in \mathcal{K}'; k = 1, \ldots, u\}$. For each $k \in K'$, we assign the passengers of groups ν_{k1}, ν_{k2}, and ν_{k3} to train k. We let train k start its operation at time 0, spend 1 time unit dwelling at each of the 9 stations that it must stop at, and complete its operation at time $6b + 11$. This assignment is feasible, because train k stops at stations $s_{2\nu_{k1}}$, $s_{2\nu_{k1}+1}$, $s_{2\nu_{k2}}$, $s_{2\nu_{k2}+1}$, $s_{2\nu_{k3}}$, and $s_{2\nu_{k3}+1}$, which are the pickup and drop-off points of passenger groups ν_{k1}, ν_{k2}, and ν_{k3}. Then, each of the passengers in groups $1, 2, \ldots, 3b$ is served by exactly one train. For each $k \in \{1, \ldots, u\} \setminus K'$, we assign a passenger of group $3b+1$ to train k, and we let train k start its operation at time $6b+11$, spend 1 time unit dwelling at each of the 9 stations that it must stop at, and complete its operation at time $12b+22$. This train schedule is feasible, because train k departs from s_{6b+2} at time $12b+20$ and arrives s_{6b+3} at time $12b+21$, enabling the passengers of group $3b+1$ to board and alight the train at the right time. The total operating cost of each train is $6b+11$, and all passengers' demands are satisfied. Hence, the total cost of this solution is Y.

Conversely, suppose that there exists a feasible solution to the constructed instance of problem **P** with a total cost of no greater than Y. Then, all passengers' demands are satisfied. Note that each train traverses $6b+2$ track segments and dwells at 9 stations. Thus, the minimum possible operating cost of each train is $6b+11$. If one of the trains begins its operation before time $6b+11$ and serves one of the passengers in group $3b+1$ (which requires the train to reach station s_{6b+3} at time $12b+21$ and completes its operation at time $12b+22$), then the total cost of the solution will exceed Y. Hence, those trains that serve the $u-b$ passengers in group $3b+1$ must start their operations no earlier than time $6b+11$. Because the latest allowed completion time of the trip of each passenger in groups $1, 2, \ldots, 3b$ is $6b+11$, at most b trains are serving passenger groups $1, 2, \ldots, 3b$. Because each of these trains stops at only 6 of stations $s_2, s_3, \ldots, s_{6b+1}$, each of them serves at most 3 passengers in groups $1, 2, \ldots, 3b$. Therefore, exactly b trains are serving passenger groups $1, 2, \ldots, 3b$, and each of them serves exactly 3 passengers in groups $1, 2, \ldots, 3b$. Let $k_1, k_2, \ldots, k_b$ denote these b trains. For $j = 1, 2, \ldots, b$, let ν_{k_j1}, ν_{k_j2}, and ν_{k_j3} be the 3 passenger groups served by train k_j. Let $\mathcal{K}' = \{\{\phi_{\nu_{k_j1}}, \phi_{\nu_{k_j2}}, \phi_{\nu_{k_j3}}\} \mid j = 1, \ldots, b\}$. Then, every element of X occurs in exactly one member of $\mathcal{K}'$. This completes the proof of the theorem. ∎

Appendix B. A Simpler Upper Bound Heuristic

The simpler upper bound heuristic mentioned in Remark 3 is as follows:

Step 1. Determine a train timetable using the basic constructive heuristic presented in Section 4.2. The output of this basic constructive heuristic is a set of x^k_{uv} values for all $k \in K$ and $u \to v \in A$. For each $u \to v \in A$, the passenger capacity of arc $u \to v$ is $\sum_{k\in K} \Gamma_k x^k_{uv}$.

Step 2. Rank the passenger groups by increasing optimal values of the $|R|$ shortest path subproblems.

Step 3. Following the ranked order determined in Step 2, for each passenger group r, construct the schedule of the group by solving a single commodity minimum cost network flow problem, and then reduce the passenger capacity of each arc by the number of passengers in group r assigned to that arc.

This upper bound heuristic is simpler than the original version presented in Section 4.2, as it does not need to solve any MCMCNF problem. To estimate the performance of this simpler heuristic, we test it using those test instances with $|K| = 70$ (i.e., instances 11–15 and 31–35 shown in Tables 5 and 7). For each test instance, we execute the overall solution procedure with this simpler upper bound heuristic for 5 hours. A comparison of the computational results of this simpler upper bound heuristic with those of the original upper bound heuristic is presented in Table 8.

Table 8: Computational results: Original upper bound heuristic vs. the simpler upper bound heuristic.

	Using the original UB heuristic				Using the simpler UB heuristic			
Instance	Avg. running time of each execution of UB heuristic (sec.)	UB value of Lagrangian relaxation heuristic	LB value of Lagrangian relaxation heuristic	Optimality gap of Lagrangian relaxation heuristic	Avg. running time of each execution of UB heuristic (sec.)	UB value of Lagrangian relaxation heuristic	LB value of Lagrangian relaxation heuristic	Optimality gap of Lagrangian relaxation heuristic
11	117.0	46716.9	37473.9	24.7%	109.0	47382.6	38341.7	23.6%
12	135.7	48629.0	38767.9	25.4%	111.8	49053.3	39040.9	25.6%
13	106.3	45661.9	36938.3	23.6%	95.7	46344.3	37491.3	23.6%
14	136.9	49833.6	38421.4	29.7%	137.0	49978.1	37850.0	32.0%
15	105.6	45495.4	38029.4	19.6%	96.6	46008.9	38102.9	20.7%
Average:	120.3	47267.4	37926.2	24.6%	110.0	47753.4	38165.4	25.1%
31	36.2	28577.2	24029.2	18.9%	38.3	28733.7	24030.4	19.6%
32	26.2	24291.8	20610.8	17.9%	30.2	24532.1	20654.0	18.8%
33	33.4	24304.0	20363.9	19.3%	35.7	24530.8	20358.0	20.5%
34	27.4	21724.4	18574.8	17.0%	29.7	21952.6	18477.4	18.8%
35	36.5	30320.8	25325.2	19.7%	38.5	30625.6	25284.0	21.1%
Average:	31.9	25843.6	21780.8	18.6%	34.5	26075.0	21760.8	19.8%

From Table 8, we observe that determining the passenger schedules using the simpler upper bound heuristic requires a comparable amount of computational time compared to the original upper bound heuristic. We also observe that the solutions generated by the Lagrangian relaxation procedure with the simpler upper bound heuristic are no better than those solutions generated by the procedure with the original upper bound heuristic in most of these test instances. This is because solving a standard MCMCNF is guaranteed no worse (and often better) than the passenger flow obtained by constructing the passenger schedules one by one.