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Direct single-step measurement of Hadamard spectrum using single-pixel optical detection

Yin Xiao, Lina Zhou, and Wen Chen

***Abstract*—Spectrum can be acquired by Hadamard transform, and the spectrum energy concentrates on the upper-left corner according to the chosen Walsh-ordering Hadamard bases. Hence, measurement of the main spectrum coefficients can facilitate the reconstruction of objects. In this Letter, we propose a method for directly calculating each Hadamard spectrum coefficient using only single-step measurement in single-pixel imaging (SPI) for high-contrast object reconstruction. The proposed method can significantly reduce the number of measurements in the SPI. In addition, an effective noise suppression strategy is further developed to recover high-contrast objects. The proposed method in the SPI is also tested in scattering environment. Experimental work is conducted to verify feasibility and effectiveness of the proposed method.**

***Index Terms*—Single-pixel imaging, Hadamard transform, single-step measurement, object reconstruction**

I. Introduction

SINGLE-pixel imaging (SPI) is an interesting imaging method in optical fields, which has attracted much attention in recent years. In comparison to charge-coupled device (CCD), single-pixel detector has advantages of low cost and high signal-to-ratio (SNR). Moreover, SPI is able to image at non-visible wavelengths and under low-light conditions. The SPI emerges from ghost imaging (GI) [1], [2]. GI utilizes the correlation algorithm to reconstruct objects, but reconstruction quality is low. Compressive sensing (CS) is a tool for reconstructing objects by using fewer measurements in the SPI [3]. The CS method achieves measurement reduction by using optimization algorithms. However, optimization algorithms employed are commonly computationally exhausted.

Since images are normally sparse in some bases, such as wavelet bases [4], discrete cosine bases [5] and Fourier bases [6], coefficients with large values after transformation can be used to reconstruct the objects. By designing illumination patterns in the SPI, these coefficients can be calculated [7]–[9]. The study in Ref. [7] realized a 4-step phase-shift Fourier spectrum method to conduct object reconstruction. Bian et al. [8] proposed a method to calculate Fourier spectrum, and the study in Ref. [9] put forward an idea to calculate discrete cosine transform coefficients in the SPI. However, the number of measurements in these SPI methods is still large.

It is recently found that Hadamard transform is also an effective way to code object [10]–[12]. Since Hadamard spectrum coefficients are real number, they are easier to be calculated compared with Fourier coefficients (complex values). In addition, sinusoidal patterns are usually designed to achieve Fourier transform in optical experiments. Sinusoidal patterns consist of continuous values, which can lead to inevitable quantization errors when digital micromirror device or spatial light modulator (SLM) is used. In contrast, Hadamard patterns have only two values (such as 0 and 1), which are much easier to be realized in optical experiments. However, differential Hadamard methods are usually used in the SPI to calculate Hadamard spectrum coefficients, which still need a large number of measurements.

In this Letter, we propose a method for directly calculating each Hadamard spectrum coefficient using only single-step measurement with single-pixel detection in the SPI. The single-step measurement takes advantage of a DC bias which has been used in optical intensity-modulation (IM) and direct-detection (DD) systems, e.g., Ref. [13]. Here, for the first time, we study and apply it to the SPI area, which can significantly reduce the number of measurements in the SPI. In addition, an effective noise-suppression strategy is further developed to recover high-contrast objects. The proposed method in the SPI is also tested in scattering environment. Experimental work is conducted to verify feasibility and effectiveness of the proposed method.

II. Principles

A two-dimensional Hadamard transform pair is defined as

$$F(u,v)=H(u,v)\,f(x,y)H(u,v), \qquad (1)$$

$$f(x,y)=\frac{1}{N^2}H(u,v)F(u,v)H(u,v), \qquad (2)$$

where $f(x,y)$ represents the object, $H(u,v)$ denotes a Hadamard matrix, $F(u,v)$ denotes Hadamard spectrum coefficients, and N denotes the dimension of horizontal or vertical direction of the object. It is worth noting that $f(x,y)$ needs to be a square matrix which should be on the order of $N=2^n$. An equivalent form of Eq. (1) can be written as

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$$F(u,v)=\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)P_{u,v}(x,y), \tag{3}$$

where $u,\ v=0,\ 1,\ \ldots,\ N-1,$ $P_{u,v}(x,y)$ denotes a basic Hadamard pattern used to realize Hadamard transform for object $f(x,y)$. Here, the Walsh-ordering Hadamard transform is studied.

The matrix element $wal(x,y)$ of $P_{u,v}(x,y)$ can be calculated by

$$wal(x,y)=(-1)^{\sum_{i=0}^{n-1}\left[g_i(u)x_i+g_i(v)y_i\right]}, \tag{4}$$

where $g_i(k)=k_{n-i}+k_{n-i-1}$ $(i=0,\ \ldots,\ n-1 \text{ and } k=u,v)$. It is worth noting that $g_0(k)$ is equal to k_{n-1}. The terms u_i, v_i, x_i and v_i are respectively binary representations of u, v, x and y, which can be expressed as

$$s=\left(s_{n-1}s_{n-2}\cdots s_1 s_0\right)_2=\sum_{i=0}^{n-1}s_i 2^i,\ \ (s=u,\ v,\ x,\ y). \tag{5}$$

A basic Hadamard pattern P is a square matrix consisting of 1 and –1, which can be a problem for intensity modulation in optical experiments. Hence, in our study, a pattern P' as follows is used, i.e.,

$$P'=\frac{1+P}{2}. \tag{6}$$

It can be seen in Eq. (6) that the pattern values contain only 0 and 1. Based on Eq. (6), a novel method is developed in this study for calculating each Hadamard spectrum coefficient with only single-step measurement in the SPI. When the pattern P' is used as illumination patterns, the SPI process can be expressed as

$$\begin{aligned}B&=\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\left|P'\bullet f\right|\\&=\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\left|\frac{1+P}{2}\bullet f\right|\\&=\frac{1}{2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\left|f+P\bullet f\right|\end{aligned}, \tag{7}$$

where B denotes the value detected by single-pixel bucket detector, and $\bullet$ denotes element-wise product between two matrices.

By calculating all elements of $P_{00}(x,y)$, it is found that all elements in $P_{00}(x,y)$ are 1. Hence, when $P_{00}(x,y)$ is used as illumination pattern, according to Eq. (3), we have

$$\begin{aligned}F(0,0)&=\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)P_{00}(x,y)\\&=\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\end{aligned}, \tag{8}$$

where $F(0,0)$ represents the first Hadamard spectrum coefficient. According to Eqs. (3) and (8), the formula in Eq. (7) can be further described by

$$B=\frac{1}{2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\left|f+P\bullet f\right|=\frac{1}{2}\left|F(0,0)+F(u,v)\right|. \tag{9}$$

Since $F(0,0)$ is positive and much larger than other values, the symbol to generate absolute value in Eq. (9) can be removed. As a result, each Hadamard spectrum coefficient can be calculated by

$$F(u,v)=2B-F(0,0). \tag{10}$$

In the proposed method, it is clearly illustrated in Eq. (10) that each Hadamard spectrum coefficient can be directly calculated using only one step in the SPI, which is different from conventional Hadamard methods. Hence, the number of measurements can be dramatically reduced in the SPI. After the Hadamard spectrum coefficients are effectively measured, object reconstruction can be further carried out by using Eq. (2).

However, in the SPI experiments, noise, such as speckle and shot noise, would greatly affect quality of reconstructed object. In a practical case, the Hadamard spectrum coefficients can be expressed as

$$F'(u,v)=F(u,v)+n_{u,v}, \tag{11}$$

where $n_{u,v}$ denotes noise existing in each measurement, and $F'(u,v)$ represents contaminated Hadamard spectrum coefficients. Here, a quantitative noise suppression strategy is further developed to suppress noise, which consists of the following steps:

(1) Do inverse Hadamard transform of Eq. (11) to get a reconstructed object.

(2) Take out the part affected by noise from the reconstructed pattern. Note that this part locates on the upper-left corner.

(3) Do Hadamard transform of the part selected in step (2) to get its transform coefficients $F_{noise}(u,v)$.

(4) Suppress noise by

$$F_{new}(u,v)=F'(u,v)-F_{noise}(u,v). \tag{12}$$

(5) Do inverse Hadamard transform of $F_{new}(u,v)$ to obtain a finally reconstructed object.

A flow chart to show the above steps (1)–(5) is illustrated in Fig. 1. It can be seen in Fig. 1 that a directly reconstructed object can be first obtained by using inverse Hadamard transform. This reconstructed object cannot visually render useful information, and needs to be further processed by using the aforementioned steps (2) and (3). In step (2), the region (only a few pixels in the proposed method, e.g., 3x3 pixels or 5x5 pixels) affected by noise can be chosen according to distribution of values in the directly reconstructed object, which is schematically illustrated in Fig. 1. The region affected by noise is taken out, and a matrix with the same dimension as original object is generated. Note that the region affected by noise still locates on the upper-left corner of the new matrix. Matrix elements beyond the noise region are zeros. Then, as

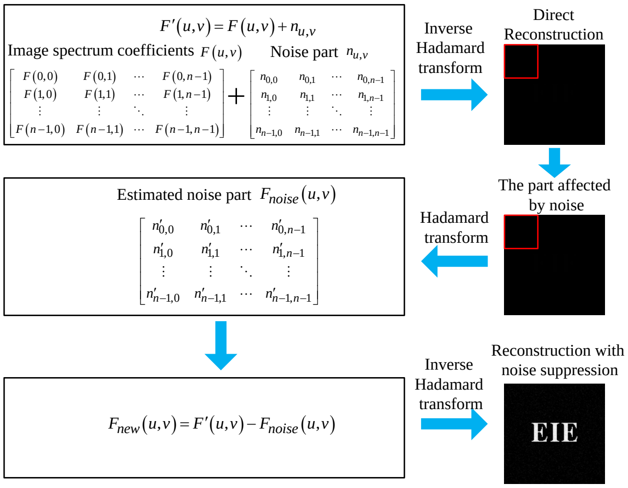


Fig. 1. Flow chart of the quantitative noise suppression strategy.

illuminated in Fig. 1, Hadamard transform is performed on this new matrix, i.e., step (3), to get an estimated noise matrix. The estimated noise matrix can effectively evaluate noise level existing in the measurements, and is further used to suppress noise, as shown in Fig. 1. In the end, a finally reconstructed object with effective suppression of noise can be correspondingly obtained.

It is worth noting that in noisy environment, there is a trade-off between conventional differential method and the proposed method. Conventional differential method needs at least two-step measurement to suppress noise. The proposed method can significantly reduce the number of measurements by half, but a noise-suppression strategy is needed.

III. Results and Discussion

The SPI experiments are conducted to illustrate feasibility and effectiveness of the proposed method. The experimental setup is shown in Fig. 2. In the SPI experiment, a SLM (Holoeye, LC-R 720) is used to generate Hadamard bases, and pixel size of the SLM is $20\ \mu m$. It is worth noting that the SLM in our experiment performs amplitude-only modulation. The illumination source used in our experiment is a He-Ne laser beam with a wavelength of $632.8\ nm$. A single-pixel detector (Newport, 918D-UV-OD3R) without spatial resolution is used, which is further connected to a power meter (Newport, 1936-R) to obtain experimental data (i.e., single-pixel sequence).

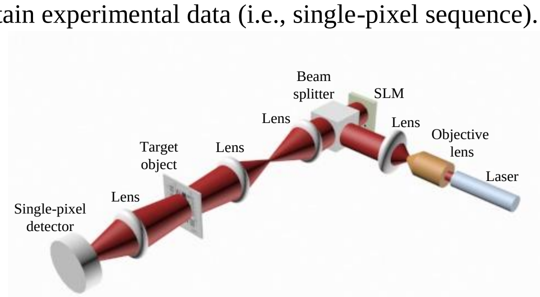


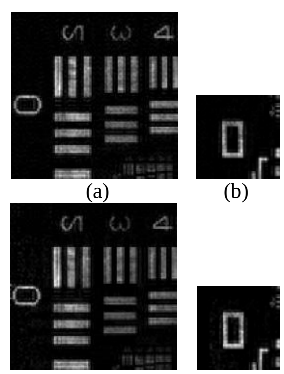
Fig. 2. Schematic of SPI experiment setup: SLM, spatial light modulator.

As shown in Fig. 2, a laser beam is expanded after propagating through an objective lens, and is collimated by a lens with $f=50\ mm$. An optical beam splitter is used to divide the laser beam into two beams and one of them illuminates the SLM. The Hadamard patterns designed here are sequentially embedded into the SLM, which can modulate the illumination light. Then, the modulated light further illuminates the target object (Edmund, negative 1951 USAF target) through a $4f$ system with $f=100\ mm$. The single-pixel detector collects the total intensity transmitting from the object by using a lens with $f=50\ mm$. Using this optical setup, a sequence of single-pixel data is correspondingly obtained.

(a) (b)

Fig. 3. Experimental results using the proposed method: (a) direct reconstruction result (256×256 pixels), (b) a reconstructed object after using the quantitative noise suppression strategy to further process that in (a).

It can be seen in Fig. 3(a) that contrast of the directly reconstructed object is low. Quantitative noise suppression is applied to Fig. 3(a) and the finally reconstructed object after noise suppression is shown in Fig. 3(b). It can be seen in Fig. 3(b) that the proposed method is feasible and effective. It is worth noting that only about 10% Hadamard spectrum coefficients (i.e., 6561 measurements) are used for recovering the high-contrast object (256×256 pixels) in the proposed method. Hence, single-step measurement of Hadamard spectrum coefficients is successfully realized by using SPI for the first time to our knowledge. To further show effectiveness of the proposed method, another two parts of the test object are further reconstructed, and a comparison between conventional method and the proposed method is shown in Fig. 4.



(a) (b) (c) (d)

Fig. 4. Experimental comparisons between conventional differential Hadamard method and the proposed method. (a) and (b) the reconstructed objects with size of 256×256 and 128×128 obtained by using conventional differential Hadamard method. (c) and (d) the reconstructed objects with size of 256×256 and 128×128 obtained by using the proposed method.

In Figs. 4(a)–4(d), reconstructed objects are obtained by using 10% Hadamard spectrum coefficients. The reconstructed objects obtained by using conventional differential method and the proposed method are of similar quality, but the number of measurements is significantly different. The number of measurements used to obtain those in Figs. 4(a) and 4(c) is 13122 and 6561, respectively. The number of measurements used to obtain those in Figs. 4(b) and 4(d) is 3200 and 1600, respectively.

We find that the proposed method is also promising and effective by using the SPI in scattering environments. The setup for the SPI through a diffuser using the proposed method is shown in Fig. 5.

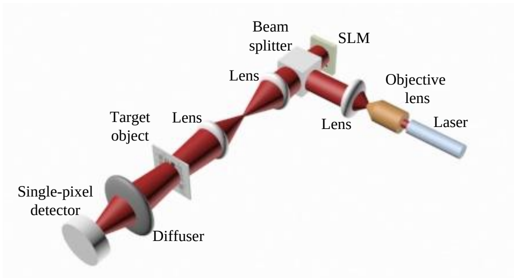


Fig. 5. Schematic of SPI experiment setup for imaging through scattering media.

As shown in Fig. 5, three different kinds of diffusers (Thorlabs, DG10-600, DG10-220, and DG10-120) are sequentially placed just before the single-pixel detector. Parameters of the diffusers to be individually used here are given in Table I.

TABLE I
Parameters of the diffusers

Diffuser Model	DG10-600	DG10-220	DG10-120
Diameter	25.4mm	25.4mm	25.4mm
Thickness	2mm	2mm	2mm
Grit	600	220	120

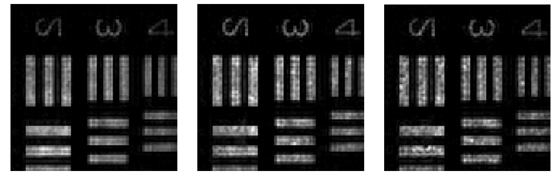
The setup before diffuser is the same as that in Fig. 2. Since light is scattered by diffuser, a lens used to collect total light intensity for the single-pixel detector is useless and removed. In this case, the single-pixel detector only detects part of total light intensity transmitting through the object. The mathematical expression can be correspondingly given by

$$B_{diffuser} = kB_{total} + noise, \quad (13)$$

where $B_{diffuser}$ represents light intensity detected by the single-pixel detector behind a diffuser, B_{total} represents the total light intensity transmitting through the object and k is a scale factor which depends on the environment condition (e.g., diffuser grit). The grits provide a range from fine to coarse scattering. A fine grit (e.g., 600) has a small diffusion pattern, and a coarser grit (e.g., 120) has a larger diffuser pattern.

In scattering environments, the proposed method is also effective, and the corresponding results of object reconstruction are shown in Fig. 6, which demonstrate feasibility and effectiveness of the proposed method in scattering environments. Here, the number of measurements used for the reconstruction is 6561, and the reconstructed objects have 256×256 pixels.

It can be found in Fig. 6 that the reconstructed object is still of high contrast in scattering environment by using the proposed method. Hence, the proposed method has a potential to be used to address imaging concerns in complex environments.



(a) (b) (c)

Fig. 6. Object reconstruction in scattering environment by using the proposed method. (a), (b) and (c) reconstructed objects obtained when diffuser model of DG10-600, DG10-220 and DG10-120 is used in the SPI setup, respectively.

IV. Conclusions

We have proposed a SPI method for high-contrast object reconstruction by using single-step Hadamard spectrum measurement, which can significantly reduce the number of measurements. A quantitative noise-suppression strategy has been further developed and applied to help achieve high-contrast object reconstruction in the proposed SPI setup. The quantitative noise-suppression strategy needs to process several pixels located on the upper-left corner to suppress noise, therefore the upper-left corner should not carry useful information. The experimental results are consistent with theoretical analyses. Moreover, it has also been found that the proposed method is feasible and effective in a scattering medium for the SPI. It is believed that the proposed method can provide a promising and effective strategy for imaging with single-pixel optical detection, and more applications can be further explored by using the proposed method, e.g., biological imaging through turbid media.

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