Order Dynamics during the Flash Crash

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Abstract

U.S. common stocks are simultaneously traded on multiple trading centers. We study quotes and trades with millisecond timestamps during the flash crash of May 6, 2010 and document new findings about order dynamics when the fragmented market is under stress. First, relative to May 5, 2010, the number of trades and the number of quotes quadrupled on May 6, 2010. Second, during the flash crash, the proportional time of a trading center offering the best bid/ask quotes substantially reduced on all trading centers, while the effectiveness of turning best quotes into trades increased on almost all trading centers. Third and most importantly, we find significant changes in the level of trade (or quote) fragmentation during the flash crash for stocks with a high or low level of fragmentation, but no significant change for stocks with a fragmentation level in the middle range. These findings together demonstrate that, despite the dramatic increase in the number of quotes, there was insufficient liquidity on all trading centers, and stocks with a medium level of trade (or quote) fragmentation were most resilient to the sudden order flow shock.

Keywords: Flash crash; Herfindahl index; market fragmentation; order imbalance

JEL Classification: G12, G14, G18

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1. Introduction

The flash crash of May 6, 2010 and a series of subsequent market glitches prompted market participants and regulators to take a critical view of the current market system in the U.S.[1] The CEO of the market research firm Tabb Group, Larry Tabb, wrote in his May 2013 report that the Regulation National Market System (Reg. NMS) "fragmented the markets, drove ever-increasing messaging rates, created order-type complexity, and arguably enabled high-frequency traders to take advantage of the very investors Reg. NMS was intended to protect, while actually making the markets less transparent for regulators."[2] Mary Jo White, the Chair of the U.S. Securities and Exchange Commission (SEC), once commented that "If we want to make good decisions about our markets, empirical evidence provides, at the very least, a starting point for a principled dialogue about what – if anything – is to be changed in our market structure."[3] In this study, we seek to provide empirical evidence on order dynamics when the fragmented market is under stress.

Regulators and researchers have examined different aspects of the flash crash. According to the joint report by the U.S. Commodities Futures Trading Commission (CFTC) and the U.S. Securities and Exchange Commission (SEC), the flash crash was triggered when a mutual fund complex initiated an automated execution algorithm to sell a large number of E-mini S&P500 Index futures contracts (valued at about $4.1 billion).[4] Their analysis focuses on the aggregate order book for the 500 constituent

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[1] Two prominent examples of these market glitches are the IPO debacle of the third-largest U.S. stock exchange, BATS, on March 23, 2012, and the huge trading loss of the market-maker firm, Knight Capital Group Inc, on Augusts 1, 2012, which led to the firm being acquired eventually.

[2] The executive summary of his report "Regulation NMS Part I: Loved or loathed and why many want it to die" is available at http://www.tabbgroup.com/PublicationDetail.aspx?PublicationID=1302.

[3] See her speech at the Security Traders Association 80th Annual Market Structure Conference on Oct. 2, 2013.

[4] On April 21, 2015, Navinder Singh Sarao, a London-based futures trader, was arrested on charges in connection to his role in the flash crash. He allegedly used an automated trading program to manipulate the market for E-Mini S&P 500 futures contracts (E-Minis) on the Chicago Mercantile Exchange (CME). His implementation of high-speed manipulative trading strategies might have triggered the Flash Crash (DoJ, 2015). Sarao pleaded guilty to fraud in November 2016.

stocks of the S&P500 Index and comparing the temporal changes in the order books for the E-mini futures contract and the S&P500 stocks. Kirilenko et al. (2011) analyze audit-trail transaction-level data for all regular transactions in the E-mini futures contract and classify trading accounts into six categories: high frequency traders (HFTs), intermediaries, fundamental buyers, fundamental sellers, noise traders, and opportunistic traders. They conclude that HFTs did not trigger the flash crash, but their trading practices exacerbated market volatility. Easley et al. (2011) analyze the E-mini futures market and find that the level of order flow toxicity (i.e., prolonged order flow imbalance) increased in the days and hours before the flash crash, which may have caused market makers to leave the market. Madhavan (2012) measures the Herfindahl index of trade and quote distribution across trading centers for a large sample of equity instruments that include 4,003 common stocks, 968 exchange-traded products (ETPs), 602 closed-end funds, 319 ADRs, and the remainder being REITs and miscellaneous equity types. He finds that stocks with greater fragmentation had worse price performance during the flash crash.

All of the above studies examine the aggregate market activities during the flash crash, and none of them examine trading centers individually. In this study, we aim to understand whether and how the sudden order flow shock during the flash crash affects each individual center differently. We study changes in trading and quoting activities in individual trading centers during the flash crash. To have a better understanding of effective liquidity on each center, we measure the proportional time of each center offering best quotes, which is defined as the ratio of the aggregate amount of time when the center offers the best bid (or ask) quote to the total amount of time within the time interval under our study. In addition, we examine each center’s effectiveness of turning best quotes to trades, which is defined as the ratio of the total number of shares that are traded on *the center* when the center is offering the best quotes, to the total number of shares that are traded on *all centers* at the same time when the center is offering the best quotes.

Moreover, we study each stock's trade fragmentation and quote fragmentation among trading centers. For each stock, we calculate the trade Herfindahl index as the sum, across all trading centers, of the squared percentage of the number of shares traded on the center. We also calculate the bid-quote (or ask-quote) Herfindahl index as the sum, across all trading centers, of the squared percentage of the total amount of time when the center offers the best bid quote (or ask quote). The quote-based Herfindahl index is a novel way of measuring fragmentation. O'Hara (2015) points out that current US stock markets are populated by computerized high-speed algorithmic traders, which means "trades are not the basic unit of market information – the underlying orders are". Quote-based measures are likely to provide new insights to market microstructure researchers.

The flash crash lasted for only a few minutes, during which stock prices dropped substantially before reaching the lowest value and rebounded quickly. To obtain a clear view of what happened during such a short event window, we use historical quote and trade data from the data vendor Nanex LLC that have a millisecond timestamp. We recognize that bid and ask quotes played different roles at the decline and rebound stages of the flash crash. When stock prices plummet, bid quotes supply liquidity; when stock prices rebound, ask quotes supply liquidity. We thus study the decline and rebound stages separately. We call the 5-minute time window immediately before a stock's price reached its lowest level on May 6 to be the 'down' period and the 5-minute time window immediately after to be the 'up' period. To control for the between-stock differences and intraday pattern in the level of liquidity, we use the same down and up periods in clock time on May 5 as the benchmark period.

Our analysis leads to several new findings about order dynamics on individual trading centers during the flash crash. First, relative to May 5, 2010, the number of trades and the number of quotes more than quadrupled on most trading centers when stock prices plummeted on May 6, and more than doubled when stock prices rebounded. Second, during the flash crash, the proportional time of a trading center offering the best bid/ask quotes substantially reduced on all trading centers, while the effectiveness of turning best quotes into trades increased on almost all trading centers. Third, we find significant changes in the level of trade (or quote) fragmentation during the flash crash for stocks with a high or low level of fragmentation, but no significant change for stocks with a fragmentation level in the middle range.

Our finding that stocks with a medium level of trade or quote fragmentation experienced no significant change in fragmentation during the flash crash is consistent with theories of investors trading behavior. On one hand, several theory studies predict that traders submit more aggressive orders when the same side of the order book becomes thicker or when the replenishment of orders on the other side of the order book is slower (see, e.g., Biais et al. (1995), Griffiths et al. (2000), Ranaldo (2004), Obizhaeva and Wang (2013)). During the flash crash, sell orders quickly accumulated while buy orders disappeared. The overwhelming buy-sell imbalance in the order book prompted investors to submit aggressive orders for faster execution. This effect could be greater for stocks with more concentrated trading because all sell orders queue up in a single order book and create a more daunting pressure. On the other hand, investors are averse to non-execution risk (see, e.g., Foucault (1999), Foucault et al. (2005), and Roşu (2009)). When trading is dispersed across multiple centers, it is uncertain which market center could provide sufficient liquidity; as a result, sell orders must be routed to several trading venues in search for the matching buy orders. A resilient market structure achieves a balance between reducing the buy-sell imbalance on each trading center and increasing the probability of execution.

The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 describes data and sample stocks with some preliminary statistics. Section 4 reports empirical findings. Section 5 concludes this paper.

2. Related literature

2.1 Debate on market consolidation versus fragmentation

One key issue of securities market design is whether a security's price should be determined by consolidating all orders in a single trading center or by allowing competitive orders on multiple trading centers. Benefits of consolidation include the economy of scale that is needed to recoup the high fixed costs of setting up and running a trading center, the network externality that lures investors to the most active market, and the lower adverse selection costs (see, e.g., Cohen, Maier, Schwartz, and Whitcomb (1982), Stoll (1982), Mendelson (1987), Chowdry and Nanda (1991), Madhavan (1995), Hasbrouck (1995), Amihud, Lauterback, and Mendelson (2003)). The downside of having a dominant trading center is that the lack of competition often fosters noncompetitive practices in the market place. Christie and Schultz (1994) find that market makers of active NASDAQ stocks rarely used odd-eighth quotes, which suggests implicit collusion among market makers to maintain bid-ask spreads of at least $0.25. Ellis, Michaely, and O'Hara (2002) find that one dealer tends to dominate trading in a NASDAQ stock and bid-ask spreads are increasing in the volume and market share of the dominant dealer. Concerns over these noncompetitive practices, together with technological advancement and changes in regulations, have shaped the current market structure, in which multiple highly automated trading centers compete for order flow (see, e.g., Barclay, Hendershott, and McCormick (2003), Goldstein, Shkilko, Van Ness, and Van Ness (2008), Hendershott and Moulton (2011)).

Many papers elaborate on the benefits of having more than one trading center. Harris (1993) argues that heterogeneous traders prefer different market mechanisms to cater to their diverse trading needs. Madhavan (1995) demonstrates in theory the coexistence of multiple trading centers that have different post-trade disclosure policy. Seppi (1997) finds that large institutional and small retail investors get better execution on hybrid markets, while investors trading intermediate-size orders may prefer a pure limit order market.

The secondary markets for U.S.-listed equities have changed dramatically in recent years because of technological advancement and regulatory development. Take the New York Stock Exchange (NYSE) as an example. In January 2005, NYSE-listed stocks were traded primarily on the floor of the NYSE in a manual fashion, and NYSE executed approximately 79.1% of the consolidated share volume in its listed stocks. Later in the same year, the SEC adopted Regulation NMS, which eliminated the trade-through protection for manual quotations. In response, NYSE began to offer its fully automated electronic trading system in October 2006. Three years later, despite its efforts, NYSE executed only 25.1% of the total volume in October 2009. The remaining volume was split among other trading centers.

According to the SEC's Concept Release on Equity Market Structure (SEC, 2010) issued in January 2010, transactions of U.S.-listed equities took place in more than 10 registered exchanges, five electronic communication networks (ECNs), about 32 dark pools, and more than 200 internalizing broker-dealers. Registered exchanges and ECNs display quotations that are widely distributed to the general public and jointly executed approximately 74.6% of the total share volume of all listed stocks in September 2009. The other 25.4% of share volume were executed by dark pools and internalizing broker-dealers that do not show their best-priced orders to the general public.

As U.S. equity trading becomes increasingly fragmented in recent years, more research focuses on the effects of trading fragmentation. Hendershott and Jones (2005) find that when the Island electronic communications network, the dominant trading center for the three most active exchange-traded funds (ETFs), stopped displaying its limit order book in the three ETFs in September 2002, trading in these ETFs became fragmented, ETF price adjustment slowed down, and trading costs increased. When Island redisplayed its limit order book in these ETFs in October 2013, Island regained a large share of trading and the reduction in fragmentation contributed to the improvement in market quality. In a cross-sectional study, O'Hara and Ye (2011) use the trade reporting facilities (TRF) data to measure the level of trading fragmentation of individual stocks, and use the Rule 605 data to measure execution quality such as effective spread, realized spread, and execution speed. They observe substantial variation in the level of fragmentation across stocks and find that more fragmented stocks have lower transaction costs and faster execution speed. SEC (2013) provides a review of other papers that study market fragmentation.

2.2. Related studies on the flash crash

In the afternoon of May 6, 2010, major U.S. stock market indices experienced a sudden drop of more than 5 percent in a matter of minutes, before rebounding almost as quickly to their “pre-crash” levels. Over 20,000 trades across more than 300 securities, including stocks and exchange traded funds, were executed at prices more than 60% away from their 2:40 p.m. prices, many of them being at prices of a penny or less, or as high as $100,000. Prices of hundreds of other securities fell 5%, 10% or even 15% before recovering most of their losses within a few minutes. In her September 7, 2010 speech, the Chairwoman of the U.S. Securities and Exchange Commission (SEC) at the time, Mary Schapiro, commented that “May 6 was clearly a market failure, and it brought to the fore concerns about our equity market structure.”

The CFTC/SEC joint report identifies that the flash crash was triggered at 2:32 p.m. when a mutual fund complex initiated an automated execution algorithm to sell a total of 75,000 June 2010 E-mini S&P 500 Index futures contracts (valued at about $4.1 billion). This sell algorithm was set to feed orders into the E-mini market for speedy execution without regard to price or time. Under the sell pressure, the price of the E-mini declined quickly between 2:32 p.m. and 2:45 p.m., and liquidity also vanished. Before trading on the E-mini was paused at 2:45:28 p.m., buy-side market depth in the E-mini fell to about $58 million or less than 1% of its depth from that morning’s level. Shortly after trading resumed at 2:45:33 p.m., the price of the E-mini began to recover.

By re-creating an aggregate order book for the 500 constituent stocks of the S&P 500 Index and comparing the temporal changes in the order books for the E-mini and the S&P 500 stocks, the report concludes that the decline in the E-mini preceded that of the stocks. A liquidity crisis occurred in the equities markets at about 2:45 p.m., when automated trading systems used by many liquidity providers temporarily paused in reaction to the sudden price declines.

In addition, the CFTC/SEC report finds that “Many over-the-counter market makers who would otherwise internally execute as principal a significant fraction of the buy and sell orders they receive from retail customers (*i.e.*, ‘internalizers’) began routing most, if not all, of these orders directly to the public exchanges where they competed with other orders for immediately available, but dwindling, liquidity.”

Kirilenko et al. (2011) present an insightful analysis of investors' trading behavior in the E-mini market on May 6. The E-mini futures contract trades exclusively on a single trading center, that is, the CME Globex trading platform. They use audit-trail transaction-level data for all regular transactions in the E-mini market to classify over 15,000 trading accounts that traded on May 6 into six categories: High Frequency Traders (HFTs), Intermediaries, Fundamental Buyers, Fundamental Sellers, Noise Traders, and Opportunistic Traders. They observe that, during May 3-5 and on May 6, Fundamental Buyers and Sellers utilize orders that consume more liquidity than the orders of HFTs and Intermediaries. This is consistent with a view that HFTs and Intermediaries generally provide liquidity while Fundamental Traders generally take liquidity.

Additionally, they find that, as the price of the E-mini futures contract dropped rapidly, Fundamental Traders were unwilling or unable to submit orders, the number of active Intermediaries dropped from 66 to 33, while most HFTs continued their trading operation but only to repeatedly buy and sell from one another, generating a "hot-potato" effect. They conclude that HFTs did not trigger the flash crash, but their trading practices exacerbated market volatility on May 6.

Easley et al. (2011) analyze the E-mini futures market and find that the flash crash is mainly due to sudden loss of liquidity. They note that in the current market structure, HFTs play an increasingly larger role as liquidity providers in the current market structure, and they are highly sensitive to intraday losses "because of low capitalization, high turnover, increased competition and small profit target". A prolonged unbalanced order flow is toxic to the liquidity providers because they may be on the wrong side of the trade with the informed investors. They find that the level of order flow toxicity increased in the days and hours before the flash crash, which may have caused market makers to leave the market.

Madhavan (2012) uses the NYSE's monthly Trade and Quote (TAQ) intraday data to measure the Herfindahl index of trade and quote distribution across trading centers for all U.S. equities from January 3, 1994 to September 12, 2011. It is observed that the level of fragmentation was quite stable in the years before 2002, then started an increasing trend in 2002 and reached the all-time high level in September 2011. He calculates the price drawdown during the flash crash as one minus the ratio of the intraday low price to the intraday high price between 1:30 PM and 4:00 PM on May 6, 2010. He then examines the effect of fragmentation during the flash crash in the regressions of the price drawdown on the average level of fragmentation in the 20 trading days prior to May 6, 2010 and other control variables. The regressions are estimated for a large sample of equity instruments that include 4,003 common stocks, 968 exchange-traded products (ETPs), 602 closed-end funds, 319 ADRs, and the remainder being REITs and miscellaneous equity types. He finds that stocks with greater fragmentation had worse price performance during the flash crash, and thus concludes that "the impact of the Flash Crash was greatest in stocks experiencing fragmentation prior to May 6."

3. Data and methodology

3.1. Data

While allowing multiple trading centers to compete for order flow, U.S. regulators emphasize that all trading centers are sufficiently linked together in a unified national market system. Of particular importance is to collect and distribute consolidated market data, so that "the public has ready access to a comprehensive, accurate, and reliable source of information for the prices and volume of any NMS stock at any time during the trading day” (SEC's Regulation NMS). Real-time information on the best-priced quotations and transactions are consolidated and distributed to the general public through three separate networks: (1) Network A for securities with their primary listing on the NYSE; (2) Network B for securities with their primary listing on exchanges other than the NYSE or NASDAQ; and (3) Network C for securities with their primary listing on NASDAQ. In addition to providing quotation and trade information to the three Networks for distribution via consolidated data feeds, many exchanges and ECNs offer individual data feeds directly to customers. The fact that direct data feeds do not need to go through the extra step of consolidation means that such data feeds can reach end-users faster than consolidated data feeds.

We obtained historical quote and trade data from Nanex LLC, whose business is to supply a real-time data feed comprising trade and quote data from all US equity, option, and futures exchanges. Nanex archives data feeds as they arrived at Nanex’s server. Data items for trades include transaction price, size, exchange code, and other information. Data items for quotes include price and size of best quotes on individual trading centers. Muravyev, Pearson, and Broussard (2013) use Nanex data to study whether option price quotes contain any information about future stock prices beyond what is already reflected in current stock prices.

Nanex puts a timestamp on every quote or trade message to 25 millisecond precision at the time when the message arrives. This time precision offers a significant advantage over the traditional monthly TAQ data, which is time-stamped to a second. Angel, Harris, and Spatt (2011) report a nearly 20-fold increase in the frequency of quotes for S&P500 stocks from 0.17 per second in May 2003 to 3.3 per second in October 2009. Hasbrouck and Saar (2009) find that over one-third of nonmarketable limit orders are cancelled within two seconds. With the 25 millisecond timestamp, we will be able to obtain a better view of what happened during the flash crash that lasted for only a few minutes.

The flash crash began with abnormal trading in the E-mini futures on the S&P500 index, according to the CFTC/SEC joint report. On May 6, 2010, the S&P500 index comprised of 96 Nasdaq-listed stocks and 404 NYSE-listed stocks. Our study focuses on the 96 Nasdaq stocks in the S&P500 index for two reasons. First, the CFTC/SEC joint report finds that there was a delay in NYSE quotes during the flash crash, which may introduce errors if NYSE quotes are used in empirical analysis. Since Nasdaq-listed stocks did not trade on NYSE on and before May 6, 2010, our empirical analysis does not involve NYSE trades and quotes, and thus avoids the potential problems. Second, by focusing on only Nasdaq-listed stocks, our findings are not affected by market structure differences between stock exchanges.

Table 1 lists the ticker symbols of the 96 Nasdaq stocks and, for each stock, the lowest value of the stock's quote midpoint (i.e., the average of the best bid price and the best ask price) on May 6th 2010 and the time in milliseconds when the lowest quote midpoint first appeared. The CFTC/SEC joint report writes that "price of the E-mini S&P500 futures began to recover after 14:45:33" and "a liquidity crisis occurred in the equities markets at about 14:45, when automated trading systems used by many liquidity providers temporarily paused in reaction to the sudden price declines." Consistent with this observation, Table 1 shows that prices of the 96 Nasdaq stocks hit the lowest value at times around 14:45:00 pm. At the earliest, the quote midpoint of the stock PDCO hit the lowest value of $25.15 at 14:43:04.025. At the latest, the quote midpoint of the stock XRAY hit the lowest value of $33.325 at 14:52:09.725.

[Table 1 is about here.]

3.2. Down and up periods during the flash crash

During the flash crash, stock prices dropped substantially before the lowest value was reached, and rebounded quickly afterwards. We call the time period immediately before the lowest value to be the 'down' period and the period immediately after that to be the 'up' period. Bid and ask quotes played a different role in these two periods: bid quotes supply liquidity during the down period, while ask quotes supply liquidity during the up period.

More specifically, we define the 'down' period to be the 5-minute period immediately before the lowest quote midpoint first appeared, and the 'up' period to be the 5-minute period immediately after that. As the time at which the lowest quote midpoint appeared differs between stocks, the down and up periods also differ between stocks. During the two 5-minute event windows, trading of these stocks was under the greatest stress.

To control for both between-stock differences and intraday variation in the level of liquidity, we use the same down and up periods in clock time on May 5 as the benchmark. For example, if the lowest quote midpoint first appeared at 14:45:27.150 on May 6, the down period is from 14:40:27.000 to 14:45:27.000 (a left-end-closed and right-end-open 5-minute interval) and the up period is from 14:45:28.000 to 14:50:28.000 on both May 5 and May 6.

Table 2 reports descriptive statistics on four measures of trades and quotes: number of trades, number of shares traded, number of bid quotes, and number of ask quotes. For each stock, we compute these measures for down and up periods on May 6 and May 5. We also calculate the ratio of a number on May 6 to its corresponding value on May 5.

Clearly, there is a significant increase in trades and quotes on May 6. In the 5-minute down period, the median number of trades in a stock is 340 on May 5, but jumped to 1,590 on May 6; the maximum number of trades in a stock is 2,912 on May 5, but surged to 22,676 on May 6. The median number of bid quotes on all trading centers for a stock is 1,891 on May 5, but 7,278 on May 6; the maximum number of bid quotes for a stock is 14,539 on May 5, but 91,481 on May 6. On average across the 96 stocks, the numbers of trades and quotes more than quadrupled in the down period, and more than doubled in the up period.

[Table 2 is about here.]

4. Empirical results

4.1. Trades on individual trading centers

Table 2 shows a significant increase in the total number of trades and quotes during the flash crash. In the following, we examine trades and quotes on individual trading centers. Trades of the 96 Nasdaq stocks in our sample are reported by a total of nine trading centers. They are the Nasdaq Exchange (NQEX), the BATS Exchange (BATS), the NYSE Arca Exchange (PACF), the Boston Stock Exchange (BOST), the International Securities Exchange (ISEX), the NYSE Trade Reporting Facility (NTRF), the Cincinnati/National Stock Exchange (CINC), the Chicago Stock Exchange (CHIC), and the Chicago Board Options Exchange (CBOE). Only eight of the nine centers distribute quotes of the 96 stocks, because NTRF is only a trade reporting facility.

Table 3 reports statistics on trades by individual centers. Panel A shows the number of stocks that have trades in each trading center. NQEX, BATS, and PACF are the three most active centers and report trades for all 96 stocks. BOST is the fourth active center and reports trades in 72 (74) stocks in the down (up) period on May 5 but a much larger number of 92 (96) stocks on May 6. CBOE appears to be the least active center and reports trades in 15 (15) stocks in the down (up) period on May 5, but 40 (7) stocks on May 6.

Panel B of Table 3 reports the total number of trades in the 96 stocks that are reported by each trading center and the associated percentage in parentheses. Most of the trades take place on the top three centers, NQEX, BATS, and PACF. Their volumes together account for more than 90 percent of the reported trades. The bottom five centers, ISEX, NTRF, CINC, CHIC, and CBOE, account for less than 5% of the total volume. In the down period on May 6, all centers experienced a significant increase in the number of trades; the most active center NQEX reports 29,135 trades on May 5 and 162,178 trades on May 6, which accounts for 54% of the total volume on May 5 and 55.7% on May 6. In the up period on May 6, most centers had a significant but relatively smaller increase; the most active center NQEX reports 33,805 trades on May 5 and 103,556 trades on May 6, which accounts for 53.3% of the total volume on May 5 and 55% on May 6. However, NTRF and CBOE experienced a large decrease in the number of trades, with only 11 trades on CBOE.

Panel C of Table 3 reports the total number of shares traded (in millions) for the 96 stocks, which presents similar patterns as the total number of trades in Panel B. The most active center NQEX has a larger share of the total volume, achieved by taking away volume from the second most active center BATS.

[Table 3 is about here.]

4.2. Quotes on individual trading centers

Next, we examine statistics on quotes by individual centers in Table 4. Panel A shows the number of stocks that have quotes in eight trading centers on May 5 and May 6, 2010. NTRF is not in Table 4 because it is a trade reporting facility and does not distribute quotes. NQEX, BATS, PACF and ISEX reported quotes in all of the 96 stocks in our sample. The least active center CHIC reported quotes in 73 stocks. CBOE clearly stands out as it reported quotes in 95 stocks in both down and up periods on May 5, 88 stocks in the down period on May 6, but only 43 stocks in the up period on May 6. This suggests either that some liquidity providers withdrew from CBOE during the flash crash or that perhaps, the CBOE was unable to process all inbound quotes.

Panel B of Table 4 shows the total number of bid quotes in the 96 stocks that are reported on each trading center and the associated percentage in parentheses. Quotes are more widely distributed among trading centers than trades. The top three centers, NQEX, BATS, and PACF, account for slightly less than 75% of the total number of quotes, compared with more than 90% of the total number of trades. The number of quotes on all centers has a significant increase in the down period on May 6. However, some centers experienced a sizable decrease in the proportion of their quotes to the total, while other centers gained in their proportions. For example, NQEX's proportion went down from 38.7% on May 5 to 36.6% on May 6, and PACF's proportion went down from 15.3% on May 5 to 9.3% on May 6. On the other hand, BATS's proportion increased from 18.8% on May 5 to 22.9% on May 6 in the down period. The total number of ask quotes exhibits the same pattern in Panel C of Table 4. The evidence indicates a dramatic change in the distribution of quotes among these trading centers, which results in a dramatic change in the supply and demand of liquidity on these centers.

[Table 4 is about here.]

The number of quotes is a rough indicator of the level of liquidity that is available on a trading center. Next, we examine another measure of liquidity that takes into account the duration when a displayed quote is the best bid or offer price. For each trading center, we compute the sum of all the time segments when the center offers the best bid (or ask) price among all centers within each time period. We define the center's proportional time of offering best quotes as the proportion of this sum to the whole time span in the period.

Table 5 reports statistics on the proportional time of offering the best bid (or ask) price by individual centers. We compare the statistics in the down and up periods between May 5 and May 6. Despite substantial increase in the number of quotes on May 6, it is clear that the proportional time of offering best bid (or ask) decreased substantially in all trading centers. On NQEX, the median proportional time of offering best bid went down from 95.2% in the down period on May 5 to 69.6% on May 6. It means that on May 5, Nasdaq offered the best bid price in the US market at 95.2% of the time. The Order Protection Rule of Regulation NMS requires that order must be executed at the best quote price in the market. A market sell order that was sent to Nasdaq on May 5 could almost certainly be executed there. However, the median proportional time of offering best bid quotes on Nasdaq was only 69.6% on May 6. Therefore, a market sell order routed to Nasdaq could not be executed there in about 30% of the time because Nasdaq's bid quote is not the best in the market. This means less liquidity on Nasdaq on May 6. We observe a similar pattern in the proportional time of offering best quotes in all centers. More dramatically, on CBOE, the median proportional time of offering best bid went down from 54.9% (59.3%) in the down (up) period on May 5 to 0.9% (0.5%) on May 6, meaning that liquidity nearly disappeared on CBOE.

[Table 5 is about here.]

A more liquid market may acquire a larger trade volume in comparison to its time at best quote prices. To capture this aspect of liquidity, we now examine a trading center’s effectiveness of turning best quotes to trades. For each trading center, when the center is offering the best bid (or ask) quote among all centers, we compute two numbers: (a) the total number of shares that are traded on *all centers* and (b) the total number of shares that are traded on *this center*. We define the center’s effectiveness of turning best quotes to trades as the ratio of (b) to (a).

Table 6 reports summary statistics on each trading center’s effectiveness. NQEX has a much greater success rate than the other centers; it captures more than 60% of the trade volume when it offers the best quotes. BATS and PACF are in the second and third places with median success rates of 17.9% and 13.6% in the down period of May 5, while the effectiveness was much lower on the other exchanges. Comparing May 6 with May 5, we find that the effectiveness increased on all centers during the flash crash. The increase in the effectiveness of BATS and PACF is much larger than that of NQEX.

[Table 6 is about here.]

4.3. Trade and quote fragmentation

The above findings suggest that during the flash crash, the demand for liquidity surged while the supply of liquidity diminished on all individual trading centers. Orders submitted to a trading center might not be executed there and would have to be routed to other centers in search of matching orders. We investigate how the sudden order flow shock would change the distribution of trades and quotes across trading centers.

We use the Herfindahl index to measure the level of trade or quote fragmentation. We calculate the trade Herfindahl index as the sum, across all trading centers, of the squared percentage of the number of shares traded on each center. Mathematically, for a given stock j in a time interval t, the trade Herfindahl index is defined as

$$H_{jt}^{v} = \sum_{i=1}^{N} \left(\frac{s_{jti}}{\sum_{i=1}^{N} s_{jti}} \right)^2 , \tag{1}$$

where N stands for the number of trading centers, s_{jti} is the number of shares traded on the *i*th center during the time interval *t*. The Herfindahl index is bounded between zero and one, and is close to one if trade volume concentrates in one center. A larger value of the Herfindahl index indicates more concentrated distribution of trades, whereas a smaller value indicates more fragmented trades.

We sort stocks into three groups by the trade Herfindahl index on May 5. Stocks in the low group have a smaller value of the trade Herfindahl index and thus a higher level of trade fragmentation than stocks in the high group. For each stock, we calculate the difference in the Herfindahl index between May 5 and May 6. We apply the t-statistic and the Wilcoxon rank-sum statistic to test whether the mean and the median of the differences are significantly different from zero. The empirical results on the trade Herfindahl index are reported in Panels A and B of Table 7.

[Table 7 is about here.]

For the whole group of the 96 stocks, there is no significant change in the mean and median Herfindahl index on May 6, relative to May 5. In the down period, the mean (median) Herfindhal index is 0.447 (0.443) on May 6, which is close to the mean (median) of 0.440 (0.424) on May 5. In the up period, the mean (median) Herfindhal index is 0.429 (0.426) on May 6, which is close to the mean (median) of 0.442 (0.426) on May 5. However, the patterns differ between subgroups. In Panel A of Table 7, the mean Herfindahl index on May 5 is 0.351 in the low group, 0.429 in the middle group and 0.542 in the high group. On May 6, the mean increased significantly for the low group, but decreased significantly for the high group. Panel B of Table 7 shows the same pattern for the medians of the three subsamples. The results show that for stocks with a high Herfindahl index, trades were more spread across trading centers during the flash crash, whereas for stocks with a low Herfindahl index, trades became more concentrated. Interestingly, stocks with a medium level of trade fragmentation did not experience a significant change in market fragmentation.

Next, we examine quote fragmentation. We study bid quotes and ask quotes separately because they play different roles during the down and up periods of the flash crash. Since the vast majority of quotes are quickly cancelled (Hasbrouck, 2013), the number of quotes is not an informative indicator of quote-related liquidity. Thus, we construct a Herfindahl index measure of quote fragmentation based on the amount of time each center being on the best bid or ask quotes. For each trading center, we compute the total amount of time when the center offers the best bid quote among all centers within a specified time interval. We then calculate the bid quote Herfindahl index as the sum, across all trading centers, of the squared percentage of the total amount of time when the center offers the best bid quote. Mathematically, for a given stock j in a period t, the bid quote Herfindahl index H_{jt}^{b} is defined as

$$H_{jt}^{b} = \sum_{i=1}^{N} \left(\frac{\tau_{jti}}{\sum_{i=1}^{N} \tau_{jti}} \right)^2 , \qquad (2)$$

where N stands for the number of trading centers, τ_{jti} measures in millisecond the aggregate amount of time when the ith center offers the best bid quote.

For each stock, we calculate the difference in the bid quote Herfindahl index between May 5 and May 6. We apply the t-statistic and the Wilcoxon rank-sum statistic to test whether the mean and the median of the differences are significantly different from zero. The empirical results on the bid quote Herfindahl index are reported in Panels A and B of Table 8. Quotes are apparently more fragmented than trades. For the whole group of 96 stocks, in the down period on May 5, the mean (median) of the bid quote Herfindahl index is 0.366 (0.351), much smaller than the mean (median) of the trade volume Herfindahl Index. Both the mean and the median of the bid quote Herfindahl Index decreased significantly on May 6, indicating that orders were more widely spread among trading centers during the flash crash.

[Table 8 is about here.]

We sort stocks into three groups according to the bid quote Herfindahl index on May 5. Stocks in the low group have a smaller index value and thus a higher level of quote fragmentation than stocks in the high group. There are significant differences between the three subgroups. For stocks in the high group, the Herfindahl indices show a significant decline in both down and up periods on May 6. This means that bid quotes were more widely spread among trading centers during the flash crash. For stocks in the low group, the Herfindahl index does not show a significant change in the down period of May 6, but increases significantly in the up period of May 6. For stocks in the middle group, the Herfindahl index decreases significantly in the down period, but does not show a significant change in the up period.

Next, we examine the level of ask quote fragmentation. For each trading center, we compute the total amount of time when the center offers the best ask quote among all centers within a specified time interval. Then, we calculate the ask quote Herfindahl index as the sum, across all trading centers, of the squared percentage of the total amount of time when the center offers the best ask quote. Mathematically, for a given stock j in a period t, the ask quote Herfindahl index H_{jt}^{a} is defined as

$$H_{jt}^{a} = \sum_{i=1}^{N} \left(\frac{\tau_{jti}}{\sum_{i=1}^{N} \tau_{jti}} \right)^2 , \qquad (3)$$

where N stands for the number of trading centers, τ_{jti} measures in millisecond the aggregate amount of time when the *i*th center offers the best ask quote.

The empirical results on the ask quote Herfindahl index are reported in Panels A and B of Table 9. For the whole group of 96 stocks, both the mean and the median of the ask quote Herfindahl Index decreased significantly on May 6, indicating that orders were more widely spread among trading centers during the flash crash. We sort stocks into three groups according to the ask quote Herfindahl index on May 5. Stocks in the low group have a smaller index value and thus a higher level of quote fragmentation than stocks in the high group. There are significant differences between the three subgroups. For stocks in the high group, the Herfindahl indices show a significant decline in both down and up periods on May 6. This means that ask quotes were more widely spread among trading centers during the flash crash. For stocks in the low group, the Herfindahl index increases significantly in the down period, and shows a marginal increase in the up period. For stocks in the middle group, there is no significant change in both down and up periods.

[Table 9 is about here.]

4.4. Robustness study

The empirical results in Section 4.3 are subject to two criticisms. First, the value of the Herfindahl Index on May 5 may not be an unbiased estimate of the degree of a stock's trade/quote fragmentation in the pre-flash-crash period. Second, the Herfindahl index does not follow a normal distribution because it must be bounded between 0 and 1. The conventional t-test is unreliable if the underlying distribution of the observations is non-normal. We conduct additional analysis in this section to address these issues.[5]

First, we examine a stock's degree of trade/quote fragmentation in a long pre-event window between March 1, 2010 and May 5, 2010 inclusively. For each stock, we calculate the trade/quote Herfindahl index in the same down and up periods on each trading day during the pre-event window. We check the normality of the distribution of each trade/quote Herfindahl index by visually inspecting the histogram and qq-plot of the daily values of each trade/quote Herfindahl index for all stocks in our sample and for all days in the pre-event window. The three panels in Figure 1 shows the histogram and qq-plots of the Herfindahl Index values for trades, bid quotes, and ask quotes, respectively.

[Figure 1 is about here.]

[5] We sincerely thank an anonymous reviewer for comments and suggestions that motivate the analysis in this section.

It is clear from Figure 1 that the Herfindahl Index is bounded between 0 and 1 and is skewed to the right. We conduct formal tests of the normality and report in Table 10 the descriptive statistics of the Herfindahl indices and the test statistics of three normality tests: the Kolmogorov-Smirnov (KS) test, the Cramer-von Mises (CM) test, and the Anderson-Darling (AD) test. All three tests reject the null hypothesis that the Herfindahl index follows a normal distribution.

[Table 10 is about here.]

Next, we carry out a random sampling test that does not assume a normal distribution of the observations. MacKinlay (1997) suggest that randomization tests can be used to overcome potential biases associated with non-normal distributions. In order to test whether the market fragmentation during the flash crash on May 6 is significantly different from the fragmentation on an average day prior to the flash crash, we conduct the randomization test as follows.

For each of the 96 Nasdaq stocks in our sample, we randomly pick one day in the pre-event window and use the stock's Herfindahl index on that day to replace the stock's Herfindhal index on May 6. This generates a pseudo sample of randomly chosen observations from the pre-event window. We repeat the random sampling procedure and create 1000 pseudo samples. We calculate the median Herfindahl index across the observations in each pseudo sample and thus have 1000 medians. Table 11 reports the 1^{st}, 5^{th}, 10^{th}, 50^{th}, and 90^{th} percentiles of the 1000 medians. If the observed median on May 6 is less than the 1^{st} (5^{th}, 10^{th}) percentile of the 1000 random samples, we conclude that the degree of trade/quote fragmentation on May 6 is significantly smaller greater than that on an average day prior to the flash crash at the 1% (5%, 10%) level.

[Table 11 is about here.]

We sort the 96 Nasdaq stocks into three equal-size subsamples by the median of each stock's daily values of the trade/quote Herfindahl index in the pre-event window.

We sort the stocks by the median, because Figure 1 and Table 10 show that the distribution of the Herfindahl index is skewed to the right and its mean is affected by extreme observations at the right tail. We apply the randomization test to the whole sample as well as the three subsamples, separately. If we find a significant outcome in the randomization test on the median, the results are convincing that market fragmentation during the flash crash on May 6 is different from normal days.

Panel A of Table 11 shows the randomization sampling distribution of the trade Herfindahl index. We examine the 1st, 5th, and 50th percentiles of the 1000 medians of randomly constructed pseudo samples for the down period first. For the whole sample of 96 stocks, the three percentiles are 0.423, 0.431, and 0.450. For stocks in the low Herfindahl index group, the three percentiles are 0.373, 0.382, and 0.406. For stocks in the middle group, the three percentiles are 0.405, 0.415, and 0.444. For stocks in the high group, the three percentiles are 0.457, 0.478, and 0.514. The 50th percentile for the low group is about the same as the 1st percentile for the middle group, while the 50th percentile for the middle group is below the 1st percentile for the high group. These numbers show that there exists considerable variation in trade fragmentation across stocks and days in the pre-event window.

For the whole sample, the median on May 6 is 0.444, which is greater than the 10th percentile but less than the 50th percentile of the randomization sampling distribution. The median on May 6 for the low group is 0.402, while the median on May 6 for the middle group is 0.441. Both are greater than the respective 10th percentile but less than the respective 50th percentile. The median on May 6 for the high group is 0.478, which is the same as the 5th percentile of the randomization sampling distribution. Hence, the randomization test shows that, in the down period of the flash crash on May 6, the degree of trade fragmentation is not significantly different from an average day in the pre-event window, except that some stocks with a very high Herfindahl index experience a significant decrease in the index on May 6.

This finding is different in some aspects from what we observe in Panel B of Table 7, where we compare the median on May 6 with that on May 5. The conclusion for the whole sample is the same, that is, there is no significant change in trade fragmentation at the median on May 6, compared with May 5. However, stocks in the low Herfindahl index group have a significant increase in the median on May 6, relative to May 5. We observe that for the whole sample, the median Herfindahl index on May 5 is 0.424, which is close to the 1st percentile (i.e. 0.423) of the randomization sampling distribution in Panel A of Table 11. This means that May 5 itself is significantly different from an average day in the pre-event window. Hence, there exist some differences between the findings with May 5 as the benchmark and the findings with reference to the randomization sampling distribution.

Next we turn to the up period in Panel A of Table 11. For the whole sample, the median on May 6 is 0.426, which is the same as the 1st percentile of the randomization sampling distribution. The median on May 6 for the low group is not significant according to the randomization test, whereas it is significant for both the middle group and the high group. Hence, in the up period, more than half of the stocks had a very low trade Herfindahl index during the flash crash, which means trades were more widely spread across trading centers.

Panel B of Table 11 presents the randomization test for the bid-quote Herfindahl index. The median index on May 6 is below the 1st percentile of the randomization sampling distribution for the whole sample and for all three subsamples. This means that the majority of stocks have a very low bid-quote Herfindahl index during the flash crash, relative to an average day in the pre-event window. The surge in the number of bid quotes during the flash crash made the distribution of bid quotes more widely spread across trading centers. We observe a similar pattern for ask quotes in Panel C of Table

11. By comparing Panel B in Table 11 with Table 8 for the bid-quote Herfindahl index and comparing Panel C in Table 11 with Table 9 for the ask-quote Herfindahl index, we find that the median of these two indices on May 5 is below the 1st percentile of the respective randomization sampling distribution.

The results in Section 4.3 show us how the flash crash affected trade/quote fragmentation relative to May 5, which is the day immediately prior to the date of the flash crash and hence is the most relevant benchmark for comparison. . The analysis in this section using the randomized sampling distribution gives the extent of variation in trade/quote fragmentation across stocks and days over a longer period of time, which is valuable for assessing the significance of the impact of the flash crash. However, what we learn by comparing May 6 with May 5 is the basis for our summary and conclusions because the findings based on a day to day comparision are relevant and informative for future research.

5. Summary and conclusion

Under the current market system, the U.S. common stocks are simultaneously traded on multiple trading centers. We study quotes and trades with millisecond timestamp on individual trading centers during the flash crash of May 6, 2010. Relative to May 5, 2010, the number of trades and the number of quotes quadrupled in the time window immediately before a stock’s worst price on May 6, 2010. However, the proportional time of offering best bid (or ask) decreased substantially in all trading centers. For example, on NQEX, the median proportional time of offering best bid decreased from 95.2% on May 5 to 69.6% on May 6; more dramatically, on CBOE, the median proportional time of offering best bid went from 54.9% on May 5 to 0.9% on May 6. Because the Order Protection Rule of Regulation NMS requires that an order must be executed at the best quote price in the whole market, the observed decline in the proportional time of offering best quotes suggests that liquidity diminished on all trading centers during the flash crash, and nearly disappeared on CBOE.

We use the Herfindahl index to measure the level of trade (or quote) fragmentation, and study the changes in trade (or quote) fragmentation during the flash crash. We find that during the flash crash, trades became more evenly spread across trading centers for stocks with high level of trade fragmentation, but more concentrated for stocks with low level of trade fragmentation.

The changes in quote fragmentation exhibit more complex patterns. Both stocks with high level of bid-quote fragmentation and stocks with high level of ask-quote fragmentation experienced significant increase in quote fragmentation in both down and up periods on May 6. However, in the down period of May 6, stocks with low bid-quote fragmentation did not have significant change in quote fragmentation, whereas stocks with low ask-quote fragmentation experienced a significant decrease in fragmentation. In the up period of May 6, stocks with low bid-quote fragmentation had a significant decrease in fragmentation. Bid quotes supply liquidity when stock price plummets, while ask quotes supply liquidity when stock price rebounds. Hence, the changes in quote fragmentation during the flash crash seem to suggest that when investors demand liquidity, they have common preference of which trading centers they should submit liquidity-demanding orders, but there is no evidence for such a common preference when they submit liquidity-supplying orders.

Last, but not least, we find that stocks with the medium level of trade/quote fragmentation did not experience any significant change in fragmentation. In other words, such stocks were most resilient to the sudden order flow shock during the flash crash.

In this study, we examine the dynamics in trade and quote activities on individual trading centers during the flash crash and find that the surge in quote and order flows during the flash crash had a significant impact on the effectiveness of trade execution on individual trading centers and the distribution of trades and quotes across centers. The randomization sampling distribution in Section 4.4 shows that the degree of trade/quote fragmentation varies considerably across stocks and days in the two months prior to the flash crash. Investors' trading behavior may change with other factors such as market volatility.[6] For example, when the market is quiescent, orders may be routed to minimize explicit trading costs; whereas, when the market is volatile, the priority is shifted to increasing the probability of execution. We call for more research to explain the temporal and cross-sectional variation in trading effectiveness and trade/quote fragmentation across trading centers.

6 We thank an anonymous reviewer for suggesting this direction of future research.

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Table 1. Lowest price of sample stocks on May 6th 2010

Our sample includes the 96 Nasdaq stocks that are constituents of the S&P500 index as of May 6th 2010. For each stock, this table reports the lowest value of the stock's quote midpoint (i.e., the average of the best bid price and the best ask price) on May 6th 2010 and the time in milliseconds when the lowest quote midpoint first appeared.

Obs.	**Ticker Symbol**	**Time (milliseconds)**	**Lowest Quote Midpoint ($)**	**Obs.**	**Ticker Symbol**	**Time (milliseconds)**	**Lowest Quote Midpoint ($)**
1	AAPL	14:46:53.800	199.655	49	INTU	14:45:28.450	34.010
2	ADBE	14:45:30.650	30.245	50	ISRG	14:46:03.400	314.610
3	ADP	14:47:38.600	30.610	51	JDSU	14:47:41.500	9.510
4	ADSK	14:46:54.525	28.660	52	KLAC	14:45:56.675	29.995
5	AKAM	14:46:27.925	34.005	53	LIFE	14:45:41.200	49.880
6	ALTR	14:46:15.150	22.470	54	LLTC	14:46:15.075	26.785
7	AMAT	14:46:08.675	12.000	55	MAT	14:45:29.100	20.555
8	AMGN	14:45:33.450	52.775	56	MCHP	14:46:45.650	26.140
9	AMZN	14:47:12.050	120.755	57	MOLX	14:45:42.100	19.715
10	APOL	14:46:05.300	54.025	58	MSFT	14:46:38.925	27.915
11	BBBY	14:46:47.250	41.240	59	MU	14:46:54.725	7.325
12	BIIB	14:46:08.625	49.470	60	MYL	14:46:15.400	20.525
13	BMC	14:47:03.700	35.545	61	NDAQ	14:46:10.650	19.530
14	BRCM	14:47:08.400	29.055	62	NOVL	14:45:21.500	5.065
15	CA	14:46:17.400	20.085	63	NTAP	14:46:36.275	30.545
16	CELG	14:49:21.875	53.715	64	NTRS	14:46:26.700	49.955
17	CEPH	14:49:37.975	58.470	65	NVDA	14:45:42.950	13.050
18	CERN	14:48:30.750	75.720	66	NVLS	14:46:07.700	23.225
19	CHRW	14:46:15.475	57.330	67	NWSA	14:45:44.175	12.910
20	CINF	14:45:44.450	26.170	68	ORCL	14:46:15.775	22.210
21	CMCSA	14:45:44.600	17.650	69	ORLY	14:45:48.575	45.240
22	CME	14:47:49.575	307.855	70	PAYX	14:46:39.000	28.115
23	COST	14:47:02.425	55.045	71	PBCT	14:47:30.875	14.070
24	CPWR	14:46:33.475	7.510	72	PCAR	14:46:11.075	39.835
25	CSCO	14:45:32.600	23.235	73	PCLN	14:47:30.400	203.075
26	CTAS	14:45:28.250	25.005	74	PDCO	14:43:04.025	25.150
27	CTSH	14:47:33.525	46.225	75	QCOM	14:46:32.575	35.565
28	CTXS	14:46:19.450	42.270	76	QLGC	14:46:00.850	17.715
29	DELL	14:46:15.100	14.285	77	ROST	14:46:40.950	50.985

Obs.	Ticker Symbol	Time (milliseconds)	Lowest Quote Midpoint ($)	Obs.	Ticker Symbol	Time (milliseconds)	Lowest Quote Midpoint ($)
30	DISCA	14:45:45.525	33.570	78	RRD	14:50:20.775	18.325
31	DTV	14:48:40.375	34.105	79	SBUX	14:47:53.875	24.420
32	EBAY	14:45:57.025	20.570	80	SHLD	14:44:58.750	102.505
33	ERTS	14:46:16.125	17.050	81	SIAL	14:46:21.225	53.490
34	ESRX	14:46:09.050	75.670	82	SNDK	14:46:46.400	33.995
35	ETFC	14:49:54.525	1.445	83	SPLS	14:45:43.275	20.650
36	EXPD	14:50:48.700	37.955	84	SRCL	14:47:05.350	54.765
37	EXPE	14:45:36.150	22.045	85	SYMC	14:46:09.025	15.955
38	FAST	14:47:05.475	48.840	86	TLAB	14:47:29.475	8.025
39	FISV	14:49:52.850	51.375	87	TROW	14:45:46.375	50.650
40	FITB	14:45:29.150	12.605	88	URBN	14:45:42.175	33.945
41	FLIR	14:46:33.950	27.195	89	VRSN	14:46:32.975	24.610
42	FSLR	14:45:20.225	118.495	90	WFMI	14:46:57.475	35.820
43	GENZ	14:50:06.975	50.945	91	WIN	14:49:52.300	6.125
44	GILD	14:45:25.575	38.015	92	WYNN	14:46:26.550	72.215
45	GOOG	14:45:36.325	462.755	93	XLNX	14:46:15.350	23.240
46	HBAN	14:46:11.725	5.670	94	XRAY	14:52:09.725	33.325
47	HCBK	14:47:25.275	12.350	95	YHOO	14:50:55.650	15.440
48	INTC	14:47:30.250	19.905	96	ZION	14:45:28.650	25.070

Table 2. Number of trades and quotes during the flash crash

The 'down' period refers to the 5-minute period immediately before the lowest quote midpoint on May 6 first appeared. The 'up' period refers to the 5-minute period immediately after the lowest quote midpoint on May 6 first appeared. Since the lowest quote midpoint appeared at different times for different stocks, the down and up periods differ between stocks. We use the same down and up periods in clock time on May 5 as the benchmark period. For example, if the lowest quote midpoint first appeared at 14:45:27.150 on May 6, the down period is from 14:40:27.000 to 14:45:27.000 (a left-end-closed and right-end-open 5-minute interval) and the up period is from 14:45:28.000 to 14:50:28.000, which are the same on May 5. This table reports descriptive statistics on number of trades, number of shares traded, number of bid quotes, and number of ask quotes. For each stock, we compute these numbers in the down and up periods on May 6 and May 5. Ratio is equal to a number on May 6th divided by its corresponding value on May 5th. The statistics are calculated cross-sectionally for the sample of 96 stocks. P5 and P95 represent the 5th and 95th percentile, and Mdn represents the median.

	Down period						**Up period**					
	Mdn	**Mean**	**Min**	**P5**	**P95**	**Max**	**Mdn**	**Mean**	**Min**	**P5**	**P95**	**Max**
Number of trades												
May 6	1590	3033	203	255	10452	22676	984	1962	106	193	5462	13302
May 5	340	562	27	92	1744	2912	434	660	17	86	1919	4629
Ratio	4.5	4.4	1.1	1.7	13.9	18.1	2.4	3.6	0.9	1.0	8.1	73.2
Number of shares traded ('000)												
May 6	278	766	25	37	3463	7393	162	529	15	24	2453	7035
May 5	52	143	3	10	562	1153	65	174	3	11	636	2019
Ratio	4.7	6.0	0.6	1.5	14.1	30.6	2.2	4.1	0.8	1.0	13.8	93.6
Number of bid quotes												
May 6	7278	14744	448	992	51264	91481	5245	7944	189	769	25728	42584
May 5	1891	2793	281	467	7813	14539	2129	3,034	279	561	8309	17018
Ratio	4.3	4.6	0.6	0.8	11.7	13.1	2.4	2.9	0.3	0.8	8.3	12.6
Number of ask quotes												
May 6	7641	14975	1145	1530	49057	89852	5392	8330	149	860	26163	44554
May 5	2189	2815	190	432	6965	12832	2256	3268	349	500	8960	17750
Ratio	4.5	4.9	0.7	1.5	11.0	12.8	2.2	2.8	0.4	0.8	6.3	16.5

Table 3. Trades on individual trading centers

Trades of the 96 stocks under our study are reported in one of the nine trading centers, which are NQEX, BATS, PACF, BOST, ISEX, NTRF, CINC, CHIC, and CBOE. Panel A reports the number of stocks that have trades in each individual trading center. Panel B reports the total number of trades for all 96 stocks in each center and its corresponding percentage in parentheses. Panel C reports the total number of shares traded (in million) for all 96 sample stocks in each trading center and its corresponding percentage in parentheses. 'May 5, Down', 'May 6, Down', 'May 5, Up', and 'May 6, Up' are the four time periods defined in Table 2.

	Total	**NQEX**	**BATS**	**PACF**	**BOST**	**ISEX**	**NTRF**	**CINC**	**CHIC**	**CBOE**
Panel A: number of stocks that have trades reported in each trading center										
May 5, Down	96	96	96	96	72	81	93	69	14	15
May 6, Down	96	96	96	96	92	96	94	81	46	40
May 5, Up	96	96	96	95	74	80	94	65	13	15
May 6, Up	96	96	96	96	96	95	89	78	42	7
Panel B: total number of trades (% of total)										
May 5, Down	53,956	29,135	11,798	7,794	2,679	690	1,442	360	25	33
	(100%)	*(54.0)*	*(21.9)*	*(14.5)*	*(5.0)*	*(1.3)*	*(2.7)*	*(0.7)*	*(0.05)*	*(0.06)*
May 6, Down	291,172	162,178	65,788	30,993	19,216	6,664	3,249	2,622	282	180
	(100%)	*(55.7)*	*(22.6)*	*(10.6)*	*(6.6)*	*(2.3)*	*(1.1)*	*(0.9)*	*(0.1)*	*(0.06)*
May 5, Up	63,405	33,805	13,884	9,507	3,389	793	1,618	365	19	25
	(100%)	*(53.3)*	*(21.9)*	*(15.0)*	*(5.3)*	*(1.3)*	*(2.6)*	*(0.6)*	*(0.03)*	*(0.04)*
May 6, Up	188,326	103,556	37,747	26,458	13,442	4,138	1,128	1,758	88	11
	(100%)	*(55.0)*	*(20.0)*	*(14.1)*	*(7.1)*	*(2.2)*	*(0.6)*	*(0.9)*	*(0.05)*	*(0.01)*
Panel C: total number of shares traded in million (% of total)										
May 5, Down	13.7	8.3	2.4	1.9	0.5	0.2	0.4	0.1	0.01	0.01
	(100%)	*(60.1)*	*(17.6)*	*(14.0)*	*(3.3)*	*(1.2)*	*(3.0)*	*(0.6)*	*(0.05)*	*(0.07)*
May 6, Down	73.5	45.0	13.9	7.7	3.5	1.6	1.2	0.6	0.07	0.02
	(100%)	*(61.1)*	*(18.9)*	*(10.4)*	*(4.8)*	*(2.2)*	*(1.6)*	*(0.8)*	*(0.1)*	*(0.03)*
May 5, Up	16.7	9.9	2.7	2.6	0.6	0.3	0.5	0.08	0.004	0.003
	(100%)	*(59.3)*	*(16.1)*	*(15.8)*	*(3.6)*	*(1.6)*	*(3.0)*	*(0.5)*	*(0.02)*	*(0.02)*
May 6, Up	50.8	29.3	9.8	6.7	2.5	1.5	0.5	0.4	0.02	0.01
	(100%)	*(57.7)*	*(19.4)*	*(13.2)*	*(4.9)*	*(2.9)*	*(1.0)*	*(0.8)*	*(0.05)*	*(0.03)*

Table 4. Quotes on individual trading centers

Quotes of the 96 stocks under our study are from one of the eight trading centers, which are NQEX, BATS, PACF, BOST, ISEX, CINC, CHIC, and CBOE. Panel A reports the number of stocks that have bid quotes in each center and, in parentheses, the number of stocks that have ask quotes. Panel B reports the total number of bid quotes for all 96 stocks in each center and its corresponding percentage in parentheses. Panel C reports the total number of ask quotes for all 96 stocks in each center and its corresponding percentage in parentheses. 'May 5, Down', 'May 6, Down', 'May 5, Up', and 'May 6, Up' are the four time periods defined in Table 2.

	Total	**NQEX**	**BATS**	**PACF**	**BOST**	**ISEX**	**CINC**	**CHIC**	**CBOE**
Panel A: number of stocks with bid quotes (number of stocks with ask quotes)									
May 5, Down	96 (96)	96 (96)	96 (96)	96 (96)	88 (89)	96 (96)	82 (83)	73 (73)	95 (95)
May 6, Down	96 (96)	96 (96)	96 (96)	96 (96)	88 (92)	96 (96)	95 (95)	73 (75)	88 (89)
May 5, Up	96 (96)	96 (96)	96 (96)	96 (96)	88 (88)	96 (96)	81 (84)	73 (73)	95 (95)
May 6, Up	96 (96)	96 (96)	96 (96)	96 (96)	94 (96)	96 (96)	88 (90)	66 (68)	43 (45)
Panel B: total number of bid quotes (% of total)									
May 5, Down	268,144	103,851	50,331	40,905	25,739	25,573	15,387	1,723	4,635
	(100%)	(38.7)	(18.8)	(15.3)	(9.6)	(9.5)	(5.7)	(0.6)	(1.7)
May 6, Down	1,415,495	517,579	324,117	132,004	170,216	121,606	123,250	8,953	17,770
	(100%)	(36.6)	(22.9)	(9.3)	(12.0)	(8.6)	(8.7)	(0.6)	(1.3)
May 5, Up	291,229	113,975	53,885	42,484	30,082	27,004	17,295	1,622	4,882
	(100%)	(39.1)	(18.5)	(14.6)	(10.3)	(9.3)	(5.9)	(0.6)	(1.7)
May 6, Up	762,610	265,841	177,639	92,506	97,504	62,633	58,658	2,994	4,835
	(100%)	(34.9)	(23.3)	(12.1)	(12.8)	(8.2)	(7.7)	(0.4)	(0.6)
Panel C: total number of ask quotes (% of total)									
May 5, Down	270,281	104,301	47,117	43,331	26,752	26,748	16,463	1,632	3,937
	(100%)	(38.6)	(17.4)	(16.0)	(9.9)	(9.9)	(6.1)	(0.6)	(1.5)
May 6, Down	1,437,637	526,150	347,578	109,993	170,987	125,853	132,302	8,788	15,986
	(100%)	(36.6)	(24.2)	(7.6)	(11.9)	(8.7)	(9.2)	(0.6)	(1.1)
May 5, Up	313,731	123,538	58,306	46,014	31,065	29,725	18,443	1,876	4,764
	(100%)	(39.4)	(18.6)	(14.7)	(9.9)	(9.5)	(5.9)	(0.6)	(1.5)
May 6, Up	799,678	255,459	185,541	109,428	112,273	62,327	66,659	3,150	4,841
	(100%)	(32.0)	(23.2)	(13.7)	(14.0)	(7.8)	(8.3)	(0.4)	(0.6)

Table 5. Proportional time of offering best quotes

For each trading center, we compute the aggregate amount of time when the center offers the best bid (or ask) quote within each time period. We define the center's proportional time of offering best quotes as the proportion of this aggregate amount to the total amount of time in the specified period. For each trading center, the table reports the mean and the median, across stocks, of the proportional time of offering best quotes (in percent). 'May 5, Down', 'May 6, Down', 'May 5, Up', and 'May 6, Up' are the four 5-minute periods defined in Table 2. The column 'N' reports the number of stocks for which a trading center has offered the best bid (or ask) quote during the period. Difference is equal to the value on May 6 minus the value on May 5 of the same stock.

Panel A: proportional time of offering best ***bid*** quote

	May 5			**May 6**			**Difference**		
	N	**Mdn**	**Mean**	**N**	**Mdn**	**Mean**	**N**	**Mdn**	**Mean**
Down period									
NQEX	96	95.2	87.7	96	69.6	68.0	96	-18.0***	-19.6***
BATS	96	85.9	71.9	96	43.8	45.2	96	-24.5***	-26.7***
PACF	96	84.8	75.0	96	42.2	45.2	96	-29.5***	-29.8***
BOST	70	54.7	49.9	91	23.4	27.7	70	-14.4***	-15.8***
ISEX	94	66.5	60.6	96	31.0	41.1	94	-17.1***	-19.1***
CINC	77	70.5	58.8	85	22.1	27.4	72	-26.2***	-28.6***
CHIC	28	31.6	38.6	67	1.7	5.0	25	-20.3***	-29.1***
CBOE	46	54.9	45.1	74	0.9	9.4	38	-22.0***	-27.7***
Up period									
NQEX	96	94.8	88.3	96	69.6	65.1	96	-21.9***	-23.2***
BATS	96	83.0	71.5	96	42.0	44.2	96	-29.6***	-27.3***
PACF	96	81.3	74.2	96	42.4	45.1	96	-29.3***	-29.1***
BOST	74	50.1	48.7	95	31.2	32.7	74	-19.8***	-12.5***
ISEX	96	54.6	57.3	96	23.8	29.5	96	-28.4***	-27.8***
CINC	76	72.5	59.6	80	27.7	28.5	69	-29.5***	-30.4***
CHIC	29	37.0	35.6	29	0.5	1.2	16	-25.7***	-28.9***
CBOE	44	59.3	47.6	66	0.5	1.8	30	-55.7***	-44.0***

***, **, * indicates significance at 1%, 5%, and 10% level, respectively.

Table 5. (continued)

Panel B: proportional time of offering best ***ask*** quote

	May 5			May 6			Difference		
	N	**Mdn**	**Mean**	**N**	**Mdn**	**Mean**	**N**	**Mdn**	**Mean**
Down period									
NQEX	96	96.8	90.4	96	71.1	70.2	96	-21.3***	-20.2***
BATS	96	89.9	74.1	96	60.7	56.0	96	-15.9***	-18.1***
PACF	96	87.3	78.7	96	39.5	41.1	96	-39.7***	-37.5***
BOST	73	59.1	53.5	91	27.7	34.1	73	-15.7***	-14.0***
ISEX	96	66.1	65.5	96	34.0	40.4	96	-19.8***	-25.1***
CINC	75	71.3	58.3	85	35.2	35.5	67	-20.6***	-19.8***
CHIC	24	44.0	50.7	25	7.4	13.6	19	-38.4***	-41.6***
CBOE	56	26.7	40.5	66	3.0	14.3	40	-11.9***	-24.5***
Up period									
NQEX	96	96.3	89.4	96	66.1	63.3	96	-25.2***	-26.1***
BATS	96	85.4	73.6	96	37.3	41.1	96	-34.2***	-32.6***
PACF	96	86.7	78.9	96	32.9	36.3	96	-43.7***	-42.7***
BOST	69	58.4	53.0	96	30.6	31.8	69	-18.6***	-17.1***
ISEX	96	62.7	64.5	96	29.8	33.3	96	-30.1***	-31.2***
CINC	77	67.4	57.0	80	25.8	30.0	68	-29.5***	-27.5***
CHIC	24	45.2	48.5	51	0.7	1.0	16	-39.9***	-44.1***
CBOE	58	10.2	35.8	50	0.6	2.5	32	-19.2***	-35.3***

***, **, * indicates significance at 1%, 5%, and 10% level, respectively.

Table 6. Effectiveness of turning best quotes to trades

For each trading center, when the center is offering the best bid (or ask) quote among all centers, we compute two metrics: (a) the total number of shares that are traded on *all centers* and (b) the total number of shares that are traded on *this center*. We define the center's effectiveness of turning best quotes to trades as the ratio of (b) to (a). For each trading center, the table reports the mean and median (in percent) of the effectiveness of turning best quotes to trades across stocks. 'May 5, Down', 'May 6, Down', 'May 5, Up', and 'May 6, Up' are the four 5-minute periods defined in Table 2. The column 'N' reports the number of stocks for which a trading center has offered the best bid (or ask) quote during the period. Difference is equal to the value on May 6 minus the value on May 5 of the same stock.

Panel A: effectiveness of turning best ***bid*** quotes to trades

	May 5			**May 6**			**Difference**		
	N	**Mdn**	**Mean**	**N**	**Mdn**	**Mean**	**N**	**Mdn**	**Mean**
Down period									
NQEX	96	62.0	62.5	96	66.9	66.3	96	2.8***	3.8***
BATS	93	17.9	18.0	96	22.3	23.1	93	5.0***	4.9***
PACF	96	13.6	15.1	96	17.8	20.2	96	4.1***	5.0***
BOST	56	4.5	7.6	82	6.7	8.2	56	0.8	-1.0
ISEX	63	1.3	2.8	93	2.5	4.0	62	1.2***	0.8
CINC	46	0.8	2.7	63	1.1	2.8	41	0.3	-0.8
CHIC	9	0.2	5.1	35	1.6	7.8	5	0.2*	0.3
CBOE	7	0.1	0.2	25	0.1	3.6	4	-0.1	-0.1
Up period									
NQEX	96	62.0	62.2	96	65.2	64.5	96	1.9	2.3*
BATS	93	16.3	17.4	96	21.6	22.5	93	4.3***	5.3***
PACF	95	13.8	15.4	95	20.5	21.4	94	6.7***	5.9***
BOST	60	5.1	5.7	94	7.5	8.1	60	0.8**	1.6**
ISEX	59	1.2	2.9	87	2.6	3.6	54	1.0**	0.2
CINC	46	0.5	1.1	60	2.0	4.4	41	0.7***	1.9***
CHIC	10	0.3	10.9	2	29.9	29.9	0	0.0	0.0
CBOE	10	0.0	0.1	4	23.4	36.7	0	0.0	0.0

***, **, * indicates significance at 1%, 5%, and 10% level, respectively.

Table 6. (continued)

Panel B: effectiveness of turning best ***ask*** quotes to trades

	May 5			May 6			Difference		
	N	**Mdn**	**Mean**	**N**	**Mdn**	**Mean**	**N**	**Mdn**	**Mean**
Down period									
NQEX	96	61.8	61.9	96	63.8	63.4	96	1.4	1.5
BATS	95	17.3	17.2	96	18.0	19.2	95	1.0	2.0
PACF	95	12.5	14.5	96	13.7	16.5	95	0.5	1.8
BOST	60	4.3	8.0	86	6.2	7.2	58	0.8	-2.0
ISEX	65	1.2	2.4	85	1.9	3.0	61	0.9***	0.6
CINC	45	0.7	2.9	64	0.9	2.0	40	0.1	-0.8
CHIC	8	0.2	10.6	13	0.3	9.4	4	0.0	0.8
CBOE	10	0.1	1.2	28	0.2	4.9	5	-0.1	-0.1
Up period									
NQEX	96	61.6	62.1	96	64.8	64.7	96	2.4	2.6**
BATS	93	16.7	17.5	96	23.0	22.7	93	4.4***	5.5***
PACF	94	13.9	15.7	95	18.9	20.0	93	4.4***	4.3***
BOST	58	4.8	6.6	93	6.5	7.8	56	1.5	0.3
ISEX	64	1.4	2.9	89	2.9	4.1	63	1.5***	0.7
CINC	42	0.5	0.9	57	1.3	2.0	35	0.7***	0.9***
CHIC	4	0.3	0.3	27	21.4	29.4	0	0.0	0.0
CBOE	11	0.1	1.4	3	0.4	16.9	0	0.0	0.0

***, **, * indicates significance at 1%, 5%, and 10% level, respectively.

Table 7. Trade fragmentation

For each stock, we calculate the trade Herfindahl index as the sum, across all trading centers, of the squared percentage of the number of shares traded on each center. We sort stocks into three groups by the level of the trade Herfindahl index on May 5. Stocks in the 'low' group have a lower value of the index, which indicates a higher level of trading fragmentation. Panels A and B report mean and median across stocks, respectively, of the trade volume Herfindahl index. The column 'Diff.' reports that mean (or median) of the same-stock difference between May 5 and May 6. We apply the t-statistic to test whether the mean of the difference is equal to zero and the Wilcoxon rank-sum statistic to test whether the median of the difference is equal to zero.

	Down period			**Up period**		
	May 5	**May 6**	**Diff.**	**May 5**	**May 6**	**Diff.**
*Panel A: the trade Herfindahl index (**mean**)*						
All (96 stocks)	0.440	0.447	0.006	0.442	0.429	-0.012
Low (32 stocks)	0.351	0.407	0.056***	0.357	0.427	0.069***
Medium (32)	0.429	0.458	0.029*	0.428	0.407	-0.022
High (32)	0.542	0.475	-0.066***	0.539	0.455	-0.084***
*Panel B: the trade Herfindahl index (**median**)*						
All (96 stocks)	0.424	0.443	0.018	0.426	0.426	-0.008
Low (32 stocks)	0.359	0.395	0.048***	0.358	0.400	0.055**
Medium (32)	0.424	0.463	0.047	0.426	0.394	-0.045
High (32)	0.516	0.463	-0.054***	0.525	0.442	-0.084***

***, **, * indicates significance at 1%, 5%, and 10% level, respectively.

Table 8. Bid-quote fragmentation

For each stock, we calculate the bid-quote Herfindahl index as the sum, across all trading centers, of the squared percentage of the total amount of time when the center offers the best bid quote. Mathematically, for a given stock j in a period t, the bid-quote Herfindahl index is defined as

$$H_{jt}^{b} = \sum_{i=1}^{N} \left(\frac{\tau_{jti}}{\sum_{i=1}^{N} \tau_{jti}} \right)^2 ,$$

where N stands for the number of trading centers, τ_{jti} measures the aggregate amount of time in millisecond when the ith center offers the best bid quote. We sort stocks into three groups by the level of the bid-quote Herfindahl index on May 5. Stocks in the 'low' group have a lower value of the index, which indicates a higher level of bid-quote fragmentation. Panels A and B report mean and median across stocks, respectively, of the bid-quote Herfindahl index. The column 'Diff.' reports that mean (or median) of the same-firm difference between May 5 and May 6. We apply the t-statistic to test whether the mean of the difference is equal to zero and the Wilcoxon rank-sum statistic to test whether the median of the difference is equal to zero.

	Down period			**Up period**		
	May 5	**May 6**	**Diff.**	**May 5**	**May 6**	**Diff.**
Panel A: the ***bid****-quote Herfindahl index (****mean****)*						
All (96 stocks)	0.366	0.296	-0.069***	0.365	0.324	-0.041**
Low (32 stocks)	0.246	0.251	0.004	0.244	0.301	0.056***
Medium (32)	0.345	0.308	-0.037**	0.327	0.360	0.033
High (32)	0.505	0.329	-0.176***	0.521	0.310	-0.211***
Panel B: the ***bid****-quote Herfindahl index (****median****)*						
All (96 stocks)	0.351	0.299	-0.049***	0.320	0.314	-0.012
Low (32 stocks)	0.251	0.259	0.007	0.253	0.275	0.033**
Medium (32)	0.351	0.318	-0.048**	0.319	0.340	0.024
High (32)	0.497	0.317	-0.147***	0.480	0.309	-0.173***

***, **, * indicates significance at 1%, 5%, and 10% level, respectively.

Table 9. Ask-quote fragmentation

For each stock, we calculate the ask-quote Herfindahl index as the sum, across all trading centers, of the squared percentage of the total amount of time when the center offers the best ask quote. Mathematically, for a given stock j in a period t, the ask-quote Herfindahl index is defined as

$$H_{jt}^{a} = \sum_{i=1}^{N} \left(\frac{\tau_{jti}}{\sum_{i=1}^{N} \tau_{jti}} \right)^2 ,$$

where N stands for the number of trading centers, τ_{jti} measures the aggregate amount of time in millisecond when the ith center offers the best ask quote. We sort stocks into three groups by the level of the ask-quote Herfindahl index on May 5. Stocks in the 'low' group have a lower value of the index, which indicates a higher level of ask-quote fragmentation. Panels A and B report mean and median across stocks, respectively, of the ask-quote Herfindahl index. The column 'Diff.' reports that mean (or median) of the same-stock difference between May 5 and May 6. We apply the t-statistic to test whether the mean of the difference is equal to zero and the Wilcoxon rank-sum statistic to test whether the median of the difference is equal to zero.

	Down period			**Up period**		
	May 5	**May 6**	**Diff.**	**May 5**	**May 6**	**Diff.**
Panel A: the ***ask****-quote Herfindahl Index (****mean****)*						
All (96 stocks)	0.375	0.326	-0.050***	0.393	0.335	-0.058***
Low (32 stocks)	0.265	0.303	0.037**	0.254	0.282	0.028**
Medium (32)	0.333	0.327	-0.006	0.356	0.331	-0.025
High (32)	0.528	0.347	-0.181***	0.568	0.391	-0.177***
Panel B: the ***ask****-quote Herfindahl Index (****median****)*						
All (96 stocks)	0.331	0.308	-0.019	0.350	0.292	-0.024
Low (32 stocks)	0.272	0.290	0.020**	0.263	0.258	0.015
Medium (32)	0.331	0.336	-0.021	0.350	0.309	-0.033
High (32)	0.472	0.320	-0.133***	0.500	0.346	-0.130***

***, **, * indicates significance at 1%, 5%, and 10% level, respectively.

Table 10. Normality test of the trade, bid-quote, and ask-quote Herfindahl indices

For each stock, we calculate the trade and quote Herfindahl indices in the same down and up periods on each trading day during a long pre-event window between March 1, 2010 and May 5, 2010 inclusively. The table below reports the descriptive statistics of the daily values of each Herfindahl index for all 96 stocks in our sample and for all days in the pre-event window. We test the null hypothesis that the daily values of each Herfindahl index follow a normal distribution with three normality tests: the Kolmogorov-Smirnov (KS) test, the Cramer-von Mises (CM) test, and the Anderson-Darling (AD) test.

	Trade Herfindahl index	Bid-quote Herfindahl index	Ask-quote Herfindahl index
# of obs.	4,512	4,245	4,243
Mean	0.4705	0.4486	0.4445
Std Dev	0.1153	0.2129	0.2123
Percentiles of daily values of each Herfindahl index			
1%	0.286	0.125	0.125
10%	0.346	0.254	0.250
25%	0.388	0.313	0.308
50%	0.450	0.389	0.388
75%	0.530	0.514	0.506
90%	0.626	0.740	0.722
99%	0.831	1.000	1.000
Normality tests			
KS	0.082***	0.138***	0.141***
CM	11.0***	27.7***	28.3***
AD	67.2***	184.2***	187.8***

*** indicates significance at 1% level, respectively.

Table 11. Randomization test for market fragmentation on May 6

We conduct a randomization test to examine whether the market fragmentation during the flash crash on May 6 is significantly different from the fragmentation on normal days. For each of the 96 Nasdaq stock in our sample, we randomly pick one day in the pre-event window and use the stock's Herfindahl index on that day to replace the stock's Herfindhal index on May 6. This generates a pseudo sample of randomly chosen observations in the pre-event window. We repeat the random sampling procedure and create 1000 pseudo samples. We calculate the median of each pseudo sample and thus have 1000 medians. The table below reports the 1st, 5th, 10th, 50th, and 90th percentiles of the 1000 medians. If the observed median on May 6 is less than the 1st (5th, 10th) percentile of the 1000 random samples, we conclude that the degree of trade/quote fragmentation on May 6 is significantly greater than an average day prior to the flash crash at the 1% (5%, 10%) level.

Panel A. Trade Herfindahl Index

		Randomization sampling distribution percentiles				
	May 6	1%	5%	10%	50%	90%
Down period						
All (96)	0.444	0.423	0.431	0.435	0.450	0.465
Low (32)	0.402	0.373	0.382	0.387	0.406	0.429
Medium (32)	0.441	0.405	0.415	0.422	0.444	0.466
High (32)	0.478*	0.457	0.478	0.484	0.514	0.542
Up period						
All (96)	0.426**	0.426	0.434	0.436	0.451	0.466
Low (32)	0.390	0.372	0.380	0.384	0.409	0.433
Medium (32)	0.411**	0.399	0.415	0.422	0.443	0.464
High (32)	0.475*	0.461	0.474	0.481	0.514	0.544

***, **, * indicates significance at 1%, 5%, and 10% level, respectively.

Panel B. Bid-quote Herfindahl Index

		Randomization sampling distribution percentiles				
	May 6	1%	5%	10%	50%	90%
Down period						
All (96)	0.299***	0.359	0.370	0.375	0.390	0.422
Low (32)	0.279***	0.298	0.310	0.318	0.345	0.378
Medium (32)	0.312***	0.347	0.360	0.370	0.397	0.439
High (32)	0.300***	0.373	0.389	0.407	0.471	0.505
Up period						
All (96)	0.311***	0.355	0.367	0.373	0.389	0.418
Low (32)	0.289***	0.293	0.309	0.318	0.343	0.379
Medium (32)	0.298***	0.336	0.352	0.361	0.392	0.433
High (32)	0.317***	0.374	0.389	0.406	0.470	0.506

***, **, * indicates significance at 1%, 5%, and 10% level, respectively.

Panel C. Ask-quote Herfindahl Index

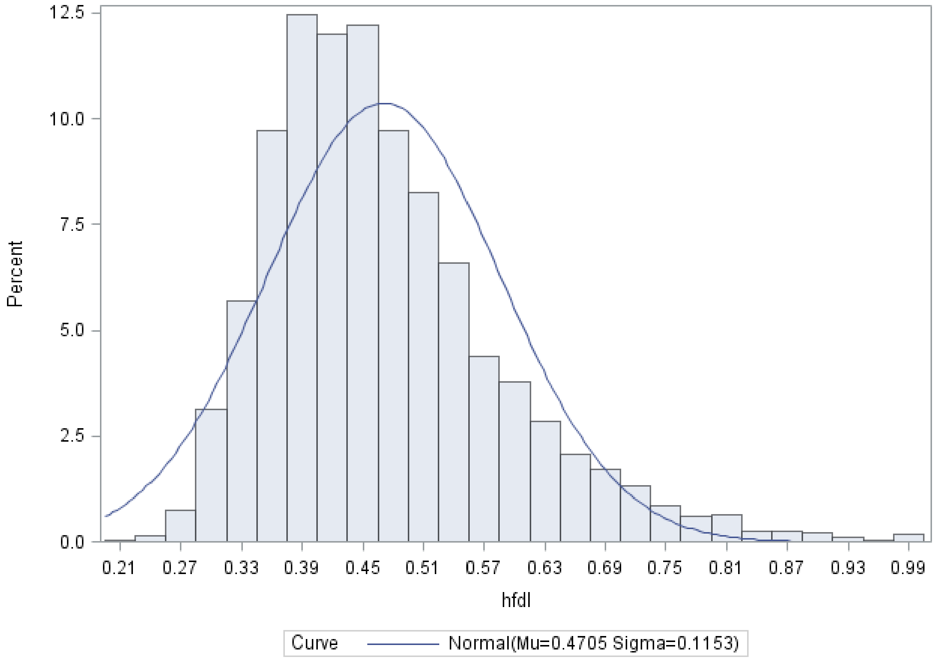
		Randomization sampling distribution percentiles				
	May 6	1%	5%	10%	50%	90%
Down period						
All (96)	0.308***	0.357	0.364	0.372	0.388	0.415
Low (32)	0.317	0.298	0.308	0.315	0.342	0.375
Medium (32)	0.313***	0.339	0.357	0.367	0.393	0.435
High (32)	0.307***	0.369	0.386	0.401	0.454	0.500
Up period						
All (96)	0.292***	0.357	0.365	0.375	0.393	0.429
Low (32)	0.270***	0.301	0.312	0.319	0.344	0.376
Medium (32)	0.363*	0.343	0.359	0.368	0.402	0.444
High (32)	0.288***	0.359	0.389	0.407	0.474	0.507

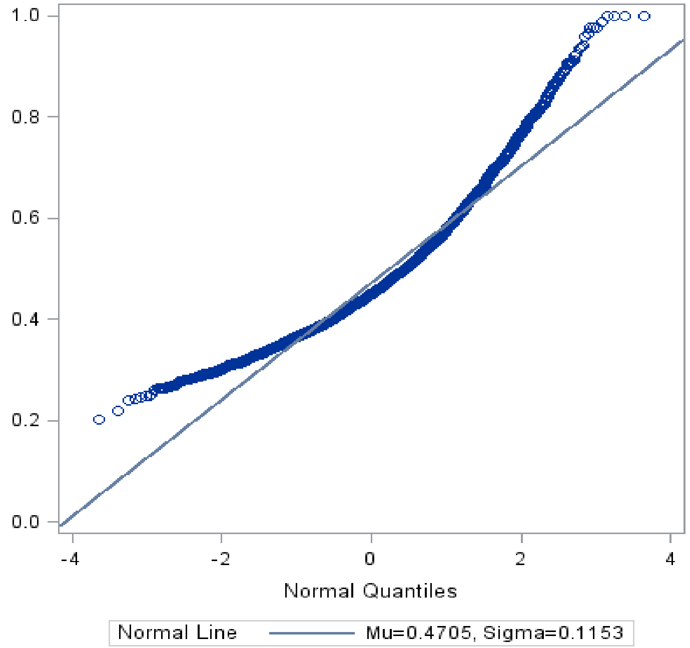
***, **, * indicates significance at 1%, 5%, and 10% level, respectively.

Figure 1. Histogram and qq-plot of trade and quote Herfindahl Index

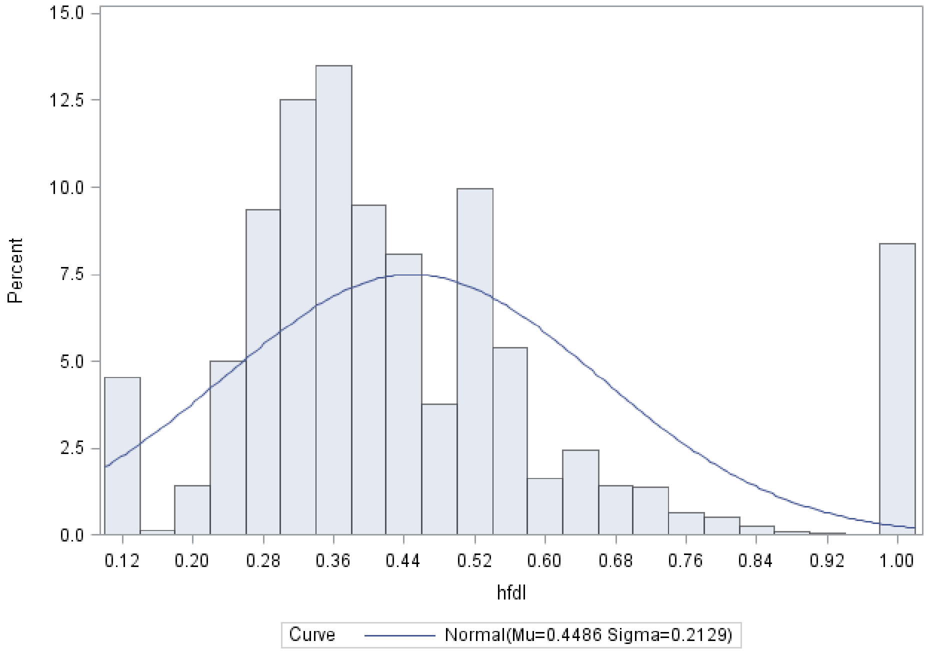
For each stock, we calculate the trade and quote Herfindahl indices in the same down and up periods on each trading day during a long pre-event window between March 1, 2010 and May 5, 2010 inclusively. We check the normality of the distribution of the Herfindahl index by visually inspecting the histogram and qq-plot of the daily values of each Herfindahl index for all stocks in our sample and for all days in the pre-event window. The three panels show the histogram and qq-plots of the Herfindahl index for trades, bid quotes, and ask quotes, respectively.

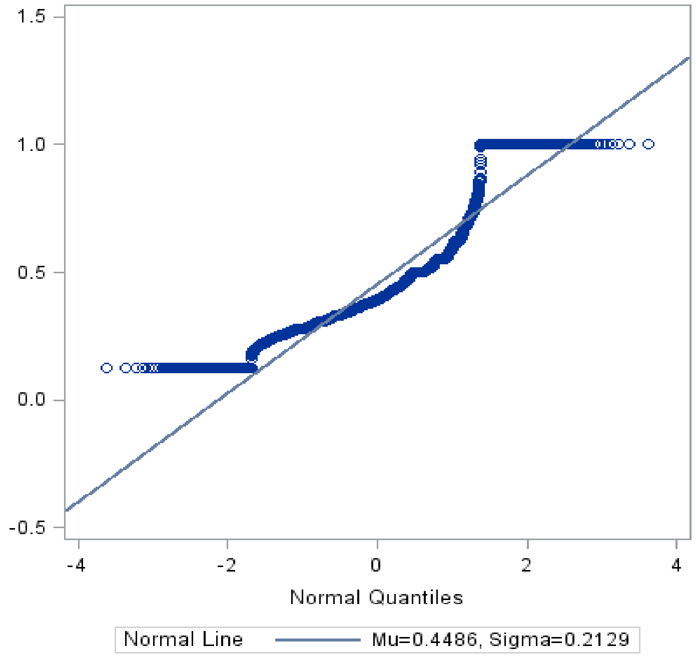
Panel A. Trade Herfinhahl Index





Panel B. Bid-quote Herfinhahl Index





Panel C. Ask-quote Herfinhahl Index