

# Single-shot in-line holographic authentication using phase and amplitude modulation

Wen Chen<sup>1,2,\*</sup>

<sup>1</sup>*Department of Electronic and Information Engineering, The Hong Kong Polytechnic University, Hong Kong, China*

<sup>2</sup>*The Hong Kong Polytechnic University Shenzhen Research Institute, Shenzhen 518057, China*

*\*Email: owen.chen@polyu.edu.hk*

## Abstract

In this paper, single-shot in-line holographic authentication using phase and amplitude modulation is presented. Amplitude modulation and phase modulation are presented and conducted at reference wave path to obtain noise-like reference wavefront and to generate the parameters (masks) at reference wave path as security keys for single-shot in-line digital holography, and its application, i.e., optical authentication, is presented to show validity of the proposed digital holographic system. A reference optical path using the cascaded random amplitude-only and phase-only masks is applied as an example. Only one in-line digital hologram is used for object reconstruction, and the recovered objects can be effectively authenticated. It is illustrated that parameters (e.g., masks) flexibly designed at reference wave path can facilitate wavefront retrieval in the CCD plane and also can act like security keys for optical information authentication. @ Elsevier, 2019.

**Keywords:** Optical security; Single-shot holography; Optical authentication; In-line holographic authentication

## 1. Introduction

Due to rapid development of modern technologies, digital holography [1,2] has attracted much attention in various applications, such as microscopy [3], optical security [4] and optical metrology [5]. Digital holography can record the whole field information related to a sample, and object reconstruction can be conducted by using numerical approaches without wet-chemical processing.

Until now, there are several types of digital holographic setups, such as off-axis [1,6], in-line [7] and parallel [8,9]. In the off-axis digital holography, a small angle between object wave and reference wave is used to generate spatial carrier for separating zero order term and twin image [1,6,10]. However, off-axis digital holographic setup is limited by low-resolution detectors, and space bandwidth is not high. In the in-line digital holography, phase-shifting algorithm [5,7] is commonly applied to extract object wavefront and is able to overcome inherent problem in the off-axis digital holography by using several exposures with phase shifts at reference wave path. However, much experimental effort should be made, and phase-shifting setup is not highly suitable for dynamic measurements. Recently, it has been found that parallel phase-shifting digital holography [8,9] can provide a new alternative. However, some devices (such as pixelated optical items) should be precisely aligned in the holographic setup, and optical components should be carefully fabricated. Single-exposure phase-shifting digital holography [11] has been developed as an effective method to overcome the above problems, and complex-valued object wavefront in the CCD plane can be extracted for the reconstruction. However, the recovered object is of very low quality, and its potential applications, e.g., optical authentication and security [12–19], have not been explored.

In this paper, single-shot in-line holographic authentication using phase and amplitude modulation is presented. Amplitude modulation and phase modulation are presented and conducted at reference wave path for single-shot in-line digital holography, and its application, i.e., optical information authentication, is further explored and presented to show validity of the proposed method. A reference optical path using the cascaded random amplitude-only and phase-only masks is applied as an example. Only one in-line digital hologram is used for object reconstruction, and the recovered objects can be effectively authenticated. It is illustrated that parameters (masks) flexibly designed at reference wave path can facilitate wavefront retrieval in the CCD plane and also can act like security keys for optical authentication.

## 2. Principles

Figure 1 shows a schematic setup for the proposed digital holography authentication method. A collimated plane wave is generated, and is divided by a beam splitter cube into object beam and reference beam. At the object wave path, object wavefront in the CCD plane can be described by [20]

$$O(\mu, \nu) = \frac{j}{\lambda} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} O(x, y) \frac{1}{\rho} \exp\left(-j \frac{2\pi}{\lambda} \rho\right) dx dy, \quad (1)$$

where  $O(\mu, \nu)$  denotes object wavefront in the CCD plane,  $(\mu, \nu)$  denotes coordinate of the CCD plane,  $O(x, y)$  denotes the test object,  $\lambda$  denotes light wavelength,  $\rho$  denotes the distance from a point in the object plane to a point in the CCD plane  $\rho = \sqrt{(x-\mu)^2 + (y-\nu)^2 + d^2}$ ,  $j = \sqrt{-1}$ , and  $d$  denotes axial distance. Here, the free-space wave propagation process is implemented by [20]

$$O(\mu, \nu) = O(x, y) \otimes \mathfrak{F}(x, y, d), \quad (2)$$

where  $\otimes$  stands for the convolution, and  $\mathfrak{I}(x, y, d)$  denotes point pulse function of the Fresnel transform described by

$$\mathfrak{I}(x, y, d) = \frac{\exp(j2\pi d/\lambda)}{jd\lambda} \exp\left[\frac{j\pi}{d\lambda}(x^2 + y^2)\right]. \quad (3)$$

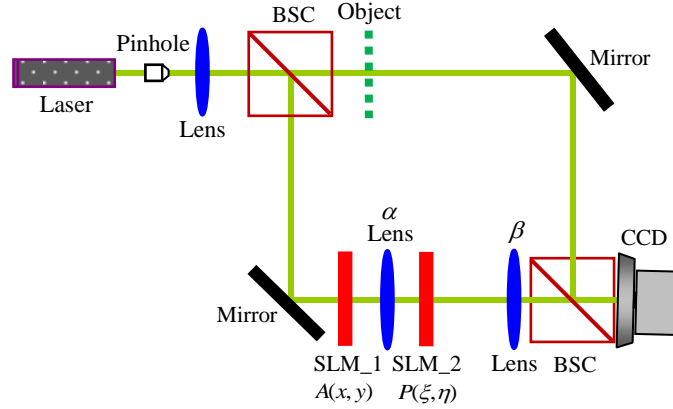


Fig. 1. A schematic setup for the proposed method: BSC, beam splitter cube; CCD, charge-coupled device; SLM, spatial light modulator. Random amplitude-only mask is embedded into SLM\_1, and random phase-only mask is embedded into SLM\_2. In practice, these two SLMs can be placed in the optical setup to reflect the propagating beam, i.e., reflective. Cascaded-mask optical arrangement is applied at reference wave path, however it is straightforward to design other optical structures for the generation of noise-like reference wavefront in the CCD plane and the control of parameters or masks (acting like security keys) at reference wave path.

At the reference wave path, optical components can be flexibly applied, and random amplitude-only and phase-only masks are cascaded and used as a typical example. Fractional Fourier transform (FrFT) [21] is employed at reference wave path. Hence, reference wavefront in the CCD plane can be described by

$$R(\mu, \nu) = \text{FrFT}_{\beta, \beta} \left( \left\{ \text{FrFT}_{\alpha, \alpha} [A(x, y)] \right\} P(\xi, \eta) \right), \quad (4)$$

where  $A(x, y)$  denotes random amplitude-only mask,  $P(\xi, \eta)$  denotes random phase-only mask, and  $\alpha$  and  $\beta$  denote FrFT function orders. The one-dimensional FrFT can be described by [21]

$$\text{FrFT}_{\alpha} [A(x)] = \int_{-\infty}^{+\infty} A(x) T_{\alpha}(\xi_{\alpha}, x) dx, \quad (5)$$

$$\text{where } T_\alpha(\xi_\alpha, x) = \begin{cases} W \exp\left\{j\pi\left[\xi_\alpha^2 \cot(\alpha\pi/2) + x^2 \cot(\alpha\pi/2) - 2\xi_\alpha x \csc(\alpha\pi/2)\right]\right\} & \text{if } \alpha \neq 2m \\ \delta(\xi_\alpha - x) & \text{if } \alpha = 4m \\ \delta(\xi_\alpha + x) & \text{if } \alpha = 4m \pm 2 \end{cases}, \quad m \text{ denotes an integer, and } W = \sqrt{1 - j \cot(\alpha\pi/2)}.$$

An in-line digital hologram  $I(\mu, \nu)$  formed by using interference principle is recorded by CCD camera, and is expressed by

$$I(\mu, \nu) = [O(\mu, \nu) + R(\mu, \nu)][O(\mu, \nu) + R(\mu, \nu)]^*, \quad (6)$$

where asterisk denotes complex conjugate.

In this study, the proposed digital holographic method is applied for information authentication, and random amplitude-only and phase-only masks  $A(x, y)$  and  $P(\xi, \eta)$  at the designed reference wave path act like security keys. When these keys and setup parameters are available to the authorized person, reference wavefront in the CCD plane can be first calculated, i.e.,  $\hat{R}(\mu, \nu) = \text{FFFT}_{\beta, \beta}(\{\text{FFFT}_{\alpha, \alpha}[A(x, y)]P(\xi, \eta)\})$ . Subsequently, extraction of complex-valued object wavefront in the CCD plane consists of the following steps:

(i) Four neighboring pixels in the digital hologram, i.e.,  $I(k, l)$ ,  $I(k, l+1)$ ,  $I(k+1, l)$  and  $I(k+1, l+1)$ , are categorized as one group, and are respectively re-denoted as  $I_1$ ,  $I_2$ ,  $I_3$  and  $I_4$ .

(ii) The corresponding four neighboring pixels of the extracted reference wavefront  $\hat{R}(\mu, \nu)$  are also considered as one group. These four pixels are respectively denoted as  $t_1 \exp(j\varphi_1)$ ,  $t_2 \exp(j\varphi_2)$ ,  $t_3 \exp(j\varphi_3)$  and  $t_4 \exp(j\varphi_4)$ , where  $t_1 - t_4$  denote amplitude parts and  $\varphi_1 - \varphi_4$  denote phase parts.

(iii) To simplify the retrieval computation, it is assumed that the corresponding four neighboring pixels of object wavefront are the same, i.e.,  $t_0 \exp(j\varphi_0)$ .

(iv) After the above conditions in steps (i)–(iii) are set, the four neighboring pixels of object wavefront can be extracted by using a method similar to phase-shifting algorithm [5,7,11,22], which is described by

$$t_0 = \frac{\sqrt{Q_1 + Q_2}}{2(Z_1 - Z_2)}, \quad (7)$$

$$\varphi_0 = \tan^{-1} \left[ \frac{\tilde{I}(t_1 \cos \varphi_1 - t_2 \cos \varphi_2) - \tilde{I}(t_3 \cos \varphi_3 - t_4 \cos \varphi_4)}{\tilde{I}(t_3 \sin \varphi_3 - t_4 \sin \varphi_4) - \tilde{I}(t_1 \sin \varphi_1 - t_2 \sin \varphi_2)} \right], \quad (8)$$

where the symbols [11,22] are respectively described by

$$Q_1 = [\tilde{I}(t_3 \sin \varphi_3 - t_4 \sin \varphi_4) - \tilde{I}(t_1 \sin \varphi_1 - t_2 \sin \varphi_2)]^2,$$

$$Q_2 = [\tilde{I}(t_1 \cos \varphi_1 - t_2 \cos \varphi_2) - \tilde{I}(t_3 \cos \varphi_3 - t_4 \cos \varphi_4)]^2,$$

$$\tilde{I} = I_1 - I_2 - (t_1^2 - t_2^2),$$

$$\tilde{I} = I_3 - I_4 - (t_3^2 - t_4^2),$$

$$Z_1 = (t_1 \cos \varphi_1 - t_2 \cos \varphi_2)(t_3 \sin \varphi_3 - t_4 \sin \varphi_4),$$

$$Z_2 = (t_1 \sin \varphi_1 - t_2 \sin \varphi_2)(t_3 \cos \varphi_3 - t_4 \cos \varphi_4).$$

(v) Similarly, other groups (i.e., each group with four neighboring pixels) in the digital hologram can also be processed, hence object wavefront  $\hat{O}(\mu, \nu)$  [i.e.,  $t0\exp(j\varphi0)$ ] in the CCD plane can be extracted. Subsequently, amplitude part of the extracted object wavefront  $\hat{O}(\mu, \nu)$  is further processed by using a median filter with a window of  $3 \times 3$  pixels to generate a filtered complex-valued object wavefront  $\hat{O}_f(\mu, \nu)$ . Finally, the test object can be reconstructed by using free-space wave back-propagation principle described by

$$\hat{O}(x, y) = \hat{O}_f(\mu, \nu) \otimes \mathfrak{I}(\mu, \nu, -d), \quad (9)$$

where  $\hat{O}(x, y)$  denotes a recovered object. Since only one in-line digital hologram is available for the recovery and grayscale object (normalized) is studied, the recovered object cannot visually render useful information. Here, correlation coefficient (CC) and mean squared error (MSE) are calculated to describe quality of recovered objects, and optical information authentication is conducted to verify the recovered object by using nonlinear correlation algorithm [23–37].

$$N(x, y) = \left| \text{IFT} \left[ \left\{ \left\{ \text{FT} [O(x, y)] \right\} \left\{ \text{FT} [\hat{O}(x, y)] \right\}^* \right\}^{v-1} \times \left\{ \text{FT} [O(x, y)] \right\} \left\{ \text{FT} [\hat{O}(x, y)] \right\}^* \right] \right|^2, \quad (10)$$

where  $N(x, y)$  denotes an authentication distribution,  $v$  denotes the strength of applied nonlinearity (i.e., 0.30), and FT and IFT denote Fourier transform and inverse Fourier transform, respectively. Original images can be separately stored [24,26], and only a platform interface is available to authorized persons without disclosure of original object information. The application objective in this study is to verify the recovered object image without visually observing original object information, i.e., acting as a novel optical security strategy. It can be considered that an additional security layer is established over optical encryption [4,13,35,38–40]. A flow chart is shown in Fig. 2 to clearly illustrate the proposed method aforementioned.

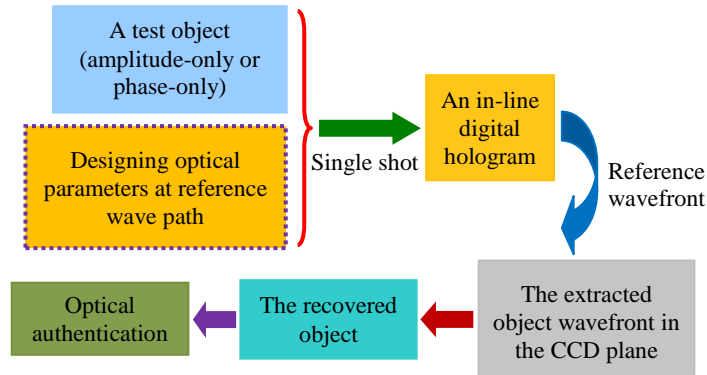


Fig. 2. A flow chart for optical authentication using the proposed digital holographic method.

### 3. Results and discussion

The setup shown in Fig. 1 is applied to numerically illustrate validity of the proposed method, and the data are processed to generate the decryption and authentication results. The plane wave (wavelength of 632.8 nm) is used, and is divided by a beam splitter cube into reference

wave and object wave. At object wave path, axial distance between the object and the CCD plane is 13.0 cm. At reference wave path, a random amplitude-only mask can be embedded into the first spatial light modulator, and a random phase-only mask can be embedded into the second spatial light modulator. The amplitude-only mask is randomly distributed in a range of  $[0, 200]$ , and the phase-only mask is randomly distributed in a range of  $[0, 2\pi]$ . In practice, the value range for amplitude-only mask can be flexibly designed to satisfy different applications. The FrFT function orders  $\alpha$  and  $\beta$  are 0.80 and 0.50, respectively. Figures 3(a) and 3(b) show amplitude and phase parts of complex-valued reference wavefront in the CCD plane, respectively. Since random masks are placed at the designed reference wave path, noise-like reference wavefront is generated. In this study, in-line digital holography is applied, and only one digital hologram is obtained (such as by using CCD camera with pixel size of  $4.65\ \mu\text{m}$  and pixel number of  $512 \times 512$ ) as shown in Fig. 3(c). In practice, the proposed method can be flexibly carried out by using optical or virtual-optics approach. For instance, SLMs can be placed in the optical setup to embed the masks and reflect the propagating beam at reference wave path, such as a reflective arrangement.

An image with  $512 \times 512$  pixels, i.e., "Baboon" (<http://sipi.usc.edu/database>), is normalized as the test object to show validity of the proposed method. Figure 4(a) shows a recovered object, when all parameters in the proposed optical system are correctly applied. CC and MSE for Fig. 4(a) are 0.10 and 0.71, respectively. It can be seen in Fig. 4(a) that the recovered object cannot visually render object information. In this study, nonlinear correlation algorithm [23–37] is applied to authenticate the recovered object, and the corresponding authentication distribution is shown in Fig. 4(b). It is illustrated that the recovered object is correctly verified, since only one remarkable peak is generated at the center of the authentication distribution. System robustness is further tested, when the recorded hologram is contaminated. Figure 4(c) shows a recovered object, when  $128 \times 128$  pixels of the hologram are occluded (i.e., set as zeros). CC and MSE for Fig. 4(c) are 0.098 and 0.64, respectively. Figure 4(d) shows an authentication distribution corresponding to Fig. 4(c). Figure 4(e) shows a recovered object, when the hologram is contaminated by additive white noise (zero mean noise with variance of 3). CC and MSE for Fig. 4(e) are 0.095 and 0.72, respectively. Figure 4(f) shows an authentication distribution corresponding to Fig. 4(e). It is illustrated in Figs. 4(d) and 4(f) that the proposed method possesses high robustness against the contaminations.

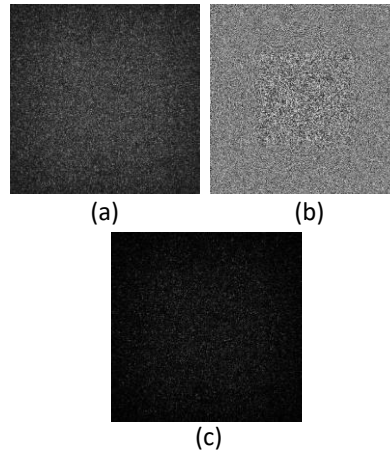


Fig. 3. (a) Amplitude part and (b) phase part of complex-valued reference wavefront in the CCD plane, and (c) an in-line digital hologram.

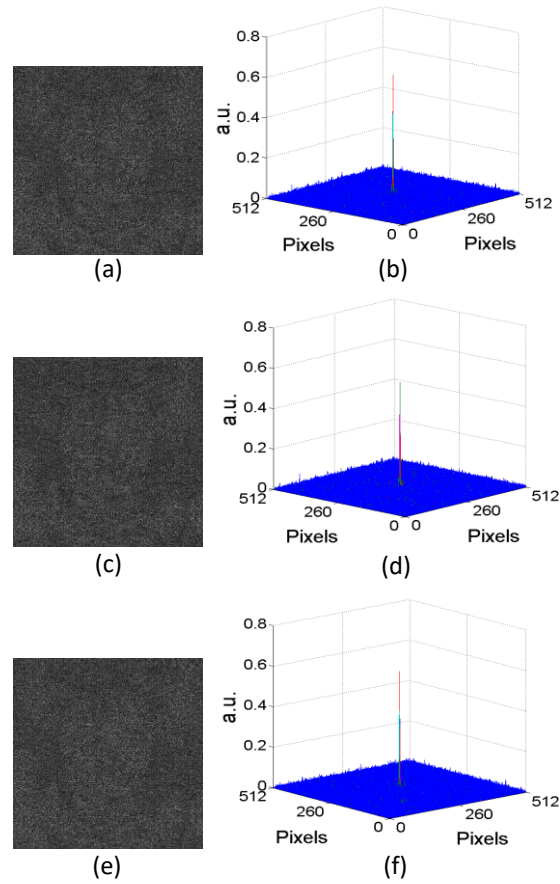


Fig. 4. (a) A recovered object obtained when all parameters in the proposed optical system are correctly applied, (b) an authentication distribution corresponding to (a); (c) a recovered object obtained when  $128 \times 128$  pixels of the hologram are occluded, (d) an authentication distribution corresponding to (c); (e) a recovered object obtained when the hologram is contaminated by additive white noise, (f) an authentication distribution corresponding to (e).

The cascaded random amplitude-only and phase-only masks are used at reference wave path, however it is straightforward to design other optical structures to generate noise-like reference wavefront in the CCD plane. This flexibility at reference wave path makes the proposed optical authentication system secure, since parameters designed at the reference wave path can act like security keys. It should be emphasized that main contribution of this study is to design complex parameters at reference wave path as security keys and apply only one in-line digital hologram for optical authentication. Figure 5(a) shows a recovered object, when amplitude-only mask is wrongly used to generate reference wave. Figure 5(b) shows an authentication distribution corresponding to Fig. 5(a). Figure 5(c) shows a recovered object, when phase-only mask is wrongly used to generate reference wave. Figure 5(d) shows an authentication distribution corresponding to Fig. 5(c). It can be seen in Figs. 5(b) and 5(d) that only noisy correlation distributions are obtained, when the designed parameters at reference wave path are wrongly used for the recovery. In this case, the wrong amplitude-only mask or wrong phase-only mask is defined as those also randomly generated within the designed

ranges. Other parameters, such as FrFT function orders, can be considered as complementary keys. Figure 5(e) shows a recovered object, when only FrFT function order  $\alpha$  contains an error of 0.01 during the generation of reference wave for the reconstruction. Figure 5(f) shows an authentication distribution corresponding to Fig. 5(e). It is worth noting that the parameters and masks used at reference wave path, such as distance (or FrFT function orders) and amplitude-only or phase-only mask, can perform like principal security keys for the proposed digital holographic authentication.

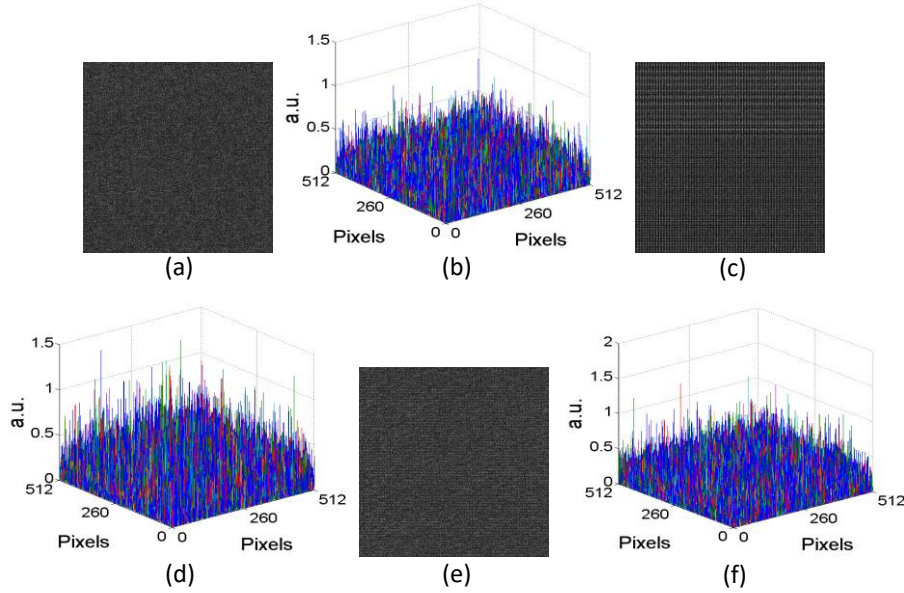


Fig. 5. (a) A recovered object obtained when amplitude-only mask is wrongly used to generate reference wave, (b) an authentication distribution corresponding to (a); (c) A recovered object obtained when phase-only mask is wrongly used, (d) an authentication distribution corresponding to (c); (e) A recovered object obtained when only FrFT function order  $\alpha$  contains an error of 0.01, (f) an authentication distribution corresponding to (e). The CC values for (a), (c) and (e) are 0.00025, -0.0048 and -0.0015, respectively. The MSE values for (a), (c) and (e) are  $7.1 \times 10^4$ ,  $3.8 \times 10^5$  and  $1.6 \times 10^5$ , respectively.

In practice, the test object can be sparsified before the recording, and some pixels of the object image can be set as zeros to enhance invisibility of the recovered objects. It is found that the recovered objects can still be verified by using the proposed method. Figure 6(a) shows a recovered object, when all parameters in the proposed optical system are correctly applied. 2.0% pixels of the object image, i.e., 5242 pixels, are set as zeros before hologram recording. CC and MSE for Fig. 6(a) are 0.088 and 0.77, respectively. Figure 6(b) shows an authentication distribution corresponding to Fig. 6(a). Figure 6(c) shows a recovered object, when all parameters in the proposed optical system are correctly applied. In this case, 5.0% pixels of the object image, i.e., 13107 pixels, are set as zeros before hologram recording. CC and MSE for Fig. 6(c) are 0.066 and 0.99, respectively. Figure 6(d) shows an authentication distribution corresponding to Fig. 6(c). It can be seen in Figs. 6(b) and 6(d) that the recovered objects can still be effectively verified, when the object image is compressed before the recording.



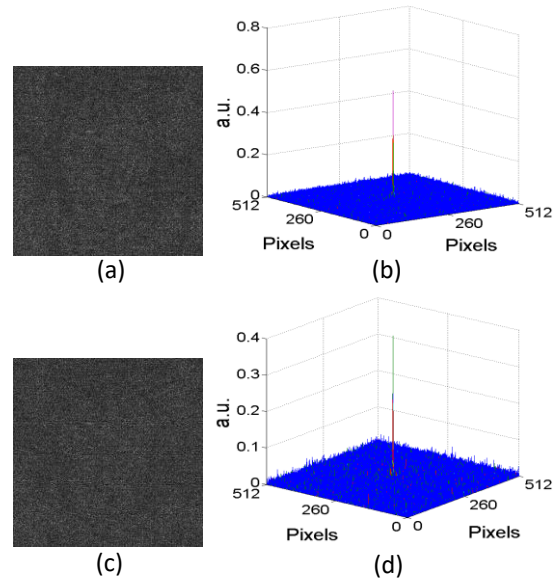


Fig. 6. (a) A recovered object obtained when all parameters in the proposed optical system are correctly applied and 2.0% pixels of the object image are set as zeros before hologram recording, (b) an authentication distribution corresponding to (a); (c) A recovered object obtained when all parameters in the proposed optical system are correctly applied and 5.0% pixels of the object image are set as zeros before hologram recording, (d) an authentication distribution corresponding to (c).

In Figs. 4–6, amplitude-only image is used as the test object, and a phase object can also be verified based on the proposed digital holographic system. Figure 7(a) shows a recovered phase object, when all parameters in the designed optical system are correctly applied. In this case, 35.0% pixels of the object image are set as zeros before converting to phase object for the recording, and other setup parameters are the same as those for Fig. 4(a). CC and MSE for Fig. 7(a) are 0.069 and 2.57, respectively. It can be seen in Fig. 7(a) that the recovered phase object does not visually provide clear information. The corresponding authentication distribution is shown in Fig. 7(b). It can be seen in Fig. 7(b) that optical information authentication can still be correctly conducted. Figure 7(c) shows a recovered phase object, when  $128 \times 128$  pixels of the hologram are occluded (i.e., set as zeros). Figure 7(d) shows an authentication distribution corresponding to Fig. 7(c). Figure 7(e) shows a recovered phase object, when the hologram is contaminated by additive white noise (zero mean noise with variance of 3). Figure 7(f) shows an authentication distribution corresponding to Fig. 7(e). It is illustrated in Figs. 7(d) and 7(f) that when phase object is studied, the proposed method still possesses high robustness against the contaminations.

Performance of the designed parameters at reference wave path is further evaluated. Figure 8(a) shows a recovered phase object, when amplitude-only mask is wrongly used to generate reference wave. Figure 8(b) shows an authentication distribution corresponding to Fig. 8(a). Figure 8(c) shows a recovered phase object, when phase-only mask is wrongly used to generate reference wave. Figure 8(d) shows an authentication distribution corresponding to Fig. 8(c). It can be seen in Figs. 8(b) and 8(d) that only noisy correlation outputs are obtained, when the designed parameters at reference wave path are wrongly used. Figure 8(e) shows a recovered phase object, when only FrFT function order  $\alpha$  contains an error of 0.01 for the generation of reference wave during object reconstruction. Figure 8(f) shows an authentication distribution corresponding to Fig. 8(e).

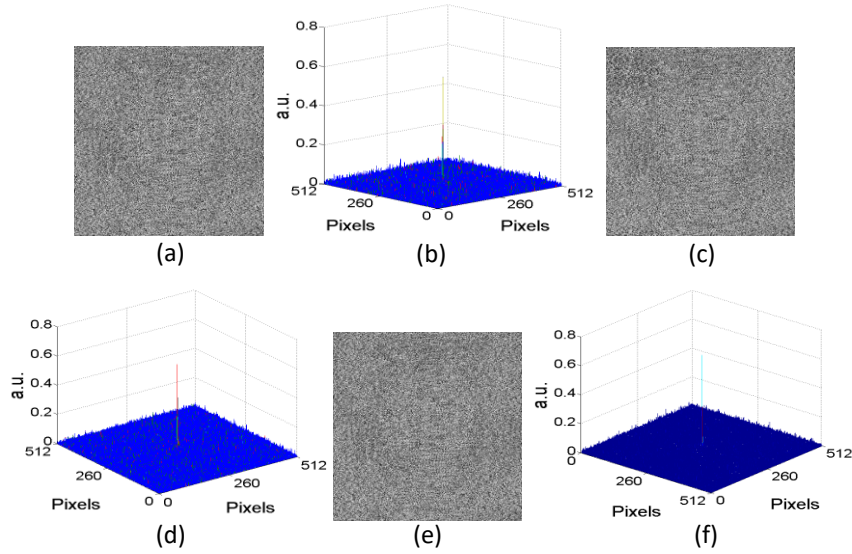


Fig. 7. Phase object case: (a) a recovered phase object obtained when all parameters in the proposed optical system are correctly applied, (b) an authentication distribution corresponding to (a); (c) a recovered phase object obtained when  $128 \times 128$  pixels of the hologram are occluded, (d) an authentication distribution corresponding to (c); (e) a recovered phase object obtained when the hologram is contaminated by additive white noise, (f) an authentication distribution corresponding to (e). The CC values for (c) and (e) are 0.069 and 0.070, respectively. The MSE values for (c) and (e) are 2.62 and 2.57, respectively.

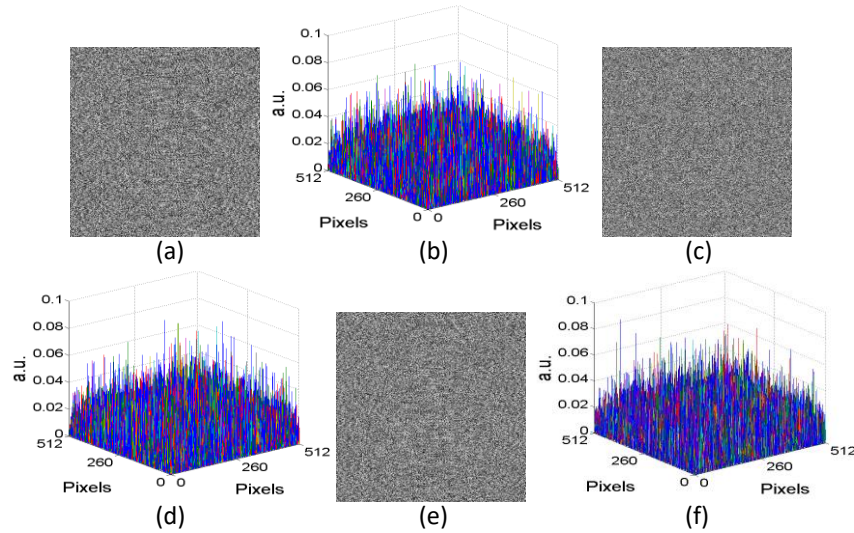


Fig. 8. Phase object case: (a) a recovered phase object obtained when amplitude-only mask is wrongly used to generate reference wave, (b) authentication distribution corresponding to (a); (c) A recovered phase object obtained when phase-only mask is wrongly used to generate reference wave, (d) authentication distribution corresponding to (c); (e) A recovered phase object obtained when only FrFT function order  $\alpha$  contains an error of 0.01 for the generation of reference wave, (f) authentication distribution corresponding to (e). The CC values for (a), (c) and (e) are 0.00047, -0.00012 and 0.0015, respectively. The MSE values for (a), (c) and (e) are 3.96, 3.95 and 3.94, respectively.

#### 4. Conclusions

Amplitude and phase modulation has been presented and conducted at reference wave path for single-shot in-line digital holography, and its application, i.e., optical information authentication, has been developed. Reference wave path is designed to generate different parameters as security keys, and only one in-line digital hologram is used for object reconstruction. The results demonstrate that the recovered objects can be effectively and correctly authenticated. It is illustrated that the proposed method possesses high robustness against contaminations, and parameters (masks) flexibly designed at reference wave path can facilitate wavefront retrieval in the CCD plane and also can act like security keys. The proposed method provides a promising strategy for digital-holography-based optical security.

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