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Why Pay for Jobs (and Not for Tasks)? [1]

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Abstract

Consider a principal who assigns a job with two tasks to two identical agents. Monitoring the agents' efforts is costly. Therefore the principal rewards the agents based on their (noisy) relative outputs. This study addresses the question of whether the principal should evaluate the outputs of each task separately and award two winner prizes, one for each task, or whether it is better to award only one winner prize to the agent who performs the best over the two tasks. There are two countervailing effects. First, there is a prize-diluting effect, because for a given budget, the prizes will be smaller when there are two winner prizes than when there is only one winner prize. The prize-diluting effect reduces the agents' incentives to invest their effort when there are two winner prizes. Second, there is a noise effect because the noisiness of the evaluation is reduced when there are two winner prizes. The main contribution of this study is to show that the prize-diluting effect dominates the noise effect. Hence, in general, principals will award prizes for combined tasks, and not for separate tasks. Several extensions are considered to test the robustness of this dominance result.

Keywords: Tournaments; contests; multi-task environments; log-concavity; head starts.
JEL Classification: D86, J33, M52.

1 Introduction

Jobs involve the repeated efforts of agents and employees to carry out the same task and the distribution of effort among different tasks. Consider academics, whose job it is to hold insightful, entertaining series of lectures, frequently publish in top academic journals, and develop and manage study programs (e.g., Gautier and Wauthy, 2007; Schnedler, 2008; DeVaro and Gürtler, 2016). Tournaments are considered an effective compensation scheme when the monitoring of effort is costly (e.g., Lazear and Rosen, 1981). The competition for promotion among executives and academics can be considered a tournament. Prizes awarded for technological advances and innovation can be considered a tournament, for example, the X-prize for private space flights (Boudreau et al., 2016). Prizes awarded for academic achievement such as for solving one of the seven mathematical Millennium Prize Problems can be considered a tournament as well.

This study addresses the question of whether the possibility of winning a big prize based on the aggregate performance of several tasks creates a stronger incentive to invest effort than the possibility of winning several smaller task-specific prizes. Should there be a big prize awarded to the first person who solves all the Millennium Prize Problems, or should the current practice be preferred, in which persons are awarded a smaller prize if they solve at least one of the Millennium Prize Problems? Alternatively, should prizes be awarded to mathematicians whose results can potentially help to solve one of the Millennium Prize Problems? This study develops insights into the incentive effect of having one tournament with a big prize versus multiple tournaments, each with smaller prizes awarded based on productivity. The case of multiple tournaments is inclusive in the sense that all agents participate in all tournaments. This is the difference between our environment and multi-winner contests where the principal can choose between a grand contest involving all contestants and a number of mutually exclusive sub-contests.[1]

The basic model involves two identical and risk-neutral agents, two identical tasks and effort costs that depend on the agent's total effort (and are therefore independent of the distribution of the agent's efforts between tasks). Our model thus captures the situation in which a reduction in winner prizes in one task may increase the incentive to invest effort in the other task, which is a useful property (e.g., Holmstrom and Milgrom, 1991). The agents' productivities are assumed to increase in effort, but they are also subject to random errors. There are three stages. In the first stage, the principal chooses between awarding multiple small and task-specific prizes or one big prize in addition to the corresponding winner and loser prizes. In the second stage, the agents choose their effort under limited liability. In the third stage, the winner and loser prizes are allocated to the agents based on their outputs.

In Section 2, the analysis begins by considering multiple tournaments, that is, one tournament for each of the two tasks . One may wonder whether agents would specialize in the sense that one agent would concentrate on one task and the other agent would concentrate on the other task to soften competition. The analysis reveals that this is not the case. That is, there is a unique effort equilibrium that can be characterized as symmetric when there is one tournament for each task and the agents are identical. The

[1] Multi-winner contests and the question of whether a grand contest provides stronger incentives than multiple exclusive sub-contests have been studied by Fu and Lu (2009) and Subhasish and Kim (2017).

analysis continues by considering one big tournament for the two tasks, in which each agent's aggregate productivity output is determined by the sum of the two task-specific outputs in Section 3. The analysis shows that a unique and symmetric effort equilibrium also exists in the case of one big tournament.

The main result, derived in Section 4, is that the big tournament outperforms the setting in which there are two task-specific tournaments under quite general stochastic conditions. More specifically, it is shown that when the distribution of task errors is log-concave, the big tournament strictly dominates the setting with multiple tournaments. This is true in the sense that, relative to multiple tournaments, the big tournament increases equilibrium effort in all tasks for any given aggregate budget of the principal. This implies that, under the conditions of log-concavity, the big tournament is strictly dominant, which is independent of whether the principal's aggregate budget is given or separately optimized for each tournament structure.

This clear-cut ranking of tournament designs, which may come as a surprise, is the consequence of two opposite effects. The first effect, called the *prize diluting effect*, captures that multiple tournaments require several winner prizes, whereas only one winner prize is needed when there is one big tournament. The second effect, called the *noise effect*, captures that aggregating outputs and noise terms over multiple tasks creates a more "noisy" signal of effort relative to the case of multiple tournaments. We develop the notion of a *budget multiplier* to show that the noise effect is dominated by the prize diluting effect when error distributions are log-concave, so that the principal always prefers the big tournament over multiple tournaments under these stochastic conditions. We further show that log-concave error distributions are plausible because they emerge when task outputs are the result of the sum of several stochastic actions and this number is sufficiently high.

Section 5 considers several extensions to test the robustness of the dominance result. It considers production functions where marginal productivities differ between tasks, even when efforts are evenly distributed between tasks and where there are cross effects in the sense that the effort invested in one task also directly affects productivity in the other task. The dominance result is robust against these changes. The intuition is simple. It is based on the notion that the budget multiplier is fully independent of production functions so long as the agents are identical. Another more complicated extension considers variations in the number of tasks. It highlights that it is never be optimal to have one tournament for each task when the agents are identical and error distributions are log-concave. It further highlights that the number of tasks is crucial in the sense that a big tournament involving all tasks dominates all other possible tournament structures if the number is exactly equal to 2 to the power of a strictly positive integer. If the number of tasks is not equal to 2 to the power of a strictly positive integer, the merging of smaller tournaments will, at some point, lead to tournaments of different sizes, measured by the number of tasks involved in each tournament. In this case, that is, when tournaments of different sizes exist, the prize-multiplier approach cannot be applied, which complicates the analysis. However, concentrating on the extreme ends of the spectrum of tournament structures, where the big tournament is compared to a situation in which there is one tournament for each task, gives the results for any number of tasks when random errors are normally distributed. In this situation, the budget multiplier is increasing in the task number, demonstrating that the relative advantage of the big tournament can be increasing in the number of tasks. A final extension covers agents with different abilities, which are captured in the form of head starts, when agents have knowledge of the head starts. This extension shows that head starts increase the relative advantage of having a big tournament when head starts are evenly distributed between agents (for instance, agent 1 has a head start in task 1 and agent 2 has a head start in task 2). Numerical simulations are used to show that the opposite is true when one agent has a head start in both tasks (for instance, agent 1 has a head start in tasks 1 and 2).

Since the seminal contributions of Lazear and Rosen (1981) and Holmstrom (1982), an extensive body of literature on labor market tournaments has been developed. See Lazear (1991), Prendergast (1999) and Kräkel (2004) for excellent literature overviews. Starting with the seminal paper by Holmstrom and Milgrom (1991), multitasking environments have been studied in the economic literature. These authors concentrate on piece-rate schemes where the agent's payment consists of a fixed element and an element that is linear in stochastic outputs, while this study concentrates on labor market tournaments where the winner and loser prizes are allocated to agents according to their ranking in stochastic outputs.[2] Franckx et al. (2004) concentrate on multiple tasks and a big tournament with one prize. They analyze how to optimally aggregate stochastic outcomes to an index used to determine the winning agent. Clark and Konrad (2007) also concentrate on a big tournament with one prize. In their scenario, the principal specifies a rule that determines in how many dimensions the winner must beat the opponent in order to become the winning agent. Gautier and Wauthy (2007) show that yardstick competition for research funds can induce better teaching quality and research. Arbatskaya and Mialon (2012) consider agents' effort invested in short- and long-term productivities in the hope of winning a big prize. DeVaro and Gürtler (2015) analyze how a big prize can distort the agents' incentives to invest in multiple tasks and encourage strategic shirking in certain task dimensions. DeVaro and Gürtler (2016) show that a multitasking environment and the absence of specialization can be used to mitigate strategic shirking in certain dimensions where there is a big prize. This study contributes to this strand of literature by examining the optimal number of tournaments in a multitasking environment and analyzing whether a big tournament or several smaller tournaments provide stronger incentives to invest effort. In the latter case of several smaller tournaments, the number of winner prizes may exceed the number of winning agents because agents may win several prizes. This feature clearly differentiates those discussed above.

Dynamic tournaments are another environment where the number of winner prizes may exceed the number of winning agents.[3] A difference between the studies concerned with dynamic tournaments and this study is the modeling of effort cost functions. This study assumes that effort costs only depend on the agent's total efforts, whereas when dynamic tournaments are studied, effort costs depend on how efforts are distributed in time and not just the total effort. The corresponding literature is mostly concerned with the

[2] Baker (2002) and Schnedler (2008) consider multitasking environments and analyze the role of distortion and risk in incentive contracts based on piece-rate schemes.

[3] A contestant can win multiple prizes in a multi-battle contest when there are intermediate prizes. Konrad and Kovenock (2009) show that one implication of intermediate prizes is pervasiveness.

role of feedback and how it can be optimized (e.g., Eriksson et al., 2009; Gershkov and Perry, 2009; Aoyagi, 2010; Ederer, 2010; Gürtler and Harbring, 2010). Klein and Schmutzler (2017) study dynamic tournaments in setting with two periods and risk-neutral agents. They show, for quadratic effort costs and normally distributed errors, that a big prize after two periods can provide better incentives than one small prize in each period when the agents are identical, the abilities are known and the principal's budget is given. The contribution of this study to this strand of literature is that it derives properties of error distribution functions that are sufficient to achieve the dominance result, where the big tournament dominates multiple tournaments without needing to refer to specific functional forms for effort costs and without the need to refer to a given principal's budget. In addition, this study provides a clear and rigorous intuition based on the countervailing prize-diluting and noise effects. Finally, this study contributes by showing how tournament structures can be used to improve incentives in the presence of head starts.

2 One tournament for each task

2.1 The setup

Consider two identical risk-neutral agents $i = 1, 2$, who perform two identical tasks $t = 1, 2$ each on behalf of a principal. Let e_{it} denote agent i's effort related to task t. Each effort e_{it} together with a random error ϵ_{it} determines agent i's output in task t, $q(e_{it}) + \epsilon_{it}$. To ensure that equilibrium efforts are positive for each task, we assume that productivity $q(e_{it})$ is strictly concave in efforts e_{it} in the sense that marginal productivity is infinite evaluated at zero (that is, $q'(0) \to \infty$) and strictly decreasing in efforts (that is, $q''(e_{it}) < 0$). Random errors ϵ_{it} have zero expected values and are identically and independently distributed with standard deviation σ.

The monitoring of efforts is costly. The principal therefore implements two simultaneous tournaments, one for each task, to incentivize the agents' effort decisions. The tournaments involve winner and loser prizes W_{1t} and W_{2t}, respectively, for each tournament $t = 1, 2$. The agents' liability is limited, that is, $W_{1t} > W_{2t} \geq 0$ for $t = 1, 2$. The principal's expected payoff can be expressed as

$$\Pi = \sum_{t=1,2} \left(q(e_{1t}) + q(e_{2t}) - W_{1t} - W_{2t}\right). \tag{1}$$

The right-hand side shows the difference between the expected total output depending on the agents' efforts and the total payments in terms of winner and loser prizes.

Agent i wins if output $q(e_{it}) + \epsilon_{it}$ exceeds output $q(e_{jt}) + \epsilon_{jt}$ (here and in the remainder, if i and j appear in the same expression, it is to be understood that $j \neq i$), that is, $q(e_{it}) + \epsilon_{it} > q(e_{jt}) + \epsilon_{jt}$, which can be rearranged as $\epsilon_{jt} - \epsilon_{it} < q(e_{it}) - q(e_{it})$. Let z_{it} be a composite error that denotes the difference of task errors among agents with $z_{it} = \epsilon_{jt} - \epsilon_{it}$ and Δq_{it} denote the difference in task-specific outputs with $\Delta q_{it} = q(e_{it}) - q(e_{jt})$. The agents' probability of winning in task t, denoted by F, can then be written as $F(\Delta q_{it}) = \Pr(z_{it} < \Delta q_{it})$, where the composite error z_{it} is a random variable with symmetric distribution around zero, denoted by $f(z_{it})$, which has a maximum at zero.

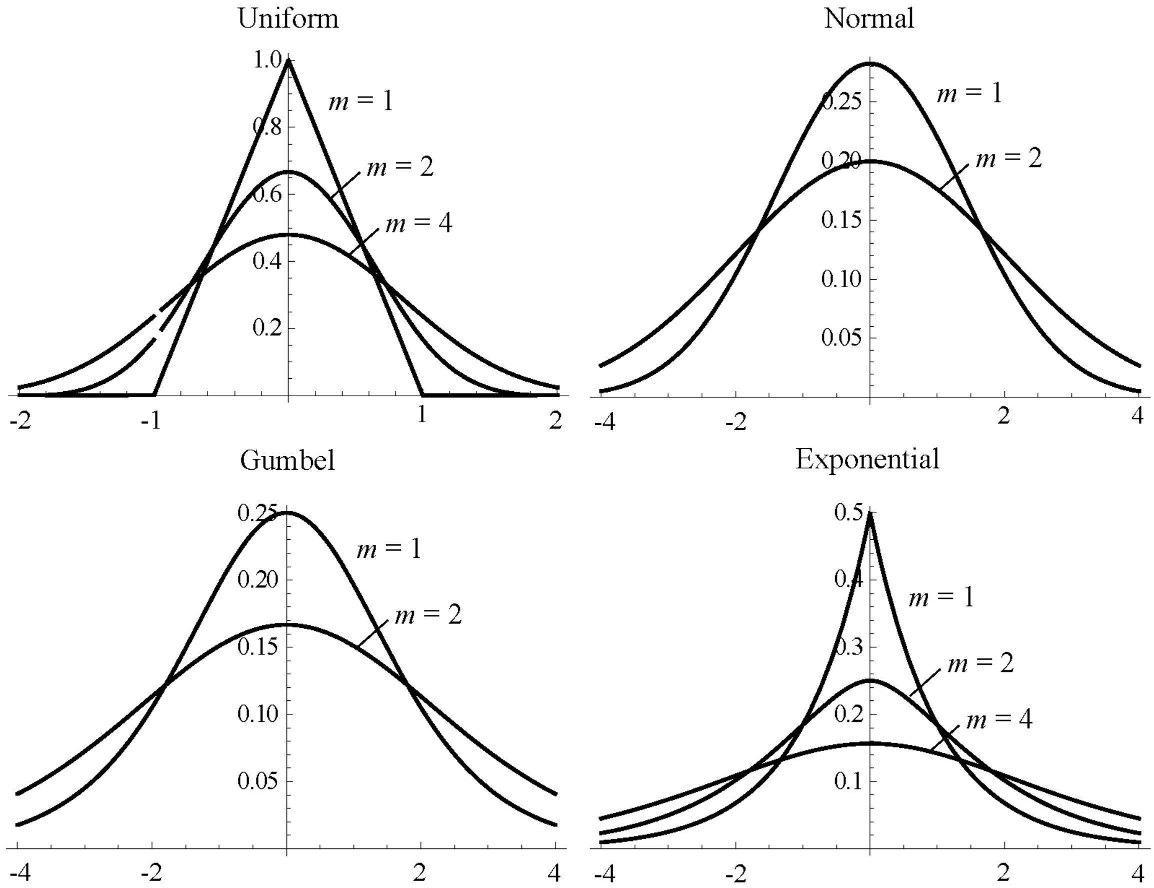

Figure 1: Distributions $f(z_{it})$ depending on actions m

As an alternative, one may assume that each task output is the result of the sum of the outputs of m actions that are unobserved by the principal. Specifically, assume that $\epsilon_{it} = \sum_{l=1}^{m} \zeta_{lt}$, where ζ_{lt} are independent and identically distributed random variables with zero expectation. The distribution $f(z_{it})$ is then continuous, unimodal and differentiable at zero if the number of actions, k, is sufficiently high, which implies that the probabilities of winning are differentiable at zero with $f'(0) = 0$. Figure 1 illustrates the effect of actions, m, on the differentiability of probability functions. If errors ζ_{lt} are uniformly distributed over the unit interval, $f(z_{it})$ is not differentiable at zero for $m = 1$, whereas the differentiability of probability functions is ensured for $m = 2$. If errors ζ_{lt} are normal or Gumbel distributed, the differentiability of probability functions is ensured for $m = 1$. If errors ζ_{lt} are exponentially distributed, the distribution is not differentiable at zero if $m = 1$, whereas the distributions approach the Normal distribution if the number of actions, m, increases.

Effort is costly for an agent. The total effort costs for agent i depend (only) on the agent's total effort denoted by e_i with $e_i = e_{i1} + e_{i2}$ and not on how efforts are distributed between tasks. Further, effort costs are strictly convex in the total effort e_i. Let e_i denote the total individual efforts, and $C(e_i)$ denote the strictly convex effort costs with $C(0) = 0$ and $C'(e_i)$, $C''(e_i) > 0$. Altogether, agent i's corresponding expected utilities can be expressed as

$$Eu_i = \sum_{t=1,2} \left(F(\Delta q_{it}) \cdot W_{1t} + (1 - F(\Delta q_{it})) \cdot W_{2t} \right) - C(e_i). \tag{2}$$

The relationship between the principal and the two agents follows a two-stage game. In stage one, the agents decide whether to work on behalf of the principal. If they both accept the competition, they simultaneously choose their efforts for each task. Together with the random errors, these efforts determine the agents' performances in stage two, and the winner and loser prizes are allocated according to their ranking in the corresponding tournament.

2.2 Equilibrium efforts

Assuming that the standard deviation σ of composite error z_{it} is large enough to ensure the existence of an interior solution for efforts (e.g., Lazear and Rosen, 1981) and letting ΔW_t denote the (strictly positive) difference in the winner and loser prizes with $\Delta W_t = W_{1t} - W_{2t}$, the best responses in terms of efforts are determined by the first-order conditions $\partial Eu_i / \partial e_{it} = 0$, which can be written as

$$\Delta W_t \cdot f(\Delta q_{it}) \cdot q'_{it}(e_{it}) - C'(e_i) = 0. \tag{3}$$

The left-hand side shows the difference between the marginal increase in the expected payment and the marginal effort costs. The first-order conditions in (3) can be used to derive the following symmetry result (the proofs of all lemmas and propositions are relegated to Appendix A):

Lemma 1 *If there is one tournament for each task, the agents' equilibrium task efforts are equal across agents in the sense that $e_{1t} = e_{2t}$ for both tasks, that is, for all $t = 1, 2$.*

This result rules out the idea that each agent specializes in one specific task to soften the rivalry in the case of two tournaments. Lemma 1 can be used to derive the following uniqueness result:

Proposition 1 *If there is one tournament for each task, there is a unique equilibrium in the agents' efforts where equilibrium task efforts are equal across agents.*

2.3 Optimal winner prizes

To derive the optimal winner prizes, it is useful to understand how equilibrium efforts change with winner prizes:

Lemma 2 *If there is one tournament for each task, the agents' equilibrium efforts in task t are increasing in the winner prize difference ΔW_t, whereas the agents' equilibrium efforts invested in the other task are reduced. The total sum of the individual efforts are, however, increased by an increase in the winner prize difference ΔW_t.*

This approach thus captures that an increase in the incentive to invest effort in one task can reduce the agents' incentive to invest effort in the other task (e.g., Holmstrom and Milgrom, 1991). Further, the result implies that the principal maximizes the agents' incentive to invest effort by minimizing the loser prize W_{2t} and setting it equal to zero, which implies $\Delta W_t = W_{1t}$.

The logic of our approach can be described as follows. Suppose that for any given aggregate expenditure on winner and loser prizes, one tournament structure always generates higher effort in each task and for every agent. In this situation, this one tournament structure clearly dominates the other from the principal's viewpoint. Importantly, this is independent of whether optimal aggregate expenditures (on wages) are different or the same for the tournament structures being considered. Our analysis uses this to identify the relative benefits of tournament structures. Specifically, we analyze the variations in the principal's budget that are required to ensure that the equilibrium efforts achieved under the big tournament are equal to those that can be achieved when there is one tournament for each task.

Let w denote the given aggregate expenditures and consider the given total efforts $e_1 = e_2$. The expected total output is maximized when $q'(e_{11}) = q'(e_{12}) = q'(e_{21}) = q'(e_{22})$. This can be achieved by winner prizes that are equal for task 1's tournament and task 2's tournament, which leads to:

Proposition 2 *If there is one tournament for each task and for a given budget w, the optimal winner and loser prizes that maximize the expected total output are given by $W_{11} = W_{12} = w/2$ and $W_{21} = W_{22} = 0$.*

The winner prizes are such that they are the same for both tasks and maximize the difference between the winner and loser prizes when the minimum loser prizes are given by zero. However, this result requires that production functions are sufficiently concave to ensure that the principal cannot be better off by setting $W_{i1} = w$ and $W_{i2} = W_{j1} = W_{j2} = 0$ hence by eliminating the agents' incentives to invest in task j. Such winner prizes could potentially increase the agents' incentives for effort investments because they eliminate one source of noise and because noise reduces the incentive to invest in effort. To justify the consideration of two tournaments, it is henceforth assumed that the production function is sufficiently concave so that the principal's expected losses from concentrating on one task would be so high that they cannot be compensated by the potential incentive gains related to reduced noise.[4]

3 One "big" tournament for both tasks

3.1 The setup

Consider the case of one "big" tournament for both tasks. In this case, there is one winner prize and one loser prize denoted by W_1 and W_2, respectively. The principal's expected payoff can be expressed as

$$\Pi^{\text{big}} = \sum_{t=1,2} (q(e_{1t}) + q(e_{2t})) - W_1 - W_2, \tag{4}$$

where the right-hand side shows, again, the difference between the expected total output, depending on the agents' efforts and the total payments in terms of the winner and loser prizes.

Agent i wins if the sum of the outputs for tasks 1 and 2, $q(e_{i1}) + q(e_{i2}) + \epsilon_{i1} + \epsilon_{i2}$, exceeds the rival's sum of the outputs for tasks 1 and 2, $q(e_{j1}) + q(e_{j2}) + \epsilon_{j1} + \epsilon_{j2}$, that is, $q(e_{i1}) + q(e_{i2}) + \epsilon_{i1} + \epsilon_{i2} >$

[4] We thank an anonymous referee who highlighted the importance of the production function's concavity on the rationalization of the two tournaments to us.

$q(e_{j1})+q(e_{j2})+\epsilon_{j1}+\epsilon_{j2}$, which can be rearranged as $\epsilon_{j1}+\epsilon_{j2}-\epsilon_{i1}-\epsilon_{i2}<q(e_{i1})+q(e_{i2})-q(e_{j1})-q(e_{j2})$. Let z_i be a composite error that denotes the sum of the differences in task errors among agents with $z_i=\epsilon_{j1}+\epsilon_{j2}-\epsilon_{i1}-\epsilon_{i2}$ and Δq_i denotes the corresponding differences in the expected outputs with $\Delta q_i=q(e_{i1})+q(e_{i2})-q(e_{j1})-q(e_{j2})$. The agents' probability of winning the tournament, denoted by F^{big}, can then be written as $F^{\text{big}}(\Delta q_i)=\Pr(z_i<\Delta q_i)$, where the composite error z_i is a random variable with symmetric distribution around zero, denoted by $f^{\text{big}}(z_i)$, a maximum at zero, and standard deviation σ^{big}. In the case of a big tournament, agent i's expected utilities can be written as

$$Eu_i^{\text{big}}=F^{\text{big}}(\Delta q_i)\cdot W_1+\Big(1-F^{\text{big}}(\Delta q_i)\Big)\cdot W_2-C(e_i). \tag{5}$$

The relationship between the principal and the two agents follows the same two-stage game structure as in the case of one tournament for each task.

3.2 Equilibrium efforts

Assuming that the standard deviation σ^{big} of composite error z_i is large enough to ensure the existence of an interior solution for efforts and letting ΔW denote the (strictly positive) difference in the winner and loser prizes with $\Delta W=W_1-W_2$, the best responses, in terms of efforts, are determined by the first-order conditions $\partial Eu_i^{\text{big}}/\partial e_{it}=0$. These first-order conditions can be written as

$$\Delta W\cdot f^{\text{big}}(\Delta q_i)\cdot q_{it}'(e_{it})-C'(e_i)=0. \tag{6}$$

The left-hand side shows the difference between the marginal increase in the expected payment and the marginal effort costs. The first-order conditions in (6) can be used to derive the following symmetry result:

Lemma 3 *If there is one big tournament for both tasks, the agents' equilibrium task efforts are equal across tasks and agents, that is,* $e_{11}=e_{21}=e_{12}=e_{22}$.

Specialization is not a sensible strategy in the context of one big tournament because only the sum of the outputs matters in the competition between agents. Lemma 3 can be used to derive the following uniqueness result:

Proposition 3 *If there is one big tournament for each task, there is a unique equilibrium in the agents' efforts, and in this equilibrium, task efforts are equal across tasks and agents.*

3.3 Optimal winner prizes

The following result can be used to derive the optimal winner prizes:

Lemma 4 *If there is one tournament for each task, each agents' equilibrium efforts in both tasks increase in the winner prize difference* ΔW.

Consider the given total efforts $e_1=e_2$. The expected total output is maximized when $q'(e_{i1})=q'(e_{i2})$ for $i=1,2$ in equilibrium. This can be achieved by winner prizes that are equal for both tournaments, which leads to:

Proposition 4 *If there is one big tournament for both tasks and for a given budget w, the optimal winner and loser prizes maximizing the expected total output are given by $W_1 = w$ and $W_2 = 0$.*

The winner prizes are, again, such that they maximize the difference between the winner prize and the loser prize with a minimum loser prize that is given by zero.

4 The budget multiplier

To identify the trade-off between the tournament structures, we introduce a *budget multiplier* denoted as γ, which determines how the principal's budget must be changed to ensure that the equilibrium efforts achieved by the big tournament can be replicated by the two tournaments. When the budget multiplier is greater than one, an increase in the aggregate winner prize is needed to ensure that the two tournaments yield the same effort as the big tournament. Hence, the principal prefers the big tournament in this situation. When the budget multiplier is smaller than one, an increase in the aggregate winner prize is needed to ensure that the big tournament yields the same effort as the two tournaments. Hence, the principal prefers two tournaments in this alternative situation.

To ensure that efforts are equal across tournament structures, the aggregate expenditure with two tournaments is $\gamma \cdot w$. Lemmas 1 and 3 imply that $\Delta q_{it} = \Delta q_i = 0$ in effort equilibrium. According to propositions 2 and 4, the optimal winner prizes can be described as $\Delta W_t = w/2$ and $\Delta W = w$, respectively. Using these results, the first-order conditions (3) and (6) can be rewritten as

$$\frac{w}{2} \cdot f(0) \cdot q'_{it}(e_{it}) - C'(e_i) = 0 \tag{7}$$

and

$$w \cdot f^{\text{big}}(0) \cdot q'_{it}(e_{it}) - C'(e_i) = 0. \tag{8}$$

The budget multiplier ensures that equilibrium efforts are equal across tournament structures. After adding the budget multiplier to the left-hand side of equation (7), the left-hand sides of equations (7) and (8) can be set equal and solved for the budget multiplier, which yields

$$\gamma = \underbrace{2}_{\text{prize-diluting effect}} \cdot \underbrace{\frac{f^{\text{big}}(0)}{f(0)}}_{\text{noise effect}}. \tag{9}$$

Two remarks are worth noting. First, the identity (9) shows that the budget multiplier is the product of two terms, which represent two effects. The first effect, which we call the *prize-diluting effect*, is determined by the number of tasks when there are two tournaments. The prize-diluting effect captures that the winner prize is split into two winner prizes when there are two tournaments, and because a reduction in the winner prizes reduces equilibrium efforts by Lemmas 2 and 4, this reduces the agent's incentives to exert effort. The second effect, which we call the *noise effect*, is determined by the ratio of density functions evaluated at zero. The noise effect captures that the standard deviation of the sum of the tasks' outcomes is an increasing function of the number of tasks such that $f^{\text{big}}(0) < f(0)$, which increases the agent's incentives to exert effort when there are two tournaments relative to the big tournament. The prize-diluting and noise effects therefore work in opposite directions.

Second, the identity (9) shows that the budget multiplier is independent of the budget w. Thus, the ranking of tournament structures is independent of the optimal budget, which will typically be different for the big tournament and the two tournaments. Suppose that the prize-diluting effect dominates the noise effect and that the budget is optimized for the case with two tournaments. However, for every budget the aggregate efforts are higher with the big tournament. Thus, even in this case the principal would be better off with the big tournament (and could further increase the expected payoff by optimizing the budget for the big tournament).

The following result shows that the big tournament is preferable to two tournaments under highly plausible conditions:

Proposition 5 *If the distribution of composite errors z_{it} is strictly log-concave, the big tournament is strictly preferred over two tournaments by the principal. This is independent of whether the principal's budget is given or optimized for each tournament regime.*

Thus, the big tournament dominates the two tournaments in the case of log-concavity error distribution functions because replicating the effort invested under the big tournament requires a strictly higher budget if there are two tournaments.

Let us now provide an important example. Given a normal error distribution, which belongs to the family of log-concave distributions, the noise effect is equal to $f^{\text{big}}(0)/f(0) = 1/\sqrt{2}$, which leads to a budget multiplier equal to $\sqrt{2}$. Figure 1 illustrates the different types of error distribution functions. Consider the upper-left diagram with uniformly distributed noise terms. A comparison of the distribution functions shows that the maximum is reduced by less than 50 percent when $m = 4$ relative to $m = 2$, which leads to a dominance of the noise effect. Similar patterns emerge for noise terms with a normal, Gumbel or exponential distribution.

The example of normally distributed errors is relevant because it implies that we can derive the dominance result from more primitive assumptions about tasks. Consider the alternative assumption: that the outcomes of tasks are the sum of a given number of random actions, k, with an *arbitrary*, but independent and identically distributed random error. This assumption makes sense for many jobs, because jobs frequently require the same type of actions each day, whereas tasks are usually evaluated on a yearly basis. For a large number of actions, the composite errors' distributions $f(z_{it})$ are then (approximately) normally distributed, which means that we obtain the same result as given in the example assumption of a normal distribution when the task consists of one action:

Corollary 1 *If the output of tasks is derived from a sufficiently high number of stochastic outcomes, the big tournament is strictly preferred over two tournaments by the principal. This is, again, independent of whether the principal's budget is given or optimized for each tournament regime.*

5 Extensions

5.1 Production functions

Consider two modifications of the initial assumption that the production function for both tasks are identical. First, allow for task-specific productivities. Second, allow for effort-productivity spillovers.

First assume that the marginal productivity of task 1 is higher than the one of task 2, $q'_{i1}(e) > q'_{i2}(e)$, which is without loss of generality. As the agents are identical, this assumption then holds for both $i = 1, 2$. The results for the benefits of the big tournament relative to two tournaments turn out to be robust with respect to those asymmetries. This is a direct consequence of the the budget multiplier, γ, being independent of the production functions when the agents are identical.

Second, assume that there are effort-productivity spillovers in the sense that a change in effort e_{it} not only increases output $q(e_{it})$, but also affects agent i's other task's output $q(e_{it'})$ with $t' \neq t$. This might occur in situations in which, for example, social competence, learning effects or effort invested in short- and long-term productivities are present. To capture the interdependencies between tasks 1 and 2, it is assumed that $q(e_{i1}, e_{i2})$ with $\partial q/\partial e_{it'} \neq 0$ for $i = 1, 2$. Again, because the budget multiplier is independent of the production functions when the agents are identical, these interdependencies do not affect the previous result for the benefit of a big tournament when the distribution functions of the noise terms are log-concave.

Altogether, this can be summarized as follows:

Proposition 6 *The budget multiplier in (9) is unchanged by the presence of task-specific productivities and effort-productivity spillovers.*

5.2 Cost functions

Consider a modification of the initial assumption that the effort cost function is independent of how effort is distributed between tasks. More specifically, assume that task effort cost functions are identical but task specific in the sense that they only depend on the effort invested in the task, that is, $C(e_{it})$ with $C'(e_{it}), C'(e_{it}) > 0$. This represents the type of cost functions commonly applied to the analysis of dynamic tournament scenarios in which task-specific effort decisions are sequential (e.g., Klein and Schmutzler, 2017). One would expect that specialization should be even less likely to occur because marginal effort costs are relatively low for the task with lower effort invested under this alternative cost function specification, whereas the marginal effort costs are independent of the tasks under the initial effort cost specification. Analogous to the other cost function specification, one can, indeed, demonstrate that a unique effort equilibrium exists where equal efforts are invested in all tasks under the alternative cost function specification. Further, because the task-specific effort cost functions are identical for the two tasks, the budget multiplier remains unchanged by this alternative effort cost function specification, which leads to:

Proposition 7 *The budget multiplier in (9) is unchanged by the presence of task-specific cost functions where task-effort costs only depend on the effort invested in the task.*

5.3 Task numbers

Assume that two identical agents perform n tasks whose outputs are given by $q(e_{it}) + \epsilon_{it}$ with $t = 1, \ldots, n$ and $n \geq 2$ for all n tasks. Consider random errors that are all independently and identically distributed in a way that ensures that the standard deviations of composite errors z_{it}, $z_{it} + z_{it'}$, $z_{it} + z_{it'} + z_{it''}, \ldots$ with $t \neq t' \neq t'' \neq \ldots$ are large enough to ensure the existence of unique interior equilibrium solutions for efforts in the case of n tournaments with winner prizes w/n and in cases where tasks t and t' form a big tournament and big tournaments are merged into even bigger tournaments.

Assume that the distribution of composite errors z_{it} is strictly log-concave. Proposition 5 implies that merging any two tournaments, where each tournament involves a single task, to form one bigger tournament involving two tasks is an improvement for the principal. This implies:

Proposition 8 *If the distribution of composite errors z_{it} is strictly log-concave, it can never be optimal to have n tournaments. This is independent of whether the principal's budget is given or optimized for each tournament regime.*

The convolution of strictly log-concave probability distributions is also strictly log-concave. This implies:

Proposition 9 *If the distribution of composite errors z_{it} is strictly log-concave, and n can be represented by 2 to the power of a strictly positive integer, that is, $n = 2^x$ with $x = 1, 2, \ldots$, then one big tournament involving all n tasks is strictly preferred over any other tournament structure composed of several smaller tournaments for subsets of tasks. This is independent of whether the principal's budget is given or optimized for each tournament regime.*

Proof. Consider a situation in which the distribution of composite errors z_{it} is strictly log-concave. Proposition 5 implies that in this situation, the principal is better off with a bigger tournament that involves two tasks than two tournaments where each of the two tournaments involves a single task. Merging the single-task tournaments leads to 2^{x-1} big tournaments where each of the big tournaments involves two tasks, say, t and t', and the distribution of the corresponding composite errors $z_{it} + z_{it'}$ is strictly log-concave. If $x = 1$, then no more mergers are possible. If $x > 1$, the log-concavity of the distributions of the composite errors implies that merging the big tournaments into bigger four-task tournaments will lead to a further improvement for the principal. These mergers lead to 2^{x-2} four-task tournaments. If $x = 2$, then no more tournament mergers are possible and it is optimal to have one big tournament involving four tasks. If $x > 2$, the log-concavity of the distributions of the corresponding composite errors implies that merging the four-task tournaments into eight-task tournaments is preferred by the principal, which leads to 2^{x-3} eight-task tournaments. If $x = 3$, then no more tournament mergers are possible and it is optimal to have one big tournament involving eight tasks, and so on. ■

If the number of tasks cannot be represented by a number of two to the power of a strictly positive integer, then identifying the optimal tournament structure will be difficult. To illustrate, consider the case of three tasks versus four tasks. With four tasks, if there are two big tournaments with each involving two tasks, then the composite error terms involved in the two tournaments will be independently and identically distributed. For instance, composite errors $z_{i1}+z_{i2}$ and $z_{i3}+z_{i4}$ are independently and identically distributed. For this reason, Proposition 5 can be applied to this case with four tasks to demonstrate the optimality of a big tournament involving four tasks (see the proof of Proposition 9). The same is not true in the case of three tasks. In this case, Proposition 5 can be used to show that a move from three tournaments, in which each involves a single task, to two tournaments, in which one involves a single task and the other involves two tasks, is an improvement for the principal. Proposition 5 can, however, not be used to show that a merger of these two tournaments would be a further improvement because the composite error terms involved are independently but not identically distributed. For instance, composite errors $z_{i1} + z_{i2}$ and z_{i3} are independently distributed but not identically distributed. However, our conjecture is that also in this case, a move to a big tournament involving all three tasks would improve the principal's expected payoff and that, more generally, log-concave error distribution functions imply that one big tournament involving all tasks is preferred by the principal over any tournament structure independent of the number of tasks.

The concept of the multiplier can be adapted to the case of n tasks. Let Δq_i^n denote the expected difference in aggregate outputs with $\Delta q_i^n = \sum_t (q(e_{it}) - q(e_{jt}))$ and z_i^n denote the corresponding composite errors with $z_i^n = \sum_t (\epsilon_{jt} - \epsilon_{it})$. The agents' probability of winning the big tournament involving n tasks is written as $F^n(\Delta q_i^n) = \Pr(z_i^n < \Delta q_i^n)$, where z_i^n is a random variable with distribution $f^n(z_i^n)$. The adapted budget multiplier, denoted by γ^n, can be written as

$$\gamma^n = n \cdot \frac{f^n(0)}{f(0)}. \tag{10}$$

For independent and identical normally distributed random errors, $\gamma^n = \sqrt{n}$, where the right-hand side increases with the number of tasks n. This indicates that the principal's benefit from a big tournament involving n tasks relative to the scenario with one tournament for each of the n tasks increases with the number of tasks.

5.4 Head starts

It is common for some agents to have certain abilities or characteristics that give them an advantage compared to their rivals from the start of the competition (e.g., Gürtler and Harbring, 2010, and Siegel, 2014). One example is job promotion, when tenure plays a role and the employee who has worked longer for the firm benefits from a head start.[5] Head starts can help develop a better understanding about how differences and asymmetries in the agents' capabilities affect the relative benefits of multiple tournaments and one big tournament.[6]

Consider agents who are advantaged in one task in the sense that for the identical effort their expected output in the task is higher than their rival's expected output in the task by a constant amount denoted by

[5] Waldmann (2003) shows that favoring internal candidates can be useful because it can help attract young workers to the firm.

[6] We thank Matthias Kräkel for pointing out to us that abilities may be modeled this way. There are several other ways to model differences in agents' abilities, see, e.g., O'Keefe et al. (1984), Bull et al. (1987) and Gürtler and Kräkel (2010).

$\phi > 0$ and when agents have knowledge about head starts. Without loss of generality, agent i's expected output in tasks i and j are written as $q(e_{ii}) + \phi$ and $q(e_{ij})$, respectively. This leads to a symmetric effort structure with $e^*_{11} = e^*_{21}$ and $e^*_{12} = e^*_{22}$ in equilibrium because the head starts do not affect the marginal probabilities of winning the tournament (e.g., Denter and Sisak, 2015, 2016). As a consequence, the big tournament's marginal probabilities are still evaluated at 0 because the aggregation of outputs means that the agents' advantages exactly cancel out, whereas the marginal probabilities are evaluated at ϕ for tasks 1 and 2 when there is one tournament for each task because cancellation does not occur in this case.[7] The corresponding budget multiplier can be written as

$$2 \cdot \frac{f^{\text{big}}(0)}{f(\phi)}, \tag{11}$$

where $f(\phi) < f(0)$. This shows that the presence of head starts can increase the value of the budget multiplier and, thus, increase the relative benefits of the big tournament relative to multiple tournaments.

Consider asymmetric agents in the sense that for identical efforts ϕ is added to one agent's expected output per task for both tasks and where agents have knowledge of the head starts. This again leads to a symmetric effort structure with $e^*_{11} = e^*_{21}$ and $e^*_{12} = e^*_{22}$ in equilibrium. As a result, the big tournament's marginal probabilities are evaluated at 2ϕ, whereas the marginal probabilities are evaluated at ϕ for tasks 1 and 2 when there is one tournament for each task. The corresponding budget multiplier, denoted as γ^A, can be written as

$$\gamma^A = 2 \cdot \frac{f^{\text{big}}(2\phi)}{f(\phi)}, \tag{12}$$

where $f^{\text{big}}(2\phi) < f(0)$ and $f(\phi) < f(0)$. The effect of head starts on the budget multiplier is ambiguous in this situation because both the numerator and the denominator are decreasing relative to the absence of head starts.

It is difficult, for us, to identify general conditions that would ensure that the prize multiplier would be an increasing or decreasing function of the head start when head starts are unevenly distributed between agents. Therefore, we rely on numerical simulations in this part. The following numerical example illustrates that head starts reduce the budget multiplier in the present scenario with asymmetric agents.

Figure 2 displays the budget multiplier γ^A for independent identical and standard normally distributed random errors depending on ϕ. The figure shows that the budget multiplier can decrease when agents are asymmetric. It is well established in the tournaments literature that differences in agents' abilities reduce their incentives to invest in their effort relative to a symmetric situation where agents' abilities are equal. The present result shows that adding up the advantages in a big tournament can further reduce incentives.

Altogether, these results can be summarized as follows:

Proposition 10 *Head starts increase the relative advantage of having a big tournament relative to one tournament for each task if they are evenly distributed between agents, and reduce the relative advantage of having a big tournament relative to one tournament for each task if they are unevenly distributed between agents.*

[7] We thank an anonymous referee who mentioned this constellation and the possibility that head starts can be canceled.

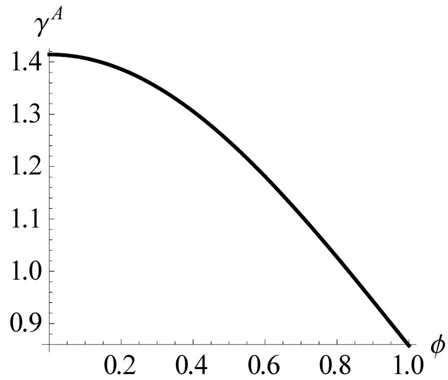

Figure 2: Prize multiplier when random errors are independent identical and standard normally distributed and agent 1 has a head start ϕ in tasks 1 and 2

6 Conclusions

This study considered a principal who assigns a job with two tasks to two identical agents. Monitoring the agents' efforts was considered costly. Therefore, the principal rewarded agents based on their (noisy) relative outputs. The principal could choose to evaluate the outputs in each task separately and award two winner prizes, one for each task, or to award only one winner prize to the agent who performs best over the two tasks. The analysis found two countervailing effects. First, there was a prize-diluting effect because, for a given budget, prizes are smaller when there are two winner prizes relative to a situation when there is only one winner prize. The prize-diluting effect reduces the agents' incentive to invest in effort when there are two winner prizes. Second, there was a noise effect, because the noisiness of the evaluation is reduced when there are two winner prizes. The major contribution of this study was to show that the prize-diluting effect dominates the noise effect in the case of log-concave error distribution functions. This dominance result was shown to persist in the presence of task-specific productivities and effort-productivity spillovers. It was further shown to persist in cases where the number of tasks could be described by two to the power of a positive integer and normally distributed errors. Finally, the dominance result was robust in the case of headways if headways were evenly distributed between agents, whereas the dominance result could not be shown for the case of headways that were unevenly distributed between agents.

There are several avenues for future research. Our conjecture was that big tournaments would outperform multiple tournaments for any number of tasks when task errors were independently and identically distributed with log-concave distribution functions. To show this, heteroskedastic error terms must be considered, which is one avenue for future research. Other avenues for future research involve asymmetric agents and risk-averse agents. The latter would be of special interest in the context of risk-taking by agents along the lines considered by, for example, Kräkel (2008).

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Appendix

A Proofs

A.1 Proof of Lemma 1

The proof is by contradiction. Suppose that $e_{i1} > e_{j1}$ and $e_{i2} > e_{j2}$. This implies $q'_{i1}(e_{i1}) < q'_{j1}(e_{j1})$ and $q'_{i2}(e_{i2}) < q'_{j2}(e_{j2})$ by the strict concavity of the production function. It further implies $e_i > e_j$ and $C'(e_i) > C'(e_j)$ by the strict convexity of the effort cost function. Altogether, this implies $q'_{it}(e_{it}) - C'(e_i) > q'_{jt}(e_{jt}) - C'(e_j)$. Using $\Delta q_{it} = q(e_{it}) - q(e_{jt}) = -\Delta q_{jt}$ and the fact that f is symmetric around zero, the first-order conditions in (3) can be rewritten as

$$\Delta W_t \cdot f(\Delta q_{it}) \cdot q'_{it}(e_{it}) - C'(e_i) = 0 \tag{13}$$

and

$$\Delta W_t \cdot f(-\Delta q_{it}) \cdot q'_{jt}(e_{jt}) - C'(e_j) = 0. \tag{14}$$

But, $q'_{it}(e_{it}) - C'(e_i) < q'_{jt}(e_{jt}) - C'(e_j)$ and thus these two conditions cannot be simultaneously satisfied.

Suppose that $e_{i1} > e_{j1}$ and $e_{i2} < e_{j2}$. This implies $q'_{i1}(e_{i1}) < q'_{j1}(e_{j1})$ and $q'_{i2}(e_{i2}) > q'_{j2}(e_{j2})$ and

$$\frac{q'_{i1}(e_{i1})}{q'_{i2}(e_{i2})} < \frac{q'_{j1}(e_{j1})}{q'_{j2}(e_{j2})}, \tag{15}$$

which is also a contradiction. To see this, note that the first-order conditions (3) imply

$$\Delta W_1 \cdot f(\Delta q_{i1}) \cdot q'_{i1}(e_{i1}) = \Delta W_2 \cdot f(\Delta q_{i2}) \cdot q'_{i2}(e_{i2}). \tag{16}$$

Rearranging yields

$$\frac{q'_{i1}(e_{i1})}{q'_{i2}(e_{i2})} = \frac{\Delta W_2 \cdot f(\Delta q_{i2})}{\Delta W_1 \cdot f(\Delta q_{i1})} \tag{17}$$

for $i = 1, 2$. Using $\Delta q_{it} = q(e_{it}) - q(e_{jt}) = -\Delta q_{jt}$ and symmetry of the distribution f around zero, this implies

$$\frac{\Delta W_2 \cdot f(\Delta q_{i2})}{\Delta W_1 \cdot f(\Delta q_{i1})} = \frac{\Delta W_2 \cdot f(-\Delta q_{j2})}{\Delta W_1 \cdot f(-\Delta q_{j1})} = \frac{\Delta W_2 \cdot f(\Delta q_{j2})}{\Delta W_1 \cdot f(\Delta q_{j1})} \tag{18}$$

and consequently

$$\frac{q'_{11}(e_{11})}{q'_{12}(e_{12})} = \frac{q'_{21}(e_{21})}{q'_{22}(e_{22})}, \tag{19}$$

which contradicts $e_{i1} > e_{j1}$ and $e_{i2} < e_{j2}$. Altogether, this shows that the agents' equilibrium efforts must be symmetric in the sense that $e_{it} = e_{jt}$ for both tasks, that is, for all $t = 1, 2$.

A.2 Proof of Proposition 1

The first-order conditions in (3) are denoted and written as

$$g_{11} = \Delta W_1 \cdot f(\Delta q_{11}) \cdot q'(e_{11}) - C'(e_1), \tag{20}$$

$$g_{12} = \Delta W_2 \cdot f(\Delta q_{12}) \cdot q'(e_{12}) - C'(e_1), \tag{21}$$

$$g_{21} = \Delta W_1 \cdot f(\Delta q_{21}) \cdot q'(e_{21}) - C'(e_2), \tag{22}$$

$$g_{22} = \Delta W_2 \cdot f(\Delta q_{22}) \cdot q'(e_{22}) - C'(e_2). \tag{23}$$

Denote $\chi_{it} = f'(\Delta q_{it}) \left(q'(e_{it})\right)^2 + f(\Delta q_{it}) q''(e_{it})$. The Jacobian of the first-order conditions can be written as

$$J = \begin{pmatrix} \frac{\partial g_{11}}{\partial e_{11}} & \frac{\partial g_{11}}{\partial e_{12}} & \frac{\partial g_{11}}{\partial e_{21}} & \frac{\partial g_{11}}{\partial e_{22}} \\ \frac{\partial g_{12}}{\partial e_{11}} & \frac{\partial g_{12}}{\partial e_{12}} & \frac{\partial g_{12}}{\partial e_{21}} & \frac{\partial g_{12}}{\partial e_{22}} \\ \frac{\partial g_{21}}{\partial e_{11}} & \frac{\partial g_{21}}{\partial e_{12}} & \frac{\partial g_{21}}{\partial e_{21}} & \frac{\partial g_{21}}{\partial e_{22}} \\ \frac{\partial g_{22}}{\partial e_{11}} & \frac{\partial g_{22}}{\partial e_{12}} & \frac{\partial g_{22}}{\partial e_{21}} & \frac{\partial g_{22}}{\partial e_{22}} \end{pmatrix} =$$

$$\begin{pmatrix} \Delta W_1 \chi_{11} - C''(e_1) & -C''(e_1) & 0 & 0 \\ -C''(e_1) & \Delta W_2 \chi_{12} - C''(e_1) & 0 & 0 \\ -\Delta W_1 f'(\Delta q_{21}) q'(e_{21}) q'(e_{11}) & 0 & 0 & 0 \\ 0 & -\Delta W_2 f'(\Delta q_{22}) q'(e_{22}) q'(e_{12}) & 0 & 0 \end{pmatrix} +$$

$$\begin{pmatrix} 0 & 0 & -\Delta W_1 f'(\Delta q_{11}) q'(e_{11}) q'(e_{21}) & 0 \\ 0 & 0 & 0 & -\Delta W_2 f'(\Delta q_{12}) q'(e_{12}) q'(e_{22}) \\ 0 & 0 & \Delta W_1 \chi_{21} - C''(e_2) & -C''(e_2) \\ 0 & 0 & -C''(e_2) & \Delta W_2 \chi_{22} - C''(e_2) \end{pmatrix} \tag{24}$$

Using symmetry and that $f'(0) = 0$, which implies $\chi_{it} = f(0) q''(e_{it})$, J reduces to

$$J =$$

$$\begin{pmatrix} \Delta W_1 f(0) q''(e_{11}) - C''(e_1) & -C''(e_1) & 0 & 0 \\ -C''(e_1) & \Delta W_2 f(0) q''(e_{12}) - C''(e_1) & 0 & 0 \\ 0 & 0 & \Delta W_1 f(0) q''(e_{21}) - C''(e_2) & -C''(e_2) \\ 0 & 0 & -C''(e_2) & \Delta W_2 f(0) q''(e_{22}) - C''(e_2) \end{pmatrix} \tag{25}$$

where the right-hand side is negative definite, which ensures the existence of a unique solution for the system of equations in (20)-(23) by the index theory approach (e.g., Vives, 1999).

A.3 Proof of Lemma 2

The first-order conditions for optimal efforts in the case of two tournaments in (3) are denoted by

$$h_{11} = \Delta W_1 \cdot f(\Delta q_{11}) \cdot q'_{11}(e_{11}) - C'(e_1) = 0, \tag{26}$$

$$h_{12} = \Delta W_2 \cdot f(\Delta q_{12}) \cdot q'_{12}(e_{12}) - C'(e_1) = 0, \tag{27}$$

$$h_{21} = \Delta W_1 \cdot f(\Delta q_{21}) \cdot q'_{21}(e_{21}) - C'(e_2) = 0, \tag{28}$$

$$h_{22} = \Delta W_2 \cdot f(\Delta q_{22}) \cdot q'_{22}(e_{22}) - C'(e_2) = 0. \tag{29}$$

Totally differentiating this system of equations with respect to effort choices and the winner prize difference ΔW_1 yields

$$dh_{11} = \frac{\partial h_{11}}{\partial e_{11}} de_{11} + \frac{\partial h_{11}}{\partial e_{12}} de_{12} + \frac{\partial h_{11}}{\partial e_{21}} de_{21} + \frac{\partial h_{11}}{\partial e_{22}} de_{22} + \frac{\partial h_{11}}{\partial \Delta W_1} d\Delta W_1 = 0, \tag{30}$$

$$dh_{12} = \frac{\partial h_{12}}{\partial e_{11}} de_{11} + \frac{\partial h_{12}}{\partial e_{12}} de_{12} + \frac{\partial h_{12}}{\partial e_{21}} de_{21} + \frac{\partial h_{12}}{\partial e_{22}} de_{22} + \frac{\partial h_{12}}{\partial \Delta W_1} d\Delta W_1 = 0. \tag{31}$$

Using symmetry, this reduces to

$$dh_{11} = \left(\frac{\partial h_{11}}{\partial e_{11}} + \frac{\partial h_{11}}{\partial e_{21}}\right) de_{11} + \left(\frac{\partial h_{11}}{\partial e_{12}} + \frac{\partial h_{11}}{\partial e_{22}}\right) de_{12} + \frac{\partial h_{11}}{\partial \Delta W_1} d\Delta W_1 = 0, \tag{32}$$

$$dh_{12} = \left(\frac{\partial h_{12}}{\partial e_{11}} + \frac{\partial h_{12}}{\partial e_{21}}\right) de_{11} + \left(\frac{\partial h_{12}}{\partial e_{12}} + \frac{\partial h_{12}}{\partial e_{22}}\right) de_{12} + \frac{\partial h_{12}}{\partial \Delta W_1} d\Delta W_1 = 0. \tag{33}$$

In matrix form, these two equations can be rewritten as

$$\begin{pmatrix} \frac{\partial h_{11}}{\partial e_{11}} + \frac{\partial h_{11}}{\partial e_{21}} & \frac{\partial h_{11}}{\partial e_{12}} + \frac{\partial h_{11}}{\partial e_{22}} \\ \frac{\partial h_{12}}{\partial e_{11}} + \frac{\partial h_{12}}{\partial e_{21}} & \frac{\partial h_{12}}{\partial e_{12}} + \frac{\partial h_{12}}{\partial e_{22}} \end{pmatrix} \begin{pmatrix} de_{11} \\ de_{12} \end{pmatrix} = \begin{pmatrix} -\frac{\partial h_{11}}{\partial \Delta W_1} \\ -\frac{\partial h_{12}}{\partial \Delta W_1} \end{pmatrix} d\Delta W_1 \tag{34}$$

with

$$\begin{pmatrix} \frac{\partial h_{11}}{\partial e_{11}} + \frac{\partial h_{11}}{\partial e_{21}} & \frac{\partial h_{11}}{\partial e_{12}} + \frac{\partial h_{11}}{\partial e_{22}} \\ \frac{\partial h_{12}}{\partial e_{11}} + \frac{\partial h_{12}}{\partial e_{21}} & \frac{\partial h_{12}}{\partial e_{12}} + \frac{\partial h_{12}}{\partial e_{22}} \end{pmatrix} = \begin{pmatrix} \Delta W_1 f(0) q''(e_{11}) - C''(e_1) & -C''(e_1) \\ -C''(e_1) & \Delta W_2 f(0) q''(e_{12}) - C''(e_1) \end{pmatrix}, \tag{35}$$

where the determinant of the right-hand side, denoted by Ψ, is strictly positive.

Cramer's rule can be applied to derive the relationship between prizes and efforts:

$$\begin{aligned} \frac{de_{11}}{d\Delta W_1} &= \frac{1}{\Psi} \det \begin{pmatrix} -\frac{\partial h_{11}}{\partial \Delta W_1} & \frac{\partial h_{11}}{\partial e_{12}} + \frac{\partial h_{11}}{\partial e_{22}} \\ -\frac{\partial h_{12}}{\partial \Delta W_1} & \frac{\partial h_{12}}{\partial e_{12}} + \frac{\partial h_{12}}{\partial e_{22}} \end{pmatrix} \\ &= \frac{1}{\Psi} \det \begin{pmatrix} -f(0) \cdot q'(e_{11}) & -C''(e_1) \\ 0 & \Delta W_2 f(0) q''(e_{12}) - C''(e_1) \end{pmatrix} \qquad (36a) \\ &= -\frac{f(0) \cdot q'(e_{11}) \left(\Delta W_2 f(0) q''(e_{12}) - C''(e_1)\right)}{\Psi}, \qquad (36b) \end{aligned}$$

where the right-hand side is positive. This implies that an increase in task one's difference in winner prizes increases the efforts of agents 1 and 2 in task 1. Further,

$$\begin{aligned} \frac{de_{12}}{d\Delta W_1} &= \frac{1}{\Psi} \det \begin{pmatrix} \frac{\partial h_{11}}{\partial e_{11}} + \frac{\partial h_{11}}{\partial e_{21}} & -\frac{\partial h_{11}}{\partial \Delta W_1} \\ \frac{\partial h_{12}}{\partial e_{11}} + \frac{\partial h_{12}}{\partial e_{21}} & -\frac{\partial h_{12}}{\partial \Delta W_1} \end{pmatrix} \\ &= \frac{1}{\Psi} \det \begin{pmatrix} \Delta W_1 f(0) q''(e_{11}) - C''(e_1) & -f(0) \cdot q'(e_{11}) \\ -C''(e_1) & 0 \end{pmatrix} \qquad (37a) \\ &= -\frac{C''(e_1) f(0) \cdot q'(e_{11})}{\Psi}, \qquad (37b) \end{aligned}$$

where the right-hand side is negative. This implies that an increase in task 1's difference in winner prizes reduces the efforts of agents 1 and 2 in task 2. The effect of the prize difference on the sum of an agent's efforts can be written as

$$\frac{d\left(e_{11} + e_{12}\right)}{d\Delta W_1} = -\frac{f(0) \cdot q'(e_{11}) \Delta W_2 f(0) q''(e_{12})}{\Psi}, \tag{38}$$

where the right-hand side is positive.

A.4 Proof of Proposition 2

If there is one tournament for each task and for a given budget, the winner prize structure that maximizes the expected total output, $q(e_{11})+q(e_{12})+q(e_{21})+q(e_{22})$, must ensure that (i) effort investments are maximized and (ii) that efforts are evenly distributed between tasks and agents. Lemma 2 implies that equilibrium effort can be increased by an increase of the winner-prize difference. Given limited liable agents, this means that loser prizes should be equal to zero. Lemma 1 ensures that equlibrium efforts are equal across agents. Because production functions are identical across tasks, winner prizes must be equal across tasks to ensure that equilibrium effort is evenly distributed between tasks. Altogether, this implies the optimal winner-prize structure $W_{11} = W_{12} = w/2$ and $W_{21} = W_{22} = 0$.

A.5 Proof of Lemma 3

The first-order conditions for the agents' effort choices can be written as

$$\Delta W \cdot f^{\text{big}}(\Delta q_i) \cdot q'(e_{i1}) - C'(e_i) = \Delta W \cdot f^{\text{big}}(\Delta q_i) \cdot q'(e_{i2}) - C'(e_i) = 0, \tag{39}$$

which implies $q'(e_{i1}) = q'(e_{i2})$ and shows that agents invest equal efforts across tasks. To see that efforts are also equal across agents, use $\Delta q_j = -\Delta q_i$ and the symmetry of distributions around zero, which implies

$$\Delta W \cdot f^{\text{big}}(\Delta q_j) \cdot q'(e_{jt}) - C'(e_j) = \Delta W \cdot f^{\text{big}}(\Delta q_i) \cdot q'(e_{jt}) - C'(e_j) = \Delta W \cdot f^{\text{big}}(\Delta q_i) \cdot q'(e_{it}) - C'(e_i) = 0. \tag{40}$$

Rearranging the equality in the middle yields

$$\Delta W \cdot f^{\text{big}}(\Delta q_i) \cdot \big(q'(e_{i1}) - q'(e_{j1})\big) = C'(e_i) - C'(e_j), \tag{41}$$

where the right-hand side decreases with e_{it} because of the concavity of the production functions, and the right-hand side increases with e_{it} with $e_i = 2e_{it}$ because of the convexity of the effort cost functions. Efforts must therefore be the same across agents and tasks because this equation is otherwise violated.

A.6 Proof of Proposition 3

The first-order conditions in (6) are denoted and written as

$$\begin{aligned}
k_{11} &= \Delta W \cdot f^{\text{big}}(\Delta q_1) \cdot q'(e_{11}) - C'(e_1), &(42)\\
k_{12} &= \Delta W \cdot f^{\text{big}}(\Delta q_1) \cdot q'(e_{12}) - C'(e_1), &(43)\\
k_{21} &= \Delta W \cdot f^{\text{big}}(\Delta q_2) \cdot q'(e_{21}) - C'(e_2), &(44)\\
k_{22} &= \Delta W \cdot f^{\text{big}}(\Delta q_2) \cdot q'(e_{22}) - C'(e_2). &(45)
\end{aligned}$$

Using symmetry and that $(f^{\text{big}})'(0) = 0$, the Jacobian of the first-order conditions can be written as

$$J = \begin{pmatrix} \frac{\partial k_{11}}{\partial e_{11}} & \frac{\partial k_{11}}{\partial e_{12}} & \frac{\partial k_{11}}{\partial e_{21}} & \frac{\partial k_{11}}{\partial e_{22}} \\ \frac{\partial k_{12}}{\partial e_{11}} & \frac{\partial k_{12}}{\partial e_{12}} & \frac{\partial k_{12}}{\partial e_{21}} & \frac{\partial k_{12}}{\partial e_{22}} \\ \frac{\partial k_{21}}{\partial e_{11}} & \frac{\partial k_{21}}{\partial e_{12}} & \frac{\partial k_{21}}{\partial e_{21}} & \frac{\partial k_{21}}{\partial e_{22}} \\ \frac{\partial k_{22}}{\partial e_{11}} & \frac{\partial k_{22}}{\partial e_{12}} & \frac{\partial k_{22}}{\partial e_{21}} & \frac{\partial k_{22}}{\partial e_{22}} \end{pmatrix} =$$

$$\begin{pmatrix} \Delta W f^{\text{big}}(0) q''(e_{11}) - C''(e_1) & -C''(e_1) & 0 & 0 \\ -C''(e_1) & \Delta W f^{\text{big}}(0) q''(e_{12}) - C''(e_1) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} +$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta W f^{\text{big}}(0)q''(e_{21}) - C''(e_2) & -C''(e_2) \\ 0 & 0 & -C''(e_2) & \Delta W f^{\text{big}}(0)q''(e_{22}) - C''(e_2) \end{pmatrix} \tag{46}$$

where the right-hand side is negative definite, which ensures the existence of a unique solution for the system of equations in (42)-(45) by the index theory approach (e.g., Vives, 1999).

A.7 Proof of Lemma 4

Totally differentiating the first-order conditions in (42)-(45) with respect to efforts and the prize difference ΔW and using symmetry yields

$$dk_{11} = \left(\frac{\partial k_{11}}{\partial e_{11}} + \frac{\partial k_{11}}{\partial e_{12}} + \frac{\partial k_{11}}{\partial e_{21}} + \frac{\partial k_{11}}{\partial e_{22}} \right) de_{11} + \frac{\partial k_{11}}{\partial \Delta W} d\Delta W = 0. \tag{47}$$

Rearranging leads to

$$\frac{de_{11}}{d\Delta W} = -\frac{\frac{\partial k_{11}}{\partial \Delta W}}{\frac{\partial k_{11}}{\partial e_{11}} + \frac{\partial k_{11}}{\partial e_{12}} + \frac{\partial k_{11}}{\partial e_{21}} + \frac{\partial k_{11}}{\partial e_{22}}} \tag{48}$$

with

$$\frac{\partial k_{11}}{\partial \Delta W} = f^{\text{big}}(0) \cdot q'(e_{11}) \tag{49}$$

and

$$\frac{\partial k_{11}}{\partial e_{11}} + \frac{\partial k_{11}}{\partial e_{12}} + \frac{\partial k_{11}}{\partial e_{21}} + \frac{\partial k_{11}}{\partial e_{22}} = -2C''(e_1) \tag{50}$$

because $(f^{\text{big}}(0))' = 0$ leading altogether to

$$\frac{de_{11}}{d\Delta W} = \frac{f^{\text{big}}(0) \cdot q'(e_{11})}{2C''(e_1)}, \tag{51}$$

where the right-hand side is positive.

A.8 Proof of Proposition 5

For insights into the relative importance of the prize-diluting and noise effects, use the formula for the density of a sum of two random variables,

$$f^{\text{big}}(x) = \int_s f(s-x) f(s) ds. \tag{52}$$

For $x = 0$, this equality reduces to

$$f^{\text{big}}(0) = \int_s f(s)^2 ds. \tag{53}$$

It is further helpful to use

$$\frac{f(0)}{2} = f(0) \int_s f(2s) ds, \tag{54}$$

which follows because $f(0) > 0$ and $\int_s f(s)ds = 1$ implies that $\int_s f(2s)ds = 1/2$. Equations (53) and (54) can be combined to show that log-concavity of the composite errors z_{it} ensures that the budget multiplier exceeds 1 as follows.

Log-concavity of the composite errors z_{it} implies

$$\lambda \log f(0) + (1-\lambda) \log f(2s) < \log f(2(1-\lambda)s) \tag{55}$$

for $\lambda \in (0,1)$. If $\lambda = 1/2$, this inequality can be written as

$$\log f(0) + \log f(2s) < 2 \log f(s). \tag{56}$$

Taking exponentials leads to

$$f(0)f(2s) < f(s)^2. \tag{57}$$

Using equations (53) and (54), this implies $f^{\text{big}}(0) > \frac{f(0)}{2}$ and $\gamma > 1$.