https://doi.org/10.1016/j.trb.2019.11.003



# Optimal Design of Transit Networks Fed by Shared Bikes

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## Abstract

Transit systems are designed in which access and egress can occur via a shared-bike service. Patrons may walk to shared-bike docking stations nearest their origins, and then cycle to their nearest transit stations where they deposit the bikes. The travel pattern is reversed when patrons cycle from their final transit stations on to their destinations. Patrons choose between this option and that of solely walking to or from transit stations. Shared bikes are priced to achieve the system-optimal assignment of the two feeder options.

Transit trunk-line networks are laid-out in hybrid fashion, as proposed in Daganzo (2010a). Transit lines thus form square grids inside city centers, and radiate outward in the peripheries. As in Daganzo (2010a) and other studies, a set of simplifying assumptions are adopted that pertain primarily to the nature of travel demand. These enable the formulation of a parsimonious, continuous model. The model produces designs that minimize total travel costs, and is ideally suited for high-level (i.e., strategic) planning. A similar model is developed for systems in which access or egress to or from transit can occur solely by walking, or by walking and riding fixed-route feeder buses in combination. The shared-bike and feeder-bus models both complement Daganzo's original model in which access and egress occur solely by walking.

Comparisons of these feeder options are drawn through numerical analyses. These are performed in parametric fashion by varying city size, travel demand, and economic conditions; and for trunk services that are provided either by ordinary buses, Bus Rapid Transit, or metro rail. Designs are produced for cases in which shared-bike and feeder-bus services are made to fit pre-existing and unchangeable trunk-line networks; and for cases in which trunk and feeder services are optimized jointly.

Outcomes reveal that shared-bike feeder systems can often reduce costs over walking alone, with cost savings as high as 7%, even when the shared bikes are made to fit a pre-existing transit network. Shared-biking often outperforms feeder-bus service as well. We further find that the joint optimization of trunk and shared-bike feeder services can reduce costs not only to users, but also to the transit agency that operates these services. Savings to the agency can be used to subsidize shared-bike services. We show that with or without this subsidy, shared-bike systems can always break even when they are suitably priced, and jointly optimized with trunk service.

*Keywords*: transit network design; bike sharing; continuous models; joint optimization; system optimal pricing

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## 1. Introduction

Public transit services can be accessed by feeder bus and, less commonly by bicycle. Both options are speedier than walking. Bicycles are less expensive than buses, however, and are more environmentally-friendly (Pucher and Buehler, 2008, 2012). And for able-bodied travelers, bikes can be convenient to use, save perhaps in hilly cities and inclement weather. Little wonder that communities that deploy bike lanes and other bicycle-friendly facilities often find that ridership grows for transit as well as for bikes (Beatley, 2014; Bonnette, 2007; Hampshire and Marla, 2012; Martens, 2007; Noland and Ishaque, 2006; Wieth-Knudsen, 2012).

Shared bicycles that are rented by transit patrons seem a particularly promising means of feeder service (Goodyear, 2014; Gutman, 2017; Liu et al., 2012; Midgley, 2011; 2013; Shaheen et al., 2009, 2010; Wang, 2013b); and one that surveys suggest is preferred by many cyclists (TNS Sofres, 2009). Sharing relieves riders of having to purchase, maintain and protect their own bikes. And it conveniently solves transit's first- and last-mile problem by enabling patrons to ride at one or both ends of their trips without carrying bicycles aboard buses or trains (Liu et al., 2012). This is something that both transit patrons and operating agencies find desirable.

In practice, bike-sharing schemes are designed to fit existing transit systems, with little or no adjustments to the latter. Policy studies on the subject presume this separate approach to design (e.g. Cheng and Liu, 2012; Li and Loo, 2016; Muñoz et al., 2016). Empirical studies reflect this approach as well (e.g. Faghih-Imani et al., 2017; Ma et al., 2015; Martens, 2007; Midgley, 2009; Nadal, 2007; Yang et al., 2015).

To our knowledge, the literature remains silent on the subject of jointly designing shared bike systems with the transit trunk-line networks they feed. This is surprising, since the higher access and egress speeds of bicycles (relative to walking) might justify trunk-line designs with greater spacings between routes and stops. These could lower operating costs for the transit agency, as well as trip times for its patrons. Both advantages were found when transit trunk networks were jointly optimized with feeder services furnished by buses (e.g. Chen and Nie, 2017a; Sivakumaran et al., 2014).

In light of the above, the present paper explores (i) how shared-bike feeder systems might best be designed to fit existing (and unalterable) trunk-line transit networks; and (ii) how these same feeders and trunk networks might be optimized jointly. In both thrusts, shared bikes are accessed via docking stations. And in both thrusts, designs are optimized without the aid of discrete models of the kind in Ibarra-Rojas et al. (2015) and Kepaptsoglou and Karlaftis (2009), since these furnish solutions that are case-specific.

We opt instead to formulate and use parsimonious, continuous models in line with Newell (1971) and Wirasinghe and Ghoneim (1981). Doing so required a host of simplifying assumptions. Most pertain to travel demands, which are assumed to be uniformly distributed over our networks, and invariant to network designs. Though these may be viewed as controversial in some circles, all of the assumptions have been adopted in previous works; e.g. see (Chen et al., 2015; Daganzo, 2010a, b; Estrada et al., 2011; Fan et al., 2018; Sivakumaran et al., 2014). The advantage of our approach lies in the general insights that it produces.

Present insights pertain to both thrusts (i) and (ii) above, and were sharpened via parametric analyses. Our case studies collectively entail trunk networks that are served by ordinary buses, by bus rapid transit (BRT) and by metro rail. Each of these forms was explored under three feeder options in which patrons: walk to and from trunk stations sans other options; choose whether or not to ride fixed-route feeder buses; and choose instead whether to ride shared bikes. We further parse each combination of trunk and feeder system by varying the city's size, its travel demand, and its economic condition.

In all these many cases, the trunk-line networks conform to the hybrid structure proposed in Daganzo (2010a), and briefly reviewed in the following section. The continuous models formulated for the three feeder options are presented in the following section as well. Parametric analyses are presented in section 3. Section 3 also explores how shared bikes can be priced so that transit agencies can always break even, with or without subsidies. Tailoring present findings to real-world environments is discussed in section 4.

## 2. Methodology

Section 2.1 reviews ideas in Daganzo (2010a) for designing transit networks accessed on foot. Our reiteration of these ideas is kept to a minimum, and is offered to justify new ideas that follow. These come in sections 2.2 and 2.3 and pertain to access via shared bicycles and feeder buses, respectively. A solution method is presented in section 2.4. Notations used throughout this section are provided in Appendix A.

### *2.1. Accessing transit on foot*

Consider the square-shaped city of size $D \times D$ (km$^2$) in Fig. 1 with a dense grid of streets throughout that are parallel to the city's boundaries. As per assumptions in Daganzo (2010a), Chang and Schonfeld (1991), and Medina et al. (2013), demand for transit travel: is exogenous and inelastic to transit service; has an hourly rate of $\lambda_p$ (trips/h/km$^2$) during the peak period of duration $t_p$ (h/day), a lower rate $\lambda_o$ (trips/h/km$^2$) during the off-peak of duration $t_o$ (h/day); and has origins and destinations that are uniformly distributed over the entire city. The average demand density $\lambda = \frac{\lambda_p t_p + \lambda_o t_o}{t_p + t_o}$ will be used as a proxy for the city's population density. The patrons' value of time, $\mu$ (\$/h), will serve as a proxy for the average hourly wage of city residents.

In further keeping with previous studies, a patron is assumed to: access and egress a transit system via the station nearest her origin and destination, respectively; arrive at her origin station randomly, regardless of the service schedule; choose the shortest-distance route; and choose between routes with equal probability, should multiple shortest routes exist. For simplicity, transit vehicles are assumed to stop at every station along a route, and dwell for time $\tau$ at each station.[1]

Transit routes collectively form the hybrid structure shown in Fig.1 in bold. In a (shaded) central area of size $\alpha D \times \alpha D$ (where $\alpha$ is a decision variable), lines evenly spaced at $S$ (km) form a grid. The lines extend (and branch as needed) in the periphery, with stations again spaced at $S$. Vehicle headways in the central area are $H_p$ (h) and $H_o$ (h) during peak and off-peak times, respectively.

In formulating the cost-minimization problem for this hybrid network, the costs born to patrons and the agency will be expressed in units of time (Chen and Nie, 2017a, b, 2018; Daganzo, 2010a, b), and with decision variables $S, H_p, H_o$, and $\alpha$. The formulation is:

$$\min_{S,H_p,H_o,\alpha} \quad \frac{t_p\lambda_p\left(UC_{W,p}+AC_p\right)+t_o\lambda_o\left(UC_{W,o}+AC_o\right)}{t_p\lambda_p+t_o\lambda_o} \tag{1a}$$

$$\text{subject to:} \quad \lambda_k SDH_k \cdot \max\left\{\frac{1-\alpha^2}{2\alpha}, \frac{3+2\alpha^2-3\alpha^4}{8\alpha}+\frac{D\left(1-\alpha^2\right)^2}{32S}\right\} \le K,\ \ k \in \{p,o\} \tag{1b}$$

$$H_k \ge H_{min},\ \ k \in \{p,o\} \tag{1c}$$

$$S > 0 \tag{1d}$$

[1] With modest additions, the model can treat the dwell time at each stop as a linear function of its boarding patrons, as in Daganzo (2010a); Estrada et al. (2011); and Fan et al. (2018).

$$S/D \leq \alpha \leq 1, \tag{1e}$$

where $UC_{W,p}$ and $UC_{W,o}$ denote the patrons' average trip costs in peak and off-peak periods, respectively; and $AC_p$ and $AC_o$ are the average peak and off-peak trip costs incurred by the transit agency. Constraint (1b) ensures that the number of patrons onboard a transit vehicle never exceeds its passenger-carrying capacity, $K$, where the left-hand-side is the maximum onboard occupancy for period $k$; see Daganzo (2010a) for the derivation. Constraint (1c) prevents headways from falling below a minimum, $H_{min}$, as determined by safety considerations or the system's vehicle-carrying capacity; and (1d) and (1e) are boundary constraints for the decision variables.

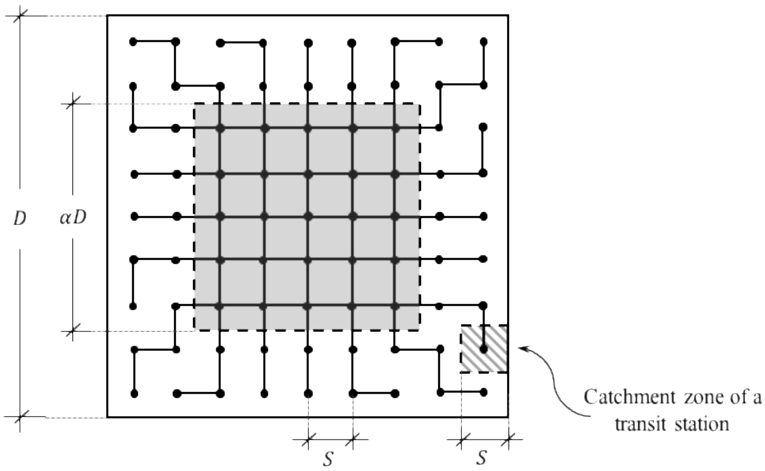



**Fig. 1.** A hybrid transit network atop a grid street network in a square city (Daganzo, 2010a).

The patrons' average trip cost in period $k$ is formulated as:

$$UC_{W,k} = \frac{S}{v_w} + E_{T,k}, \;\; k \in \{p, o\} \tag{2a}$$

$$E_{T,k} = H_k \left(\frac{2+\alpha^3}{3\alpha} + \frac{\left(1-\alpha^2\right)^2}{4}\right) + \delta \left(1 + \frac{\left(1-\alpha^2\right)^2}{2}\right) + \frac{D}{12}\left(\frac{1}{v} + \frac{\tau}{S}\right)\left(12 - 7\alpha + 5\alpha^3 - 3\alpha^5 + \alpha^7\right), \;\; k \in \{p, o\} \tag{2b}$$

where $v_w$ is walking speed, such that $\frac{S}{v_w}$ is the average time spent accessing and then egressing transit stations; $\delta$ is the penalty cost per transfer (in hours); and $v$ is the transit vehicle's cruise speed. The $E_{T,k}$ is the sum of: (i) average wait time per trip at the origin and transfer stations in period $k$; (ii) average transfer penalty per trip; and (iii) average in-vehicle travel time per trip.

The agency cost consists of the infrastructure cost for the lines (rails or bus lanes) and the stations, and the operating costs based on vehicle-kms traveled (e.g. fuel) and vehicle-hours traveled (e.g. driver wages). The average agency cost per trip in period $k$ is formulated as:

$$AC_k = \frac{1}{\mu\lambda_k}\left(\frac{\left(1+\alpha^2\right)C_I}{S} + \frac{C_S}{S^2} + \frac{2\left(3\alpha-\alpha^2\right)C_{VD}}{SH_k} + \frac{2\left(3\alpha-\alpha^2\right)C_{VT}}{SH_k}\left(\frac{1}{v} + \frac{\tau}{S}\right)\right), \;\; k \in \{p, o\}, \tag{3}$$

where $C_I, C_S, C_{VD}, C_{VT}$ are unit cost parameters: $C_I$ (\$/h/km) denotes the amortized monetary cost per km of transit line per hour of operation; $C_S$ (\$/h/station) the amortized cost per station per hour; $C_{VD}$ (\$/km/vehicle) the unit distance-based operating cost; and $C_{VT}$ (\$/h/vehicle) the unit time-based operating cost. In the interest of brevity, further explanation and derivation of (2a-3) are omitted here. Readers can refer to Daganzo (2010a) for details.

*2.2. Access via shared bikes*

For the sake of simplicity, ignore the possibility that some travelers (e.g. those with short commutes) might use bicycles for their entire trips, and assume instead that shared bikes are used solely for accessing and egressing transit. The assumption is conservative because it over-estimates the costs incurred by some short-distance travelers, and therefore obscures the full benefits of shared-biking.

Our first order of business is to lay-out the docking stations where patrons check-out and return bicycles. Two types of stations are used: large docking stations that are placed next to transit stations to facilitate transit access and egress; and smaller docking stations that are uniformly distributed over a city at a density $P$ (station/km$^2$). Layout of the latter stations is done per our first proposition below, with a proof relegated to Appendix B.

**Proposition 1**. For a given $P$, the diamond-grid layout of small docking stations shown in Fig. 2 (where the black dots represent the docking stations) minimizes the average walking distance between an average patron's origin or destination and the nearest docking station. The resulting average walking distance is $d_w = \sqrt{\frac{2}{9P}}$.

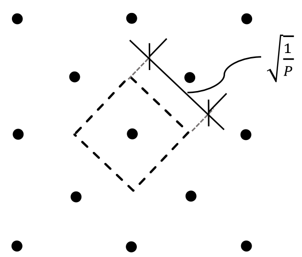



**Fig. 2.** The optimal layout of small bike docking stations.

The joint optimization of the transit network and docking stations takes five decision variables and is formulated as follows:

$$\min_{S,H_p,H_o,\alpha,P} \quad \frac{t_p\lambda_p(UC_{B,p}+AC_p)+t_o\lambda_o(UC_{B,o}+AC_o)}{t_p\lambda_p+t_o\lambda_o} + AC_B \tag{4}$$

subject to: (1b-e) and $P \geq 0$,

where $UC_{B,k}$ ($k \in \{p, o\}$) denotes the average patron's cost in period $k$; and $AC_B$ the bike-sharing agency cost per trip, to be defined in due course. The $AC_k$ ($k \in \{p, o\}$) is the same as defined in (3). The $UC_{B,k}$ is given by:

$$UC_{B,k} = E_{B,k} + E_{T,k}, \text{for } k \in \{p, o\}. \tag{5}$$

Note that a trip's average access and egress time by walking, $\frac{S}{v_w}$ in (2a), is replaced in (5) by $E_{B,k}$ to account for the costs of cycling. These costs depend on how patrons choose between walking and renting shared bikes. To model these choices, assume that only a proportion, $\beta$, of transit patrons are able-bodied and thus consider biking as a feeder option. These patrons choose between walking and cycling so as to lower their costs. The $\beta$ reflects patrons' willingness to cycle, and can be used to capture the long-term effects of weather, terrain and the presence or absence of bike-friendly facilities and policies.[2] The remaining $(1-\beta)$ of patrons access transit solely by walking (Nurworsoo et al., 2012).

[2] More detailed choice models, such as probit or logit (Hausman and Wise, 1978; Taylor and Mahmassani, 1996; Wen and Koppleman, 2001) can be incorporated into our modeling framework too.

Consider an able-bodied patron in period $k \in \{p, o\}$. Define $d$ as the access distance from that patron's origin to her nearest transit station, or as the egress distance between the patron's destination and the transit station nearest that destination.[3] Access or egress cost by riding a shared bike, $u_{Bk}(d)$, or by solely walking, $u_{Wk}(d)$, are formulated as:

$$u_{Bk}(d) = \frac{d}{v_b} + t_w + t_{dp} + t_{dd} + t_f + \frac{\varphi_k(d)}{\mu} \tag{6a}$$

$$u_{Wk}(d) = \frac{d}{v_w}, \tag{6b}$$

where $v_b$ denotes the cycling speed; $t_w$ the walking time from the patron's origin to the nearest bike station (from Proposition 1, we have $t_w \approx d_w/v_w = \sqrt{\frac{2}{9P}}/v_w$); $t_{dp}$ and $t_{dd}$ the times for picking-up and dropping-off a bike at a docking station, respectively; $t_f$ the intermodal transfer penalty between the transit station and the nearby bike station; and $\varphi_k(d)$ the distance-based bike rental fee in period $k$. We present the following two propositions concerning a patron's choice of access or egress mode.

**Proposition 2**. At system optimum, there exists a critical distance $d_{ck} > 0$ for each period $k \in \{p, o\}$, such that if a patron's access or egress distance $d < d_{ck}$, she will choose to walk to or from the transit station, and if an able-bodied patron's access or egress distance $d > d_{ck}$, she will choose to ride a shared bike.

**Proposition 3**. The above system optimum can be attained by appropriately pricing the bike rental fee.

Proofs of the above two propositions are relegated to Appendices C and D. Appendix D also presents a scheme that entails the system optimum mode choices, in which the bike rental fee increases linearly with distance. The derivation of the critical distance $d_{ck}$ ($k \in \{p, o\}$) is furnished in Appendix E. Understanding the proofs and derivations in the appendices requires additional notations regarding the agency cost of shared bikes to be defined. Specifically, we denote: $C_B$ (\$/bike/day) and $C_D$ (\$/dock/day) as the purchase, maintenance, and operating costs for each bike and each dock, respectively. These costs are amortized over the lifecycles of a bike and a dock. Further denote: $\xi$ as the fixed ratio between the numbers of docks and bikes for a bike-sharing system, which usually takes a value of 1.5~1.7 for real-world business solutions (Gauthier et al., 2013; Gleason and Miskimins, 2012; Tang et al., 2011; Yang et al, 2015); and $\rho$ as the bikes' peak-period utilization ratio, i.e., the average proportion of time when a bike is in use during peak periods ($0 < \rho \leq 1$).

Parameter $\rho$ indicates how fast the bikes are circulated during peak hours, which is affected by the demand imbalance, randomness, and the performance of bike redistribution strategies. When $\rho = 1$, each bike will be checked out immediately after someone returns it to a docking station, as may occur when the incoming and outgoing demands at each station are perfectly balanced and deterministic. Low values of $\rho$ can be used to represent cases where the incoming and outgoing demands are highly stochastic and imbalanced between stations, and where no efficient bike redistribution strategy is implemented. Using low values of $\rho$ would be conservative because more bikes and docks are needed to satisfy the demand, entailing a higher agency cost. For simplicity, detailed modeling of the bike redistribution strategy is omitted in this paper, and its cost is assumed to be factored into the amortized costs for the bikes and the stations (Gleason and Miskimins, 2012; Wang, 2013a; Yang et al., 2015).

Following Proposition 2, denote the part of a catchment zone that is defined by $d < d_{ck}$ as the "walk-only" region, and the remaining catchment zone as the "cycling" region. We then have the following corollary of Proposition 2:

**Corollary 1**. The $E_{B,k}$ under the system-optimal choices of access modes is given by:

[3] An able-bodied patron may choose to ride a shared bike to access transit, or to egress transit, or to do both.

$$E_{B,k} = (1-\beta)\frac{S}{v_w} + 2\beta\left(\frac{A_{bw,k}}{S^2}\cdot\frac{d_{bin,k}}{v_w} + \left(1-\frac{A_{bw,k}}{S^2}\right)\left(\frac{d_{bout,k}}{v_b} + t_w + t_{dp} + t_{dd} + t_f\right)\right), k \in \{p, o\}, \qquad (7)$$

where $A_{bw,k}$ and $d_{bin,k}$ are the area and the average access distance of the walk-only region in period $k$; and $d_{bout,k}$ is the average access distance in the cycling region in that period.

Proof of Corollary 1 and derivations for $A_{bw,k}$, $d_{bin,k}$ and $d_{bout,k}$ are also presented in Appendix E.[4]

Finally, following the price structures of real-world bike-sharing service vendors (e.g. Gauthier et al., 2013; Gleason and Miskimins, 2012), we formulate the bike-sharing agency cost $AC_B$ as:

$$AC_B = \frac{C_P\cdot\left(P+\frac{1}{S^2}\right)+(C_B+\xi C_D)\frac{n_B\lambda_p}{\rho}}{\mu(\lambda_p t_p+\lambda_o t_o)}, \qquad (8)$$

where $C_P$ (\$/station/day) denotes the fixed cost rate for the purchase, installation, and maintenance of a docking station, amortized over its lifecycle. For simplicity, we use the same unit cost rate for both large docking stations deployed near transit stations, and small docking stations distributed evenly over the city.[5] The $\frac{n_B\lambda_p}{\rho}$ is the number of bikes needed per km$^2$ of service area; and $n_B$ denotes the average bike-hours used per patron during peak periods. The $n_B$ depends on the proportion of the cycling region and the average time that a bike user occupies a bike in peak periods. Derivation of $n_B$ is also furnished in Appendix E.

Although the bike rental fee affects a patron's choice to walk or cycle, this fee is not a part of the generalized cost because it is a transfer of money from the bike users to the operating agency.

### *2.3. Access via fixed-route feeder buses*

Consider the trunk and fixed-route feeder-bus network proposed in Sivakumaran et al. (2014), and shown in Fig. 3. The large, dark circle is a transit trunk station with a catchment zone bounded by dashed lines. The thick solid lines represent trunk lines as they would be laid-out in a grid network. (Note that in the peripheral area of a hybrid trunk-line network, only part of the two trunk lines shown in Fig. 3 may exist). The thinner solid lines with arrowheads (shown for illustration in the lower-right portion of the catchment zone) are feeder-bus lines. The small squares are feeder-bus stops.

We use the continuous cost model formulated in Sivakumaran et al. (2014) to design the trunk and feeder network, but with two modifications. These (i) accommodate the hybrid trunk network previously shown in Fig. 1; and (ii) enable transit patrons to choose between walking and riding a feeder bus to and from trunk stations. The formulation has seven decision variables $(S, H_p, H_o, \alpha, S_f, H_{f,p}, H_{f,o})$ and takes the form:

$$\min_{S,H_p,H_o,\alpha,S_f,H_{f,p},H_{f,o}} \quad \frac{t_p\lambda_p(UC_{F,p}+AC_p+AC_{F,p})+t_o\lambda_o(UC_{F,o}+AC_o+AC_{F,o})}{t_p\lambda_p+t_o\lambda_o} \qquad (9a)$$

$$\text{subject to:} \quad \frac{\left(1-\frac{A_{fw,k}}{S^2}\right)\cdot\lambda_k\cdot S_f\cdot S\cdot H_{f,k}}{4} \le K_f, \quad k \in \{p, o\} \qquad (9b)$$

$$H_{f,k} \ge H_{fmin}, \quad k \in \{p, o\} \qquad (9c)$$

$$S_f > 0 \qquad (9d)$$

[4] A similar method was used in Chen and Nie (2017a) for the access mode assignment between walking and riding via a flexible-route feeder service.

[5] The size of each docking station can be determined from the proportion of incoming and outgoing bike flows. When space is limited, large docking stations can be designed as underground bike parking facilities, as in the Netherlands (Bicycle Dutch, 2018).

$$\frac{S}{S_f} \in \{1,2,3, \dots\} \tag{9e}$$

(1b-e),

where $UC_{F,k}$ is the patrons' average trip cost for period $k$; $AC_{F,k}$ is the agency cost for feeder-bus service; $A_{fw,k}$ is the area within a trunk station's catchment zone where patrons access and egress trunk service on foot (see Appendix F for the derivation); $H_{f,k}$ is the feeder-bus headway; $S_f$ is the spacing between feeder-bus lines and stops, which are assumed equal for simplicity; $K_f$ is a feeder bus's passenger-carrying capacity; and $H_{fmin}$ is the minimum headway for a feeder-bus line. The agency cost for trunk-line service in period $k$, $AC_k$ ($k \in \{p, o\}$), is the same as in (3). Constraint (9b) reflects the limits in feeder-bus carrying capacity, where the left-hand-side is the maximum number of onboard passengers allowed for period $k$. Constraint (9c) specifies the minimum headway for feeder buses. Constraint (9e) requires trunk line spacing to be an integer multiple of feeder line spacing.

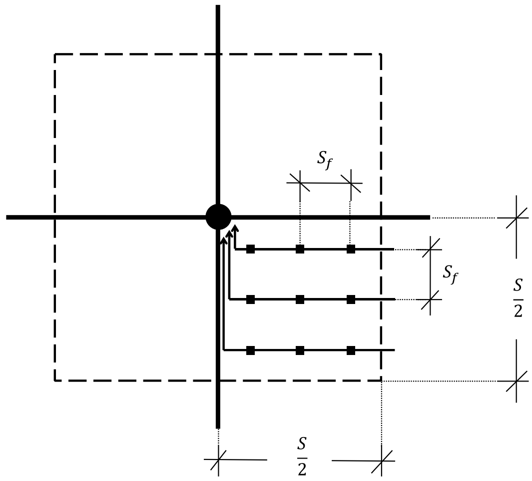



**Fig. 3.** The feeder bus network in Sivakumaran et al. (2014).

The user cost for feeder-bus service, $UC_{F,k}$, is formulated as:

$$UC_{F,k} = E_{F,k} + E_{T,k}, \;\; k \in \{p, o\}, \tag{10}$$

where $E_{F,k}$ is the average access and egress cost per trip for period $k$. The $E_{F,k}$ depends on how patrons choose between walking and riding a feeder bus, and is derived in a manner similar to $E_{B,k}$ in section 2.2. The detailed derivation is relegated to Appendix F.

The feeder agency cost, $AC_{F,k}$ ($k \in \{p, o\}$), is formulated as:

$$AC_{F,k} = \frac{1}{\mu \lambda_k S_f}\left(C_{fI} + \frac{C_{fS}}{S_f} + \frac{3C_{fVD}}{H_{f,k}} + \frac{C_{fVT}}{H_{f,k}}\left(\frac{3}{v_f} + \frac{2\tau_f}{S_f}\right)\right), \;\; k \in \{p, o\}, \tag{11}$$

where $C_{fI}$ (\$/h/km) denotes the amortized hourly cost per km of feeder line infrastructure; $C_{fS}$ (\$/h/stop) the amortized hourly cost for constructing and maintaining a feeder bus stop; $C_{fVD}$ (\$/km/bus) and $C_{fVT}$ (\$/h/bus) are the unit distance-based and time-based feeder bus operating costs, respectively; $v_f$ the cruise speed of feeder buses; and $\tau_f$ the feeder bus dwell time at a stop. Refer to Sivakumaran et al. (2014) for the derivation of (11).

*2.4. Solution method*

We first derive the closed-form optimal solution for trunk-line headway, $H_k$ ($k \in \{p, o\}$), in (1), (4) and (9). The solution is the same for these three mathematical programs because the parts of their objective functions and the constraints related to $H_k$ are the same for all three. All three programs are convex in $H_k$, and when the constraints are ignored the first-order condition with respect to $H_k$ yields:

$$\widetilde{H}_k = \sqrt{\frac{2(3\alpha-\alpha^2)\left[C_{VD}+C_{VT}\left(\frac{1}{v}+\frac{\tau}{S}\right)\right]}{\mu\lambda_k S\left(\frac{2+\alpha^3}{3\alpha}+\frac{(1-\alpha^2)^2}{4}\right)}},\ k \in \{p, o\}. \tag{12}$$

Constraints (1b-c) specify that $H_k$ is bounded from above and below by $\frac{K}{\lambda_k SD\cdot\max\left\{\frac{1-\alpha^2}{2\alpha},\frac{3+2\alpha^2-3\alpha^4}{8\alpha}+\frac{D(1-\alpha^2)^2}{32S}\right\}}$ and $H_{min}$, respectively. Thus, the closed-form solution for $H_k$ can be written as a function of $\alpha$ and $S$ as follows:

$$H_k^* = \mathrm{mid}\left(\widetilde{H}_k, H_{min}, \frac{K}{\lambda_k SD\cdot\max\left\{\frac{1-\alpha^2}{2\alpha},\frac{3+2\alpha^2-3\alpha^4}{8\alpha}+\frac{D(1-\alpha^2)^2}{32S}\right\}}\right),\ k \in \{p, o\}, \tag{13}$$

where the function $\mathrm{mid}(\cdot)$ takes the middle value among the three arguments.

With (13), the number of decision variables is reduced to: two for walk-only access $(S, \alpha)$; three for bike-sharing access $(S, \alpha, P)$; and five for feeder-bus access $\left(S, \alpha, S_f, H_{f,p}, H_{f,o}\right)$. Thanks to the small number of decision variables, these reduced optimization models can be solved by a number of commercial solvers. We employ the "fmincon" tool with the sequential quadratic programming algorithm in MATLAB R2016b.[6]

The above method cannot guarantee a globally-optimal solution, owing to the non-convex nature of the programs. Thus, we repeated the procedure 10 times for each case study examined in the paper. Each time the optimization started with an initial solution randomly selected from the feasible ranges of decision variables, which were defined as: $S \in [0.05, 2.5]$ km, $\alpha \in \left[\frac{S}{D}, 1\right]$, $P \in [10, 1000]$ station/km$^2$, $S_f \in [0.05, 0.5]$ km, and $H_{f,p}, H_{f,o} \in \left[\frac{1}{60}, \frac{1}{3}\right]$ h. We found that each repetition of the solution procedure always produced the same final solution, and are therefore confident in our solution.

## 3. Numerical analysis

Parameter values used in our numerical tests are presented in section 3.1. Feeder systems are designed to suit pre-existing transit networks in section 3.2. Trunk and feeder systems are optimized jointly in section 3.3. We examine how bike-sharing fees can ensure that agencies break even on fare revenues and costs in section 3.4.

### *3.1. Parameter values*

We borrow from Daganzo (2010a) and Chen et al. (2015) and specify that the square city's length (and width), $D \in [10, 30]$ km; demand density, $\lambda \in [200, 3000]$ trips/h/km$^2$; peak-period duration, $t_p = 4$ h; off-peak duration, $t_o = 14$ h; and $\lambda_p = 2.5\lambda$. A low walking speed, $v_w = 2$ km/h is used to account for delays at street junctions and for the inconvenience of walking (Daganzo, 2010a). A value of time $\mu = 5$ \$/h is used for low-wage cities, and $\mu = 25$ \$/h for high-wage ones.

[6] We solve the program for feeder-bus access by first ignoring the integer constraint (9e). If in the solution $\frac{S}{S_f} = \kappa$ is not an integer, we specify that $\frac{S}{S_f}$ equals each of $\kappa$'s two neighboring integers; separately solve the programs for both integer neighbors; and take the lower-cost solution to be optimal (see Chen and Nie, 2017a; Fan et al., 2018; Nourbakhsh and Ouyang, 2012).

Cost and operating parameters for ordinary buses, BRT and metro rail trunk-line systems are furnished in Table 1. These are borrowed from Daganzo (2010a), Gu et al. (2016), Sivakumaran et al. (2014) and Fan et al. (2018). The $C_{VT}$, $C_I$, and $C_S$ are formulated as linear functions of wage rate, $\mu$, to capture labor costs; see Gu et al. (2016) for details.

**Table 1** Operating and cost parameters for three transit technologies: bus, BRT, and rail.

| | Operating Parameters | | | | | | Cost Parameters | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Operating cost rates | | Infrastructure cost rates | |
| | $\tau$ (s) | $t_f$ (s) | $v$ (km/h) | $\delta$ (h) | $K$ (passenger/veh) | $H_{min}$ (min) | $C_{VD}$ ($/km) | $C_{VT}$ ($/h) | $C_I$ ($/h/km) | $C_S$ ($/h/station) |
| Bus (including feeder bus) | 30 | 30 | 25 | 0.015 | 80 | 1 | 0.59 | $2.66 + 3\mu$ | $6 + 0.2\mu$ | $0.42 + 0.014\mu$ |
| BRT | 30 | 30 | 40 | 0.015 | 160 | 1 | 0.66 | $3.81 + 4\mu$ | $162 + 5.4\mu$ | $4.2 + 0.14\mu$ |
| Rail | 45 | 60 | 60 | 0.1 | 3000 | 1.5 | 2.20 | $101 + 5\mu$ | $594 + 19.8\mu$ | $294 + 9.8\mu$ |

Values for $C_B$, $C_D$ and $C_P$ are furnished in Table 2 for bike-sharing systems in low- and high-wage cities. These cost rates are derived in Appendix G. Table 2 also presents the values used for $t_{dp}$, $t_{dd}$, $v_b$, and $\xi$, along with two values for $\beta$ to represent walk- and bike-friendly cities, and two values for $\rho$ to reflect low and high bike utilizations. All values were taken from the literature, as cited in the table.

**Table 2** Operating and cost parameters for bike-sharing systems.

| Operating Parameters | | | | | | | |
|---|---|---|---|---|---|---|---|
| $t_{dp}$ | $t_{dd}$ | $v_b$ | $\xi$[#] | $\beta$[※] | | $\rho$[§] | |
| (s) | (s) | (km/h) | | Low (walk-friendly) | High (bike-friendly) | Low bike utilization | High bike utilization |
| 30 | 30 | 12 | 1.5 | 0.3 | 0.5 | 0.3 | 0.5 |

| Cost Parameters | | | | | |
|---|---|---|---|---|---|
| $C_B$ ($/bike/day) | | $C_D$ ($/dock/day) | | $C_P$ ($/station/day) | |
| Low-wage | High-wage | Low-wage | High-wage | Low-wage | High-wage |
| 0.44 | 2.58 | 0.08 | 0.65 | 9.36 | 19.10 |

[#] This value is taken from Gleason and Miskimins (2012).
[※] The two values of $\beta$ are selected according to the bicycle ownership and mode share data in Gunn (2018) and Oke et al. (2015).
[§] The two values of $\rho$ are selected conservatively by referring to the empirical data of bike usage found in Hampshire and Marla (2012), Lane (2015) and Suzuki and Nakamura (2017).

We devised a large set of case studies by endowing cities with the eight possible combinations of $\mu$, $\beta$ and $\rho$. Each of these eight city types was separately served by ordinary buses, BRT and metro rail. In separate analyses, the first of these trunk-line systems was accessed and egressed by walking (only) and by riding shared bikes. The latter two trunk-line systems were separately fed by all three options (walking, riding bikes and riding feeder buses). These 64 combinations were separately examined under ranges of $D$ and $\lambda$.

### *3.2. Pre-existing transit service*

We explore whether shared-bikes or feeder-buses can reduce the costs of existing transit systems. Trunk-line networks were optimized to serve access on foot; i.e., the $S$, $\alpha$, and $H_k$ $(k = p, o)$ were obtained by solving (1). The resulting lines and stations are assumed to be immovable. This gives foot-access an advantage when drawing comparisons against the two other feeder options. For the shared-bike option, the density of small docking stations, $P$, is optimized by solving (4) when $S$, $\alpha$, and $H_k$ are fixed and determined by (1). The $S_f$, $\alpha$ and $H_{f,k}$ ($k = p, o$) are optimized for feeder-bus service using (9) in similar fashion.

Consider first cities with a high wage of $\mu = 25$ \$/h, and that are more favorable to walking than to biking, such that $\beta = \rho = 0.3$. We find that shared-bikes can reduce generalized costs in most cases when transit service is provided by ordinary buses, save for those where $D$ and $\lambda$ are both high. The contour lines in Fig. 4a show the percent reductions in costs for wide ranges of $D$ and $\lambda$. Given our choices for $\beta$ and $\rho$, the savings are modest and never reach 3%. They diminish as $D$ or $\lambda$ increases. This is because the transit vehicle capacity constraint (1b) is binding, and thus the optimal line and stop spacing, $S$, decreases as $D$ or $\lambda$ grows. The shorter-spaced transit stations are better accessed via walking; see the top-right corner demarcated by the boldface contour line in the figure.

Shared-bike access was also found to produce lowest costs for a greater range of $D$ and $\lambda$ when trunk-line services in these cities were provided by BRT. Fig. 4b shows that shared bikes result in lower costs than does walk-only access, save for the top-right corner, which is again demarcated by a boldface contour line. These savings are slightly greater than those when trunk-line transit service is provided by buses, because a BRT network has greater line and stop spacings than does a bus network under the same demand level, which favors access by bike.[7]

Shared-bike access is invariably the lowest-cost means when trunk-line service is provided by rail. Thanks to the large line and station spacings required of rail, the cost savings brought by adding bikes can exceed 6%; see Fig. 4c. In all three figures, feeder-bus access is never the lower-cost option.

Shared bikes can produce greater cost savings in low-wage cities. This becomes clear by visually comparing Fig. 5a with Fig. 4b. Savings grow to over 7% when circumstances are friendlier to cycling; i.e., under higher values of $\beta$ and $\rho$. This becomes clear by comparing Fig. 5b with Fig. 4b.

### *3.3. Systems designed from the scratch*

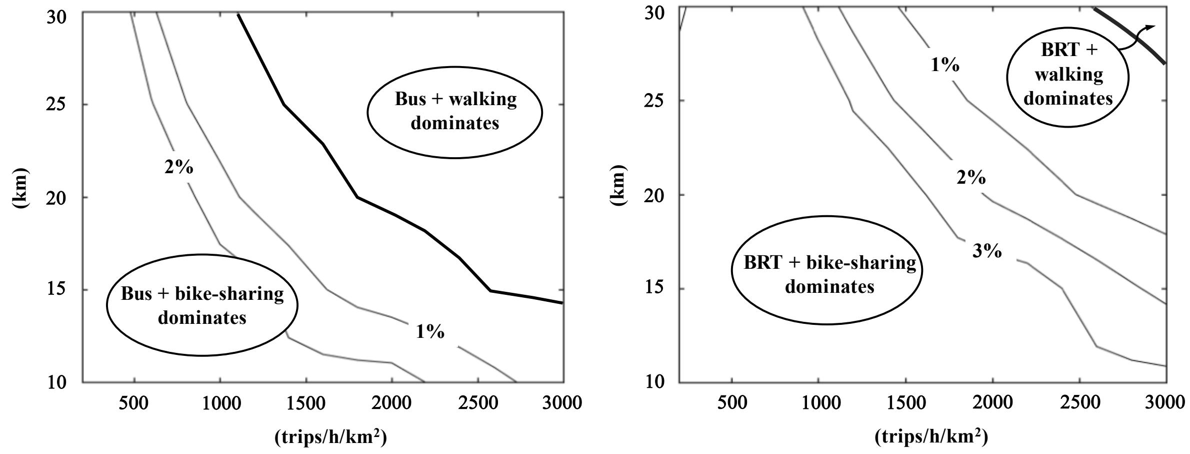
Generalized costs diminished when trunk and feeder systems were optimally designed in joint fashion. When trunk services were provided by ordinary buses and BRT, shared bikes continued to be a lower-cost feeder option than walking for the majority of the $D$ and $\lambda$ examined. The cost savings were slightly greater as compared against those estimated in section 3.2. This was true for both low- and high-wage cities. Of note, in jointly-optimized designs, substantial savings were often achieved in the agency cost of transit service due to the increased transit line and station spacings. The transit agency cost savings can offset a large portion (and sometimes all) of the added agency cost for providing the bike-sharing service.

When trunk services were instead provided by rail, however, feeder buses become the lowest-cost option in most cases studied. Shared-bike access wins only when $D$ and $\lambda$ are both small; see Fig. 6. Higher-speed buses better suit the larger line and station spacings that metro-rail engenders. And economies of trip density are enjoyed by focusing higher demands onto those buses. Cost savings were substantial relative to access by walking; e.g. differences reached 20% for large $D$. As a practical matter, however, bike sharing might still be judged a preferred feeder option, since it imparts a lower cost to transit agencies.

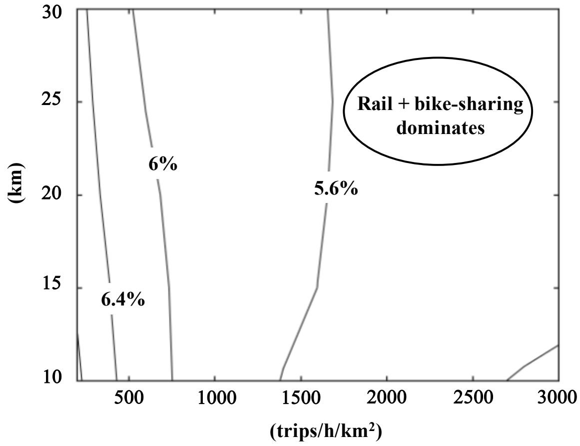
We think it of further interest to examine how $D$ and $\lambda$ affect lowest-cost designs when all 8 combinations of trunk and feeder options are in play. To this end, Fig. 7a and 7b show outcomes for low- and high-wage cities that are not especially favorable to cycling; i.e., we set $\beta = \rho = 0.3$. The

[7] Before the capacity constraint becomes binding, the cost saving would increase with $D$. This is because longer trips in big cities require larger spacings between BRT lines and stations, which are better accessed by fast-moving bikes. This was observed when $D \leq 10$ km, which is not shown in Fig. 4b.

contour lines demarcate cases in which certain trunk-feeder combinations produced the lowest generalized costs among all 8 options.

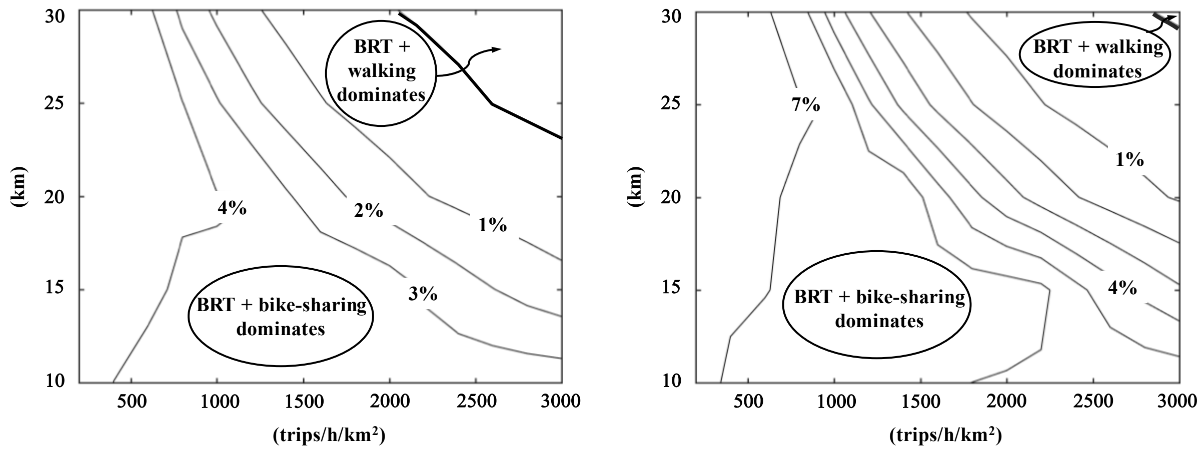



(a) pre-existing ordinary bus networks (b) pre-existing BRT networks




(c) pre-existing metro-rail networks

**Fig. 4.** Percentage savings in generalized costs by adding share-bikes or feeder-buses to fit pre-existing transit networks in high-wage, walk-friendly cities with low bike utilization ($\mu = 25$ \$/h, $\beta = \rho = 0.3$).




(a) low-wage, walk-friendly cities with low bike utilization ($\mu = 5$ \$/h, $\beta = \rho = 0.3$) (b) high-wage, bike-friendly cities with high bike utilization ($\mu = 25$ \$/h, $\beta = \rho = 0.5$)

**Fig. 5.** Percentage savings in generalized costs by adding shared-bikes to feed existing BRT networks.

Tellingly, access by walking is never the winner. Nor are rail and shared-bike combinations preferred. Fig. 7a and 7b show instead that ordinary buses and BRT fed by shared bikes are the low-cost options in smaller, less-populated cities. In large cities with high demand density, metro-rail fed by buses produces the lowest cost. BRT and rail as trunk options are more favorable in rich cities where patrons place a higher premium on their time.

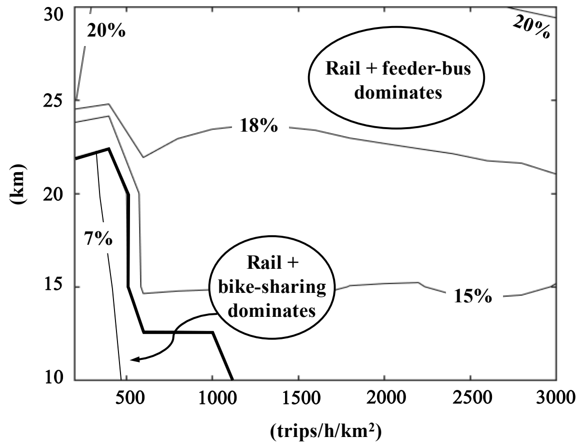
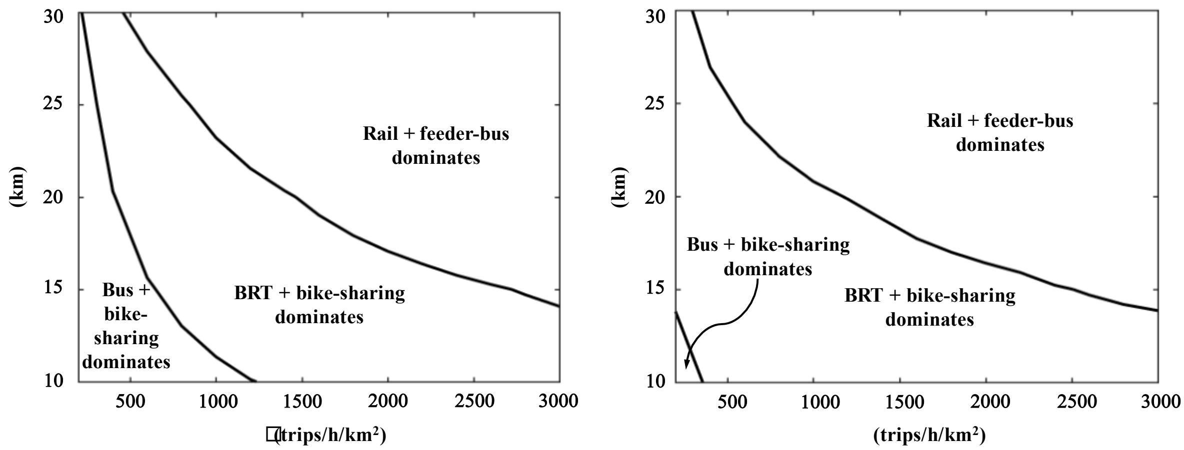



**Fig. 6.** Percentage savings in generalized costs for jointly-optimized metro-rail systems in high-wage, walk-friendly cities with low bike utilization ($\mu = 25$ \$/h, $\beta = \rho = 0.3$).




(a) low-wage cities with $\mu = 5$\$/h (b) high-wage cities with $\mu = 25$\$/h

**Fig. 7.** Lowest-cost designs for walk-friendly cities with low bike utilization ($\beta = \rho = 0.3$).

### *3.4. Break-even fee schemes for the bike-sharing system*

This section explores how revenues generated by bike-sharing feeder systems can match their costs under system-optimal conditions. To look for insights, we continue to assume that travel demand, $\lambda$, is exogenous to features of our trunk and feeder services; and focus on two piecewise-linear distance-based fee rates in (D4) of Appendix D. These rates are $\gamma_p$ and $\gamma_o$ (\$/km) for peak and off-peak periods, respectively. We assume that transit trunk and bike-sharing feeder systems are jointly optimized, and examine cases in which the cost savings brought by bike sharing are, and are not, used to subsidize feeder service. In both cases, bike-rental revenue, $R$ (\$/day/km$^2$), is calculated as:

$$R = \beta\left(\lambda_p t_p \oiint_{d\sigma \in A_{b,p}} \varphi_p\left(d;\gamma_p\right) d\sigma + \lambda_o t_o \oiint_{d\sigma \in A_{b,o}} \varphi_o(d;\gamma_o) d\sigma\right), \tag{14}$$

where: the two surface integrals in parentheses are integrated over the cycling regions, $A_{b,p}$ and $A_{b,o}$ (see Appendix E), and $\varphi_p\left(d;\gamma_p\right)$ and $\varphi_o(d;\gamma_o)$ are given by (D4).

In the absence of subsidies, the system-optimal range of $(\gamma_p, \gamma_o)$ is given by $0 \leq \gamma_p, \gamma_o < \mu\left(\frac{1}{v_w} - \frac{1}{v_b}\right)$; see Appendix D. This range is plotted as a rectangle in Fig. 8 for a low-wage city with $\mu = 5$ \$/h that is small ($D = 10$ km) and walk-friendly ($\beta = \rho = 0.3$), and has trunk service provided by BRT. The break-even fee schemes are plotted as dashed, solid and dotted lines for the $\lambda$ shown in the figure's legend. For a given $\lambda$, the agency can presumably reap a profit by pricing the bike-sharing service at a point above the corresponding break-even line, and suffer a loss by setting the fee below that line.

Interestingly, the figure shows that break-even fees increase as $\lambda$ grows. This is because optimal spacings between trunk stations decrease under larger $\lambda$, and the mode share for bikes diminishes owing to the smaller access and egress distances. Similarly, since trunk-station spacings increase with diminishing $\mu$, break-even fees decrease accordingly. In contrast, break-even fees for small, high-wage cities could be unacceptably high, especially in walk-friendly cities with small $\beta$ and $\rho$ and in those with large $\lambda$. Figures for these results are not shown for brevity.

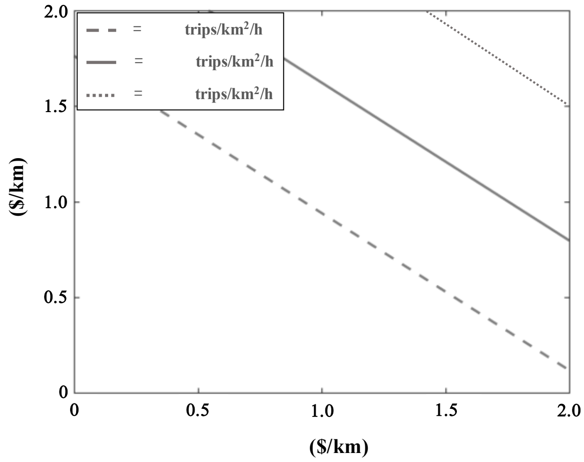



**Fig. 8.** Feasible ranges of break-even bike-rental fees for BRT trunk-feeder systems in a low-wage, small, walk-friendly city with low bike utilization ($\mu = 5$ \$/h, $D = 10$ km, $\beta = \rho = 0.3$)

Break-even fees diminish in the presence of subsidies. In these cases, we find that the entire system-optimal range of $(\gamma_p, \gamma_o)$ is profitable in nearly all cases studied. Bike-rental fees can thus be set as low as a fixed rate $\varphi_p(d) = \frac{c_B + \xi c_D}{\rho t_p}\left(\frac{d_{cp}}{v_b} + t_{dp} + t_{dd}\right)$ during the peak periods, and $\varphi_o(d) = 0$ during the off-peak. (The former entails only \$0.023 per bike trip in a low-wage, walk-friendly, small city with low bike utilization, $D = 10$km and $\lambda = 2000$ trips/h/km$^2$.) Those lowest fees are obtained by setting $\gamma_p = \gamma_o = 0$ in (D4), though details of this are omitted from the present paper again in the interest of brevity.

## 4. Conclusions

A battery of tests has revealed that shared bikes can be a cost-effective means to access or egress public transit. When designed to fit pre-existing transit networks, shared-bike feeder systems reduced generalized costs by as much as 7% over networks accessed on foot. Bike sharing turned out to be the lowest-cost feeder option for the lion's share of cases studied. The walk-only feeder option won-out only in large-sized cities with high travel demands that were served by ordinary buses or BRT, even though the pre-existing transit networks were designed to suit access and egress on foot. The feeder-bus option never attained the lowest cost among the cases examined.

Not surprisingly, greater benefits could be achieved by optimizing trunk and feeder systems jointly. In these cases, shared bikes continued to be the lowest-cost feeder option for ordinary bus systems, and for BRT systems, in small-sized or low-demand cities. Feeder buses were winners in large-sized or high-demand cities with metro-rail systems. It bears noting that shared-bike feeder service might still be preferable in these latter cases, as it often imposed lower costs to the transit agency than did feeder buses. Savings in transit-agency cost could be used to subsidize shared-bike systems. In practice, subsidies could be achieved by discounting transit fares to shared-bike riders.

The numerical results presented in this paper are limited, partly due to the large number of parameters in our models. Still, we believe that the experiments are sufficient to show that accessing transit via shared bikes is often worth considering. For a specific city, the applicable scope and benefits of a transit network fed by shared bikes can be derived from our model, should key parameter values (e.g., the intermodal transfer penalty $t_f$) of that city be available.

Of further note, the present findings came by means of simplified models for idealized cases. In some instances, the simplifications are conservative; e.g. recall that some short-distance commuters might enjoy greater cost savings by using shared bikes to cover their entire trips. This might justify trunk designs with larger line and station spacings (to serve longer-distance trips). The benefits might trigger favorable modal shifts, which could benefit transit, and was likewise not considered in the present study. New models are presently being developed to explore optimal system designs under more realistic operating scenarios that account for mode choice and spatially heterogeneous demand. Consideration of demand heterogeneity may entail the use of continuum approximation techniques (e.g. Chien and Schonfeld, 1997; Ouyang et al., 2014). In addition, our modeling framework can be tuned to analyze transit systems fed by other access modes, e.g., scooters, e-bikes, and autonomous cars. Work in this regard is also underway.

## Acknowledgments

The research was supported by General Research Funds (Project No.: **15217415** and 15280116) provided by the Research Grants Council of Hong Kong, a startup fund provided by the Hong Kong Polytechnic University (Project ID: **P0001008**), and by the National Natural Science Foundation of China (Project No. 51608455). The authors are indebted to Professor Carlos F. Daganzo, University of California, Berkeley, for his valuable insights in the early stages of this study. We also thank Mr. Shize Xiao, Tsinghua University, Beijing, China, for assembling the cost data of shared bikes.

## Appendix A. Table of notation

| **Notation** | **Description** | **Unit** |
|---|---|---|
| **Decision variables** | | |
| $\alpha$ | Ratio between the sides of central square and the city in the hybrid transit network | - |
| $P$ | Density of small bike docking stations | station/km$^2$ |
| $H_k$ | Trunk-line transit headway in period $k$, $k \in \{p, o\}$ | h |
| $H_{f,k}$ | Feeder-bus headway in period $k$, $k \in \{p, o\}$ | h |
| $S$ | Trunk-line transit station spacing | km |
| $S_f$ | Feeder-bus line and stop spacing | km |
| **Other variables and parameters** | | |
| $\beta$ | Percentage of able-bodied persons in the city's transit patrons | - |
| $\rho$ | Bike utilization ratio during peak periods | - |
| $\mu$ | Patrons' value of time | \$/h |
| $t_k$ | Peak/off-peak period duration of a day, $k \in \{p, o\}$ | h |
| $\lambda$ | Average demand density during the service hours of a day | trips/h/km$^2$ |

| Notation | Description | Unit |
|---|---|---|
| $D$ | Length (and width) of the square city | km |
| $v$ | Trunk-line transit vehicle cruise speed | km/h |
| $v_f$ | Feeder-bus cruise speed | km/h |
| $v_w$ | Walking speed | km/h |
| $v_b$ | Cycling speed | km/h |
| $\delta$ | Equivalent walking time for a transfer between two perpendicular trunk transit lines | h |
| $t_f$ | Intermodal transfer penalty | h |
| $\tau_f$ | Feeder-bus dwell time per stop | h |
| $\tau$ | Trunk-line transit vehicle dwell time per station | h |
| $t_{dp}$ | Bike pick-up delay at the origin docking station | h |
| $t_{dd}$ | Bike drop-off delay at the destination docking station | h |
| $C_I$ | Amortized hourly cost rate of trunk-line transit infrastructure | $/h/km |
| $C_S$ | Amortized hourly cost rate of trunk-line transit station | $/h/station |
| $C_{VD}$ | Distance-based operating cost rate of trunk-line transit | $/vehicle-km |
| $C_{VT}$ | Time-based operating cost rate of trunk-line transit | $/vehicle-h |
| $C_{fI}$ | Amortized hourly cost rate of feeder-bus line infrastructure | $/h/km |
| $C_{fS}$ | Amortized hourly cost rate of a feeder-bus stop | $/h/stop |
| $C_{fVD}$ | Distance-based operating cost rate of feeder bus | $/bus-km |
| $C_{fVT}$ | Time-based operating cost rate of feeder bus | $/bus-h |
| $C_B$ | Daily cost per bike | $/bike/day |
| $C_D$ | Daily cost per dock | $/dock/day |
| $C_P$ | Daily cost per docking station | $/station/day |
| $n_B$ | Average bike hours used per patron during peak periods | h |
| $\xi$ | Ratio between the numbers of docks and bikes | - |
| $UC_{W,k}$ | Average patron cost per trip in period $k$ for access solely by walking, $k \in \{p, o\}$ | h |
| $UC_{B,k}$ | Average patron cost per trip in period $k$ for access by cycling and walking, $k \in \{p, o\}$ | h |
| $UC_{F,k}$ | Average patron cost per trip in period $k$ for access by feeder buses and walking, $k \in \{p, o\}$ | h |
| $E_{T,k}$ | Sum of average wait, in-vehicle travel time and transfer penalty per trip in period $k$, $k \in \{p, o\}$ | h |
| $t_w$ | Average walking time to the nearest bike docking station for each cycling trip | h |
| $t_r$ | Average in-vehicle travel time by feeder bus for each feeder-bus trip | h |
| $E_{B,k}$ | Average access and egress time per trip via bike in period $k$, $k \in \{p, o\}$ | h |
| $E_{F,k}$ | Average access and egress time per trip via feeder bus in period $k$, $k \in \{p, o\}$ | h |
| $AC_k$ | Average trunk-line transit agency cost per trip in period $k$, $k \in \{p, o\}$ | h |
| $AC_B$ | Average bike-sharing agency cost per trip | h |
| $AC_{F,k}$ | Average feeder-bus agency cost per trip in period $k$, $k \in \{p, o\}$ | h |
| $K$ | Trunk-line transit vehicle's passenger-carrying capacity | passenger/vehicle |
| $K_f$ | Feeder bus's passenger-carrying capacity | passenger/bus |
| $MC_{B-W}$ | Marginal generalized cost when a patron switches from walking to cycling at one end of her trip | h |
| $MAC_B$ | Marginal bike-sharing agency cost when a patron switches from walking to cycling at one end of her trip | $ |
| $MC_{F-W}$ | Marginal generalized cost when a patron switches from walking to feeder bus at one end of her trip | h |
| $MAC_F$ | Marginal feeder-bus agency cost when a patron switches from walking to feeder bus at one end of her trip | $ |

| Notation | Description | Unit |
|---|---|---|
| $d_w$ | Average distance to the nearest bike docking station | km |
| $d_{ck}$ | Critical distance between walking and cycling in period $k$, $k \in \{p, o\}$ | km |
| $d_{ck1}$ | Critical distance between walking and taking feeder bus in the non-stop direction in period $k$, $k \in \{p, o\}$ | km |
| $d_{ck2}$ | Critical distance between walking and taking feeder bus in the passenger-collection direction in period $k$, $k \in \{p, o\}$ | km |
| $\varphi_k$ | Distance-based bike rental fee in period $k$, $k \in \{p, o\}$ | $ |
| $\gamma_k$ | Bike rental rate in period $k$, $k \in \{p, o\}$ | $/km |
| $A_{fw,k}$ | Area of the walk-only region in period $k$ for access by feeder buses and walking, $k \in \{p, o\}$ | km$^2$ |
| $A_{bw,k}$ | Area of the walk-only region in period $k$ for access by cycling and walking, $k \in \{p, o\}$ | km$^2$ |
| $d_{bin,k}$ | Average access distance in the walk-only region in period $k$ for access by cycling and walking, $k \in \{p, o\}$ | km |
| $d_{bout,k}$ | Average access distance in the cycling region in period $k$, $k \in \{p, o\}$ | km |
| $d_{fin,k}$ | Average access distance in the walk-only region in period $k$ for access by feeder buses and walking, $k \in \{p, o\}$ | km |
| $d_{fout1,k}$ | Average access distance in the non-stop direction in the feeder-bus region in period $k$, $k \in \{p, o\}$ | km |
| $d_{fout2,k}$ | Average access distance in the passenger-collection direction in the feeder-bus region in period $k$, $k \in \{p, o\}$ | km |
| $R$ | Bike rental revenue | $/day/km$^2$ |

## Appendix B. Proof of Proposition 1

First note that when small docking stations are placed at the grid points of a diamond-grid layout, a station's catchment zone has a diamond shape as shown in Fig. B1. In this case, the average access distance is equal to the $L_1$-distance (Manhattan distance) between the docking station and the centroid of the shaded triangle (i.e. a quarter of the catchment zone) in the figure. One can easily verify that this average access distance is $\sqrt{\frac{2}{9P}}$. (Recall that the streets are parallel to the city's boundaries.)

To see why this diamond grid layout is optimal, note in Fig. B2 that under this layout the boundary of a station's catchment zone are the isodistance lines in the $L_1$-metric for the distance $\sqrt{\frac{1}{2P}}$. Thus, a catchment zone of the same area ($\frac{1}{P}$ km$^2$) but with a different shape (see the one enclosed by a solid black boundary in the figure) will always have a larger average access distance to the docking station. Note that the cross-hatched parts in Fig. B2, which belong to an arbitrary-shaped catchment zone but not to the diamond zone, are located outside of the isodistance lines, and thus have an average access distance greater than $\sqrt{\frac{1}{2P}}$. In contrast, the linear-hatched parts, which belong to the diamond zone but not to the arbitrary-shaped one, have an average access distance less than $\sqrt{\frac{1}{2P}}$. ■

## Appendix C. Proof of Proposition 2

Consider an able-bodied patron whose access distance is $d$ ($0 \le d \le S$). The marginal generalized cost to the system when the patron switches from walking to cycling is:

$$MC_{B-W} = \left(\frac{d}{v_b} + t_w + t_{dp} + t_{dd} + t_f + \frac{MAC_B}{\mu}\right) - \frac{d}{v_w}, \tag{C1}$$

where $\frac{d}{v_b} + t_w + t_{dp} + t_{dd} + t_f$ is the patron's access time by bike; and $MAC_B$ denotes the marginal bike-sharing agency cost (in \$) for serving this additional patron. The $MAC_B$ is formulated as:

$$MAC_B = \begin{cases} (C_B + \xi C_D)\frac{1}{\rho \cdot t_p}\left(\frac{d}{v_b} + t_{dp} + t_{dd}\right), & \text{for a peak-period patron} \\ 0, & \text{for an off-peak-period patron.} \end{cases} \tag{C2}$$

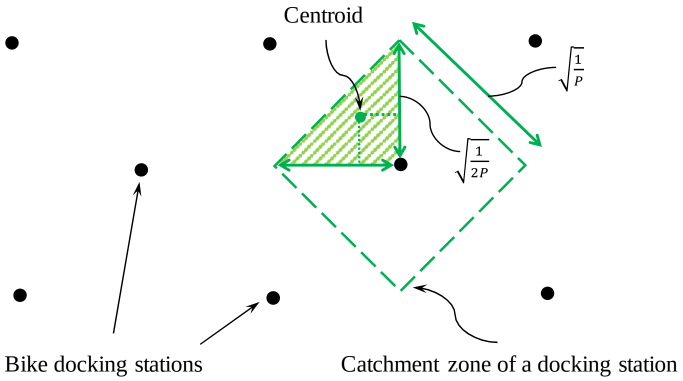



**Fig. B1.** A diamond grid network of bike docking stations.

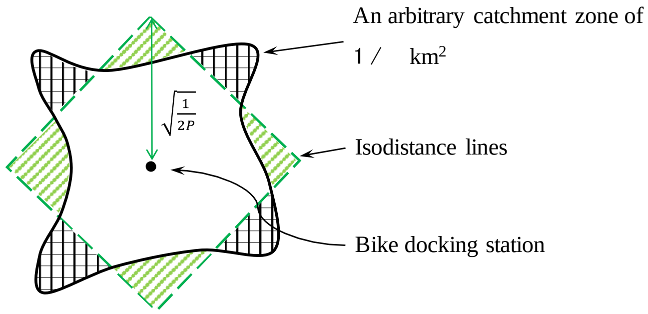



**Fig. B2.** An arbitrary catchment zone versus a diamond zone with the same area.

Recall that $\rho$ is the bike utilization ratio during peak periods. Thus, $\frac{1}{\rho \cdot t_p}\left(\frac{d}{v_b} + t_{dp} + t_{dd}\right)$ is the number (fraction) of bikes the additional cycling patron occupies during peak hours. The marginal bike-sharing agency cost is zero during off-peak hours because there are always redundant bikes.

Note now that $MC_{B-W}$ is a linear function of $d$. Thus, the equation $MC_{B-W} = 0$ has a unique solution of $d$ for each $k \in \{p, o\}$. We denote this solution as $d_{ck}$ (the critical distance) for period $k$, and have:

(i) if $d < d_{ck}$, then $MC_{B-W} > 0$, and thus the able-bodied patron will choose to walk to or from the nearest transit station;

(ii) if $d > d_{ck}$, then $MC_{B-W} < 0$, and the able-bodied patron will choose to rent a bike; and

(iii) if $d = d_{ck}$, then $MC_{B-W} = 0$, and the able-bodied patron is indifferent between walking and riding a shared bike. ■

## Appendix D. Proof of Proposition 3

Equations (6a) and (6b) reveal that an able-bodied patron in period $k \in \{p, o\}$ with access distance $d$ will choose walking if $u_{Bk}(d) - u_{Wk}(d) = \left(\frac{d}{v_b} + t_w + t_{dp} + t_{dd} + t_f + \frac{\varphi_k(d)}{\mu}\right) - \frac{d}{v_w} > 0$, and will choose cycling otherwise. From Proposition 2, we know the following conditions should be satisfied for system-optimal pricing:

$$\begin{cases} \frac{d}{v_w} < \left(\frac{d}{v_b} + t_w + t_{dp} + t_{dd} + t_f\right) + \frac{\varphi_k(d)}{\mu}, & if\ 0 \le d < d_{ck} \\ \frac{d_{ck}}{v_w} = \left(\frac{d_{ck}}{v_b} + t_w + t_{dp} + t_{dd} + t_f\right) + \frac{\varphi_k(d_{ck})}{\mu}, & if\ d = d_{ck} \qquad k \in \{p, o\}. \\ \frac{d}{v_w} > \left(\frac{d}{v_b} + t_w + t_{dp} + t_{dd} + t_f\right) + \frac{\varphi_k(d)}{\mu}, & if\ d_{ck} < d \le S \end{cases} \tag{D1}$$

Mathematically, the middle equation of (D1) does not need to hold for a system-optimal pricing scheme; i.e., at the critical distance, a patron can choose either walking or cycling, and the costs of the two access modes do not have to be equal. However, we keep this equation for the simplicity of derivation. Since $d_{ck}$ is the root of $MC_{B-W} = 0$ (see Appendix C), we have:

$$\varphi_k(d_{ck}) = MAC_B = \begin{cases} \frac{C_B + \xi C_D}{\rho t_p}\left(\frac{d_{ck}}{v_b} + t_{dp} + t_{dd}\right), & \text{for } k = p \\ 0, & \text{for } k = o. \end{cases} \tag{D2}$$

By subtracting the middle equation of (D1) from the first and third inequalities of (D1), we have:

$$\begin{cases} \frac{d - d_{ck}}{v_w} < \frac{d - d_{ck}}{v_b} + \frac{\varphi_k(d) - \varphi(d_{ck})}{\mu} & \text{if } 0 \le d < d_{ck} \\ \frac{d - d_{ck}}{v_w} > \frac{d - d_{ck}}{v_b} + \frac{\varphi_k(d) - \varphi(d_{ck})}{\mu} & \text{if } d_{ck} < d \le S. \end{cases} \tag{D3}$$

We only need to show that there exists $\varphi_k(d)$ for period $k \in \{p, o\}$ that satisfies (D2) and (D3). In addition, a feasible fee scheme, $\varphi_k(d)$, should generally be: (i) non-negative for all the $d \in [0, S]$; and (ii) non-decreasing as $d$ increases. To show the existence of a feasible system-optimal fee scheme, we consider a special case: a scheme where the fee increases linearly with the distance traveled. This linear fee scheme is expressed by $\varphi_k(d) - \varphi_k(d_{ck}) = \gamma_k(d - d_{ck})$ $(k \in \{p, o\})$, where $\gamma_k$ is a non-negative constant rate for period $k$, and $\varphi_k(d_{ck})$ is given by (D2). This linear fee scheme is non-decreasing as $d$ grows and satisfies (D3) if $0 \le \gamma_k < \mu\left(\frac{1}{v_w} - \frac{1}{v_b}\right)$. To ensure $\varphi_k(d)$ is non-negative for all the $d \in [0, S]$, we modify the definition of $\varphi_k(d)$ to the following:

$$\varphi_k(d) = \max\{0, \gamma_k(d - d_{ck}) + \varphi_k(d_{ck})\},\ 0 \le \gamma_k < \mu\left(\frac{1}{v_w} - \frac{1}{v_b}\right),\ k \in \{p, o\}. \tag{D4}$$

The above modification will not alter any patron's access choice, because a negative $\gamma_k(d - d_{ck}) + \varphi_k(d_{ck})$ may occur only when $d < d_{ck}$ (i.e. in the walk-only region). ■

## Appendix E. Derivation of $d_{ck}$, $E_{B,k}$ $(k \in \{p, o\})$ and $n_B$

We first derive the critical distance, $d_{ck}$ $(k \in \{p, o\})$, by solving the equation $MC_{B-W} = 0$. From (C1) and (C2), we know that for off-peak periods:

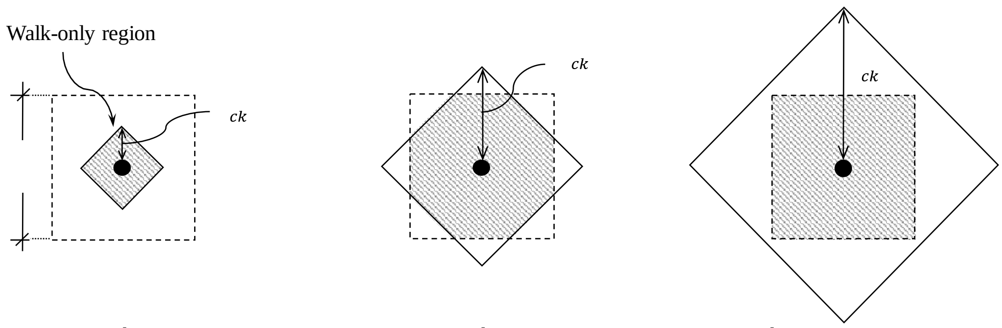
$$d_{co} \equiv \frac{\frac{1}{v_w}\sqrt{\frac{2}{9P}} + t_{dp} + t_{dd} + t_f}{\frac{1}{v_w} - \frac{1}{v_b}}; \tag{E1}$$

and for peak periods when $\frac{1}{v_w} > \frac{1}{v_b} + \frac{C_B + \xi C_D}{\mu \rho t_p v_b}$:

$$d_{cp} = \frac{\frac{C_B + \xi C_D}{\mu \rho t_p}(t_{dp} + t_{dd}) + \frac{1}{v_w}\sqrt{\frac{2}{9P}} + t_{dp} + t_{dd} + t_f}{\frac{1}{v_w} - \frac{1}{v_b} - \frac{C_B + \xi C_D}{\mu \rho t_p v_b}}. \tag{E2}$$

For peak periods when $\frac{1}{v_w} \le \frac{1}{v_b} + \frac{C_B + \xi C_D}{\mu \rho t_p v_b}$, $MC_{B-W} > 0$ for all non-negative values of $d$. In this case, we set $d_{cp} = S$.

In each period (peak or off-peak), the isodistance lines at $d_{ck}$ divide the catchment zone of a transit station into the walk-only region ($d \le d_{ck}$) and the cycling region ($d > d_{ck}$). Only the able-bodied patrons originating in or destined for the cycling region will access or egress transit via shared bikes. The rest of the patrons will choose walking. Depending on the value of $\frac{d_{ck}}{S}$, the walk-only region can take one of the three shapes as shown in Fig. E1a-c. In each figure: the black dot represents the transit station; the solid lines are the isodistance lines at $d_{ck}$; the dashed lines are the boundary of the catchment zone; and the walk-only region is marked by shading.




(a) when $\frac{d_{ck}}{S} \le \frac{1}{2}$ (b) when $\frac{1}{2} < \frac{d_{ck}}{S} < 1$ (c) when $\frac{d_{ck}}{S} \ge 1$

**Fig. E1.** The walk-only region in the catchment zone of a transit station in period $k \in \{p, o\}$.

For each of the three cases shown in Fig. E1a-c, we define $A_{bw,k}$ as the area of the walk-only region in period $k$, $d_{bin,k}$ as the average access distance for the origins and destinations in the walk-only region, and $d_{bout,k}$ as the average access distance in the cycling region in period $k$. The $E_{B,k}$ can be calculated by averaging the access and egress costs for the walkers and the cyclists:

$$E_{B,k} = (1-\beta)\frac{S}{v_w} + 2\beta\left(\frac{A_{bw,k}}{S^2}\cdot\frac{d_{bin,k}}{v_w} + \left(1-\frac{A_{bw,k}}{S^2}\right)\left(\frac{d_{bout,k}}{v_b} + t_w + t_{dp} + t_{dd} + t_f\right)\right), k \in \{p, o\} \quad \text{(E3)}$$

The average bike-hours used per peak-period patron, $n_B$, is calculated as follows:

$$n_B = 2\beta\left(1-\frac{A_{bw,k}}{S^2}\right)\left(\frac{d_{bout,k}}{v_b} + t_{dp} + t_{dd}\right). \quad \text{(E4)}$$

The $A_{bw,k}$ and $d_{bin,k}$ ($k \in \{p, o\}$) are developed for each case in Fig. E1a-c as follows:

(i) When $\frac{d_{ck}}{S} \le \frac{1}{2}$, the walk-only region has a diamond shape (see Fig. E1a). Thus we have:

$A_{bw,k} = 2d_{ck}^2$;

$$d_{bin,k} = \frac{4\int_0^{d_{ck}} dx \int_0^{d_{ck}-x}(x+y)dy}{A_{bw,k}} = \frac{2}{3}d_{ck}.$$

(ii) When $\frac{1}{2} < \frac{d_{ck}}{S} < 1$, the walk-only region is an octagon (see Fig. E1b). By geometry, we have:

$$A_{bw,k} = 2d_{ck}^2 - 4\left(d_{ck} - \frac{S}{2}\right)^2;$$

$$d_{bin,k} = \frac{\frac{2}{3}d_{ck}\cdot 2d_{ck}^2 - 8\int_0^{d_{ck}-\frac{S}{2}} dx \int_{\frac{S}{2}}^{d_{ck}-x}(x+y)dy}{A_{bw,k}} = \frac{\frac{2}{3}d_{ck}\cdot 2d_{ck}^2 - 8\left(\frac{2}{3}d_{ck} + \frac{S}{6}\right)\cdot\frac{1}{2}\left(d_{ck} - \frac{S}{2}\right)^2}{A_{bw,k}};$$

(iii) When $\frac{d_{ck}}{S} \ge 1$, the walk-only region fills up the entire catchment zone (see Fig. E1c). Thus, $A_{bw,k} = S^2$; $d_{bin,k} = \frac{S}{2}$.

In all the three cases, $d_{bout,k} = \frac{\frac{S}{2} \cdot S^2 - d_{bin,k} \cdot A_{bw,k}}{S^2 - A_{bw,k}}$. Specifically, in case (iii), $d_{bout,k} = 0$.

## Appendix F. Derivation of $A_{fw,k}$ and $E_{F,k}$ ($k \in \{p, o\}$)

We again consider a patron whose access distance is $d$ ($0 \le d \le S$). The marginal generalized cost incurred to the system when the patron switches from walking to riding a feeder bus is:

$$MC_{F-W} = \left(\frac{s_f}{2v_w} + \frac{H_{f,k}}{2} + t_r + t_f + \frac{MAC_F}{\mu}\right) - \frac{d}{v_w}, \tag{F1}$$

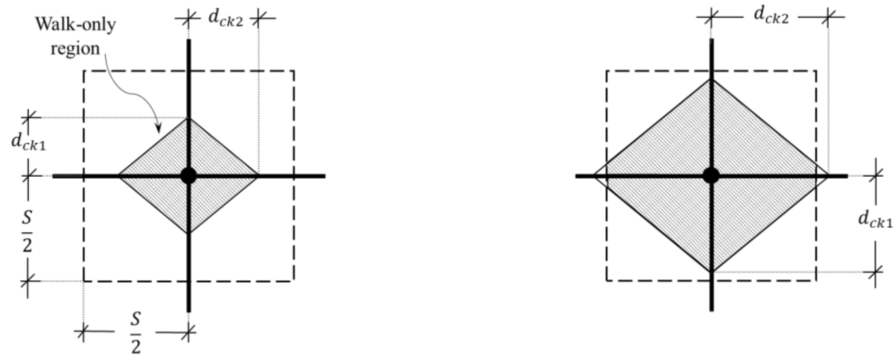
where $\frac{s_f}{2v_w}$ denotes the (average) walking time from the patron's origin to the nearest feeder bus station; $\frac{H_{f,k}}{2}$ the (average) time spent to wait for a feeder bus at the origin station; $t_r$ the travel time in the feeder bus; $t_f$ the intermodal transfer penalty between feeder bus and trunk transit; and $MAC_F$ the marginal feeder bus agency cost (in \$) added to the system for serving this additional feeder passenger. At the system optimum, the patron will choose a feeder bus if and only if $MC_{F-W} < 0$, and will choose walking otherwise. Therefore, we can again obtain the system-optimal access mode assignment by solving $MC_{F-W} = 0$. To solve this equation, we need to derive $MAC_F$ and $t_r$.

To simplify the derivation of $MAC_F$, we assume that a feeder bus always has sufficient capacity to accommodate its patrons. This is usually true because a feeder bus serves a small local zone only (and we have verified in all the numerical instances in this paper that the feeder bus capacity constraint (9b) is never binding). Under this assumption, adding a new passenger to the feeder network will not incur any extra agency cost, which means $MAC_F = 0$. Note that this also means the system-optimal feeder-bus fare can be set to zero.

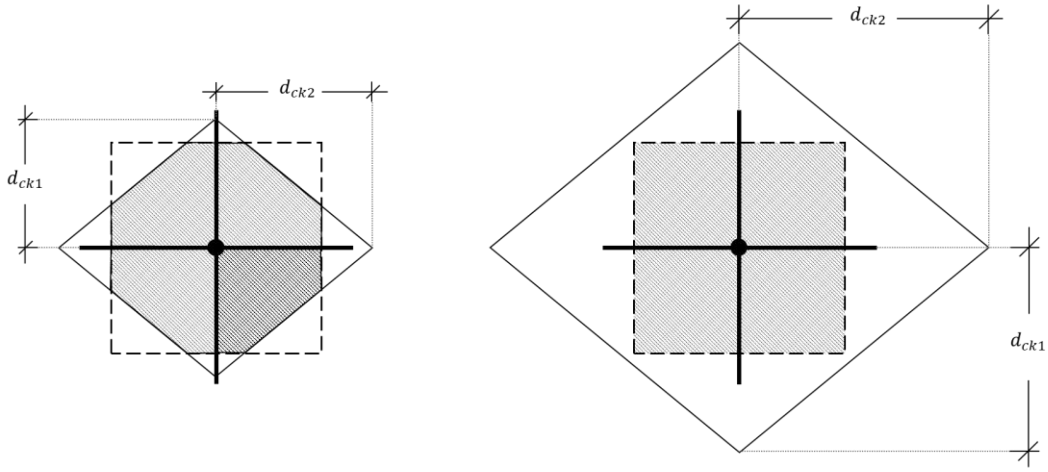
The $t_r$ is the sum of two parts: the in-vehicle travel time along the non-stop route segment, $t_{r1} = \frac{d_1}{v_f}$, and the in-vehicle travel time along the route segment when collecting passengers, $t_{r2} = d_2\left(\frac{1}{v_f} + \frac{\tau_f}{s_f}\right)$. Here $d_1$ and $d_2$ are the patron's access distances along the two perpendicular segments, respectively; $v_f$ denotes the feeder bus's cruise speed; and $\tau_f$ denotes the bus dwell time at a feeder bus stop. Hence, two critical distances will be developed by solving $MC_{F-W} = 0$: by setting $d_2 = 0$, we find the critical distance, $d_{ck1}$ ($k \in \{p, o\}$), in the non-stop travel direction; and by setting $d_1 = 0$, we find the critical distance, $d_{ck2}$ ($k \in \{p, o\}$), in the passenger-collection direction. They are:

$$\begin{cases} d_{ck1} = \dfrac{\frac{s_f}{2v_w} + \frac{H_{f,k}}{2} + t_f}{\frac{1}{v_w} - \frac{1}{v_f}} \\ d_{ck2} = \dfrac{\frac{s_f}{2v_w} + \frac{H_{f,k}}{2} + t_f}{\frac{1}{v_w} - \frac{1}{v_f} - \frac{\tau_f}{s_f}} \end{cases}, \quad k \in \{p, o\}. \tag{F2}$$

This means that the isodistance lines from the trunk station form an anisotropic diamond, as shown by the thin, solid lines in Fig. F1a-d. (Note that $d_{ck1}$ is always smaller than $d_{ck2}$.) As a result, four cases may arise regarding the shape of the walk-only region, as illustrated in Fig. F1a-d. They are: when $\frac{d_{ck1}}{S}, \frac{d_{ck2}}{S} \le \frac{1}{2}$; when $\frac{d_{ck1}}{S} \le \frac{1}{2}, \frac{d_{ck2}}{S} > \frac{1}{2}$; when $\frac{d_{ck1}}{S}, \frac{d_{ck2}}{S} > \frac{1}{2}$ and $\frac{d_{ck1} d_{ck2}}{d_{ck1} + d_{ck2}} \le \frac{S}{2}$; and when $\frac{d_{ck1} d_{ck2}}{d_{ck1} + d_{ck2}} > \frac{S}{2}$. In each figure, the trunk station is marked by the black dot and its catchment zone is bounded by the dashed square; the thick solid lines represent the trunk lines as they would be laid-out in a grid network; and the walk-only region is shaded.




(a) when $\frac{d_{ck1}}{S}, \frac{d_{ck2}}{S} \leq \frac{1}{2}$ (b) when $\frac{d_{ck1}}{S} \leq \frac{1}{2}, \frac{d_{ck2}}{S} > \frac{1}{2}$




(c) when $\frac{d_{ck1}}{S}, \frac{d_{ck2}}{S} > \frac{1}{2}$ and $\frac{d_{ck1}d_{ck2}}{d_{ck1}+d_{ck2}} \leq \frac{S}{2}$ (d) when $\frac{d_{ck1}d_{ck2}}{d_{ck1}+d_{ck2}} > \frac{S}{2}$

**Fig. F1.** The walk-only region in the catchment zone of a trunk station in period $k \in \{p, o\}$.

For each of the four cases shown in Fig. F1a-d, we define $A_{fw,k}$ ($k \in \{p, o\}$) as the area of the walk-only region in period $k$; and $d_{fin,k}$ as the average access distance in that region during $k$. In the feeder-bus region, we define two average access distances, $d_{fout1,k}$ and $d_{fout2,k}$, for the non-stop trip portion and the passenger-collection portion, respectively. The $E_{F,k}$ is calculated as:

$$E_{F,k} = 2\left(\frac{A_{fw,k}}{S^2} \cdot \frac{d_{fin,k}}{v_w} + \left(1 - \frac{A_{fw,k}}{S^2}\right)\left(\frac{d_{fout1,k}}{v_f} + d_{fout2,k} \cdot \left(\frac{1}{v_f} + \frac{\tau_f}{S_f}\right) + \frac{S_f}{2} + \frac{H_{f,k}}{2} + t_f\right)\right), \ k \in \{p, o\}. \quad \text{(F3)}$$

The $A_{fw,k}$, $d_{fin,k}$, $d_{fout1,k}$ and $d_{fout2,k}$ ($k \in \{p, o\}$) are developed for each case shown in Fig. F1a-d as follows:

(i) When $\frac{d_{ck1}}{S}, \frac{d_{ck2}}{S} \leq \frac{1}{2}$, the walk-only region has a diamond shape (see Fig. F1a). Thus, we have:

$A_{fw,k} = 2d_{ck1}d_{ck2}$;

$d_{fin,k} = \frac{4\int_0^{d_{ck2}} dx \int_0^{d_{ck1} - \frac{d_{ck1}}{d_{ck2}}x} (x+y)dy}{A_{fw,k}} = \frac{d_{ck1}+d_{ck2}}{3}$;

$d_{fout1,k} = d_{fout2,k} = \frac{S^2 + 2d_{ck1}d_{ck2}}{4S}$.

(ii) When $\frac{d_{ck1}}{S} \leq \frac{1}{2}, \frac{d_{ck2}}{S} > \frac{1}{2}$, the walk-only region is a hexagon (see Fig. F1b). Thus, we have:

$A_{fw,k} = S \cdot \left(2d_{ck1} - \frac{d_{ck1} \cdot S}{2d_{ck2}}\right)$;

$$d_{fin,k} = \frac{2d_{ck1}d_{ck2}\cdot\left(\frac{d_{ck1}+d_{ck2}}{3}\right) - 4\int_{\frac{S}{2}}^{d_{ck2}-\frac{S}{2}} dx \int_0^{d_{ck1}-\frac{d_{ck1}}{d_{ck2}}x} (x+y)dy}{A_{fw,k}}$$

$$= \frac{2d_{ck2}(d_{ck1}+S) - (d_{ck2}-S)\left(S - \frac{d_{ck1}}{2d_{ck2}}S - \frac{6d_{ck2}^2}{S} + \frac{6d_{ck1}d_{ck2}}{S}\right)}{3d_{ck2}\left(2 - \frac{S}{2d_{ck2}}\right)};$$

$$d_{fout1,k} = \frac{\frac{\left(d_{ck1}+\frac{S}{2}\right)}{2} + \frac{\left(d_{ck1} - \frac{d_{ck1}}{d_{ck2}}\cdot\frac{S}{2} + \frac{S}{2}\right)}{2}}{2} = \frac{2d_{ck1} - \frac{d_{ck1}}{d_{ck2}}\cdot\frac{S}{2} + S}{4};$$

$$d_{fout2,k} = \frac{\left(\frac{S}{2}+\frac{S}{4}\right)}{2} \cdot \frac{\frac{d_{ck1}}{d_{ck2}}\cdot\frac{S}{2}}{\left(\frac{S}{2} + \frac{d_{ck1}}{d_{ck2}}\cdot\frac{S}{2} - d_{ck1}\right)} + \frac{S}{4} \cdot \frac{\left(\frac{S}{2} - d_{ck1}\right)}{\left(\frac{S}{2} + \frac{d_{ck1}}{d_{ck2}}\cdot\frac{S}{2} - d_{ck1}\right)}$$

$$= \frac{\frac{3S^2\cdot d_{ck1}}{16d_{ck2}} + \frac{S^2}{8} - \frac{S}{4}\cdot d_{ck1}}{\left(\frac{S}{2} + \frac{d_{ck1}}{d_{ck2}}\cdot\frac{S}{2} - d_{ck1}\right)}.$$

(iii) When $\frac{d_{ck1}}{S}, \frac{d_{ck2}}{S} > \frac{1}{2}$ and $\frac{d_{ck1}d_{ck2}}{d_{ck1}+d_{ck2}} \leq \frac{S}{2}$, the walk-only region is an octagon (see Fig. F1c). Thus, we have:

$$A_{fw,k} = 2d_{ck1}\cdot S - \frac{d_{ck1}\cdot S^2}{2d_{ck2}} + 2d_{ck2}\cdot S - \frac{d_{ck2}\cdot S^2}{2d_{ck1}} - 2d_{ck1}d_{ck2};$$

$$d_{fin,k} = \frac{2d_{ck1}d_{ck2}\cdot\left(\frac{d_{ck1}+d_{ck2}}{3}\right) - 4\int_{\frac{S}{2}}^{d_{ck2}-\frac{S}{2}} dx \int_0^{d_{ck1}-\frac{d_{ck1}}{d_{ck2}}x} (x+y)dy - 4\int_0^{d_{ck2}-\frac{d_{ck2}\cdot S}{2d_{ck1}}} dx \int_{\frac{S}{2}}^{d_{ck1}-\frac{d_{ck1}}{d_{ck2}}x} (x+y)dy}{A_{fw,k}}$$

$$= \frac{d_{ck1}d_{ck2}(4S - 4d_{ck1} - 2d_{ck2} + 3d_{ck2}\cdot S) + S\left(2d_{ck1}{}^2 + 2d_{ck1}\cdot S - 3d_{ck2}{}^2\cdot S\right) + \frac{d_{ck1}\cdot S}{d_{ck2}}\left(\frac{d_{ck1}\cdot S}{d_{ck2}} - d_{ck1} - S^2\right) + \frac{d_{ck2}\cdot S^2}{4d_{ck1}}(d_{ck2}\cdot S + 6d_{ck2} - S)}{3\left(2d_{ck1}\cdot S - \frac{d_{ck1}\cdot S^2}{2d_{ck2}} + 2d_{ck2}\cdot S - \frac{d_{ck2}\cdot S^2}{2d_{ck1}} - 2d_{ck1}d_{ck2}\right)};$$

$$d_{fout1,k} = \frac{\frac{S}{2} + \frac{\left(\frac{\left(d_{ck1}-\frac{S}{2}\right)\cdot d_{ck2}}{d_{ck1}} + \frac{S}{2}\right)}{2}}{2} = \frac{\frac{3S}{2}\cdot d_{ck1} + \left(d_{ck1} - \frac{S}{2}\right)\cdot d_{ck2}}{4d_{ck1}};$$

$$d_{fout2,k} = \frac{\frac{S}{2} + \frac{\left(\frac{S}{2} - \frac{d_{ck1}}{d_{ck2}}\cdot\frac{S}{2} + d_{ck1}\right)}{2}}{2} = \frac{3S - \frac{d_{ck1}}{d_{ck2}}\cdot S + 2d_{ck1}}{8}.$$

(iv) When $\frac{d_{ck1}d_{ck2}}{d_{ck1}+d_{ck2}} > \frac{S}{2}$, the walk-only region fills up the entire catchment zone of the transit station (see Fig. F1d). Thus, we have: $A_{fw,k} = S^2$, $d_{fin,k} = \frac{S}{2}$ and $d_{fout1,k} = d_{fout2,k} = 0$.

## Appendix G. Bike-sharing cost rates

Cost rates for bike-sharing systems were derived by considering both the capital and the operating costs. The former include the purchase and installation fees for bikes, individual docks, and bike docking stations; and the latter consist of maintenance, repair and replacement, system management (including bike redistribution), and insurance fees for bikes and docking stations (Gleason and Miskimins, 2012). In this paper, we provide cost rates for low- and high-wage cities.

We derive these rates by combining data from multiple sources. Capital cost rates for high-wage cities were calculated by fitting a linear regression model to real-world data obtained from the B-cycle systems in 14 US cities, and from the Capital Bikeshare system in Arlington, Virginia (Arlington, 2010). Operating cost rates for high-wage cities were calculated using financial analysis data collected from the Nice Ride public bike-share program in Minnesota's Twin Cities (City of Minneapolis, 2008). Capital and operating cost rates for low-wage cities were taken from the Hangzhou (China) public bike system (Wikipedia, 2017). The above cost parameters are summarized in Table G1.

We then calculated the daily costs per bike, per dock, and per docking station for both high- and low-wage cities. The daily cost for each item is the sum of the capital cost amortized over the item's lifecycle (assumed to be 5 years) and the operating cost. We assumed that each year had 365 days. Calculation results are also shown in Table G1.

**Table G1** Cost rate breakdown for bike-sharing systems.

| | | High-wage cities | Low-wage cities |
|---|---|---|---|
| Capital cost ($/item) | Bike | 1,118 | 57 |
| | Dock | 1,195 | 149 |
| | Docking station | 19,434 | 10,401 |
| Operating cost ($/item/year) | Bike | 719.6 | 148.6 |
| | Dock | - | - |
| | Docking station | 3,084 | 1,337 |
| $C_B$ ($/bike/day) | | $\frac{(1118/5)+719.6}{365} = 2.58$ | $\frac{(57/5)+148.6}{365} = 0.44$ |
| $C_D$ ($/dock/day) | | $\frac{(1195/5)}{365} = 0.65$ | $\frac{(149/5)}{365} = 0.08$ |
| $C_P$ ($/bike/day) | | $\frac{(19434/5)+3084}{365} = 19.10$ | $\frac{(10401/5)+1337}{365} = 9.36$ |

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