Optimal Global Liner Service Procurement by Utilizing Liner Service Schedules

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Abstract

For global shippers (i.e., multinational manufacturers or retailers), strategic sourcing of container liner shipping services from ocean carriers is an important element of successful global supply chain management. In this paper, we study how to utilize information about different ocean carriers' liner service schedules so that shippers can optimize their sourcing decisions over the selection of carriers to transport container cargoes around the world through multiple shipping lanes. We first introduce a deterministic optimization problem, where cargo demands are given in advance and in which inventory holding costs are included to capture the impacts of liner service schedules on the total operating cost. We formulate the deterministic optimization problem as a mixed integer linear program, which can be simplified by exploiting a totally unimodular property, and can thus be solved directly by a general optimization solver. To further capture the impact of liner service schedules on shippers' resilience to the uncertainties in cargo demands, we then introduce a two-stage robust optimization counterpart of the deterministic problem based on a probability-free demand uncertainty set. As the two-stage robust optimization problem is challenging to solve, we derive some novel reformulations that can enable us to develop an effective solution method. By exploiting the information about liner service schedules, we show how our proposed models and solution method can help shippers to optimize their liner service procurement decisions.

Keywords: Liner service procurement; Service schedules; Demand uncertainty; (Mixed) integer linear programming; Robust optimization.

1 Introduction

Nowadays, international trade among more than 85% of the world's countries is supported by liner shipping services, through which containerships rotate among seaports with regular liner service schedules to transport container cargoes of multitrillion USD per year (UNCTAD, 2019). The key players in the liner shipping market are "shippers" and "ocean carriers". Shippers (i.e., manufacturers or retailers) need to move container cargoes along lanes defined by origin-destination pairs. Ocean carriers are shipping companies who provide liner services to move the container cargoes by containerships through different lanes. Every year, numerous shippers spend countless US dollars on liner services for cargoes to be moved along different lanes around the world. Hence, strategic sourcing of liner services is an important aspect of supply chain management. Successful management of global supply chains has to be cost-effective, and it has to meet the Triple-A Supply Chain standard of being Agile, Adaptable, and Aligned (Lee, 2004). Being in the digital age, many shippers today are exploiting information and analytics to optimize liner service sourcing decisions.

In this work, we study the problem on how to utilize information about liner service schedules for a shipper to optimize its liner service sourcing decisions. Specifically, the shipper needs to select carriers and allocate container cargoes for the selected carriers to transport, with the aim of minimizing the total operating cost. This problem is motivated by a current huge gap between practical needs and existing research works regarding carriers' liner service schedules in shippers' liner service procurement. In practice, when selecting carriers to transport its container cargoes on a long-term contract, a shipper usually wants to take into consideration not only shipping rates, but also factors relevant to the carriers' liner service schedules (e.g., departure days and transit times), which are provided by the carriers (see Figure 1 for an example). As pointed out to us by a large Dutch electronics manufacturer, liner service schedules have a significant impact on a shipper's transportation costs, inventory costs, and resilience to demand uncertainties.

To see the impact of carriers' liner service schedules, consider the example shown in Figure 1, where a shipper needs to transport 10 containers of a product every week over a lane from an origin (Hong Kong) to a destination (Rotterdam) by sea, and the cargo needs to be picked up at Rotterdam every Monday. The shipper thus requests service from three liner carriers, A, B and C,

B	C	D	E	F	G	H	I	J	K	L	M	N
Carrier	Origin Port	Destination Port	Projected Volume (TEU/week)	Freight Rates USD/TEU	Departure Day							Transit Time (days)
					Mon	*Tue*	*Wed*	*Thu*	*Fri*	*Sat*	*Sun*	
A	HONG KONG	ROTTERDAM	10	1020	X							28
B	HONG KONG	ROTTERDAM	10	1000			X					35
C	HONG KONG	ROTTERDAM	10	1020					X			28

Figure 1: Excerpt from a sample Excel file that a large Dutch electronics manufacturer used to summarize carriers' quotations for cargo from Hong Kong to Rotterdam, where TEU refers to twenty-foot equivalent unit for container capacity, and data are modified for confidential reasons.

who bid on shipping rates of USD 1020, 1000 and 1020 per container, with a departure from Hong Kong once a week respectively on Monday, Wednesday, and Friday, and with a transit time of 4 weeks, 5 weeks, and 4 weeks, respectively. The service capacity for each carrier is to transport at most 10 containers per week.

Without taking different carriers' liner service schedules (i.e., departure days and transit times) into consideration, the shipper will select carrier B due to it having the lowest shipping rate. However, since carriers A and C offer transit times one week shorter than carrier B, choosing either A or C instead of B will reduce the in-transit inventory holding cost. Since the cargo shipped by carrier A arrives at its destination every Monday, which is the same as the cargo's pickup day, choosing carrier A instead of B or C will reduce the inventory holding cost at Rotterdam to zero.

Moreover, if both carriers A and C are chosen, the frequency of transportation will increase, which can enhance the resilience of the shipper against demand uncertainties. To see this, consider a scenario where there is an unexpected cargo of two additional containers that need to be shipped to and picked up at Rotterdam on Friday of a week during a peak season. The unexpected cargo, together with the regular cargo of 10 containers in that week, exceeds the capacity of each individual carrier. Accordingly, choosing both carriers A and C instead of only carrier A allows the shipper to use carrier C to transport this unexpected cargo for arrival at the destination on its pickup day, which not only ensures sufficient capacity for shipping this cargo, but also reduces the inventory holding cost of this cargo at its destination to zero. However, the shipper may not actually be able to choose both carriers A and C, as carriers often require a commitment to a minimum quantity of cargo as a condition of service. Hence, the optimal decisions depend on the trade-off between transportation and inventory costs, the uncertainties of cargo demands, and the practical requirements from carriers and shippers.

The above problem is very common in practice, but a well-developed solution is lacking. The problem becomes even more complicated when involving multiple lanes (i.e., origin-destination pairs), multiple departure days per carrier, and various additional requirements from the shipper and carriers. Thus, it would be very rewarding in practice if new theoretical insights and optimization methods can be derived and implemented to guide shippers on how to utilize the information about carriers' liner service schedules in liner service procurement.

Despite the shippers' needs, very limited work in the literature on liner service procurement has to date considered information relating to liner service schedules. Almost all the existing works aim to minimize a shipper's transportation cost based solely on carriers' shipping rates. In these works (see, e.g., Lim et al., 2006, 2008; Xu and Lai, 2015), the problems of liner service procurement are often formulated as (mixed) integer linear programming models, where cargo demands are assumed to be deterministic and known in advance. These models consist of basic decisions that include the selection of carriers and the allocation of cargo to the selected carriers, so as to satisfy the transportation of the cargoes as well as other basic constraints required by shippers and carriers, such as shippers' restrictions on the maximum and the minimum numbers of selected carriers, and carriers' limits on shipping capacities and requests for minimum quantity commitments.

The only existing works that take into account liner service schedules in liner service procurement are Hu et al. (2016) and Lu et al. (2017); **?**. In Hu et al. (2016), only the transit times of liner service schedules are taken into account, and a bi-objective integer linear programming model is used to accommodate two objectives, where one minimizing the total transportation cost and the other minimizing total transit time are formulated. In their model, the impact of liner service schedules on the shipper's inventory holding costs is not considered, and cargo demands are all assumed to be deterministic. Lu et al. (2017) study a liner service procurement problem for a newsvendor-type shipper who transports and sells cargoes of perishable products to an overseas market, which is then extended by **?** to investigate how the shipper may benefit from the competition between two carriers on shipping rates and transit times. In these two works, although liner service schedules are taken into account for their impact on inventory holding costs under uncertain cargo demands, the analytical results derived are applicable to only one shipping lane, most of the basic practical constraints are not considered, and the volume of a cargo is allowed to be a fraction of a container. In some other works where liner service schedules are not considered, uncertain cargo demands are incorporated but only for studies on the design of contracts, so as to facilitate the coordination between carriers and shippers for risk and cost sharing (see, e.g., Lee et al., 2015; Yang et al., 2019).

Besides liner services, the problem of selecting carriers for transporting cargoes for a shipper has also been studied for trucking services, in which carriers often bid shipping rates for bundles of lanes, and the resulting integer linear programming formulations of the problem are set-partitioning-like models (see, e.g., Caplice and Sheffi, 2005). Most of these studies take into account only carriers' shipping rates, without incorporating their liner service schedules, for deterministic cargo demand models (Song and Regan, 2005) and for uncertain cargo demand models (Remli et al., 2019). In other studies (see, e.g., Xu and Huang, 2017), the design of a bidding mechanism is studied for trucking service procurement, where transit times of carrier services are taken into account, but as a separated objective or constraint, without considering their impacts on inventory holding costs.

In this work, we attempt to fill the gap described above between the actual needs of industrial practitioners and existing literature studies, particularly regarding how to utilize liner service schedule information within a shipper's global liner service procurement decisions for multiple lanes worldwide. To achieve this, we first introduce a deterministic version of the problem, where cargo demands are known in advance, that incorporates liner service schedules and their impacts on both the transportation costs and inventory holding costs. This deterministic problem is formulated as an integer linear programming model, for which a totally unimodular property is utilized to reformulate the problem as a mixed integer linear programming model containing less integer decision variables. These models can be solved directly for the shipper by using general optimization solvers.

Based on the deterministic version of the problem, we then derive a two-stage robust optimization counterpart of the problem that incorporates uncertainties of cargo demands in utilizing liner service schedules. In this study, we adopt a robust optimization approach instead of a stochastic programming approach, so as to ensure the shipper's resilience against unexpected scenarios in the worst case, as well as to relax the need for actual information on demand probability, which is hard to obtain in practice. See Bertsimas et al. (2011) for general theories and applications of robust optimization. Our robust optimization formulation has two stages. The first stage decisions include the selections of carriers for each lane, these being made before cargo demands are realized. The second stage decisions include the allocations of cargo to the selected carriers for each lane, which are recourse actions, being made after cargo demands are realized, so as to be adaptive to the demand uncertainties. Such a two-stage robust optimization approach has been applied to various applications in the literature (see, e.g., Remli and Rekik, 2013; Wang and Qi, 2020), and is known to produce less conservative and more cost effective solutions than the basic single-stage approach, due to its second stage recourse actions (Atamtürk and Zhang, 2007).

For the two-stage robust optimization counterpart of the problem studied in this work, it is a min-max-min problem, which is hard to tackle directly. By exploiting the totally unimodular property identified in the deterministic version of the problem, we are able to reformulate the two-stage problem as a mixed integer linear programming model. Based on this and by a constraint generation approach, we develop a solution method that can be implemented to help shippers utilize the information about liner service schedules for liner service procurement under uncertain cargo demands. Extensive computational experiments have been conducted to demonstrate the effectiveness of the proposed models and solution method.

The remainder of this paper is organized as follows. In Section 2, we describe and formulate the deterministic version of the problem. In Section 3, we first extend the deterministic version of the problem to obtain a two-stage robust optimization counterpart of the problem, where uncertainties of cargo demands are taken into account, for which we then derive reformulations and develop a solution method. Computational experiments are reported in Section 4, followed by the conclusion in Section 5. All proofs are contained in Appendix A of the online companion to this paper.

2 Mathematical Models for the Deterministic Problem

In this section we detail a deterministic version of the global liner service procurement problem with liner service schedules, where cargo demands for all lanes are known in advance. We present the description of the deterministic problem in Section 2.1, followed by an integer linear programming (ILP) model and its mixed integer linear programming (MILP) reformulation in Section 2.2, which can be solved directly by general optimization solvers.

2.1 Description of the Deterministic Problem (Problem D)

Let $\mathbb{N}_0$ indicate the set of non-negative integers. Consider that a shipper needs to purchase liner services from a set I of carriers for a set J of lanes over a planning horizon of T periods indexed by $t = 1, 2, \ldots, T$. The shipper first translates an aggregated forecast of its products into its cargo demands, which are represented by non-negative integers $u_{jt} \in \mathbb{N}_0$ for $j \in J$ and $t \in \{1, 2, \ldots, T\}$, with each u_{jt} indicating the *number of containers* that need to be transported from the original port of lane j to its destination port, and that need to be picked up from the destination port in period t. The shipper then announces these cargo demands, for which each carrier $i \in I$ responds with a quotation of shipping rates $r_{ij} \geq 0$ for $j \in J$, with each r_{ij} indicating the cost to ship a container through lane j by carrier i.

In order to consider liner service schedules, the shipper also needs each carrier $i \in I$ to provide a list of liner service schedules for each lane $j \in J$. Let n_{ij} indicate the number of available liner service schedules provided by carrier i for lane j. For each $k = 1, 2, \ldots, n_{ij}$, the k-th liner service schedule of carrier i for lane j, which is referred to as schedule (i, j, k), contains information on its capacity, departure time, transit time, and arrival time. Let $s_{ijk} \in \mathbb{N}_0$ denote the service capacity of schedule (i, j, k), indicating that schedule (i, j, k) can only carry at most s_{ijk} containers. Let $d_{ijk} \in \{1, 2, \ldots, T\}$, $\tau_{ijk} \in \mathbb{N}_0$, and $a_{ijk} \in \{1, 2, \ldots, T\}$ denote the departure time at the original port, the transit time, and the arrival time at the destination port of schedule (i, j, k), respectively, where $a_{ijk} = d_{ijk} + \tau_{ijk}$. They indicate that under schedule (i, j, k), a containership departs from the original port of lane j at the end of period d_{ijk}, carries containers through lane j for τ_{ijk} periods, and arrives at the destination port of lane j at the beginning of period a_{ijk}. With such information, for each $j \in J$ and $t \in \{1, 2, \ldots, T\}$, the shipper can obtain a set S_{jt} of the schedules (i, j, k) with $i \in I$, $k \in \{1, 2, \ldots, n_{ij}\}$, and $a_{ijk} = t$, which includes all the available liner service schedules that transport cargoes for lane j and arrive at the destination port of lane j in period t.

For cargoes that cannot meet the pickup times by using the selected carriers' liner services, the shipper has to transport them by some alternative shipping services from the spot-market, which are usually much more costly. For this, let e_{jt} denote the spot-market shipping rate, i.e., the cost per container for using the spot-market shipping services to transport cargoes for demands of lane j that need to be picked up at the destination port in period t, which can be very costly when shipping modes other than by sea transport have to be adopted.

The shipper then needs to make decisions on the selection of carriers and the allocation of cargoes to the liner service schedules of the selected carriers. In addition to satisfying all the cargo demands and following all the carriers' schedules, the shipper's selection of carriers and allocation of cargoes are often restricted by the following basic practical constraints known in the literature (see, e.g. Hu et al., 2016). For the shipper, the total number of carriers selected from I cannot exceed a threshold m. For each $j \in J$, the total number of carriers selected to serve lane j must fall in a given range between L_j and U_j. For each $i \in I$, the total number of containers allocated for carrier i to ship cannot be above a given overall capacity c_i, or below a minimum quantity commitment b_i. Here, m, L_i, U_j, c_i, and b_i are all non-negative integers given as input data.

Accordingly, *the deterministic version of the shipper's decision-making problem, referred to as* **problem D** *in short, is to select carriers and allocate cargoes to liner service schedules of the selected carriers so as to satisfy all the practical constraints, with the total cost minimized.* In problem D, the total cost needs to include not only transportation costs, such as the liner shipping costs and the spot-market shipping costs, but also inventory holding costs. To quantify inventory holding costs, the shipper needs information about initial inventory levels and unit holding costs for the cargoes of different lanes. Accordingly, for each $j \in J$, let integer H_{j0} indicate the initial inventory level of cargoes of lane j held at the destination port of lane j before the start of the planning horizon. For each $i \in I$ and $j \in J$, let h'_{ij} and h_j indicate the unit inventory holding costs (per container) of those in transit by carrier i and at the destination port of lane j, respectively, for cargoes of lane j.

2.2 ILP and MILP Models for the Deterministic Problem (Problem D)

Problem D can be formulated as an ILP model based on the following decision variables. For each $i \in I$, let binary variable y_i indicate whether or not carrier i is selected. For each $i \in I$ and $j \in J$, let binary variable z_{ij} indicate whether or not carrier i serves lane j. For each $i \in I$, $j \in J$, and $k \in \{1, 2, \dots, n_{ij}\}$, let integer variable x_{ijk} indicate the number of containers shipped by carrier i for lane j through schedule (i, j, k). For each $j \in J$ and $t \in \{1, 2, \dots, T\}$, let integer variable H_{jt} indicate the inventory of cargoes of lane j held at the destination port of lane j in period t, and let integer variable w_{jt} indicate the number of containers that need to be shipped by spot-market carriers for cargoes of lane j and that need to be picked in period t. Accordingly, the ILP model of problem D can be obtained as follows, and is referred to as model ILP_D:

$$(\text{ILP}_\text{D}) \qquad \min \quad \sum_{i\in I}\sum_{j\in J}\sum_{k=1}^{n_{ij}}(r_{ij} + h'_{ij}\tau_{ijk})x_{ijk} + \sum_{t=1}^{T}\sum_{j\in J} h_j H_{jt} + \sum_{t=1}^{T}\sum_{j\in J} e_{jt} w_{jt} \tag{1}$$

$$\text{s.t.} \quad H_{jt} = H_{j,t-1} - u_{jt} + \sum_{(i,j,k)\in S_{jt}} x_{ijk} + w_{jt}, \ \forall j \in J, t \in \{1, 2, \dots, T\}, \tag{2}$$

$$\sum_{i\in I} y_i \le m, \tag{3}$$

$$b_i y_i \le \sum_{j\in J}\sum_{k=1}^{n_{ij}} x_{ijk}, \ \forall i \in I, \tag{4}$$

$$\sum_{j\in J}\sum_{k=1}^{n_{ij}} x_{ijk} \le c_i y_i, \ \forall i \in I, \tag{5}$$

$$\sum_{j\in J} z_{ij} \le |J| y_i, \ \forall i \in I, \tag{6}$$

$$L_j \le \sum_{i\in I} z_{ij} \le U_j, \ \forall j \in J, \tag{7}$$

$$x_{ijk} \le s_{ijk} z_{ij}, \ \forall i \in I, j \in J, k \in \{1, 2, \dots, n_{ij}\}, \tag{8}$$

$$b_i y_i \le \sum_{j\in J}\sum_{k=1}^{n_{ij}} s_{ijk} z_{ij}, \ \forall i \in I, \tag{9}$$

$$x_{ijk} \in \mathbb{N}_0, \ \forall i \in I, j \in J, k \in \{1, 2, \dots, n_{ij}\}, \tag{10}$$

$$y_i \in \{0, 1\}, \ \forall i \in I, \tag{11}$$

$$z_{ij} \in \{0, 1\}, \ \forall i \in I, j \in J, \tag{12}$$

$$H_{jt} \in \mathbb{N}_0, w_{jt} \in \mathbb{N}_0, \ \forall j \in J, t \in \{1, 2, \ldots, T\}. \tag{13}$$

In model $\mathrm{ILP_D}$ above, the objective in (1) is to minimize the sum of liner shipping costs, in-transit holding costs, inventory holding costs of cargoes at destinations, and spot-market shipping costs. Constraints (2) are balanced constraints for inventories at the destination port of each lane for each period. Constraint (3) ensures that at most m carriers can be selected. Constraints (4) and Constraints (5) ensure that the total number of containers of cargoes assigned to each carrier $i \in I$ satisfies the minimum quantity commitment b_i and does not exceed the overall capacity limit c_i if carrier i is selected. Constraints (6) ensure that only the selected carriers can serve each lane. Constraints (7) ensure that the number of carriers that serve lane $j \in J$ is not below a minimum number L_j or above a maximum number U_j. Constraints (8) ensure that the service capacity s_{ijk} of each schedule (i, j, k) operated by carrier $i \in I$ is not exceeded. Constraints (9) are valid inequalities derived from (4) and (8), which are included here to tighten the linear programming relaxation of the model, and which are also consistent with our robust optimization formulation (studied later in Section 3). Constraints (10)–(13) are non-negative integral or binary constraints, where the non-negative constraints on H_{jt} for $j \in J$ and $t \in \{1, 2, \ldots, T\}$ ensure that all cargoes can be picked up on time at their destinations, so that cargo demands are all satisfied.

According to Theorem 1 below, the integral constraints (10) and (13) on variables x_{ijk}, H_{jt}, and w_{jt} in model $\mathrm{ILP_D}$ can be relaxed to $x_{ijk} \geq 0$, $H_{jt} \geq 0$, and $w_{jt} \geq 0$, so that *model* $\mathrm{ILP_D}$ *of problem D can be reformulated as a MILP model, which is referred to as* **model $\mathbf{MILP_D}$**, where the number of integer variables are reduced by as many as $(\sum_{i \in I} \sum_{j \in J} n_{ij} + 2\,|J| \cdot T)$.

Theorem 1. *When $y_i \in \{0, 1\}$ and $z_{ij} \in \{0, 1\}$ are given for each $i \in I$ and $j \in J$, model* $\mathrm{ILP_D}$ *is totally unimodular and can be reformulated as a minimum cost network flow problem. Thus, integral constraints on variables x_{ijk}, H_{jt}, and w_{jt} can be relaxed in model* $\mathrm{ILP_D}$.

3 Robust Optimization for Demand Uncertainty

In this section, we study a robust optimization version of the global liner service procurement problem with liner service schedules, where cargo demands for all lanes are uncertain. For this, in Section 3.1, we present a two-stage robust optimization formulation of the problem, which is followed by its reformulations and solution method in Section 3.2 and Section 3.3, respectively.

3.1 Two-Stage Robust Optimization Problem (Problem R)

Unlike the deterministic problem (problem D) studied in Section 2, we now take into account uncertainties of cargo demands. Let $\tilde{\mathcal{U}}$ indicate a complete scenario set that contains all possible values of cargo demands $\mathbf{u} = (u_{jt})_{j \in J, t \in \{1,2,\ldots,T\}}$. To incorporate uncertainties of cargo demands, it is natural to formulate the problem as a stochastic programming model by extending the deterministic model $\mathrm{ILP_D}$, so as to minimize the expected total cost (see Appendix B of the online companion for the details). However, the stochastic programming model requires actual information on the probability of each possible value of cargo demands $\mathbf{u} \in \tilde{\mathcal{U}}$, which is hard to obtain. It is also known that solving the stochastic programming model is very challenging, as the model involves a large number of decision variables that are proportional to the size of the complete scenario set $\tilde{\mathcal{U}}$ and can be exponentially large.

Instead, we adopt a robust optimization approach to incorporate the uncertainties of cargo demands for global liner service procurement with liner service schedules. Unlike the stochastic programming approach, the robust optimization approach aims to minimize the total cost for the worst case, i.e., to minimize the maximum possible value of the total cost for all possible values of cargo demands in a given uncertainty set. As a result, this not only ensures the shipper's resilience against unexpected scenarios, but also relaxes the requirement for actual information on demand probability, and is thus easier to be applied in practice. Moreover, in the robust optimization formulation that we are going to present below, decisions are made in two stages, similar to the stochastic programming approach. The selection of carriers for each lane is made in the first stage before cargo demands are realized, and the allocation of cargoes to the liner service schedules of selected carriers for each lane is made in the second stage as recourse decisions after demands are realized. Such a "wait-and-see" approach reflects actual practice, and ensures that the solution produced is not only robust but also adaptive to demand uncertainties.

We can now extend the deterministic model $\mathrm{ILP_D}$ to formulate a two-stage robust optimization problem as follows for global liner service procurement with liner service schedules under uncertainties of cargo demands, in which possible values of cargo demands belong to an uncertainty set. To define the uncertainty set, we assume that for each lane $j \in J$ and period $t \in \{1, 2, \ldots, T\}$, the uncertain cargo demand u_{jt} is a non-negative integer that lies within the interval $[\bar{u}_{jt} - \hat{u}_{jt}, \bar{u}_{jt} + \hat{u}_{jt}]$, where $\bar{u}_{jt} \in \mathbb{N}_0$ is the nominal demand and $\hat{u}_{jt} \in \mathbb{N}_0$ is the maximum deviation with $\bar{u}_{jt} - \hat{u}_{jt} \geq 0$, which are both given as input data. As a result, the cargo demand u_{jt} can be formulated as

$$u_{jt} = \bar{u}_{jt} + \hat{u}_{jt}\sigma_{jt} \text{ and } u_{jt} \in \mathbb{N}_0, \tag{14}$$

where $-1 \leq \sigma_{jt} \leq 1$.

Let integer $\Gamma \in \mathbb{N}_0$ with $\Gamma \leq |J| \cdot T$ indicate a given **budget of uncertainty**, which can be used to adjust the robustness against the conservatism level. We define $\mathcal{U}(\Gamma)$ as an uncertainty set that consists of all the possible values of cargo demands $\mathbf{u} = (u_{jt})_{j \in J, t \in \{1,2,\ldots,T\}}$ to be considered, with the total deviation of all lanes with respect to their nominal demands not exceeding Γ, which can be formulated as follows:

$$\mathcal{U}(\Gamma) = \left\{ \mathbf{u} : u_{jt} = \bar{u}_{jt} + \hat{u}_{jt}\sigma_{jt}, u_{jt} \in \mathbb{N}_0, \sigma_{jt} \in [-1, 1], \forall j \in J, t \in \{1, 2, \ldots, T\}, \sum_{j \in J} \sum_{t=1}^{T} |\sigma_{jt}| \leq \Gamma \right\}. \tag{15}$$

As in model ILP_D of problem D, we still use binary variables y_i to indicate whether or not carrier i is selected, and binary variables z_{ij} to indicate whether or not carrier i serves lane j, for $i \in I$ and $j \in J$, which are determined in the first stage of problem R before the cargo demands are realized. We still use integer variables x_{ijk} to indicate the number of containers shipped by carrier i for lane j through schedule (i, j, k) for $i \in I$, $j \in J$, and $k \in \{1, 2, \ldots, n_{ij}\}$, the integer variables H_{jt} to indicate the inventory of cargoes of lane j held at the destination port of lane j in period t, and integer variables w_{jt} to indicate the number of containers that need to be shipped by spot-market carriers for cargoes of lane j to be picked up in period t, for $j \in J$ and $t \in \{1, 2, \ldots, T\}$. Given the values of variables y_i and z_{ij}, denoted by $(\mathbf{y}, \mathbf{z})$, the second stage of problem R optimizes the values of variables x_{ijk}, H_{jt}, and w_{jt}, denoted by $(\mathbf{x}, \mathbf{H}, \mathbf{w})$, as recourse decisions, so as to minimize the total cost for each realization of cargo demands in the uncertainty set under (15).

Accordingly, *the two-stage robust optimization problem for global liner service procurement with liner service schedules, referred to as* **problem R** *for short, aims to select carriers for each lane* *(as the first stage decisions) before cargo demands are realized, and then allocate cargoes to the liner service schedules of the selected carriers for each lane (as the second stage decisions) after cargo demands are realized, with all the practical constraints satisfied, so as to minimize the total cost for the worst case in the uncertainty set.* It can be formulated as follows by extending model $\mathrm{ILP_D}$:

$$\text{(R)} \quad \min_{(\mathbf{y},\mathbf{z})} \quad F_{\mathrm{RP}}(\mathbf{y},\mathbf{z}) \tag{16}$$

$$\text{s.t.} \quad \text{constraints (3), (6), (7), (9), (11), and (12),} \tag{17}$$

where function $F_{\mathrm{RP}}(\mathbf{y},\mathbf{z})$ denotes the optimal objective value of the second-stage recourse problem:

$$F_{\mathrm{RP}}(\mathbf{y},\mathbf{z}) = \max_{\mathbf{u}\in\mathcal{U}(\Gamma)} \min_{(\mathbf{x},\mathbf{H},\mathbf{w})} \quad \sum_{i\in I}\sum_{j\in J}\sum_{k=1}^{n_{ij}}(r_{ij} + h'_{ij}\tau_{ijk})x_{ijk} + \sum_{t=1}^{T}\sum_{j\in J}(h_j H_{jt} + e_{jt}w_{jt}) \tag{18}$$

$$\text{s.t.} \quad \text{constraints (2), (4), (5), (8), (10), and (13).} \tag{19}$$

In the above two-stage robust optimization formulation of problem R, the objective function (16) is to minimize the total cost of the worst case in the uncertainty set, which is incurred during the second stage. Constraints in (17) of the first stage are the same as those constraints in the deterministic model $\mathrm{ILP_D}$, which are imposed on decisions $(\mathbf{y},\mathbf{z})$. Given decisions $(\mathbf{y},\mathbf{z})$, the second stage is a max-min problem, which, as shown in (18), aims to compute the total cost of the worst case in the uncertainty set. Specifically, for each realization $\mathbf{u}$ of cargo demands in the uncertainty set $\mathcal{U}(\Gamma)$, it optimizes decisions $(\mathbf{x},\mathbf{H},\mathbf{w})$ to minimize the total cost. It then maximizes the minimum total cost among all the possible realizations $\mathbf{u}$ in the uncertainty set to identify the worst case. Constraints in (19) of the second stage are the same as those constraints in the deterministic model $\mathrm{ILP_D}$, which are imposed on decisions $(\mathbf{x},\mathbf{H},\mathbf{w})$. It is worth noting that constraints (9) in (17) of the first stage problem are not only valid inequalities but also essential constraints for problem R, because this guarantees the existence of feasible solutions for the second-stage recourse problem in (18)–(19), as shown later in Section 3.2 (in the proof of Lemma 1) for our model analysis.

3.2 Model Analysis and Reformulations for Problem R

As shown in the two-stage robust formulation (16)–(19), Problem R aims to minimize function $F_{\mathrm{RP}}(\mathbf{y},\mathbf{z})$ over the binary decisions $\mathbf{y}$ and $\mathbf{z}$ of the first stage, where $F_{\mathrm{RP}}(\mathbf{y},\mathbf{z})$ represents the optimal objective value of the second-stage recourse problem, which, as shown in (18), is a max-min problem. As a result, problem R is a min-max-min problem, which is hard to tackle directly.

We now show that the second-stage recourse problem of problem R can be reformulated as a maximization MILP, so that problem R can be reformulated as a minimization MILP. These results provide a foundation for the development of solution method for problem R in Section 3.3.

For any given first-stage decisions $\mathbf{y}$ and $\mathbf{z}$ that satisfy constraints in (17) of problem R, and for any given cargo demands $\mathbf{u} \in \mathcal{U}(\Gamma)$, consider the following inner minimization problem of the second stage max-min problem, defined in (18)–(19), of problem R:

$$\min_{(\mathbf{x},\mathbf{H},\mathbf{w})} \quad \sum_{i\in I}\sum_{j\in J}\sum_{k=1}^{n_{ij}}(r_{ij}+h'_{ij}\tau_{ijk})x_{ijk}+\sum_{t=1}^{T}\sum_{j\in J}(h_jH_{jt}+e_{jt}w_{jt}) \tag{20}$$

$$\text{s.t.} \quad \text{constraints (2), (4), (5), (8), (10), and (13).} \tag{21}$$

Lemma 1 below indicates that this inner minimization problem always has a feasible solution.

Lemma 1. *Given any* $\mathbf{y}$ *and* $\mathbf{z}$ *that satisfy constraints in (17), and any* $\mathbf{u} \in \mathcal{U}(\Gamma)$*, the inner minimization problem of the second stage problem of problem R always has a feasible solution.*

It can also be seen that the inner minimization problem of the second stage of problem R has the same optimum objective value as its LP relaxation. This is because constraints in (21) are the same as those constraints of model ILP_D imposed on decisions $(\mathbf{x},\mathbf{H},\mathbf{w})$, and the objective function in (20) is the same as that of model ILP_D, which, together with Theorem 1, imply that integral decisions (10) and (13) in (21) can be relaxed to $x_{ijk} \ge 0$ for $i \in I$, $j \in J$, and $t \in \{1, 2, \ldots, T\}$, and to $w_{jt} \ge 0$ and $H_{jt} \ge 0$ for $j \in J$ and $t \in \{1, 2, \ldots, T\}$, respectively.

By Lemma 1, the LP relaxation of the inner minimization problem of the second stage of problem R must also have a feasible solution. Accordingly, we impose the dual variables α_{jt}, β_i, γ_i, and λ_{ijk} for constraints (2), (4), (5), and (8) in (21), respectively. By the strong duality theorem, we can obtain that the LP relaxation of the inner minimization problem of the second stage of problem R has the same objective value as its dual problem, which is given as follows:

$$\max_{(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\lambda})} \quad -\sum_{j\in J}\sum_{t=1}^{T}u_{jt}\alpha_{jt}+\sum_{i\in I}b_iy_i\beta_i-\sum_{i\in I}c_iy_i\gamma_i-\sum_{i\in I}\sum_{j\in J}\sum_{k=1}^{n_{ij}}s_{ijk}z_{ij}\lambda_{ijk}+\sum_{j\in J}H_{j0}\alpha_{j1} \tag{22}$$

$$\text{s.t.} \quad \alpha_{jt}-\alpha_{j,t+1} \le h_j,\ \forall j \in J, t \in \{1,2,\ldots,T-1\}, \tag{23}$$

$$\alpha_{jT} \leq h_j, \ \forall j \in J, \tag{24}$$

$$-\alpha_{j,a_{ijk}} + \beta_i - \gamma_i - \lambda_{ijk} \leq r_{ij} + h'_{ij}\tau_{ijk}, \ \forall i \in I, j \in J, k \in \{1, 2, \ldots, n_{ij}\}, \tag{25}$$

$$-\alpha_{jt} \leq e_{jt}, \ \forall j \in J, t \in \{1, 2, \ldots, T\}, \tag{26}$$

$$\beta_i \geq 0, \gamma_i \geq 0, \ \forall i \in I. \tag{27}$$

$$\lambda_{ijk} \geq 0, \ \forall i \in I, j \in J, k \in \{1, 2, \ldots, n_{ij}\}. \tag{28}$$

This, together with the definition of $\mathcal{U}(\Gamma)$ in (15), implies that the second stage max-min problem of problem R, which defines function $F_{\mathrm{RP}}(\mathbf{y}, \mathbf{z})$ in (18)–(19), can be reformulated as the following maximization problem:

$$F_{\mathrm{RP}}(\mathbf{y}, \mathbf{z}) \quad = \quad \max_{(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \mathbf{u}, \boldsymbol{\sigma})} - \sum_{j \in J} \sum_{t=1}^{T} u_{jt}\alpha_{jt} + \sum_{i \in I} b_i y_i \beta_i - \sum_{i \in I} c_i y_i \gamma_i - \sum_{i \in I} \sum_{j \in J} \sum_{k=1}^{n_{ij}} s_{ijk} z_{ij} \lambda_{ijk} + \sum_{j \in J} H_{j0}\alpha_{j1} \tag{29}$$

$$\text{s.t.} \qquad \text{constraints (23)–(28)}, \tag{30}$$

$$u_{jt} = \bar{u}_{jt} + \hat{u}_{jt}\sigma_{jt}, \ \forall j \in J, t \in \{1, 2, \ldots, T\}, \tag{31}$$

$$u_{jt} \in \mathbb{N}_0, \ \forall j \in J, t \in \{1, 2, \ldots, T\}, \tag{32}$$

$$-1 \leq \sigma_{jt} \leq 1, \ \forall j \in J, t \in \{1, 2, \ldots, T\}, \tag{33}$$

$$\sum_{j \in J} \sum_{t=1}^{T} |\sigma_{jt}| \leq \Gamma. \tag{34}$$

Due to the term $-\sum_{j \in J} \sum_{t=1}^{T} u_{jt}\alpha_{jt}$, the above reformulation for $F_{\mathrm{RP}}(\mathbf{y}, \mathbf{z})$ is a nonlinear maximization problem. It can further be reformulated as a MILP based on Theorem 2 below.

Theorem 2. *There exists an optimal solution to the nonlinear maximization problem defined in (29)–(34) such that $\sigma_{jt} \in \{-1, 0, 1\}$ for $j \in J$ and $t \in \{1, 2, \ldots, T\}$.*

By Theorem 2, constraints (33) can be replaced with $\sigma_{jt} \in \{-1, 0, 1\}$ for $j \in J$ and $t \in \{1, 2, \ldots, T\}$. Accordingly, (31) implies that $u_{jt} \in \{\bar{u}_{jt} + \hat{u}_{jt}, \bar{u}_{jt}, \bar{u}_{jt} - \hat{u}_{jt}\}$, which, together with $\bar{u}_{jt} \in \mathbb{N}_0$, $\hat{u}_{jt} \in \mathbb{N}_0$, and $\bar{u}_{jt} \geq \hat{u}_{jt}$, implies that $u_{jt} \in \mathbb{N}_0$. Thus, constraints (32) can be removed. Moreover, we can also replace each nonlinear term $u_{jt}\alpha_{jt}$ with a new variable ξ_{jt}, and replace each integer variable σ_{jt} with three new binary variables $\theta_{jt,-1}$, $\theta_{jt,0}$, and $\theta_{jt,1}$, for $j \in J$ and

$t \in \{1, 2, \ldots, T\}$, which are used to indicate whether σ_{jt} equals -1, 0, and 1, respectively. This, together with the following new linear constraints,

$$\theta_{jt,-1} + \theta_{jt,0} + \theta_{jt,1} = 1 \tag{35}$$

$$\bar{u}_{jt}\alpha_{jt} - M(1 - \theta_{jt,0}) \leq \xi_{jt} \leq \bar{u}_{jt}\alpha_{jt} + M(1 - \theta_{jt,0}) \tag{36}$$

$$(\bar{u}_{jt} + \hat{u}_{jt})\alpha_{jt} - M(1 - \theta_{jt,1}) \leq \xi_{jt} \leq (\bar{u}_{jt} + \hat{u}_{jt})\alpha_{jt} + M(1 - \theta_{jt,1}) \tag{37}$$

$$(\bar{u}_{jt} - \hat{u}_{jt})\alpha_{jt} - M(1 - \theta_{jt,-1}) \leq \xi_{jt} \leq (\bar{u}_{jt} - \hat{u}_{jt})\alpha_{jt} + M(1 - \theta_{jt,-1}) \tag{38}$$

$$\theta_{jt,-1} \in \{0, 1\}, \theta_{jt,0} \in \{0, 1\}, \theta_{jt,1} \in \{0, 1\}, \tag{39}$$

where M is a sufficiently large constant, ensures $\xi_{jt} = (\bar{u}_{jt} + \hat{u}_{jt})\alpha_{jt}$ only when $\theta_{jt,1} = 1$ (i.e., $\sigma_{jt} = 1$), ensures $\xi_{jt} = (\bar{u}_{jt} - \hat{u}_{jt})\alpha_{jt}$ only when $\theta_{jt,-1} = 1$ (i.e., $\sigma_{jt} = -1$), and ensures $\xi_{jt} = \bar{u}_{jt}\alpha_{jt}$ only when $\theta_{jt,0} = 1$ (i.e., $\sigma_{jt} = 0$). Accordingly, constraints (34) can be replaced with

$$\sum_{j\in J}\sum_{t=1}^{T}(\theta_{jt,-1} + \theta_{jt,1}) \leq \Gamma. \tag{40}$$

Decision variables u_{jt} and constraints (31) now become redundant, and thus can be removed.

Accordingly, the second-stage max-min problem of problem R, which defines function $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$ in (18)–(19), can be reformulated as the following maximization MILP:

$$F_{\text{RP}}(\mathbf{y}, \mathbf{z}) = \max_{(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\lambda},\boldsymbol{\xi},\boldsymbol{\theta})} -\sum_{j\in J}\sum_{t=1}^{T}\xi_{jt} + \sum_{i\in I} b_i y_i \beta_i - \sum_{i\in I} c_i y_i \gamma_i - \sum_{i\in I}\sum_{j\in J}\sum_{k=1}^{n_{ij}} s_{ijk} z_{ij} \lambda_{ijk} + \sum_{j\in J} H_{j0}\alpha_{j1} \tag{41}$$

s.t. constraints (23)–(28), (35)–(39), and (40).

Let $\mathcal{S}$ indicate the set of all feasible solutions $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\xi}, \boldsymbol{\theta})$ of the reformulation (41) for $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$. Let ϕ indicate the value of $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$. By (41), we know that ϕ cannot be less than $-\sum_{j\in J}\sum_{t=1}^{T}\xi_{jt} + \sum_{i\in I} b_i y_i \beta_i - \sum_{i\in I} c_i y_i \gamma_i - \sum_{i\in I}\sum_{j\in J}\sum_{k=1}^{n_{ij}} s_{ijk} z_{ij} \lambda_{ijk} + \sum_{j\in J} H_{j0}\alpha_{j1}$ for every feasible solution $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\xi}, \boldsymbol{\theta})$ of the reformulation (41) for $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$. Hence, we can reformulate problem R as the following minimization MILP, which is referred to as MILP_{R}:

$$(\text{MILP}_{\text{R}}) \qquad \min_{(\mathbf{y},\mathbf{z})} \; \phi \tag{42}$$

$$\text{s.t.} \quad \phi \geq -\sum_{j\in J}\sum_{t=1}^{T}\xi_{jt} + \sum_{i\in I} b_i y_i \beta_i - \sum_{i\in I} c_i y_i \gamma_i - \sum_{i\in I}\sum_{j\in J}\sum_{k=1}^{n_{ij}} s_{ijk} z_{ij} \lambda_{ijk} + \sum_{j\in J} H_{j0}\alpha_{j1},$$

$$\forall(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\lambda},\boldsymbol{\xi},\boldsymbol{\theta}) \in \mathcal{S}, \quad (43)$$

$$\text{constraints, (3), (6), (7), (9), (11), and (12).} \quad (44)$$

3.3 Solution Method for Problem R

Based on the reformulations in Section 3.2, we develop a solution method for problem R, by a constraint generation approach proposed in Remli and Rekik (2013). Let $\mathcal{S}' \subseteq \mathcal{S}$ indicate a subset of values of variables $(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\lambda},\boldsymbol{\xi},\boldsymbol{\theta})$ that satisfy constraints (23)-(28) and (35)-(39) of the MILP reformulation (41) for $F_{\mathrm{RP}}(\mathbf{y},\mathbf{z})$, denoted by $(\boldsymbol{\alpha}^{(p)},\boldsymbol{\beta}^{(p)},\boldsymbol{\gamma}^{(p)},\boldsymbol{\lambda}^{(p)},\boldsymbol{\xi}^{(p)},\boldsymbol{\theta}^{(p)})$ for $p = 1, 2, \ldots, |\mathcal{S}'|$. From the MILP reformulation (model $\mathrm{MILP_R}$) of problem R in (42)–(44), we obtain the following relaxation of problem R, which is also a MILP and referred to as the master problem (MP):

$$\text{(MP)} \quad \min_{(\mathbf{y},\mathbf{z})} \ \phi \quad (45)$$

$$\text{s.t.} \quad \phi \geq -\sum_{j\in J}\sum_{t=1}^{T}\xi_{jt}^{(p)} + \sum_{i\in I} b_i y_i \beta_i^{(p)} - \sum_{i\in I} c_i y_i \gamma_i^{(p)} - \sum_{i\in I}\sum_{j\in J}\sum_{k=1}^{n_{ij}} s_{ijk} z_{ij} \lambda_{ijk}^{(p)} + \sum_{j\in J} H_{j0}\alpha_{j1}^{(p)},$$
$$\forall(\boldsymbol{\alpha}^{(p)},\boldsymbol{\beta}^{(p)},\boldsymbol{\gamma}^{(p)},\boldsymbol{\lambda}^{(p)},\boldsymbol{\xi}^{(p)},\boldsymbol{\theta}^{(p)}) \in \mathcal{S}', \quad (46)$$

$$\text{constraints (3), (6), (7), (9), (11), and (12).} \quad (47)$$

When being applied to solve problem R, the constraint generation method iteratively extends the set $\mathcal{S}'$ until the optimal solution is found or a given time limit is exceeded. In each iteration $l = 1, 2, \ldots$, it goes through the following three steps:

1. Solve the relaxed model MP with $\mathcal{S}' = \{(\boldsymbol{\alpha}^{(p)},\boldsymbol{\beta}^{(p)},\boldsymbol{\gamma}^{(p)},\boldsymbol{\lambda}^{(p)},\boldsymbol{\xi}^{(p)},\boldsymbol{\theta}^{(p)}) : p = 1, 2, \ldots, l-1\}$ to obtain its optimal solution $(\mathbf{y},\mathbf{z})$ together with the optimal objective value ϕ^*;

2. Solve the MILP reformulation (41) for $F_{\mathrm{RP}}(\mathbf{y},\mathbf{z})$ to obtain its optimal solution $(\boldsymbol{\alpha}^{(l)},\boldsymbol{\beta}^{(l)},\boldsymbol{\gamma}^{(l)},\boldsymbol{\lambda}^{(l)},$ $\boldsymbol{\xi}^{(l)},\boldsymbol{\theta}^{(l)})$;

3. If $\phi^* \geq F_{\mathrm{RP}}(\mathbf{y},\mathbf{z})$, then ϕ^* must be the optimal objective value of problem R, and return $(\mathbf{y},\mathbf{z})$ as the optimal solution of problem R; otherwise add $(\boldsymbol{\alpha}^{(l)},\boldsymbol{\beta}^{(l)},\boldsymbol{\gamma}^{(l)},\boldsymbol{\lambda}^{(l)},\boldsymbol{\xi}^{(l)},\boldsymbol{\theta}^{(l)})$ to $\mathcal{S}'$ so that a new valid constraint, $\phi \geq F_{\mathrm{RP}}(\mathbf{y},\mathbf{z})$, i.e., $\phi \geq -\sum_{j\in J}\sum_{t=1}^{T}\xi_{jt}^{(l)} + \sum_{i\in I} b_i y_i \beta_i^{(l)} - \sum_{i\in I} c_i y_i \gamma_i^{(l)} - \sum_{i\in I}\sum_{j\in J}\sum_{k=1}^{n_{ij}} s_{ijk} z_{ij} \lambda_{ijk}^{(l)} + \sum_{j\in J} H_{j0}\alpha_{j1}^{(l)}$, is added to (46) of MP.

When the iteration stops, the optimal objective value of the relaxed model provides a valid lower bound for problem R, and the minimum value of $F_{\mathrm{RP}}(\mathbf{y}, \mathbf{z})$ over all $(\mathbf{y}, \mathbf{z})$ found in Step 1 of the iterations provides a valid upper bound for problem R. See Appendix C of the online companion for more details of our implementation of this method, where we illustrate how initial values of $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\xi}, \boldsymbol{\theta})$ are generated for set $\mathcal{S}'$, and how speedup techniques from existing literature on two-stage robust optimization (see, e.g., Remli et al., 2019) are applied and extended.

4 Computational Experiments

In this section, we report on the computational experiments carried out to test the effectiveness of the newly proposed models and solution method for a shipper to utilize liner service schedules in liner service procurement. We apply the constraint generation method described in Section 3.3 to solve model $\mathrm{MILP}_{\mathrm{R}}$ in (42)–(44) of the two-stage robust optimization problem R, which is referred to as robust optimization model $\mathrm{MILP}_{\mathrm{R}}$ in this section. Our experiments thus have two main objectives, including (i) to test the performance of the constraint generation method in solving the robust optimization model $\mathrm{MILP}_{\mathrm{R}}$; and (ii) to evaluate the advantage of solutions obtained from the robust optimization model $\mathrm{MILP}_{\mathrm{R}}$ over solutions obtained from other benchmark models, showing the value of utilizing liner service schedules in the liner service procurement, as well as the robustness of solutions against demand uncertainties.

We illustrate the generation of test instances in Section 4.1, the benchmark models for comparison in Section 4.2, and discuss the computational results in Section 4.3. All our experiments were coded in Java, and run on a computer with an Intel(R) Core(TM) i7-8700 CPU clocked at 3.20 GHz and 64 GB RAM, using the optimization solver Gurobi (version 8.1.1) to solve linear programs and (mixed) integer linear programs.

4.1 Instance Generation

To thoroughly test the models and solution method, we generate random test instances of different sizes, with parameters that are representative in practice. The number of periods T of the planning horizon is fixed to be 420 days (60 weeks), which include 56 days (8 weeks) of buffering periods, followed by 364 days (52 weeks, i.e., about a year) of demand periods, for the following reason: Due to the transit times of liner services, which can be as large as 8 weeks in practice, demands for cargo pickups at their destinations during the beginning of the demand periods need to be satisfied by liner services that depart from their origins before the demand periods. Thus, we include 56 days (8 weeks) of buffering periods at the beginning of the planning horizon, during which time only liner service schedules need to be taken into account, while the shipper's cargo pickups do not (since they shall be satisfied in the last planning horizon). Moreover, the number of carriers $|I|$ is fixed to be 10. Instance sizes vary as the number of lanes $|J|$ is taken to be 5, 10, 20, 40, 80, and 160. Such configurations cover many situations in practice, which are planned for a year, having about 10 carriers and tens of lanes. For each size of J, we generate a set of five instances randomly, for which the detailed parameter settings are illustrated in Appendix D of the online companion.

4.2 Benchmark Models for Comparison

In our experiments, we compare solutions obtained from the robust optimization model MILP_R with the following two benchmark models:

(i) Deterministic MILP model MILP_D defined in Section 2.2, where carriers' liner service schedules are taken into account, but cargo demands u_{jt} for each lane $j \in J$ and period $t \in \{1, 2, \ldots, T\}$ are deterministic and are given by their norm values $\bar{u}_{jt}$;

(ii) Deterministic MILP model MILP_DNS defined in Appendix E of the online companion, where carriers' liner service schedules are not taken into account, and cargo demands u_j for lane j are deterministic and are given by their norm values $\sum_{t=1}^{T} \bar{u}_{jt}$.

It can be seen that the values of decision variables $(\mathbf{y}, \mathbf{z})$ in any feasible solutions of these two benchmark models are also feasible in the first stage problem of the two-stage robust optimization problem R. By comparing the second-stage optimal objective value of problem R for solutions obtained from model MILP_R with solutions obtained from these two benchmark models, MILP_D and MILP_DNS, we can demonstrate the value of utilizing liner service schedules in liner service procurement, as well as the robustness of solutions against demand uncertainties.

4.3 Results and Discussions

For each randomly generated instance, we apply the constraint generation method to the robust optimization model $\mathrm{MILP_R}$ with a time limit of 10 hours, and use UB and LB denote the best upper bound and the best lower bound found. The budget of uncertainty Γ is set to be $\epsilon \times 52 \times |J|$ with $\epsilon = 0.5$. We compute an **optimality gap**, defined as $(\mathrm{UB} - \mathrm{LB})/\mathrm{LB} \times 100\%$, representing the gap between the upper and lower bound, which measures the quality of the upper and the lower bounds as a performance indicator of the constraint generation method.

For each instance, we also apply the optimization solver to solve the two benchmark models, $\mathrm{MILP_D}$ and $\mathrm{MILP_{DNS}}$, with the same time limit of 10 hours. For each benchmark model, we then compute an **improvement ratio**, defined as $(F_{\mathrm{RP}}(\mathbf{y}, \mathbf{z}) - \mathrm{UB})/\mathrm{UB} \times 100\%$, where $(\mathbf{y}, \mathbf{z})$ is obtained from the best solution found for the benchmark model in our experiment, and $F_{\mathrm{RP}}(\mathbf{y}, \mathbf{z})$ is computed by solving the MILP reformulation (41). The ratio indicates the improvement of the best solution found for problem R against $(\mathbf{y}, \mathbf{z})$ with regard to their objective values in problem R. A positive value of the ratio implies that the robust optimization model $\mathrm{MILP_R}$ has an advantage in producing better solutions with regard to the total cost for the worst case in the demand uncertainty set.

The computational results are presented in Table 1. For the optimality gap of model $\mathrm{MILP_R}$, and the improvement ratios of model $\mathrm{MILP_R}$ over models $\mathrm{MILP_D}$ and $\mathrm{MILP_{DNS}}$, their maximum, average, and minimum values over the five randomly generated instances of each instance set are shown in different columns. From the optimality gap of model $\mathrm{MILP_R}$, we can see that the constraint generation method is effective in producing optimal or close to optimal solutions for small-sized and median-sized instances of up to 80 lanes, with the optimality gap not exceeding 3.9% at maximum. For the large-sized instances of 160 lanes, the algorithm is still effective in producing solutions of good qualities, with the optimality gap varying from 5.4% to 11.1%.

From the improvement ratios, we can see that solutions generated from model $\mathrm{MILP_R}$ outperform the two benchmark models, with improvements ranging from 6.5% to 19.6% when compared with model $\mathrm{MILP_D}$, and from 14.2% to 40.9% when compared with model $\mathrm{MILP_{DNS}}$. *This demon-*

Table 1: Comparative results of robust optimization model MILP_R, deterministic model MILP_D (with liner service schedules), and deterministic model MILP_DNS (without liner service schedules).

Instance Set			MILP_R : Optimality gap			MILP_R vs. MILP_D : Improvement ratio			MILP_R vs. MILP_DNS : Improvement ratio		
$\|J\|$	$\|I\|$	T	min	avg	max	min	avg	max	min	avg	max
5	10	420	0.0%	0.0%	0.0%	11.2%	12.9%	15.9%	14.2%	29.8%	40.9%
10	10	420	0.0%	0.0%	0.0%	9.1%	12.6%	17.2%	30.5%	33.8%	38.9%
20	10	420	0.0%	0.4%	1.4%	6.5%	12.7%	19.6%	23.2%	30.1%	35.3%
40	10	420	0.9%	1.6%	2.6%	10.1%	12.5%	15.1%	26.2%	28.5%	30.7%
80	10	420	2.7%	3.3%	3.9%	10.6%	11.6%	12.7%	26.6%	29.9%	32.2%
160	10	420	5.4%	7.5%	11.1%	9.6%	10.7%	13.5%	24.5%	27.0%	29.9%

strates that by considering liner service schedules and demand uncertainties, our robust optimization model and its solution method can help shippers make robust decisions with a considerable total cost reduction in sourcing liner services without precise information about demand probabilities.

Moreover, we compare the solutions from model MILP_R and the two benchmark models under different levels of uncertainty budget Γ as well as maximum demand deviations $\hat{u}_{jt}$. The results are presented in Appendix F of the online companion, demonstrating that the trends of improvements made by the solutions from model MILP_R grow with increasing levels of Γ and $\hat{u}_{jt}$, respectively.

Finally, for comparative results of the robust optimization model MILP_R and the stochastic programming model SP, see Appendix G of the online companion. The results show that by choosing Γ properly, the robust optimal solution obtained from model MILP_R can help shippers achieve cost effectiveness with respect to both the worst-case criterion and the expected criterion.

5 Conclusions

This work studies how to utilize liner service schedules for shippers to optimize their sourcing decisions on selecting ocean carriers to transport container cargoes through multiple shipping lanes all around the world. Information on liner service schedules is available to shippers, and is widely recognized to have a significant impact on shippers' sourcing of liner services. However, this has rarely been studied in existing literature. To fill this research gap, we first introduce a deterministic optimization problem, with cargo demands given in advance, and with inventory holding costs included so as to capture the impact of liner service schedules on shippers' operating costs. We formulate the problem as a mixed integer linear program, which is then simplified by exploiting a totally unimodular property, which can thus be solved directly by a general optimization solver. To further capture the impact of liner service schedules on shippers' resilience against unexpected cargo demands, we then study a two-stage robust optimization problem, using a probability-free uncertainty set to enclose all the possible demand realizations. To tackle the challenge of solving this two-stage robust optimization problem, we derive novel reformulations of the problem, and based on them develop a solution method that adopts a constraint-generation approach. Extensive numerical experiments have been conducted to demonstrate the effectiveness of the newly proposed models and solution method, as well as the value of utilizing liner service schedules in shippers' liner service procurement, together with the robustness of decision making against demand uncertainties.

This work has opened up several directions for future research. First, although the solution method developed in this work for the two-stage robust optimization problem can produce good quality solutions for instances of more than a hundred shipping lanes, it may not be efficient when thousands of lanes are involved, which does occur in some situations, though not often. It is thus of great interest to develop efficient heuristic methods for solving very large scaled problem instances. Second, in this work, although we only take into account the uncertainty of cargo demands, the models and solution method proposed can also be extended to incorporate uncertainty of transit times. Shippers' resilience to this is affected by carriers' liner service schedules as well. However, when transit time uncertainty is also taken into account, the formulations of the two-stage robust optimization problem will contain many more decision variables and constraints, so that more efficient solution methods need to be developed in the future. Finally, in addition to logistics between seaports, ocean carriers' liner service schedules also affect shippers' logistics between seaports and inland facilities, such as factories and stores. It would be beneficial for shippers to jointly optimize decision making for both the sourcing of liner shipping services and the planning of inland logistics, which is more complicated to solve and deserves future studies.

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Online Companion to "Optimal Global Liner Service Procurement by Utilizing Liner Service Schedules"

Chung-Yee Lee, Shengnan Shu, and Zhou Xu

A Proofs

A.1 Proof of Theorem 1

Given any $y_i \in \{0,1\}$ and $z_{ij} \in \{0,1\}$ for $i \in I$ and $j \in J$, to reformulate model ILP$_\text{D}$ defined in (1)–(13) as a network flow model, consider a digraph $G = (V, A)$. The node set V of G includes (i) nodes η_i for each carrier $i \in I$; (ii) nodes ρ_{jt} for each lane $j \in J$ and period $t \in \{1, 2, \ldots, T\}$; and (iii) node s and s' as the source and sink nodes. The arc set A of G includes (i) arcs (s, η_i) for each carrier $i \in I$; (ii) arcs $\zeta_{ijk} = (\eta_i, \rho_{jt})$ for each schedule $(i, j, k) \in S_{jt}$ for some $t \in \{1, 2, \ldots, T\}$; (iii) arcs $(\rho_{jt}, \rho_{j,t+1})$ for each lane $j \in J$ and period $t \in \{1, 2, \ldots, T-1\}$; (iv) arcs (s, ρ_{jt}) for each lane $j \in J$ and period $t \in \{1, 2, \ldots, T\}$; (v) arcs (s', s) and (ρ_{jT}, s') for each lane $j \in J$. See Figure A.1 for an illustration.

Accordingly, for each $i \in I$, the flow over arc (s, η_i) of G, denoted by $f_{(s,\eta_i)}$, corresponds to the total number of containers shipped by carrier i, which equals $\sum_{j \in J} \sum_{k=1}^{n_{ij}} x_{ijk}$ of model ILP$_\text{D}$. Thus, from (5), we know that the flow $f_{(s,\eta_i)}$ needs to satisfy a bounding constraint, i.e., $b_i y_i \le f_{(s,\eta_i)} \le c_i y_i$. From (1), we know that the unit flow cost over arc (s, η_i) of G equals 0.

For each schedule $(i, j, k) \in S_{jt}$ for some $t \in \{1, 2, \ldots, T\}$, the flow over arc $\zeta_{ijk} = (\eta_i, \rho_{jt})$ of G, denoted by $f_{\zeta_{ijk}}$, corresponds to the total number of containers shipped by carrier i through schedule (i, j, k), which arrives at the destination port of lane j in period t, and it equals x_{ijk} of model ILP$_\text{D}$. Thus, from (8), we know that the flow $f_{\zeta_{ijk}}$ needs to satisfy an upper bounding (capacity) constraint, i.e., $f_{\zeta_{ijk}} \le s_{ijk}$. From (1), we know that the unit flow cost over arc ζ_{ijk} of

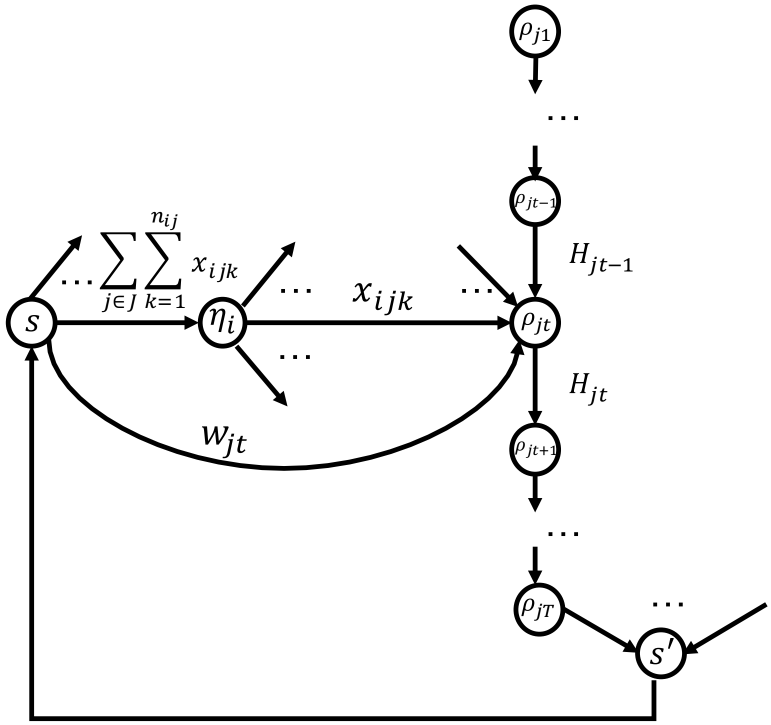

Figure A.1: Illustration of the minimum cost flow problem constructed for the proof of Theorem 1.

G equals $(r_{ij} + h'_{ij}\tau_{ijk})$.

For each $j \in J$ and $t \in \{1, \ldots, T-1\}$, the flow over arc $(\rho_{jt}, \rho_{j,t+1})$, denoted by $f_{(\rho_{jt},\rho_{jt+1})}$, corresponds to the inventory of cargo of lane j held in the destination port in period t, which equals H_{jt} of model $\mathrm{ILP_D}$. From (1) we know that the unit flow cost over arc (ρ_{jt}, ρ_{jt+1}) equals h_j.

For each $j \in J$ and $t \in \{1, \ldots, T\}$, the flow over arc (s, ρ_{jt}), denoted by $f_{(s,\rho_{jt})}$, corresponds to the total number of containers that need to be shipped by spot-market shipping services, and equals w_{jt} of model $\mathrm{ILP_D}$. From (1) we know that the unit flow cost over arc (s, ρ_{jt}) equals e_{jt}.

Flows over arcs (s', s) and (ρ_{jT}, s') for $j \in J$ are for balancing and have zero costs. Moreover, from (2), we know that node ρ_{j1} has a demand of $(u_{j1} - H_{j0})$, and that each other node ρ_{jt} for $j \in J$ and $t \in \{2, 3, \ldots, T\}$ has a demand of u_{jt}. For the source node s, its demand is $-\sum_{j\in J}\sum_{t=1}^{T} u_{jt} + \sum_{j\in J} H_{j0}$. For other nodes, their demands are all zero.

Hence, given any $y_i \in \{0, 1\}$ and $z_{ij} \in \{0, 1\}$ for $i \in I$ and $j \in J$, we obtain that every feasible solution of variables x_{ijk}, H_{jt}, and w_{jt} to model $\mathrm{ILP_D}$ can be transformed to a feasible solution of flows $f_{(s,\eta_i)}$, $f_{\zeta_{ijk}}$, $f_{(\rho_{jt},\rho_{j,t+1})}$, $f_{(s,\rho_{jt})}$, $f_{(s',s)}$, and $f_{(\rho_{jT},s')}$ to the minimum cost network flow problem constructed above, with the same total cost, and vice versa. Thus, model $\mathrm{ILP_D}$ can be reformulated as a minimum cost network flow model. Since the ILP formulation of the minimum cost network flow model is known to be totally unimodular, it has the same optimal objective value as its LP relaxation. Therefore, the integral constraints on flow variables of the minimum cost network flow model can be relaxed, and thus, the integral constraints on variables x_{ijk}, H_{jt}, and w_{jt} of model $\mathrm{ILP_D}$ can also be relaxed. Therefore, model $\mathrm{ILP_D}$ is also totally unimodular. Theorem 1 is proved.

A.2 Proof of Lemma 1

Consider any given $\mathbf{y}$ and $\mathbf{z}$ that satisfy constraints in (17), and any given cargo demand $\mathbf{u} \in \mathcal{U}(\Gamma)$. To prove Lemma 1, we first label the lanes in J by $1, 2, \ldots, |J|$, and then we show that the following setting of decisions $(\mathbf{x}, \mathbf{H}, \mathbf{w})$ is a feasible solution to the inner minimization problem of the second stage problem of problem R defined in (20)-(21):

- For each carrier $i \in I$, and each lane $j = 1, 2, \ldots, |J|$, sequentially, we construct values of x_{ijk} as follows for $k \in \{1, 2, \ldots, n_{ij}\}$ (based on values of y_i and z_{ij} given in $\mathbf{y}$ and $\mathbf{z}$): If $y_i = 0$ or $z_{ij} = 0$, which means that carrier i is not selected to serve lane j, then we accordingly set $x_{ijk} = 0$ for all $k \in \{1, 2, \ldots, n_{ij}\}$. Otherwise, for each $k = 1, 2, 3, \ldots, n_{ij}$, , sequentially, we set $x_{ijk} = \min\{s_{ijk}, c_i - \sum_{j'=1}^{j-1} \sum_{k'=1}^{n_{ij'}} x_{ij'k'} - \sum_{k'=1}^{k-1} x_{ijk'}\}$, which is the largest possible value for x_{ijk} without exceeding the schedule capacity s_{ijk} and the total capacity c_i for carrier i.

- For each lane $j \in J$, and for each period $t = 1, 2, \ldots, T$, sequentially, we construct values of w_{jt} and H_{jt} as follows (based on values of u_{jt} given in $\mathbf{u}$): if $H_{j,t-1} - u_{jt} + \sum_{(i,j,k) \in S_{jt}} x_{ijk} < 0$ then we set $w_{jt} = -(H_{j,t-1} - u_{jt} + \sum_{(i,j,k) \in S_{jt}} x_{ijk})$ and $H_{jt} = 0$, so that cargoes that cannot be shipped by the selected carriers' services are shipped by spot-market carriers. Otherwise, no spot-market carriers are needed, and thus, we set $w_{jt} = 0$ and $H_{jt} = H_{j,t-1} - u_{jt} + \sum_{(i,j,k) \in S_{jt}} x_{ijk}$.

It can be seen that $\sum_{j \in J} \sum_{k=1}^{n_{ij}} x_{ijk} \leq c_i y_i$ for $i \in I$, and that x_{ijk} for $i \in I$ and $j \in J$ and $k \in \{1, 2, \ldots, n_{ij}\}$ are all integers and satisfy that $0 \leq x_{ijk} \leq s_{ijk} z_{ij}$. These imply that constraints (5) and (8) in (21) are satisfied. It can also be seen that for each $i \in I$, the setting above satisfies that $\sum_{j\in J}\sum_{k=1}^{n_{ij}} x_{ijk} = \min\{\sum_{j\in J}\sum_{k=1}^{n_{ij}} s_{ijk}z_{ij}, c_i\}$. Thus, since $b_i \le c_i$, and since $b_iy_i \le \sum_{j\in J}\sum_{k=1}^{n_{ij}} s_{ij}z_{ij}$, which is due to constraints (9) in (17), we obtain that $b_iy_i \le \sum_{j\in J}\sum_{k=1}^{n_{ij}} x_{ijk}$, which implies that constraints (4) in (21) are also satisfied. Moreover, it can be seen that $H_{jt} = H_{j,t-1} - u_{jt} + \sum_{(i,j,k)\in S_{jt}} x_{ijk} + w_{jt}$ for $j \in J$ and $t \in \{1,2,\ldots,T\}$, and thus, constraints (2) in (21) are satisfied. Hence, the above setting of decisions $(\mathbf{x},\mathbf{H},\mathbf{w})$ forms a feasible solution to the inner minimization problem of the second stage problem of problem R, which completes the proof of Lemma 1.

A.3 Proof of Theorem 2

Consider a relaxation of the nonlinear maximization problem defined in (29)–(34) with the integral constraints (32) of $\mathbf{u}$ being relaxed, which is referred to as relaxation A in this proof. Consider any optimal solution, denoted by $(\boldsymbol{\alpha}^*,\boldsymbol{\beta}^*,\boldsymbol{\gamma}^*,\boldsymbol{\lambda}^*,\mathbf{u}^*,\boldsymbol{\sigma}^*)$, of relaxation A. By fixing $\boldsymbol{\alpha}=\boldsymbol{\alpha}^*$, $\boldsymbol{\beta}=\boldsymbol{\beta}^*$, $\boldsymbol{\gamma}=\boldsymbol{\gamma}^*$, $\boldsymbol{\lambda}=\boldsymbol{\lambda}^*$, and $\mathbf{u}=\mathbf{u}^*$, relaxation A becomes a maximization problem over only decisions $\boldsymbol{\sigma}$, which, according to (29)–(34), is equivalent to the following LP:

$$\max_{\boldsymbol{\sigma}} \quad -\sum_{j\in J}\sum_{t=1}^{T}(\hat{u}_{jt}\alpha_{jt}^*)\sigma_{jt}, \tag{A.1}$$

$$\text{s.t.} \quad -1 \le \sigma_{jt} \le 1,\ \forall j \in J, t \in \{1,2,\ldots,T\}, \tag{A.2}$$

$$\sum_{j\in J}\sum_{t=1}^{T} |\sigma_{jt}| \le \Gamma, \tag{A.3}$$

where constraint (A.3), which defines a polyhedron of $\boldsymbol{\sigma}$, can be replaced by a finite set of linear constraints. Since $(\boldsymbol{\alpha}^*,\boldsymbol{\beta}^*,\boldsymbol{\gamma}^*,\boldsymbol{\lambda}^*,\mathbf{u}^*,\boldsymbol{\sigma}^*)$ is an optimal solution to relaxation A, $\boldsymbol{\sigma}^*$ is an optimal solution to the LP above in (A.1)–(A.3). Moreover, for any optimal solution $\boldsymbol{\sigma}$ to the LP above, $(\boldsymbol{\alpha}^*,\boldsymbol{\beta}^*,\boldsymbol{\gamma}^*,\boldsymbol{\lambda}^*,\mathbf{u},\boldsymbol{\sigma})$ with $u_{jt} = \bar{u}_{jt} + \hat{u}_{jt}\sigma_{jt}$ for $j \in J$ and $t \in \{1,2,\ldots,T\}$ forms a feasible solution to relaxation A of the same objective value as that of $(\boldsymbol{\alpha}^*,\boldsymbol{\beta}^*,\boldsymbol{\gamma}^*,\boldsymbol{\lambda}^*,\mathbf{u}^*,\boldsymbol{\sigma}^*)$. Thus, such $(\boldsymbol{\alpha}^*,\boldsymbol{\beta}^*,\boldsymbol{\gamma}^*,\boldsymbol{\lambda}^*,\mathbf{u},\boldsymbol{\sigma})$ is an optimal solution to relaxation A.

It is known that for any LP, there exists an optimal solution that is an extreme point of its feasible region. Consider such an optimal solution $\boldsymbol{\sigma}$ to the LP above in (A.1)–(A.3). Since $\boldsymbol{\sigma}$ is an extreme point of the feasible region defined in (A.2) and (A.3), we have $\sigma_{jt} \in \{-1,0,1\}$. Since both $\bar{u}_{jt}$ and $\hat{u}_{jt}$ are non-negative integers with $\bar{u}_{jt} \ge \hat{u}_{jt}$ for all $j \in J$ and $t \in \{1,2,\ldots,T\}$, we know that $u_{jt} = \bar{u}_{jt} + \hat{u}_{jt}\sigma_{jt}$ for $j \in J$ and $t \in \{1, 2, \ldots, T\}$ are all non-negative integers. Thus, we obtain that $(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*, \boldsymbol{\gamma}^*, \boldsymbol{\lambda}^*, \mathbf{u}, \boldsymbol{\sigma})$, which is an optimal solution to relaxation A, is also an optimal solution to the nonlinear maximization problem defined in (29)–(34). Theorem 2 is proved.

B Stochastic Programming Model

The model MILP_D of problem D can be extended to formulate a stochastic programming model for the problem under stochastic cargo demands, where $\tilde{\mathcal{U}}$ indicates a scenario set of all possible values of cargo demands $\mathbf{u} = (u_{jt})_{j \in J, t \in \{1,2,\ldots,T\}}$, and $\text{Prob}(\mathbf{u})$ indicates the probability of each possible value of cargo demands $\mathbf{u} \in \tilde{\mathcal{U}}$, aiming to minimize the expected value of the total cost:

$$\text{(SP)}\ \ \min \quad \sum_{\mathbf{u} \in \tilde{\mathcal{U}}} \text{Prob}(\mathbf{u}) \Big[\sum_{i \in I} \sum_{j \in J} \sum_{k=1}^{n_{ij}} (r_{ij} + h'_{ij}\tau_{ijk}) x_{ijk}(\mathbf{u}) + \sum_{t=1}^{T} \sum_{j \in J} h_j H_{jt}(\mathbf{u}) + \sum_{t=1}^{T} \sum_{j \in J} e_{jt} w_{jt}(\mathbf{u}) \Big] \tag{B.1}$$

$$\text{s.t.} \quad H_{jt}(\mathbf{u}) = H_{j,t-1}(\mathbf{u}) - u_{jt} + \sum_{(i,j,k) \in S_{jt}} x_{ijk}(\mathbf{u}) + w_{jt}(\mathbf{u}), \quad \forall j \in J, t \in \{1, 2, \ldots, T\}, \mathbf{u} \in \tilde{\mathcal{U}}, \tag{B.2}$$

$$\sum_{i \in I} y_i \leq m, \tag{B.3}$$

$$b_i y_i \leq \sum_{j \in J} \sum_{k=1}^{n_{ij}} x_{ijk}(\mathbf{u}),\ \forall i \in I, \mathbf{u} \in \tilde{\mathcal{U}}, \tag{B.4}$$

$$\sum_{j \in J} \sum_{k=1}^{n_{ij}} x_{ijk}(\mathbf{u}) \leq c_i y_i,\ \forall i \in I, \mathbf{u} \in \tilde{\mathcal{U}}, \tag{B.5}$$

$$\sum_{j \in J} z_{ij} \leq |J| y_i,\ \forall i \in I, \tag{B.6}$$

$$L_j \leq \sum_{i \in I} z_{ij} \leq U_j,\ \forall j \in J, \tag{B.7}$$

$$x_{ijk}(\mathbf{u}) \leq s_{ijk} z_{ij},\ \forall i \in I, j \in J, k \in \{1, 2, \ldots, n_{ij}\}, \mathbf{u} \in \tilde{\mathcal{U}}, \tag{B.8}$$

$$b_i y_i \leq \sum_{j \in J} \sum_{k=1}^{n_{ij}} s_{ijk} z_{ij},\ \forall i \in I, \tag{B.9}$$

$$x_{ijk}(\mathbf{u}) \geq 0,\ \forall i \in I, j \in J, k \in \{1, 2, \ldots, n_{ij}\}, \mathbf{u} \in \tilde{\mathcal{U}}, \tag{B.10}$$

$$y_i \in \{0, 1\},\ \forall i \in I, \tag{B.11}$$

$$z_{ij} \in \{0, 1\},\ \forall i \in I, j \in J, \tag{B.12}$$

$$H_{jt}(\mathbf{u}) \geq 0, w_{jt}(\mathbf{u}) \geq 0, \ \forall j \in J, t \in \{1, 2, \ldots, T\}, \mathbf{u} \in \tilde{\mathcal{U}}. \tag{B.13}$$

Model SP is a MILP. Following model MILP_D of problem D, model SP still uses binary variables y_i to indicate whether or not carrier i is selected, and binary variables z_{ij} to indicate whether or not carrier i serves lane j, for $i \in I$ and $j \in J$. Integer decision variables $x_{ijk}(\mathbf{u})$ for $i \in I$, $j \in J$, and $k \in \{1, 2, \ldots, n_{ij}\}$, as well as $H_{jt}(\mathbf{u})$ and $w_{jt}(\mathbf{u})$ for $j \in J$ and $t \in \{1, 2, \ldots, T\}$, represent recourse actions made after demands $\mathbf{u}$ are realized, and as in MILP_D of problem D, they indicate the number of containers allocated to schedule (i, j, k), the inventory level of cargo for lane j at its destination in period t, and the number of containers that need to be shipped by spot-market carriers for cargoes of lane j and that need to be picked up in period t. Integral constraints on variables $x_{ijk}(\mathbf{u})$, $H_{jt}(\mathbf{u})$, and $w_{jt}(\mathbf{u})$ are relaxed as in (B.10) and (B.13) by following an argument similar to the proof of Theorem 1.

Moreover, model SP can be reformulated as the following two-stage optimization model:

$$\min_{(\mathbf{y},\mathbf{z})} \quad F_{\text{SP}}(\mathbf{y}, \mathbf{z}) \tag{B.14}$$

$$\text{s.t.} \quad \text{constraints (B.3), (B.6), (B.7), (B.9), (B.11), and (B.12),} \tag{B.15}$$

where $F_{\text{SP}}(\mathbf{y}, \mathbf{z})$ represents the optimum objective value of the second stage problem given below:

$$F_{\text{SP}}(\mathbf{y}, \mathbf{z}) = \min_{(\mathbf{x}(\mathbf{u}),\mathbf{H}(\mathbf{u}),\mathbf{w}(\mathbf{u}))_{\mathbf{u}\in\tilde{\mathcal{U}}}} \sum_{\mathbf{u}\in\tilde{\mathcal{U}}} \text{Prob}(\mathbf{u}) \Bigg[\sum_{i\in I} \sum_{j\in J} \sum_{k=1}^{n_{ij}} (r_{ij} + h'_{ij}\tau_{ijk}) x_{ijk}(\mathbf{u}) + \\ + \sum_{t=1}^{T} \sum_{j\in J} h_j H_{jt}(\mathbf{u}) + \sum_{t=1}^{T} \sum_{j\in J} e_{jt} w_{jt}(\mathbf{u}) \Bigg] \tag{B.16}$$

$$\text{s.t.} \quad \text{constraints (B.2), (B.4), (B.5), (B.8), (B.10), and (B.13).} \tag{B.17}$$

C Implementation Details of the Constraint Generation Method for Problem R

At the beginning of the constraint generation method (described in Section 3.3), we need some initial values of $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\xi}, \boldsymbol{\theta})$ for the set $\mathcal{S}'$. To enhance the efficiency of the method, it is preferable to generate these initial values from demands $\mathbf{u}$ that are more likely to be the worst case in the uncertainty set $\mathcal{U}(\Gamma)$ for problem R. Specifically, we first sort pairs (j, t) for $j \in J$ and

$t \in \{1, 2, \ldots, T\}$ by non-increasing order of maximum deviations $\hat{u}_{jt}$, and choose the top Γ pairs of (j, t) with the largest maximum deviations $\hat{u}_{jt}$ to form a set Ω. We then construct $\mathbf{u}$ by setting $u_{jt} = \bar{u}_{jt} + \hat{u}_{jt}$ if $(j, t) \in \Omega$, and setting $u_{jt} = \bar{u}_{jt}$ otherwise, which represents the case of $\mathbf{u}$ in $\mathcal{U}(\Gamma)$ with the maximum total non-negative deviation of demands. For the constructed demand $\mathbf{u}$, we solve model MILP_D of the deterministic problem D, so as to obtain values of $(\mathbf{y}, \mathbf{z})$ from some of its best solutions found. We use these values of $(\mathbf{y}, \mathbf{z})$ to generate some initial values of $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\xi}, \boldsymbol{\theta})$ as in Step 2 and Step 3 of the constraint generation method, and add them to $\mathcal{S}'$.

During the iterations of the constraint generation method, we use LB and UB to indicate the best lower and upper bounds found, respectively, for problem R. When the relaxed model MP is solved in Step 1, the best lower bound found on ϕ^* is used to increase LB if it is larger than LB. When the MILP reformulation (41) for $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$ is solved in Step 2 for some $(\mathbf{y}, \mathbf{z})$, the best upper bound found on $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$ is used to reduce UB if it is smaller than UB.

In our implementation of the constraint generation method, we also apply the following two speedup techniques. First, in Step 1 of the method, we set a time limit L_1 for a general optimization solver to solve the relaxed model MP, and the solver returns the best solution of $(\mathbf{y}, \mathbf{z})$ found when it stops. For each solution $(\mathbf{y}, \mathbf{z})$ returned in Step 1, we then apply the optimization solver in Step 2 to solve the MILP reformulation (41) for $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$ with a time limit L_2, which returns the best B solutions of $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\xi}, \boldsymbol{\theta})$ that are found when the solver stops. Here L_1, L_2, and B are all given parameters. If the best upper bound found on ϕ^* is less than the best lower bound found on $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$, then we know $\phi^* < F_{\text{RP}}(\mathbf{y}, \mathbf{z})$, and accordingly we add all these B solutions of $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\xi}, \boldsymbol{\theta})$ to $\mathcal{S}'$ in Step 3 of the constraint generation method, so as to enhance its efficiency.

Second, when the optimization solver solves the MILP reformulation (41) for $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$ in Step 2, if it finds a lower bound on $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$ that exceeds UB, we know that any further search of the solution to the MILP reformulation (41) for $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$ will not lead to any decrease in UB, and that the best upper bound found on ϕ^*, which is less than or equal to UB, must be less than this lower bound found on $F_{\text{RP}}(\mathbf{y}, \mathbf{z})$, implying that $\phi^* < F_{\text{RP}}(\mathbf{y}, \mathbf{z})$. Thus, in this case, we terminate the solver, select the best B solutions of $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\xi}, \boldsymbol{\theta})$ found by the solver, and add them to $\mathcal{S}'$ as in Step 3 of the constraint generation method.

We also set an overall time limit L_0, given as a parameter. After each iteration, if the running time exceeds L_0, the constraint generation method stops. For the solution $(\mathbf{y}, \mathbf{z})$ obtained in the last iteration of the method, we use the optimization solver to compute $F_{\mathrm{RP}}(\mathbf{y}, \mathbf{z})$ again, so as to update UB with an accurate value of $F_{\mathrm{RP}}(\mathbf{y}, \mathbf{z})$, which, together with LB, is returned as the best lower bound and the best upper bound found on the optimal objective value of problem R.

In our numerical experiments reported in Section 4, we set parameters L_1 to be unlimited, set B to be 50 for instances with $|J| < 80$, and 200 for instances with $|J| \geq 80$, and set L_2 and L_0 to be 3 hours and 10 hours, respectively.

D Parameter Settings for Generation of Random Test Instances

For each size of $|J|$ in $\{5, 10, 20, 40, 80, 160\}$, given $T = 420$ and $|I| = 10$, we generate a set of five instances randomly with parameter settings illustrated below, most of which follow the setting in Hu et al. (2016), to reflect typical patterns and characteristics in some real data.

For each lane $j \in J$, its cargo pickup time is randomly chosen from $\{\text{Sunday}, \text{Monday}, \ldots, \text{Saturday}\}$ of each week with equal probability. For the cargo demand u_{jt} of lane j and pickup time t, its norm value $\bar{u}_{jt}$ is chosen randomly from a uniform distribution in $\{0, 1, \ldots, 50\}$, and its deviation $\hat{u}_{jt}$ is set to be $\mu_0 \times \bar{u}_{jt}$ with $\mu_0 = 0.9$. The budget of uncertainty Γ is set to be $\epsilon \times 52 \times |J|$ with $\epsilon = 0.5$.

To construct liner service schedules for each carrier $i \in I$ and each lane $j \in J$, we consider two types of regular services, including weekly services with 80% probability, and biweekly services with 20% probability, for which their departure times are randomly chosen from $\{\text{Sunday}, \text{Monday}, \ldots,$ $\text{Saturday}\}$ of each week with equal probability. By following this, we can determine the number of liner service schedules n_{ijk} provided by carrier i for lane j during the T periods of the planning horizon, as well as the departure time d_{ijk} for each schedule (i, j, k). We set the transit times t_{ijk} of schedules (i, j, k) with the same i and j all equal to $t_{ij} = \lceil \nu_{ij}^{(1)} \times \bar{t}_j \rceil$, where $\nu_{ij}^{(1)}$ and $\bar{t}_j$ are randomly chosen from uniform distributions in $[1.0 - \mu_j^{(1)}, 1.0 + \mu_j^{(1)}]$ and $[10, 50]$, respectively, and where $\mu_j^{(1)}$ is randomly chosen from a uniform distribution in $[0.05, 0.15]$. We then set the capacity s_{ijk} of each schedule (i, j, k) equal to s_{ij}/n_{ij}, where s_{ij} is randomly chosen from a uniform distribution in either $[0.2\sum_{t=1}^{T}\bar{u}_{jt}, 0.6\sum_{t=1}^{T}\bar{u}_{jt}]$, representing capacities offered by small-sized carriers, or $[0.8\sum_{t=1}^{T}\bar{u}_{jt}, 1.2\sum_{t=1}^{T}\bar{u}_{jt}]$, representing capacities offered by large-sized carriers. In or experiment, the proportions of small-sized and large-sized carriers are 60% and 40%, respectively.

The remaining parameters are determined as follows. For each lane $j \in J$, we fix $L_j = 1, U_j = 4$. For each carrier $i \in I$, we fix its total capacity $c_i = \sum_{j\in J} s_{ij}$, and randomly select the minimum quantity commitment b_i from a uniform distribution in $[0.1\sum_{j\in J}\sum_{t=1}^{T}\bar{u}_{jt}, 0.2\sum_{j\in J}\sum_{t=1}^{T}\bar{u}_{jt}]$. For each lane $j \in J$ and carrier $i \in I$, we set the shipping rate r_{ij} equal to $\nu_{ij}^{(2)}\bar{r}_j$ where $\bar{r}_j$ is randomly chosen from a uniform distribution in $[20, 100]$, and where $\nu_{ij}^{(2)}$ is randomly chosen from a uniform distribution either in $[1.0-\mu_j^{(2)}, 1.0]$, representing a low shipping rate, or in $[1.0, 1.0+\mu_j^{(2)}]$, representing a high shipping rate, where $\mu_j^{(2)}$ is randomly chosen from a uniform distribution in $[0.05, 0.3]$. In our experiment, each small-sized carrier has a 90% chance of offering a low shipping rate, and each large-sized carrier has a 90% chance of offering a high shipping rate. We then fix the spot-market shipping rate $e_{jt} = 3.0 \cdot \bar{r}_j$ for $j \in J$ and $t \in \{1, 2, \dots, T\}$, and set holding costs $h'_{ij} = 0.3 \times (r_{ij}/t_{ij})$ and $h_j = \nu_j^{(3)} \times \sum_{i\in I}(r_{ij}/t_{ij})/|I|$, for $i \in I$ and $j \in J$, where $\nu_j^{(3)}$ is randomly chosen from a uniform distribution in $[0.1, 0.5]$.

E Deterministic Model without Service Schedules

The model MILP_{DNS} of problem D can be modified as follows to formulate the problem without taking into account liner service schedules:

$$
\begin{aligned}
(\text{MILP}_{\text{DNS}}) \qquad \min \quad & \sum_{i\in I}\sum_{j\in J} r_{ij}x_{ij} + \sum_{j\in J} e_j w_j && \text{(E.1)}\\
\text{s.t.} \quad & \sum_{i\in I} x_{ij} + w_j = u_j,\ \forall j \in J, && \text{(E.2)}\\
& b_i y_i \le \sum_{j\in J} x_{ij},\ \forall i \in I, && \text{(E.3)}\\
& \sum_{j\in J} x_{ij} \le c_i y_i,\ \forall i \in I, && \text{(E.4)}\\
& \sum_{i\in I} y_i \le m, && \text{(E.5)}\\
& \sum_{j\in J} z_{ij} \le |J| y_i,\ \forall i \in I, && \text{(E.6)}
\end{aligned}
$$

$$L_j \leq \sum_{i \in I} z_{ij} \leq U_j, \ \forall j \in J, \tag{E.7}$$

$$x_{ij} \leq s_{ij} z_{ij}, \ \forall i \in I, j \in J, \tag{E.8}$$

$$b_i y_i \leq \sum_{j \in J} s_{ij} z_{ij}, \ \forall i \in I, \tag{E.9}$$

$$x_{ij} \geq 0, \ \forall i \in I, j \in J, \tag{E.10}$$

$$y_i \in \{0, 1\}, \ \forall i \in I, \tag{E.11}$$

$$z_{ij} \in \{0, 1\}, \ \forall i \in I, j \in J, \tag{E.12}$$

$$w_j \geq 0, \ \forall j \in J. \tag{E.13}$$

As in MILP_D of problem D, model MILP_DNS uses binary variables y_i to indicate whether or not carrier i is selected, and binary variables z_{ij} to indicate whether or not carrier i serves lane j, for $i \in I$ and $j \in J$. Since liner service schedules are not taken into account, integer variables x_{ij} are used to indicate the number of containers shipped by carrier i for lane j where $i \in I$ and $J \in J$, and integer variables w_j are used to indicate the number of containers that need to be shipped by spot-market carriers for cargoes of lane j where $j \in J$. Integral constraints on variables x_{ij} and w_j are relaxed as in (E.10) and (E.13), by following an argument similar to the proof of Theorem 1.

Moreover, due to the exclusion of liner service schedules, notation for some input data is also changed. Specifically, we use an integer s_{ij} to represent the capacity of the service provided by each carrier $i \in I$ for each lane $J \in J$, use an integer u_j to represent the cargo demands of each lane $j \in J$, and use a positive number e_j to indicate the spot-market shipping rate for each lane $j \in J$. In our computational experiment, we set s_{ij} to be $\sum_{k=1}^{n_{ij}} s_{ijk}$, u_j to be $\sum_{t=1}^{T} u_{jt}$, and e_j to be $\sum_{t=1}^{T} e_{jt}/T$, for $i \in I$ and $j \in J$.

F Computational Results under Different Values of Γ and $\hat{u}_{jt}$

As explained in Appendix D, we set the budget of uncertainty $\Gamma = \epsilon \times 52 \times |J|$, and the maximum demand deviations $\hat{u}_{jt} = \mu_0 \times \bar{u}_{jt}$, where ϵ and μ_0 are the level parameters of Γ and $\hat{u}_{jt}$, respectively. To evaluate the impacts of Γ and $\hat{u}_{jt}$ on the improvement of solutions generated from the robust optimization model MILP_R against the solutions from the benchmark models MILP_D and MILP_DNS

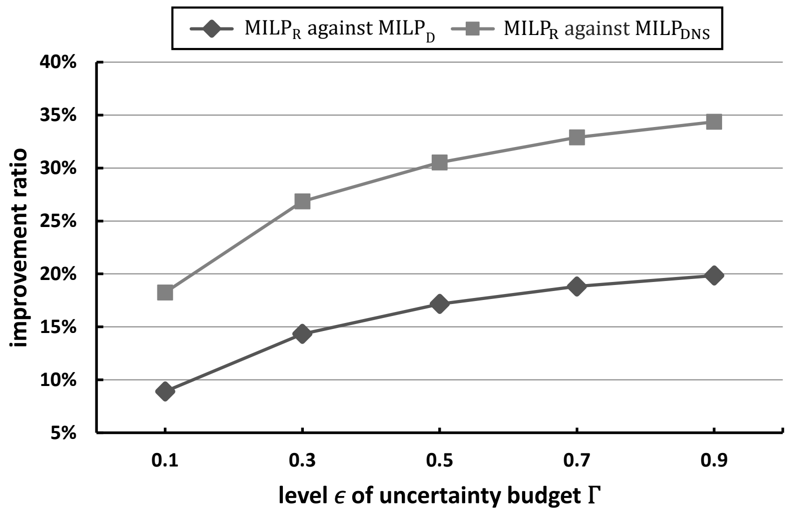

Figure F.1: Comparing the improvements of the solutions obtained from models MILP_R against the solutions from models MILP_D and MILP_DNS, for different levels ϵ of uncertainty budget Γ, where $\Gamma = \epsilon \times 52 \times |J|$, $|J| = 10$, and the level μ_0 of maximum demand deviations is fixed to be 0.9.

(described in Section 4.2), we arbitrarily choose a test instance of 10 lanes (i.e., $|J| = 10$), with level ϵ varying from 0.1, 0.3, 0.5, 0.7, to 0.9 (for μ_0 fixed to be 0.9), and with level μ_0 varying from 0.1, 0.3, 0.5, 0.7, to 0.9 (for ϵ fixed to be 0.5), so that uncertainty budget Γ varies from $0.1 \times 52 \times |J| = 52$ to $0.9 \times 52 \times |J| = 468$, and maximum demand deviations $\hat{u}_{jt}$ vary from $0.1 \times \bar{u}_{jt}$ to $0.9 \times \bar{u}_{jt}$, respectively. For all of these instances, problem R can be solved to optimum by applying the constraint generation method to model MILP_R. For the chosen instance, Figure F.1 and Figure F.2 plot values of the improvement ratios (defined in Section 4.3) for different ϵ and μ_0, respectively, so as to compare improvements of the solutions obtained from model MILP_R against solutions from models MILP_D and MILP_DNS. The plots demonstrate that the trends of such improvements grow with increasing ϵ and μ_0, and thus with increasing Γ and $\hat{u}_{jt}$, respectively. Similar trends are consistently observed in other test instances.

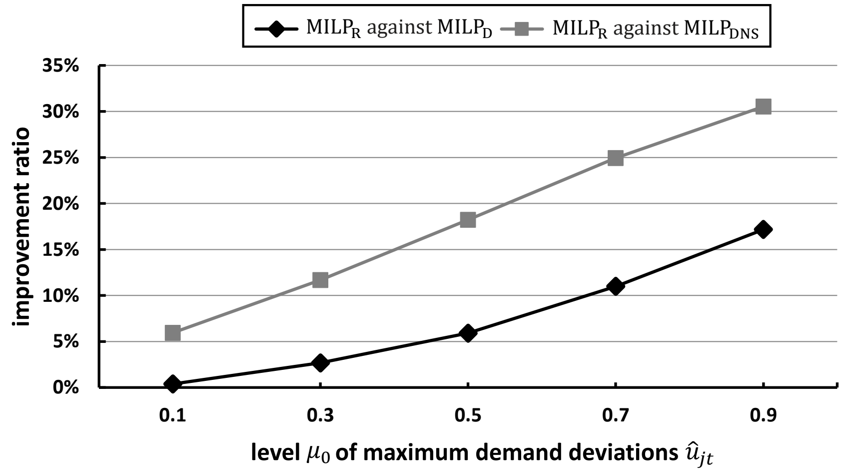

Figure F.2: Comparing the improvements of the solutions obtained from models $\mathrm{MILP_R}$ against the solutions from models $\mathrm{MILP_D}$ and $\mathrm{MILP_{DNS}}$, for different levels μ_0 of maximum demand deviations $\hat{u}_{jt}$, where $\hat{u}_{jt} = \mu_0 \times \bar{u}_{jt}$, and the level ϵ of uncertainty budget is fixed to be 0.5.

G Computational Results for Comparing models $\mathrm{MILP_R}$ and SP

We have conducted some additional experiments to compare solutions obtained from the robust optimization model $\mathrm{MILP_R}$ and the stochastic programming model SP (defined in Appendix B). For this, we arbitrarily pick a test instance of 10 lanes, and generate a set $\tilde{\mathcal{U}}$ of 200 scenarios of cargo demands u_{jt} which are assumed to be uniformly distributed in $[\bar{u}_{jt} - \hat{u}_{jt}, \bar{u}_{jt} + \hat{u}_{jt}]$. We apply the optimization solver to solve model SP with a time limit of 10 hours, and obtain the best solution found from model SP as well as the best lower bound found for the minimum expected total cost of model SP. Next, for each uncertainty budget $\Gamma \in \{0, 5, 10, \ldots, 50\}$, we solve model $\mathrm{MILP_R}$ to optimum, which can all be done within 10 hours, and we refer to its optimal solution as the Γ-robust optimal solution. Figure G.1 plots the expected total cost and the maximum (worst-case) total cost of each solution obtained over the 200 scenarios of cargo demand realizations in $\tilde{\mathcal{U}}$, as well as the lower bound found for the minimum expected total cost of model SP. We can see that while the best solution found from model SP leads to relatively high total costs in some worst-case scenarios, the expected total cost realized by the Γ-robust optimal solutions with small Γ is close to the lower bound on the minimum of model SP, outperforming the best solution found from

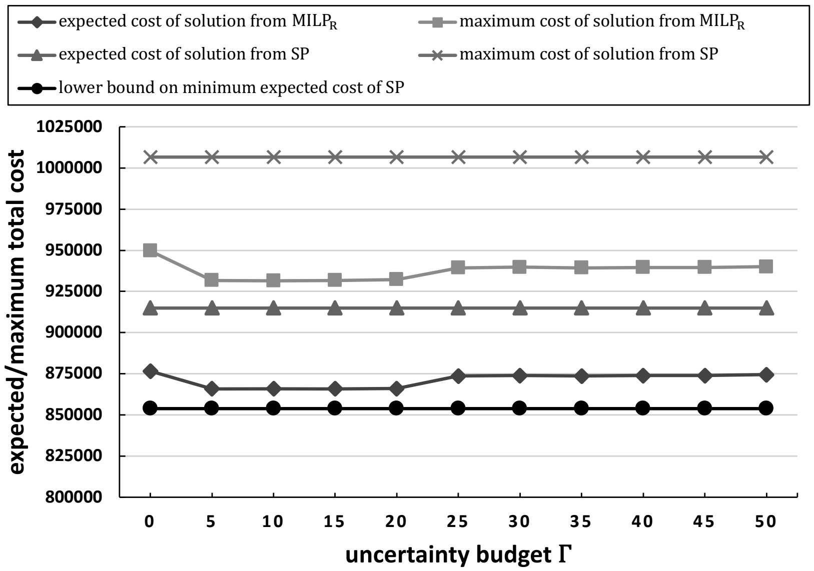

Figure G.1: Comparing expected and maximum total costs under the scenario set $\tilde{\mathcal{U}}$, with respect to the best solutions found from model SP and the Γ-robust optimal solutions found from model MILP_R with $\Gamma = 0, 5, 10, \ldots, 50$, where the maximum demand deviation $\hat{u}_{jt}$ is fixed to be $0.9\bar{u}_{jt}$.

model SP. Similar findings are consistently observed in other test instances. Hence, by choosing Γ properly, the robust optimal solution obtained from model MILP_R can help shippers achieve cost effectiveness with respect not only to the worst-case criterion but also to the expected criterion.

References

Hu, Q., Z. Zhang, A. Lim. 2016. Transportation service procurement problem with transit time. *Transportation Research Part B: Methodological*, 86 19–36.