ARTICLE TEMPLATE

Coordinating Inventory Sharing with Retailer's Return in the Consignment Contracts

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ABSTRACT
Hospitals purchase medical supplies from the dealer on consignment contracts. Dealer provides return policy for unused inventory but charging a return fee. Two hospitals could share inventory which reduces the amount of return to the dealer. Motivated by this consignment contract policy for the medical supply chain, we develop a common dealer and two independent retailers framework that considers retailers' sharing action and return problem. We aim at developing a coordinating mechanism to manage the retailers' sharing and return action that benefits both the dealer and retailers. The dealer-dominated sharing and retailer-dominated sharing are compared from the perspective of sharing performance and expected profits. We also analyze the condition that dealer is better off from retailers' sharing when the dealer has the power to encourage retailers' sharing, and the dealer's trading preference for a non-cooperative retailer or cooperative retailers when the dealer has no power to encourage retailers' sharing. Numerical experiments are conducted to examine the sensitivity of retailers' sharing decisions, retailers' and dealer's profit to the return price.

KEYWORDS
consignment stock; inventory sharing; return policy; supply chain coordination; healthcare logistics;

1. Introduction

Hospitals or clinics apply various purchasing or inventory policies to manage medicine and disposable medical items, but the shortage or waste of expired medical inventory exists as the difficulty of predicting patients' demand. These healthcare organizations pay a high price to manage such medical inventory. For example, a small private clinic needs to dispose of an average of 750$ worth of drugs and disposable medical items per month in Hong Kong. For reducing such inventory management costs, large hospitals adopt consignment contracts to manage medicine and disposable medical inventories. Consignment is a business model that a dealer places products at the retailer's warehouse but receives the payment until these items are sold. In general, long-lead-time disposable medical items have strong consignment potentials, such as intraocular lenses and orthopedic implants. Taking the intraocular lenses as an exam-

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ple, it is widely used for treating cataracts and has multiple models and sizes for fitting different patients. If all sizes of intraocular lenses are purchased and stocked in the hospital, the inventory cost would be very high. Therefore, the hospital prefers to allow the dealer to consign these items in the hospital, rather than purchase and keep all sizes of intraocular lenses. Besides, these items have a long manufacturing lead time, the dealer cannot replenish the hospital in a short time. By applying the consignment inventory policy, hospitals are benefited twofold. Firstly, consignment allows the hospital with more flexible use of medical items, especially for emergent surgical items, consignment inventory guarantees immediate supply when urgent needs occur, so the patients face less risk of treatment delay. Secondly, the cash flow of the hospital can be significantly improved since the used medical products can be charged to patients before the hospital pays the dealer (Ballard 1991).

However, although the consignment contract reduces the waste of medical inventory and mitigates the risk of overstock for hospitals, there are still some issues in the consignment contracts. The first issue is the increasing return of unused medical items. As the technology advancement of the medical item category, high inventory cost, and demand uncertainty, hospitals would rather return unsold/unused (we use these alternatively in the following sections) items to the dealer than keep them in their warehouses. To dominate the market, the dealer provides a return policy for hospitals, under which the hospital is allowed to return unsold items at the end of a selling season. Consequently, the dealer needs to dispose of the returned medical items. Different from traditional commodities (such as toys or clothes), returned medical items are generally easily-expired, have a low salvage value, and cannot be traded in the secondary market. Therefore, the dealer wants to reduce the hospital's return with a more reliable return policy which is acceptable to the hospitals. For example, "Cardinal Health", an integrated healthcare services and products company, provides consignment contracts to their medical items. It charges the hospital a 10% of invoice price as a restocking fee when the hospital returns unsold products in a saleable condition, and charges the hospital 25% of the invoice price as a restocking fee for returns not in a saleable condition. That means although the hospital can return the unsold products to the medical dealer at the end of a selling season, the dealer still charges some fees for restocking according to the condition of the returns. Such a consignment contract with return issues also exists in industrial practices. For instance, some big retailers (e.g. Walmart, Target, Meijer) sell seasonal items (e.g. Christmas decorations or toys) by the consignment contract (Lee and Wai 2005). They will return unsold items to the vendor at the end of selling season (Hu et al. 2014). Besides, in the book trading industry, the wholesalers and retailers are allowed to return unsold books to the publisher at full price minus the shipping fee and handling fee (Rungtusanatham et al. 2007). Therefore, the return problem and policy in consignment contracts worth more academic and practical concerns.

The second issue is about resource utilization, traditional consignment focuses on a one-to-one contract, such as one dealer to one hospital. In practice, a large medical company serves two or more hospitals. hospital's inventory information is shared with the common dealer but not shared among hospitals. Therefore, for the same medical item, one hospital might return a large amount of unsold medical items while another hospital might face a stockout of the same medical item. Under the case, inventory sharing provides a better solution to increase the overall resource utilization rate. Inventory sharing between retailers or dealers has been widely applied in industry and has drawn scholars' attention (Shao et al. 2011). In recent years, hospitals and other healthcare organizations also consider inventory sharing as a new inventory management approach to reduce the stockout of medical supplies. For instance, more than 50% of U.S. hospitals join in the multi-hospital system, this type of hospital consolidation encourages medical resource sharing (i.e. medical items, vaccines, and blood) (Cutler et al. 2011; Burns et al. 2015). It is investigated that the multi-hospital system is beneficial to reduce the medical supply chain risk (Zepeda et al. 2016). Although the application of revenue-sharing contract in the consignment design has obtained some concentration in academia (Bart 2020; Heydari 2020), there still lacks the analytical work on the consignment contracts with inventory sharing and the consideration of retailers' return problem.

Motivated by the above-discussed problems of retailers' consignment contract, we aim at developing a sharing and return framework for two retailers and one common dealer, exploring the interactive effects among retailers' sharing decision, return decision, and dealer's pricing decision, and providing insights for a dealer to choose a non-cooperative retailer or cooperative retailers, for retailers to be individual or cooperative.

This paper tackles the following three research problems:

(1) Considering retailers' sharing action, how the dealer makes the pricing decision, and the return policy.
(2) How the dealer and retailers benefit from the sharing policy in a dealer-dominated case and a retailer-dominated case respectively.
(3) How the dealer's return policy affects the retailer's sharing action, retailers' profits, and the dealer's profit.

Our paper provides a twofold contribution to production research. First is the novel model design, we construct a framework that includes a common dealer and two retailers, in which we consider the retailers' inventory sharing action and return action. To our knowledge, we are the first to investigate the interactive effects among inventory sharing, return decision, and pricing decision in the consignment contracts. Second is the practical application. Our results provide some managerial insights for production practice in real life. For instance, if the dealer is powerful, he benefits from retailers' sharing by reducing the refund products while if the dealer is weak, he is better off by transacting with the individual retailer rather than cooperative retailers (such as retailers of chain stores). Furthermore, increasing the return price that retailers need to pay for refund product cannot encourage retailers' inventory sharing, retailers prefer sharing excess inventory with a decreasing return price counter-intuitively.

The remaining sections of this paper are arranged as follows: Section 2 reviews three streams of relevant literature and identifies the main difference between the previous and ours. In Section 3, we develop a two-echelon model between the dealer and two retailers, which captures the feature of sharing and return in consignment contracts. Section 4 analyzes dealer's pricing & return policy and retailer's ordering decision without sharing option. Section 5 compares the performance of the dealer and retailers under two different cases: the dealer controls retailers' inventory sharing and retailers control their sharing. This section also identifies the effects of return price on sharing decisions as well as expected profits. Section 6 summarizes the main findings of the paper. All proofs and results of supplemental numerical experiments are presented in the Appendix.

2. Literature review

As our research explores the effects of the return policy on consignment inventory sharing/transshipment decision, we review the following three streams of literature: inventory return problem, consignment contract, and inventory sharing. Considering the generality of our model, we review literature that covers one or two of the above features in the setting of industry or enterprise. Furthermore, as our research is motivated by the practice of the hospital return problem in the consignment contract, we also review the literature on hospital or healthcare inventory management.

The first category of relevant literature tackles the pricing and order problem in the supply chain when the return policy is considered. There are two categories of return problems in the traditional supply chain, channel return, and customer return (Hu et al. 2014). Channel return policies are offered by suppliers/dealers to retailers for returning unsold products, while customer return policies are offered by retailers to customers for not-fitting taste or expectations. Gümü et al. (2013) also denotes the channel return as intrachannel return and the customer return as extrachannel return respectively. Yao et al. (2008) explore the effects of price-sensitivity on the return policy and find that when the price-sensitivity is low, the return policy can better coordinate the channel profits than the wholesale-price-only contract. Chen and Bell (2009) adopt a return function to measure the customer return quantity, which depends on the retail price and selling quantity. Chen and Bell (2011) further consider both channel return and customer return in their research. They suggest two categories of buyback prices for unsold return and customer return respectively. Buyback policy is an approach that the manufacturer buys unsold products with a buyback price from the retailer. They also find that retailers need to make a joint decision on price and order quantity when facing price-sensitive stochastic demand. Xiao et al. (2010) develop a coordination mechanism between a manufacturer and a retailer when considering the customer return. They examine the effects of full refund policy, partial refund policy, and no refund policy on profitability. Chen (2011) explores whether the retailer shares customer return information to the manufacturer will affect the retailer's order decision and manufacturer's pricing decision. Liu et al. (2014) investigate the impacts of buyback policy on supply chain coordination. They find that when the return quantity is decided exogenously, buyback policy can coordinate the manufacturer and retailer while coordination is not induced when return quantity is a decision variable.

The secondary category of literature concerns the return issue in the consignment contracts. Regarding the consignment contracts, scholars have done a wide range of studies, such as the research on channel performance of consignment contracts, the comparison of the Vendor Management Inventory (VMI), and the Retailer Management Inventory (RMI), pricing & ordering decision in consignment, and consignment contract design. Under the consignment contract, Wang et al. (2004) examine the channel performance and the individual performance of firms in a centralized system and decentralized system respectively. Ru and Wang (2010) compare the performance between Vendor Management Consignment Inventory mechanism (VMCI) and the Retailer Management Inventory mechanism (RMCI). It is found that the former is always better than the latter for both suppliers and retailers. Some scholars focus on the specific features of consignment contract design. For instance, Yang et al. (2019) explore the manufacturer-retailer matching policy with the consideration of revenue sharing and slotting fee in the consignment agreement. Sen et al. (2020) investigate the effect of warehouse space constraints of both the consignor and consignee on supply chain performance. They also consider the deterioration effect of the product in the consignment contract design. However, the literature on return policy in the consignment contract is scarce. In the consignment contracts, to capture a larger market share, the supplier/dealer offers a free return policy for unsold products to the retailer, which puts the supplier to a low-power position. To reduce profit loss caused by free return, the supplier/dealer charges retailers some additional fee for restocking or handling unsold products. Based on the model setting of Ru and Wang (2010), Hu et al. (2014) consider the salvage value of returns, incorporate the customer return and channel return in consignment contract, and show that whether the return policy is offered, VMCI mechanism is always more beneficial than RMCI mechanism.

The third stream of literature focuses on the effect of the return policy on inventory decisions when inventory sharing/transshipment exists. Shao et al. (2011) explore a similar sharing inventory mode with two retailers and a common dealer and identify the difference between dealer-control sharing and retailer-control sharing. However, they do not incorporate the product-return option. Our research considers the downstream sharing policy when the return option is available in a consignment contract, additionally, we identify the effects of the return policy on sharing decisions. Dan et al. (2016) construct a two-period model with a manufacturer and retailers where the preventative manufacturer-dominated inventory transshipment policy happens in retailers. Under this setting, they also investigate the impacts of return constraint on pricing and ordering decisions. Although this work is similar to our study, there is still much difference between their work and ours. The first difference is that they explore the return issue in a regular wholesale-price contract while we focus on the consignment contracts. The second difference is the type of transshipment, they apply the preventative inventory transshipment (before stockout) while the sharing policy in our model is reactive (when stockout happens). The last difference lay in the return policy. In their work, return price is a parameter, which represents the refund price that manufacturer refunds to the retailer for unsold products, but in our model setting, return price is a decision variable and denotes the additional restocking fee that dealer charges retailer for unsold inventory.

Besides, we review some research on hospital inventory management. Saha and Ray (2019) provide a detailed review that classifies the modeling methodologies and solution methods about healthcare inventory management. Based on their paper, we find that the difference in inventory management approaches in hospitals mainly lies in the replenishment policies. There are two traditional replenishment policies that have been widely used in the industrial setting: periodic review policy (the inventory is replenished at the beginning of the replenishment period) and continuous review policy (the inventory is replenished when the warehouse is empty) (Rosales 2015). Bijvank and Vis (2012) explore the application of two types of periodic review policy in hospital inventory replenishment. The first model maximizes the hospital service level with a capacity constraint while the second model minimizes the hospital capacity with a service-level constraint. Saedi et al. (2016) employ continuous review policy to mitigate the drug shortage when considering the uncertain demand in the hospital. However, Rosales (2014) and Rosales (2015) find that the hybrid inventory management policy which combines the low-cost periodic-review replenishment policy and the high-cost continuous-review replenishment policy is more profitable for hospitals. Furthermore, Wang et al. (2015) propose an innovative dynamic replenishment policy where the buffer size is adjusted according to the dynamic demand.

After conducting an exhaustive analysis of related literature, we summarize the major differences between the representative literature and our research in Table 1. In short, the main contribution of our paper for production research literature is twofold. First, our paper makes up the research gap on consignment contract considering inventory sharing and return policy, we develop a two-echelon (upstream dealer and downstream retailers) model with downstream inventory sharing as well as downstream return to the upstream dealer. This framework in the consignment contract has not been considered in the supply chain or other settings. Second, we identify the complex interaction among upstream dealer's pricing decisions, return policy, and downstream retailers' sharing decisions. The findings in this paper could provide some theoretical supports for future research on consignment contract design with the interactive effect of inventory sharing and return policy.

Table 1. The difference among several representative literature and ours.

	Centralized	Decentralized	Channel return	Customer return	Consignment contract	Wholesale price contract	Sharing/ transshipment
Yao et al. (2008)		◇	◇			◇	
Xiao et al. (2010)	◇	◇	◇	◇		◇	
Chen (2011)		◇	◇	◇		◇	
Hu et al. (2014)	◇	◇	◇	◇	◇		
Dan et al. (2016)		◇	◇	◇		◇	◇
Wu et al. (2016)	◇	◇	◇	◇	◇		
Shao et al. (2011)	◇	◇				◇	◇
ours	◇	◇	◇		◇		◇

3. Model description

We consider a single-period model consisting of one common dealer and two retailers. The dealer (he) sells a kind of disposable item to two independent retailers (she) by consignment contract. The dealer decides the unit consignment price to two retailers for sold products and the unit return price (restocking price, we use them alternatively in the following sections) for retailers' unsold returns. Before the demands are realized, two retailers decide their order quantity simultaneously and the dealer delivers the order to the consignment warehouse of the retailer at the beginning of a consignment cycle. Considering the long lead time of the product, no replenishment is provided during the selling season. The dealer's unit transaction cost (handling cost) is c for processing the retailer's order. Retailers can fulfill the demand of customers by consignment inventory. The stochastic demand of retailer i is denoted by D_i, where $i = 1, 2$ represents the retailer, F_i and f_i denote the distribution function and density function of D_i respectively.

When a consignment cycle begins, the dealer determines a consignment price w per unit item, and a return price r charged to retailers for unsold returns at the end of the cycle. The retailer charges the customer p per unit item, and the retail price is determined by the market. We assume that the sharing transportation cost is 0. The agreed sharing price w_s is predetermined by their common dealer or set by retailers (Shao et al. 2011). Also, the sharing amount from retailer i to retailer j is denoted by T_i, which equals the minimum of the excess inventory of retailer i and the excess demand of retailer j ($T_i = min\{(Q_i - D_i)^+, (D_j - Q_j)^+\}$). At the end of the consignment cycle, the retailer pays the dealer the net used order amount and return the unused inventory to the dealer with r. For the dealer, the salvage value per unit return is s. We assume that $s + r < w$ to prevent the arbitrage opportunity. $c > s$ is also a critical assumption which allows the dealer to charge some return fee to retailers in consignment contract.

Under the above framework, the sequence of events under the sharing mechanism

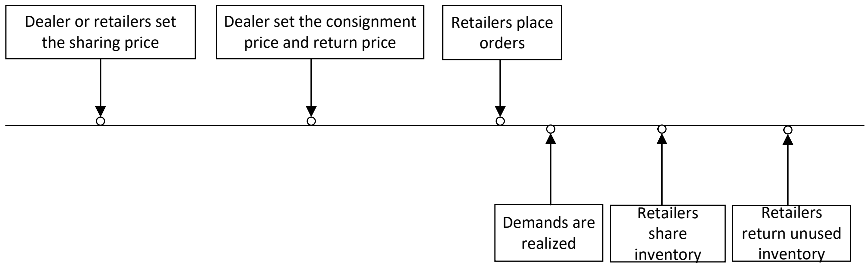


Figure 1. The timeline of events

is illustrated as follows (also shown in Figure 1):

Step 1. The powerful dealer sets the sharing price or retailers set a sharing price w_s.
Step 2. The dealer decides the consignment price w charged to the retailer and the return price r for unsold return at the end of the consignment cycle.
Step 3. Two retailers decide the order quantity Q_i simultaneously and receive the order at the beginning of the consignment cycle. For such a long-lead-time item, we assume that no emergent or additional replenishment during the consignment cycle.
Step 4. After the demand is realized, the retailer charges customers the market price p for each unit item.
Step 5. When one retailer anticipates stockout and another retailer has excess inventory, they share inventory with the given sharing price w_s.
Step 6. At the end of the consignment cycle, two retailers pay the dealer the net used amount, if there are unsold items, the retailer refunds them with per unit return price r.

The sequence of events under no sharing case can ignore step 1 and step 5. As Figure 2 shows, "O" denotes the retailer places an order from the dealer, "R" denotes the retailer returns the unused inventory to the dealer, while "S" represents the inventory sharing action between retailers and "A" means that retailers form a cooperative alliance. We first consider the no sharing case as a benchmark and explore the difference between individual-retailers-sharing and cooperative-retailers-sharing (Figure 2a). Then further investigate the impacts of retailers' sharing action on the profit of the dealer and retailers respectively. Besides, we analyze the performance of the dealer and retailers when they control the sharing price respectively, and a retailer's preference for being individual or cooperative.

4. No sharing case

When the inventory sharing option is not considered, the decision sequence of the events is as follows: the dealer decides consignment price w and the return price r charged to the retailer according to the sold quantity and unsold quantity respectively. Then the retailer decides the order quantity Q_i for the whole consignment cycle. We aim at obtaining the Nash equilibrium by backward induction.

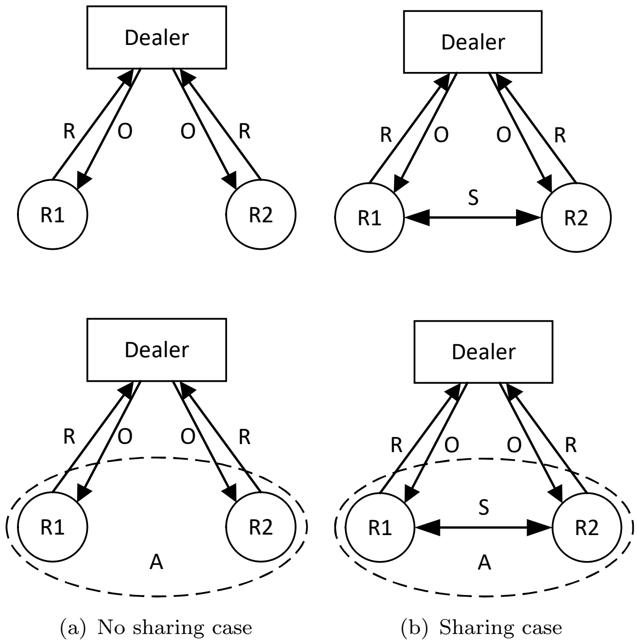


Figure 2. The operation of supply chain without retailers' sharing & with retailers' sharing.

4.1. *The ordering policy of retailer*

Considering the ordering decision of the retailer i, the expected profit is denoted by $E[\Pi_N^{h_i}(Q_i)]$,

$$E[\Pi_N^{h_i}] = (p - w)\min(D_i, Q_i) - r(Q_i - D_i)^+ \tag{1}$$

The profit of retailer i equals to the net profit of sold quantity minus return fee of unsold refunds. Defining $\Lambda(Q_i) = \int_A^{Q_i}(Q_i - D_i)f_i(D_i)dD_i$ and $\Theta(Q_i) = \int_{Q_i}^{B}(D_i - Q_i)f_i(D_i)dD_i$, we obtain the unique Q_i (denoted by Q_N^i) that maximizes the expected profit of retailer i when w, r and p are given.

$$Q_N^i = F_i^{-1}(\frac{p - w}{p - w + r}) \tag{2}$$

4.2. *The pricing policy of dealer*

The dealer's expected profit is denoted by $E[\Pi_N^d]$,

$$E[\Pi_N^d] = \sum_{i=1}^{2}[w\min(D_i, Q_i) + (r + s)(Q_i - D_i)^+ - cQ_i] \tag{3}$$

Considering the centralized system as a benchmark, we obtain the total expected profit of the dealer and two retailers, which is denoted by $E[\Pi_N^c]$,

$$E[\Pi_N^c] = \sum_{i=1}^{2}[(p\min(D_i, Q_i) + s(Q_i - D_i)^+ - cQ_i] \tag{4}$$

Let the first order condition $\frac{\partial E[\Pi_N^c]}{\partial Q_i} = 0$, we obtain the expression of Q_N^{ci}, which maximizes the total expected profit of the centralized system.

$$Q_N^{ci} = F_i^{-1}(\frac{p-c}{p-s}) \tag{5}$$

To motivate the retailer places Q_N^{ci} under decentralized system, the dealer offers specific pricing and return policies to retailers.

Proposition 4.1. *When the dealer and retailers are coordinated without inventory sharing option, the dealer provides a return policy* (w, r) *for retailers, in which, the range of return price is denoted as* (r_N^l, r_N^h)*, where*

$$r_N^l = 0 \tag{6}$$

$$r_N^h = c - s \tag{7}$$

$$w = p - \frac{(p-c)r}{c-s} \tag{8}$$

Proposition 4.1 indicates that $w(r)$ is a decreasing function of r according to $c - s > 0$. Without retailers' sharing option, the dealer offers a return policy $r \in (r_N^l, r_N^h)$ that ensures both of the dealer and retailers earn a positive profit. If the dealer charges a higher return price r to retailers ($r > r_N^h$), the dealer earns a negative expected profit since the consignment price is lower than c. If the dealer charges a lower return price ($r < r_N^l$), retailers will make a negative profit because the consignment price provided by the dealer is higher than the retail price charged to customers.

5. Sharing case

In this section, we consider two retailers i and j and their common dealer. With the retailers' sharing option, we apply backward induction and first analyze the ordering decision of retailers, then explore how the dealer decides consignment price and return price with the consideration of retailers' sharing action. Finally, we investigate the setting of sharing price, and mainly consider two cases: the dealer controls retailers' sharing and retailers control their sharing.

5.1. *The ordering policy of the retailer*

By backward induction, we first consider how the retailers make order decisions with the sharing option. The decision process is similar to the general Newsvendor model when the sharing amount between two retailers are decided.

Let $E[\Pi_S^{h_i}]$ denotes the total expected profit of retailer i,

$$E[\Pi_S^{h_i}] = (p-w)\min(D_i, Q_i) + (w_s - w)T_i + (p - w_s)T_j - r(Q_i - D_i - T_i + T_j)^+ \tag{9}$$

Take the first derivative with Q_i, we obtain the unique Q_i (denoted as Q_S^i) that maximizes retailer i's expected profit.

$$F_i(Q_S^i) = 1 + \frac{w_s - w}{p - w}\frac{\partial T_i}{\partial Q_i} + \frac{p - w_s}{p - w}\frac{\partial T_j}{\partial Q_i} - \frac{r}{p-w}F_i(Q_i - T_i + T_j) \tag{10}$$

5.2. *The pricing policy of dealer*

Under the sharing mechanism, retailers' inventory sharing reduces the return quantity for the retailer with excess inventory. Dealer's expected profit is affected by retailers' sharing. Therefore, the dealer needs to consider the possible effects of sharing action when he determines the consignment price and return price.

Let $E[\Pi_N^d(w,r)]$ denote the total expected profit of dealer from two retailers,

$$E[\Pi_S^d] = \sum_{i=1,j\neq i}^{2} \{w[\min(D_i, Q_i) + T_i] + (r+s)(Q_i - D_i - T_i + T_j)^+ - cQ_i\} \tag{11}$$

Before tackling the optimal pricing decision of the dealer, we also consider the centralized system of the dealer and retailers as a benchmark. By adding (9) and (11), we obtain the total expected profit of the centralized system as follows,

$$E[\Pi_S^c] = \sum_{i=1,j\neq i}^{2} \{p\min(D_i, Q_i) + w_sT_i + (p-w_s)T_j + s(Q_i - D_i - T_i + T_j)^+ - cQ_i\} \tag{12}$$

Solving the first condition $\frac{\partial E[\Pi_S^c]}{\partial Q_i} = 0$, we find that there exists a unique Q_S^{ci} that maximizes the total expected profit of the system. The corresponding Q_S^{ci} satisfies the following condition:

$$F_i(Q_S^{ci}) = \frac{p-c}{p} + \frac{w_s}{p}\frac{\partial T_i}{\partial Q_i} + \frac{p-w_s}{p}\frac{\partial T_j}{\partial Q_i} + \frac{s}{p}F_i(Q_i - T_i + T_j) \tag{13}$$

for any i.

To encourage the coordination among the dealer and two retailers in the decentralized system, the dealer propose a specific return and pricing policy (w, r) with the consideration of retailers' sharing as follows.

Proposition 5.1. *When the dealer and retailers are coordinated with the inventory sharing option, the dealer provides a return policy* (w, r) *for retailers, in which, the range of return price is denoted as* (r_S^l, r_S^h)*, where*

$$r_S^l = 0 \tag{14}$$

$$r_S^h = c - s \tag{15}$$

$$w = \frac{p[c-(r+s)F_i(Q_i - T_i + T_j)]}{c - sF_i(Q_i - T_i + T_j) - (p-w_s)(\frac{\partial T_j}{\partial Q_i} - \frac{\partial T_i}{\partial Q_i})} \tag{16}$$

Considering retailers' sharing action, Proposition 5.1 indicates that the dealer offers such a (w, r) policy that ensures the dealer and retailers make positive profits, where $r \in (r_S^l, r_S^h)$. Under the sharing mechanism, the range of r is independent of sharing parameters, while the consignment price w decreases as return price r increases, and increases as sharing price w_s increases.

5.3. ***Who dominates the sharing price***

In this section, we will analyze the effect of sharing parameters on retailers' sharing performance and profits. The sharing action affects the ordering decision of retailers. First, for retailer i, if she decreases the order quantity from Q_N^i, she will send a sharing request to retailer j, pays the sharing price w_s, and uses these units for patients by charging a market price p. So the retailer i can obtain a margin $p - w_s$. Second, if retailer i increases the order quantity from Q_N^i, she will share more units to retailer j by charging w_s, rather than return to the dealer and pays the return fee r to the dealer.

Lemma 5.2. *w_s increases, then Q_S^i increases.*

Lemma 5.2 identifies the impact of sharing price w_s on the retailer's ordering quantity under the sharing case. When consignment price and return price are given, if w_s increases, the retailer with excess inventory is willing to increase the order quantity such that she can share out. Simultaneously, as w_s increases, the retailer would order more and keep enough inventory to fulfill demand, since the margin of sharing request $(p - w_s)$ decreases.

Proposition 5.3.

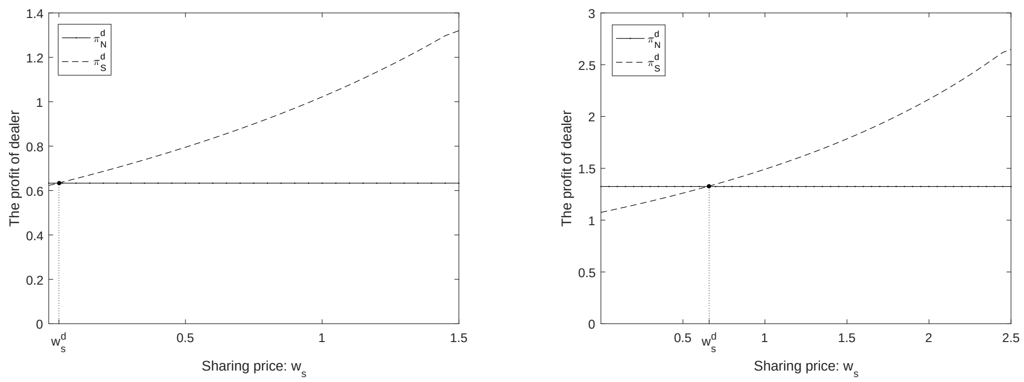
i. For the retailer i, when the consignment price and return price are given, $Q_N^i > Q_S^i$ at $w_s = 0$; $Q_N^i < Q_S^i$ at $w_s = p$.
ii. For the dealer, when he sets the optimal consignment price and return price, his profit increases as the sharing price w_s increases. The dealer makes a lower profit under inventory sharing mechanism at $w_s = 0$ and a higher profit at $w_s = p$.

Proposition 5.3 shows that the sharing price w_s incentivizes or stunts the sharing action. Although the dealer determines the optimal consignment price and the return price that benefits both of dealer and the retailer, he still prefers a higher sharing price under the sharing mechanism. For retailers, the sharing price determines their inventory choices: share excess inventory to another retailer or return to the dealer. Hence, the determination of sharing price is critical.

In the following section, we consider two possible cases in the transaction process: first, the dealer dominates the inventory sharing in the supply chain. Under the case, the dealer will offer a sharing price that benefits himself and can be accepted by retailers. Second, retailers negotiate a sharing price that is profitable for themselves. We design numerical experiments to analyze the effect of w_s on dealer's profit, retailer's profit, and the profit of retailers' alliance with and without sharing action. Let $r = 0.1$, $c = 0.5$, $s = 0.3$, and $D_i \sim U(0, 1), i = 1, 2$. We compare the results by setting $p = 1.5$ and $p = 2.5$ sequentially. Furthermore, we show the above results with the demand submits to the normal distribution in Appendix A. In the experiments, we use Π_N^{hi}, Π_N^{hj}, Π_N^h, and Π_N^d to denote the profit of retailer i, the profit of retailer j, the profit of retailers-alliance, and dealer's profit under no sharing case, while Π_S^{hi}, Π_S^{hj}, Π_S^h, and Π_S^d denote the profit that under sharing case respectively.

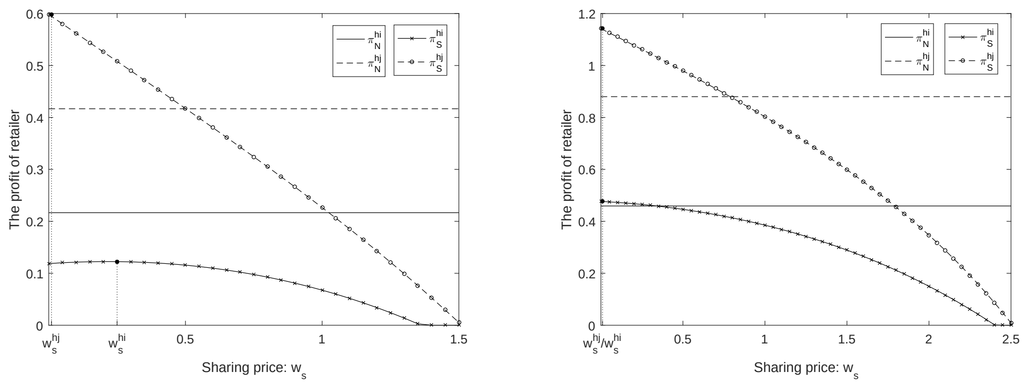
5.3.1. The dealer dominates the sharing price

Proposition 5.4. *When the dealer dominates the sharing price, he is better off with retailers' sharing and makes more profits as the sharing price increases.*



(a) $p = 1.5$ (b) $p = 2.5$

Figure 3. Dealer's profit.



(a) $p = 1.5$ (b) $p = 2.5$

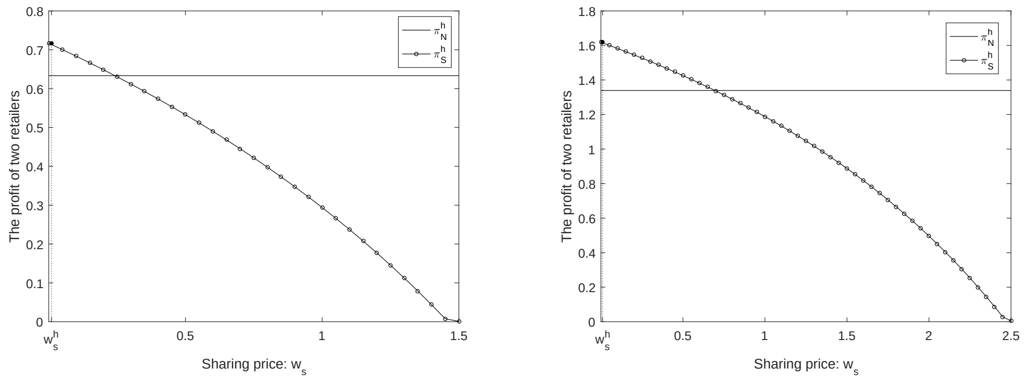
Figure 4. Retailers' profits when they are individual.

Observation 5.5. *When the dealer dominates retailers' sharing action, retailers may be worse off with sharing.*

If the dealer dominates retailers' sharing action, that means the dealer can set a sharing price for retailers and has the power to induce retailers to share. The Figure 3 shows that the dealer is better off with retailers' sharing if $w_s > w_s^d$. Therefore, as Proposition 5.4 indicates, the dealer always prefers setting a higher w_s. Besides, we observe that retailers may worse off because of sharing. Although the profit of retailer i suffers an increasing trend before decreases, it is still lower than the profit without sharing (Figure 4a). Retailer j with excess demand benefits from sharing when w_s is lower (around < 0.5). Therefore, when the dealer dominates sharing price, he prefers w_s increases to p, while retailers are worse off under the case.

5.3.2. Retailers dominate the sharing price

Proposition 5.6. *When retailers dominate their sharing action, they set the sharing price $w_s^{\hat{h}}$, dealer is better off with retailers' sharing action if $w_s^{\hat{h}} > w_s^d$, worse off if $w_s^{\hat{h}} < w_s^d$, and indifferent if $w_s^{\hat{h}} = w_s^d$.*



(a) $p = 1.5$ (b) $p = 2.5$

Figure 5. Retailers' profits when they form an alliance.

Observation 5.7. *When retailers dominate their sharing action, if $w_s^{\hat{h}} > w_s^h$, the dealer prefers consigning with individual retailers; otherwise, individual retailers and cooperative retailers-alliance are indifferent to the dealer.*

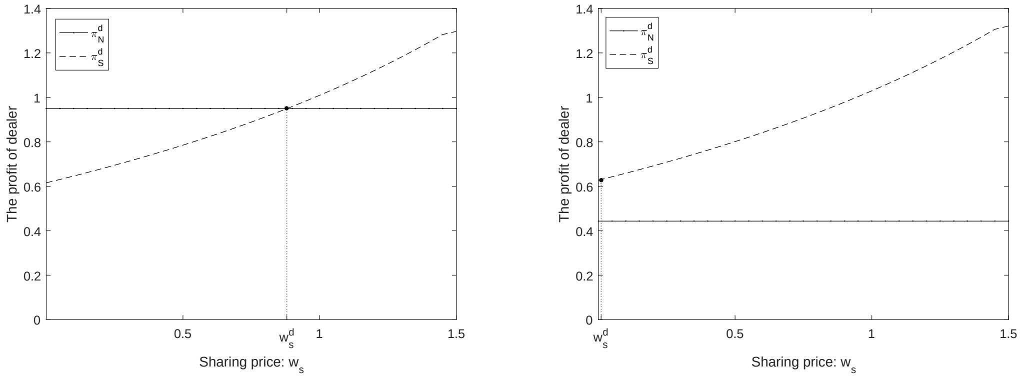
When retailers dominate their sharing price, they decide the sharing price without dealer's interference. Two retailers aim to maximize their profit respectively if they prefer to be independent. w_s^{hi} and w_s^{hj} denotes retailer's preferred sharing prices, under which retailer i and j are most profitable respectively. Under the non-alliance case, two retailers will negotiate and decide a sharing price $w_s^{\hat{h}}$ in the range of (w_s^{hj},w_s^{hi}). Dealer's profit is affected by retailers' sharing decisions. Figure 3a shows that the dealer benefits from retailers' sharing when $w_s^{\hat{h}} > w_s^d$ and hurts by sharing when $w_s^{\hat{h}} < w_s^d$. But in the case of Figure 3b, the dealer is always better off from sharing because retailer will set $w_s^{\hat{h}} = 0$ and $w_h^d > 0$.

Then we focus on the dealer's preference for retailers' cooperation. Whey two retailers negotiate to form a cooperative alliance, they aim to maximize their total profit but still make the order decision independently. w_s^h is denoted as the sharing price that is determined by the retailers' alliance. From Figure 5, retailers set the sharing price $w_s^h = 0$ and obtain the highest total profit. Therefore, for the dealer, he makes more profits if retailers are individual than cooperative ($w_s^{\hat{h}} > w_s^h$). But we also observe that the dealer is indifferent to retailers' cooperation when $w_s^{\hat{h}} = w_s^h$.

5.4. *The impacts of the return policy on sharing performance*

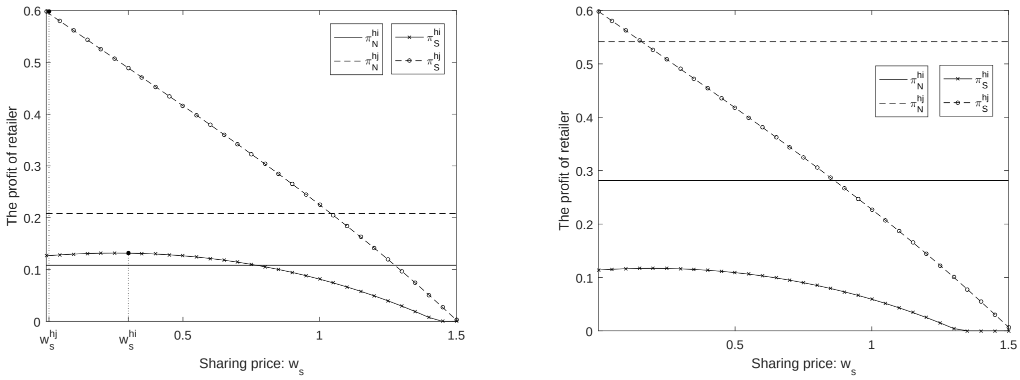
The decision of sharing price depends on the consignment price, the return price for unused refunds, the given market price, and the uncertain demand. Return price is critical because it also affects the consignment price. In this section, we aim at exploring the effects of return price on retailers' sharing performance and profits as well as the dealer's profit. Let $p = 1.5$, $c = 0.5$, $s = 0.3$, $D_i \sim U(0,1), i = 1,2$. We obtain the dealer's profit, retailer's profit, and the profit of retailers' alliance with and without sharing action when $r = 0.05$ and $r = 0.13$ sequentially. Additionally, we show the above results with the demand submits to the normal distribution in Appendix A.

Proposition 5.8.



(a) $r = 0.05$ (b) $r = 0.13$

Figure 6. Dealer's profit.



(a) $r = 0.05$ (b) $r = 0.13$

Figure 7. Retailers' profits when they are individual.

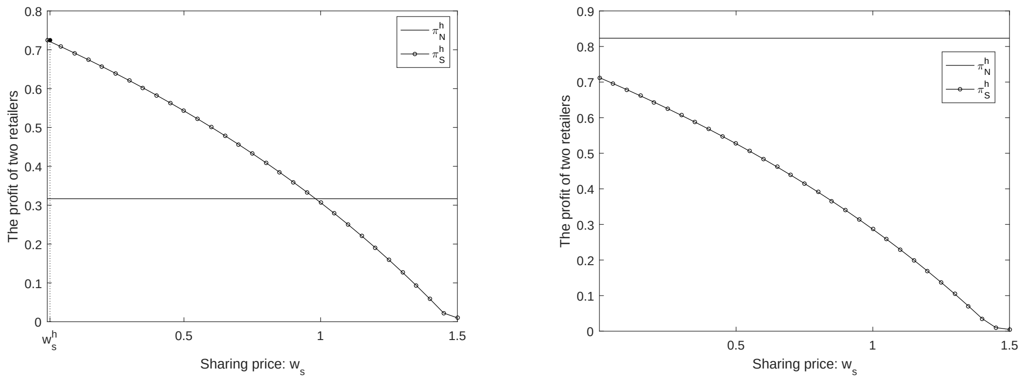
i. When the dealer increases return price r,

- *if the dealer dominates sharing action, the dealer is always better off with retailers' sharing.*
- *if retailers dominate sharing action, they reject the sharing action. Dealer is always worse off without retailers' sharing.*

ii. When the dealer decreases return price r,

- *if the dealer dominates sharing action, he is better off with retailers' sharing when $w_s > w_s^d$.*
- *if retailers dominate sharing action, they set the sharing price w_s^h. The dealer is better off with retailers' sharing action if $w_s^{\hat{h}} > w_s^d$, worse off if $w_s^{\hat{h}} < w_s^d$, and indifferent if $w_s^{\hat{h}} = w_s^d$. In addition, the dealer prefers that retailers are individual.*

The Proposition 5.8 shows the return price makes significant effects on retailers' sharing decisions, retailers' profits, and dealer's profit. According to the results of numerical experiments, when the dealer increases the return price ($r = 0.13$), we consider the sharing performance under two cases. First, if the dealer dominates sharing action, that means retailers submit to the dealer's decision. Figure 6b shows that the dealer always benefits from retailers' sharing action, and he prefers a high sharing



(a) $r = 0.05$ (b) $r = 0.13$

Figure 8. Retailers' profits when they form an alliance.

price ($0 < w_s \leq p$). But in this case, sharing is not profitable for retailers. Second, if retailers have the power to dominate the sharing action, retailers reject sharing, since there is no motivation to share (Figure 7b and 8b). Retailers are almost worse off with sharing action (except the retailer j benefits from sharing when w_s is in a small range). Under the case, the dealer is always worse off without retailers' sharing because the increase of r leads to the decrease of consignment price w. Therefore, for the dealer, to increase the return price is not always available to encourage retailers' sharing, especially when retailers dominate their sharing action.

When the dealer decreases the return price ($r = 0.05$), if the dealer dominates retailers' sharing, the dealer is better off with sharing when $w_s > w_s^d$ (Figure 6a). If retailers dominate their sharing action, they will negotiate and decide a sharing price between w_s^{hi} and w_s^{hj}, where w_s^{hi} and w_s^{hj} represents that retailer i and j make the highest profit respectively. For retailer i with excess inventory, the decrease of r leads to an increase of w, the profit of retailer i decreases $(p - w)$. So as the w_s increases, retailer i begins to benefit from share out. Simultaneously, as r decreases, the profit of retailer j with excess demand decreases than that of a higher r case, so retailer j is better off sharing in a wider w_s range (Figure 7a). Considering the total profit maximization, it is possible that retailers form an alliance and set the sharing price $w_s^h = 0$, and then reallocate the profit. In this paper, we do not consider the details of profit reallocation. When retailers dominate sharing action, the dealer prefers they are individual but not a cooperative alliance.

6. Conclusions

This paper constructs a framework that contains a common dealer and two retailers with the consideration of the retailer's sharing action and return action to the dealer. Retailers place orders from the dealer by a consignment contract, which means they only pay for used items at the end of the consignment cycle. Retailers can choose to return the unused items to the dealer by paying a return fee or share with another retailer after demand realization. We aim at developing an approach to coordinate retailer's sharing with the retailer's return. Also, we consider two cases: dealer-dominated sharing and retailer-dominated sharing. We further explore the effect of the retailer's return policy on sharing performance and profits. Some managerial insights are concluded as follows:

1. (1) Retailers share excess inventory and reduce the return amount to the dealer. It seems that the sharing action benefits retailers and the dealer especially when the salvage value of refunds is very low. However, the benefit-effect depends on who controls the sharing action. If the dealer has complete power in the consignment contract (the dealer domains the market), he sets a higher sharing price and retailers are worse off with sharing, retailers prefer to return excess inventory to the dealer rather than share with each other. When retailers have complete power (multiple dealers provide the same products), retailers prefer setting a lower sharing price and choose to form a cooperative alliance to achieve inventory sharing. In this case, the dealer is worse off with retailers' sharing most time.
2. (2) Dealer can choose to trade with individual retailers or cooperative retailers-alliance. When the dealer cannot dominate retailers' sharing, he prefers trading with the individual retailer such that he makes more profit under the sharing mechanism, because individual retailers set a higher sharing price than cooperative retailers.
3. (3) The return price has significant effects on the retailer's sharing action, retailers' profit, and dealer's profit. When the dealer has no power to dominate sharing price, he may set a higher return price to encourage retailers' sharing. But it does not work, since a higher return price increases consignment price, which causes retailers worse off under sharing. Therefore, retailers will choose the no-sharing mechanism and the dealer is worse off without sharing. On contrary, a decreasing return price will encourage retailers' sharing because retailers begin to better off with sharing when the consignment price is higher.

We also propose two new research directions based on this paper. First, considering the interaction between OM and marketing in this topic, for example, model the market share of retailers and the dealer in the decision process, which may provide some marketing managerial insights in the dealer's pricing decision. Second, when the dealer decides to trade with individual retailers or cooperative retailers, he trusts the retailers when the information is complete. But information asymmetry will induce the dealer to consider the possibility of the information truth. The dealer may adjust his pricing and return policy for retailers. Therefore, we will extend the work by considering information asymmetry in future research.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix A

Proof of Proposition 4.1

Proof. Under decentralized system, the condition $Q_N^i = Q_N^{ci}$ is satisfied when the dealer and retailers are coordinated. Therefore, we obtain $w(r)$ in Proposition 4.1. Plugging (8) into (3) and take the first derivative with r, we obtain that for $i = 1, 2$,

$$\frac{\partial E[\Pi_N^d]}{\partial r} = -\frac{p-c}{c-s}(Q_i - \Lambda(Q_i)) + \Lambda(Q_i) < 0 \tag{A.1}$$

It shows that the $E[\Pi_N^d]$ is a decreasing function of r, hence, $E[\Pi_N^{h_i}]$ is an increasing function of r as $E[\Pi_N^c]$ is independent of r. Let $E[\Pi_N^{h_i}] = 0$ and $E[\Pi_d] = 0$, we obtain r_l and r_h respectively.

We consider the dealer firstly, and take the first derivative with Q_i for (3):

$$\frac{\partial E[\Pi_N^d]}{\partial Q_i} = w(1 - F_i(Q_i)) + (r+s)F_i(Q_i) - c \tag{A.2}$$

which represents the expected marginal profit of the dealer for selling a product, and it should be positive. Therefore, we obtain $w(r)(1 - F_i(Q_i)) + (r+s)F_i(Q_i) - c > 0$. Replacing w with $w(r)$ in (3):

$$E[\Pi_N^d] = \sum_{i=1, j\neq i}^{2} [w(r)(Q_i - \Lambda(Q_i)) + (r+s)\Lambda(Q_i) - cQ_i] \tag{A.3}$$

when $E[\Pi_N^d] = 0$, $r = r_N^h = c - s$.

Then consider the retailer, we also obtain $w(r) = p - r\frac{F_i(Q_i)}{1-F_i(Q_i)}$ by solving the first order condition:

$$\frac{\partial E[\Pi_N^{h_i}]}{\partial Q_i} = (p_i - w)(1 - F_i(Q_i)) - rF_i(Q_i) = 0 \tag{A.4}$$

then replacing w with $w(r)$ in (1):

$$E[\Pi_N^{h_i}] = r\frac{F_i(Q_i)}{1 - F_i(Q_i)}(Q_i - \Lambda(Q_i)) - r\Lambda(Q_i) \tag{A.5}$$

when $E[\Pi_N^{h_i}] = 0$, $r = r_N^l = 0$ and $r_N^h > r_N^l$ is obvious with the assumption $c > s$. $\square$

Proof of Proposition 5.1

Proof. We obtain the $w(r)$ by coordinating the dealer and two retailers in decentralized system. Then plugging (16) into (11) and taking the first derivative with r,

$$\frac{\partial E[\Pi_S^d]}{\partial r} = \frac{(Q_i - \Lambda(Q_i) + T_i)p\lambda}{c - s\lambda - (p - w_s)(\beta - \alpha)} + \Lambda(Q_i - T_i + T_j) < 0 \tag{A.6}$$

where $\alpha = \frac{\partial T_i}{\partial Q_i}$, $\beta = \frac{\partial T_j}{\partial Q_i}$, and $\lambda = F_i(Q_i - T_i + T_j)$. We can observe that $E[\Pi_S^d]$ is a decreasing function of r under coordination case. $E[\Pi_S^{h_i}]$ increases as r increases since $E[\Pi_S^c]$ is independent of r. Then let $E[\Pi_S^{h_i}] = 0$ and $E[\Pi_S^d] = 0$, we obtain the r_S^l and r_S^h respectively.

Considering the dealer first and taking the first derivative with Q_i for (11),

$$\begin{aligned}\frac{\partial E[\Pi_S^d]}{\partial Q_i} &= \sum_i^2[w(1 - F_i(Q_i) + \frac{\partial T_i}{\partial Q_i}) + (r + s)F_i(Q_i - T_i + T_j) - c] \\ &\geq [(w - (r + s))(1 - F_i(Q_i - T_i + T_j)) + w\frac{\partial T_i}{\partial Q_i} + (r + s - c)] > 0\end{aligned} \tag{A.7}$$

when $r = r_S^h = c - s$, $E[\Pi_S^d]$ has the minimum value.

When considering the retailer, we obtain $w(r) = p - \frac{(p-w_s)(\frac{\partial T_i}{\partial Q_i} - \frac{\partial T_j}{\partial Q_i}) - rF_i(Q_i - T_i + T_j)}{1 - F_i(Q_i) + \frac{\partial T_i}{\partial Q_i}}$ by solving the first order condition:

$$\frac{\partial E[\Pi_S^{h_i}]}{\partial Q_i} = (p - w)(1 - F_i(Q_i)) + (w_s - w)\frac{\partial T_i}{\partial Q_i} + (p - w_s)\frac{\partial T_j}{\partial Q_i} - rF_i(Q_i - T_i + T_j) \tag{A.8}$$

then replacing w with $w(r)$ in (9):

$$E[\Pi_S^{h_i}(w(r))] = (p - w(r))(Q_i - \Lambda(Q_i)) + (w_s - w(r))T_i + (p - w_s)T_j - r\Lambda(Q_i - T_i + T_j) - 0 \tag{A.9}$$

for each retailer i. When $E[\Pi_S^{h_i}(w(r))] = 0$,

$r = r_S^l = \frac{(p-w_s)[\frac{\partial T_i}{\partial Q_i}(Q_i-\Lambda(Q_i)+T_j)-\frac{\partial T_j}{\partial Q_i}(Q_i-\Lambda(Q_i)+T_i)-(T_i-T_j)(1-F_i(Q_i))]}{\Lambda(Q_i-T_i+T_j)(1-F_i(Q_i)+\frac{\partial T_i}{\partial Q_i}-F_i(Q_i-T_i+T_j)(Q_i-\Lambda(Q_i)+T_i)}$
when w_s increases to p, and r decreases to 0, then $E[\Pi_S^{h_i}] = 0$. Therefore, under retailers' sharing option, the dealer still provides a return policy $r_S^l = 0$ and $r_S^h = c - s$ to make both of the dealer and retailers profitable. □

Proof of Proposition 5.3

Proof. Under no sharing case, the ordering quantity of retailer i satisfies $F_i(Q_N^i) = \frac{p-w}{p-w+r}$. Under sharing case, when the consignment price w and return price r are determined by the dealer, we consider the ordering quantity of the retailer under two extreme cases: $w_s = 0$ and $w_s = p$. When $w_s = 0$, $F_i(Q_S^i) = 1 + \frac{p}{p-w}\frac{\partial T_j}{\partial Q_S^i} - \frac{r}{p-w}F_i(Q_S^i - T_i + T_j)$, we obtain that $F_i(Q_S^i) \leq \frac{p-w}{p-w+r} + \frac{p}{p-w+r}\frac{\partial T_j}{\partial Q_S^i}$. Because $\frac{\partial T_j}{\partial Q_S^i} < 0$ and the monotonicity of $F_i(\cdot)$, $Q_N^i > Q_S^i$ at $w_s = 0$. Therefore, when the dealer sets the optimal consignment price and return price under no sharing case, he will make a higher profit than under sharing case at $w_s = 0$.

When $w_s = p$, $F_i(Q_S^i) = 1 + \frac{\partial T_i}{\partial Q_S^i} - \frac{r}{p-w}F_i(Q_S^i - T_i + T_j)$, we observe that $F_i(Q_S^i) \geq \frac{p-w}{p-w+r} + \frac{\partial T_i}{\partial Q_S^i}\frac{p-w}{p-w+r}$. Because $\frac{\partial T_i}{\partial Q_S^i} > 0$ and the monotonicity of $F_i(\cdot)$, $Q_N^i < Q_S^i$ at $w_s = p$. The dealer makes a higher profit under sharing case when $w_s = p$. □

Proof of Proposition 5.4

Proof. According to the monotonicity of dealer's profit, the Proposition 5.4 can be obtained. □

Proof of Proposition 5.6

Proof. The proof of Proposition 5.6 is similar with Proposition 5.4. □

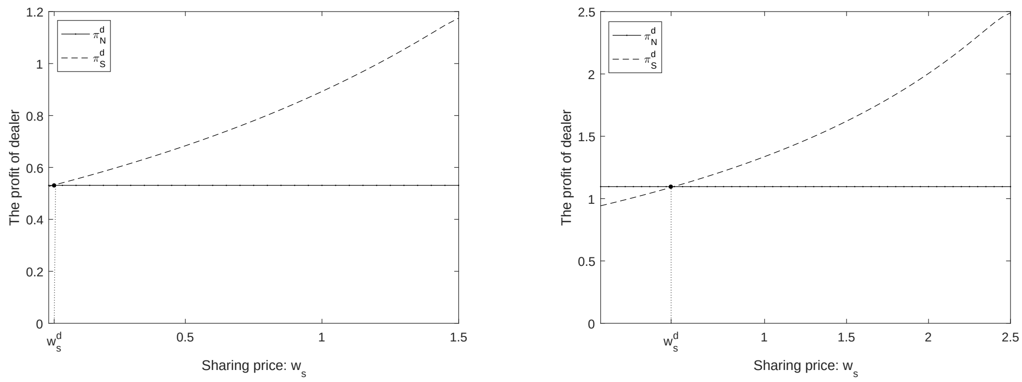
Proof of Proposition 5.8

Proof. According to Observation 5.5, Observation 5.7 and Proposition 5.6, the Proposition 5.8 can be obtained. □

Appendix A. Supplemental numerical experiments

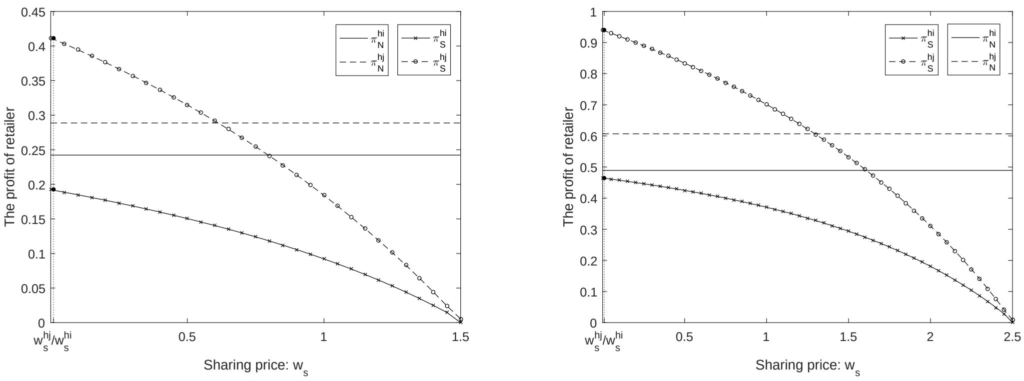
To examine the robustness of the conclusions about the effects of sharing price, we conduct another group of numerical experiments where the demand submits to the normal distribution. Let $r = 0.1$, $c = 0.5$, $s = 0.3$, $D_i \sim N(0.5, 0.08), i = 1, 2$. We obtain the dealer's profit, the retailer's profit and the profit of retailers' alliance with and without sharing action when $p = 1.5$ and $p = 2.5$ sequentially. The results also validate the conclusions in dealer-dominated sharing case and retailers-dominated sharing case.

To examine the robustness of the conclusions about the effects of return price, we conduct another group of numerical experiments where the demand submits to the normal distribution. Let $p = 1.5$, $c = 0.5$, $s = 0.3$, $D_i \sim N(0.5, 0.08), i = 1, 2$. We obtain the dealer's profit, the retailer's profit and the profit of retailers' alliance with



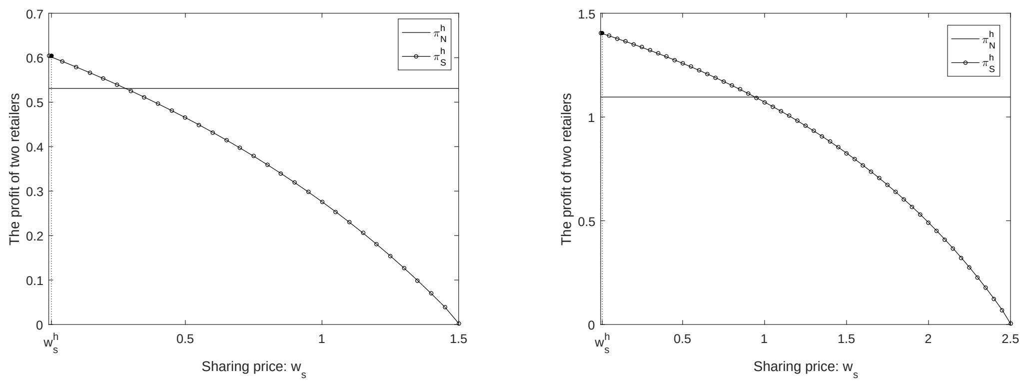
(a) $p = 1.5$ (b) $p = 2.5$

Figure A1. Dealer's profit.



(a) $p = 1.5$ (b) $p = 2.5$

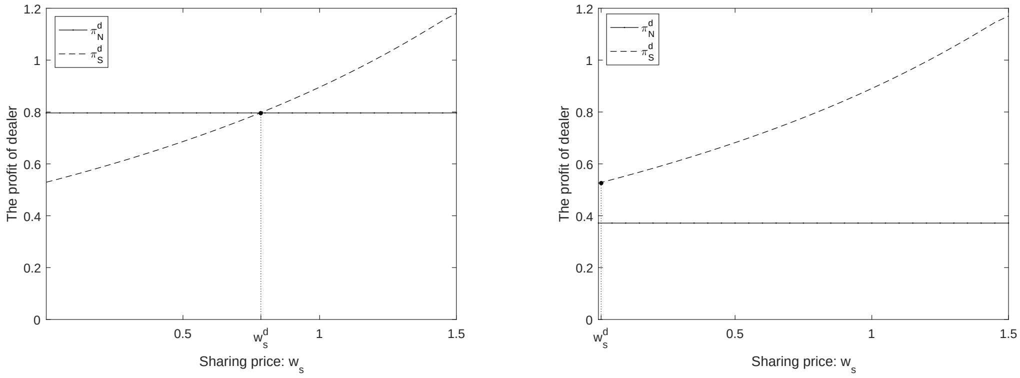
Figure A2. Retailers' profits when they are individual.



(a) $p = 1.5$ (b) $p = 2.5$

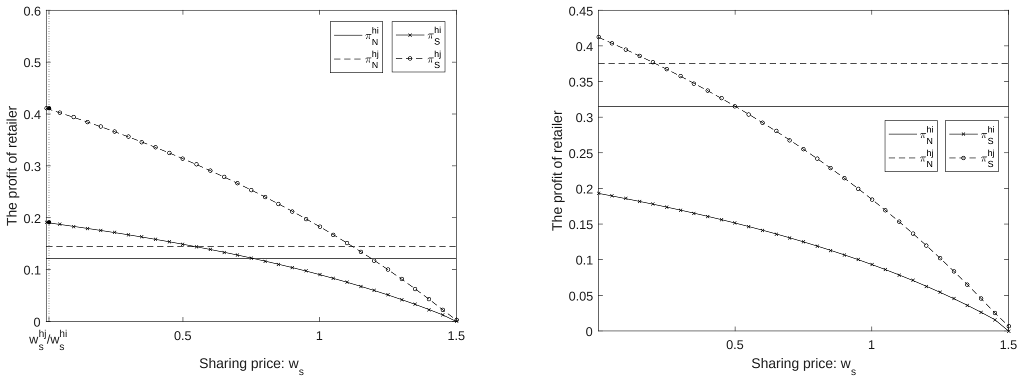
Figure A3. Retailers' profits when they form an alliance.

and without sharing action when $r = 0.05$ and $r = 0.13$ sequentially. The results also validate the conclusions in Proposition 5.8.



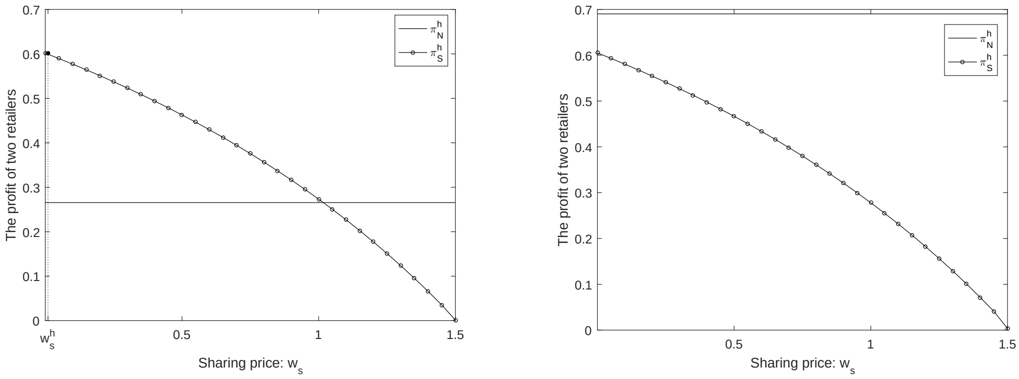
(a) $r = 0.05$ (b) $r = 0.13$

Figure A4. Dealer's profit.



(a) $r = 0.05$ (b) $r = 0.13$

Figure A5. Retailers' profits when they are individual.



(a) $r = 0.05$

(b) $r = 0.13$

Figure A6. Retailers' profits when they form an alliance.