

1        **Development of a two-stage ship fuel consumption prediction and**  
2                                **reduction model for a dry bulk ship**

3

4        **Abstract**

5            Shipping industry is the backbone of global trade. However, the large quantities of  
6 greenhouse gas emissions from shipping, such as carbon dioxide (CO<sub>2</sub>), cannot be  
7 ignored. In order to comply with the international environmental regulations as well as  
8 to increase commercial profits, shipping companies have stronger motivations to  
9 improve ship energy efficiency. In this study, a two-stage ship fuel consumption  
10 prediction and reduction model is proposed for a dry bulk ship. At the first stage, a fuel  
11 consumption prediction model based on random forest regressor is proposed and  
12 validated. The prediction model takes into account ship sailing speed, total cargo weight,  
13 and sea and weather conditions and then predicts the hourly fuel consumption of the  
14 main engine. The mean absolute percentage error of the random forest regressor is  
15 7.91%. At the second stage, a speed optimization model is developed based on the  
16 prediction model proposed at the first stage while guaranteeing the estimated arrival  
17 time to the destination port. Numerical experiment on two consecutive-8-day voyages  
18 shows that the proposed model can reduce ship fuel consumption by 2% to 7%. The  
19 reduction in ship fuel consumption will also lead to lower CO<sub>2</sub> emissions.

20

21        *Keywords:* Fuel consumption prediction, Ship fuel efficiency, Ship speed optimization,  
22        Random forest regressor; Machine learning

23 **1. Introduction**

24 In the past few years, improving ship energy efficiency has received wide attention  
25 not only from governmental and non-governmental organizations, but also from  
26 shipping companies (Yang et al., 2019). Although shipping is a vital component of  
27 global economy, air pollutants and greenhouse gas emissions from shipping industry  
28 caused by fuel consumption cannot be ignored. Regarding greenhouse gas emissions,  
29 such as CO<sub>2</sub>, it is reported by the International Maritime Organization (IMO) that CO<sub>2</sub>  
30 emissions from shipping constitute 3.1% of global emissions, while international  
31 shipping emissions take up for 2.6% of the global emissions during 2007 to 2012 (IMO  
32 2014). Thus, an increasing number of international regulations have been focused on  
33 improving ship energy efficiency. The first related international regulation is the  
34 amendments of the International Convention for the Prevention of Pollution from Ships  
35 (MARPOL) Annex VI proposed by Marine Environment Protection Committee (MEPC)  
36 in 2011 (IMO 2011). In addition, an approach named “Energy Efficiency Operational  
37 Indicator (EEOI)” was also proposed as a monitoring tool to manage ship and fleet  
38 efficiency performance. In 2016, amendments to MARPOL Annex VI mandatorily  
39 required ships to record and report their fuel oil consumption.

40 For shipping companies, due to high fuel prices, fuel costs have become the  
41 dominant factor of ship operational costs (Du et al., 2019). It is estimated that ship fuel  
42 costs constitute 20% to 50% of the total ship operating costs (Leifsson, 2008; Hasselaar,  
43 2011). For a large container ship, fuel costs can reach about three-quarters of its  
44 operating costs when the fuel prices are high. In addition, the costs can be higher if the  
45 container ship chooses to use cleaner fuel (Ronen, 2011). In shipping industry, slow  
46 steaming is a commonly used countermeasure to reduce fuel consumption, but on-time  
47 delivery may not be guaranteed (Lee et al., 2015). Thus, in order to conform to the  
48 international environmental protection regulations as well as to increase revenue and  
49 enhance competitiveness, shipping companies are developing stronger motivations to  
50 propose practicable measures to increase ship energy efficiency.

51 For the existing ships, it can be hard to change their structure to reduce fuel  
52 consumption. Thus, finely planning ship voyages, e.g., adopting weather routing and  
53 optimizing sailing speed are more popular measures. For the fixed sailing routes over a  
54 voyage, one main duty for the shipping company is to plan the daily sailing speeds of  
55 the ships in advance to minimize fuel consumption over the voyage while guaranteeing  
56 on-time arrival. Sailing speed optimization requires predicting ship fuel consumption  
57 in different situations. However, there are several challenges in making accurate  
58 prediction. First, inaccuracy exists in ship sailing data that can be used to construct fuel  
59 consumption prediction models, as these datasets mainly come from manually filled  
60 ship log data, such as noon reports. Second, factors influencing ship fuel consumption

61 are high-dimensional. Although it is widely believed that ship sailing speed is the most  
62 important influencing factor on ship fuel consumption (Fagerholt et al., 2010; Corbett  
63 et al, 2009; Psaraftis and Knotovas, 2013, Bialystocki and Konovessis, 2016), other  
64 factors can also have impacts. These factors include but are not limited to trim condition,  
65 displacement and draft conditions, weather and sea conditions, and hull and propeller  
66 roughness (Andersen et al., 2005; IMO, 2011; Bialystocki and Konovessis, 2016).  
67 Nevertheless, it is hard to have detailed information on all the influencing factors on  
68 fuel consumption, which prevents classic regression models from making accurate fuel  
69 consumption prediction. Third, as different ships have different properties and  
70 structures, one fuel consumption prediction model cannot be universally applied  
71 (Banawan et al., 2013). Alternatively, a tailored prediction model should be developed  
72 for each single ship to achieve more satisfactory prediction performance. Developing  
73 tailored machine learning models is a desirable and promising way to deal with these  
74 challenges. Machine learning models have the ability to handle multi-dimensional input  
75 data and to extract hidden information from complex datasets. In addition, they usually  
76 have better ability to deal with noisy data. Compared with traditional statistical  
77 regression models, machine learning models can address higher dimensional data (e.g.,  
78 ship displacement conditions, sea and weather conditions, trim conditions, and sailing  
79 speed) and make much more accurate predictions, and thus provide a more reliable  
80 foundation on developing tailored ship fuel consumption reduction models.

81 The purpose of this study is to propose a two-stage ship sailing speed optimization  
82 model for a dry bulk ship which contains two steps: in step 1, a machine learning model  
83 performing regression task (i.e. a random forest regression model) with high accuracy  
84 is proposed to make predictions on ship fuel consumption under different sailing speeds  
85 as well as cargo, weather, and sea conditions; in step 2, a sailing speed optimization  
86 model is proposed based on the prediction results in step 1 to minimize ship total fuel  
87 consumption over a voyage.

88

## 89 **2. Literature review**

### 90 **2.1 Research on ship fuel consumption prediction**

91 During the last few years, there has been an increasing amount of literature on  
92 prediction of ship fuel consumption (Zhao and Yang, 2018; Yang et al., 2019). The  
93 pioneering, basic, and commonly used models are deterministic models, which are also  
94 called white box models. In a deterministic model, the ship behavior of hull resistance,  
95 propeller propulsion, and main engine performance are described (Yang et al., 2019).  
96 Typical and pioneering studies include Holtrop (1977, 1978), Holtrop and Mennen  
97 (1978), and modern studies include Kristensen and Lützen (2012). Apart from the  
98 deterministic model, two types of models are also widely used in more recent research:

99 statistical models and machine learning models. Regarding the development of  
100 statistical models for fuel consumption prediction, Bocchetti et al. (2013) proposed a  
101 multiple linear regression analysis model, which took ship sailed distance and  
102 displacement as well as wind speed conditions into account to predict fuel consumption  
103 and CO<sub>2</sub> emissions of a cruise ship. Bocchetti et al. (2015) then developed another  
104 multiple linear regression model for a cruise ship by containing more influencing  
105 factors. Erto et al. (2015) also developed a multiple linear regression model for a cruise  
106 ship by taking ship operational factors and wind condition into consideration. As the  
107 foundation of a ship fuel consumption analysis system, Kee et al. (2018) proposed a  
108 multiple linear regression method to estimate fuel consumption of two tugboats.  
109 Although statistical models are intuitive and interpretable, there can be some drawbacks.  
110 First, parametric statistical models require making assumptions on data distributions  
111 before developing models, and this may bring bias. In addition, even if the log-log  
112 model can express the power function of speed and fuel consumption, the linear  
113 regression models usually cannot perform well when dealing with complicated data and  
114 multicollinearity data. Moreover, they are easily influenced by noisy data (Neter et al.,  
115 1996; Goldstein, 2011).

116 Over the past years, a growing body of innovative literature has focused on  
117 developing machine learning methods for ship fuel consumption prediction. The most  
118 popular method is Artificial Neural Networks (ANNs) model. Pedersen and Larsen  
119 (2009) proposed an ANN model for predicting propulsion power of a tanker based on  
120 ship noon report data. They also found that by combining sea and wind information,  
121 the performance of ANN model could be significantly improved. Beşikçi et al. (2016)  
122 developed a decision support system (DSS) for improving energy efficiency of an oil  
123 tanker. The decision system contained two parts: an ANN model for fuel consumption  
124 prediction under various operational conditions and a DSS based on the prediction  
125 results for energy-efficient ship operations. In comparison studies, they reported that  
126 the performance of the ANN model was superior to multiple regression analysis based  
127 on their dataset. Petersen and Jacobsen (2012a) compared the performance of ANN and  
128 Gaussian processes (GP) models when applied to predict fuel consumption of a  
129 domestic ferry. The result indicated that the performance of ANN was a little superior  
130 than the GP in all the tests. Petersen et al. (2012b) proposed tapped-delay neural  
131 network model for fuel consumption prediction of a tanker, which was then applied to  
132 trim optimization of the tanker. Petursson (2009) developed five machine learning  
133 models for fuel consumption prediction of a passenger ship: support vector regression  
134 (SVR), k-nearest neighbor (kNN), ANN, classification and regression trees (CART)  
135 and bagging. They found that the SVR and kNN outperformed the other models on their  
136 dataset. Other types of machine learning models are also adopted for ship fuel

137 consumption prediction. A least absolute shrinkage and selection operator (LASSO)  
138 regression model, which contained sea and weather conditions, was adopted to predict  
139 fuel consumption of a container ship (Wang et al., 2018). Soner et al. (2018) developed  
140 three tree-based models: bagging, random forest, and bootstrap based on the log dataset  
141 of a ferry ship that was also used by Petersen et al. (2012b). They identified that the  
142 performance of tree-based prediction models had higher prediction accuracy. Grey-box  
143 models, which is in between the white-box model and black-box model, were also  
144 developed. More specifically, one type of the grey-box model structure is built based  
145 on basic principles of ship propulsion and the unknown parameters are estimated by  
146 statistical regression models, such as Journée et al. (1987), Lu et al. (2013), Meng et al.  
147 (2016) and Yang et al. (2019). The other type of grey-box model combines white-box  
148 model, which describes some components of resistance or fuel consumption, and black-  
149 box model, such as machine learning and statistical models, for the remaining parts.  
150 This type of grey-box model can be seen in Leifsson et al. (2008), Coraddu et al. (2015),  
151 Haranen et al. (2016), and Coraddu et al. (2017). The advantage of grey-box models is  
152 they are able to integrate mechanistic knowledge with data analysis methods.

153 Machine learning models are capable of dealing with high-dimensional data and  
154 making more accurate predictions on complicated data than traditional regression  
155 models. In addition, no human interventions are needed when learning the models  
156 (Bishop, 2006; Alpaydin, 2009). Several studies have shown that the machine learning  
157 models outperform statistical models (Petersen and Jacobsen, 2012a; Wang et al., 2018;  
158 Du et al., 2019).

159 Regarding the factors that influence ship fuel consumption, almost all the above-  
160 mentioned studies, either based on statistical regression methods or machine learning  
161 methods, show that ship sailing speed is the dominant factor for ship fuel consumption  
162 prediction (Bocchetti et al., 2013, 2015; Petersen and Jacobsen, 2012a; Meng et al.,  
163 2016). Actually, the “cubic law” between ship sailing speed and fuel consumption, i.e.,  
164 the bunker consumption of a ship in one time unit is proportional to the sailing speed  
165 to the power of three, is widely-believed and adopted in shipping industry and maritime  
166 studies (Meng et al., 2016). Apart from sailing speed, ship displacement, such as total  
167 weight of the ship, cargo conditions, and ballast water, can also have an influence on  
168 fuel consumption based on vessel dynamics. Sea conditions, such as ocean currents (Lo  
169 and McCord, 1995), sea waves and swell (Lu et al., 2015; MAN Diesel & Turbo, 2011),  
170 are also proved to be influential to ship fuel consumption. Moreover, weather conditions  
171 are regarded as relevant to ship fuel consumption. For example, Kwon (1981) and  
172 Townsin and Kwon (1993) investigated weather conditions on ship performance and a  
173 group of regression models were proposed. Recently, models incorporating sea and  
174 weather information, including wind direction and force, sea wave direction and height,

175 and sea water temperature have exhibited high accuracy in ship fuel consumption  
176 prediction, such as the models proposed by Wang et al. (2016), Lee et al. (2018), Meng  
177 et al. (2016), and Du et al. (2019). Combining ship sailing related features together with  
178 sea and weather conditions have shown great potential in ship fuel consumption  
179 prediction and management.

## 180 181 **2.2 Research on improving ship energy efficiency**

182 Much of the current literature on ship energy efficiency pays particular attention to  
183 finding viable measures to reduce ship fuel consumption. As suggested by SEEMP,  
184 there are several effective ways to save ship fuel consumption from management  
185 perspective, which mainly include speed optimization, weather routing, efficient cargo  
186 operation, and trim optimization. As sailing speed is the most significant influencing  
187 factor, a considerable amount of literature has been focused on optimizing ship sailing  
188 speed to reduce fuel consumption, such as Fagerholt et al. (2010), Norstad et al. (2011),  
189 Yao et al. (2012), Wang and Meng (2012), Wang et al. (2013), Lee et al. (2015),  
190 Lindstad and Eskeland (2015), Song et al. (2015), Wang (2016) and Wang and Wang  
191 (2016). Weather routing helps ships to locally avoid rough sea and weather conditions  
192 in order to guarantee sailing safety as well as reduce fuel consumption. Studies on  
193 designing ship routes over a voyage based on weather information to realize fuel  
194 consumption reduction include Takashima et al. (2009), Shao et al. (2012), and Lin et  
195 al. (2013). IMO reported that trim optimization could reduce the main engine fuel  
196 consumption for most ship types by 0.5% to 3.0% (IMO, 2019). There is also research  
197 on developing trim optimization schemes for ship fuel consumption reduction, such as  
198 Reichel et al. (2014), Sherbaz and Duan (2014), Perera et al. (2015), and Moustafa et  
199 al. (2015). Proposing efficient ship cargo operation is often combined with fleet  
200 deployment and speed optimization, e.g., Xia et al. (2015) and Wang et al. (2015).

201 Over the period from 2016 through 2019, much more attention has been focused  
202 on developing two-phase optimization models for ship energy efficiency improvement.  
203 Generally, in the first phase, one or more models are developed for fuel consumption  
204 or weather conditions prediction under different situations; in the second phase, an  
205 optimization model is proposed for ship fuel consumption reduction over a voyage.  
206 Some typical two-phase models are presented as follows. Wang et al. (2016) proposed  
207 a real-time optimization model for a cruise ship which contained prediction of weather  
208 condition based on wavelet neural network (WNN) and determining the optimal engine  
209 speed based on the calculated ship resistance. Coraddu et al. (2017) developed a vessel  
210 trim optimization model for a tanker ship. The model included two parts: in the first  
211 part, a grey box model, which contained both mechanistic knowledge and historical  
212 data analysis, was proposed to predict the fuel consumption; in the second part, trim

213 optimization techniques were proposed. Lee et al. (2018) proposed a way to explore  
214 weather archive big data and optimize sailing speed for a container ship. First, the  
215 impact of weather conditions on ship fuel consumption was figured out by data mining  
216 methods. Then, speed optimization model was developed for the container. Du et al.  
217 (2019) presented a two-phase model for speed and trim optimization for a container. In  
218 the first phase, an ANN model was developed for estimating ship fuel consumption in  
219 different conditions. In the second phase, three countermeasures were put forward for  
220 reducing fuel consumption, including speed optimization, trim optimization as well as  
221 speed and trim optimization.

222 Although there are a growing number of studies on predicting and reducing ship  
223 fuel consumption, there are still considerable gaps existing in current literature. First,  
224 the literature has studied tankers, container ships, ferries, tugboats, and passenger ships.  
225 However, to the best of our knowledge, no model containing machine learning  
226 techniques for fuel consumption prediction and speed optimization are proposed for dry  
227 bulk ships. As the fuel prediction and optimization models are not universally applied  
228 (Lee et al., 2018; Banawan et al., 2013), it is necessary to develop such tailored model  
229 for a specific dry bulk carrier. Second, a large number of studies only include ship  
230 sailing speed as the input feature to predict ship fuel consumption. Actually, the  
231 determinants of fuel consumption are varied, including ship displacement and trim  
232 conditions as well as sea and weather conditions, but there is only a small number of  
233 studies considering these factors. Even if some studies take sea and weather information  
234 into account, the information is taken just from the noon report. Few studies have  
235 combined ship noon report with weather forecast, which could provide more  
236 comprehensive and accurate data. Third, most of the proposed machine learning models  
237 for ship fuel consumption prediction are based on ANNs. However, the development of  
238 ANN models usually requires a large number of training samples, and their structures  
239 are largely based on experience. In addition, tuning the parameters in ANNs can be  
240 difficult, and the prediction results are lack of interpretability. Moreover, the  
241 influencing degree of each input variable on the output variable is hard to figure out.  
242 Fourth, there are only a few pioneering studies on combining ship fuel prediction  
243 models and optimization models that can be put into practice to reduce fuel  
244 consumption and CO<sub>2</sub> emissions.

245 To bridge the gaps, we propose a two-stage model for a dry bulk ship based on ship  
246 noon report data and weather forecast data that contains (i) prediction of ship fuel  
247 consumption under different sailing speed, cargo, wind, swell, wind waves, and current  
248 conditions by adopting a random forest regressor, which is an ensemble learning  
249 method for regression based on multiple decision trees, and (ii) development of a speed  
250 optimization model to minimize ship fuel consumption over a voyage while

251 guaranteeing the estimated arrival time to the destination port based on the prediction  
252 results at the first stage. Compared with traditional statistical regression models, the  
253 advantages of using random forest regressors are that they are able to deal with high-  
254 dimensional data and make more accurate predictions. Compared to some other  
255 machine learning models, including ANNs, they are easier and faster to be implemented  
256 with more interpretable results and the influence degree of the features on the target  
257 variable can be generated, which can be used for feature selection.

### 259 **3. Data description**

#### 260 **3.1 Ship noon report**

261 Noon report of a ship is a ship voyage report data sheet prepared by the ship's  
262 captain on a daily basis (usually at noon). Many attributes of the ship's sailing behavior  
263 are recorded, such as ship geographic location, distance travelled since last report,  
264 average propeller revolutions per minute (RPM), engine speed, sailing speed, total hold  
265 cargo and total deck cargo. In addition, sea and weather conditions of the recording  
266 time are also comprised, e.g., information on sea swell direction (coming direction of  
267 sea swell), sea swell height, sea current value (depth of sea current), sea current type  
268 (coming direction of sea current), wind force, wind direction (coming direction of wind),  
269 and sea temperature. It should be noted that although noon report data is the main source  
270 for ship fuel consumption and optimization research, the features contained in the report  
271 are limited and may vary among different reports. The factors used in other studies that  
272 also choose noon report as the data source for ship fuel consumption management are  
273 similar, such as "wind speed and direction, sea water temperature, air temperature,  
274 water depth, and wave height and direction" in the model calibrated by Pedersen and  
275 Larsen (2009), "displacement, wave direction and height, and wind force" in the model  
276 proposed by Meng et al. (2016), "displacement, wave direction, wind force and  
277 direction, sea current direction, sea water temperature, and trim" in the model  
278 developed by Du et al. (2019), "forward draft, aft draft, wind direction and wind  
279 Beaufort number" in the model presented by Yang et al. (2019).

#### 280 **3.2 Description of ship voyage data**

281 The voyage data used in this study is the noon report data of a handy-size dry bulk  
282 ship with propeller diameter 5450mm, which was provided by an international shipping  
283 company. Time range of the voyage data is from 11<sup>th</sup> September 2017 to 27<sup>th</sup> February  
284 2019. Initially, the voyage report data for the ship contains 738 data entries. To start  
285 with, we filter the data entries by choosing the records with ship conditions as "sailing  
286 at sea", "with cargo loaded" and sailing speed value no less than 5 knots. After  
287 preprocessing, there are 242 selected entries left in the entire case dataset. Then, we use  
288 the hourly fuel consumption of the ship as the target variable (which is calculated by

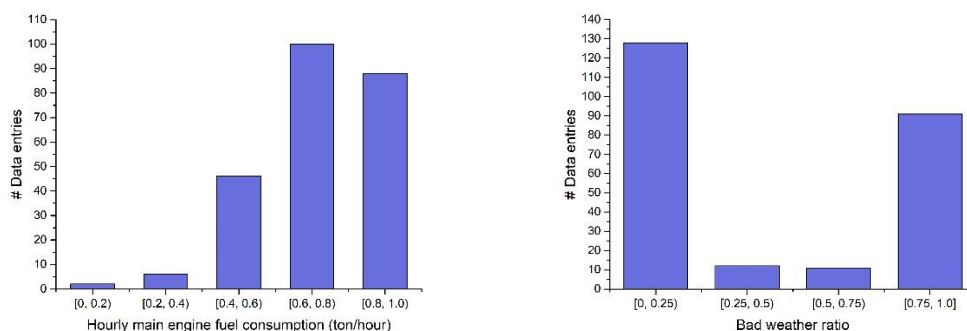


289 dividing the total fuel consumption by the total steaming hours) and delete the variables  
 290 that are not suitable to be the input of the fuel consumption prediction model. Finally,  
 291 9 input variables are selected from the attributes in the voyage data, namely, bad  
 292 weather ratio, ship sailing speed (knots), relative sea swell direction to ship's heading  
 293 ( $^{\circ}$ ), sea swell height (m), sea current type, sea current value (m), relative wind direction  
 294 to ship's heading ( $^{\circ}$ ), wind force (Beaufort force number), and total cargo weight  
 295 (metric ton). Based on the recording time and location (longitude and latitude) provided  
 296 by the noon report, we include two more attribute variables: height of combined wind  
 297 waves and swell (m) and relative wind wave direction to ship's heading ( $^{\circ}$ ) downloaded  
 298 from the European Centre for Medium-Range Weather Forecasts (ECMWF) (ECMWF,  
 299 2019). Eventually, the dataset contains 11 features as the input. Table 1 presents the  
 300 statistical information of the variables in the case dataset. Figure 1 illustrates the  
 301 distributions of the 242 data entries for the selected ship against the 11 input variables  
 302 and the output variable.

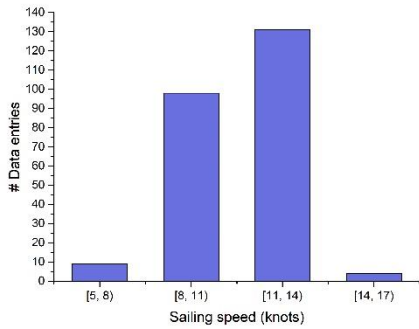
303 Table 1. Description of the variables in the entire dataset

Variable name	Meaning	Unit	Max value	Min value	Mean value
Hourly main engine fuel consumption	Fuel consumption of main engine in an hour	MT/h	0.9275	0.4139	0.7064
Bad weather ratio	Steaming time in bad weather/total steaming time	\	1	0	0.4139
Sailing speed	Ship average sailing speed	knots	14.2	5.3	11.1021
Relative sea swell direction	Direction of sea swell relative to ship's heading degree (-1 for no swell)	degree	180	1	87.3440
Sea swell height	Height of sea swell	meter	4	0	1.9959
Sea current type	Sea current against ship's heading (-1), with ship's heading (+1), no current (0)	\	1	-1	-0.3843
Sea current value	Depth of sea current	meter	4	0	0.4628
Relative wind direction	Direction of wind relative to ship's heading degree (-1 for no wind)	degree	180	0	99.8333
Wind force	Measure of wind speed	Beaufort force number	8	0	4.9008
Total cargo weight	Sum of the weights of on-deck cargo and under-deck cargo	MT	32741.5	11260	28685.06
Combined wind waves and swell height	Height of the combination of wind waves and swell (-1 for no wave and swell)	meter	9	0.1	2.1406
Relative wind wave direction	Direction of wind wave relative to ship's heading degree (-1 for no wave)	degree	179	0	81.1972

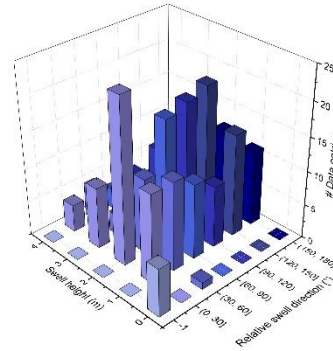
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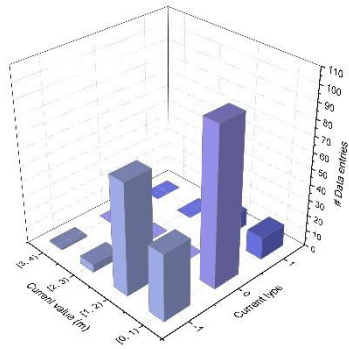
(a) Distribution of hourly main engine fuel consumption



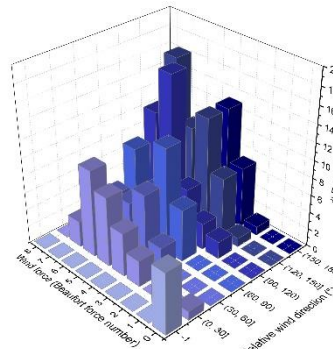
(b) Distribution of bad weather ratio



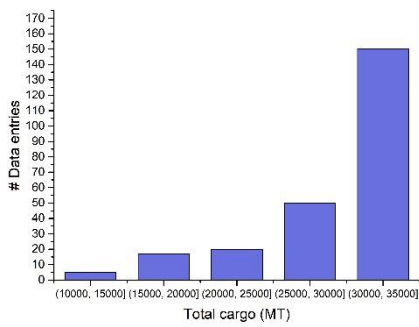
(c) Distribution of ship sailing speed



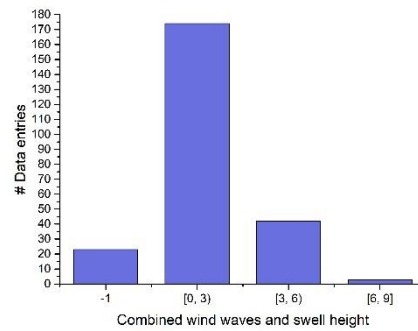
(d) Distribution of relative sea swell direction and sea swell height



(e) Distribution of sea current type and sea current value

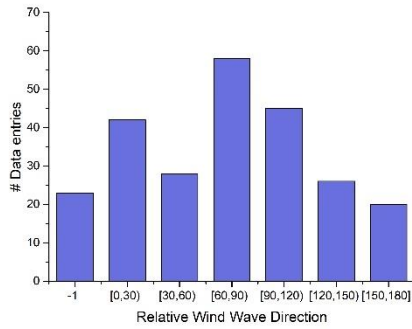


(f) Distribution of relative wind direction and wind force



(g) Distribution of total cargo weight

(h) Distribution of combined wind waves and swell height



(j) Distribution of relative wind wave direction

Figure 1. Distribution of data entries in the entire dataset

In order to validate the performance of the proposed two-stage speed optimization model for ship fuel consumption reduction, we randomly select two 8-day continuous noon report data from the entire dataset which is used for numerical experiments. The noon report records from 16 Jan 2018 to 23 Jan 2018 and from 24 Dec 2018 to 31 Dec 2018 are selected, respectively. For the remaining 226 data entries, 80% of them are randomly selected to constitute the training set to develop the regression models, while the remaining 20% form the test set.

## 4. Development of tree-based models for ship fuel prediction

### 4.1 Introduction of Decision tree (DT) regression model

A decision tree (DT) is a supervised and tree-like decision support model which is widely used to predict both discrete valued output (*classification tree*) and continuous valued output (*regression tree*) (Myles et al., 2004). There are several nodes in a decision tree, and each node contains a certain number of input data entries. The output value of a node is the average output of all the comprised data entries. A decision tree consists of three types of nodes: root node (the topmost node), leaf node (which gives final prediction output), and internal node (node except for root and leaf node). The process of dividing a node into two successive nodes is called *splitting*. A feature and one of its corresponding value are chosen to split a node, and each splitting of a node requires finding out the *best split* based on some splitting criteria. In DT classifiers, common splitting criteria are *Gini impurity* and *information gain*. In DT regressor, common splitting criterion is *mean squared error* (MSE) (Friedman et al., 2001). A node being split is called parent node while the successive nodes are called child nodes.

Three widely used DT generation algorithms are ID3, C4.5, and CART (Classification and Regression Tree) (Loh, 2014). As both input and output data contain continuous valued data, we construct a DT regressor by adopting CART algorithm (Breiman et al., 1984). CART algorithm requires recursively and binarily splitting the

333 nodes, and a binary decision tree will be built. Originally, the construction process  
334 terminates when all the leaf nodes contain the data entries of the same output value.  
335 However, this usually means that the tree is extremely large and is heavily overfitted.  
336 To alleviate overfitting, termination criteria are preset to control tree dimension. Three  
337 commonly used criteria are presented as follows. It should be noted that the values of  
338 these decision tree parameters may vary from different training sets.

339 (a) The maximum depth of a tree (denoted by *max\_depth*). The depth of a node in a  
340 decision tree is the number of nodes on a route from the root node to its parent node  
341 (the depth of root node is 0). The maximum depth of the tree is the maximum depth of  
342 all the nodes. A node cannot be further split if it reaches the maximum depth.

343 (b) The minimum number of data entries required to split a node (denoted by  
344 *min\_samples\_split*). If and only if a node contains data entries no less than  
345 *min\_samples\_split* can this node be further split.

346 (c) The minimum number of data entries required to be at a leaf node (denoted by  
347 *min\_samples\_leaf*). If and only if the number of data entries contained in both of the  
348 successive nodes split by the *best split* is no less than *min\_samples\_leaf* can the node  
349 be split.

350 Learning an optimal decision tree is known as an NP-complete problem (Laurent  
351 and Rivest, 1976; Naumov, 1991). Starting from splitting the root node, successive  
352 nodes are split in a depth-first manner until one of the termination criteria has been  
353 reached. Then, the next node for splitting is determined by retrospectively search for a  
354 node that can be further split. The algorithm terminates until there is no node that can  
355 be split. Main steps to generate a decision tree are described in Appendix A (Friedman  
356 et al., 2001; Harrington, 2012; Breiman, 2017).

## 357 **4.2 Introduction of random forest (RF) regression model**

358 Although the DT models are simple, intuitive, and interpretable, the main  
359 drawbacks of a single decision tree are that they are easy to get overfit (i.e., creating  
360 over-complex trees with poor generalization ability) and lack of robustness (i.e., small  
361 variations in the training data might result in a completely different tree being generated)  
362 (Ahmad et al., 2017). Ensemble learning is one of the popular ways to improve DT  
363 regressor performance. Ensemble methods contain multiple learning algorithms (called  
364 *weak learners*) and can obtain more desirable predictive performance than any of the  
365 constituent learning algorithms alone (Opitz and Maclin, 1999). There are two popular  
366 ensemble methods based on decision trees: *boosting* and *bagging*. In boosting,  
367 successive trees are dependent on the earlier trees, while in bagging, the trees are  
368 constructed using bootstrap sample of the training set (i.e., randomly selecting a certain  
369 number of samples from all the training samples with replacement) and the trees are  
370 independent on the other trees. Based on the bagging method, Breiman (2001) proposed

371 random forests by adding another layer of randomness: instead of considering all the  
 372 data features to split the nodes in each DT included in the forest, a randomly generated  
 373 subset of candidate features is used. Thus, apart from the abovementioned three  
 374 parameters in DT regressor, there are two more parameters in RF regressor:

375 (d) The number of decision trees contained in the forest (denoted by  $n\_estimators$ ).  
 376 Breiman (2001) proposed that adding more trees in the RF regressor will not suffer  
 377 from overfitting. Instead, more trees have the ability to limit the value of generalization  
 378 error.

379 (e) The number of features to consider when finding the *best split* of a node in each  
 380 decision tree (denoted by  $max\_features$ ). The value of  $max\_features$  should less than  
 381 the total number of data features and the certain number of features are randomly  
 382 selected at each splitting.

383 If CART based decision trees are the weak learners in a RF regressor, the main  
 384 differences between constructing a DT regressor and a single decision tree in the RF  
 385 regressor are twofold. (i) For a decision tree in RF regressor, bootstrap sampling from  
 386 the entire training set to form a new training set is required; for a normal DT regressor,  
 387 all the entries in the training set are used. (ii) For a decision tree in RF regressor,  
 388 randomly selecting a subset of data features for splitting the nodes in each decision tree  
 389 is required; for a normal DT regressor, all the features are considered when splitting  
 390 each node. After a certain number of DTs are constructed, the RF regressor requires  
 391 averaging the output values of all the trees as the prediction results (Liaw and Wiener,  
 392 2002). Compared with DT regressor, RF regressor has the advantages of robustness and  
 393 lower variance (Siroky, 2009). For the detailed process of constructing an RF regressor,  
 394 please refer to Breiman (2001), Biau and Scornet (2016), and Breiman (2017).

### 395 4.3 Metrics for model validation

396 In order to demonstrate the model performance in the test set, four typical regressor  
 397 performance measures are adopted: *mean squared error* (MSE), *root mean squared*  
 398 *error* (RMSE), *mean absolute error* (MAE), and *mean absolute percentage error*  
 399 (MAPE). Denote the input variable vector by  $x_e$ , the predicted output value by  $f(x_e)$ ,  
 400 and the real output value by  $y_e$ . The total number of data entries in the test set is  $N$ .  
 401 The definitions of MSE, RMSE, MAE, and MAPE are as follows:

$$402 \quad MSE = \frac{1}{N} \sum_{e=1}^N [f(x_e) - y_e]^2 \quad (1)$$

$$403 \quad RMSE = \sqrt{\frac{1}{N} \sum_{e=1}^N [f(x_e) - y_e]^2} \quad (2)$$

$$404 \quad MAE = \frac{1}{N} \sum_{e=1}^N |f(x_e) - y_e| \quad (3)$$

405

$$MAPE = \frac{100\%}{N} \sum_{e=1}^N \left| \frac{f(x_e) - y_e}{y_e} \right|. \quad (4)$$

406

#### 4.4 Construction and prediction results of DT and RF regression models

407

We adopt the scikit-learn machine learning library for Python to implement DT regressor and RF regressor based on CART algorithm (Pedregosa et al., 2011). The parameters for DT and RF regressors are set based on grid search method with five-fold cross validation as presented in Table 2. Except for those parameters, all the other parameters are set as the default values in scikit-learn library.

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Table 2. Parameters used in DT and RF regressors

Parameter	Decision tree regressor	Random forest regressor
<i>max_depth</i>	4	11
<i>min_samples_split</i>	5	2
<i>min_samples_leaf</i>	10	1
<i>n_estimators</i>	/	1000
<i>max_features</i>	/	4

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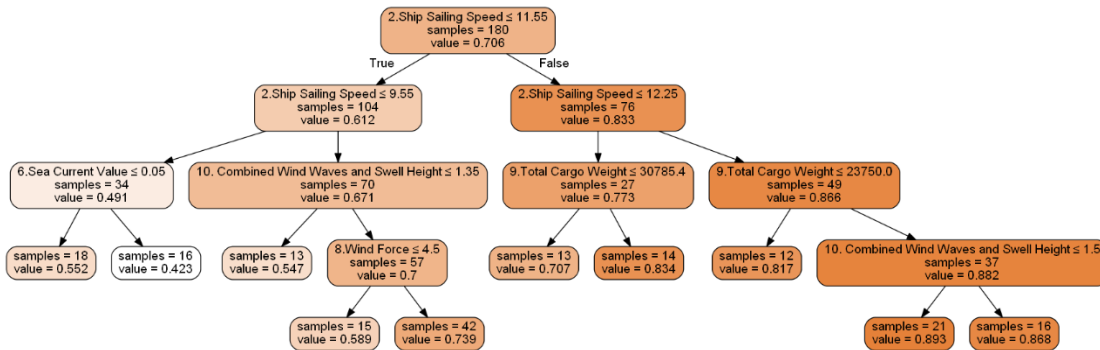
The DT regressor model is visualized in Figure 2. For each root and internal node in the figure, the first row indicates the selected splitting variable and the splitting value. The second row shows the number of samples contained in the node. The third row is the output value of this node. For each leaf node, the first row is the number of samples contained in the node, and the second row is the final output value.

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Figure 2. Visualization of the DT model for ship fuel prediction

421

The prediction performance of the two proposed regression models has also been compared with the popular machine learning based fuel consumption prediction methods in current literature. Three typical and popular regression models are selected for comparison: artificial neural network (ANN), least absolute shrinkage and selection operator (LASSO) regression, and support vector regression (SVR). ANN is a widely used machine learning model which contains a large number of highly interdependent processing elements called neurons. Usually, a typical ANN contains three layers of neurons: input layer, hidden layer, and output layer. LASSO is a linear regression

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429 analysis method that can perform variable selection and regularization in order to  
 430 improve regression performance. SVR is an application of support vector machine  
 431 (SVM) to regression problems. The datasets used for training and testing the models  
 432 are the same as those are used for the DT and RF regressors. It is worth mentioning that  
 433 the LASSO and SVR models are implemented by adopting scikit-learn machine  
 434 learning library for Python, and the parameters for the two models are tuned by grid  
 435 search method with five-fold cross validation. The construction of the ANN model is  
 436 similar to Du et al. (2019): five ANN models are constructed in MATLAB R2017a and  
 437 the average of the outputs of the five ANN models is the prediction output. The  
 438 prediction performance of the DT regressor, RF regressor, ANN, LASSO, and SVR  
 439 models are on a daily basis, i.e., hourly fuel consumption data has been converted to  
 440 fuel consumption for a day by considering steaming hours, are shown in Table 3. It can  
 441 be seen that both of the tree-based regressors perform well on our test set and the RF  
 442 regressor performs the best. Moreover, the RF regressor outperforms the DT regressor  
 443 regarding every metric.

444 Table 3. Performance of the five regression models on test set

Model/Metric	MSE	RMSE	MAE	MAPE
DT regressor	6.16	2.48	1.74	11.33%
<b>RF regressor</b>	<b>3.17</b>	<b>1.78</b>	<b>1.21</b>	<b>7.91%</b>
ANN	5.67	2.38	1.68	11.95%
LASSO	5.60	2.37	1.72	11.51%
SVR	9.37	3.06	2.55	15.47%

445

## 446 5. Development of ship speed optimization model

447

### 447 5.1 Problem description

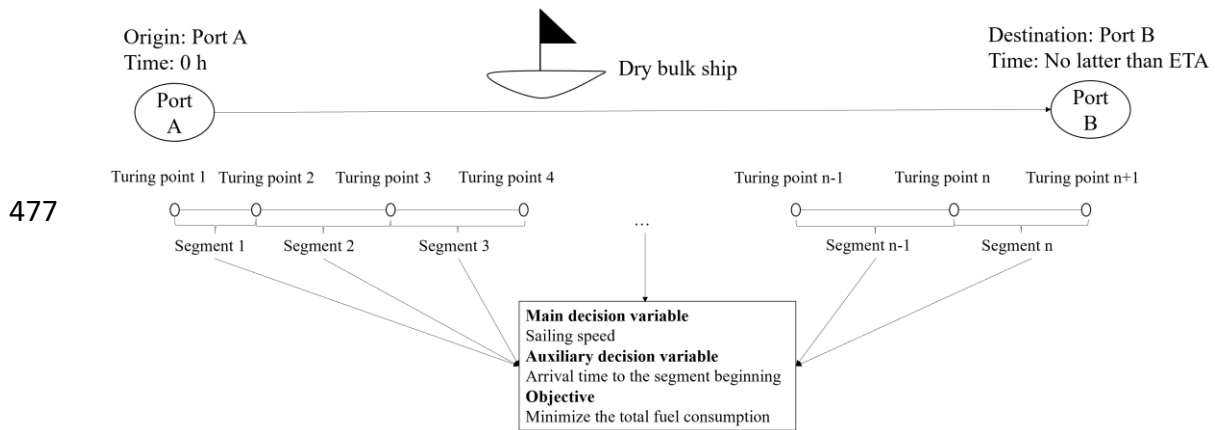
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448 As mentioned in the introduction part, shipping companies have a strong  
 449 motivation to carefully plan ship sailing speed during a voyage to reduce bunker  
 450 consumption and comply with the environmental protection conventions. Based on the  
 451 proposed RF regressor, which is able to predict fuel consumption of the dry bulk ship  
 452 under different sailing speeds, total cargo weight and sea and weather conditions, we  
 453 develop a speed optimization model between two ports while guaranteeing ship  
 454 estimated time of arrival (ETA) to the destination port.

455

455 To develop the sailing speed optimization model, we consider a situation when this  
 456 dry bulk ship sails from an origin port A to a destination port B along a fixed path which  
 457 the captain is quite familiar with. The loaded cargo of this ship is pre-determined and  
 458 fixed during the voyage and the sea and weather conditions can be obtained via  
 459 forecasts 5 to 7 days in advance. Due to the dynamic conditions at sea, the whole path  
 460 can be divided into several segments, and in each segment, we assume that the

461 international conventions that the ship needs to obey as well as sea and weather  
 462 conditions can be viewed as identical. A lower bound and an upper bound of permitted  
 463 ship sailing speed are also the same in one segment. The range of the allowable sailing  
 464 speed is determined by many factors, especially the sea and weather conditions in the  
 465 segment. For example, Tsou and Cheng (2013) adopted a formula to calculate a ship's  
 466 allowable maximum speed while navigation in storm conditions based on wave height  
 467 and wave direction to ensure navigational safety. The ship departure time from port A  
 468 is 0, and the ETA of port B is no later than the latest allowable arrival time. Two  
 469 questions need to be addressed: when to adjust the sailing speed (referred to as the time  
 470 of speed turning point) and what speed should be adjusted to (referred to as adjusted  
 471 speed). The objective of the model is to minimize the total fuel consumption of this dry  
 472 bulk ship over the whole voyage by determining the sailing speed in each segment.  
 473 Except for sailing speed, external factors that influence ship fuel consumption are the  
 474 same in one segment and thus the optimal speed should be the same in a segment. Thus,  
 475 it can be justified that the speed turning points can only occur at the beginning of a  
 476 segment. An illustration of the optimization problem is presented in Figure 3.



478 Figure 3. An illustration of the problem

479 **5.2 Development of a mathematical model**

480 The notation of the mathematical model is defined as follows.

---

Sets and indices

---

$n$	Total number of path segments
$i$	Index of a path segment, $i \in \{1, \dots, n+1\}$ . Segment $n+1$ presents the end of segment $n$ , i.e., port B
$I$	Set of all path segments, $I = \{1, \dots, n\}$

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481

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Parameters

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$c_i$	Ship total loaded cargo and sea and weather conditions in segment $i$
$v_0$	Ship speed before departure
$v_{i,c_i}^{\max}$	Maximum allowable speed when sailing in segment $i$ with the loaded



	cargo and sea and weather conditions as $c_i$ (knots)
$v_{i,c_i}^{\min}$	Minimum allowable speed when sailing in segment $i$ with the loaded cargo and sea and weather conditions as $c_i$ (knots)
$f^{RF}(v,c)$	Predicted ship fuel consumption (ton/hour) by using the proposed RF model when sailing speed is $v$ and ship total loaded cargo and sea and weather conditions are $c$
$L_i$	Path length of segment $i$ (nm)
$T_{\max}$	Latest allowable arrival time to the destination port

482

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Main decision variables

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$v_i$  Ship sailing speed in segment  $i$  (knots)

---

Auxiliary decision variable

---

$t_i$  Arrival time to the beginning of segment  $i$ ,  $t_1 = 0$ .  $t_{n+1}$  is the arrival time to the end of segment  $n$ , i.e., the arrival time of port B.

---

483 The speed optimization problem can be formulated by using Model **M1** based on  
484 the parameters and decision variables.

485 **[M1]**

486 
$$\min \sum_{i=1}^n (f^{RF}(v_i, c_i) \times \frac{L_i}{v_i}) \quad (5)$$

487 subject to:

488 
$$t_{i+1} = t_i + \frac{L_i}{v_i}, \forall i \in I \quad (6)$$

489 
$$t_{n+1} \leq T_{\max} \quad (7)$$

490 
$$v_0 = 0 \quad (8)$$

491 
$$v_{i,c_i}^{\min} \leq v_i \leq v_{i,c_i}^{\max}, \forall i \in I \quad (9)$$

492 
$$t_i \geq 0, \forall i \in I \cup \{n+1\}. \quad (10)$$

493 Objective (5) minimizes ship fuel consumption over the voyage. Constraint (6)  
494 indicates the relationship between the arrival time to the beginning of the previous  
495 segment and that of the next segment. Constraint (7) ensures the ship arrival time to the  
496 destination port is no later than the allowable arrival time. Constraint (8) ensures the  
497 sailing speed before departure is 0. Constraint (9) guarantees the lower and upper  
498 bounds of the sailing speed in each segment. Constraint (10) grants that the arrival  
499 time to the beginning of every segment is nonnegative. M1 cannot be solved directly  
500 by the off-the-shelf optimizers, thus we linearize the model in the next section.

501 **5.3 Linearization of model M1**

502 Given the maximum and minimum allowable sailing speeds, we can discretize the  
 503 speed values with 0.1 knot as an interval. Specifically, given  $v_{i,c_i}^{\max}$  and  $v_{i,c_i}^{\min}$  in segment  
 504  $i$  respectively, as we discretize the sailing speeds with 0.1 as an interval, we have the  
 505 sailing speed parameters  $v_i^1 = v_{i,c_i}^{\min}$ ,  $v_i^2 = v_i^1 + 0.1, \dots, v_i^{u_i} = v_{i,c_i}^{\max}$  and a specific sailing  
 506 speed as  $v_i^u = \{v_i^1, v_i^2, \dots, v_i^{u_i}\}$ . We further introduce a binary decision variable  $y_i^u \in \{0,1\}$ ,  
 507 and if  $v_i = v_i^u$ ,  $y_i^u = 1$ ; otherwise  $y_i^u = 0$ . The new main decision variable is  $y_i^u$ , and the  
 508 auxiliary decision variable is  $t_i$ . Based on the new parameters and decision variables,  
 509 we can convert model M1 to model M2.

510 [M2]

511 
$$\min \sum_{i=1}^n \sum_{u=1}^{u_i} [y_i^u \times (f^{RF}(v_i^u, c_i) \times \frac{L_t}{v_i^u})] \quad (11)$$

512 subject to:

513 
$$t_{i+1} = t_i + \sum_{u=1}^{u_i} (\frac{L_t}{v_i^u} \times y_i^u), \forall i \in I \quad (12)$$

514 
$$t_{n+1} \leq T_{\max} \quad (13)$$

515 
$$v_0 = 0 \quad (14)$$

516 
$$y_i^u \in \{0,1\}, \forall i \in I, u \in \{1, 2, \dots, u_i\} \quad (15)$$

517 
$$\sum_{u=1}^{u_i} y_i^u = 1, \forall i \in I \quad (16)$$

518 
$$t_i \geq 0, \forall i \in I \cup \{n+1\} \quad (17)$$

519 Model M2 is equivalent to M1 and is a mixed-integer linear programming (MIP)  
 520 model, which can be solved by the off-the-shelf optimizers, such as CPLEX.

522 **6. Computational experiments**

523 **6.1 Prediction of ship fuel consumption**

524 In this section, we adopt the RF regressor developed in Section 4 to predict fuel  
 525 consumption of the dry bulk ship during two continuous 8-day sailing voyages (denoted  
 526 by voyage 1 and voyage 2, respectively). The total sailing distance of voyage 1 is 2001.2  
 527 nautical miles with the total sailing time as 195 hours, and the total fuel consumption  
 528 is 126.36 tons. The total sailing distance of voyage 2 is 1946.4 nautical miles with the  
 529 total sailing time as 192 hours, and the total fuel consumption is 114.92 tons. The total  
 530 cargo weight as well as sea and weather conditions in each sailing segment of each  
 531 voyage are presented in Table 4 and Table 5.

532

Table 4. Initial ship sailing information of voyage 1

Day	Steaming hour (h)	Total fuel (tons)	Speed (knots)	Hourly fuel consumption (tons/h)	Bad weather ratio	Relative sea swell direction (degree)	Sea swell height (m)	Sea current type	Sea current value	Wind force (Beaufort force number)	Relative wind direction (degree)	Total cargo weight (MT)	Combined Wind Waves and Swell Height (m)	Relative Wind Wave Direction (degree)
1	24	15.69	10	0.65375	1	160	2	0	0	6	160	26605	1.7	136
2	24	15.42	9.9	0.64250	1	89	2	0	0	6	111.5	26605	3.0	92.3
3	25	16.22	9.8	0.64880	0.40	44	1	0	0	4	89	26605	1.2	6.5
4	24	15.43	10.2	0.64292	0.25	82	2	0	0	4	8	26605	2.4	52.8
5	25	16.10	10.4	0.64400	0.80	170	2	0	0	5	170	26605	0.8	170.3
6	24	15.45	10.3	0.64375	1	106	2	0	0	6	128.5	26605	3.3	80.5
7	24	15.54	10.9	0.64750	0.50	89	2	0	0	5	91	26605	1.2	28.1
8	25	16.51	10.6	0.66040	1	99	2	0	0	7	144	26605	2.3	90.3

533

534

Table 5. Initial ship sailing information of voyage 2

Day	Steaming hour (h)	Total fuel (tons)	Speed (knots)	Hourly fuel consumption (tons/h)	Bad weather ratio	Relative sea swell direction (degree)	Sea swell height (m)	Sea current type	Sea current value	Wind force (Beaufort force number)	Relative wind direction (degree)	Total cargo weight (MT)	Combined Wind Waves and Swell Height (m)	Relative Wind Wave Direction (degree)
1	24	14.45	11.2	0.60208	0	113	1	-1	0.6	3	90.5	31948	1.9	93.0
2	24	14.41	10.8	0.60042	0	87	1	-1	0.6	4	154.5	31948	2.0	76.9
3	24	14.35	9.2	0.59792	0.75	132	2	-1	1	6	177	31948	1.9	106.4
4	24	14.36	8.9	0.59833	1	177	2	-1	1.5	6	160.5	31948	1.6	121.6
5	24	14.33	9.7	0.59708	0.17	95	2	-1	1.5	4	117.5	31948	1.5	111.8
6	24	14.36	10.5	0.59833	0	100	2	-1	1	4	100	31948	1.8	104.6
7	24	14.45	11.5	0.60208	0.04	145	1	0	0	5	100	31948	2.3	106.1
8	24	14.21	9.3	0.59208	1	145	2	-1	1.5	6	100	31948	3.0	113.0

536

We use the total 226 data entries (i.e., all the data entries in the entire dataset except for the 16 records used to validate the optimization model) to construct the RF regressor. Due to the lack of extreme valued data, predicting fuel consumption under too large or too small speed values is highly likely to suffer from inaccuracy (Freidman et al., 2001). Thus, we make predictions on fuel consumption with speed values ranging between 10% and 90% from small to large in the training set, i.e., we exclude the 10% smallest and 10% largest speed values. The selected speed values are from 8.9 to 13.3 knots. The fuel consumption prediction results are presented in Figure 4. For the 16 validation records, we only have the real output value under the given speed. The performance of the RF regressor on predicting the fuel consumption under the given speed of the 16 records are given in Table 6. It can be seen that the predicted fuel consumption under the given speed is higher than the real fuel consumption.

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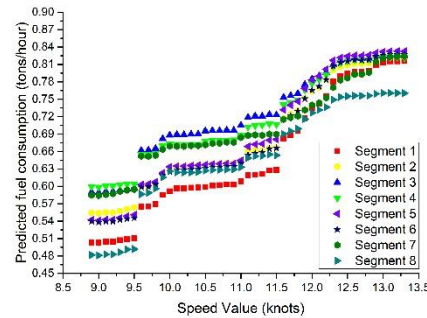
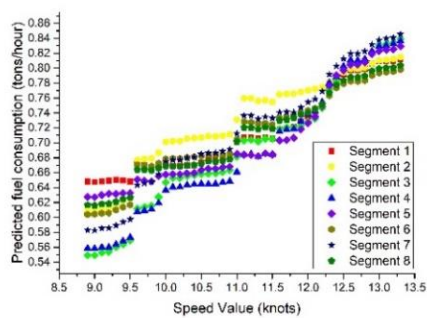
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(a) Prediction results of voyage 1 (b) Prediction results of voyage 2

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Figure 4. Fuel consumption prediction results of the voyages

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Table 6. RF regressor performance on predicting fuel consumption of the two voyages

555

Path NO	Real total fuel consumption	Predicted total fuel consumption	MSE	RMSE	MAE	MAPE
1	126.36	129.6335	0.5186	0.7201	0.6285	4.00%
2	114.92	116.6830	1.5702	1.2531	0.8950	6.24%

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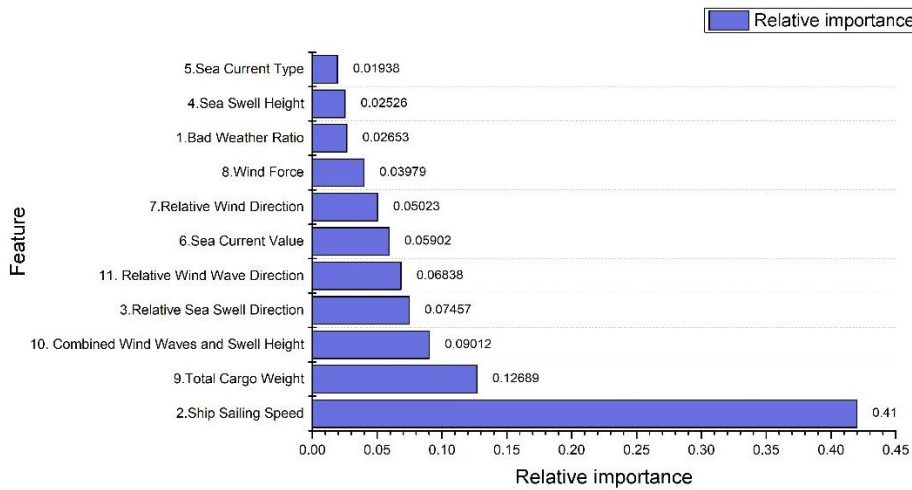
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Apart from predicting fuel consumption under different speed values in each segment, the RF regressor is also able to illustrate the feature importance of the input variables when predicting fuel consumption. The feature importance generated by the 230 data entries is shown in Figure 5.

562



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Figure 5. Relative importance of the input features

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Figure 5 indicates that ship sailing speed is the most significant influencing factor on ship fuel consumption, which allies with most of the current literature. Total cargo weight also has a great influence on fuel consumption. Regarding sea and weather conditions, combined wind wave and swell height, relative sea swell, and relative wind wave direction can have more impact, while sea current type has the least impact on ship fuel consumption.

570

### 6.2 Validation of speed optimization model

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574

We validate the performance of the proposed mathematical model by adopting CPLEX optimizer to find the optimal sailing speed in each segment which can minimize the total fuel consumption. A laptop (Intel Core i7, 2.20GHz, Memory 16.0G) is used to conduct the experiment with the programming language C++. The selected sailing

575 speed, sailing time, and fuel consumption in each segment of the two voyages are  
 576 shown in Table 7 and Table 8.

577 Table 7. Ship sailing information in voyage 1 after optimization

Day	Sailing hours (h)	Sailing speed (knots)	Hourly fuel consumption (tons/h)	Total fuel consumptions (tons)
1	18.0451	13.3	0.812160	14.6555
2	17.8647	13.3	0.814638	14.5532
3	25.7895	9.5	0.569093	14.6766
4	22.4587	10.9	0.648483	14.5641
5	22.6087	11.5	0.684186	15.4685
6	18.5865	13.3	0.798043	14.8328
7	21.6198	12.1	0.758690	16.4027
8	19.9248	13.3	0.803543	16.0104
Total	166.898	/	/	121.1638 (-4.11%)

579 Table 8. Ship sailing information in voyage 2 after optimization

Day	Sailing hours (h)	Sailing speed (knots)	Hourly fuel consumption (tons/h)	Total fuel consumptions (tons)
1	28.2947	9.5	0.510766	14.4520
2	22.5391	11.5	0.666378	15.0196
3	16.6015	13.3	0.825215	13.6998
4	18.5739	11.5	0.706721	13.1266
5	24.5053	9.5	0.550893	13.4998
6	26.5263	9.5	0.545664	14.4745
7	24	11.5	0.689559	16.5494
8	23.4947	9.5	0.491945	11.5581
Total	184.536	/	/	112.380 (-2.21%)

581 It takes only 0.05 and 0.03 second respectively to find the optimal solutions for  
 582 case 1 and case 2. After speed optimization, the dry bulk ship consumes 121.1638 tons  
 583 of fuel to complete voyage 1 and 112.38 tons of fuel to complete voyage 2 while  
 584 guaranteeing the arrival time to the destination. Compared with the real fuel  
 585 consumption of 126.36 and 114.92 tons in voyage 1 and 2, the ship can save 4.11% and  
 586 2.21% of total fuel consumption after speed optimization. It should be noted that the  
 587 predicted fuel consumption is a little higher than the real fuel consumption. If we  
 588 compare the fuel consumption after speed optimization and the predicted total fuel  
 589 consumption in these two voyages (i.e. 129.6335 and 116.6830), we can conclude that  
 590 6.53% and 3.69% of fuel consumption can be reduced, respectively. We can then  
 591 conclude that the two-stage fuel consumption prediction and speed optimization model  
 592 can help the bulk carrier ship to save 2% to 7% fuel to complete an 8-day voyage. Note  
 593 that as the data for training the RF regressor is limited and there can be inaccuracy in  
 594 fuel consumption prediction, the savings in fuel consumption may have variations.

595  
 596 **7. Extension and future research**

597 Although numerous studies on ship fuel management are conducted based on ship  
598 noon report, the data of ship sailing information and sea and weather conditions  
599 provided by the noon report is actually limited (as discussed in Section 3.1) and the  
600 time resolution is low: usually only one record for 24 hours. In order to make precise  
601 fuel consumption prediction, which is the foundation of efficient ship fuel management,  
602 one possible way is to incorporate more data features from other data sources, such as  
603 sea and weather data from weather forecast website. For example, water depth can be  
604 included for considering the influence of shallow water on fuel consumption. Besides,  
605 temperature and salinity of water can also have an effect. Another possible way is to  
606 combine noon report with ship sensory data, which can provide more ship sailing  
607 features such as trim and draft condition with much lower time interval. Combining  
608 sensory data can also help to develop sailing speed optimization model. If more data  
609 can be obtained for a day, the division of sailing segment can be more flexible, e.g., by  
610 the length of a fixed time such as 3h, 6h, or 12h voyage at the calm water set speed. In  
611 addition, if more ship sailing information is accessible, it is easier to combine sea and  
612 weather data with ship sailing data. In the current noon report, the sailing distance of  
613 one day is usually more than 200 nm. As only one record is generated for each day, the  
614 sea and weather conditions are viewed as identical in the whole sailing distance covered  
615 by a whole day. However, sea and weather data can usually have a given resolution. For  
616 example, if the resolution of sea and weather forecasting data is  $0.5^\circ$ , the associated arc  
617 length is 30 nm. In addition, the weather forecast usually renews every few hours. For  
618 example, the weather forecast provided by ECMWF renews every 6 hours. If we can  
619 have 4–6 reports each day, the ship sailing data and sea and weather data can be  
620 combined. Therefore, accurate and practical fuel consumption prediction model and  
621 sailing speed optimization model can be proposed.

## 623 **8. Conclusion**

624 Shipping companies are developing stronger motivations to improve ship fuel  
625 energy efficiency for the purpose of complying with environmental conventions and  
626 increasing their profits. This study proposes a two-stage fuel consumption prediction  
627 and reduction model for a dry bulk ship to improve its energy efficiency based on the  
628 noon report data. More specifically, at the first stage, a random forest regression model  
629 is developed to predict the dry bulk ship's fuel consumption under different total carried  
630 cargo, sea, and weather conditions. It is also validated that the proposed RF regressor  
631 outperforms the widely used machine learning models such as ANN, SVR, and LASSO  
632 for ship fuel consumption prediction. At the second stage, a speed optimization model  
633 is proposed based on the fuel consumption prediction results at the first stage. The  
634 objective of the optimization model is to minimize the total fuel consumption of the dry

635 bulk ship over a voyage which contains several segments by deciding the sailing speed  
636 in each segment. The model is a mixed integer programming model which can be  
637 efficiently solved by CPLEX. In the computational experiments, we use two 8-day  
638 sailing voyage reports to test the performance of the two-stage model. The results show  
639 that the proposed model can save fuel consumption to 2%–7% compared with the real  
640 situation, which can also lead to significant CO<sub>2</sub> emissions reduction. In addition, the  
641 influence degree of the input features on the total fuel consumption is also generated.  
642 Similar to other related studies, it is indicated that ship sailing speed is the dominant  
643 factor of ship fuel consumption, then followed by total carried cargo. Regarding sea  
644 and weather conditions, combined wind wave and swell height, relative sea swell and  
645 wind wave directions can also have remarkable impact, while the current type has the  
646 least influence on ship fuel consumption. **This paper considers the relationship between  
647 ship sailing speed and fuel consumption rate in a non-analytical form, which improves  
648 the common understanding about fuel consumption management.** The proposed model  
649 is one of the pioneering models which combine a machine learning model with an  
650 optimization model in ship fuel consumption prediction and reduction. Based on the  
651 model, shipping companies are able to finer plan the daily sailing speed of their ships  
652 in order to reduce fuel consumption and CO<sub>2</sub> emissions.

653

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663

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868 **Appendix A. Construction of decision tree regressor (Friedman et al., 2001;**  
869 **Harrington, 2012; Breiman, 2017)**

870 The input information for decision tree construction contains the training set and  
871 termination conditions. We denote the set of  $J$  input features as  $(x_1, x_2, \dots, x_J)$ . An input  
872 feature is denoted by  $x_j$ , and the value range of this  $J$ -feature is  $[x_j^{\min}, x_j^{\max}]$ . A  
873 specific value of this feature is denoted by  $s_j$ ,  $s_j \in [x_j^{\min}, x_j^{\max}]$ . In addition, we denote  
874 the training set containing  $N$  data entries as  $D = \{(x^1, y^1), (x^2, y^2), \dots, (x^N, y^N)\}$ . A data  
875 entry is denoted by  $(x^e, y^e)$  with  $e = 1, \dots, N$ , where  $x^e = (x^{e1}, x^{e2}, \dots, x^{ej}, \dots, x^{eJ})$  is a  
876 dimensional vector containing  $J$  features and  $y^e$  is a one dimensional output value.  
877 The construction process of a regression decision tree based on CART algorithm  
878 requires finding the *best split* pair  $(j^*, s_{j^*})$ ,  $s_{j^*} \in [x_{j^*}^{\min}, x_{j^*}^{\max}]$  of the nodes when splitting.  
879 Denote termination condition (a) to (c) as  $T_a$ ,  $T_b$ , and  $T_c$ . The main steps to construct  
880 a CART decision tree are presented as follows:

---

*Procedure 1: Construction of CART decision tree*

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<i>Input</i>	Training set $D$ and termination conditions $T_a$ , $T_b$ , and $T_c$ .
<i>Output</i>	Regression tree $f^{DT}(x)$ .
<i>Step 1</i>	Find the <i>best split</i> pair $(j^*, s_{j^*})$ of the current splitting node by solving the following formula: $(j^*, s_{j^*}) \in \arg \min_{\substack{j \in \{1, \dots, J\} \\ s_j \in [x_j^{\min}, x_j^{\max}]}} \left[ \sum_{e \in R_{m1}(j, s_j)} \left( y^e - \frac{1}{ R_{m1}(j, s_j) } \sum_{e \in R_{m1}(j, s_j)} y^{e1} \right)^2 + \sum_{e \in R_{m2}(j, s_j)} \left( y^e - \frac{1}{ R_{m2}(j, s_j) } \sum_{e \in R_{m2}(j, s_j)} y^{e2} \right)^2 \right],$ where $R_{m1}(j, s_j) = \{e = 1, \dots, N \mid x^{ej} \leq s_j\}$ and $R_{m2}(j, s_j) = \{e = 1, \dots, N \mid x^{ej} > s_j\}$ .
<i>Step 2</i>	Use the <i>best split</i> $(j^*, s_{j^*})$ to split the current node into two nodes that contain two sub datasets $R_{m1}(j^*, s_{j^*}) = \{e = 1, \dots, N \mid x^{ej} \leq s_{j^*}\}$ and $R_{m2}(j^*, s_{j^*}) = \{e = 1, \dots, N \mid x^{ej} > s_{j^*}\}$ with output values as $c_1 = \frac{1}{ R_{m1}(j^*, s_{j^*}) } \sum_{e_1 \in R_{m1}(j^*, s_{j^*})} y^{e1}$ and $c_2 = \frac{1}{ R_{m2}(j^*, s_{j^*}) } \sum_{e_2 \in R_{m2}(j^*, s_{j^*})} y^{e2}$ , respectively.
<i>Step 3</i>	Repeat step 1 and step 2 in a depth-first manner until coming to a node that reaches one of the preset termination conditions. Then, this node becomes a leaf node and a new node for splitting is found by backtracking.
<i>Step 4</i>	Repeat Step 3 until there is no more nodes that can be split. Finally, the total training set is separated into $M$ mutually exclusive sub-sets $R_1, R_2, \dots, R_M$ , and a sub-set is denoted by $R_m$ . The decision tree model can be presented by

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$$f^{DT}(x) = \sum_{m=1}^M c_m I(x \in R_m), \text{ where } I(x \in R_m) = \begin{cases} 1, & x \in R_m \\ 0, & x \notin R_m \end{cases}.$$

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