This is the Pre-Published Version.
This is the peer reviewed version of the following article: Guan, X., Wang, Y., Yi, Z. and Chen, Y.-J. (2020), Inducing Consumer Online Reviews Via Disclosure. Prod Oper Manag, 29: 1956-1971, which has been published in final form at https://doi.org/10.1111/poms.13199. This article may be used for non-commercial purposes in accordance with Wiley Terms and Conditions for Use of Self-Archived Versions.

Inducing Consumer Online Reviews via Disclosure

Xu Guan

School of Management, Huazhong University of Science and Technology, Wuhan, 430074 China, guanxu@hust.edu.cn

Yulan Wang*[1]

Faculty of Business, The Hong Kong Polytechnic University, Hong Kong, yulan.wang@polyu.edu.hk

Zelong Yi

Department of Transportation Economics and Logistics Management, College of Economics, Shenzhen University, Shenzhen, Guangdong, China, yizl@szu.edu.cn

Ying-Ju Chen

School of Business and Management & School of Engineering, Hong Kong University of Science and Technology, Kowloon, Hong Kong, imchen@ust.hk

---

In this paper, we investigate a seller's voluntary disclosure strategy when serving two groups of consumers who arrive sequentially and are reference dependent with respect to product quality. Consumers may be naive or sophisticated, depending on whether they can make rational inferences from the seller's disclosure behavior and an experienced consumer's quality review. We show that when consumers are naive, the seller can strategically withhold high quality information and disclose low quality information to boost the reference-dependent early adopter's subjective quality review, which in turn enhances the follower's quality expectation and allows the seller to extract more surplus in the second period. However, when consumers are sophisticated, it is difficult for the seller to enhance their quality expectations by designing his disclosure strategy to manipulate the early adopter's quality review. The seller discloses all the quality information to consumers. When the market contains both naive and sophisticated consumers, the seller is able to withhold relatively low quality information in advance. In such a situation, the seller exclusively serves naive customers in the first period by charging a high retail price. The above results are quite robust, regardless of whether the review rating is bounded or whether consumers possess heterogeneous preferences with respect to product quality.

Keywords: Information Disclosure; Reference-Dependent Preferences; Sequential Selling

---

[1]Corresponding author.

1 Introduction

A seller often needs to decide whether to disclose his or her product information to prospective consumers. To disclose quality information, the seller may invest in informative advertisements, provide free samples or employ product labels. In contrast, to withhold such information, the seller can reduce investment in advertising or limit the content presented in product packaging (Branco et al., 2015). The logic behind these two decisions seems straightforward. When the product has high quality, a seller should work diligently to promote his or her product to attract more consumers, while if the product has low quality, the seller needs to conceal such information to prevent consumers from forming negative impressions. However, as indicated by many scholars (e.g., Jovanovic (1982), Shavell (1994), Guo (2009)), unless consumers are naive or information disclosure is very costly, it is difficult for a seller to withhold his or her negative quality information. This is because according to the so-called "unraveling" result (Grossman and Hart, 1980; Milgrom, 1981), a rational consumer will infer the seller's non-disclosure behavior as indicating the lowest product quality.

Nonetheless, recent years have witnessed the success of an "information withholding" strategy in many companies. For example, Spanx, an American hosiery company, GoPro, a high-def camera company, and Krispy Kreme Doughnuts Inc have all succeeded without investing or investing little in advertising.[1] One underlying argument is that with the rapid growth of information technology and the popularity of social media, a consumer can easily obtain product information through the comments of other experienced consumers. Consequently, a firm should rely more on *word-of-mouth* marketing to advocate its products. Compared with traditional advertising, this option is also effective and saves significant amounts of money (Dellarocas, 2003). In addition to this cost-driven consideration, some scholars also believe that the *information non-disclosure* strategy could be driven by the seller's intention to manipulate consumers' word of mouth (Dellarocas, 2006).

Many empirical studies have verified that a consumer's quality assessment inevitably involves behavioral factors such as self-selection bias and reference-dependent preferences that can be influenced by the seller's marketing strategy (Li and Hitt, 2008; Dellarocas, 2006). For example, a consumer's overall post-consumption quality evaluation normally hinges not only on the true quality level but also on how much it contrasts with a reference quality level (Anderson and Sullivan, 1993; Tversky and Kahneman, 1991;

[1] Please refer to Forbes: "How Spanx Became A Billion-Dollar Business Without Advertising", and ABC News: "Some Brands Thrive Without Advertising".

Kőszegi and Rabin, 2006; Gneezy et al., 2014). That is, customers are often reference dependent and become more excited and positive if they find that a product's quality exceeds their initial expectation (which serves as the reference point) (Gneezy et al., 2014). In his best selling book about *word-of-mouth* marketing, Bueno (2007) has indicated that "people love to share what surprised them," and so "it is much better to let the consumer discover the best thing about the product instead of hearing the seller to shout it from the rooftops."[2] Similarly, if a seller strategically withholds high quality information, the quality assessment of early-arriving reference-dependent consumers may exceed the product's true quality level, which in turn encourages more late-arriving consumers to buy the product.

The above discussion reveals that a seller's decision regarding information disclosure needs to take into account the product's inherent quality level, consumer responses to the seller's information disclosure/withholding, the impact of such information disclosure/withholding on consumers' post-consumption quality evaluations and how late-arriving consumers interpret such quality evaluations. Notably, how consumers interpret the quality assessments posted by experienced consumers and the seller's disclosure behavior varies according to their types. In particular, a sophisticated consumer can make a rational inference about product quality from the seller's disclosure behavior, while a naive consumer cannot (Jovanovic, 1982; Shavell, 1994; Gabaix and Laibson, 2006; Taylor, 2004). Additionally, when reading consumers' subjective product quality reviews, a sophisticated consumer could strategically interpret them to form his or her own quality expectation, while a naive consumer simply takes those reviews as his or her quality expectation.

Our aim in this paper is to investigate how consumers' reference-dependent preferences and the interplay among the aforementioned factors affect a firm's information disclosure decision. Specifically, we are interested in addressing the following research questions that have not been adequately investigated in the existing literature:

- How will a seller's information disclosure behavior affect a reference-dependent consumer's quality expectation and purchasing decision?
- How will the quality evaluations of experienced and reference-dependent consumers affect a new consumer's quality expectation and corresponding purchasing decision?

[2]For more information, please refer to the book "Why We Talk: Seven Reasons Your Customers Will - Or Will Not - Talk About Your Brand".

- How will a consumer's type and his or her reference-dependent preferences jointly affect a seller's voluntary information disclosure strategy?
- What is the impact of consumer type and reference-dependent preferences on the seller's performance?

To address the preceding questions, we consider a monopolistic seller (he) who sells a type of search product to two representative reference-dependent consumers, an early adopter and a follower, who arrive sequentially in two periods. The consumer (she) can be either naive or sophisticated, depending on whether she can make rational inferences based on the seller's disclosure behavior and the quality assessment made by the previous consumer. At the beginning of the selling season, the seller privately observes his own product quality and decides whether to disclose this information to the early adopter. After observing the seller's disclosure behavior, the early adopter forms her initial quality expectation (that is, her quality reference point) and accordingly decides whether to buy the product. After consuming the product, the early adopter then generates a subjective review by comparing the experienced quality (the real quality level) with her reference quality, which is also bounded within a limited range. Note that the bounded review system is very common in practice and adopted by many companies; see the examples from hotel/food/movie websites such as Tripadvisor, Booking.com and Yelp. If the early adopter finds that the true quality level is higher than her reference quality, she forms a very positive product quality evaluation. On the contrary, if the product quality falls short of her reference quality, she comments more negatively on the product's quality. Finally, the review is realized and becomes public information at the beginning of the second period. The follower observes this quality assessment and then makes her purchasing decision.

We show that when consumers are naive, the seller can strategically manipulate the consumer's quality evaluation/review by fine-tuning his disclosure strategy. That is, the seller deliberately withholds the relatively high quality information to boost the early adopter's quality review and then relies on this word-of-mouth effect to increase the follower's quality expectation. When the magnitude of reference effect is high, such strategic information withholding allows the seller to extract more surplus from the follower that surpasses his loss in the first period due to charging a relatively low price. As a result, the seller in equilibrium can disclose the sufficiently high quality information but withhold the low quality information and his expected payoff monotonically increases in the magnitude of reference effect.

In contrast, when consumers are sophisticated, their quality expectation can be hardly augmented even when the seller manipulates the early adopter's quality review. This is because sophisticated consumers can make rational inference upon observing the seller's disclosure behavior and the early adopter' reference-dependent review. Consequently, the seller has to disclose all the quality information. A close look at the equilibrium outcomes under the aforementioned two consumer types reveals that the seller discloses his quality information in more scenarios but obtains a lower payoff when facing sophisticated consumers than when facing naive consumers.

We then consider a scenario in which, in each period, the market contains both naive and sophisticated consumers. We show that in equilibrium, the seller would not choose to initially withhold high quality information to boost the early adopter's quality review. The underlying reason is that sophisticated followers' quality expectations are not manipulated by the early adopter's subjective review or the seller's information withholding behavior. Consequently, the seller can extract almost no additional surplus This, however, is not profitable as long as the proportion of naive consumers is not sufficiently large. In contrast, the seller is still able to withhold sufficiently low quality information. In such a situation, the seller charges a high retail price and exclusively serves naive consumers in the first period.

We further consider several extensions to examine the robustness of our results. First, we extend the homogeneous consumers in the baseline model to heterogeneous consumers with personal preferences. We show that diversified consumer reviews further facilitate the seller's efforts to utilize the reference effect to better tailor his disclosure strategy. Second, we investigate how the bounds of review ratings affect the seller's disclosure strategy. We show that when the review is unbounded and consumers are naive, the seller can withhold the highest quality information to boost the early adopter's quality review because the rating is no longer capped. All the derived results remain qualitatively intact.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. In Section 3, we present the model setting. The seller's equilibrium disclosure and pricing strategies are analyzed in Section 4. Section 5 discusses the extensions. Concluding remarks are provided in Section 6. All the proofs are relegated to online Appendix A.

2 Literature Review

Our work belongs to the research stream that investigates how consumers' reference-dependent preferences affect a firm's strategic decisions (Bell, 1985; Kőszegi and Rabin, 2006; Fibich et al., 2007; Nasiry and Popescu, 2011; Delquié and Cillo, 2006). Using data from a winery, Gneezy et al. (2014) build a complex reference-dependent model and derive an intriguing relationship between product quality and pricing. Baron et al. (2015) investigate a situation in which a newsvendor sells to strategic customers who have stochastic reference points with respect to both price and product availability. Popescu and Wu (2007) consider how reference price effects influence a firm's pricing decisions when customers remember prices from the past and form reference prices according to a simple heuristic rule. In an advance selling setting, Nasiry and Popescu (2012) study how anticipated regret impacts customer purchasing behavior, and Liu and Shum (2013) investigate a firm's optimal dynamic pricing and rationing decisions when consumers have psychological elation and disappointment. Some scholars study this issue in a competitive environment (Heidhues and Kőszegi, 2008; Zhou, 2011). For example, Heidhues and Kőszegi (2008) investigate price competition and verify the existence of a focal price equilibrium in the presence of consumer reference effects. Karle and Peitz (2014) also examine price competition by assuming that a proportion of customers are reference dependent. Yang et al. (2018) consider service pricing when consumers are reference-dependent towards both waiting time and price. They show that a service provider can obtain a higher profit in a duopoly market than that in a monopoly market when consumers are reference-dependent and loss-averse. Different from the aforementioned studies, our paper considers a monopolistic seller with private quality information selling to reference-dependent consumers (who can be either naive or sophisticated) under a two-period setting. We show that the seller can fine-tune his information disclosure strategy to manipulate consumers' reference-dependent subjective review to improve his payoff under certain conditions.

Besides consumer reviews, another way that consumers can learn the product information is through the seller's voluntary information provision strategy. Some papers discuss how the seller can help consumers to learn their fitness/preference information (Lewis, 1994; Gu and Xie, 2013; Kuksov and Lin, 2010). For example, in a competitive environment, Gu and Xie (2013) investigate the firms' disclosure strategies on whether to assist consumers in finding their fit information towards the products. Kuksov and Lin (2010) further investigate the firms' information provision strategy when both the product quality and consumer's preference are uncertain to consumers initially. Differently, other papers investigate how the seller can convey his private quality information to the unknown consumers. For example, the seller can signal his quality information via advertising (Nelson, 1974; Milgrom and Roberts, 1986), money back guarantee (Moorthy and Srinivasan, 1995) or quality disclosure.

Regarding voluntary information disclosure,[3] one prominent result is the "unraveling" theory (Grossman and Hart, 1980; Milgrom, 1981) that a firm should always reveal any private product information as long as disclosure is costless and the consumers are strategic. Otherwise, if disclosure is costly, a firms's equilibrium disclosure strategy exhibits a threshold-type structure (Jovanovic, 1982; Matthews and Postlewaite, 1985; Shavell, 1994; Guo and Zhao, 2009; Guo, 2009; Guan et al., 2020). Our paper contributes to the literature on voluntary disclosure in the following aspects. First, we extend the firm's strategic decisions from a single-period setting to a multi-period setting. That is, the seller's disclosure decision would influence two groups of consumers who arrive sequentially. Second, we consider two quality signals that can influence a consumer's purchasing decision. One is the seller's voluntary disclosure decision and the other is the quality assessment of the experienced consumer. Last and most importantly, we show that the classic unraveling result no longer holds once the reference effect is sufficiently high.

This paper is also related to the literature on social learning and strategic consumer behavior. In our model setting, consumers who come later learn the quality perception from the predecessors' word of mouth (e.g., on-line review). Note that this social learning process (word-of-mouth) is supported by both practice and academic literature (Ellison and Fudenberg, 1995; Villas-Boas, 2004; Trusov et al., 2009; Dellarocas, 2003). Papanastasiou et al. (2013) investigate the implications of social learning on a monopolist seller's joint pricing and inventory decisions. Ottaviani (1999) shows how a seller should design his pricing to influence the consumers' learning about the true product quality. Besides, we divide consumers into two types: naive and sophisticated, depending on whether they can make rational reference from the seller's disclosure behavior as well as from other consumers' quality assessments. The existence of both naive and strategic consumers has been widely recognized in the literature (e.g., Assuncao and Meyer (1993), Erdem and Keane (1996), Su (2007), Levin et al. (2010), and Lim and Tang (2013)). We have shown that consumer type is one critical driving force that determines the seller's equilibrium disclosure strategy, and the systematic comparison under two customer types leads to several non-trivial implications.

[3] As indicated by Dranove and Jin (2010), the definition of "voluntary disclosure" distinguishes itself from broader marketing efforts where sellers do not provide verifiable product information, such as non-informative advertising.

3 Model Setup

Consider a dynamic model in which a monopolistic seller (he) sells a type of search products to reference-dependent consumers over two consecutive periods. Without loss of generality, we normalize the number of consumers in the first period to 1 and that in the second period to m. In each period i, $i = 1, 2$, a representative consumer (she) is adopted to denote the mass of consumers, that is, an early adopter in the first period and a follower in the second period. Each consumer in period i demands at most one unit of the product from the seller, and her surplus from purchasing is

$$U_i(q, p) = q - p_i, i = 1, 2,$$

where q denotes the quality level of the seller's product and p_i is the retail price charged by the seller in period i. Here, we use the term "quality" to refer to all of the relevant aspects regarding a product such as performance, reliability and features that affect the product's perceived desirability (Guo, 2009; Biyalogorsky and Koenigsberg, 2014). The product's quality level is a random variable, the value of which can be observed only by the seller.[4] Consumers maintain a prior belief that q follows a uniform distribution between zero (the lowest quality level) and one (the highest quality level): $q \sim U[0, 1]$.

At the beginning of the first period, the seller privately observes the true quality level q and decides whether to disclose this quality information to the early adopter. As the seller can make the disclosure decision after observing the true quality, this is termed the *ex post information disclosure* strategy. If the seller discloses the quality information, the disclosed information must be truthful (Grossman and Hart, 1980). The truthful revelation can be enforced by third-party verification or the hard evidence that is needed to confirm the information. For example, a newly opened hotel may use a five-star rating or ISO 9000 certification to demonstrate its super standard to attract uninformed consumers. We also assume that the cost of disclosure is zero, because advances in information technology have made information disclosure almost costless. More critically, this assumption eliminates the possible effect of disclosure costs (Jovanovic, 1982; Guo, 2009; Guo and Zhao, 2009) and allows us to focus exclusively on the strategic impact of consumers' reference-dependent preferences on the seller's voluntary disclosure strategy.

We consider two types of consumers: one is the sophisticated consumer (denoted as s), and the other is the naive consumer (denoted as n). In particular, if the seller discloses his quality information, then the consumer, regardless of her type (sophisticated or

[4]This setting is prevalent in the vast literature on voluntary disclosure (Jovanovic, 1982; Guo, 2009; Guo and Zhao, 2009), which represents the reality that true product quality is normally influenced by many random and uncontrollable factors and that the seller, who is involved in the production process, can employ a series of methods to identify it before putting the product on the market.

naive), can observe this information and confirm the product's exact quality level. However, when the seller withholds his quality information, a sophisticated consumer can rationally make inferences based on the seller's non-disclosure behavior and update her quality expectation, while a naive consumer is insensitive to such non-disclosure behavior and maintains her original prior quality belief. That is, one key difference between these two consumer types is whether the consumer can make a rational inference about the product's quality when observing the seller's non-disclosure behavior. Such consumer type classification has also been adopted by Taylor (2004) and Gabaix and Laibson (2006).

Let q_1^t be the quality expectation of the type-t early adopter, $t = s, n$. Then, it can be inferred that if the early adopter is naive, her quality expectation upon non-disclosure q_1^n can be derived as $q_1^n = E[q|0 \leq q \leq 1] = \frac{1}{2}$ based on her prior belief. However, if the early adopter is sophisticated, her quality expectation q_1^s hinges on the seller's disclosure decision. This quality expectation q_1^t ($t = n, s$) subsequently sets up the early adopter's *reference quality point* and facilitates her purchasing decision. After consumption, if the exact quality level q (observed upon consumption) exceeds her reference point q_1^t, a sense of enjoyment arises, and the early adopter generates a quality evaluation higher than the true quality level. However, if the exact quality level falls short of the reference point, the early adopter experiences disappointment and will then comment negatively on the product's quality. This is the well known *reference-dependent preference*: a consumer's overall quality evaluation depends not only on the product's true quality but also on its comparison to a reference point (Tversky and Kahneman, 1991; Kőszegi and Rabin, 2006; Gneezy et al., 2014).

Based on the reference quality point (the quality expectation in our context) and the observed true quality upon consumption, the type-t early adopter makes the following ex post *subjective quality assessment* (**SQA**), q_r^t:

$$q_r^t = \max\{0, \min\{1, q + \alpha(q - q_1^t)\}\} = \min\{((1+\alpha)q - \alpha q_1^t)^+, 1\}, t = n, s, \tag{1}$$

where $x^+ = \max\{0, x\}$. In (1), $\alpha \in [0, 1]$ measures the magnitude of consumer reference dependence and $\alpha = 0$ indicates that consumers are reference independent. The term $\alpha(q - q_1^t)$ is the reference-dependent perceived quality increase/decrease caused by the gap between the true quality level and the reference quality point. The consumer's finalized SQA is the true quality level plus this perceived term. Note that the value of q_r^t is restricted to $[0, 1]$, which implies that the SQA cannot fall below the quality level's lower bound 0 or exceed its upper bound 1. Such SQA is realistic and prevalent in both the literature and practice (Gao et al., 2013). Consider a hotel rating website (e.g., Tripadvisor or Booking.com) that invites experienced consumers to evaluate hotels by scoring them from 1 (lowest rating) to 10 (highest rating). An individual's rating is certainly affected by her personal living experience and initial expectation about the hotel, and she normally issues a rating higher than the hotel's exact level if the hotel exceeds her initial expectation. However, if a hotel is already perfect (e.g., $q = 10$), the consumer could never rate above 10 even though it is far better than her initial expectation. Similarly, the consumer cannot issue a score below zero regardless of how awful she may feel during her stay (e.g., $q = 0$).

At the beginning of the second period, the type-t early adopter's SQA q_r^t is released to the public (e.g., by word of mouth) and becomes observable to both the seller and the follower. It will guide the follower in making her purchasing decision. Undoubtedly, the follower will react differently to the early adopter's SQA depending on whether she is sophisticated or naive. Denote q_2^t as the type-t follower's quality expectation, $t = n, s$. In particular, if the follower is naive, she will simply trust the early adopter's SQA and take q_r^t as her quality expectation when making her purchasing decision. However, if the follower is sophisticated, she knows that q_r^t would be a biased quality assessment due to the existence of the reference effect and will rationally infer the true quality level.

It is worth mentioning that in our baseline model, the consumer review only reflects the search product's quality rather than the consumer's personal preference. Specifically, the quality review from the representative consumer can be viewed as an aggregation of all the individual consumers' quality assessments in a market, such as a hotel rating on Ctrip or a product rating on Amazon or Taobao. Although each consumer may have her own preference towards the product, combining all the consumer reviews can neutralize the impact of personal preferences on a product's overall assessment. Consequently, it is the product quality that determines the representative consumer's review. Having said the above, we also consider an extension wherein the consumer review can reflect both product quality and the consumer's personal preference in Section 5.1, and we show that our main results remain robust in this case.

Disclosure Timing. Figure 1 illustrates our decision sequence. Note that we restrict the seller to make his disclosure decision only at the beginning of the first period. The underlying reason is that, compared to advocacy by the seller, a potential buyer would rely more on the experienced consumer's comments/reviews in making her purchasing decision.[5] In other words, disclosure would become ineffective once the product's reputation (i.e., SQA) has been accumulated. Thus, in the second period, even though bias exists in the early adopter's SQA, the seller would be unable to rectify it by disclosing

[5]As indicated by Chevalier and Mayzlin (2006) and Moretti (2011), potential customers seek relevant information by consulting the opinions of early buyers, and these opinions play a pivotal role in potential customers' purchasing decisions.

his true quality level. It should be noted that although the seller sequentially decides his retail prices at the beginning of each period, the optimization of these two sequential pricing decisions can be done jointly in advance. This is because there exists no consumer spillover between the two periods. Thus, the seller's equilibrium pricing decisions remain intact under either of the sequences, solving sequentially or simultaneously in advance.

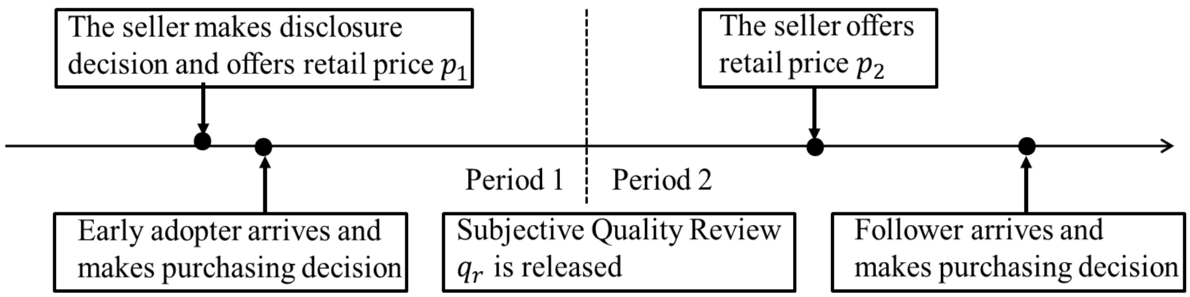

Figure 1: An illustration of the decision sequence.

Both the seller and consumers are risk neutral and seek to maximize their own surpluses. We normalize the seller's marginal operating cost to zero, and there is no time discounting for the seller's payoffs over the two periods. All the parties' utilities upon no trade are zero. To focus on the interplay between the effect of voluntary disclosure and consumers' reference-dependent preferences, we also exclude the possible signaling role of prices in our model. Since the game involves multiple rounds of strategic interactions, backward induction is adopted to ensure subgame perfection. Table 1 summarizes the frequently used notation.

Table 1: Summary of Frequently Used Notation

Notation	Explanation
m	Market size in the second period
n	Naive consumer
s	Sophisticated consumer
α	Magnitude of consumer reference-dependent preferences
d/nd	The seller's disclosure strategy: disclosure/non-disclosure
q	Product quality
$\overline{q}$	Quality expectation upon non-disclosure
q_r^t	Type-t early adopter's subjective review, $t = n, s$
p_i^k	Retail price in period i when the seller adopts the disclosure strategy k, $i = 1, 2$, $k = d, nd$
$\pi^{t_1 t_2}$	Seller's ex post payoff after observing q when facing a type-t_1 early adopter and type-t_2 follower
$\Pi^{t_1 t_2}$	Seller's ex ante expected payoff when facing a type-t_1 early adopter and type-t_2 follower

4 Equilibrium Analysis

In this section, we investigate the seller's equilibrium disclosure decision when facing reference-dependent consumers. Depending on whether consumers are naive or sophisticated, three scenarios will be analyzed. In the first scenario, both the early adopter and the follower are naive, whereas in the second scenario, they are sophisticated. In the third scenario, we consider that in each period, the market contains both sophisticated and naive consumers. Let π_{nd}^{tt} and π_{d}^{tt} be the seller's ex post payoff with type-t consumers when the firm adopts the *non-disclosure* strategy (denoted as nd) via withholding his private quality information and the *disclosure* strategy (denoted as d) via disclosing his private quality information, respectively, where $t \in \{n, s\}$. Similarly, denote p_i^{nd} and p_i^d as the optimal retail price in period i when the firm adopts the *non-disclosure* and *disclosure* strategy, respectively, $i = 1, 2$.

4.1 Naive Consumers

When consumers in both periods are naive, the early adopter is insensitive to the seller's non-disclosure decision while the follower simply believes in the early adopter's SQA.

In the first period, if the seller withholds his quality information, the naive early adopter then expects the product quality to be $q_1^n = E[q|0 \leq q \leq 1] = \frac{1}{2}$. She will purchase the product if and only if $q_1^n - p_1 \geq 0$. If the early adopter decides to buy the product, after consumption, she forms her SQA q_r^n as stated in (1), which hinges on the difference between the true quality level and her initial expectation and can be written as

$$q_r^n = \min\left\{\left((1+\alpha)q - \frac{\alpha}{2}\right)^+, 1\right\}.$$

At the beginning of the second period, the naive follower observes the SQA posted by the early adopter and takes it as her quality expectation. That is, $q_2^n = q_r^n$. The naive follower purchases the product if and only if $q_2^n - p_2 = q_r^n - p_2 \geq 0$. Anticipating the naive consumer's purchasing behavior, the information-withholding seller then determines the retail prices over the two periods to maximize his ex post payoff:[6]

$$\pi_{nd}^{nn} = \mathbf{1}(q_1^n \geq p_1)p_1 + \mathbf{1}(q_2^n \geq p_2)mp_2.$$

It can be easily shown that the optimal retail prices are

$$p_1^{nd} = \frac{1}{2} \text{ and } p_2^{nd} = q_r^n = \min\left\{\left((1+\alpha)q - \frac{\alpha}{2}\right)^+, 1\right\}.$$

[6]Herein, we simultaneously derive the seller's equilibrium prices over the two periods. As discussed in §3, under our setting, simultaneous optimization leads to the same outcome as that under sequential optimization (with backward induction).

Thus, the seller's ex post equilibrium payoff under non-disclosure can be derived as

$$\pi_{nd}^{nn} = \begin{cases} \dfrac{1}{2}, & \text{if } q \le \dfrac{\alpha}{2(1+\alpha)}; \\ m(1+\alpha)q + \dfrac{1-m\alpha}{2}, & \text{if } \dfrac{\alpha}{2(1+\alpha)} < q \le \dfrac{2+\alpha}{2(1+\alpha)}; \\ \dfrac{1}{2} + m, & \text{otherwise.} \end{cases} \tag{2}$$

In contrast, if the seller initially reveals his private quality information q, this subsequently eliminates the possibility that the consumer's quality evaluation deviates from the true quality level. As a result, the ex ante quality expectation of both the early adopter and the follower is $q_1^n = q_2^n = q$, and the information-disclosing seller's equilibrium pricing and payoff can easily be derived as

$$p_1^d = p_2^d = q \text{ and } \pi_d^{nn} = (1+m)q. \tag{3}$$

The seller then compares his payoffs under the disclosure and non-disclosure strategies to decide whether to disclose his quality information as follows:

$$\pi_d^{nn} - \pi_{nd}^{nn} = \begin{cases} (1+m)q - \dfrac{1}{2}, & \text{if } q \le \dfrac{\alpha}{2(1+\alpha)}; \\ (1-m\alpha)\left(q - \dfrac{1}{2}\right), & \text{if } \dfrac{\alpha}{2(1+\alpha)} < q < \dfrac{2+\alpha}{2(1+\alpha)}; \\ (1+m)q - \left(\dfrac{1}{2} + m\right), & \text{otherwise.} \end{cases} \tag{4}$$

Proposition 1. *When the consumers are naive, in equilibrium,*

(1) if $m\alpha < 1$, the seller voluntarily discloses his private quality information when the true product quality $q > \frac{1}{2}$; otherwise, he prefers non-disclosure.

(2) if $m\alpha \ge 1$, the seller voluntarily discloses his private quality information when the true product quality $q \in \left(\frac{1}{2(1+m)}, \frac{1}{2}\right) \cup \left(\frac{1+2m}{2(1+m)}, 1\right]$; otherwise, he prefers non-disclosure.

Proposition 1 indicates that the seller can tailor his disclosure strategy when facing naive consumers who are unable to make rational inferences about the product's quality if he withholds the quality information. It is evident that the seller's equilibrium disclosure strategy is jointly determined by the second-period market size m and the reference-dependence parameter α. To that end, we use the product term $m\alpha$ as a *proxy* in measuring *the magnitude of the reference effect*. When the reference effect is weak ($m\alpha < 1$), the seller's disclosure strategy exhibits a traditional cutoff structure: disclosure is adopted only if the product quality q is higher than a threshold $\frac{1}{2}$. That is, when the reference effect is weak, the seller's potential gain from disclosing high quality information (i.e., $q > \frac{1}{2}$) in advance can outweigh the corresponding gain from non-disclosure and taking advantage of the consumer's SQA in the second period.

Interestingly, when the reference effect is strong ($m\alpha \geq 1$), the seller prefers to disclose his quality information when the true quality level falls within two isolated intervals: quality is either relatively low or sufficiently high. Specifically, disclosing relatively low quality information (i.e., $q \in \left(\frac{1}{2(1+m)}, \frac{1}{2}\right)$) can safeguard the seller from suffering a large potential loss in the second period. This is because if the seller withholds this low quality information in the first period, a high reference effect would seriously lower the early adopter's SQA, leading it to deviate dramatically from the true quality level, thereby harming the seller's payoff. Nonetheless, the impact of the high reference effect becomes marginal when the true quality level is sufficiently high. That is, given the upper bound on the consumer's quality assessment, i.e., the SQA cannot exceed 1, the benefit of withholding extremely high quality information to boost the early adapter's SQA is limited.[7] As a result, the seller would prefer to disclose his extremely high quality information in the first period. See Figure 2 for an illustration of the seller's equilibrium disclosure strategy when consumers are all naive.

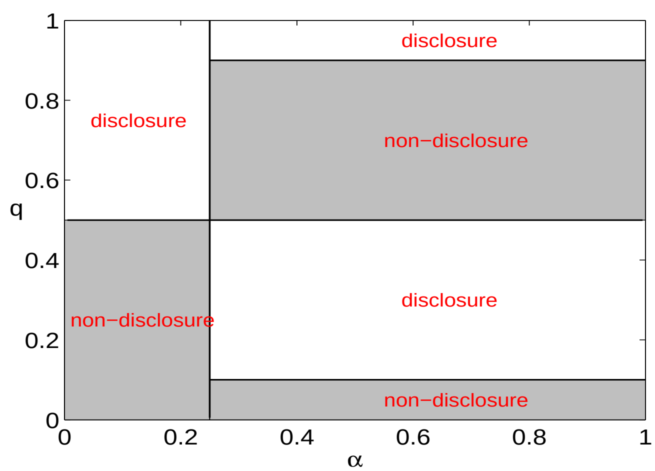

Figure 2: Equilibrium disclosure strategy when consumers are naive: $m = 4$.

Building upon the seller's equilibrium disclosure strategy, we next derive the seller's

[7]Consider the extreme case of $q = 1$. If the seller withholds this quality information in the first period, the early adopter's SQA q_r^n is still capped at 1. Thus, the reference effect vanishes in this case.

ex ante expected payoff.[8] When the reference effect is weak ($m\alpha < 1$), the seller's ex ante expected payoff can be written as

$$\Pi^{nn}_{m\alpha<1} = \underbrace{\int_0^{\frac{\alpha}{2(1+\alpha)}} \frac{1}{2}dq + \int_{\frac{\alpha}{2(1+\alpha)}}^{\frac{1}{2}} \left(m(1+\alpha)q + \frac{1-m\alpha}{2}\right) dq}_{\text{Non-disclosure}} + \underbrace{\int_{\frac{1}{2}}^{1} (1+m)qdq}_{\text{Disclosure}}. \quad (5)$$

Similarly, we can derive the seller's ex ante expected payoff when the reference effect is strong ($m\alpha \geq 1$) as follows:

$$\Pi^{nn}_{m\alpha\geq 1} = \underbrace{\int_0^{\frac{1}{2(1+m)}} \frac{1}{2}dq + \int_{\frac{1}{2}}^{\frac{2+\alpha}{2(1+\alpha)}} \left(m(1+\alpha)q + \frac{1-m\alpha}{2}\right) dq + \int_{\frac{2+\alpha}{2(1+\alpha)}}^{\frac{1+2m}{2(1+m)}} \frac{1+2m}{2}dq}_{\text{Non-disclosure}}$$
$$+ \underbrace{\int_{\frac{1}{2(1+m)}}^{\frac{1}{2}} (1+m)qdq + \int_{\frac{1+2m}{2(1+m)}}^{1} (1+m)qdq}_{\text{Disclosure}}. \quad (6)$$

Corollary 1. *When both the early adopter and the follower are naive, in equilibrium, the seller's ex ante expected payoff first decreases in the reference-dependence parameter α for $\alpha < \frac{1}{m}$ and then increases in it.*

Corollary 1 shows that the seller's ex ante expected payoff exhibits a U-shaped relationship with respect to the reference-dependence parameter α when the consumers are all naive. When α is small ($\alpha < \frac{1}{m}$), the seller would withhold all the quality information that is lower than the consumer's ex ante quality expectation. Then, due to consumers' reference-dependent preferences, non-disclosure undoubtedly pulls down the quality assessment of the early adopter, and this negative impact is amplified as α increases. Consequently, the seller's ex ante expected payoff monotonically decreases in α when $\alpha < \frac{1}{m}$. In contrast, when α is large ($\alpha \geq \frac{1}{m}$), it motivates the seller to strategically withhold his quality information when the true quality level is higher than the consumer's ex ante quality expectation (i.e., $q \in \left(\frac{1+2m}{2(1+m)}, 1\right]$). In such a case, an increase in α further magnifies the reference-dependent quality assessment and benefits the seller.

4.2 Sophisticated Consumers

We now consider the scenario in which consumers in both periods are sophisticated. That is, consumers are all smart enough to make rational quality inferences based on both the seller's disclosure behavior and the consumer's SQA.

[8] Note that ex ante expected payoff is the seller's expected payoff before the quality information is realized, since the product quality is a random variable determined by nature and can be privately observed by the seller. The same concept has been adopted by the related literature like Guo (2009), Guo and Zhao (2009) and Guan et al (2020).

Again, if the seller discloses his quality information in advance, the equilibrium prices are $p_1^d = p_2^d = q$ and the ex post payoff is $\pi_d^{ss} = (1+m)q$. Otherwise, if the seller withholds his quality information, both the early adopter and the follower infer that the true product quality falls into certain range(s), and they initially form the same ex ante quality expectation $\bar{q}$. After consumption, the early adopter's SQA becomes $q_r^s = \min\{((1+\alpha)q - \alpha\bar{q})^+, 1\}$, which is observable to the follower. The follower now makes the rational inference of the true product quality based on the following *two* rationales. One, the follower knows that the review q_r^s is biased due to the presence of the reference-dependent preferences and she would rectify this biased review q_r^s backward to q. Two, the follower knows that under such a circumstance, the seller is better off withholding his private quality information; otherwise, the seller would have already disclosed such information. Building upon the consumers' quality inference, we can then derive the seller's equilibrium disclosure strategy, which is summarized in the following proposition.

Proposition 2. *When both the early adopter and the follower are sophisticated, in equilibrium, the seller always discloses his private quality information in advance.*

Proposition 2 reveals that the seller cannot withhold any private quality information when both the early adopter and the follower are sophisticated. This result is consistent with the classic unraveling theory but is in striking contrast to that of Proposition 1 in which consumers are naive. The intuition is that if the follower is able to strategically rectify the biased quality review to its original level, the seller has no incentive to withhold the high quality information to boost the early adopter's SQA. Moreover, if the seller chooses non-disclosure at the beginning, the sophisticated early adopter can infer that the product quality must be sufficiently low. Given the sophisticated consumers' quality inference and that disclosure is costless, in equilibrium, the seller has no choice but to disclose all the quality information. Consequently, the reference effect vanishes.

The seller's ex ante expected payoff in this scenario can be easily derived as follows:

$$\Pi^{ss} = \underbrace{\int_0^1 (1+m)q\,dq}_{\text{Disclosure}} = \frac{1+m}{2}. \tag{7}$$

Obviously, it is independent of the reference effect parameter α.

Corollary 2. *When both the early adopter and the follower are sophisticated, the seller ex ante discloses more quality information but obtains a lower payoff than when the consumers are naive.*

Note that the ex ante disclosure probability has been adopted in the literature to represent the likelihood of quality disclosure (Guo and Zhao, 2009). Thus, it explicitly shows that when consumers are sophisticated, the seller has no choice but to ex post disclose his private quality information, leading to the highest ex ante disclosure probability of 1. In contrast, when consumers are naive, the seller would withhold his quality information when the true quality is either sufficiently low or relatively high to craft the naive consumer's quality expectation. Moreover, given this strategic withholding of information from naive consumers, the seller is also able to extract more surplus from them than in the scenario in which the consumers are sophisticated, as illustrated in Figure 3.

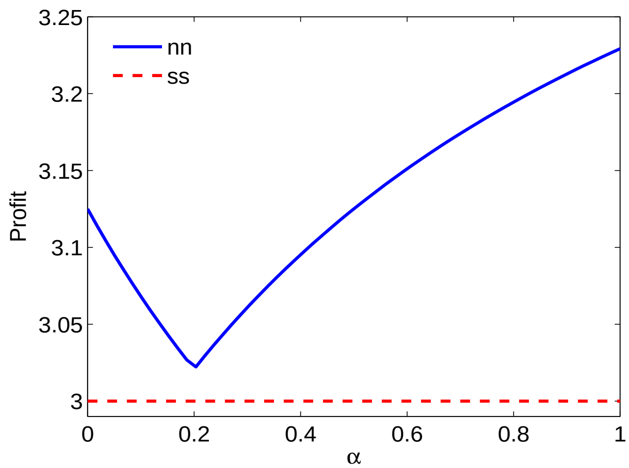

Figure 3: Equilibrium payoffs under nn and ss: $m = 5$.

The above discussion indicates that the consumers' types have a strategic impact on the seller's disclosure strategies and payoffs, the transparency of product quality information, and consumer surplus. In a more-developed market, consumers are more likely to be well educated and sophisticated. Then, the seller must disclose his quality information more, leading to a higher degree of product information transparency, which benefits consumers. In contrast, in a less-developed market, consumers are more likely to lack business knowledge and be naive. Then, the seller can withhold his quality information, resulting in a lower degree of product information transparency, which harms consumers. In such a situation, the government and policy makers may need to take various actions such as implementing mandatory disclosure on the firm side and educating consumers to protect consumers' interests.

4.3 A Mixture of Naive and Sophisticated Consumers

In the aforementioned discussion, there is only one type of consumer, either naive or sophisticated, in the market. However, in practice, it is possible that the market consists of both naive and sophisticated consumers. In such a case, how should the seller design his disclosure strategy to induce a consumer quality review that allows him to obtain more surplus? This is the question we intend to examine here.

Assume that the market is composed of both naive and sophisticated consumers (denoted as *mix*). In each period, among all consumers, a fraction γ of them are naive while the rest are sophisticated, where γ is assumed to be less than $\frac{2+\alpha}{2(1+\alpha)}$. The composition of these two consumer types (i.e., γ) is common knowledge and observable by the seller and consumers. Note that when the market contains both naive and sophisticated consumers, the interaction between the seller and consumers becomes much more complicated because naive and sophisticated consumers may hold potentially different quality beliefs. We elaborate on this in the following.

In the first period, if the seller discloses quality information in advance, then both naive and sophisticated consumers can observe this information and confirm that the product's true quality level is q. Consequently, in equilibrium, the seller would charge retail prices $p_1^d = p_2^d = q$. All consumers, regardless of their type, purchase in both periods. The seller's total payoff is thus $\pi_d^{mix} = (1+m)q$.

In contrast, if the seller initially withholds the quality information, then naive early adopters maintain their prior quality belief that $q_1^n = \frac{1}{2}$, but sophisticated early adopters rationally update their quality belief to $q_1^s = \overline{q}$. Evidently, these two quality beliefs are different, leading to two possible pricing strategies for the seller. One pricing strategy is that the seller sets a low retail price $p_1 = \min\{\overline{q}, \frac{1}{2}\}$ to serve both naive and sophisticated early adopters. Accordingly, after consumption, a naive early adopter generates the SQA $q_r^n = \min\left\{\left((1+\alpha)q - \frac{\alpha}{2}\right)^+, 1\right\}$, while a sophisticated early adopter generates the SQA $q_r^s = \min\{((1+\alpha)q - \alpha\overline{q})^+, 1\}$. Given the composition of these two types of consumers in the market, the aggregate average quality rating R^{mix} can be derived as

$$R^{mix} = \gamma q_r^n + (1-\gamma) q_r^s.$$

In the second period, after observing the early adopters' average quality rating R^{mix}, the naive follower would believe it, and thus her quality expectation would be $q_2^n = R^{mix}$. However, the sophisticated follower would further make an inference about the product's quality.

The other pricing strategy is that the seller sets a high price $p_1 = \max\{\overline{q}, \frac{1}{2}\}$ to serve only one type of early adopters. Then, the consumer review R^{mix} would reflect only that type of early adopters' SQA. In the second period, naive followers believe R^{mix} and take it as their quality expectation, while sophisticated followers make further inferences and update their quality expectations. Under either pricing strategy, naive and sophisticated followers hold different quality beliefs. Again, the seller needs to decide whether to serve both types of followers by charging a low price or serve only one type of follower by charging a high price.

Overall, one can see that when there is a mixture of naive and sophisticated consumers in the market and the seller initially withholds his quality information, in each period, the seller has two possible pricing strategies that lead to different review outcomes, quality expectations and payoffs. A combination of these two-period pricing strategies leads to four possible scenarios, and the derivation of the equilibrium pricing strategies becomes quite complicated because it involves various possible inter-results. Below, we analyze this information-withholding scenario by adopting backward induction.

We first consider the situation wherein the seller initially withholds the quality information and charges a high first-period retail price to solely serve naive early adopters. Given this first-period pricing strategy, in the second period, naive followers observe the naive early adopters' SQA and take it as their quality expectation: $q_2^n = q_r^n = \min\left\{\left((1+\alpha)q - \frac{\alpha}{2}\right)^+, 1\right\}$. The sophisticated followers further update their quality belief, and we provide the related result in the following lemma.

Lemma 1. *When the fraction of naive consumers in the market* $\gamma < \frac{2+\alpha}{2(1+\alpha)}$ *and the seller initially withholds his quality information, then we have the following:*

(1) If the early adopters' SQA $0 < q_r^n < 1$*, the sophisticated follower can rationally infer the true quality level; that is,* $q_2^s = q$*.*

(2) If $q_r^n = 0$*, the sophisticated follower believes that the true quality level is uniformly distributed over the range* $[0, q_l]$*, where* $q_l = \min\left\{\frac{\gamma}{m+m\gamma+2}, \frac{\alpha}{2(1+\alpha)}\right\}$ *is the cutoff point at which the seller is indifferent between disclosing and withholding his quality information.*

(3) If $q_r^n = 1$*, the sophisticated follower believes that the true product quality level is* $\frac{2+\alpha}{2(1+\alpha)}$*.*

Note that a sophisticated follower rationally infers the product quality based on two rationales. One, she knows that the review q_r^n is biased due to the naive consumers' reference-dependent preferences and that the reference quality level is $\frac{1}{2}$, so she would rectify this biased review. For example, if the follower observes a review rating $0 < q_r^n < 1$, she then forms the belief that $(1+\alpha)q - \frac{\alpha}{2} = q_r^n$ and infers through backward induction that $q = \frac{2q_r^n+\alpha}{2(1+\alpha)}$. Two, when the consumer review q_r^n reaches the boundary point (i.e.,

$q_r^n = 0$ or $q_r^n = 1$) such that a sophisticated follower fails to infer the exact quality level, she further infers the product's quality based on the rationale that in such a case, the seller is better off withholding his private quality information; otherwise, the seller would have already disclosed such information in advance. For the sake of brevity, we move the detailed discussion of this inference process to the proof of Lemma 1 in Online Appendix A.

We then derive the seller's optimal selling strategy and have the following result.

Lemma 2. *When the fraction of naive consumers in the market $\gamma < \frac{2+\alpha}{2(1+\alpha)}$, if the seller initially withholds the quality information, then in the first period, it is in his best interest to only serve naive consumers by setting the retail price $p_1^{nd} = \frac{1}{2}$.*

Lemma 2 shows that in the first period, the seller would only serve naive consumers when withholding his quality information if the proportion of naive consumers γ is lower than a threshold $\frac{2+\alpha}{2(1+\alpha)}$. Note that since the reference-dependence parameter $\alpha \in [0,1]$, the threshold $\frac{2+\alpha}{2(1+\alpha)}$ falls within the range $[\frac{3}{4}, 1]$ and is relatively large. This implies that the result stated in Lemma 2 generally holds as long as the proportion of naive consumers in the market is not too large. In this situation, if the seller wants to serve both naive and sophisticated consumers while withholding his quality information, then the first-period retail price has to be pushed down to zero given that the sophisticated early adopters would infer the seller has the lowest quality level. The rationale here is similar to that stated in §4.2. Thus, the seller is better off charging a high price to serve only naive early adopters.

At the beginning of the second period, since naive followers and sophisticated followers hold different quality expectations ($q_2^n \neq q_2^s$), the seller needs to make a tradeoff between serving both types of followers by charging a low price $p_2 = \min\{q_2^n, q_2^s\}$ and serving only one type of follower by charging a high price $p_2 = \max\{q_2^n, q_2^s\}$. Thus, when $\gamma < \frac{2+\alpha}{2(1+\alpha)}$, the seller's total payoff under non-disclosure can be written as

$$\pi_{nd}^{mix} = \frac{1}{2}\gamma + m \max\{\min\{q_2^n, q_2^s\}, \gamma q_2^n, (1-\gamma) q_2^s\}.$$

The further comparison between the seller's payoffs under disclosure and non-disclosure then leads to his equilibrium disclosure strategy. To avoid tedious discussion, in the following, we provide one example to illustrate the seller's equilibrium disclosure strategy by assuming that $\gamma = \frac{1}{2}$; however, the insights obtained from this case can be applied to more general cases when $\gamma < \frac{2+\alpha}{2(1+\alpha)}$.

Proposition 3. *When the market contains both naive and sophisticated consumers and half of them are naive ($\gamma = \frac{1}{2}$), in equilibrium, the seller withholds his quality information when*

$$q \in \begin{cases} \left(0, \frac{2m\alpha-1}{4(m\alpha-1)}\right), & \text{if } \alpha \leq \dfrac{1}{2(1+m)}; \\ \left(0, \frac{1}{2(2+m)}\right), & \text{if } \dfrac{1}{2(1+m)} < \alpha < \dfrac{2}{2+3m}; \\ \left(0, \frac{1}{4+3m}\right) \cup \left(\frac{\alpha}{2(1+\alpha)}, \frac{1}{2(2+m)}\right), & \text{if } \dfrac{2}{2+3m} \leq \alpha \leq \dfrac{1}{1+m}; \\ \left(0, \frac{1}{4+3m}\right), & \text{otherwise.} \end{cases}$$

Otherwise, the seller prefers disclosure.

Proposition 3 characterizes the seller's equilibrium disclosure strategy when the market contains half naive consumers and half sophisticated consumers. It shows that the seller can still strategically withhold his quality information but mostly for those in the sufficiently low range. This is in contrast to that of Proposition 1 in which the seller would strategically withhold high quality information when all consumers are naive. Recall that when the market solely contains naive consumers, withholding high quality information would result in an extremely positive review that allows the seller to charge naive followers a very high price in the second period. However, when the market contains both naive and sophisticated consumers, sophisticated followers are not manipulated by the early adopter's subjective quality review. Thus, the seller can either charge a high price to serve only naive consumers or charge a low price to serve both naive and sophisticated consumers. When the proportion of naive consumers is not large, forgoing serving sophisticated consumers is too costly for the seller. In this situation, the benefit of withholding the high quality information vanishes and so does the seller's incentive to withhold high quality information.

Nonetheless, the seller is still able to withhold sufficiently low quality information in the first period. As indicated above, this can be achieved only if the seller charges a high price to solely serve naive early adopters and exclude sophisticated early adopters from the market. Moreover, one can verify that the seller's incentive to withhold low quality information (e.g., the range of the non-disclosure zone) decreases as the magnitude of the reference effect increases; see Figure 4 for an illustration. This is because with a higher magnitude of the reference effect, non-disclosure would result in a more negative quality review and thus reduce naive followers' quality expectations (although it has no impact on the sophisticated consumers' quality expectations). To mitigate such a negative impact on naive consumers, the seller has to disclose more quality information in advance.

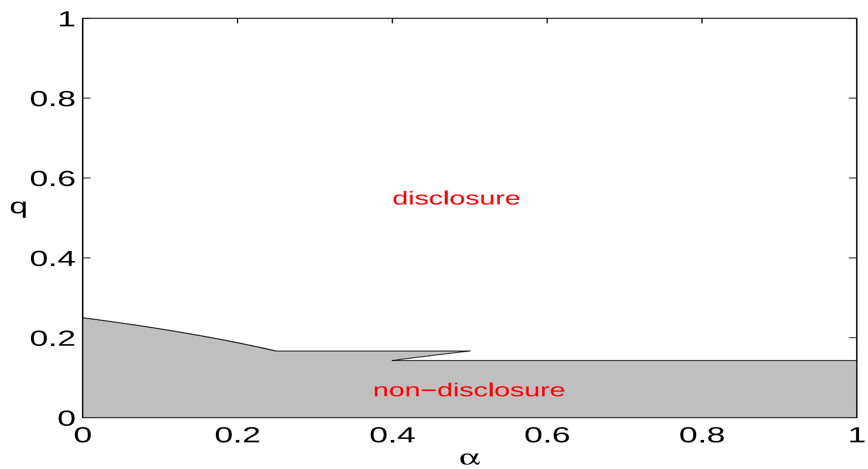

Figure 4: Equilibrium disclosure strategy when the market contains both naive and sophisticated consumers: $m = 1$ and $\gamma = 0.5$.

For the general case where the proportion of naive consumers $\gamma \in \left[0, \frac{2+\alpha}{2(1+\alpha)}\right)$, we conduct numerical experiments and find that as γ decreases, the seller is less likely to withhold the relatively low quality information; that is, the non-disclosure zone depicted in Figure 4 shrinks. This is because the profit gain from non-disclosure can be achievable mainly based on a condition that the seller serves solely naive consumers in the first period. As the proportion of sophisticated consumers in the market increases, such profit gain from non-disclosure diminishes. This subsequently dampens the seller's information withholding incentive. When the market is mainly composed of sophisticated consumers (e.g., when $\liminf \gamma = 0$), the seller would disclose all the quality information, a result same as that stated in Proposition 2 when all consumers are sophisticated.

In Proposition 3, we have shown that when the market is composed of half naive consumers and half sophisticated consumers, it is never in the seller's best interest to withhold the high quality information to boost the early adopters' reference-dependent review. The underlying reason is that sophisticated followers' quality expectation can be hardly manipulated as they can make the rational inferences about the true product quality. Thus, as long as the proportion of sophisticated/naive consumers is not too small/large (i.e., $\gamma < \frac{2+\alpha}{2(1+\alpha)}$), the seller cannot charge a high retail price when it has to serve both consumer types in the second period. Nonetheless, when the proportion of naive consumers continues to increase (e.g., when $\gamma \geq \frac{2+\alpha}{2(1+\alpha)}$), then it may become profitable for the seller to charge the high price to only serve the naive followers, rather than charging the relatively low price to serve both consumer types. As such, it becomes more likely for the seller to withhold some high quality information because the gain for serving only naive consumers can outweigh the loss from not serving sophisticated consumers. Also, one can infer that when the market is mainly composed of naive consumers (e.g., $\limsup \gamma = 1$), the seller's equilibrium disclosure strategy shall be the same as that stated in Proposition 1 when all consumers are naive.[9]

The above elaborations indicate that the composition of consumer types hugely impacts the seller's information disclosure strategy. In practice, many firms have chosen to withhold their own product information to influence/manipulate consumer review; see our motivating examples stated in the Introduction. Our analytical results help the firms to have a deeper understanding of when such withholding strategy is desirable. To make the right information disclosure strategy, it is necessary for the firm to first understand the market characteristics, i.e., the composition of consumer types in the market. Non-disclosure information strategy can be effective only when the market contains not so many sophisticated consumers and the seller focuses on a partial market instead of the entire market by serving naive consumers exclusively.

5 Discussion

Below, we extend our baseline model to the following two scenarios: one, consumers are heterogeneous in their personal preference toward the product; two, the ratings used in reviews are unbounded. We would like to examine whether our results still hold in these settings.

5.1 When Consumers Have Heterogenous Preferences

In certain situations, a consumer's utility from purchasing is not only affected by the product's quality level but also by her personal preference/fitness/taste regarding the product. This is particularly prevalent for those product categories such as fashion products and entertainment goods. In such a case, consumer's review is not merely a reflection of her perceived quality but also contains her personal preference. When reference-dependent consumers have an independent and private personal preference toward the product, how will this affect the seller's voluntary disclosure strategy? This is the research question we aim to address here. Assume that two groups of consumers arrive sequentially: early adopters in the first period and followers in the second period.[10] A

[9] We refer the interested readers to Section SC presented in the online Appendix B for a more detailed discussion on this point.

[10] Note that due to consumer heterogeneity, it is no longer suitable to merely consider a representative consumer in each period.

consumer's surplus from purchasing is now given by

$$U(q,p) = q + v - p,$$

where v represents her preference/fitness/taste regarding the product. In particular, we assume that consumers' personal preferences are uniformly distributed between $-\frac{1}{2}$ and $\frac{1}{2}$, indicating that a consumer can hold either a positive or a negative preference towards the product. Moreover, we require that a consumer can only confirm her exact personal preference after consumption, while before consumption, she holds an expected preference $\overline{v} = E[v| -\frac{1}{2} < v < \frac{1}{2}] = 0$, based on which she makes her purchasing decision.

Suppose that a type-t ($t \in \{n, s\}$) early adopter has purchased the product. She then makes the following ex post SQA $q_r^{t,v}$ based on her type (i.e., whether she is sophisticated or naive), her reference quality point (quality expectation q_1^t), the observed quality q and her personal preference v,

$$q_r^{t,v} = \max\left\{0, \min\{1, q + \alpha(q - q_1^t) + v\}\right\}, t = n, s, \tag{8}$$

where a consumer's subjective review $q_r^{t,v}$ is bounded between 0 and 1. However, because consumers now possess heterogeneous personal preferences, their reviews differ. To that end, we assume that all the type-t preference-heterogenous early adopters' SQAs finally form an aggregate average quality rating (AQR) R^t as follows:

$$R^t = \int_{-1/2}^{1/2} q_r^{t,v} dv.$$

This rating is then released online and becomes observable to both the seller and the followers at the beginning of the second period. The other settings remain the same as those in the baseline model. We then investigate the seller's disclosure strategies under the two scenarios to examine the impact of consumer personal preference heterogeneity.

First, we consider the scenario in which both the early adopters and followers are naive. In such a situation, early adopters hold a prior belief that the expected product quality $\overline{q} = \frac{1}{2}$, and followers simply rely on the early adopters' AQR, i.e., R^n, to make their purchasing decisions. If the seller discloses his private quality information in advance, consumers in period i observe the true quality level and purchase the product only when $q - p_i - E[v] = q - p_i \geq 0$, $i = 1, 2$. This is exactly the same as in the baseline model. Thus, the seller's payoff is $\pi_d^{nn} = (1 + m)q$.

When the seller withholds his quality information, the early adopters ex ante hold a quality expectation $q_1^n = \frac{1}{2}$. Accordingly, the seller's optimal first-period retail price under non-disclosure can be derived as $p_1^{nd} = q_1^n = \frac{1}{2}$. After consumption, each early adopter confirms her personal preference v and posts her SQA as follows: $q_r^{n,v} = \max\{0, \min\{1, q_r^n + v\}\}$, where $q_r^n = q + \alpha\left(q - \frac{1}{2}\right)$. Then, the AQR R^n of naive early adopters can be written as

$$R^n = \begin{cases} \int_{-\frac{1}{2}}^{1-q_r^n}(v + q_r^n)dv + \int_{1-q_r^n}^{\frac{1}{2}}(1)dv = -\frac{1}{2}(q_r^n)^2 + \frac{3}{2}q_r^n - \frac{1}{8}, & \text{if } q_r^n = q + \alpha\left(q - \frac{1}{2}\right) > \frac{1}{2}; \\ \int_{-\frac{1}{2}}^{-q_r^n}(0)dv + \int_{-q_r^n}^{\frac{1}{2}}(v + q_r^n)dv = \frac{1}{2}(q_r^n)^2 + \frac{1}{2}q_r^n + \frac{1}{8}, & \text{if } q_r^n = q + \alpha\left(q - \frac{1}{2}\right) < \frac{1}{2}. \end{cases}$$

It can be easily shown that the seller's optimal second-period retail price is $p_2^{nd} = R^n$. Correspondingly, the seller's ex post payoff under non-disclosure is $\pi_{nd}^n = \frac{1}{2} + mR^n$. A comparison of the seller's ex post payoffs under disclosure and non-disclosure leads to the following result.

Proposition 4. *When consumers are naive and heterogeneous in their personal preference, in equilibrium, we have the following:*

(1) If $m\alpha < 1$, the seller voluntarily discloses his private quality information in advance when the true product quality $q > \frac{1}{2}$; otherwise, he prefers non-disclosure.

(2) If $m\alpha \geq 1$, the seller voluntarily discloses his private quality information in advance when the true product quality $q \in \left(\frac{1}{2} - \frac{2(m\alpha-1)}{m(1+\alpha)^2}, \frac{1}{2}\right) \cup \left(\frac{1}{2} + \frac{2(m\alpha-1)}{m(1+\alpha)^2}, 1\right)$; otherwise, he prefers non-disclosure.

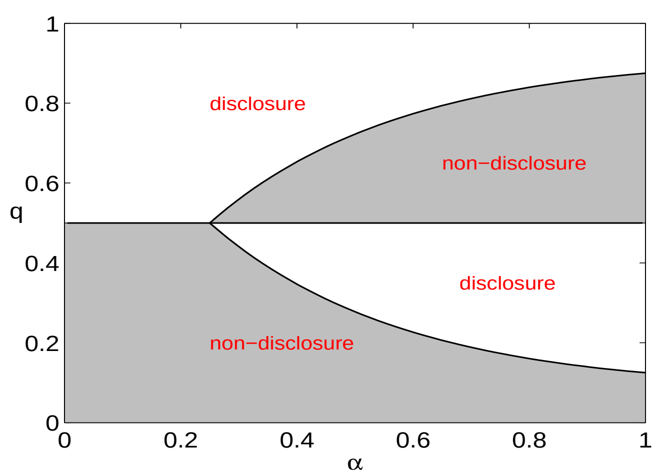

Figure 5: Equilibrium disclosure strategy when consumers are naive and have heterogeneous preferences: $m = 4$.

Proposition 4 indicates that the seller's equilibrium disclosure strategy when serving naive but heterogeneous consumers remains qualitatively similar to that in the baseline model when serving naive and homogeneous consumers (stated in Proposition 1); see Figure 5 for the illustration. Moreover, a closer examination of Propositions 1 and 4 implies that consumer heterogeneity makes the seller less likely to benefit from withholding high quality information and disclosing low quality information. A potential explanation is that consumers' heterogeneous preferences would diversify consumers' SQAs and discount the impact of the reference effect on the AQR R^n. In particular, when the true quality level is above $\frac{1}{2}$, although withholding such quality information can lead to an AQR higher than the true quality level, this AQR is still lower than the rating posted by the consumers when they do not have personal preferences, i.e., $R^n < q_r^n$. This subsequently dampens the positive effect of withholding high quality information. Consequently, the seller discloses more high quality information when consumers are heterogeneous. Similarly, when the true quality level is below $\frac{1}{2}$, withholding such quality information can lead to an AQR higher than the corresponding rating posted by the consumers when they do not have personal preferences. This actually mitigates the negative effect of withholding low quality information, and thus the seller does not need to reveal too much low quality information.

Regarding the scenario with sophisticated consumers, more diversified consumer reviews actually help followers better speculate about product quality. Thus, in equilibrium, the seller would still disclose all of his quality information given that strategic information withholding is no longer useful for attracting sophisticated consumers. In this sense, the results in the basic model remain robust when consumers have heterogeneous preferences. As the diversity of consumer reviews would dampen the impact of reference effect, it may be in the seller's interest to reduce such review diversity so as to better exploit consumers' reference-dependent preferences. Note that in practice, this can be achieved by narrowing down consumers' review score choices. For example, consumers can assess the product quality only as either "positive" or "negative" rather than a specific score between 0 and 1.

5.2 When Consumer Reviews Are Unbounded

In our baseline model, the quality review/assessment is bounded over an interval, a commonly observed rating practice. Here, we consider another setting in which reviews are unbounded. This may represent the scenarios in which the consumer review is communicated in the format of descriptions instead of scores. In such a setting, the type-t early adopter's SQA after consumption becomes

$$q_r^t = q + \alpha(q - q_1^t), t = n, s.$$

Similar to that studied in §4, we first derive the seller's equilibrium pricing and payoff given the seller's disclosure decision. We then compare the seller's payoffs under the two disclosure options to derive the equilibrium disclosure strategy. Note that in this situation, if the follower is sophisticated, she can perfectly rectify the early adopter's SQA to the true quality level, and the subsequent analysis is quite straightforward. Thus, below, we only focus on the scenarios in which the consumers are naive. Regarding the scenario in which the consumers are sophisticated, the boundary on reviews has no impact given that the seller would fully disclose the quality information. For the sake of brevity, we omit the detailed mathematical derivation here. We simply present the results in the following proposition; see Figure 6 for the illustration.

Proposition 5. *When consumer reviews are unbounded and both the early adopter and the follower are naive, in equilibrium, the seller discloses his quality information in advance either when* $q > \frac{1}{2}$ *and* $m\alpha < 1$ *or when* $q \in \left(\frac{1}{2(1+m)}, \frac{1}{2}\right)$ *and* $m\alpha \geq 1$.

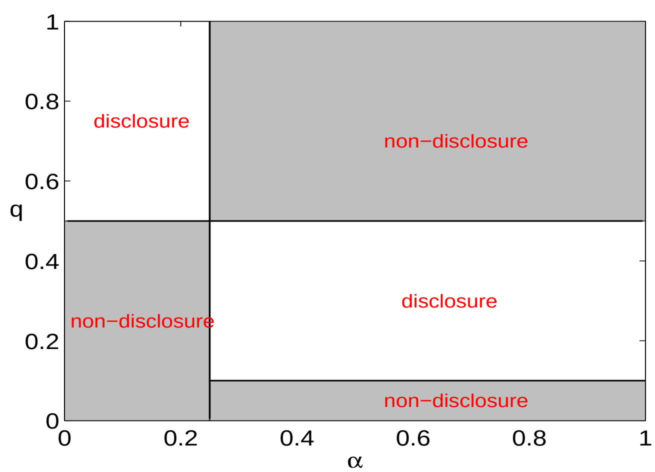

Figure 6: Equilibrium disclosure strategy when consumer reviews are unbounded given that both the early adopter and the follower are naive: $m = 4$.

The strategic impact of review bounds on the seller's voluntary information disclosure strategy can be identified by comparing Propositions 1 and 5. Specifically, when the consumers are naive, regardless of whether consumer reviews are bounded, the seller's disclosure strategy remains the same when the magnitude of the reference effect is small ($m\alpha < 1$). However, when the reference effect is strong ($m\alpha \geq 1$), the seller no longer discloses his extremely high quality information when consumer reviews are unbounded, a result different from that when consumer reviews are bounded. Recall from our baseline model that the review bounds would restrain the positive effect of withholding sufficiently high quality information, since the consumer's reference-dependent SQA is capped at the review's upper bound. However, when there is no upper bound, the seller can increase the early adopter's SQA by withholding extremely high quality information, thereby deriving more surplus from the naive follower. That is, when consumer reviews are unbounded, the seller would only disclose intermediate or relatively low quality information to avoid the negative impact of the reference effect but would withhold both high and low quality information.

6 Conclusion

In this paper, we investigate the strategic interactions between a monopolistic seller holding private quality information and two groups of reference-dependent consumers who arrive in two periods (an early adopter and a follower). The seller needs to decide whether to voluntarily disclose his quality information in advance. Consumers can be either sophisticated or naive, depending on whether they can make rational inferences based on the seller's disclosure behavior and the reference-dependent SQA posted by the early adopter.

We show that the seller's equilibrium disclosure strategy substantially hinges on the consumers' types and the magnitude of the reference effect. Specifically, when consumers are naive and the magnitude of the reference effect is small, the seller should disclose his private quality information only if the product quality is higher than a threshold value. When the magnitude of the reference effect is large, the seller switches between disclosing and withholding his quality information as the true quality level decreases. By doing so, the seller can boost the reference-dependent early adopter's subjective quality review and in turn enhance the follower's quality expectation to extract more surplus in the second period. In strict contrast, when consumers are sophisticated, it is difficult for the seller to enhance their quality expectation by manipulating the quality reviews, so in equilibrium, the seller discloses all the quality information to consumers. Our analysis reveals how consumer type could influence the seller's equilibrium disclosure strategy and performance. Specifically, the seller has a higher ex ante disclosure probability but obtains a lower payoff when consumers are sophisticated than when consumers are naive. Moreover, it shows that a seller can benefit from withholding certain quality information although disclosure itself is costless, which may explain why *information withholding* is popular in business practice.

When the market contains both naive and sophisticated consumers, we show that the seller is still able to withhold low quality information in advance. In such a situation, in the first period, the seller charges a high retail price and serves only naive consumers.

When consumers exhibit personal preferences towards the product, we show that the existence of such heterogeneous preferences alleviates the impact of the reference effect on consumer reviews. However, it is still possible for the seller to ex post tailor his disclosure decision to take advantage of the consumer's reference-dependent SQA to extract more surplus. When consumer reviews are unbounded, the seller can withhold extremely high quality information when consumers are naive.

In this paper, we examine how consumers' reference-dependent preferences affect their SQA and the seller's disclosure strategy. We assume that consumers in each period are ex ante homogeneous such that their purchasing decisions would be the same. However, it might be worthwhile to assume that consumers hold ex ante heterogeneous preferences before making their purchasing decision. In such a case, the seller could use the price to manipulate consumer reviews. For example, he could charge a high price in the first period to only serve consumers with a high preference to induce a high SQA. In the article, to bypass the possible price signaling issue, we assume that early adopters are one-time consumers. If the early adopters were repeat consumers, the related analysis would be very challenging, at topic that we leave for future research.

In our paper, the consumer type remains the same across the two periods. In reality, it is possible that consumer type changes over different periods. For example, the early adopter is sophisticated, but the follower is naive. The underlying justification for such a scenario is that a sophisticated consumer is clever enough to infer the quality level from the seller's disclosure behavior and thus arrives in the first period. However, a naive consumer has to wait for the experienced consumers' reviews to infer product quality and thus can only arrive in the second period. In online Appendix B, we further consider the scenarios in which consumer types change across the two periods. We show that our main insights remain intact.

Finally, in this paper, we only focus on the adoption of informative advertising, through which the seller can truthfully disclose his private quality information. Nonetheless, there exists another approach to advertising, uninformative advertising, through which the seller can enhance consumer awareness or expand his market potential. How would this type of advertising influence consumer reviews? Would the seller still adopt the non-disclosure strategy in certain circumstances? Further investigation in this direction may lead to more fruitful insights.

Acknowledgments

We are grateful to the departmental editor (Professor Asoo Vakharia), an anonymous senior editor, and two anonymous referees for very helpful comments and suggestions. The first author Xu Guan's work was supported by the National Natural Science Foundation of China (NSFC) (No. 71922010, 7182100, 71871167, 71961160735). The corresponding author Yulan Wang acknowledges the financial supports from the Research Grants Council of Hong Kong (RGC Reference Number: PolyU 15502917). The third author Zelong Yi was supported by the NSFC (No. 71801155). The fourth author Ying-Ju Chen acknowledges the financial support by the NSFC/RGC (# N_HKUST615/19).

References

Anderson, E. W. and M. W. Sullivan (1993). The antecedents and consequences of customer satisfaction for firms. *Marketing Science 12*(2), 125–143.

Assuncao, J. L. and R. J. Meyer (1993). The rational effect of price promotions on sales and consumption. *Management Science 39*(5), 517–535.

Baron, O., M. Hu, S. Najafi-Asadolahi, and Q. Qian (2015). Newsvendor selling to loss-averse consumers with stochastic reference points. *Manufacturing & Service Operations Management 17*(4), 456–469.

Bell, D. E. (1985). Disappointment in decision making under uncertainty. *Operations Research 33*(1), 1–27.

Biyalogorsky, E. and O. Koenigsberg (2014). The design and introduction of product lines when consumer valuations are uncertain. *Production and Operations Management 23*(9), 1539–1548.

Branco, F., M. Sun, and J. M. Villas-Boas (2015). Too much information? information provision and search costs. *Marketing Science, accepted and forthcoming*.

Bueno, B. J. (2007). *Why We Talk: Seven Reasons Your Customers Will - Or Will Not - Talk About Your Brand*. Creative Crayon Publishers.

Chevalier, J. A. and D. Mayzlin (2006). The effect of word of mouth on sales: Online book reviews. *Journal of Marketing Research 43*(3), 345–354.

Dellarocas, C. (2003). The digitization of word of mouth: Promise and challenges of online feedback mechanisms. *Management Science 49*(10), 1407–1424.

Dellarocas, C. (2006). Strategic manipulation of internet opinion forums: Implications for consumers and firms. *Management Science 52*(10), 1577–1593.

Delquié, P. and A. Cillo (2006). Expectations, disappointment, and rank-dependent probability weighting. *Theory and Decision 60*(2), 193–206.

Dranove, D. and G. Jin (2010). Quality disclosure and certification: theory and practice. Technical report, National Bureau of Economic Research.

Ellison, G. and D. Fudenberg (1995). Word-of-mouth communication and social learning. *The Quarterly Journal of Economics*, 93–125.

Erdem, T. and M. P. Keane (1996). Decision-making under uncertainty: Capturing dynamic brand choice processes in turbulent consumer goods markets. *Marketing Science 15*(1), 1–20.

Fibich, G., A. Gavious, and O. Lowengart (2007). Optimal price promotion in the presence of asymmetric reference-price effects. *Managerial and Decision Economics 28*(6), 569–577.

Gabaix, X. and D. Laibson (2006). Shrouded attributes, consumer myopia, and information suppression in competitive markets. *Quarterly Journal of Economics 121*(2).

Gao, G. G., B. N. Greenwood, J. McCullough, and R. Agarwal (2013). Vocal minority and silent majority: How do online ratings reflect population perceptions of quality? *Working Paper*.

Gneezy, A., U. Gneezy, and D. O. Lauga (2014). A reference-dependent model of the price-quality heuristic. *Journal of Marketing Research 51*(2), 153–164.

Grossman, S. and O. Hart (1980). Disclosure laws and takeover bids. *The Journal of Finance 35*(2), 323–334.

Gu, Z. J. and Y. Xie (2013). Facilitating fit revelation in the competitive market. *Management Science 59*(5), 1196–1212.

Guan, X., B. Liu, Y.-j. Chen, and H. Wang (2020). Inducing supply chain transparency through supplier encroachment. *Production and Operations Management*.

Guo, L. (2009). Quality disclosure formats in a distribution channel. *Management Science 55*(9), 1513–1526.

Guo, L. and Y. Zhao (2009). Voluntary quality disclosure and market interaction. *Marketing Science 28*(3), 488–501.

Heidhues, P. and B. Kőszegi (2008). Competition and price variation when consumers are loss averse. *The American Economic Review*, 1245–1268.

Jovanovic, B. (1982). Truthful disclosure of information. *The Bell Journal of Economics 13*(1), 36–44.

Karle, H. and M. Peitz (2014). Competition under consumer loss aversion. *The RAND Journal of Economics 45*(1), 1–31.

Kőszegi, B. and M. Rabin (2006). A model of reference-dependent preferences. *The Quarterly Journal of Economics*, 1133–1165.

Kuksov, D. and Y. Lin (2010). Information provision in a vertically differentiated competitive marketplace. *Marketing Science 29*(1), 122–138.

Levin, Y., J. McGill, and M. Nediak (2010). Optimal dynamic pricing of perishable items by a monopolist facing strategic consumers. *Production and Operations Management 19*(1), 40–60.

Lewis, T. R. (1994). Supplying information to facilitate price discrimination. *International Economic Review 35*(2), 309–327.

Li, X. and L. M. Hitt (2008). Self-selection and information role of online product reviews. *Information Systems Research 19*(4), 456–474.

Lim, W. S. and C. S. Tang (2013). Advance selling in the presence of speculators and forward-looking consumers. *Production and Operations Management 22*(3), 571–587.

Liu, Q. and S. Shum (2013). Pricing and capacity rationing with customer disappointment aversion. *Production and Operations Management 22*(5), 1269–1286.

Matthews, S. and A. Postlewaite (1985). Quality testing and disclosure. *The RAND Journal of Economics 16*(3), 328–340.

Milgrom, P. (1981). Good news and bad news: Representation theorems and applications. *The Bell Journal of Economics 12*(2), 380–391.

Milgrom, P. and J. Roberts (1986). Price and advertising signals of product quality. *Journal of Political Economy 94*(4), 796–821.

Moorthy, S. and K. Srinivasan (1995). Signaling quality with a money-back guarantee: The role of transaction costs. *Marketing Science 14*(4), 442–466.

Moretti, E. (2011). Social learning and peer effects in consumption: Evidence from movie sales. *The Review of Economic Studies 78*(1), 356–393.

Nasiry, J. and I. Popescu (2011). Dynamic pricing with loss averse consumers and peak-end anchoring. *Operations Research 59*(6), 1361–1368.

Nasiry, J. and I. Popescu (2012). Advance selling when consumers regret. *Management Science 58*(6), 1160–1177.

Nelson, P. (1974). Advertising as information. *Journal of Political Economy 82*(4), 729–754.

Ottaviani, M. (1999). Monopoly pricing with social learning. *Working Paper*.

Papanastasiou, Y., N. Bakshi, and N. Savva (2013). Social learning from early buyer reviews: Implications for new product launch. *History*.

Popescu, I. and Y. Wu (2007). Dynamic pricing strategies with reference effects. *Operations Research 55*(3), 413–429.

Shavell, S. (1994). Acquisition and disclosure of information prior to sale. *The RAND Journal of Economics 25*(1), 20–36.

Su, X. (2007). Intertemporal pricing with strategic customer behavior. *Management Science 53*(5), 726–741.

Taylor, C. R. (2004). Consumer privacy and the market for customer information. *RAND Journal of Economics*, 631–650.

Trusov, M., R. E. Bucklin, and K. Pauwels (2009). Effects of word-of-mouth versus traditional marketing: Findings from an internet social networking site. *Journal of Marketing 73*(5), 90–102.

Tversky, A. and D. Kahneman (1991). Loss aversion in riskless choice: A reference-dependent model. *Quarterly Journal of Economics 106*(4), 1039–1061.

Villas-Boas, J. M. (2004). Consumer learning, brand loyalty, and competition. *Marketing Science 23*(1), 134–145.

Yang, L., P. Guo, and Y. Wang (2018). Service pricing with loss-averse customers. *Operations research 66*(3), 761–777.

Zhou, J. (2011). Reference dependence and market competition. *Journal of Economics & Management Strategy 20*(4), 1073–1097.

Online Appendix
"Inducing Consumer Online Review via Disclosure"

Appendix A: Proofs

Proof of Proposition 1: The seller chooses disclosure when the right-hand side of (4) is positive. We first consider the case of $m\alpha < 1$. If $q \leq \frac{\alpha}{2(1+\alpha)}$, we have $(1+m)q - \frac{1}{2} < 0$, because $(1+m)q - \frac{1}{2} \leq (1+m)\frac{\alpha}{2(1+\alpha)} - \frac{1}{2} = \frac{m\alpha-1}{2(1+\alpha)} < 0$. Thus, the seller chooses non-disclosure when $q \leq \frac{\alpha}{2(1+\alpha)}$. If $\frac{\alpha}{2(1+\alpha)} < q < \frac{2+\alpha}{2(1+\alpha)}$, it can be shown that $(1 - m\alpha)(q - \frac{1}{2}) > 0$ when $q \in (\frac{1}{2}, \frac{2+\alpha}{2(1+\alpha)})$ and $(1-m\alpha)\left(q - \frac{1}{2}\right) \leq 0$ when $q \in (\frac{\alpha}{2(1+\alpha)}, \frac{1}{2}]$. Thus, the seller adopts disclosure strategy when $q \in (\frac{1}{2}, \frac{2+\alpha}{2(1+\alpha)})$. If $q \geq \frac{2+\alpha}{2(1+\alpha)}$, we have $(1+m)q - \frac{1+2m}{2} \geq (1+m)\frac{2+\alpha}{2(1+\alpha)} - \frac{1+2m}{2} = \frac{1-m\alpha}{2(1+\alpha)} > 0$, which implies that the seller is better off with disclosure. Combining the three conditions, the seller prefers disclosing the quality information when $q > \frac{1}{2}$ and withholding otherwise.

We next consider the case of $m\alpha \geq 1$. Similarly, if $q \leq \frac{\alpha}{2(1+\alpha)}$, we have $(1+m)q - \frac{1}{2} > 0$ when $q \in (\frac{1}{2(1+m)}, \frac{\alpha}{2(1+\alpha)}]$; if $\frac{\alpha}{2(1+\alpha)} < q < \frac{2+\alpha}{2(1+\alpha)}$, $(1-m\alpha)\left(q-\frac{1}{2}\right) > 0$ when $q \in (\frac{\alpha}{2(1+\alpha)}, \frac{1}{2})$; if $q \geq \frac{2+\alpha}{2(1+\alpha)}$, $(1+m)q - \frac{1+2m}{2} > 0$ when $q \in (\frac{1+2m}{2(1+m)}, 1]$. Under any of these conditions, disclosing quality information leads to a higher payoff for the seller, and the seller's equilibrium disclosure strategy stated in Proposition 1 can be obtained. □

Proof of Corollary 1: Based on (5) and (6), after some algebra operations, we can further simplify the seller's payoff as follows:

$$\begin{aligned}
\Pi^{nn}_{m\alpha<1} &= \frac{4m + 5\alpha + 3m\alpha + 5}{8(1+\alpha)};\\
\Pi^{nn}_{m\alpha\geq 1} &= \frac{7m + 5\alpha + 8m\alpha + 5m^2\alpha + 4m^2 + 5}{8(1+\alpha)(1+m)}.
\end{aligned}$$

Taking their first-order derivatives with respect to α, we can show that

$$\frac{\partial \Pi^{nn}_{m\alpha<1}}{\partial \alpha} = -\frac{m}{8(1+\alpha)^2} < 0 \text{ and } \frac{\partial \Pi^{nn}_{m\alpha\geq 1}}{\partial \alpha} = \frac{m}{8(1+\alpha)^2} > 0.$$

Thus, the seller's payoff first decreases in α when $\alpha < \frac{1}{m}$ and then increases in α when $\alpha \geq \frac{1}{m}$. □

Proof of Proposition 2: We first examine how the sophisticated follower infer the quality information from the early adopter's quality review. If the follower observes $q^s_r = 0$, she can infer that $(1+\alpha)q - \alpha\bar{q} \leq 0$, that is, $q \leq \frac{\alpha\bar{q}}{1+\alpha}$. Noting that the seller is better off withholding this quality information than disclosing it upfront, she also believes that the product quality q must fall into the range of $\left[0, q^s_l\right]$ $(q^s_l \leq \frac{\alpha\bar{q}}{1+\alpha})$. Thus, the follower's quality expectation should be $q_2^s = \frac{q_l^s}{2}$, and at $q = q_l^s$ the seller is indifferent between disclosure and non-disclosure: $(1+m)q_l^s = \overline{q} + m\frac{q_l^s}{2}$. Thus, in equilibrium, we have $q_l^s = \min\{\frac{2\overline{q}}{2+m}, \frac{\alpha\overline{q}}{1+\alpha}\}$.

If the follower observes $q_r^s = 1$, she can infer that $(1+\alpha)q - \alpha\overline{q} \geq 1$, that is, $q \geq \frac{1+\alpha\overline{q}}{1+\alpha}$. Noting that the seller is better off withholding this quality information than disclosing it upfront, she also believes that the product quality q must fall into the range of $\left[\frac{1+\alpha\overline{q}}{1+\alpha}, q_h^s\right]$ ($\frac{1+\alpha\overline{q}}{1+\alpha} < q_h^s \leq 1$). Thus, the follower's quality expectation should be $q_2^s = \frac{1}{2}\left(\frac{1+\alpha\overline{q}}{1+\alpha} + q_h^s\right)$, and at $q = q_h^s$ the seller should be indifferent between disclosure and non-disclosure. However, given that $q_h^s > \frac{1+\alpha\overline{q}}{1+\alpha} > \overline{q}$, we can easily show that the seller is always worse off by withholding the quality information at q_h^s since under such a scenario, we can derive that $q_h^s(1+m) - \overline{q} - \frac{m}{2}\left(\frac{1+\alpha\overline{q}}{1+\alpha} + q_h^s\right) > 0$. Similar to Lemma S1, this again contradicts our prior assumption the seller is better off withholding his quality information when $q \in \left[\frac{1+\alpha\overline{q}}{1+\alpha}, q_h^s\right]$. Thus. this range does not exist and we simply set $q_2^s = q_h^s = \frac{1+\alpha\overline{q}}{1+\alpha}$. And $q_r^s = 1$ can never arise in equilibrium.

The last case is that $0 < q_r^s < 1$, in which the follower can always infer the true quality level that $q_2^s = q$.

Then, we compare the seller's payoffs under two disclosure options. Note that $q_l = \frac{\alpha\overline{q}}{1+\alpha}$ if $m\alpha < 2$ and $q_l = \frac{2\overline{q}}{m+2}$ if $m\alpha \geq 2$. First, if $m\alpha < 2$ and $q_l = \frac{\alpha\overline{q}}{1+\alpha}$, we have

$$\pi_d^{ss} - \pi_{nd}^{ss} = \begin{cases} (1+m)q - \overline{q} - m\dfrac{\alpha\overline{q}}{2(1+\alpha)}, & \text{if } q \leq \dfrac{\alpha\overline{q}}{1+\alpha}; \\ q - \overline{q}, & \text{if } \dfrac{\alpha\overline{q}}{1+\alpha} < q \leq \dfrac{1+\alpha\overline{q}}{1+\alpha}; \\ (1+m)q - \overline{q} - m\dfrac{1+\alpha\overline{q}}{1+\alpha}, & \text{otherwise.} \end{cases}$$

Second, if $m\alpha \geq 2$ and $q_l = \frac{2\overline{q}}{2+m}$, we have

$$\pi_d^{ss} - \pi_{nd}^{ss} = \begin{cases} (1+m)q - \overline{q} - \dfrac{m\overline{q}}{2+m}, & \text{if } q \leq \dfrac{2\overline{q}}{2+m}; \\ q - \overline{q}, & \text{if } \dfrac{2\overline{q}}{2+m} < q \leq \dfrac{1+\alpha\overline{q}}{1+\alpha}; \\ (1+m)q - \overline{q} - m\dfrac{1+\alpha\overline{q}}{1+\alpha}, & \text{otherwise.} \end{cases}$$

It is evident that for any quality above $\overline{q}$, the seller is better off disclosing it upfront. Therefore, according to the unraveling theory, in equilibrium the seller discloses all the quality information above $\overline{q}$, which subsequently squeezes $\overline{q} = 0$. □

Proof of Corollary 2: When both the early adopter and the follower are sophisticated, the seller always discloses quality information and hence his ex-ante disclosure probability is higher than that when the consumers are naive.

By Corollary 1, we have

$$\Pi^{nn}_{m\alpha<1} = \frac{4m+5\alpha+3m\alpha+5}{8(1+\alpha)};$$
$$\Pi^{nn}_{m\alpha\geq 1} = \frac{7m+5\alpha+8m\alpha+5m^2\alpha+4m^2+5}{8(1+\alpha)(1+m)}.$$

By equation (7), we have $\Pi^{ss} = \frac{1+m}{2}$. Then,

$$\Pi^{nn}_{m\alpha<1} - \Pi^{ss} = \frac{4m+5\alpha+3m\alpha+5}{8(1+\alpha)} - \frac{1+m}{2} = \frac{\alpha-m\alpha+1}{8(1+\alpha)} > 0;$$
$$\Pi^{nn}_{m\alpha\geq 1} - \Pi^{ss} = \frac{7m+5\alpha+8m\alpha+5m^2\alpha+4m^2+5}{8(1+\alpha)(1+m)} - \frac{1+m}{2} = \frac{m^2\alpha-m+\alpha+1}{8(1+\alpha)(1+m)} > 0.$$

Then, we have $\Pi^{nn} > \Pi^{ss}$ and the result follows. □

Proof of Lemma 1: Note that the sophisticated followers can confirm that the seller withholds the quality information and only serves the naive consumers in the first period if he does so. Therefore, given that $q^n_r = \min\{((1+\alpha)q - \frac{\alpha}{2})^+, 1\}$, if the sophisticated follower observes a quality assessment $q^n_r \in (0,1)$, she can infer that $(1+\alpha)q - \frac{\alpha}{2} = q^n_r$. Thus, the sophisticated follower can backward induce that $q = \frac{2q^n_r+\alpha}{2(1+\alpha)}$.

If the sophisticated follower observes $q^n_r = 0$, she can infer that $(1+\alpha)q - \frac{\alpha}{2} \leq 0$; that is, $q \leq \frac{\alpha}{2(1+\alpha)}$. The sophisticated follower cannot obtain the exact quality level, and instead she can infer that $q \in [0, \frac{\alpha}{2(1+\alpha)}]$. Noting that the seller is better off withholding this quality information than disclosing it upfront, she also believes that the product quality q must fall into the range of $[0, q_l]$, where $q_l \leq \frac{\alpha}{2(1+\alpha)}$. Thus, the sophisticated follower's quality expectation should be $q^s_2 = \frac{q_l}{2}$, and at $q = q_l$ the seller is indifferent between disclosure and non-disclosure: $(1+m)q_l = \frac{1}{2}\gamma + m\frac{q_l}{2}(1-\gamma)$. Note that if the seller withholds the quality information, in the first period she only serves the naive consumers whose quality expectation is $q^n_1 = \frac{1}{2}$; while in the second period, the seller can only serve the sophisticated consumers whose quality expectation is $q^s_2 = \frac{q_l}{2}$. Combining them together, the seller's payoff under non-disclosure is given by $\pi^{mix}_{nd} = \frac{1}{2}\gamma + m\frac{q_l}{2}(1-\gamma)$. Thus, in equilibrium, we have $q_l = \min\{\frac{\gamma}{m+m\gamma+2}, \frac{\alpha}{2(1+\alpha)}\}$.

If the sophisticated follower observes $q^n_r = 1$, she can infer that $(1+\alpha)q - \frac{\alpha}{2} \geq 1$, which implies that the product level q is no less than $\frac{2+\alpha}{2(1+\alpha)}$. Under this circumstance, the sophisticated follower cannot infer the exact quality level. Instead, she believes that the product quality q falls into the range of $\left[\frac{2+\alpha}{2(1+\alpha)}, q_h\right]$, where $\frac{2+\alpha}{2(1+\alpha)} \leq q_h \leq 1$, and the seller is better off withholding this quality information than disclosing it upfront in this quality range. This subsequently leads to the sophisticated follower's quality expectation $q_2^s = \frac{1}{2}\left(\frac{2+\alpha}{2(1+\alpha)} + q_h\right)$.

Thus, at q_h the seller should be indifferent between disclosure and non-disclosure. Note that when the seller withholds the quality information, the naive followers' quality expectation $q_2^n = 1$ and the sophisticated followers' quality expectation $q_2^s \geq \frac{2+\alpha}{2(1+\alpha)}$. Since we have assumed that $\gamma < \frac{2+\alpha}{2(1+\alpha)}$, it is always better off for the seller to serve both consumer types in the second period, i.e., $mq_2^s > m\gamma q_2^n$. Therefore, the seller's profit under non-disclosure is $\pi_{nd}^{mix} = \frac{1}{2}\gamma + mq_2^s$. Comparing to her payoff under disclosure $\pi_d^{mix} = (1+m)q_h$, one can see that the seller's payoff under disclosure is always higher as $(1+m)q_h > \frac{1}{2}\gamma + mq_2^s$. This contradicts our assumption that the seller is better off withholding his quality information when $q \in \left[\frac{2+\alpha}{2(1+\alpha)}, q_h\right]$. Thus, this range does not exist. For technical tractability, we just set $q_h = \frac{2+\alpha}{2(1+\alpha)}$ (its lowest value) and the sophisticated follower generates the lowest quality expectation that $q_2^s = \frac{2+\alpha}{2(1+\alpha)}$ after observing $q_r^n = 1$. As will be shown later, this assumption is consistent with the seller's equilibrium disclosure strategy, which ensures that $q_r^n = 1$ can never arise in the equilibrium. □

Proof of Lemma 2: Herein, we prove that in equilibrium the seller would never serve both consumer types under non-disclosure in a market with a mixture of customer types. We prove this via a two-steps process. First, we derive the seller's payoff when it withholds the quality information and serves both consumer types. Second, we demonstrate that when $\gamma < \frac{2+\alpha}{2(1+\alpha)}$, if he does so, his payoff is always lower than that when he discloses the quality information upfront. Thus, when $\gamma < \frac{2+\alpha}{2(1+\alpha)}$, withholding the quality information and serving both consumer types is never an equilibrium solution in our setting.

To start, we assume that the seller withholds the quality information upfront. Then, the naive consumers' quality expectation $q_1^n = \frac{1}{2}$ and the sophisticated consumers' quality expectation $q_1^s = \overline{q}$. It can be easily shown that $\overline{q} \leq \frac{1}{2}$. Note that if $\overline{q} > \frac{1}{2}$, then the seller can certainly derive more surplus by withholding all the quality information that is below $\frac{1}{2}$. By doing so, he can surprise the reference-dependent early adopter and boost the SQA, which in turn enhances the follower's quality expectation and benefits the seller. As a result, a sophisticated consumer can never form a quality expectation that is above $\frac{1}{2}$ under such a circumstance. This contradicts the assumption $\overline{q} > \frac{1}{2}$. Thus, to serve both consumer types, the seller's retail price in first period shall be $p_1 = \overline{q}$. After consumption, the aggregate average quality rating R^{mix} can be derived as

$$R^{mix} = \gamma q_r^n + (1-\gamma)q_r^s, \text{ where}$$

$$R^{mix} = \begin{cases} 0, & \text{if } q \leq \dfrac{\alpha\bar{q}}{1+\alpha}; \\ (1-\gamma)\left((1+\alpha)q - \alpha\bar{q}\right), & \text{if } \dfrac{\alpha\bar{q}}{1+\alpha} < q \leq \dfrac{\alpha}{2(1+\alpha)}; \\ (1-\gamma)\left((1+\alpha)q - \alpha\bar{q}\right) + \gamma\left((1+\alpha)q - \dfrac{\alpha}{2}\right), & \text{if } \dfrac{\alpha}{2(1+\alpha)} < q \leq \dfrac{1+\alpha\bar{q}}{1+\alpha}; \\ (1-\gamma) + \gamma\left((1+\alpha)q - \dfrac{\alpha}{2}\right), & \text{if } \dfrac{1+\alpha\bar{q}}{1+\alpha} < q \leq \dfrac{2+\alpha}{2(1+\alpha)}; \\ 1, & \text{otherwise.} \end{cases} \tag{9}$$

Given R^{mix}, the naive followers would take it as their quality expectation $q_2^n = R^{mix}$ while the sophisticated followers would further update their quality expectation, whose process is essentially similar to Lemma 1. That is, when $0 < R^{mix} < 1$, the sophisticated followers can confirm the true quality information via backward induction, $q_2^n = q$. If $R^{mix} = 1$, the sophisticated follower believes that the product quality q falls into the range of $\left[\frac{2+\alpha}{2(1+\alpha)}, q_h\right]$, where $\frac{2+\alpha}{2(1+\alpha)} \leq q_h \leq 1$, and the seller is better off withholding this quality information than disclosing it upfront in this quality range.

Still, we can demonstrate that this quality range does not exist, since for any $q_h \geq \frac{2+\alpha}{2(1+\alpha)}$, the seller can obtain a higher payoff by disclosing the quality information than withholding it. Thus, we simply assume that $q_h = \frac{2+\alpha}{2(1+\alpha)}$ for technical tractability. This is exactly the same as that in Lemma 1. Last, if $R^{mix} = 0$, the sophisticated follower believes that the product quality q falls into the range of $[0, q_l]$, where $q_l \leq \frac{\alpha\bar{q}}{1+\alpha}$, and the seller is better off withholding this quality information than disclosing it upfront in this quality range, where $q_l = \min\{\frac{2\bar{q}}{m+m\gamma+2}, \frac{\alpha\bar{q}}{1+\alpha}\}$.

Building upon this, we can derive the seller's payoffs under disclosure and non-disclosure. Note that according to the unraveling theory, if the seller's payoff under disclosure is always higher than that under non-disclosure ($\pi_d^{mix} \geq \pi_{nd}^{mix}$) when $q > \bar{q}$, then in equilibrium $\bar{q}$ will be pushed down to zero, i.e., $\bar{q} = 0$. Below, we compare the seller's payoffs under disclosure and non-disclosure when $q > \bar{q}$. Then, we have the following four exclusive and exhaustive cases.

First, if $q > \frac{2+\alpha}{2(1+\alpha)}$, because $q_2^s = \frac{2+\alpha}{2(1+\alpha)} > \gamma$, the seller would charge q_2^s and serve all followers in the second period. Thus, $\pi_{nd}^{mix} = \bar{q} + m\frac{2+\alpha}{2(1+\alpha)}$. We can easily show that $\pi_d^{mix} = (1+m)q > \pi_{nd}^{mix}$.

Second, if $\frac{1+\alpha\bar{q}}{1+\alpha} < q \leq \frac{2+\alpha}{2(1+\alpha)}$, we have

$$\left[R^{mix} - q\right]\Big|_{q=\frac{2+\alpha}{2(1+\alpha)}} = \left[(1-\gamma) + \gamma((1+\alpha)q - \frac{\alpha}{2}) - q\right]\Big|_{q=\frac{2+\alpha}{2(1+\alpha)}} = \frac{\alpha}{2(1+\alpha)} > 0;$$

and

$$\left[R^{mix}-q\right]\Big|_{q=\frac{1+\alpha\overline{q}}{1+\alpha}}=-\frac{\alpha(2\overline{q}+\gamma-2\overline{q}\gamma+\alpha\gamma-2\overline{q}\alpha\gamma-2)}{2(1+\alpha)}>0,$$

as it can be verified that $2\overline{q}+\gamma-2\overline{q}\gamma+\alpha\gamma-2\overline{q}\alpha\gamma-2<0$ for any $\overline{q}$ within $[0,\frac{1}{2}]$. Thus, we have $R^{mix}>q$. Then, the seller's optimal pricing decision in the second period can be drawn by comparing γR^{mix} and q. For the comparison between γR^{mix} and q, we have

$$\left[\gamma R^{mix}-q\right]\Big|_{q=\frac{2+\alpha}{2(1+\alpha)}}=-\frac{\alpha-2\gamma-2\alpha\gamma+2}{2\,(1+\alpha)}<0 \text{ since } \gamma<\frac{2+\alpha}{2(1+\alpha)};$$

$$\left[\gamma R^{mix}-q\right]\Big|_{q=\frac{1+\alpha\overline{q}}{1+\alpha}}=\frac{2\gamma-2\overline{q}\alpha-\alpha^2\gamma^2+2\alpha\gamma-\alpha\gamma^2+2\overline{q}\alpha^2\gamma^2+2\overline{q}\alpha\gamma^2-2}{2\,(1+\alpha)}<0$$

since it can be verified that $2\gamma-2\overline{q}\alpha-\alpha^2\gamma^2+2\alpha\gamma-\alpha\gamma^2+2\overline{q}\alpha^2\gamma^2+2\overline{q}\alpha\gamma^2-2<0$ for any $\overline{q}\in[0,\frac{1}{2}]$ and $\gamma<\frac{2+\alpha}{2(1+\alpha)}$. Thus, $\gamma R^{mix}<q$. The seller shall serve both types of followers by setting the price $p_2^{nd}=q$ and his payoff under non-disclosure is thus $\pi_{nd}^{mix}=\overline{q}+mq$, which is lower than her payoff under disclosure, $\pi_d^{mix}=(1+m)q$.

Third, if $\max\{\overline{q},\frac{\alpha}{2(1+\alpha)}\}<q\leq\frac{1+\alpha\overline{q}}{1+\alpha}$, we have

$$\begin{aligned}\left[R^{mix}-q\right]\Big|_{q=\frac{1+\alpha\overline{q}}{1+\alpha}} &= \left[(1-\gamma)\left((1+\alpha)q-\alpha\overline{q}\right)+\gamma((1+\alpha)q-\frac{\alpha}{2})-q\right]\Big|_{q=\frac{1+\alpha\overline{q}}{1+\alpha}}\\ &= -\frac{\alpha(2\overline{q}+\gamma-2\overline{q}\gamma+\alpha\gamma-2\overline{q}\alpha\gamma-2)}{2(1+\alpha)}>0;\end{aligned}$$

$$\left[R^{mix}-q\right]\Big|_{q=\overline{q}}=\left[(1-\gamma)\left((1+\alpha)q-\alpha\overline{q}\right)+\gamma((1+\alpha)q-\frac{\alpha}{2})-q\right]\Big|_{q=\overline{q}}=\frac{1}{2}\alpha\gamma\left(2\overline{q}-1\right)<0;$$

$$\begin{aligned}\left[R^{mix}-q\right]\Big|_{q=\frac{\alpha}{2(1+\alpha)}} &= \left[(1-\gamma)\left((1+\alpha)q-\alpha\overline{q}\right)+\gamma((1+\alpha)q-\frac{\alpha}{2})-q\right]\Big|_{q=\frac{\alpha}{2(1+\alpha)}}\\ &= -\frac{\alpha\left(2\overline{q}-\alpha+\gamma+2\overline{q}\alpha-2\overline{q}\gamma+\alpha\gamma-2\overline{q}\alpha\gamma\right)}{2(1+\alpha)};\end{aligned}$$

and

$$\frac{\partial\left[R^{mix}-q\right]}{\partial q}=\alpha>0.$$

It can be verified that $2\overline{q}-\alpha+\gamma+2\overline{q}\alpha-2\overline{q}\gamma+\alpha\gamma-2\overline{q}\alpha\gamma$ can be either greater or less than zero. Thus, generally, there exists a unique q^* that $R^{mix}<q$ when $q\in\left[\max\{\overline{q},\frac{\alpha}{2(1+\alpha)}\},q^*\right]$ and $R^{mix}>q$ when $q\in\left[q^*,\frac{1+\alpha\overline{q}}{1+\alpha}\right]$. Note that if $R^{mix}<q$, $\pi_{nd}^{mix}=\overline{q}+m\max\left\{(1-\gamma)q,R^{mix}\right\}$, which is always lower than $\pi_d^{mix}=(1+m)q$.

If $R^{mix}>q$, let us define

$$G=\gamma R^{mix}-q=\gamma\left((1-\gamma)\left((1+\alpha)q-\alpha\overline{q}\right)+\gamma((1+\alpha)q-\frac{\alpha}{2})\right)-q.$$

As $\frac{\partial G}{\partial q} = (1+\alpha)\gamma - 1$, G increases (decreases) in q when $\gamma \geq \frac{1}{1+\alpha}$ ($\gamma < \frac{1}{1+\alpha}$). We can verify that $G\big|_{q=\overline{q}} = \overline{q}\gamma - \overline{q} - \frac{1}{2}\alpha\gamma^2 + \overline{q}\alpha\gamma^2 < 0$ when $\overline{q} \leq \frac{1}{2}$. Thus, if $\gamma < \frac{1}{1+\alpha}$, we have $G < 0$ and $\gamma R^{mix} - q < 0$ for $q > \overline{q}$. If $\frac{1}{1+\alpha} < \gamma < \frac{2+\alpha}{2(1+\alpha)}$, the highest value of G can be attained at $q = \frac{1+\alpha\overline{q}}{1+\alpha}$ and $\gamma = \frac{2+\alpha}{2(1+\alpha)}$, since

$$\frac{\partial\left[G\Big|_{q=\frac{1+\alpha\overline{q}}{1+\alpha}}\right]}{\partial\gamma} = 2\overline{q}\alpha\gamma - \alpha\gamma + 1 > 0.$$

Furthermore, since $\overline{q} \leq \frac{1}{2}$, we have

$$\begin{aligned} G\Big|_{q=\frac{1+\alpha\overline{q}}{1+\alpha},\gamma=\frac{2+\alpha}{2(1+\alpha)}} &= \left[\left[\gamma\left((1-\gamma)\left((1+\alpha)q - \alpha\overline{q}\right) + \gamma((1+\alpha)q - \frac{\alpha}{2})\right) - q\right]\Big|_{q=\frac{1+\alpha\overline{q}}{1+\alpha}}\right]\Big|_{\gamma=\frac{2+\alpha}{2(1+\alpha)}} \\ &= \frac{\alpha^3\left(2\overline{q}-1\right)}{8\left(1+\alpha\right)^2} \leq 0. \end{aligned}$$

This implies that $\gamma R^{mix} - q < 0$ for any $q \in \left[q^*, \frac{1+\alpha\overline{q}}{1+\alpha}\right]$. Thus, The seller shall serve both types of followers by setting the price $p_2^{nd} = q$, and the seller's payoff under non-disclosure $\pi_{nd}^{mix} = \overline{q} + mq < \pi_d^{mix} = (1+m)q$.

Last, if $\overline{q} < q \leq \frac{\alpha}{2(1+\alpha)}$, following a similar logic, we can demonstrate that if $R^{mix} < q$, the seller's payoff under non-disclosure is lower than that under disclosure, i.e., $\pi_{nd}^{mix} = \overline{q} + \max\{(1-\gamma)q, R^{mix}\} < \pi_d^{mix} = (1+m)q$. If $R^{mix} > q$, the seller's payoff under non-disclosure is $\pi_{nd}^{mix} = \overline{q} + \max\{\gamma R^{mix}, q\}$. Then, we need to compare γR^{mix} and q. Let us define

$$G = \gamma R^{mix} - q = \gamma(1-\gamma)\left((1+\alpha)q - \alpha\overline{q}\right) - q.$$

Then, we have

$$G\Big|_{q=\overline{q}} = \left[\gamma(1-\gamma)\left((1+\alpha)q - \alpha\overline{q}\right) - q\right]\Big|_{q=\overline{q}} = -\overline{q}\left(\gamma^2 - \gamma + 1\right) < 0;$$

and

$$G\Big|_{q=\frac{\alpha}{2(1+\alpha)}} = \frac{-\alpha\left(2\gamma(1-\gamma)(1+\alpha)\overline{q} + 1 - \gamma(1-\gamma)(1+\alpha)\right)}{2(1+\alpha)} < 0,$$

since we can verify that $1 - \gamma(1-\gamma)(1+\alpha) > 0$ for all $\gamma \in (0, \frac{2+\alpha}{2(1+\alpha)})$. Thus, if $R^{mix} > q$, the seller's payoff under non-disclosure $\pi_{nd}^{mix} = \overline{q} + \max\{\gamma R^{mix}, q\} < \overline{q} + mq < \pi_d^{mix} = (1+m)q$.

Based on the above analysis, we have shown that when $q > \overline{q}$, $\pi_d^{mix} > \pi_{nd}^{mix}$ always holds. Then, according to the unraveling theory, we have $\overline{q} = 0$ in equilibrium (a result consistent with our premise). This implies that withholding the quality information and serving both consumer types can never be the equilibrium solution in our setting. □

Proof of Proposition 3: This proof has two steps. First, note that $\gamma = \frac{1}{2}$ is smaller than $\frac{2+\alpha}{2(1+\alpha)}$. Thus, the seller would set the first-period retailer price $p_1 = \frac{1}{2}$ when withholding his quality information upfront. We then identify the seller's disclosure strategy. Next, we compare the seller's payoffs under disclosure and non-disclosure to derive his equilibrium disclosure strategy.

Consider the situation where the seller chooses non-disclosure. Then, he charges the first-period retail price $p_1 = \frac{1}{2}$ and serves naive early adopters only. According to Lemma 1, we can divide our problem into two sub-cases depending on the sophisticated follower's quality belief upon non-disclosure. First, if $\alpha < \frac{2\gamma}{m(1+\gamma)+2(1-\gamma)} = \frac{2}{2+3m}$, we have $\frac{\gamma}{m+m\gamma+2} = \frac{1}{4+3m} > \frac{\alpha}{2(1+\alpha)}$ and hence $q_l = \frac{\alpha}{2(1+\alpha)}$. Consequently, after consumption, the naive early-adopters' AQR R^{mix} and the quality expectations of naive and sophisticated followers are given in the following table.

Quality level	AQR R^{mix}	q_2^n	q_2^s
$q < \frac{\alpha}{2(1+\alpha)}$	0	0	$\frac{\alpha}{4(1+\alpha)}$
$\frac{\alpha}{2(1+\alpha)} < q < \frac{2+\alpha}{2(1+\alpha)}$	$(1+\alpha)q - \frac{\alpha}{2}$	$(1+\alpha)q - \frac{\alpha}{2}$	q
$\frac{2+\alpha}{2(1+\alpha)} < q < 1$	1	1	$\frac{2+\alpha}{2(1+\alpha)}$

Accordingly, when the seller adopts non-disclosure, in the second period, he has two possible prices to charge, either $\min\{q_2^n, q_2^s\}$ to serve both consumer types or $\frac{1}{2}\max\{q_2^n, q_2^s\}$ to serve only one consumer type. Comparing the payoffs under these two pricing strategies leads to the following seller's optimal payoff under non-disclosure:

$$\pi_{nd}^{mix} = \begin{cases} \frac{1}{4} + m\frac{\alpha}{8(1+\alpha)}, & \text{if } q \leq \frac{\alpha}{2(1+\alpha)}; \\ \frac{1}{4} + m\frac{q}{2}, & \text{if } \frac{\alpha}{2(1+\alpha)} < q < \frac{\alpha}{1+2\alpha}; \\ \frac{1}{4} + m((1+\alpha)q - \frac{\alpha}{2}), & \text{if } \frac{\alpha}{1+2\alpha} \leq q \leq \frac{1}{2}; \\ \frac{1}{4} + mq, & \text{if } \frac{1}{2} < q \leq \frac{2+\alpha}{2(1+\alpha)}; \\ \frac{1}{4} + m\frac{2+\alpha}{2(1+\alpha)}, & \text{otherwise.} \end{cases}$$

We now compare the seller's payoffs under non-disclosure π_{nd}^{mix} with that under disclosure π_d^{mix}. The seller chooses non-disclosure only when $\pi_{nd}^{mix} > \pi_d^{mix}$. Then, we have the following results:

(1). When $q > \frac{2+\alpha}{2(1+\alpha)}$, $\pi_{nd}^{mix} - \pi_d^{mix} = \left[\frac{1}{4} + m\frac{2+\alpha}{2(1+\alpha)} - (1+m)q\right]_{q > \frac{2+\alpha}{2(1+\alpha)}} < 0$, the seller should choose disclosure.

(2). When $\frac{1}{2} < q \le \frac{2+\alpha}{2(1+\alpha)}$, $\pi_{nd}^{mix} - \pi_{d}^{mix} = \left[\frac{1}{4} + mq - (1+m)q\right]_{q>\frac{1}{2}} < 0$, the seller should choose disclosure.

(3). When $\frac{\alpha}{1+2\alpha} \le q \le \frac{1}{2}$, $\pi_{nd}^{mix} - \pi_{d}^{mix} = m\alpha q - \frac{1}{2}m\alpha - q + \frac{1}{4}$. If $2\alpha + 2m\alpha - 1 > 0$, $\pi_{nd}^{mix} - \pi_{d}^{mix} < 0$ always holds and thus the seller should choose disclosure. However, if $2\alpha + 2m\alpha - 1 < 0$, we have $\pi_{nd}^{mix} - \pi_{d}^{mix} > 0$ when $\frac{\alpha}{1+2\alpha} \le q < \frac{2m\alpha-1}{4(m\alpha-1)}$ and the seller should choose non-disclosure while $\pi_{nd}^{mix} - \pi_{d}^{mix} < 0$ when $\frac{2m\alpha-1}{4(m\alpha-1)} < q \le \frac{1}{2}$ and the seller should choose disclosure.

(4). When $\frac{\alpha}{2(1+\alpha)} < q < \frac{\alpha}{1+2\alpha}$, $\pi_{nd}^{mix} - \pi_{d}^{mix} = \frac{1}{4} - \frac{1}{2}mq - q$. If $2\alpha + 2m\alpha - 1 < 0$, $\pi_{nd}^{mix} - \pi_{d}^{mix} > 0$ always holds and thus the seller should choose non-disclosure. If $\alpha + m\alpha - 1 < 0 < 2\alpha + 2m\alpha - 1$, $\pi_{nd}^{mix} - \pi_{d}^{mix} > 0$ when $\frac{\alpha}{2(1+\alpha)} < q < \frac{1}{2(2+m)}$ and the seller should choose non-disclosure while $\pi_{nd}^{mix} - \pi_{d}^{mix} < 0$ when $\frac{1}{2(2+m)} < q < \frac{\alpha}{1+2\alpha}$ and the seller should choose disclosure. If $\alpha + m\alpha - 1 > 0$, $\pi_{nd}^{mix} - \pi_{d}^{mix} < 0$ always holds and thus the seller should choose disclosure.

(5). When $q \le \frac{\alpha}{2(1+\alpha)}$, $\pi_{nd}^{mix} - \pi_{d}^{mix} = \frac{1}{4} + m\frac{\alpha}{8(1+\alpha)} - (1+m)q$. Because $2\alpha + 3m\alpha - 2 < 0$, we have $\pi_{nd}^{mix} - \pi_{d}^{mix} > 0$ always holds so that the seller should choose non-disclosure.

Second, differently, if $\alpha \ge \frac{2}{2+3m}$ and the seller chooses non-disclosure upfront, we have $\frac{\gamma}{m+m\gamma+2} = \frac{1}{4+3m} < \frac{\alpha}{2(1+\alpha)}$ and $q_l = \frac{\gamma}{m+m\gamma+2} = \frac{1}{4+3m}$. Then, the naive early-adopters' AQR R^{mix} and the quality expectations of naive and sophisticated followers are given in the following table.

Quality level	AQR R^{mix}	q_2^n	q_2^s
$q < \frac{\alpha}{2(1+\alpha)}$	0	0	$\frac{1}{2(4+3m)}$
$\frac{\alpha}{2(1+\alpha)} < q < \frac{2+\alpha}{2(1+\alpha)}$	$(1+\alpha)q - \frac{\alpha}{2}$	$(1+\alpha)q - \frac{\alpha}{2}$	q
$\frac{2+\alpha}{2(1+\alpha)} < q < 1$	1	1	$\frac{2+\alpha}{2(1+\alpha)}$

Then, we have

$$\pi_{nd}^{mix} = \begin{cases} \frac{1}{4} + m\frac{1}{4(4+3m)}, & \text{if } q \le \frac{\alpha}{2(1+\alpha)}; \\ \frac{1}{4} + m\frac{q}{2}, & \text{if } \frac{\alpha}{2(1+\alpha)} < q < \frac{\alpha}{1+2\alpha}; \\ \frac{1}{4} + m((1+\alpha)q - \frac{\alpha}{2}), & \text{if } \frac{\alpha}{1+2\alpha} \le q \le \frac{1}{2}; \\ \frac{1}{4} + mq, & \text{if } \frac{1}{2} < q \le \frac{2+\alpha}{2(1+\alpha)}; \\ \frac{1}{4} + m\frac{2+\alpha}{2(1+\alpha)}, & \text{otherwise.} \end{cases}$$

The comparison analysis is very similar to that when $\alpha < \frac{2}{2+3m}$. The only difference is that now $2\alpha + 2m\alpha - 1 > 0$ always holds as $\alpha \geq \frac{2}{2+3m}$. Thus, we have:

(1). When $\frac{\alpha}{1+2\alpha} \leq q \leq \frac{1}{2}$, $\pi_{nd}^{mix} - \pi_d^{mix} < 0$ always holds and the seller should choose disclosure.

(2). When $\frac{\alpha}{2(1+\alpha)} < q < \frac{\alpha}{1+2\alpha}$, $\pi_{nd}^{mix} - \pi_d^{mix} = \frac{1}{4} - \frac{1}{2}mq - q$. If $\alpha + m\alpha - 1 < 0$, $\pi_{nd}^{mix} - \pi_d^{mix} > 0$ when $\frac{\alpha}{2(1+\alpha)} < q < \frac{1}{2(2+m)}$ and the seller should choose non-disclosure while $\pi_{nd}^{mix} - \pi_d^{mix} < 0$ when $\frac{1}{2(2+m)} < q < \frac{\alpha}{1+2\alpha}$ and the seller should choose disclosure. If $\alpha + m\alpha - 1 > 0$, $\pi_{nd}^{mix} - \pi_d^{mix} < 0$ always holds and thus the seller should choose disclosure.

(3). When $q \leq \frac{\alpha}{2(1+\alpha)}$, $\pi_{nd}^{mix} - \pi_d^{mix} = \frac{1}{4} + m\frac{1}{4(4+3m)} - (1+m)q$. Thus, $\pi_{nd}^{mix} > \pi_d^{mix}$ when $q < \frac{1}{4+3m}$. Otherwise, if $\frac{1}{4+3m} < q < \frac{\alpha}{2(1+\alpha)}$, $\pi_{nd}^{mix} < \pi_d^{mix}$ and the seller is better off disclosing the quality information.

Combining all the above cases leads to the equilibrium disclosure strategy stated in Proposition 3. □

Proof of Proposition 4: We compare the seller's payoffs under two disclosure options. Note that if the seller chooses disclosure upfront, his payoff $\pi_d^{nn} = (1+m)q$. Replacing $q = \frac{q_r^n + \alpha/2}{1+\alpha}$, we can obtain that

$$\pi_d^{nn} = \frac{q_r^n + \alpha/2}{1+\alpha}(1+m).$$

Based on the above, for the gap between two disclosure options, we have

$$\begin{aligned} \text{if } q_r^n &> \frac{1}{2}, \ \pi_d^{nn} - \pi_{nd}^{nn} = \frac{(2q_r^n - 1)(2mq_r^n - 5m\alpha - m + 2m\alpha q_r^n + 4)}{8(1+\alpha)}; \\ \text{if } q_r^n &< \frac{1}{2}, \ \pi_d^{nn} - \pi_{nd}^{nn} = \frac{(1 - 2q_r^n)(3m\alpha - m + 2mq_r^n + 2m\alpha q_r^n - 4)}{8(1+\alpha)}. \end{aligned}$$

When $q_r^n > \frac{1}{2}$, $2q_r^n - 1 > 0$. Note that $2mq_r^n - 5m\alpha - m + 2m\alpha q_r^n + 4 > 0$ if $q_r^n > \frac{m+5m\alpha-4}{2m(1+\alpha)}$. Thus, $\pi_d^{nn} - \pi_{nd}^{nn} < 0$ when $q_r^n \in (\frac{1}{2}, \frac{m+5m\alpha-4}{2m(1+\alpha)})$ but $\pi_d^{nn} - \pi_{nd}^{nn} > 0$ when $q_r^n \in (\frac{m+5m\alpha-4}{2m(1+\alpha)}, 1 + \frac{\alpha}{2}]$, where the upper bound of q_r^n is attained at $q = 1$. Also, note that the existence of the range $(\frac{1}{2}, \frac{m+5m\alpha-4}{2m(1+\alpha)})$ requires that $m\alpha \geq 1$. Therefore, if $m\alpha < 1$, $\pi_d^{nn} - \pi_{nd}^{nn} > 0$ for all $q_r^n > \frac{1}{2}$ (or $q > \frac{1}{2}$).

When $q_r^n < \frac{1}{2}$, $1 - 2q_r^n > 0$. Note that $3m\alpha - m + 2mq_r^n + 2m\alpha q_r^n - 4 > 0$ if $q_r^n > \frac{m-3m\alpha+4}{2m(1+\alpha)}$. Thus, $\pi_d^{nn} - \pi_{nd}^{nn} > 0$ when $q_r^n \in (\frac{m-3m\alpha+4}{2m(1+\alpha)}, \frac{1}{2})$ but $\pi_d^{nn} - \pi_{nd}^{nn} < 0$ when $q_r^n \in (-\frac{\alpha}{2}, \frac{m-3m\alpha+4}{2m(1+\alpha)})$, where the lower bound of q_r^n is attained at $q = 0$. Also, note that the existence of the range $(\frac{m-3m\alpha+4}{2m(1+\alpha)}, \frac{1}{2})$ requires that $m\alpha \geq 1$. Therefore, if $m\alpha < 1$, $\pi_d^{nn} - \pi_{nd}^{nn} < 0$ for all $q_r^n > \frac{1}{2}$.

Finally, we can replace q_r^n by q according to $q = \frac{q_r^n + \alpha/2}{1+\alpha}$. The seller chooses disclosure (non-disclosure) only if $\pi_d^{nn} - \pi_{nd}^{nn} > 0$ ($\pi_d^{nn} - \pi_{nd}^{nn} < 0$). This completes our proof for Proposition 4. □

Proof of Proposition 5: Note that the consumers are naive and the seller chooses non-disclosure. The early adopter's quality review would be $q_r^n = (1+\alpha)q - \frac{\alpha}{2}$, which subsequently sets the follower's quality expectation, $q_2^n = (1+\alpha)q - \frac{\alpha}{2}$. Thus, the seller's equilibrium prices are $p_1 = \frac{1}{2}$ and $p_2 = \max\{0, q_2^n\}$ and the payoff under non-disclosure is

$$\pi_{nd}^{nn} = \begin{cases} \dfrac{1}{2}, & \text{if } q \leq \dfrac{\alpha}{2(1+\alpha)}; \\ \\ \dfrac{1}{2} + m\left((1+\alpha)q - \dfrac{\alpha}{2}\right), & \text{if } \dfrac{\alpha}{2(1+\alpha)} < q \leq 1. \end{cases}$$

The seller's equilibrium prices and payoff under disclosure remain the same as the baseline model, where $\pi_d^{nn} = (1+m)q$. Thus, we can compare the seller's payoffs under two disclosure options to identify the equilibrium disclosure strategy, where

$$\pi_d^{nn} - \pi_{nd}^{nn} = \begin{cases} (1+m)q - \dfrac{1}{2}, & \text{if } q \leq \dfrac{\alpha}{2(1+\alpha)}; \\ \\ -\dfrac{1}{2}\left(2q-1\right)\left(m\alpha - 1\right), & \text{if } \dfrac{\alpha}{2(1+\alpha)} < q \leq 1. \end{cases}$$

Note that if $m\alpha < 1$, $\left[(1+m)q - \frac{1}{2}\right]\Big|_{q=\frac{\alpha}{2(1+\alpha)}} = \frac{1}{2(1+\alpha)}\left(m\alpha - 1\right) < 0$, and the seller chooses disclosure when $q > \frac{1}{2}$. While if $m\alpha \geq 1$, $(1+m)q - \frac{1}{2} > 0$ when $q \in (\frac{1}{2(1+m)}, \frac{\alpha}{2(1+\alpha)}]$ and $-\frac{1}{2}\left(2q-1\right)\left(m\alpha - 1\right) > 0$ when $q < \frac{1}{2}$. Thus, the seller chooses disclosure when $q \in (\frac{1}{2(1+m)}, \frac{1}{2})$. □

Appendix B: Supplementary Discussion

SA Naive Early Adopter and Sophisticated Follower

In the baseline model, we have investigated two scenarios where customers are of the same type, either naive or sophisticated, across two periods. Here, we further consider the scenarios in which the consumer type changes across the periods. Specifically, two new scenarios will be examined: one, the naive early adopter and the sophisticated follower; two, the sophisticated early adopter and the naive follower. We will show that our central finding that the seller can strategically withhold the quality information to manipulate the reference-dependent early adopter's subjective quality review also holds under these two scenarios.

First, we consider that the early adopter is naive while the follower is sophisticated (named the *ns* scenario). In comparison to that in §4.1, now the follower is able to rationally infer product quality after observing the seller's disclosure behavior and the early adopter's SQA, q_r^n. It is evident that if the seller discloses quality information upfront, consumers in both periods know the true product quality q. Therefore, the seller's equilibrium price and ex-post payoff remain the same as that stated in §4.1, that is, $p_1^d = p_2^d = q$ and $\pi_d^{ns} = (1+m)q$.

However, if the seller withholds the quality information, the naive early adopter still believes that the product quality is uniformly distributed over the range $[0,1]$. Thus, her quality expectation $q_1^n = \frac{1}{2}$, and she buys the product only if $q_1^n \geq p_1$. This leads to the seller's optimal first-period price $p_1^{nd} = \frac{1}{2}$. After consumption, the early adopter generates her SQA as follows:

$$q_r^n = \min\left\{\left((1+\alpha)q - \frac{\alpha}{2}\right)^+, 1\right\}.$$

This biased review is publicly announced at the beginning of the second period, and the sophisticated follower then makes her rational inference of the true product quality based on the following *two rationales*. One, the follower knows that the review q_r^n is biased due to the presence of the reference-dependent preference and the reference quality level is $\frac{1}{2}$, and she would rectify this biased review. For example, if the follower observes a review rating $0 < q_r^n < 1$, she then forms that $(1+\alpha)q - \frac{\alpha}{2} = q_r^n$ and backward infers that $q = \frac{2q_r^n + \alpha}{2(1+\alpha)}$. Two, when the review reaches the boundary (e.g., $q_r^n = 0$ or $q_r^n = 1$) that the follower fails to infer the exact quality level, she then further infer the quality based on the rational that under such a circumstance, the seller is better off withholding his private quality information; otherwise, the seller should have already disclosed such information upfront. The following lemma summarizes the follower's quality inference result.

Lemma S1. *When the early adopter is naive and the follower is sophisticated, if the seller adopts the non-disclosure strategy, in equilibrium,*

(1) if the early adopter's SQA $0 < q_r^n < 1$*, the follower can rationally infer the true quality level; that is,* $q_2^s = q$*.*

(2) if the early adopter's SQA $q_r^n = 0$*, the follower believes that the true quality level is uniformly distributed over the range* $[0, q_l]$*, where* $q_l = \min\left\{\frac{1}{2+m}, \frac{\alpha}{2(1+\alpha)}\right\}$ *is the cutoff point that the seller is indifferent between disclosing and withholding his quality information.*

(3) if the early adopter's SQA $q_r^n = 1$*, the follower believes that the true product quality level is* $\frac{2+\alpha}{2(1+\alpha)}$*.*

Lemma S1 indicates the sophisticated follower's rational quality inference process after observing the naive consumer's quality review. This process is essentially similar to that of Lemma 2, where the seller would only serve naive consumers in the first period when withholding the quality information. In particular, when $0 < q_r^n < 1$ the sophisticated follower can exactly infer the true product quality. Nonetheless, the follower is unable to infer the exact quality level when the SQA of the naive early adopter reaches the lower bound, $q_r^n = 0$. As the reference effect of the early adopter is restrained by the bounds of the quality assessment, it hampers the follower's inference on the exact true quality.

Given the follower's quality expectation stated in Lemma S1, the non-disclosure seller's optimal price in the second period can be derived as $p_2^{nd} = q_2^s$. Then, his ex-post payoff under non-disclosure can be derived as

$$\pi_{nd}^{ns} = \begin{cases} \frac{1}{2} + m\frac{q_l}{2}, & \text{if } q \leq q_l; \\ \frac{1}{2} + mq, & \text{if } q_l < q \leq \frac{2+\alpha}{2(1+\alpha)}; \\ \frac{1}{2} + m\frac{2+\alpha}{2(1+\alpha)}, & \text{otherwise.} \end{cases}$$

We then compare the seller's payoff under non-disclosure π_{nd}^{ns} with that under disclosure π_d^{ns} to derive the seller's equilibrium disclosure strategy.

Proposition S1. *When the early adopter is naive but the follower is sophisticated, in equilibrium,*

(1) if $m\alpha < 2$*, the seller voluntarily discloses his private quality information when the true product quality* $q > \frac{1}{2}$*; otherwise, he prefers non-disclosure.*

(2) *if $m\alpha \geq 2$, the seller voluntarily discloses his private quality information when the true product quality $q \in \left(\frac{1}{2+m}, \frac{\alpha}{2(1+\alpha)}\right] \cup \left(\frac{1}{2}, 1\right]$; otherwise, he prefers non-disclosure.*

Proposition S1 shows that the seller's disclosure strategy when facing the naive early adopter and the sophisticated follower is radically different from that stated in Proposition 1 when consumers are all naive; see Figure S1 for the illustration. Now, the seller always discloses his product quality when it is above $\frac{1}{2}$; while where the consumers are all naive, the seller may deliberately withhold such information. This is driven by the fact that a sophisticated follower can rationally infer the true product quality based on the early adopter's biased review. Therefore, she is no longer fooled by the reference-effect-induced extremely high quality assessment. This subsequently leaves no incentive for the seller to withhold his high quality information, and he would prefer disclosing it upfront so as to earn more surplus from the naive early adopter. Interestingly, Proposition S1 indicates that the seller would actively disclose some low quality information ($\frac{1}{2+m} < q \leq \frac{\alpha}{2(1+\alpha)}$) when the reference effect is strong ($m\alpha \geq 2$). In this quality region, if the seller chooses non-disclosure, the reference effect would induce the early consumer's SQA to be zero ($q_r^s = 0$). This not only hampers the sophisticated follower's quality inference but also lowers her quality expectation relative to the true one. Thus, the seller has no choice but to disclose such low quality upfront. Overall, we can conclude that if the follower is sophisticated, the impact of reference effect is substantially alleviated by the consumer's rational inference in the second period.

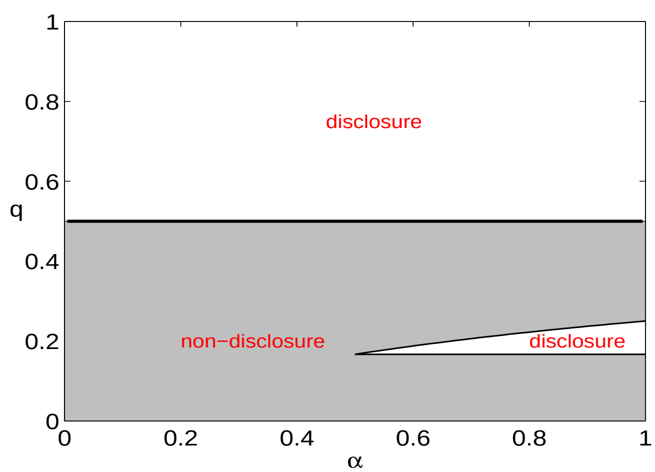

Figure S1: Equilibrium disclosure strategy when the early adopter is naive and the follower is sophisticated: $m = 4$.

Next, we examine how the reference effect affects the seller's ex-ante expected payoff. Again, the seller's ex-ante expected payoff function takes two forms depending on the magnitude of reference effect, where

$$\Pi^{ns}_{m\alpha<2} = \underbrace{\int_0^{\frac{1}{2}} \left(\frac{1}{2} + \frac{m}{4}\right) dq}_{\text{Non-disclosure}} + \underbrace{\int_{\frac{1}{2}}^1 (1+m)qdq}_{\text{Disclosure}}; \tag{S1}$$

$$\Pi^{ns}_{m\alpha\geq 2} = \underbrace{\int_0^{\frac{1}{2+m}} \left(\frac{1}{2} + m\frac{\alpha}{4(1+\alpha)}\right) dq + \int_{\frac{\alpha}{2(1+\alpha)}}^{\frac{1}{2}} \left(\frac{1}{2} + mq\right) dq}_{\text{Non-disclosure}} + \underbrace{\int_{\frac{1}{2+m}}^{\frac{\alpha}{2(1+\alpha)}} (1+m)qdq + \int_{\frac{1}{2}}^1 (1+m)qdq}_{\text{Disclosure}}. \tag{S2}$$

Corollary S1. *When the early adopter is naive but the follower is sophisticated, in equilibrium, the seller's ex-ante expected payoff is independent of the reference-dependent parameter α when $\alpha < \frac{2}{m}$ and decreases in α otherwise.*

Corollary S1 shows that when the follower is sophisticated, the existence of reference-dependent preference has no impact on the seller's ex-ante payoff when its magnitude is small and hurts the seller's ex-ante payoff when its magnitude is large, a result dramatically different from that when all consumers are naive as stated in Corollary 1. This implies that the overall impact of the consumer's reference-dependent preference critically hinges on the consumers' types in these two periods. Note that when the follower is sophisticated, she can make the rational inference about the true quality based on the early-arriving consumer's SQA and the seller's disclosure behavior. Thus, the seller is unable to utilize the reference effect to manipulate the follower's quality expectation to derive more surplus. Moreover, in certain conditions, the seller needs to announce his low quality level upfront to avoid the consequence that the early adopter's reference-dependent quality review is so low that it pulls down the follower's quality expectation. This downside of reference effect becomes stronger when the magnitude of reference effect becomes larger, because the seller has to reveal more low quality information upfront under such a circumstance.

SB Sophisticated Early Adopter and Naive Follower

In this subsection, we investigate the scenario where the early adopter is sophisticated but the follower is naive (named the *sn* scenario). Under such a circumstance, the early adopter first rationally infers the product quality after observing the seller's disclosure behavior, and then makes the corresponding purchasing decision. Without loss of generality, we assume that the early adopter generates a quality expectation $q_1^s = \widetilde{q} := (q, \overline{q})$, wherein $\widetilde{q} = q$ if the seller discloses his quality upfront and $\widetilde{q} = \overline{q}$ if the seller withholds his quality information. After consumption, the early adopter forms an reference-dependent SQA as follows:

$$q_r^s = \min\left\{ \left((1+\alpha)q - \alpha\widetilde{q}\right)^+, 1 \right\}.$$

This assessment is released to the naive follower at the beginning of the second period, which becomes her quality expectation, i.e., $q_2^n = q_r^s$.

We now derive the seller's equilibrium pricing and ex-post payoffs under both disclosure and non-disclosure strategies. Again, it is evident that if the seller chooses disclosure, his optimal prices are $p_1^d = p_2^d = q$ and the corresponding ex-post payoff is $\pi_d^{sn} = (1+m)q$. However, if the seller withholds his quality information upfront, his first-period retail price p_1^{nd} shall be equal to the sophisticated early adopter's quality expectation $\overline{q}$, whose value will be rigorously derived later. For the naive follower, her quality expectation equals the early adopter's SQA. Consequently, the seller's optimal second-period price shall be $p_2^{nd} = q_r^s$. Then, the seller's ex-post payoff after observing his quality level q can be written as

$$\pi_{nd}^{sn} = \begin{cases} \overline{q}, & \text{if } q \le \dfrac{\alpha\overline{q}}{1+\alpha}; \\ m(1+\alpha)q + \overline{q}(1-m\alpha), & \text{if } \dfrac{\alpha\overline{q}}{1+\alpha} < q \le \dfrac{1+\alpha\overline{q}}{1+\alpha}; \\ \overline{q} + m, & \text{otherwise.} \end{cases} \tag{S3}$$

From (S3), we know that if the seller withholds his quality information but the true product quality is far below the early adopter's quality expectation, i.e., $q \le \frac{\alpha\overline{q}}{1+\alpha}$, the early adopter's reference-dependent SQA after consumption reaches the lower bound 0, and the seller cannot extract any surplus from the follower. Meanwhile, if the true quality level is far above the early adopter's quality expectation, i.e., $q > \frac{1+\alpha\overline{q}}{1+\alpha}$, the early adopter's SQA reaches the upper bound 1, and the seller obtains the maximum second-period payoff. Comparing the seller's payoffs under the two disclosure options, π_d^{sn} and π_{nd}^{sn}, leads to the following result.

Proposition S2. *When the early adopter is sophisticated but the follower is naive, in equilibrium,*

(1) if $m\alpha < 1$, the seller always discloses his private quality information at the beginning of the first period.

(2) if $m\alpha \ge 1$, the seller discloses his quality information when the true product quality $q \in \left(\frac{\overline{q}}{1+m}, \overline{q}\right) \cup \left(\frac{\overline{q}+m}{1+m}, 1\right]$, where

$$\overline{q} = \frac{m}{m+\sqrt{1+2m}}.$$

Proposition S2 implies that when the early adopter is sophisticated and the follower is naive, the seller can tailor-make his disclosure strategy only when the magnitude of reference effect is large; see Figure S2. As the naive follower believes in the early adopter's SQA, the seller now can make his information disclosure decision by utilizing the reference effect to extract more surplus from the follower. To be specific, when the magnitude of reference effect is large ($m\alpha \geq 1$), the seller can deliberately withhold the relatively high quality information $q \in \left(\frac{\bar{q}}{1+m}, \bar{q}\right)$ but disclose the relatively low quality information $q \in \left(\frac{\bar{q}}{1+m}, \bar{q}\right)$ upfront to adjust the early adopter's SQA so as to derive more surplus in the second period. This disclosure strategy is structurally similar to that discussed in §4.1 where the early adopter is also naive. Likewise, the crux here is the tradeoff between the gain from boosting the reference-dependent SQA via information withholding and the loss from information withholding.

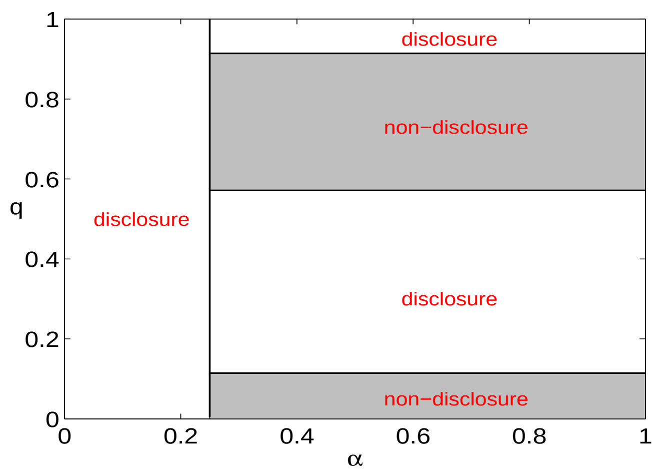

Figure S2: Equilibrium disclosure strategy with the sophisticated early adopter and the naive follower: $m = 4$.

Nonetheless, when the magnitude of reference effect is small ($m\alpha < 1$), the seller always discloses his quality information to the sophisticated early adopter upfront. This is structurally different from that in §4.1 where all consumers are naive. This is because the sophisticated early adopter is smart enough to infer the product quality after observing the seller's disclosure strategy rather than hold a naive and fixed quality expectation at the point $\frac{1}{2}$. Thus, if the sophisticated early adopter anticipates that the seller should disclose his high quality information upfront to extract more surplus in the first period, then any non-disclosure behavior leads to a negative quality inference from the early adopter. Because disclosure is costless, in equilibrium the seller should always disclose his quality information that is above the early adopter's quality expectation. This ultimately pushes the sophisticated early adopter's quality expectation upon non-disclosure to be zero. Consequently, the disclosure strategy is always adopted by the seller when the reference effect is low.

It is worth mentioning how the early adopter forms her quality inference when the reference effect is strong and the seller withholds the quality information upfront. Under such a circumstance, the early adopter can infer that the seller's true quality should be either low ($0 < q < \frac{\overline{q}}{1+m}$) or relatively high ($\overline{q} < q < \frac{\overline{q}+m}{1+m}$), leading to her quality expectation that is consistent with her inference:

$$\overline{q}\left(\frac{\overline{q}+m}{1+m} - \overline{q} + \frac{\overline{q}}{1+m}\right) = \int_0^{\frac{\overline{q}}{1+m}} q dq + \int_{\overline{q}}^{\frac{\overline{q}+m}{1+m}} q dq.$$

Moreover, it is evident that the early adopter's quality expectation upon non-disclosure only increases in the second-period market size ($\frac{\partial \overline{q}}{\partial m} > 0$). When it increases, the seller has a stronger incentive to withhold his high quality information and leads to a higher quality expectation $\overline{q}$.

We can derive the seller's ex-ante expected payoff under this scenario as follows, which depends on the magnitude of reference effect:

$$\Pi^{sn}_{m\alpha<1} = \underbrace{\int_0^1 (1+m) q dq;}_{\text{Disclosure}} \tag{S4}$$

$$\Pi^{sn}_{m\alpha\geq 1} = \underbrace{\int_0^{\frac{\overline{q}}{1+m}} \overline{q} dq + \int_{\overline{q}}^{\frac{\overline{q}+m}{1+m}} \left(m(1+\alpha)q + \overline{q}(1-m\alpha)\right) dq}_{\text{Non-disclosure}} + \underbrace{\int_{\frac{\overline{q}}{1+m}}^{\overline{q}} (1+m) q dq + \int_{\frac{\overline{q}+m}{1+m}}^{1} (1+m) q dq.}_{\text{Disclosure}} \tag{S5}$$

Corollary S2. *When the early adopter is sophisticated but the follower is naive, in equilibrium, the seller's ex-ante expected payoff is independent of the reference-dependent parameter α when $\alpha < \frac{1}{m}$ and increases in α otherwise.*

Corollary S2 shows that reference effect is always (weakly) beneficial to the seller in terms of the expected payoff when the early adopter is sophisticated and the follower is naive. This result is in striking contrast to that of Corollary S1 where the early adopter is naive and the follower is sophisticated. The underlying reason here is that if the follower is naive, the seller can manipulate the early adopter's SQA and thus the follower's quality expectation by adjusting his disclosure strategy. However, if the follower is sophisticated, the seller can hardly uplift the follower's quality expectation via utilizing the reference effect. Thus, if the magnitude of reference effect becomes larger, the benefit from strategically manipulating consumers' review (via the disclosure strategy) also increases.

Corollary S2 together with Corollary S1 indicate that the impact of reference-dependent preference on the seller's performance is affected by the specific arriving sequence of consumer types in the two periods, i.e., which type of consumers arrives first.

Proof of Lemma S1: The proof procedure is similar to that of Proposition 2 and Lemma 2. So we omit its details. □

Proof of Proposition S1: We first derive the gap between the seller's payoffs under two disclosure options when facing the naive early adopter and sophisticated follower. Note that $q_l = \frac{\alpha}{2(1+\alpha)}$ if $m\alpha < 2$ and $q_l = \frac{1}{2+m}$ if $m\alpha \geq 2$. First, if $m\alpha < 2$ and $q_l = \frac{\alpha}{2(1+\alpha)}$, we have

$$\pi_d^{ns} - \pi_{nd}^{ns} = \begin{cases} (1+m)q - \frac{1}{2} - m\frac{\alpha}{4(1+\alpha)}, & \text{if } q \leq \frac{\alpha}{2(1+\alpha)}; \\ q - \frac{1}{2}, & \text{if } \frac{\alpha}{2(1+\alpha)} < q \leq \frac{2+\alpha}{2(1+\alpha)}; \\ (1+m)q - \frac{1}{2} - m\frac{2+\alpha}{2(1+\alpha)}, & \text{otherwise.} \end{cases}$$

Then, when $q \in [0, \frac{\alpha}{2(1+\alpha)}]$, $\Pi_d^{ns} - \Pi_{nd}^{ns} \leq \left[(1+m)q - \frac{1}{2} - m\frac{\alpha}{4(1+\alpha)}\right]\Big|_{q=\frac{\alpha}{2(1+\alpha)}} = \frac{1}{4(1+\alpha)}(m\alpha - 2) < 0$. When $q \in (\frac{\alpha}{2(1+\alpha)}, \frac{2+\alpha}{2(1+\alpha)}]$, $\pi_d^{ns} - \pi_{nd}^{ns} > 0$ if $q \in (\frac{1}{2}, \frac{2+\alpha}{2(1+\alpha)}]$ and $\pi_d^{ns} - \pi_{nd}^{ns} \leq 0$ if $q \in [\frac{\alpha}{2(1+\alpha)}, \frac{1}{2}]$. When $q \in (\frac{2+\alpha}{2(1+\alpha)}, 1]$, $\Pi_d^{ns} - \Pi_{nd}^{ns} \geq \left[(1+m)q - \left(\frac{1}{2} + m\frac{2+\alpha}{2(1+\alpha)}\right)\right]\Big|_{q=\frac{2+\alpha}{2(1+\alpha)}} = \frac{1}{2(1+\alpha)} > 0$. Combining them together, it is evident that the seller can obtain a higher payoff from disclosure only if $q > \frac{1}{2}$. Otherwise, the seller should withhold the quality information.

Second, if $m\alpha \geq 2$ and $q_l = \frac{1}{2+m}$, we have

$$\pi_d^{ns} - \pi_{nd}^{ns} = \begin{cases} (1+m)q - \frac{1}{2} - m\frac{1}{2(2+m)}, & \text{if } q \leq \frac{\alpha}{2(1+\alpha)}; \\ q - \frac{1}{2}, & \text{if } \frac{\alpha}{2(1+\alpha)} < q \leq \frac{2+\alpha}{2(1+\alpha)}; \\ (1+m)q - \frac{1}{2} - m\frac{2+\alpha}{2(1+\alpha)}, & \text{otherwise.} \end{cases}$$

Thus, the only difference compared with the first case arises in the region of $q \in [0, \frac{\alpha}{2(1+\alpha)}]$, in which $\pi_d^{ns} - \pi_{nd}^{ns} > 0$ when $q \in (\frac{1}{2+m}, \frac{\alpha}{2(1+\alpha)}]$ and $\pi_d^{ns} - \pi_{nd}^{ns} \leq 0$ when $q \in [0, \frac{1}{2+m}]$. Combining all the cases together, we show that now the seller should choose disclosure when $q \in (\frac{1}{2+m}, \frac{\alpha}{2(1+\alpha)}] \cup (\frac{1}{2}, 1]$. Otherwise, the seller should remain silent. □

Proof of Corollary S1: The results can be derived by obtaining the first-order conditions of equations (S1) and (S2) with respect to α. Note that when $m\alpha < 2$, $\Pi_{m\alpha<2}^{ns}$ can be simplified to $\Pi^{ns}_{m\alpha<2} = \frac{1}{2}m + \frac{5}{8}$, which is independent of α. When $m\alpha \geq 2$, $\Pi^{ns}_{m\alpha\geq 2}$ can be simplified to

$$\Pi^{ns}_{m\alpha\geq 2} = \frac{\left(4m^3\alpha^2 + 8m^3\alpha + 4m^3 + 22m^2\alpha^2 + 42m^2\alpha + 21m^2 + 36m\alpha^2 + 68m\alpha + 36m + 20\alpha^2 + 40\alpha + 24\right)}{8\left(1+\alpha\right)^2\left(2+m\right)^2}.$$

Thus, $\frac{\partial \Pi^{ns}_{m\alpha\geq 2}}{\partial \alpha} = \frac{m\alpha - 2}{4(1+\alpha)^3(2+m)} \geq 0$ when $m\alpha \geq 2$. □

Proof of Proposition S2: The seller's payoff with disclosure is $\Pi^{sn}_d = (1+m)q$. Thus, we present the payoff gap between two disclosure options, where

$$\pi^{sn}_d - \pi^{sn}_{nd} = \begin{cases} (1+m)q - \bar{q}, & \text{if } q \leq \dfrac{\alpha\bar{q}}{1+\alpha}; \\ (q-\bar{q})\left(1 - m\alpha\right), & \text{if } \dfrac{\alpha\bar{q}}{1+\alpha} < q \leq \dfrac{1+\alpha\bar{q}}{1+\alpha}; \\ (1+m)q - (\bar{q}+m), & \text{otherwise.} \end{cases}$$

Accordingly, we can identify the seller's disclosure strategy in each region:

(1) $q \leq \frac{\alpha\bar{q}}{1+\alpha}$: When $q \in (\frac{\bar{q}}{1+m}, \frac{\alpha\bar{q}}{1+\alpha})$, $\pi^{sn}_d - \pi^{sn}_{nd} > 0$ and the seller chooses disclosure; while when $q \in (0, \frac{\bar{q}}{1+m})$, $\pi^{sn}_d - \pi^{sn}_{nd} < 0$ and the seller chooses non-disclosure. However, the range $q \in (\frac{\bar{q}}{1+m}, \frac{\alpha\bar{q}}{1+\alpha})$ exists only when $m\alpha \geq 1$. Thus, when $m\alpha < 1$, $\pi^{sn}_d - \pi^{sn}_{nd} < 0$ always holds and the seller chooses non-disclosure.

(2) $\frac{\alpha\bar{q}}{1+\alpha} < q \leq \frac{1+\alpha\bar{q}}{1+\alpha}$: If $m\alpha < 1$, $\pi^{sn}_d - \pi^{sn}_{nd} > 0$ and the seller chooses disclosure when $q \in (\bar{q}, \frac{1+\alpha\bar{q}}{1+\alpha})$ while $\pi^{sn}_d - \pi^{sn}_{nd} < 0$ and the seller chooses non-disclosure when $q \in (\frac{\alpha\bar{q}}{1+\alpha}, \bar{q})$. If $m\alpha \geq 1$, $\pi^{sn}_d - \pi^{sn}_{nd} < 0$ and the seller chooses non-disclosure when $q \in (\bar{q}, \frac{1+\alpha\bar{q}}{1+\alpha})$ while $\pi^{sn}_d - \pi^{sn}_{nd} > 0$ and the seller chooses disclosure when $q \in (\frac{\alpha\bar{q}}{1+\alpha}, \bar{q})$.

(3) $q > \frac{1+\alpha\bar{q}}{1+\alpha}$: When $q \in (\frac{1+\alpha\bar{q}}{1+\alpha}, \frac{\bar{q}+m}{1+m})$, $\pi^{sn}_d - \pi^{sn}_{nd} < 0$ and the seller chooses non-disclosure; while when $q \in (\frac{\bar{q}+m}{1+m}, 1]$, $\pi^{sn}_d - \pi^{sn}_{nd} > 0$ and the seller chooses disclosure. Note that the range $(\frac{1+\alpha\bar{q}}{1+\alpha}, \frac{\bar{q}+m}{1+m})$ exists only when $m\alpha \geq 1$. Thus, when $m\alpha < 1$, the seller always chooses disclosure.

Combining the above cases, if $m\alpha < 1$, the seller chooses disclosure when $q > \bar{q}$. Because the early adopter is sophisticated, according to the unraveling theory, in equilibrium, $\bar{q} = 0$. While if $m\alpha \geq 1$, the seller chooses non-disclosure at two quality zones: $(\bar{q}, \frac{\bar{q}+m}{1+m})$ and $(0, \frac{\bar{q}}{1+m})$. Because the quality expectation should be consistent with the ranges of non-disclosure, in equilibrium, we have

$$\frac{1}{2}\left(\left(\frac{\bar{q}+m}{1+m}\right)^2 - \bar{q}^2 + \left(\frac{\bar{q}}{1+m}\right)^2\right) = \bar{q}\left(\frac{\bar{q}+m}{1+m} - \bar{q} + \frac{\bar{q}}{1+m}\right).$$

This leads to $\bar{q} = \frac{m}{m+\sqrt{1+2m}}$. □

Proof of Corollary S2: The results can be derived by obtaining the first-order conditions of equations (S4) and (S5) with respect to α. Note that when $m\alpha < 1$, $\Pi^{sn}_{m\alpha<1}$ is independent of α. When $m\alpha \geq 1$, we can show that $\Pi^{sn}_{m\alpha\geq 1}$ can be simplified and we have

$$\frac{\partial \Pi^{sn}_{m\alpha\geq 1}}{\partial \alpha} = \frac{m^3(1-\overline{q})^2}{2(1+m)^2} > 0.$$

Then, the proof completes. □

SC When the Market Contains Both Consumer Types and $\gamma > \frac{2+\alpha}{2(1+\alpha)}$

In this section, we consider a situation wherein the market contains γ proportion of naive consumers and $1-\gamma$ proportion of sophisticated consumers, where $\gamma \in \left[\frac{2+\alpha}{2(1+\alpha)}, 1\right)$. In particular, we would like to examine whether the seller has incentive to withhold the high quality information to boost the early adopters' subjective review. Again, we need to consider two scenarios depending on whether the seller would only serve naive early adopters or serve both consumers types when withholding the quality information in the first period. After obtaining the equilibrium outcomes in each case, we then compare the seller's optimal profits under the two scenarios to derive the equilibrium disclosure strategy.

We first consider the scenario in which the seller serves naive early adopters only when withholding the quality information. Similar to that in the proof of Proposition 3, we first derive the naive early adopters' AQR R^{mix} and the quality expectations of naive and sophisticated followers, which are summarized in the following table.

Quality level	AQR R^{mix}	q_2^n	q_2^s
$q < \frac{\alpha}{2(1+\alpha)}$	0	0	$\min\left\{\frac{\gamma}{2(m+m\gamma+2)}, \frac{\alpha}{4(1+\alpha)}\right\}$
$\frac{\alpha}{2(1+\alpha)} < q < \frac{2+\alpha}{2(1+\alpha)}$	$(1+\alpha)q - \frac{\alpha}{2}$	$(1+\alpha)q - \frac{\alpha}{2}$	q
$\frac{2+\alpha}{2(1+\alpha)} < q < 1$	1	1	$\frac{2+\alpha}{2(1+\alpha)}$

Again, when the seller adopts non-disclosure, in the second period, he has two possible prices to charge, either $\min\{q_2^n, q_2^s\}$ to serve both consumer types or $\max\{q_2^n, q_2^s\}$ to serve only one consumer type. Comparing the payoffs under these two pricing strategies leads to the following seller's optimal payoff under non-disclosure.

When the reference-dependent parameter $\alpha \in \left(\min\left\{\frac{2(1-\gamma)}{2\gamma-1}, 1\right\}, 1\right]$ (only under which the condition $\gamma > \frac{2+\alpha}{2(1+\alpha)}$ is satisfied), we have

$$\pi_{nd}^{mix} = \begin{cases} \frac{\gamma}{2} + \min\left\{\frac{m\gamma}{4(m+m\gamma+2)}, \frac{m\alpha}{8(1+\alpha)}\right\}, & \text{if } q \leq \frac{\alpha}{2(1+\alpha)}; \\ \frac{\gamma}{2} + m(1-r)q, & \text{if } \frac{\alpha}{2(1+\alpha)} < q < \frac{\alpha r}{2(2r+r\alpha-1)}; \\ \frac{\gamma}{2} + m\left((1+\alpha)q - \frac{\alpha}{2}\right), & \text{if } \frac{\alpha r}{2(2r+r\alpha-1)} \leq q \leq \frac{1}{2}; \\ \frac{\gamma}{2} + mq, & \text{if } \frac{1}{2} < q \leq \frac{\alpha\gamma}{2(r+r\alpha-1)}; \\ \frac{\gamma}{2} + m\gamma\left((1+\alpha)q - \frac{\alpha}{2}\right), & \text{if } \frac{\alpha\gamma}{2(r+r\alpha-1)} < q \leq \frac{2+\alpha}{2(1+\alpha)}; \\ \frac{\gamma}{2} + m\gamma, & \text{otherwise.} \end{cases}$$

A close comparison of the aforementioned result with those stated in the proofs of Lemma 2 and Proposition 3 indicates that the key difference arises only when the product quality is relatively high, i.e, when $q > \frac{1}{2}$. When $\frac{1}{2} < q \leq \frac{2+\alpha}{2(1+\alpha)}$, $q_2^n = (1+\alpha)q - \frac{\alpha}{2} > q_2^s = q$ and the seller's payoff in the second period equals $max\{\gamma((1+\alpha)q - \frac{\alpha}{2}), q\}$. Thus, the seller would only serve naive followers when $q \in [\frac{\alpha\gamma}{2(r+r\alpha-1)}, \frac{2+\alpha}{2(1+\alpha)}]$ while serve both consumer types when $q \in [\frac{1}{2}, \frac{\alpha\gamma}{2(r+r\alpha-1)}]$. When $q > \frac{2+\alpha}{2(1+\alpha)}$, $q_2^n = 1 > q_2^s = \frac{2+\alpha}{2(1+\alpha)}$ and the seller's payoff in the second period equals $max\{\gamma, \frac{2+\alpha}{2(1+\alpha)}\}$. Because $\gamma > \frac{2+\alpha}{2(1+\alpha)}$, the seller would only serve naive followers.

Next, we compare the seller's equilibrium payoffs under disclosure and non-disclosure. Recall that the seller's payoff under disclosure is $\pi_d^{mix} = (1+m)q$. We can show that when the magnitude of reference effect $\alpha \in \left(\min\left\{\frac{2(1-\gamma)}{2\gamma-1}, 1\right\}, 1\right]$, $\pi_{nd}^{mix} > \pi_d^{mix}$ when $\alpha \in \left(\min\left\{\frac{2+2m-\gamma-2m\gamma}{\gamma+2m\gamma-m-1}, 1\right\}, 1\right]$ and $q \in \left[\frac{(m\alpha-1)\gamma}{2m(1+\alpha)\gamma-2(1+m)}, \frac{(1+2m)\gamma}{2(1+m)}\right]$. This implies that when the proportion of naive consumers is sufficiently large and the magnitude of reference effect is also sufficiently high, the seller can be better off withholding the relatively high quality information in a certain quality range. Moreover, one can easily check that as the proportion of naive consumers γ increases, this high-quality-information-nondisclosure zone increases, i.e., $\partial\left(\frac{(1+2m)\gamma}{2(1+m)} - \frac{(m\alpha-1)\gamma}{2m(1+\alpha)\gamma-2(1+m)}\right)/\partial\gamma > 0$ while the required lower bound magnitude regarding the reference effect parameter α decreases, i.e., $\partial(\frac{2+2m-\gamma-2m\gamma}{\gamma+2m\gamma-m-1})/\partial\gamma < 0$. In particular, when $\limsup \gamma = 1$, one can easily find that the seller's optimal disclosure strategy becomes exactly the same as that stated in Section 4.1 (see Proposition 1), in which the seller would withhold the high quality information $q \in \left[\frac{1}{2}, \frac{1+2m}{2(1+m)}\right]$ when $\alpha > \frac{1}{m}$.

For the scenario that the seller serves both consumer types in the first period when withholding the quality information, the analysis becomes very complicated that requires sophisticated consumers' rational quality reference to be consistent with the seller's optimal disclosure strategy. Thus, one may rely on numerical experiments to examine this issue. Similar to the above scenario, it can be verified that when the proportion of naive consumers is sufficiently high, the seller may prefer withholding the high quality information. Under such a circumstance, the seller would only serve naive followers in the second period. In the extreme case that $\gamma \to 1$, the sophisticated consumer can infer that the seller would withhold both the relatively high and extremely low quality information so that her quality expectation upon non-disclosure $\overline{q} \to \frac{1}{2}$. In this case, the seller's optimal disclosure strategy and payoff becomes the same as that discussed in Section 4.1.

Note that we still need to compare the seller's optimal payoffs under the above two scenarios to derive the equilibrium disclosure strategy. However, given that the seller's payoff under disclosure remains unchanged under both scenarios and in the first scenario the seller can obtain a higher payoff by withholding the relatively high quality information, we can claim that in equilibrium the seller would *at least* withhold the high quality information in the range of $\left[\frac{(m\alpha-1)\gamma}{2m(1+\alpha)\gamma-2(1+m)}, \frac{(1+2m)\gamma}{2(1+m)}\right]$ when the reference effect is sufficiently high. This confirms that the seller does have incentive to withhold the high quality information to boost the early adopters' reference-dependent reviews when the market contains both naive and sophisticated consumers and the proportion of naive consumers is relatively large.