Role of Risk Aversion in Price Postponement under Supply Random Yield

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Price postponement is an effective mechanism to hedge against the adverse effect of supply random yield. However, its effectiveness and the resulting production decisions have not been studied for risk averse firms. In this paper, we investigate the impact of price postponement and risk aversion under supply yield risk. Specifically, we study a risk averse monopoly firm's production and pricing decisions under supply random yield with two distinct pricing schemes: (1) ex ante pricing - the firm simultaneously makes the sales price and sourcing decisions before production takes place; and (2) responsive pricing - the pricing decision is postponed until after the production yield realization. We adopt the conditional-value-at-risk (CVaR) as the risk aversion measurement, and investigate the impact of the firm's risk aversion level on its optimal decisions and the corresponding profit. Among other results, we show that for each pricing scheme, there exists a unique risk aversion threshold, under which the firm chooses not to produce. Interestingly, price postponement has no impact on the risk aversion threshold, as the cutoff values under both pricing schemes are the same. We further show that the value of CVaR improvement from responsive pricing may not be monotonic in the firm's risk aversion level. Consequently, our results indicate that although price postponement induces operational flexibility by better matching demand with available supply, whether the firm should adopt responsive pricing needs to be carefully evaluated, as the benefit may not justify the potential fixed cost associated with price postponement, especially for a highly risk averse firm. In addition, we show that responsive pricing, albeit its ex post revenue maximization behavior, benefits the end market consumers in equilibrium. Finally, we conduct extensive numerical studies to check and confirm the robustness of our results.

Key words: random yield, price postponement, risk aversion, consumer surplus

1. Introduction

Supply risk is prevalent in today's global economy and is listed among the top three most significant risks that firms need to battle against (Gyorey et al. 2010). Among various supply risks, production yield risk is a significant one for many industries, such as seminconductor industry, agribusiness, and vaccine manufactuering (see, e.g., Tang and Kouvelis 2014, Deo and Corbett 2009). In recent years, we have witnessed many supply chain shortages caused by yield uncertainty. For example, in December 2010, a cold weather blasted through Florida's key citrus-growing areas, slashed the orange crop, and resulted in a smaller-than-expected crop yield. The orange supply from Florida was down by 12% from the previous year (Sellen 2010). As another example, egg companies and turkey producers suffered substantial losses in the U.S. bird-flu outbreak in 2015. The extermination of 32 million egg laying hens accounted for 10% of the U.S. egg-laying flock. Turkey production took a hit as well, with 2.5% of the nation's flock affected (Gee 2015). Such supply yield risks not only prevail in many industries, but also bring significant financial losses. In the above bird-flu example, Chad Gregory, president of the United Egg Producers, anticipated that some egg companies were unlikely to survive because of the cost of culling animals and having to go months without revenue while facilities were cleaned and repopulated with hens, even though the government had earmarked nearly $400 million to help compensate farmers for culled birds, cleanup and disease testing (Gee 2015). Both the prevalence and the costly consequences of supply uncertainty highlight the importance of supply risk management for researchers and practitioners.

There are many effective risk mitigating mechanisms that can be used to alleviate the adverse effects of supply random yield, with pricing undoubtedly serving as both a revenue generating and risk mitigating tool. Even though a fixed stable pricing strategy may be frequently observed in practice, it is not uncommon when facing supply yield risk, a price-setting firm could postpone the pricing decision after the yield realization to better match demand with supply and maximize the revenue from the available inventory (see, e.g., Li et al. 2013, 2017). For example, Etienne et al. (2017) document that many agri-producers have the option to choose between forward and spot sales of their products, e.g., corn, oranges. While forward sale with sales price determined ex ante upon production is a primary market tool for crop farmers to maintain the stability of their output price, spot sale with output price postponed until after harvest, partially with the purpose to better utilize the available supply in case of shortage, is also commonly seen in practice (Davis et al. 2005, Shao and Wu 2019). For example, when facing Florida's low orange yield in 2010, Tropicana Inc. raised orange juice price by 8% for pure premium orange juice. At the same time, to better utilize the limited supply while keeping its announced retail price unchanged, the company shrank its most popular product, the 64-ounce container, by about 8 percent (The Associated Press 2010).

The above examples clearly illustrate that, when endowed with the power to determine pricing timing, firms may postpone the sales price to partially mitigate the adverse impact of random yield. Price postponement strategies have been mostly discussed for risk neutral firms (Van Mieghem and Dada 1999, Tang and Yin 2007, Li et al. 2017). However, firms, especially small producers and local processors, usually exhibit certain degrees of risk aversion and amplify the concern of low profit realizations when making their operational decisions. Such exhibition of risk aversion towards downside profit shocks may change the firm's production and pricing decisions, and affect the potential adoption and the corresponding value of price postponement. For instance, the adoption of price postponement in the above examples may depend on many factors, such as the characteristics of the firm and its product, and the cost associated with delayed price announcement. We are also interested in understanding at what level of risk aversion, which we refer to as "risk aversion threshold", the firm decides not to produce. The main objective of this paper is to understand the strategic interactions between risk aversion and price postponement under supply yield risk from both the firm's and the consumers' perspectives. Particularly, we would like to investigate the following research questions: How does price postponement change a firm's risk aversion threshold? How does risk aversion alter the value and the operational execution of price postponement? And how does the pricing scheme choice of a risk averse firm affect its consumers' total welfare?

Specifically, we study a monopoly firm's pricing and production decision under supply random yield. Depending on the timing of pricing decision, we study two distinct pricing schemes: (1) ex ante pricing scheme, under which the firm sets price and quantity simultaneously before production takes place; and (2) responsive pricing scheme, under which the firm postpones the pricing decision until after the yield realization. We study each pricing scheme under both risk neutrality and risk aversion of the decision makers, and adopt the conditional value at risk (CVaR) criterion as the risk aversion measurement. Proposed by Rockafellar and Uryasev (1999), the CVaR criterion is a coherent risk measure for quantifying risk, which emphasizes the lower tail of the profit realizations and measures the average value of the profit falling below a certain quantile level. The CVaR criterion quantifies the firm's risk aversion attitude towards low profit by a single parameter, and enables us to characterize the impact of risk aversion level upon the optimal decisions under each pricing scheme. Moreover, we compare the results of the two pricing schemes to understand the impact of risk aversion on the value and operational execution of price postponement as well as their joint effect on the end market consumers' total welfare. We also discuss alternative risk aversion measurements, such as the expected utility and the mean-variance criteria, as model extensions.

For each pricing scheme, we fully characterize the firm's optimal pricing and production decisions and investigate the corresponding impacts of risk aversion. Among other results, we show that under ex ante pricing, a more risk averse firm sets a higher price and obtains a lower CVaR. However, the impact of risk aversion on the production quantity is non-monotonic. When the production cost is low, the production quantity first increases and then decreases as the firm becomes less risk averse. For responsive pricing, although the firm produces more [less] than the riskless quantity when the production cost is low [high], the impact of risk aversion on the production quantity may still be non-monotonic, and shares a similar pattern as that under ex ante pricing.

The main focus of our paper is to understand the strategic impact of risk aversion on the value and operational adoption of price postponement strategy in mitigating supply yield risk. Our analysis focuses on four interrelated issues: (1) impact of price postponement on the firm's risk aversion threshold; (2) impact of risk aversion on the operational execution of price postponement; (3) impact of risk aversion on the value of price postponement; and (4) impact of a risk averse firm's pricing scheme choice on consumers' total welfare. First, we find that there exists a unique threshold risk aversion level in each pricing scheme, below which the firm chooses not to produce. Intuitively, one may conjecture that responsive pricing should allow a more risk averse firm to produce, as it partially mitigates supply risk by reactive pricing. However, our results show that, contrary to the intuition, the flexibility of price postponement has no impact on the firm's risk aversion threshold, i.e., the two pricing schemes share a common risk aversion cutoff level. Second, we find that the firm always produces more [less] under responsive pricing when the production cost is high [low], and the region in which the firm produces more under responsive pricing shrinks as it becomes less risk averse. In addition, we show that the value of CVaR improvement from responsive pricing may not be monotonic in the firm's risk aversion level. As such, our results imply that although price postponement induces operational flexibility by better matching demand with available supply, whether the firm should adopt such strategy needs to be carefully evaluated since the CVaR gain may not necessarily cover the associated cost of implementing the price postponement. Finally, we show that responsive pricing improves consumer surplus in equilibrium. That is, consumers benefit from price postponement even though the firm may strategically set price to better match supply with demand through ex post revenue maximization.

To conclude, we overview our contributions to the existing literature. Although price postponement is recognized as an effective tool to mitigate supply yield risk, its value and operational execution for a risk averse firm have not been analyzed. Our work contributes by explicitly evaluating the impact of risk aversion on the value and adoption of responsive pricing strategy. Furthermore, our results indicate that price postponement, albeit its ex post revenue maximizing behavior, could improve consumers' overall surplus. However, such an operational flexibility does not enhance a firm' risk aversion threshold, and the CVaR gain for a risk averse firm may not necessarily cover the associated cost of delayed price announcement. As a result, although responsive pricing could potentially lead to a win-win outcome for both the firm and its consumers, such a Pareto improvement critically depends on the firm's risk aversion level and the potential fixed implementation cost associated with postponing sales price.

The rest of this paper is organized as follows. We position our paper in the related literature in Section 2. Section 3 sets up the model. Section 4 analyzes the two pricing schemes. Section 5 investigate the impact of price postponement and risk aversion. We conduct numerical analysis to check the robustness of our results in Section 6, and provide additional discussions in Section 7. Section 8 concludes the paper. All proofs and additional results are relegated to the appendices.

2. Literature Review

Our paper mainly contributes to four streams of literature: (1) operations management under risk aversion; (2) supply risk management; (3) value of price postponement; and (4) joint pricing and inventory management.

First, our work is related to the literature of operations management under risk aversion. There are different ways to model a decision maker's risk aversion attitude. For example, the expected utility criterion models a decision maker's risk aversion attitude through a general concave utility function, under which decisions are optimized by considering all possible realizations of the underlying uncertainty (see Chen et al. 2007, Agrawal and Seshadri 2000, Kazaz and Webster 2015). The mean-variance criterion focuses on both the expected value and the variability of the objective function under uncertainty (see Chen and Federgruen 2000, Dong and Liu 2007). Different from the previous criteria, the conditional value-at-risk (CVaR) criterion focuses on the average value of the objective function falling below a certain quantile level (VaR), which takes into consideration both the reward and the risk (see Ahmed et al. 2007, Choi and Ruszczynski 2008, Chen et al. 2009). We also adopt the CVaR criterion as the risk aversion measurement for our pricing problems, and investigate the impact of risk aversion level upon the firms' optimal decisions under random yield.

Second, our paper contributes to the extensive literature on supply risk management, see Yano and Lee (1995) for a comprehensive review. The primary focus in this line of research is how to design operational strategies to effectively mitigate supply risk. For example, firms can inflate production and/or hold extra inventory to hedge against yield risk (see Henig and Gerchak 1990), diversify their supply base to enjoy the risk pooling effect and reduce supply output variability (see Anupindi and Akella 1993, Dada et al. 2007, Federgruen and Yang 2008, 2009, 2011, 2014, Tang and Kouvelis 2011, Feng and Shi 2012, Li et al. 2013, Hu and Kostamis 2014, Dong et al. 2015, Tan et al. 2016, Feng et al. 2018, Dong et al. 2020), and exert effort to improve their suppliers' production reliabilities (see Wang et al. 2009, Tang et al. 2014). Despite the extensive literature on supply risk management, little research has been conducted on the impact of risk aversion on the adoption of supply risk mitigation strategies, and especially price postponement.

Another stream of research investigates the value of price postponement in mitigating the mismatch of supply and demand. Van Mieghem and Dada (1999) are the first to study the impact of the timing of price setting on the inventory decision when demand is uncertain. Chod and Rudi (2005) study the impact of responsive pricing on the usage of flexible resources. Granot and Yin (2008) investigate the impact of order and price postponement in a decentralized supply chain with a price-setting retailer. With uncertain yield, Tang and Yin (2007) demonstrate that responsive pricing improves the profit of a firm. Li et al. (2017) study the impact of price postponement on a price-setting firm's supply diversification decision under supply random capacity.[1] Different from the above papers, our work contributes by investigating how a firm's risk aversion affects the value and operational execution of price postponement in mitigating supply yield risk.

Finally, our paper is also related to the literature on joint pricing and inventory management. For multiple period settings, see Yano and Gilbert (2003) and Chen and Simchi-Levi (2012) for a comprehensive review. Federgruen and Heching (1999) provide a general treatment for this type of problem and show that a base-stock/list-price policy is optimal. In subsequent research, Chen and Simchi-Levi (2004a,b, 2006) study inventory control and pricing strategies with fixed setup costs and show the optimality of (s, S, p) policy for the finite horizon, the infinite horizon and the continuous review models. Feng et al. (2014) investigate the dynamic pricing and inventory management problem under a general demand function, which includes both additive and multiplicative random demand as special cases. Li and Zheng (2006) and Feng (2010) show that a reorder-point/list-price policy is optimal under supply random yield and random capacity, respectively.

For single period settings, Petruzzi and Dada (1999) provide a comprehensive review on the price-setting newsvendor problem. With a multiplicative or additive demand model, Yao et al. (2006) show that if the mean price-induced demand satisfies the IPE property and the random term follows an IGFR distribution, then the equilibrium price and quantity can be uniquely determined. Kocabıyıkoğlu and Popescu (2011) provide general conditions to analyze the price-setting newsvendor problem with a general demand function. Lu and Simchi-Levi (2013) characterize a general set of conditions to ensure the unimodality of the profit function under a multiplicative and additive demand function. For supply side uncertainty, Pan and So (2010) study an assembler's pricing and inventory decisions with random yield. Xu and Lu (2013) analyze the impact of supply yield uncertainty on a price-setting newsvendor with a backup supplier. Our work contributes by studying the price-setting newsvendor problem of a risk averse firm with two pricing schemes, ex ante and responsive, under random yield. We model risk control under the CVaR criterion, and characterize the impact of the firm's risk aversion level upon the optimal decisions.

3. Model Setup

Consider a monopoly firm, which produces a single product and sells to a market with deterministic price sensitive demand $d(p)$. We assume $d(p)$ strictly decreases in p with inverse demand function $p(d) := d^{-1}(p)$. Let $p_0 \leq \infty$ be the maximal allowable price that induces 0 demand, i.e., $d(p_0) = 0$. The firm's production suffers from yield uncertainty, i.e., the output quantity is only a random fraction ξ of the input quantity. We assume that the yield factor, ξ, is a continuous random variable with support on the interval $[l, u]$, where $0 \leq l < u \leq 1$. Let $g(\xi)$, $G(\xi)$, $\bar{G}(\xi)$, and μ

[1] The detailed discussions between random yield and random capacity models are provided in Section 7.3.

denote the probability density function (p.d.f.), the cumulative distribution function (c.d.f.), the complementary cumulative distribution function (c.c.d.f.), and the mean yield, respectively.

Throughout the analysis, we focus on the scenario where our focal firm is a vertically integrated firm with its own internal production facility. As such, it is responsible for all the production inputs regardless of the yield realization. Let c denote the unit production cost incurred for every unit of the input production quantity and equivalently, the effective unit production cost per usable unit is c/μ. To avoid the trivial solution of no production, we assume that $c \leq p_0\mu$. We remark that for our centralized setup, it could be practically possible for the firm to incur additional cost for each successfully produced (and delivered) final product after yield realization, including product handling and packaging cost, distribution/delivery cost, etc (see Deo and Corbett 2009, Tang and Kouvelis 2014). Such a general cost structure will be discussed in Section 7.2. In addition, we assume, without loss of generality, that there is no salvage value or goodwill cost, as these costs can be easily incorporated without affecting our main results qualitatively.

We consider both the risk neutral and the risk averse cases. For modeling risk aversion, we are particularly interested in the case where the firm amplifies the concern of low profit realization and adopt the conditional value-at-risk (CVaR) criterion, which is proposed by Rockafellar and Uryasev (1999) as a coherent risk measurement for quantifying risk. In its standard definition, CVaR deals with a random loss function to quantify its average above a certain quantile level (i.e., right tail) to avoid a large loss. To adopt in a newsvendor setting, we apply CVaR to a random profit function and measure the average profit falling below a certain quantile level (i.e., left tail) to emphasize low profit realization cases (see Chen et al. 2007). Next, we provide formal definition of CVaR applied to a random profit function and its equivalent maximization formula. Let π be a random profit function and $\eta \in (0,1]$ be a quantile level. The value-at-risk (VaR) of π at quantile level η, denoted as $\mathrm{VaR}_\eta(\pi)$, is the η percentile of π, i.e., $\mathrm{VaR}_\eta(\pi) = \inf\{v \in \mathbb{R} : \mathrm{P}(\pi \leq v) \geq \eta\}$. The CVaR at quantile level η is defined as the average value falling below the corresponding VaR value: $\mathrm{CVaR}_\eta(\pi) = \mathrm{E}[\pi | \pi \leq \mathrm{VaR}_\eta(\pi)]$. Although having clear managerial implication, the conditional expectation definition is usually not straightforward to calculate. This is because, to get CVaR through the above definition, we need to first derive the VaR value of the random profit, which can be lack of nice mathematical property and more complicated itself (Rockafellar and Uryasev 1999). To resolve such technical inconvenience, Rockafellar and Uryasev (1999, 2002) propose an alternative formula of CVaR as the maximization of the following auxiliary function:[2]

$$\mathrm{CVaR}_\eta(\pi) = \max_{v \in \mathrm{R}} \{v + \frac{1}{\eta}\mathrm{E}\min\{\pi - v, 0\}\}.$$

[2] In Rockafellar and Uryasev (2002), CVaR is applied to random loss and the alternative formula is to minimize an auxiliary function. In our setting, we focus on random profit, and the alternative formula is to maximize a revised auxiliary function. Such an alternative formula is heavily adopted in the Operations Management literature to simplify the analysis. We further prove the equivalence relationship between the two formulas in Property 1 of Appendix EC.3.

The above maximization formula possesses nice functional property as it is concave in v, and reveals that CVaR can be calculated bypassing the calculation of the corresponding VaR value, on which its original definition depends. In addition, being the optimal solution of the aforementioned optimization formula, the VaR value can also be obtained simultaneously as a byproduct. This maximization formula for CVaR calculation is heavily adopted in the related literature with parameter η reflecting a decision maker's risk aversion level to the downside profit realizations, see Chen et al. (2007) for example. We will adopt this alternative CVaR formula in our analysis.

Next, we detail the firm's activities. Our main objective is to study and compare the firm's production decisions under two pricing schemes: (1) the ex ante pricing scheme, under which the firm makes the pricing and production decisions simultaneously before yield uncertainty realizes; and (2) the responsive pricing scheme, under which the firm postpones the pricing decision after yield uncertainty is resolved. We introduce the problem formulation under each pricing scheme.

Ex ante pricing scheme. The firm faces a one-stage problem of setting price and production quantity simultaneously before yield realizes. Let p_a and q_a be the market price and the quantity, respectively. Based on previous discussion, we adopt the maximization formula. The firm's CVaR is given by Equation (1) below, where the subscript "a" represents the ex ante pricing scheme.

$$\Pi_a(p_a,q_a) := \mathrm{CVaR}_\eta(\pi_a(p_a,q_a)) = \max_{v\in R}\left\{v + \frac{1}{\eta}\mathrm{E}_\xi \min\{\pi_a(p_a,q_a)-v,0\}\right\}, \tag{1}$$
$$\text{where} \quad \pi_a(p_a,q_a) := p_a \min\{d(p_a), q_a\xi\} - cq_a.$$

Note that $\pi_a(p_a,q_a)$ is the firm's ex post profit when the yield realization is ξ, and $\eta\in(0,1]$ can be interpreted as the risk aversion parameter. The CVaR criterion measures the average value of the profit below η quantile level, and emphasizes low profit realization cases. The larger the η, the less risk averse the firm is. When $\eta = 1$, the model reduces to the risk neutral case.

Responsive pricing scheme. The firm engages in a two-stage decision making process. At the first stage, the firm decides the production quantity q_r. At the second stage, after yield realization, the firm sets market price p_r to maximize its revenue. The firm's CVaR is given by Equation (2) below, where the subscript "r" represents the responsive pricing scheme.

$$\Pi_r(q_r) := \mathrm{CVaR}_\eta(\pi_r(q_r)) = \max_{v\in R}\left\{v + \frac{1}{\eta}\mathrm{E}_\xi \min\{\pi_r(q_r)-v,0\}\right\}, \tag{2}$$
$$\text{where} \quad \pi_r(q_r) := \max_{p_r\in[0,p_0]}\{p_r \min\{d(p_r), q_r\xi\}\} - cq_r.$$

Note that $\pi_r(q_r)$ is the firm's ex post profit given the yield realization ξ and the firm's revenue maximizing pricing decision p_r. In the first stage, the firm's ex post profit $\pi_r(q_r)$ is a random function depending on the production quantity q_r and the realization of the production yield ξ, albeit the firm's revenue maximizing pricing decision. In this case, the CVaR criterion continues to emphasize the low yield realization cases even though the firm has additional flexibility to better match demand with supply through postponed pricing.

Given the above objective functions, we proceed to characterize the firm's optimal production and pricing decisions for each pricing scheme and compare them to investigate the impacts of risk aversion and price postponement on the firm's optimal decisions and corresponding profits. We further remark that the term "risk aversion" in the subsequent analysis specifically refers to the case where the firm amplifies the concern of low profit realizations and focuses on the average profit falling below certain quantile level as modeled through the CVaR criterion. The impacts of alternative risk aversion criteria are discussed in Section 7.1.

4. Model Analysis

4.1. Ex ante Pricing

In this section, we characterize the firm's optimal pricing and production decisions under the ex ante pricing scheme. In general, the objective function $\Pi_a(p_a, q_a)$ is not jointly concave in (p_a, q_a). Hence, we adopt the sequential optimization approach. For any given sales price p_a, the problem is a standard newsvendor-type decision problem under random yield and CVaR criterion. The following proposition characterizes the firm's optimal production quantity.

PROPOSITION 1. *For any $\eta \in (0,1]$ and $p_a \in [c/\mu, p_0]$, there exists a threshold sales price $\tilde{p} := \min\{p_0, \frac{c\eta}{\int_l^{G^{-1}(\eta)} \xi dG(\xi)}\} \geq \frac{c}{\mu}$. The optimal production quantity and objective function value are:*

$$q_a^*(p_a) = \begin{cases} \frac{d(p_a)}{\delta} & \text{if } p_a \in [\tilde{p}, p_0] \\ 0 & \text{if } p_a \in \left[\frac{c}{\mu}, \tilde{p}\right) \end{cases} \text{ and } \Pi_a(p_a, q_a^*(p_a)) = \begin{cases} p_a d(p_a)(1 - \frac{G(\delta)}{\eta}) & \text{if } p_a \in [\tilde{p}, p_0] \\ 0 & \text{if } p_a \in \left[\frac{c}{\mu}, \tilde{p}\right) \end{cases},$$

respectively, where the inflation factor δ satisfies $\int_l^{\delta} \xi dG(\xi) = c\eta/p_a$. Moreover, δ decreases in p_a and increases in η, and $q_a^*(p_a)$ decreases in η.

Proposition 1 shows that the firm produces if and only if the sales price is higher than the threshold sales price $\tilde{p}$. When the sales price and the corresponding margin are sufficiently high (i.e., $p > \tilde{p}$), the optimal production quantity is a linear inflation of the price induced demand, i.e., $q_a^* = d(p_a)/\delta$. Moreover, the optimal production quantity decreases in η, which implies that given that the firm decides to produce, the more risk averse firm tends to produce more. This is because the CVaR criterion emphasizes the low profit realization cases, which occur when yield realization is low. To mitigate the yield uncertainty, the more risk averse firm tends to produce more to hedge against underage risk. In contrast, when the margin is thin (i.e., $p < \tilde{p}$), the gain from satisfying demand through aggressive production inflation does not justify the increased production cost. Thus, the risk averse firm optimally chooses not to produce.

Next, we proceed to analyze the firm's optimal pricing decision by optimizing $\Pi_a(p_a, q_a^*(p_a))$ directly. To facilitate the analysis, we impose the following set of assumptions on both the yield distribution and the demand function.

Assumption 1. *(a)* $\frac{x\bar{G}(x)}{\int_l^x \xi g(\xi)d\xi}$ *decreases in* $x \in [l, u]$. *(b)* $d(p)$ *satisfies the increasing price elasticity property, i.e.,* $\tau(p) = -\frac{pd'(p)}{d(p)}$ *increases in* p.

Assumption 1 is first proposed in Kouvelis et al. (2018) to analyze the risk neutral price-setting newsvendor problem under supply random yield. Assumption 1(a) is quite general and is satisfied by all continuous yield distributions with an increasing length-biased hazard rate, which is also known as increasing generalized failure rate (IGFR) in the Operations Management literature. This property is satisfied by many commonly used distributions, including Uniform, Beta, Normal, Gamma distributions as well as their truncations. Assumption 1(b) is satisfied by most of the demand functions in the literature, e.g., linear, iso-elastic, concave, and log-concave demand functions. We show that Assumption 1 can be directly applied to the risk averse case under CVaR criterion, and is used to establish the unimodality of $\Pi_a(p_a, q_a^*(p_a))$ in the following proposition.

Proposition 2. *Assume Assumption 1 holds. For any* c *and* p_0 *with* $p_0 \geq c/\mu$*, there exists a risk aversion threshold* $\eta_a^*(p_0, c) \in [0, 1]$ *such that:*

(i) If $\eta < \eta_a^*(p_0, c)$*, it is optimal for the firm not to produce.*

(ii) If $\eta \geq \eta_a^*(p_0, c)$*,* $\Pi_a(p_a, q_a^*(p_a))$ *is quasi-concave with the unique optimal price* p_a^* *defined by the first order condition (FOC), which is equivalent to* $1 + \frac{\int_l^\delta \xi g(\xi)d\xi}{\delta(\eta - G(\delta))} + \frac{p_a d'(p_a)}{d(p_a)} = 0$.

Given the cost structure (p_0, c), Proposition 2 shows that there exists a threshold risk aversion level, $\eta_a^*(p_0, c)$. If $\eta < \eta_a^*(p_0, c)$, the firm exhibits strongly risk averse attitude towards low profit realizations. In this case, the gain from satisfying demand does not justify the production cost associated with the aggressive production inflation, so the firm optimally chooses not to produce. Otherwise, for $\eta \geq \eta_a^*(p_0, c)$, it is optimal for the firm to produce. In addition, the optimal sales price and production quantity can be uniquely determined under Assumption 1. Note that when $\eta = 1$, the firm is risk-neutral. Since $\eta_a^*(p_0, c) \leq 1$, it is always optimal for a risk-neutral firm to produce and its corresponding optimal price can be uniquely characterized via the FOC.

We now provide some economic intuition for Assumption 1(a). For risk neutral firms, the overproduction probability is defined as $P(d(p_a) < q^*(p_a)\xi) = P(d(p_a) < d(p_a)\frac{\xi}{\delta}) = \bar{G}(\delta)$, where the inflation factor δ satisfies $\int_l^\delta \xi g(\xi)d\xi = c/p_a$. For risk averse firms, we inflate the underproduction probability by $1/\eta$ to capture the impact of the risk aversion level, and define the modified overproduction probability as $1 - \frac{1}{\eta}G(\delta)$, where δ satisfies $\int_l^\delta \xi g(\xi)d\xi = \eta c/p_a$. Note that, on the feasible price range with positive production quantity (i.e., $p_a \in (\tilde{p}, p_0])$, $G(\delta) \in [0, \eta)$ and the modified overproduction probability is well-defined. The price elasticity of the modified overproduction probability is:

$$e(p_a, \eta) := \frac{p_a \frac{\partial\left(1-\frac{1}{\eta}G(\delta)\right)}{\partial p_a}}{1-\frac{1}{\eta}G(\delta)} = \frac{\int_l^\delta \xi g(\xi) d\xi}{\delta(\eta - G(\delta))} = \frac{\int_l^\delta \xi g(\xi) d\xi}{\delta \bar{G}(\delta)} \frac{1-G(\delta)}{\eta - G(\delta)},$$

which increases in δ under Assumption 1(a) when $\delta \leq G^{-1}(\eta)$ and, thus, decreases in p_a on the feasible price range. The economic interpretation is that as the sales price increases, the modified overproduction probability becomes less sensitive to the sales price change, i.e., the same percentage increase in the sales price leads to a smaller percentage increase in the modified overproduction probability. Moreover, an increase in the sales price has two effects: (1) it decreases the market demand; and (2) it increases the inflation rate $1/\delta$ and, thus, the modified overproduction probability. Recall that the optimal sales price p_a^* satisfies the FOC, which pins down to $1 + \frac{\int_l^\delta \xi g(\xi) d\xi}{\delta(\eta - G(\delta))} + \frac{p_a d'(p_a)}{d(p_a)} = 1 + e(p_a, \eta) - \tau(p_a) = 0$. This shows that p_a^* is charged by striking a balance between the price elasticity of demand and that of the modified overproduction probability.

In what follows, we study the impacts of supply uncertainty and risk aversion upon the optimal sales price and the corresponding optimal profit. As a benchmark, we define the optimal riskless price p_a^d as the optimal sales price without supply uncertainty (i.e., the yield is deterministic with mean μ): $p_a^d := \arg\max_{p \in [c, p_0]} \Pi_a^d(p)$, where $\Pi_a^d(p) = (p - \frac{c}{\mu}) d(p)$. Corollary 1 below shows that the firm charges a higher sales price under supply uncertainty and risk aversion, i.e., $p_a^* \geq p_a^d$, and earns a lower CVaR, i.e, $\Pi_a(p_a^*) \leq \Pi_a^d(p_a^d)$. Moreover, as the firm becomes more risk averse, the optimal price becomes higher, which leads to a smaller demand and a lower corresponding CVaR.

Corollary 1. *Under Assumption 1, for any $\eta \in [\eta_a^*(p_0, c), 1]$: (i) $p_a^* \geq p_a^d$ and $\Pi_a(p_a^*) \leq \Pi_a^d(p_a^d)$. (ii) p_a^* decreases in η and $\Pi_a(p_a^*)$ increases in η.*

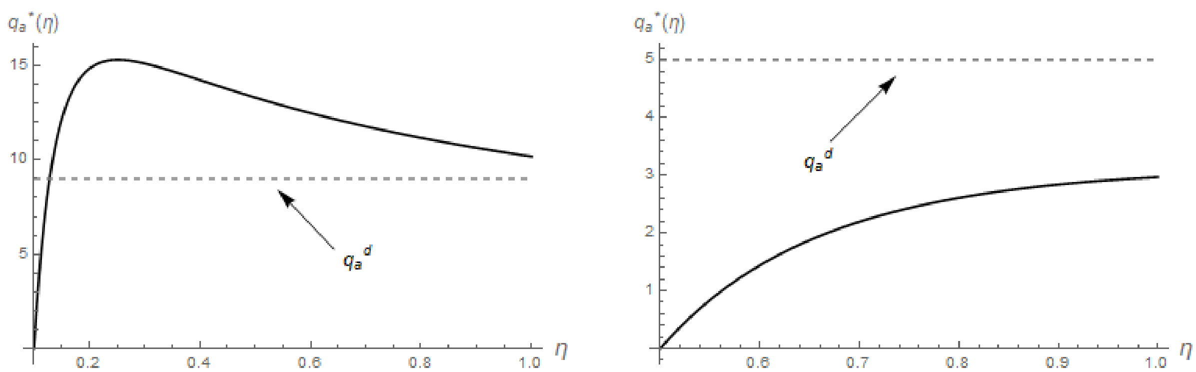
The CVaR criterion emphasizes the lower tail of the profit realization, which occurs when the yield realization is low. As such, Corollary 1(ii) indicates that the more risk averse firm sets a higher sales price to limit demand and mitigate the underage risk. Finally, we conclude this section by investigating the impact of risk aversion on the optimal production quantity $q_a^* = \frac{d(p_a^*)}{\delta}$ and compare it with the optimal riskless quantity q_a^d under linear demand and uniform yield distribution.

Corollary 2. *Assume $d(p) = a - bp$ and $\xi \sim$ Uniform $[0, 1]$. For any $c \in [0, \frac{a}{2b}]$ and $\eta \in [\eta_a^*(p_0, c), 1]$: (i) There exists a threshold cost $\hat{c}_a$ independent from η. If $c < \hat{c}_a$, $q_a^*(\eta)$ first increases and then decreases in η; otherwise, $q_a^*(\eta)$ increases in η. (ii) There exist threshold values $\bar{c}_a$ and $\bar{\eta}_a^1 < \bar{\eta}_a^2$ such that if $c < \bar{c}_a$ and $\eta \in [\bar{\eta}_a^1, \min\{\bar{\eta}_a^2, 1\}]$, $q_a^*(\eta) \geq q_a^d$; otherwise, $q_a^*(\eta) < q_a^d$.*

The level of risk aversion affects the optimal production quantity $\frac{d(p_a^*)}{\delta}$ in two opposite ways. As the firm becomes less risk averse (i.e., η increases), it reduces the price p_a^* to induce a larger demand.

Meanwhile, it also reduces the production inflation by increasing δ. The two effects counterbalance each other and may result in the non-monotonic behavior of the optimal production quantity, as shown by Corollary 2(i). In particular, if the production cost is high and/or when the firm is moderately risk averse, then hedging low profit realization through production inflation becomes too risky, and the more risk averse firm prefers to produce less (i.e., $q_a^*(\eta)$ increases in η). In contrast, if the production cost is low and the firm is not too risk averse, then the firm may prefer to hedge low yield realization through a higher production quantity as it becomes more concerned about low profit shock (i.e., $q_a^*(\eta)$ could decrease in η). See Figure 1 for a detailed depiction.

Part (ii) compares q_a^* with the optimal riskless quantity q_a^d. Although the firm charges a higher sales price under yield uncertainty and risk aversion by Corollary 1(i), it may not necessarily produce less. The comparison can be twofold and depends on both the firm's risk aversion level and the production cost. Specifically, when the production cost is low, the firm may produce a larger quantity (i.e., $q_a^*(\eta) \geq q_a^d$) given that it exhibits intermediate level of risk aversion (i.e., $\eta \in [\bar{\eta}_a^1, \min\{\bar{\eta}_a^2, 1\}]$). This is because for moderately risk averse firms, avoiding low profit realization through conservative operations is not the primary concern. As such, the firm reduces the sales price to stimulate the demand and copes with the inflation behavior, resulting in a larger overall production quantity.



(a) $c = 0.5$ (b) $c = 2.5$

Figure 1 Impact of Risk Aversion on $q_a^*(\eta)$ under Ex ante Pricing: $d(p) = 10 - p$, $\xi \sim$ Uniform $[0,1]$

4.2. Responsive Pricing

In this section, we characterize the firm's optimal pricing and production decisions under responsive pricing. Recall that $d(p)$ strictly decreases in p with inverse demand function $p(d) := d^{-1}(p)$. To facilitate the analysis, we make the following assumption on the revenue function $p(d)d$:

ASSUMPTION 2. *The revenue function $p(d)d$ is concave and twice differentiable in d.*

Assumption 2 is a standard assumption in joint pricing and inventory management literature and is satisfied by many commonly used demand functions (see, e.g., Federgruen and Heching 1999). The concavity of $p(d)d$ in d suggests a decreasing marginal revenue with respect to demand. With Assumption 2, we are able to solve the two-stage decision making problem in Equation (2) through backward induction. Let d^* be the unconstrained revenue maximizing demand, i.e., $p'(d^*)d^* + p(d^*) = 0$. The second stage optimal pricing decision is given by the following lemma.

Lemma 1. *Under Assumption 2, given on-hand inventory $q_r\xi$, the firm's optimal retail price is $p_r^* = p(\min\{q_r\xi, d^*\})$ and its optimal second stage revenue is $\min\{q_r\xi, d^*\}p(\min\{q_r\xi, d^*\})$.*

Lemma 1 shows that when supply is sufficient, the firm sets price to sell just d^* to achieve the unconstrained revenue maximization albeit left with surplus inventory; when supply is scarce, it sets the inventory clearing price to make better use of the available inventory. Such a pricing strategy is independent from the risk aversion level, as there is no risk involved ex post after yield realization. Back to the first stage, the firm decides the optimal production quantity q_r^* to maximize its CVaR, and the result is characterized by the following proposition.

Proposition 3. *Assume Assumption 2 holds. For any production cost c and maximal allowable price p_0 with $p_0 \geq c/\mu$, $\Pi_r(q_r)$ is continuously differentiable and concave in q_r. There exists a risk aversion threshold $\eta_r^*(p_0, c) \in [0,1]$ such that:*

(i) If $\eta < \eta_r^*(p_0, c)$, it is optimal for the firm not to produce.

(ii) If $\eta \geq \eta_r^*(p_0, c)$, let $\bar{c}_r := \frac{1}{\eta}\int_l^{G^{-1}(\eta)}\left[p\left(\frac{\xi d^*}{G^{-1}(\eta)}\right) + \left(\frac{\xi d^*}{G^{-1}(\eta)}\right)p'\left(\frac{\xi d^*}{G^{-1}(\eta)}\right)\right]\xi dG(\xi)$, then there exists an optimal production quantity q_r^* satisfying the following equations:

$$q_r^* \textit{ solves} \begin{cases} \frac{1}{\eta}\int_l^{G^{-1}(\eta)}\left[p'(q_r^*\xi)(q_r^*\xi) + p(q_r^*\xi)\right]\xi dG(\xi) = c & \textit{if } c \in [\bar{c}_r, \max\{\bar{c}_r, p_0\mu\}], \\ \frac{1}{\eta}\int_l^{\frac{d^*}{q_r^*}}\left[p'(q_r^*\xi)(q_r^*\xi) + p(q_r^*\xi)\right]\xi dG(\xi) = c & \textit{if } c \in [0, \bar{c}_r]. \end{cases}$$

Similar to its counterpart under ex ante pricing, Proposition 3 shows that, for any given feasible parameters (p_0, c), there always exists a risk aversion threshold, $\eta_r^*(p_0, c)$, below which the firm chooses not to produce as the gain from selling does not justify the associated cost of inflated production quantity due to random yield. On the other hand, if the firm is less risk averse than the risk aversion cutoff level (i.e., $\eta \geq \eta_r^*(p_0, c)$), it is always optimal to produce. Moreover, there exists a unique cost threshold $\bar{c}_r$, at which the firm produces $q_r^* = \frac{d^*}{G^{-1}(\eta)}$. The quantity $\frac{d^*}{G^{-1}(\eta)}$ has an interesting implication, as it is the revenue maximizing quantity adjusted for the firm's risk aversion level: If the firm is risk-neutral (i.e., $\eta = 1$), $\frac{d^*}{G^{-1}(\eta)} = \frac{d^*}{u}$, which is the largest quantity that will be cleared for sure in the market under any yield realization. If the firm is risk-averse (i.e., $\eta < 1$), the revenue maximizing quantity d^* is modified to $\frac{d^*}{G^{-1}(\eta)}$, taking into account of the firm's risk aversion level. As shown in part (ii), when the production cost is high [low], the firm produces less [more] than the risk-aversion adjusted revenue maximizing quantity $\frac{d^*}{G^{-1}(\eta)}$.

Next, we investigate the impact of yield uncertainty and risk aversion on the firm's optimal production decision and the corresponding profit. As a benchmark, consider the riskless production quantity q_r^d under deterministic yield case (i.e., the yield is deterministic with fixed rate μ), $q_r^d := \arg\max_{q\in[0,\infty)} \Pi_r^d(q)$, where $\Pi_r^d(q) = p(q\mu)q\mu - cq$. The results are given in the following corollary.

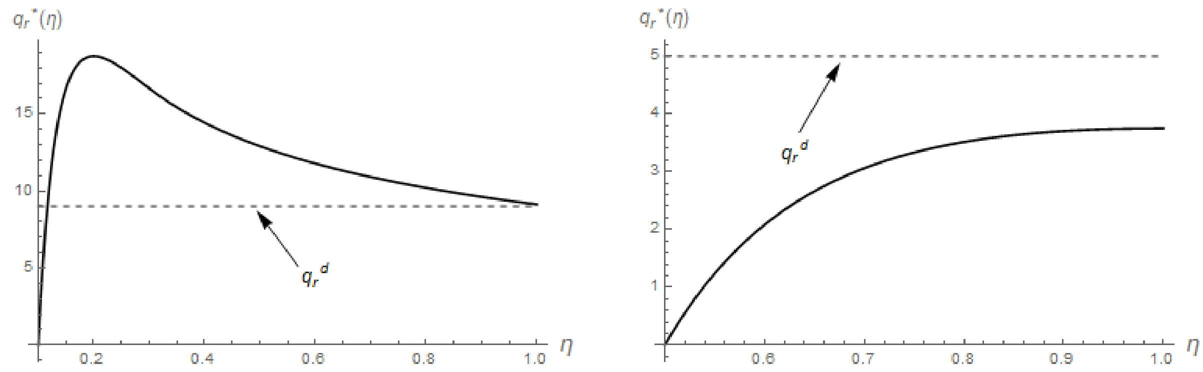
COROLLARY 3. *Under Assumption 2, for any $\eta \in [\eta_r^*(p_0,c),1]$: (i) There exists $\delta_i > 0$, $i = 1,2$, such that $q_r^* > q_r^d$ if $c \in [0,\delta_1)$, and $q_r^* < q_r^d$ if $c \in (p_0\mu - \delta_2, p_0\mu)$. Moreover, $\Pi_r(q_r^*) \le \Pi_r^d(q_r^d)$. (ii) When $c \le \bar{c}_r$, q_r^* decreases in η. When $c > \bar{c}_r$, q_r^* may not be monotonic in η. (iii) $\Pi_r(q_r^*)$ increases in η.*

Corollary 3(i) indicates that, compared to the case under deterministic yield, the firm produces more [less] under yield uncertainty and risk aversion when c is sufficiently low [high].[3] Nevertheless, yield uncertainty always leads to a lower CVaR for the firm. Parts (ii) and (iii) investigate the impact of risk aversion on the firm's optimal production quantity and its corresponding optimal CVaR. While the more risk averse firm always earns a lower CVaR, the effect of η on q_r^* is again not straightforward. As the CVaR criterion emphasizes low profit realization caused by low yield realization, one may conjecture that the more risk averse firm produces more to hedge against underproduction. This intuition is true when the firm produces more than the risk-aversion adjusted revenue maximizing quantity $\frac{d^*}{G^{-1}(\eta)}$ (i.e., when $c \le \bar{c}_r$), but is not necessarily true when the firm produces less than $\frac{d^*}{G^{-1}(\eta)}$ (i.e., when $c > \bar{c}_r$). In this case, the more risk averse firm may produce less as production quantity inflation is too risky under high production cost. In addition, as $\bar{c}_r$ depends on η, the overall impact of η on q_r^* may not be monotonic, as illustrated in Corollary 4.

COROLLARY 4. *Assume $d(p) = a - bp$ and $\xi \sim$ Uniform $[0,1]$. For any $c \in [0, \frac{a}{2b}]$ and $\eta \in [\eta_r^*(p_0,c),1]$, there exists a threshold cost $\hat{c}_r$ independent from η. If $c < \hat{c}_r$, $q_r^*(\eta)$ first increases and then decreases in η; otherwise, $q_r^*(\eta)$ increases in η.*

Corollary 4 confirms that the overall effect of risk aversion on $q_r^*(\eta)$ is qualitatively the same as that under ex ante pricing: When production cost is low, the impact is non-monotonic, whereas when production cost is high, the production quantity increases as the firm becomes less risk averse. The underlying tradeoff is again to carefully strike a balance between costly production inflation and low yield realization. A depiction of Corollary 4 for responsive pricing is given in Figure 2.

[3] With linear demand function $d(p) = a - bp$ and Uniform$[0,1]$ yield distribution, we show that there exists a unique threshold cost $\hat{c}$ such that $q_r^* > q_r^d$ holds if and only if $c \in [0,\hat{c})$. See the proof of Corollary 3(i) for details.



(a) $c = 0.5$ (b) $c = 2.5$

Figure 2 Impact of Risk Aversion on $q_r^*(\eta)$ under Responsive Pricing: $d(p) = 10 - p$, $\xi \sim$ **Uniform**$[0, 1]$

5. Impacts of Risk Aversion and Price Postponement

In this section, we compare the two pricing schemes to investigate our main research question: the strategic interactions between risk aversion and price postponement under supply random yield. We proceed by answering the following four questions: (1) how price postponement affects a firm's risk aversion threshold; (2) how risk aversion changes the operational execution of price postponement; (3) how the value of price postponement gets affected by the level of risk aversion; and (4) how a risk averse firm's pricing scheme choice affects the overall consumer surplus.

When comparing the two pricing schemes, we impose consistent assumptions to regulate the CVaR objectives under both models. For the yield distribution, Assumption 1(a) alone is sufficient. For the demand functions, we assume that both Assumptions 1(b) and 2 hold to ensure the unimodality of the objectives. Lemma 2 shows that most of the commonly used demand functions satisfy these assumptions. For the rest of this section, without loss of generality, we assume both Assumptions 1 and 2 hold. Whenever investigation under those general assumptions is intractable, we adopt Uniform $[0, 1]$ yield distribution and linear demand function $d(p) = a - bp$ to derive insights, and confirm the robustness of our results through numerical experiments in Section 6.

LEMMA 2. *Both Assumptions 1(b) and 2 hold under: (i)* $d(p) = a - bp^k$, $a > 0$, $b > 0$, $k \geq 1$*; (ii)* $d(p) = (1 + p)^{-a}$, $a > 1$*, (iii)* $d(p) = ap^{-b}$, $a > 0$, $b > 1$*; and (iv)* $d(p)$ *is concave.*

5.1. Impact on Risk Aversion Threshold

We start our discussion by investigating how price postponement affects a firm's risk aversion threshold. Recall from Propositions 2 and 3 that, under both pricing schemes, there always exists a respective risk aversion threshold, below which the risk averse firm chooses not to produce. The risk aversion threshold is a characteristic of the decision making environment, which depends on parameters c and p_0 and the underlying yield uncertainty. Lowering this threshold implies that it is more likely for risk averse firms to produce in this environment, because more firms will have η larger than this cutoff level. The following proposition compares the two key threshold values.

PROPOSITION 4. *Assume Assumptions 1 and 2 hold. For any c and p_0 with $p_0 \geq c/\mu$: (i) $\eta_a^*(p_0, c) = \eta_r^*(p_0, c) := \eta^*$. (ii) η^* increases in c and decreases in p_0. (iii) Let ξ_1 be stochastically larger than ξ_2, i.e., $\xi_1 \geq_{st} \xi_2$, then $\eta_1^* \leq \eta_2^*$. (iv) Let $\xi \sim$ Uniform $[\mu - \sigma, \mu + \sigma] \subseteq [0, 1]$, then η^* increases in σ.*

Intuitively, price postponement provides a firm with flexibility to better match demand with the realized supply, and thus partially mitigates the adverse effect of yield uncertainty. Hence, one may conjecture that responsive pricing enhances the risk aversion threshold of the decision making environment, i.e., responsive pricing allows more risk averse firms to produce. Proposition 4(i) shows that this conjecture is not true and price postponement has no impact on the risk aversion threshold. The underlying intuition can be explained as follows: When deciding whether to produce or not, the risk averse firm always compares the marginal revenue with the associated marginal cost. Due to the unimodality of the CVaR objective, the firm only needs to evaluate such comparison at a marginal production quantity. That is, if producing an infinitely small quantity would result in a positive marginal profit, then it implies that the firm will choose to produce optimally. As such, the investigation pins down to understanding whether the firm would have incentive to produce a marginal quantity. Clearly, with a marginal production quantity, the firm will set price optimally as p_0 under both pricing schemes. In this case, the marginal benefit is the same as $\frac{p_0 \int_l^{G^{-1}(\eta)} \xi dG(\xi)}{\eta}$ for both pricing schemes, which leads to the identical risk aversion threshold. Part (ii) shows that as the production cost decreases or the pricing power (as measured by p_0) increases, the risk aversion threshold η^* decreases, allowing a more risk averse firm to produce. Part (iii) investigates the effect of yield distribution on the risk aversion threshold. As the yield distribution becomes stochastically larger, the risk aversion threshold decreases. This implies it is more likely for a risk averse firm to produce when the underlying yield distribution becomes more reliable. Finally, part (iv) shows that under a uniformly distributed yield, the risk aversion threshold decreases as yield variability decreases. Section 6 further confirms this finding numerically with general yield distributions: As the yield decreases in convex order, the risk aversion threshold decreases, creating a decision making environment with binary production decisions (to produce or not) less impacted by risk aversion.

5.2. Impact on Production Quantity

Next, we compare the optimal production quantity decisions. When the firm chooses to produce, it is not intuitively clear how the flexibility of price postponement affects the firm's production quantity and how such impact varies with the firm's risk aversion level. We address these questions under the linear demand function and uniform yield distribution in the following proposition.

PROPOSITION 5. *Assume $d(p) = a - bp$ and $\xi \sim$ Uniform $[0,1]$. For any $c \in [0, \frac{a}{2b}]$ and $\eta \in [\eta^*, 1]$: (i) There exists a threshold cost $\hat{c} \in (0, \frac{a}{2b})$, s.t., $q_r^* \le q_a^*$ if $c \le \hat{c}$ and $q_r^* > q_a^*$ otherwise. Moreover, the threshold $\hat{c}$ increases in η. (ii) $q_r^* - q_a^*$ may not be monotonic in η.*

Proposition 5 characterizes the effects of price postponement and risk aversion on the firm's optimal production decision under the linear demand function and uniform yield distribution. For any feasible risk aversion level, part (i) shows that there always exists a cost threshold such that the firm produces more under responsive pricing if and only if its production cost exceeds this threshold. The underlying intuition is that when the cost is high [low], the firm produces less [more]. After yield realization, it is more likely for the responsive pricing firm to set inventory clearing [revenue maximizing] price, which enhances [reduces] the marginal value of inventory and hence increases [decreases] the optimal quantity under responsive pricing. In addition, the threshold cost $\hat{c}$ increases in η. This indicates that as the responsive pricing firm gets less risk averse, it is less likely for the firm to produce more than that of an ex ante pricing firm, because $\hat{c}$ increases. Part (ii) further characterizes the impact of risk aversion on the difference of the optimal production quantities under the two pricing schemes. As the firm becomes less risk averse (i.e., η increases), the firm may either increase or decrease the additional quantity produced under responsive pricing (i.e., $q_r^* - q_a^*$), which could be negative if $q_r^* < q_a^*$. To further elaborate such non-monotonic trends, our numerical study in Section 6 illustrates that when the production cost is low, $q_r^* - q_a^*$ first increases and then decreases in η; when the production cost is high, $q_r^* - q_a^*$ increases in η. This echos the respective impacts of risk aversion on the optimal quantity under each pricing scheme. See column (a) of Figure 3 for details.

To sum up, our results imply that whether to postpone pricing has a profound impact on a firm's production decision, which in turn affects the overall supply of the product and the corresponding ex post price. Such an impact further interacts with the firm's risk aversion level in a non-monotonic way, and intricately influences both the firm and its consumers as shown in the subsequent sections.

5.3. Impact on the Value of Price Postponement

We now investigate the impact of risk aversion on the value of price postponement. Clearly, price postponement creates additional flexibility in mitigating supply risk and hence improves the firm's CVaR. However, it is not clear how the benefit of responsive pricing gets affected (amplified or diminished) by the risk aversion level. In practice, when deploying the price postponement strategy, firms may incur a fixed implementation cost, which captures the cost associated with goodwill/reputation loss, delayed announcement, etc. The adoption of the responsive pricing scheme for a risk averse firm critically depends on whether the CVaR improvement could justify the additional fixed cost. We are interested in quantifying the value of price postponement for firms with different risk aversion levels and characterizing when adopting price postponement is worthwhile. To this purpose, we define the value of price postponement (VPP) as $\Pi_r^* - \Pi_a^*$, which clearly is non-negative. The following proposition characterizes the impact of risk aversion on VPP under the linear demand function and uniform yield distribution.

PROPOSITION 6. *Assume $d(p) = a - bp$ and $\xi \sim$ Uniform $[0,1]$. For any $c \in [0, \frac{a}{2b}]$ and $\eta \in [\eta^*, 1]$, there exists a threshold cost $\tilde{c}$ independent from η. If $c < \tilde{c}$, VPP first increases and then decreases in η. Otherwise, VPP increases in η.*

As responsive pricing better matches supply with demand, one may conjecture that postponement will be more beneficial for a more risk averse firm and induce a larger CVaR improvement with higher risk aversion (i.e., lower η). However, Proposition 6 shows that the VPP always increases in η when the production cost is high, and is non-monotonic in η otherwise. In other words, the CVaR improvement from price postponement may increase as the firm becomes less risk averse. To better understand Proposition 6, we decompose the VPP into the following two parts:

$$\text{VPP} = \underbrace{\Pi_r(q_r^*) - \Pi_r(q_a^*)}_{\text{Quantity Effect}} + \underbrace{\Pi_r(q_a^*) - \Pi_a(q_a^*, p_a^*)}_{\text{Pricing Effect}}.$$

The first part, i.e., $\Pi_r(q_r^*) - \Pi_r(q_a^*)$, measures the CVaR difference for a responsive pricing firm due to the difference of the optimal production quantities under the two pricing schemes, which captures the *quantity effect* of price postponement. The second part, i.e., $\Pi_r(q_a^*) - \Pi_a(q_a^*, p_a^*)$, measures the CVaR difference under the two pricing schemes without adjusting the production quantity q_a^*, which teases out the *pricing effect* of price postponement. Clearly, both the quantity and the pricing effects are nonnegative, and jointly contribute to the VPP. Due to the unimodality of $\Pi_r(q_r)$, the magnitude of the quantity effect depends on how close q_a^* is relative to q_r^*, and may not be monotonic as η changes. However, we numerically observe that, compared to the pricing effect, the impact of the quantity effect on the VPP is relatively small in magnitude. Hence, the shape of the VPP with respect to risk aversion parameter η is primarily driven by the impact of η on the pricing effect. That is, η affects the VPP in a similar way as it affects the pricing effect. We further note that the pricing effect quantifies the value of price postponement without adjusting the production quantity, which is essentially the revenue difference between the two pricing schemes evaluated at quantity q_a^*. For any yield realization, a larger q_a^* provides the responsive pricing firm more operational flexibility in choosing the ex post sales price from a weakly larger feasible set $[\max\{p(d^*), p(q_a^*)\}, p_0]$. Therefore, a larger optimal quantity q_a^* is more likely to induce a larger pricing effect. In addition, Corollary 4 shows that the effect of risk aversion on the optimal production quantity q_a^* is non-monotonic [increasing] when production cost c is low [high]. Combining the above discussions, we explain why the VPP is non-monotonic in η when the production cost is low, but increases in η otherwise. In Appendix EC.2, we further provide two numerical illustrations to show that the pricing effect primarily contributes to the VPP and is affected by η in a qualitatively similar way as that for the optimal production quantity q_a^*.

Proposition 6 provides useful managerial insights: Although price postponement creates operational flexibility, whether a risk averse firm should adopt responsive pricing needs to be carefully evaluated, especially when postponing pricing decision incurs a fixed implementation cost. When the production cost is high (i.e., $c > \tilde{c}$), a less risk averse firm is more likely to deploy the price postponement strategy. On the other hand, when the production cost is low, due to the unimodality of the VPP, there exist some fixed implementation costs such that postponing the sales price may be worthwhile only for firms with medium risk aversion level, and too costly for firms with either high or low risk aversion level. This finding could be potentially helpful in understanding why different firms may have different attitudes towards price postponement. For example, with uncertain crop yields, some agri-producers may opt to sign forward contracts at the time of production, which require them deliver the harvest at the predetermined price regardless of the yield realization. In contrast, others may find it optimal not to sign forward contracts but to make price decisions responsively after harvest (Davis et al. 2005). The robustness of our findings is further confirmed in Section 6 by using Beta yield distribution family.

5.4. Impact on Consumer Surplus

We conclude this section by studying how the firm's decisions, jointly driven by price postponement and its risk aversion level, affect its end market consumers' total surplus. To this purpose, we first define consumer surplus for each of the two pricing schemes under supply random yield. Recall that $p(d)$ represents the inverse demand function $d^{-1}(p)$. Under the ex ante pricing scheme, for any given sales price p_a and order quantity q_a, the consumer surplus, CS_a, is defined by:

$$CS_a(p_a, q_a) = E_\xi \left(\int_0^{\min\{d(p_a), q_a\xi\}} (p(x) - p_a)dx \right). \tag{3}$$

Note that the above definition of $CS_a(p_a, q_a)$ corresponds to the definition of consumer surplus under demand uncertainty with H rule allocation from Cohen et al. (2018). That is, when shortage happens, available supply is allocated with priority to the consumers who have higher valuations. Clearly, this allocation rule leads to the highest possible consumer surplus under the ex ante pricing scheme. Next, under the responsive pricing scheme, for any given q_r together with the revenue maximizing pricing decision captured in Lemma 1, the consumer surplus, CS_r, is defined by:

$$CS_r(q_r) = E_\xi \left(\mathbb{1}_{\{q_r\xi > d^*\}} \int_0^{d^*} (p(x) - p(d^*))dx + \mathbb{1}_{\{q_r\xi < d^*\}} \int_0^{q_r\xi} (p(x) - p(q_r\xi))dx \right), \tag{4}$$

where $\mathbb{1}_{\{\}}$ is the indicator function and d^* is the revenue maximizing quantity. Plugging the firm's optimal decisions under each pricing scheme, we have the equilibrium consumer surplus as $CS_a^* = CS_a(p_a^*, q_a^*)$ and $CS_r^* = CS_r(q_r^*)$, respectively. To quantify the joint impact of price postponement and risk aversion on consumer surplus, we define the change of consumer surplus (CCS) as $CS_r^* - CS_a^*$. Proposition 7 characterizes the impact of risk aversion on CCS under the linear demand function and uniform yield distribution, and its robustness is confirmed in Section 6 numerically.

PROPOSITION 7. *Assume $d(p) = a - bp$ and $\xi \sim$ Uniform $[0,1]$. For any $c \in [0, \frac{a}{2b}]$ and $\eta \in [\eta^*, 1]$: (i) CS_a^* increases in η. CS_r^* increases in $\eta \in [\eta^*, \min\{\frac{4bc}{a}, 1\}]$ and decreases otherwise. (ii) $CCS > 0$.*

Note that the higher the sales price, the lower the consumer surplus, because both individual consumer's gain and total sales decrease in sales price. Such intuition is helpful in explaining the results stated in Proposition 7. Specifically, part (i) shows that risk aversion may have different impacts on consumer surplus under different pricing schemes. On the one hand, for ex ante pricing, Corollary 1(ii) shows that p_a^* always decreases in η, which leads to a higher consumer surplus as η increases. On the other hand, for responsive pricing, the sales price is undetermined ex ante but is closely related to the firm's production quantity. A lower [higher] production quantity is more likely to result in a higher [lower] ex post sales price after any yield realization, which, in return, decreases [increases] consumer surplus. Corollary 4 shows that when the production cost is low, the impact of risk aversion on a responsive pricing firm's optimal production quantity is non-monotonic with larger quantities produced when the firm has a medium risk aversion level, leading to possibly lower ex post sales prices and higher consumer surplus for moderately risk averse firms. In contrast, when production cost is high, the responsive pricing firm produces more as it becomes less risk averse, which potentially leads to a lower ex post price and higher consumer surplus.

Next, we investigate how the pricing scheme choice of a risk averse firm may affect its consumers' overall surplus. Since the responsive pricing scheme better matches supply with demand through ex post revenue maximization with potential inventory withholding, one may, intuitively, conjecture that such a pricing flexibility further squeezes consumers' net welfare and results in a lower overall surplus. However, Proposition 7(ii) shows that, contrary to the intuition, responsive pricing always benefits the consumers in equilibrium, regardless of the firm's risk aversion level. The underlying rationale for this interesting result can be explained as follows: On the one hand, Corollary 1(i) shows that the ex ante pricing firm always charges a price higher than the optimal riskless (i.e., deterministic supply) price, which is captured as $p_a^* \geq p_a^d = \frac{a+c/\mu}{2b}$ under the linear demand function $d(p) = a - bp$. This clearly implies that the ex ante pricing firm charges a price higher than the revenue maximizing price charged by the responsive pricing firm, i.e., $p_a^* \geq \frac{a}{2b}$. Consequently, the ex post consumer surplus under responsive pricing is larger when the revenue maximizing price is charged. On the other hand, when the realized supply is insufficient, the responsive pricing firm will charge the inventory clearance price, which is more likely to happen when the initial production quantity is low under a high production cost. Proposition 5(i) shows that with a high production cost, the ex ante pricing firm charges a high sales price to pass the cost to the consumers and produces an even smaller quantity than that under responsive pricing, which may lead to a lower ex post consumer surplus. In addition, we further remark that, given CS_r^* is unimodal in η when c is low, CCS may not be monotonic in η, especially with a low production cost.

Indeed, the fact that CCS>0 continues to hold under any other allocation rule. Note that CS_a in Equation (3) allocates the available supply with priority to the consumers with higher valuations in case of shortage, and represents the largest possible consumer surplus among all allocation rules. In contrast, supply shortage never happens under responsive pricing because the firm either sets the inventory clearance price or withholds part of the available supply. In both cases, the demand is fully satisfied, implying the irrelevance of the allocation rule. Hence, CCS>0 continues to hold.

Although price postponement may lead to a high ex post sales price when facing low yield realization, its overall impact on consumer surplus remains beneficial. This is because the firm may strategically adjust the production quantity in accordance to the chosen pricing scheme, and such adjustment is further governed by the firm's risk aversion level in an intricate way. An immediate managerial implication of Propositions 6 and 7 is that the responsive pricing scheme may lead to a win-win outcome for both the risk averse firm and its consumers. Such a Pareto improvement is more likely to occur when the production cost is low [high] and the firm's risk aversion level is medium [low]. However, the win-win outcome may not be readily induced for a firm with high risk aversion level, especially when price postponement may incur some fixed implementation cost.

6. Numerical Analysis

We now conduct numerical study to confirm the robustness of our main results and obtain additional managerial insights. We focus on linear demand function $d(p)=10-p$ and present our results using Beta yield distribution as it has various kinds of shapes, which represent yields with distinct natures. Specifically, we choose Beta distribution $B(\alpha,\beta)$ with $\alpha=\beta=k$, where $k\in\{0.5,1,1.5,2\}$. The mean of the Beta distribution is equal to 0.5, and variances are $\{0.125,0.0833,0.0625,0.05\}$, respectively. As k increases, the distribution $B(k,k)$ decreases in convex order, i.e., the mean keeps unchanged and the variance decreases. Moreover, the shape of the distribution changes from U-shape to Uniform and then to unimodal with decreasing variance as k increases. To avoid the trivial case of no production, we require $c\le\frac{a}{b}\mu=5$ and pick the production cost $c\in\{0.5,1.5,2.5,3.5,4.5\}$. In what follows, we uncover the impacts of the risk aversion parameter η on the changes of optimal production quantity $q_r^* - q_a^*$, optimal profit $\Pi_r^* - \Pi_a^*$, and optimal consumer surplus $CS_r^* - CS_a^*$, respectively, and investigate how these impacts get affected by the production cost.[4]

We first investigate the impact of risk aversion on $q_r^* - q_a^*$, which is depicted in column (a) of Figure 3. There are several interesting observations. First, consistent with the results in Proposition 5(ii), $q_r^* - q_a^*$ could be non-monotonic in η, which happens when the production cost c is relatively low. In contrast, when c is high, the quantity difference increases when the firm becomes less risk averse. Second, recall from Proposition 5(i), under Uniform yield distribution, there exists a threshold cost $\hat{c}$ such that $q_r^* < q_a^*$ iff $c < \hat{c}$. Since $\hat{c}$ increases in η, the relation between c and $\hat{c}$ may switch as η changes. Figure 3(a1) shows that the quantity difference changes from positive to negative when c is small, whereas Figures 3(a2)-(a5) show that the quantity difference always remains positive when c is large. Finally, Proposition 4 shows that the risk aversion threshold is the same between the two pricing schemes, and decreases as the yield becomes more reliable under uniform yield distribution. Figure 3 complements our theoretical results by showing that as the yield decreases in convex order, the risk aversion threshold also decreases. That is, as shown in Figures 3(a1)-(a5), η^* obtained in $B(k,k)$ distribution decreases as k increases regardless of c.

Next, we study the impacts of risk aversion on the VPP (i.e., $\Pi_r^* - \Pi_a^*$), which are presented in column (b) of Figure 3. Consistent with the result in Proposition 6(i), Figure 3(b1) shows that when c is relatively small (i.e., $c = 0.5$), VPP first increases and then decreases in η. This indicates that, for some fixed implementation cost, only firms with intermediate level of risk aversion may find it worthwhile to deploy the price postponement strategy. In contrast, Figures 3(b2)-(b5) show that when c is large (i.e., $c \in \{1.5, 2.5, 3.5, 4.5\}$), VPP always increases in η, indicating that the CVaR improvement from price postponement is always more significant when the firm is less risk averse. Moreover, although each firm obtains a higher CVaR when the yield becomes less variable, the impact of yield variability on VPP remains undetermined, especially when c is low. In contrast, for a high production cost, Figures 3(b2)-(b5) imply that the benefit of responsive pricing tends to be more significant when the yield distribution is less variable. That is, a lower yield variability strengthens the value of responsive pricing and leads to a higher CVaR improvement for any η.

Finally, we investigate the impact of risk aversion on the CCS (i.e., $CS_r^* - CS_a^*$) in column (c) of Figure 3. The trends observed are consistent with our analytical result in Proposition 7(ii): the effect of risk aversion on CCS could be non-monotonic, especially when c is low. In contrast, with a high cost, Figure 3(c5) shows that the consumers benefit more from responsive pricing as the firm becomes less risk averse. Moreover, Figures 3(c1)-(c5) further confirm that CCS is positive, which implies that price postponement always improves consumer surplus. Finally, the impact of

[4] Note that the observations presented in this section are robust to different model parameter combinations and yield distributions, including Uniform, truncated Normal, etc.

yield variability on CCS is involved when c is low (i.e., $c \in \{0.5, 1.5\}$). However, when c is high (i.e., $c \in \{2.5, 3.5, 4.5\}$), a lower yield variability always strengthens consumer surplus gain.

7. Additional Discussions

Now, we provide discussions on some model assumptions and potential extensions. Specifically, we discuss alternative risk aversion criteria in Section 7.1, general cost structure in Section 7.2, random capacity model in Section 7.3, and demand uncertainty in Section 7.4, respectively.

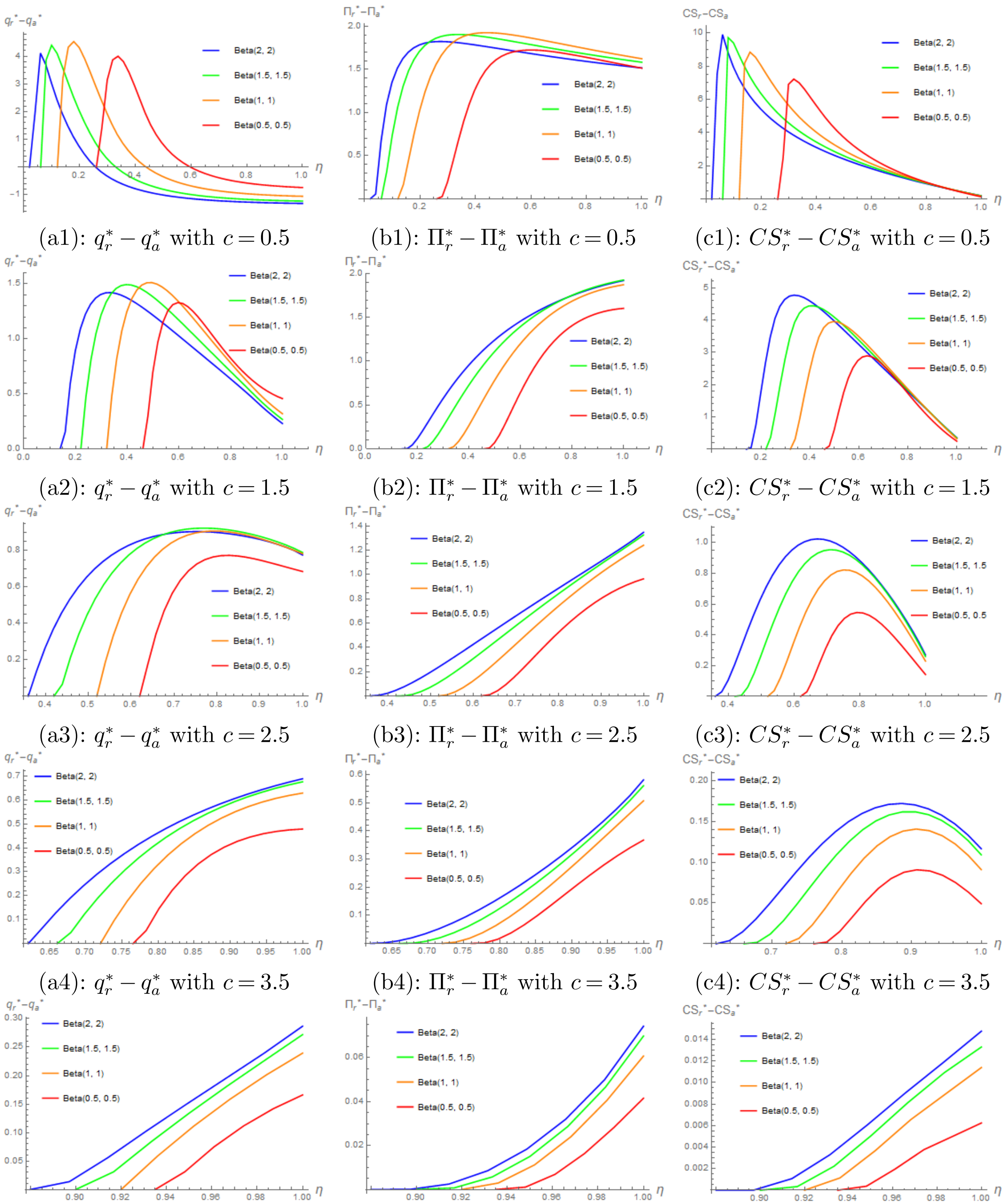
7.1. Alternative Risk Aversion Criteria

We adopt the CVaR objective in our model, which is only one way to capture a decision maker's risk aversion attitude. There are many alternative measures, among which the expected utility and the mean-variance criteria are widely adopted. Different from the CVaR criterion, which ignores the profit above the η-quantile level, the expected utility criterion considers the whole spectrum of the profit with amplified concerns on the low realizations. In contrast, the mean-variance criterion shifts the focus to balancing the expected value and variability of the profit. In the remainder of this subsection, we discuss how the choice of risk aversion criterion affects our main results.

The expected utility criterion: Let $U(\pi)$ be the firm's utility as a function of its ex-post profit π, which satisfies the properties that $U' > 0$ and $U'' \leq 0$. The firm optimizes its decisions by maximizing the expected utility $E_\xi U(\pi_i)$, where ex-post profit π_i, $i \in \{a, r\}$, is given in Equations (1) and (2), respectively. On the one hand, for ex-ante pricing, Kazaz and Webster (2015) provide a sophisticated condition for the objective to be jointly concave. They show that, if the objective is supermodular, then a risk averse firm charges a higher price and produces a larger quantity than those under risk neutrality, which is consistent with our prediction in Corollary 1(i) for the pricing part and Corollary 2(i) for the quantity part with high cost. They further remark that without such regulation condition, the impact of risk aversion can go either way due to the complexity of the problem. On the other hand, for responsive pricing, Lemma 3 shows that the problem is well behaved and the firm always produces given $p_0 \geq \frac{c}{\mu}$. In addition, the comparison of the optimal quantity with that in the riskless case shares the same trend as the one under the CVaR criterion.

LEMMA 3. *For the responsive pricing scheme, $EU(\pi_r(q_r))$ is continuously differentiable and concave in q_r with $q_r^* \geq 0$ given $p_0 \geq \frac{c}{\mu}$. In addition, Corollary 3(i) continues to hold.*

To investigate the impact of price postponement and risk aversion, we conduct numerical analysis using the exponential utility function family $U(\pi) = \frac{1-e^{-a\pi}}{a}$, in which the firm is risk averse [risk seeking] when $a > 0$ [when $a < 0$] and is risk neutral when $a \to 0$. As we only focus on the risk averse situations, the utility formula can be simplified to $U(\pi) = 1 - e^{-a\pi}$, which has constant absolute risk aversion level, i.e., $\frac{-U(\pi)''}{U(\pi)'} = a$ and the firm exhibits a higher absolute risk aversion level as



(a1): $q_r^* - q_a^*$ with $c = 0.5$ (b1): $\Pi_r^* - \Pi_a^*$ with $c = 0.5$ (c1): $CS_r^* - CS_a^*$ with $c = 0.5$

(a2): $q_r^* - q_a^*$ with $c = 1.5$ (b2): $\Pi_r^* - \Pi_a^*$ with $c = 1.5$ (c2): $CS_r^* - CS_a^*$ with $c = 1.5$

(a3): $q_r^* - q_a^*$ with $c = 2.5$ (b3): $\Pi_r^* - \Pi_a^*$ with $c = 2.5$ (c3): $CS_r^* - CS_a^*$ with $c = 2.5$

(a4): $q_r^* - q_a^*$ with $c = 3.5$ (b4): $\Pi_r^* - \Pi_a^*$ with $c = 3.5$ (c4): $CS_r^* - CS_a^*$ with $c = 3.5$

(a5): $q_r^* - q_a^*$ with $c = 4.5$ (b5): $\Pi_r^* - \Pi_a^*$ with $c = 4.5$ (c5): $CS_r^* - CS_a^*$ with $c = 4.5$

Figure 3 **Impact of Risk Aversion Parameter η on $q_r^* - q_a^*$ (Column a), $\Pi_r^* - \Pi_a^*$ (Column b) and $CS_r^* - CS_a^*$ (Column c): c increases in $\{0.5, 1.5, 2.5, 3.5, 4.5\}$ from top to bottom.**

a increases. We adopt a similar setup as that in Section 6 with $d(p) = 10 - p$, $\xi \sim$ Uniform$[0,1]$, $c \in \{0.5, 1.5, 2.5, 3.5, 4.5\}$, and $a \in [0.05, 1]$. The results are listed in Figure EC.4 of Appendix EC.4.

We briefly highlight the similarities and the major differences between the expected utility and the CVaR criteria. Consistent with our observations from Figure 3, $q_r^* - q_a^*$, $U_r^* - U_a^*$ and $CS_r^* - CS_a^*$ depicted in Figure EC.4 all show some non-monotonic trends as the firm's absolute risk aversion level a changes, with the peak typically achieved when a is in some intermediate range. In addition, recall that the firm is less risk averse when η becomes larger [a becomes smaller] in the CVaR criterion [the expected utility criterion]. As such, the impacts of risk aversion exhibit qualitatively the same nature under both the CVaR and the expected utility criteria. The only major difference observed is that: When the cost is high and/or the firm exhibits a high absolute risk aversion level, $CS_r^* - CS_a^* < 0$ could happen under the expected utility criterion. In this case, the firm orders a small quantity under both pricing schemes and the quantity difference is almost negligible (i.e., as a decreases from 1, columne (a) in Figure EC.4 shows that $q_r^* - q_a^*$ increases from 0 very slowly). As the optimal quantity is almost the same, the responsive pricing firm could potentially charge a higher inventory clearance price after yield realization, which decreases consumer surplus. This is in contrast to the finding obtained under the CVaR criterion, in which the quantity difference is noticeably changed under the corresponding scenarios (i.e., in column (a) of Figure 3, as η increases from η^*, $q_r^* - q_a^*$ increases sharply from 0), leading to a larger expected consumer surplus under responsive pricing. This difference is mainly driven by the choice of risk aversion measurement: For the expected utility criterion, the firm always produces given $p_0 > \frac{c}{\mu}$ but may produce a minimum amount when it is too risk averse and production inflation is very costly. In contrast, under the CVaR criterion, the firm simply forgoes those unattractive scenarios without producing.

To sum up, the impacts of price postponement under risk aversion are qualitatively consistent under the CVaR and expected utility criteria, as they both amplify the concern of the low profit realizations. On the other hand, they also exhibit some quantitative differences mainly resulting from the facts that the CVaR criterion purely focuses on low profit realizations whereas the expected utility considers the whole spectrum of the profit with primary weights on low profit realizations.

The mean-variance criterion: Let $U(\pi_i) = E\pi_i - \lambda Var(\pi_i)$ be the firm's mean-variance utility function given ex-post profit π_i, $i \in \{a, r\}$, where $\lambda \geq 0$ is the risk aversion parameter measuring the firm's attitude towards profit variability. Rather than amplifying the concern of low profit realizations, the firm optimizes its decisions by striking a balance between the expected profit and the induced profit variability. Such a distinct focus has a profound impact on the firm's production decision. Specifically, under the CVaR criterion, the firm pays particular attention to low profit realizations, which occur when the supply yield is low. To mitigate the underage risk, the firm always inflates the production quantity. In contrast, under the mean-variance criterion, the firm could be conservative to production inflation, since it may amplify the profit variability due to the multiplicative nature of random yield. We show in Lemma 4 that the firm's optimal production may exhibit deflation behavior to control the profit variability, which is in sharp contrast to the inflation behavior uncovered under the CVaR criterion. As such, it is not surprising that the firm under the mean-variance criterion may opt to different operational decisions, leading to distinct implications of price postponement under risk aversion. On the other hand, Lemma 4 shows that the firm's mean-variance objective may not be well behaved under random yield, since the variance term is non-monotonic in the production quantity. We leave the full exploration for future research.

LEMMA 4. *For any given sales price p and demand $d(p)$, the firm's utility $U(\pi(q))$ is continuously differentiable but may not be unimodal, where $\pi(q) = p\min\{d(p), q\xi\} - cq$. Given $p_0 \geq \frac{c}{\mu}$, there exists a $q^* \geq 0$ satisfying the first order condition. In addition, $q^* < d(p)$ may hold.*

7.2. General Cost Structure

Our main model assumes that the firm pays for the production inputs at a unit cost c. As discussed in Section 3, it is possible that the firm may incur additional cost for each successfully produced product after the yield realization. Suppose the firm needs to pay a per unit cost $\bar{c}$ for each delivered product. Then, the firm's total ex-post cost incurred for a production quantity q contingent upon yield realization ξ is $C(q|\xi) = cq + \bar{c}q\xi$ (Deo and Corbett 2009). This general payment scheme is also adopted in a procurement setting, in which the buying firm pays a fraction for the total order submitted and the remaining upon successful delivery (Tang and Kouvelis 2014, Li et al. 2017).

For a risk neutral firm (i.e., $\eta = 1$), this general payment scheme does not affect our analysis and all the results continue to hold. This is because, under risk neutrality, the firm only cares about the expected total cost $E_\xi C(q|\xi) = (c + \bar{c}\mu)q$, which solely depends on the yield mean μ rather than individual yield realization ξ. As such, we can define a new unit production cost $\tilde{c} := (c + \bar{c}\mu)$ and transfer the problem back to the original one, in which the firm pays for all the production inputs at the unit cost $\tilde{c}$ (i.e., $E_\xi C(q|\xi) = \tilde{c}q$). This linear transformation technique has been adopted in the random yield literature to demonstrate the equivalence relationship among various commonly used payment schemes (see, e.g., Li and Zheng 2006, Tang and Kouvelis 2014, for detailed discussions).

In contrast, for a risk averse firm (i.e., $\eta < 1$), when the production cost is contingent upon the yield realization, the firm's problem becomes more involved as the above linear transformation no longer applies. This is because the component $\bar{c}q\xi$ interacts with the revenue term in the CVaR objective and affects the firm's ex-post profit realization. As such, the linear production inflation rule in Proposition 1 may not hold. Though technically challenging, we manage to conduct some preliminary analysis in Appendix EC.5, and show in Lemma 5 that the firm opts to produce nothing if its risk aversion parameter is below a threshold level. Moreover, we show that price postponement does not affect the firm's risk aversion threshold under this general cost structure.

LEMMA 5. *For the general cost structure, there exists a unique risk aversion threshold $\eta_i^* \in (0,1]$ for pricing scheme $i \in \{a,r\}$ such that the firm produces if and only if $\eta \geq \eta_i^*$. Moreover, $\eta_a^* = \eta_r^*$.*

7.3. Random Capacity

In practice, supply shortage may also be driven by other factors rather than random yield. An commonly adopted alternative modeling choice is random capacity, in which the final delivery of the order quantity q is capped by a random capacity level K, i.e., $\min\{q,K\}$ (see Li et al. 2017, for example). In what follows, we demonstrate some subtle differences between the two modeling frameworks, and show that they may lead to distinct managerial implications.

For a risk neutral firm (i.e., $\eta = 1$), we discuss how price postponement affects the optimal ordering policies. For the random yield model, the random factor ξ affects the production quantity q in a multiplicative way. As such, the firm can strategically inflate the production quantity above the price-induced demand $d(p_a^*)$ [revenue maximizing demand d^*] under ex ante [responsive] pricing to partially mitigate the supply risk. Such inflation behavior introduces overage risk and affects the marginal revenue of production quantity, leading to a two-fold impact of price postponement. That is, as shown in Proposition 5(i), responsive pricing reduces [increases] production quantity when the cost is low [high]. In contrast, for the random capacity model, the random capacity K affects q via an additive way and there is no need for production inflation. As shown in Li et al. (2017), the firm produces the price-induced demand under ex ante pricing, and no larger than the revenue maximizing demand under responsive pricing. In this case, the overage risk is absent, and the responsive pricing firm sets the inventory clearance price to better match supply with demand. Such a pricing flexibility always enhances the marginal value of inventory and induces a larger production quantity, which contrasts the result from the random yield model in Proposition 5(i).

In contrast, for a risk averse firm (i.e., $\eta < 1$), subtle differences on the impacts of risk aversion exist under the two supply models. To illustrate, consider the setting in Proposition 1 with supply risk replaced by random capacity K. The firm chooses production quantity q to maximize its CVaR given the price-induced demand $d(p)$, where $\text{CVaR}(\pi(q)) = \max_{v \in R}\{v + \frac{1}{\eta} E \min\{\pi(q) - v, 0\}\}$ and $\pi(q) = p\min\{q, d(p), K\} - cq$. The firm's optimal ordering decision is given by the following lemma.

LEMMA 6. *Given sales price $p > c$ and price-induced demand $d(p)$, for any $\eta \in (0,1]$, the firm's optimal production quantity is:*

$$q^*(p) = \begin{cases} d(p) & \textit{if} \;\; d(p) \leq F^{-1}(\frac{\eta(p-c)}{p}), \\ F^{-1}(\frac{\eta(p-c)}{p}) & \textit{otherwise} \; , \end{cases}$$

where $F(K)$ is the cdf of random capacity K. Clearly, $q^*(p) \leq d(p)$ and increases in η.

Lemma 6 draws some interesting comparisons with its counterpart under the random yield model. For the random capacity model, given a positive margin (i.e., $p > c$), the firm always chooses to produce regardless of its risk aversion level η. Moreover, the firm always produces no larger than the price-induced demand $d(p)$ and the less risk averse firm produces more. In contrast, for the random yield model, Proposition 1 shows that the firm produces if and only if the sales price is high enough, or equivalently, the firm is not too risk averse. In addition, whenever the firm chooses to produce, it always inflates its production over $d(p)$ and the less risk averse firm produces less. The above differences are driven by the source of supply uncertainty and the fact that the CVaR criterion focuses on the low profit realizations. For the random capacity [yield] model, low profit occurs when the realized capacity [yield] is low, so a more risk averse firm reduces [increases] production quantity to avoid excess spending [hedge against shortage]. To sum up, the above discussions clearly demonstrate the fundamental differences between the two supply uncertainty models. We leave the full exploration of the random capacity model as a future research direction.

7.4. Random Demand

Our model focuses on supply random yield and treats demand as deterministic, which helps us single out the unique impact of yield uncertainty on a risk averse firm's operational decisions. In contrast, demand uncertainty is also a common source of risk in many sectors. In the literature, a risk averse firm's pricing and production decision under demand uncertainty has been investigated, e.g., Agrawal and Seshadri (2000) and Kazaz and Webster (2015) for a general concave utility function with and without emergent supply, respectively, and Chen et al. (2009) for a CVaR objective. We now compare our findings and the corresponding ones under demand uncertainty and explain how the uncertainty source interacts with risk aversion and affects the firm's decisions.

For demand uncertainty, under mild regulation conditions, a risk averse firm charges a lower sales price and/or orders less quantity compared to those under risk neutrality (Agrawal and Seshadri 2000 for the additive model, Kazaz and Webster 2015). Under the CVaR criterion, Chen et al. (2009) show that a risk averse firm always produces given a positive margin and a more risk averse firm charges a lower sales price. However, the impact of risk aversion on the optimal production quantity is involved, as it depends on how demand uncertainty is modeled (multiplicative vs. additive). In contrast, for yield uncertainty, we show that, under the CVaR criterion, only the firm with relatively low risk averse level chooses to produce. Compared to the riskless case, the firm charges a higher sales price and produces more [less] if it is less [more] risk averse. Moreover, as the firm becomes less risk averse, it reduces the sales price but may increase the production quantity.

The above comparisons show some subtle differences between demand and yield uncertainties. The underlying intuition is: Both the CVaR and the expected utility criteria amplify the concern on low profit realization and generate the decision rules to alleviate such concern. For demand uncertainty, low profit occurs when the demand realization is low, which reduces revenue and generates potential excess inventory. As such, the risk averse firm mitigates the overage risk by boosting demand via a low sales price and/or reducing the production. Moreover, given positive margin, the firm always opts to produce regardless of its risk aversion level. In contrast, for yield uncertainty, low yield realization leads to downside profit shock due to insufficient supply, which generates low revenue and potential lost sales. Consequently, a more risk averse firm would charge a higher sales price to limit demand. In addition, given the multiplicative nature of random yield, the firm would inflate the production above the price-induced demand to hedge against underage risk, and a more risk averse firm favors a larger inflation rate $\frac{1}{\delta}$. Such inflation behavior may intensify the overage risk due to high yield realization and result in inventory disposal. Hence, some firms forgo this costly endeavor by producing nothing, especially for those who are highly risk averse. In sum, the source of uncertainty has a profound impact on a risk averse firm's operational decisions.

8. Conclusion

Price postponement is an effective mechanism to hedge against the adverse effect of supply random yield. However, its value and the resulting operational decisions have not been studied for risk averse firms. We study the interaction between risk aversion and price postponement under supply yield risk. Among other results, we show that price postponement does not affect a firm's risk aversion threshold. That is, all else being equal, there exists a unique risk aversion threshold under which firms with higher risk aversion (i.e., lower η) choose not to produce under both pricing schemes. We further show that the value of CVaR improvement due to price postponement may not be monotonic in the firm's risk aversion level, which implies that a risk averse firm should be cautious when implementing the price postponement strategy as the gain may not necessarily cover the fixed postponement cost. In addition, our results show that responsive pricing, albeit its ex post revenue maximizing behavior, benefits the end market consumers in equilibrium.

To conclude, our work can be further explored in several other directions. First, it would be interesting to understand the impact of risk aversion on the value and adoption of other supply risk mitigation tools such as supply diversification. Second, including an independent supplier could help understand the effect of double marginalization on price postponement for risk averse firms.

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Appendices to "Role of Risk Aversion in Price Postponement under Supply Random Yield"

The appendices consist of five parts. Appendix EC.1 provides the proofs for all the results in the main paper. Appendix EC.2 discusses both the pricing and the quantity effects uncovered in Section 5.3. Appendix EC.3 presents some fundamental properties of the CVaR criterion that will be used to facilitate the analysis. Appendix EC.4 illustrates the numerical results from the expected utility criterion discussed in Section 7.1. Finally, appendix EC.5 provides additional analysis for the general cost structure from Section 7.2.

EC.1. Proofs of Statements

Proof of Proposition 1: We prove **part (i)** and **part (ii)** together. For any given sales price p_a and production quantity q_a, define $F(v)=v+\frac{1}{\eta}E_\xi \min\{\pi_a(p_a,q_a)-v,0\}$. There are three cases to consider.

Case 1: If $q_a \le \frac{d(p_a)}{u}$, regardless of the realization of ξ, we have $\pi_a(p_a,q_a)=(p_a\xi-c)q_a$. In this case, $\pi_a(p_a,q_a)-v\le 0$ if and only if (iff) $\xi \le \frac{cq_a+v}{p_aq_a}$. Then, we have:

$$F(v)=\begin{cases} v, & \text{if } v\in(-\infty,(p_al-c)q_a], \\ F_1^1(v):=v+\frac{1}{\eta}\int_l^{\frac{cq_a+v}{p_aq_a}}[(p_a\xi-c)q_a-v]dG(\xi), & \text{if } v\in((p_al-c)q_a,(p_au-c)q_a], \\ F_1^2(v):=v+\frac{1}{\eta}\int_l^{u}[(p_a\xi-c)q_a-v]dG(\xi), & \text{if } v\in((p_au-c)q_a,\infty]. \end{cases}$$

Easy to check that $F(v)$ is continuously differentiable and concave in v, with $\frac{\partial}{\partial v}F(v)|_{v=(p_al-c)q_a}=1>0$ and $\frac{\partial}{\partial v}F(v)|_{v=(p_au-c)q_a}=1-\frac{1}{\eta}\le 0$. Thus, v^* is defined by the first order condition (FOC), i.e., $\frac{\partial}{\partial v}F(v)=0$, which implies that $v^*=(p_aG^{-1}(\eta)-c)q_a$ and $F(v^*)=\frac{1}{\eta}\int_l^{G^{-1}(\eta)}(p_a\xi-c)q_adG(\xi)$.

Case 2: If $q_a\in\left(\frac{d(p_a)}{u},\frac{d(p_a)}{l}\right]$, we can write $F(v)$ using similar comparisons as in Case 1:

$$F(v)=\begin{cases} v, & \text{if } v\in(-\infty,(p_al-c)q_a], \\ F_2^1(v):=v+\frac{1}{\eta}\int_l^{\frac{cq_a+v}{p_aq_a}}[(p_a\xi-c)q_a-v]dG(\xi), & \text{if } v\in((p_al-c)q_a,p_ad(p_a)-cq_a], \\ F_2^2(v):=v+\frac{1}{\eta}(\int_l^{\frac{d(p_a)}{q_a}}[(p_a\xi-c)q_a-v]dG(\xi)+\int_{\frac{d(p_a)}{q_a}}^u[p_ad(p_a)-cq_a-v]dG(\xi)), & \text{if } v\in(p_ad(p_a)-cq_a,\infty]. \end{cases}$$

Easy to check that $F(v)$ is continuous. Moreover, $\frac{\partial}{\partial v}F_2^1(v)|_{v=p_ad(p_a)-cq_a}=1-\frac{1}{\eta}\int_l^{\frac{d(p_a)}{q_a}}dG(\xi)>\frac{\partial}{\partial v}F_2^2(v)|_{v=p_ad(p_a)-cq_a}=1-\frac{1}{\eta}$. Morover, $\frac{\partial}{\partial v}F_2^2(v)=1-\frac{1}{\eta}<0$, $\forall v\in(p_ad(p_a)-cq_a,\infty]$. Hence, the optimal v^* belongs to the interval $((p_al-c)q_a,p_ad(p_a)-cq_a]$. Easy to see $\frac{\partial}{\partial v}F_2^1(v)|_{v=p_ad(p_a)-cq_a}$ increases in q_a, thus, there exists a unique $\bar{q}_a:=\frac{d(p_a)}{G^{-1}(\eta)}$ such that $\frac{\partial}{\partial v}F_2^1(v)|_{v=p_ad(p_a)-c\bar{q}_a}=0$, with $\bar{q}_a\in[\frac{d(p_a)}{u},\frac{d(p_a)}{l})$. When $q_a<\bar{q}_a$, v^* is defined by $\frac{\partial}{\partial v}F_2^1(v)=0$, which implies $v^*=(p_aG^{-1}(\eta)-c)q_a$ and $F(v^*):=F_2^1(v^*)=\frac{1}{\eta}\int_l^{G^{-1}(\eta)}(p_a\xi-c)q_adG(\xi)$. When $q_a>\bar{q}_a$, $v^*=p_ad(p_a)-cq_a$ and $F(v^*):=F_2^2(v^*)=p_ad(p_a)-cq_a+\frac{1}{\eta}\int_l^{\frac{d(p_a)}{q_a}}[p_a\xi q_a-p_ad(p_a)]dG(\xi)$.

Case 3: If $q_a \in \left(\frac{d(p_a)}{l}, \infty\right]$, then regardless of the realization of ξ, we have $\pi_a(p_a, q_a) = p_a d(p_a) - cq_a$. $\pi_a(p_a, q_a) - v \leq 0$ if and only if (iff) $v \geq p_a d(p_a) - cq_a$. Easy to calculate in this case $v^* = p_a d(p_a) - cq_a$ and $F(v^*) = p_a d(p_a) - cq_a$ decreases in q_a.

Combining all above cases, for any given sales price p_a, we can express the firm's CVaR as a function of q_a as follow:

$$\Pi_a(q_a|p_a) := F(v^*) = \begin{cases} \frac{1}{\eta}\int_l^{G^{-1}(\eta)}(p_a\xi - c)q_a dG(\xi), & \text{if } q_a \in (-\infty, \bar{q}_a], \\ p_a d(p_a) - cq_a + \frac{1}{\eta}\int_l^{\frac{d(p_a)}{q_a}}[p_a\xi q_a - p_a d(p_a)]dG(\xi), & \text{if } q_a \in \left(\bar{q}_a, \frac{d(p_a)}{l}\right], \\ p_a d(p_a) - cq_a, & \text{if } q_a \in \left(\frac{d(p_a)}{l}, \infty\right). \end{cases}$$

When $q_a = \bar{q}_a$, we have $p_a d(p_a) - c\bar{q}_a = G^{-1}(\eta)p_a\bar{q}_a - c\bar{q}_a$ and, thus, $\lim_{q_a \to \bar{q}_a^-}\Pi_a(q_a|p_a) = \lim_{q_a \to \bar{q}_a^+}\Pi_a(q_a|p_a)$. When $q_a = \frac{d(p_a)}{l}$, we have $p_a d(p_a) - c\bar{q}_a = G^{-1}(\eta)p_a\bar{q}_a - c\bar{q}_a$ and, thus, $\lim_{q_a \to \frac{d(p_a)}{l}^-}\Pi_a(q_a|p_a) = \lim_{q_a \to \frac{d(p_a)}{l}^+}\Pi_a(q_a|p_a)$. Consequently, $\Pi_a(q_a|p_a)$ is continuous in q_a. Moreover, $\lim_{q_a \to \bar{q}_a^-}\frac{\partial}{\partial q_a}\Pi_a(q_a|p_a) = \frac{1}{\eta}\int_l^{G^{-1}(\eta)}(p_a\xi - c)dG(\xi) = -c + \frac{p_a}{\eta}\int_l^{\frac{d(p_a)}{\bar{q}_a}}\xi dG(\xi) = \lim_{q_a \to \bar{q}_a^+}\frac{\partial}{\partial q_a}\Pi_a(q_a|p_a)$, and $\lim_{q_a \to \frac{d(p_a)}{l}^-}\frac{\partial}{\partial q_a}\Pi_a(q_a|p_a) = -c = \lim_{q_a \to \frac{d(p_a)}{l}^+}\frac{\partial}{\partial q_a}\Pi_a(q_a|p_a)$. Hence, $\Pi_a(q_a|p_a)$ is continuously differentiable.

When $q_a > \bar{q}_a$, $\Pi_a(q_a|p_a)$ is concave since $\frac{\partial^2}{\partial q^2}\Pi_a(q_a|p_a) = -\frac{p_a d(p_a)^2}{\eta q_a^3}g(\frac{d(p_a)}{q_a}) < 0$. And $\lim_{q_a \to \infty}\frac{\partial}{\partial q_a}\Pi_a(q_a|p_a) = -c < 0$. When $q_a \leq \bar{q}_a$, $\frac{\partial}{\partial q_a}\Pi_a(q_a|p_a) = \frac{1}{\eta}\int_l^{G^{-1}(\eta)}(p_a\xi - c)dG(\xi)$, which is a constant independent of q_a, and its sign determines the optimal production quantity q_a^*. In particularly, if $\frac{1}{\eta}\int_l^{G^{-1}(\eta)}(p_a\xi - c)dG(\xi) \leq 0$ or $p_a \leq \frac{c\eta}{\int_l^{G^{-1}(\eta)}\xi dG(\xi)}$, $\Pi_a(q_a|p_a)$ decreases in q_a with $q_a^* = 0$. If $\frac{1}{\eta}\int_l^{G^{-1}(\eta)}(p_a\xi - c)dG(\xi) > 0$ or $p_a > \frac{c\eta}{\int_l^{G^{-1}(\eta)}\xi dG(\xi)}$, $\Pi_a(q_a|p_a)$ is concave in q_a with $q_a^* \in [\bar{q}_a, \frac{d(p_a)}{l}]$ defined by $q_a^* = \frac{d(p_a)}{\delta}$, where δ satisfies $\int_l^{\delta}\xi dG(\xi) = \frac{c\eta}{p_a}$. Let $\tilde{p} := \min\{p_0, \frac{c\eta}{\int_l^{G^{-1}(\eta)}\xi dG(\xi)}\}$ and plug q_a^* into $\Pi_a(p_a, q_a)$, we have:

$$\Pi_a(p_a, q_a^*(p_a)) = \begin{cases} p_a d(p_a)(1 - \frac{G(\delta)}{\eta}) & \text{if } \; p_a \in [\tilde{p}, p_0] \\ 0 & \text{if } \; p_a \in \left[\frac{c}{\mu}, \tilde{p}\right). \end{cases}$$

At last, since δ satisfies $\int_l^{\delta}\xi dG(\xi) = \frac{c\eta}{p_a}$, it is immediate that δ strictly increases in η and strictly decreases in p_a. Moreover, $q_a^* = \frac{d(p_a)}{\delta}$ strictly decreases in η. □

Proof of Proposition 2: We prove **Parts (i) and (ii)** together. From Proposition 1, we have

$$\Pi_a(p_a|q_a^*(p_a)) = \begin{cases} p_a d(p_a)(1 - \frac{G(\frac{d(p_a)}{q_a^*(p_a)})}{\eta}) & \text{if } p_a \in \left(\min\left\{p_0, \frac{c\eta}{\int_l^{G^{-1}(\eta)}\xi dG(\xi)}\right\}, p_0\right] \\ 0 & \textit{otherwise.} \end{cases}$$

We first focus on the case when $\frac{c\eta}{\int_l^{G^{-1}(\eta)}\xi dG(\xi)} < p_0$ (at the end of the proof we will provide condition under which this holds). Let $\delta = \frac{d(p_a)}{q_a^*(p_a)}$ and from the proof of Proposition 1, it is clear that $\delta \in$

$[l,u]$. When $p \in \left(\frac{c\eta}{\int_l^{G^{-1}(\eta)} \xi dG(\xi)}, p_0\right]$, we have $\int_l^\delta \xi dG(\xi) = \frac{\eta c}{p_a} < \int_l^{G^{-1}(\eta)} \xi dG(\xi)$, which implies that $\delta < G^{-1}(\eta)$ or $G(\delta) < \eta$. In this case, $\Pi_a(p_a|q_a^*(p_a)) = p_a d(p_a)(1 - \frac{1}{\eta}G(\delta))$, and taking derivative, we have:

$$\begin{aligned}
\frac{\partial}{\partial p_a}\Pi_a(p_a|q_a^*(p_a)) &= [p_a d'(p_a) + d(p_a)]\left(1 - \frac{G(\delta)}{\eta}\right) + p_a d(p_a)\left(-\frac{g(\delta)}{\eta}\right)\frac{\partial \delta}{\partial p_a} \\
&= [p_a d'(p_a) + d(p_a)]\left(1 - \frac{G(\delta)}{\eta}\right) + d(p_a)\frac{\int_l^\delta \xi g(\xi) d\xi}{\delta \eta} \\
&= d(p_a)\left(1 - \frac{G(\delta)}{\eta}\right)\left(\frac{p_a d'(p_a)}{d(p_a)} + 1 + \frac{\int_l^\delta \xi g(\xi) d\xi}{\delta(\eta - G(\delta))}\right)
\end{aligned}$$

When $p_a = \frac{c\eta}{\int_l^{G^{-1}(\eta)} \xi dG(\xi)}$, we have $\delta = G^{-1}(\eta)$ and $\frac{\partial}{\partial p_a}\Pi_a(p_a|q_a^*(p_a))|_{p_a = \frac{c\eta}{\int_l^{G^{-1}(\eta)} \xi dG(\xi)}} = d(p_a)\frac{\int_l^\delta \xi g(\xi) d\xi}{\delta \eta} > 0$. When $p_a = p_0$, we have $d(p_a) = 0$ and $\frac{\partial}{\partial p_a}\Pi_a(p_a|q_a^*(p_a))|_{p_a = p_0} = p_a d'(p_a)\left(1 - \frac{G(\delta)}{\eta}\right) < 0$. Due to continuity, there must exist at least one $p_a^* \in \left(\frac{c\eta}{\int_l^{G^{-1}(\eta)} \xi dG(\xi)}, p_0\right)$ such that $\frac{\partial}{\partial p_a}\Pi_a(p_a|q_a^*(p_a))|_{p_a = p_a^*} = 0$. Moreover, $p_a^* < p_0$ implies $d(p_a^*) > 0$ and $p_a^* > \frac{c\eta}{\int_l^{G^{-1}(\eta)} \xi dG(\xi)}$ implies $1 - \frac{G(x)}{\eta}|_{p_a = p^*} > 0$. Thus, p_a^* must satisfy $\frac{p_a d'(p_a)}{d(p_a)} + 1 + \frac{\int_l^x \xi g(\xi) d\xi}{x(\eta - G(x))} = 0$. Evaluating the second order derivative at $p_a = p_a^*$, we have

$$\frac{\partial^2}{\partial p_a^2}\Pi_a(p_a|q_a^*(p_a))|_{p_a = p_a^*} = d(p_a^*)\left(1 - \frac{G(x(p_a^*))}{\eta}\right)\left(\frac{\partial}{\partial p_a}\frac{p_a d'(p_a)}{d(p_a)}|_{p_a = p_a^*} + \frac{\partial}{\partial p_a}\frac{\int_l^x \xi g(\xi) d\xi}{x(\eta - G(x))}|_{p_a = p_a^*}\right)$$

When demand has the IPE property, $\frac{\partial}{\partial p_a}\frac{p_a d'(p_a)}{d(p_a)} \le 0$ for any p_a. Thus, to show $\frac{\partial^2}{\partial p_a^2}\Pi_a(p_a|q_a^*(p_a))|_{p_a = p_a^*} < 0$, it is sufficient to show $\frac{\partial}{\partial p_a}\frac{\int_l^\delta \xi g(\xi) d\xi}{\delta(\eta - G(\delta))}|_{p_a = p_a^*} < 0$, which is equivalent to $\frac{\partial}{\partial \delta}\frac{\int_l^\delta \xi g(\xi) d\xi}{\delta(\eta - G(\delta))}|_{\delta = \delta^*} > 0$ since δ decreases in p_a. Next, we show that if the yield distribution satisfies Assumption 1(a), i.e., $\frac{\int_l^x \xi g(\xi) d\xi}{x(1 - G(x))}$ increases in $x \in [l,u]$, then $\frac{\int_l^x \xi g(\xi) d\xi}{x(\eta - G(x))}$ increases in $x \in [l, G^{-1}(\eta)]$. Taking derivative, we have

$$\frac{\partial}{\partial x}\frac{\int_l^x \xi g(\xi) d\xi}{x(\eta - G(x))} = \frac{(\eta - G(x))\left(x^2 g(x) - \left(1 - \frac{x g(x)}{\eta - G(x)}\right)\int_l^x \xi dG(\xi)\right)}{x^2(\eta - G(x))^2}$$

Define $A(\eta) = x^2 g(x) - \left(1 - \frac{x g(x)}{\eta - G(x)}\right)\int_l^x \xi dG(\xi)$, which decreases in η. $\forall \eta \in (0,1]$, $A(\eta) > A(1) > 0$, where $A(1) > 0$ comes from Assumption 1(a). Thus, $\frac{\int_l^x \xi g(\xi) d\xi}{x(\eta - G(x))}$ increases in $x \in [l, G^{-1}(\eta)]$, which implies that $\frac{\partial}{\partial p_a}\frac{\int_l^\delta \xi g(\xi) d\xi}{\delta(\eta - G(\delta))} < 0$. As a consequence, $\frac{\partial^2}{\partial p_a^2}\Pi_a(p_a|q_a^*(p_a))|_{p_a = p_a^*} < 0$, and $\Pi_a(p_a|q_a^*(p_a))$ is quasi-concave in p_a with the unique p_a^* defined by the FOC.

Finally, we need to check when $\frac{c\eta}{\int_l^{G^{-1}(\eta)} \xi dG(\xi)} < p_0$ holds. Define $g(\eta) := \frac{\int_l^{G^{-1}(\eta)} \xi dG(\xi)}{\eta}$. Easy to see $g'(\eta) = \frac{\eta G^{-1}(\eta) - \int_l^{G^{-1}(\eta)} \xi dG(\xi)}{\eta^2} > 0$. Moreover, $g(1) = \mu \ge \frac{c}{p_0}$ and $g(0) = G^{-1}(0) = l$ by applying the L' Hospital's rule. If $l \le \frac{c}{p_0}$, there exists a unique $\eta_a^*(p_0, c) > 0$ such that $\frac{c\eta}{\int_l^{G^{-1}(\eta)} \xi dG(\xi)} < p_0$ holds iff $\eta > \eta_a^*(p_0, c)$. And if $l > \frac{c}{p_0}$, $\forall \eta > 0 := \eta_r^*(p_0, c)$, $\frac{c\eta}{\int_l^{G^{-1}(\eta)} \xi dG(\xi)} < p_0$ holds. Combining all the above arguments, parts (i) and (ii) of Proposition 2 are proved. □

Proof of Corollary 1: Part (i): under the IPE assumption, $(p-c/\mu)d(p)$ is quasi-concave (see Chen et al. 2009). Thus, $p=p_a^d$ is defined by $(p-c/\mu)d'(p)+d(p)=0$. If $p_a^d<\tilde{p}$, $p_a^d<p_a^*$ automatically holds. Otherwise, p_a^d satisfies $\frac{c}{p_a^d}=\frac{\mu(p_a^d d'(p_a^d)+d(p_a^d))}{p_a^d d'(p_a^d)}$, and we have $G(\delta_{p_a^d})<\eta$, since $p_a^d>\tilde{p}$. From the proof of Proposition 2, the FOC is:

$$\frac{\partial}{\partial p_a}\Pi_a(p_a|q_a^*(p_a))=d(p_a)\left(1-\frac{G(\delta)}{\eta}\right)\left(\frac{p_a d'(p_a)}{d(p_a)}+1+\frac{\int_l^\delta \xi dG(\xi)}{\delta(\eta-G(\delta))}\right).$$

Thus, plugging p_a^d into $\frac{\partial}{\partial p_a}\Pi_a(p_a|q_a^*(p_a))$, we only need to check the sign of $A(p_a):=\frac{p_a d'(p_a)}{d(p_a)}+1+\frac{\int_l^\delta \xi dG(\xi)}{\delta(\eta-G(\delta))}$ at point $p_a=p_a^d$. For simplicity, we denote δ_d as satisfying $\int_l^{\delta_d}\xi dG(\xi)=\frac{c\eta}{p_a^d}$, which is a function of p_a^d in the following analysis.

$$\begin{aligned}A(p_a^d)&=\frac{p_a^d d'(p_a^d)}{d(p_a^d)}+1+\frac{\int_l^{\delta_d}\xi dG(\xi)}{\delta_d(\eta-G(\delta_d))}=\frac{p_a^d d'(p_a^d)}{d(p_a^d)}+1+\frac{c\eta}{p_a^d\delta_d(\eta-G(\delta_d))}\\&=\frac{(p_a^d d'(p_a^d)+d(p_a^d))\left(p_a^d d'(p_a^d)\delta_d\left(1-\frac{G(\delta_d)}{\eta}\right)+\mu d(p_a^d)\right)}{d(p_a^d)p_a^d d'(p_a^d)\delta_d\left(1-\frac{G(\delta_d)}{\eta}\right)},\end{aligned}$$

where the last equality comes from the fact that $\frac{c}{p_a^d}=\frac{\mu(p_a^d d'(p_a^d)+d(p_a^d))}{p_a^d d'(p_a^d)}$. When $p=p_a^d$, we have $p_a^d d'(p_a^d)+d(p_a^d)=\frac{c}{\mu}d'(p_a^d)<0$. We can write $A(p_a^d)=K\left(p_a^d d'(p_a^d)\delta_d\left(1-\frac{G(\delta_d)}{\eta}\right)+d(p_a^d)\right)$, where $K:=\frac{(p_a^d d'(p_a^d)+d(p_a^d))}{d(p_a^d)p_a^d d'(p_a^d)\delta_d\left(1-\frac{G(\delta_d)}{\eta}\right)}>0$. Next, since $\frac{c\eta}{p_a^d}=\int_l^{\delta_d}\xi dG(\xi)=\delta_d G(\delta_d)-\int_l^{\delta_d}G(\xi)d\xi$. Plugging into $A(p_a^d)$, it is immediate that

$$A(p_a^d)=p_a^d d'(p_a^d)\left(\delta_d-\mu-\frac{1}{\eta}\int_l^{\delta_d}G(\xi)d\xi\right).$$

Thus, we only need to check the sign of $M:=\delta_d-\mu-\frac{1}{\eta}\int_l^{\delta_d}G(\xi)d\xi$. Recall from the proof of Proposition 2, we have $\mu=g(1)\geq g(\eta):=\frac{1}{\eta}\int_l^{G^{-1}(\eta)}\xi dG(\xi)=\frac{1}{\eta}\left(G^{-1}(\eta)\eta-\int_l^{G^{-1}}G(\xi)d\xi\right)$. Plugging into the express of M, we have

$$\begin{aligned}M&\leq\frac{1}{\eta}\left(\delta_d\eta-\int_l^{\delta_d}G(\xi)d\xi-\eta G^{-1}(\eta)+\int_l^{G^{-1}(\eta)}G(\xi)d\xi\right)\\&=(\delta_d-G^{-1}(\eta))+\frac{1}{\eta}\left(\int_{\delta_d}^{G^{-1}(\eta)}G(\xi)d\xi\right)\leq(\delta_d-G^{-1}(\eta))+\frac{1}{\eta}\left(\int_{\delta_d}^{G^{-1}(\eta)}\eta d\xi\right)=0,\end{aligned}$$

where the first equality comes from the fact that $G(\delta_d)<\eta$ and the second inequality comes from the fact that $G(\xi)$ increases in ξ. Thus, $\frac{\partial}{\partial p_a}\Pi_a(p_a|q_a^*(p_a))|_{p_a=p_a^d}\geq 0$ and $p_a^d\leq p_a^*$, due to the unimodality of $\Pi_a(p_a|q_a^*(p_a))$. Next, we show $\Pi_a(p_a^*):=\Pi_a(p_a^*|q_a^*(p_a^*))\leq\Pi_a^d(p_a^d)$. By part (ii), since $\Pi_a(p_a^*)$ increases in η, it is clear that $\Pi_a(p_a^*)|_{\eta<1}<\Pi_a(p_a^*)|_{\eta=1}$. When $\eta=1$, the firm is risk neutral with expected profit concave in ξ. Thus, due to convex order, $\Pi_a(p_a^*)|_{\eta=1} \le \Pi_a^d(p_a^d)$, which concludes the proof.

Part (ii): Let $A(p_a) := \frac{p_a d'(p_a)}{d(p_a)} + 1 + \frac{\int_l^\delta \xi dG(\xi)}{\delta(\eta - G(\delta))}$, where δ is defined by $\int_l^\delta \xi dG(\xi) = \frac{\eta c}{p_a}$. From the proof of Proposition 2, p_a^* is defined by $A(p^*) = 0$ and $\frac{\partial A(p)}{\partial p_a}|_{p_a = p_a^*} < 0$. Thus, the sign of $\frac{\partial p_a^*}{\partial \eta} = -\frac{\frac{\partial A}{\partial \eta}}{\frac{\partial A}{\partial p_a}}|_{p_a = p_a^*}$ is the same as that of $\frac{\partial A}{\partial \eta}|_{p_a = p_a^*}$. For any p_a, $\frac{\partial A(p_a)}{\partial \eta} = \frac{\partial}{\partial \eta} \frac{\int_l^\delta \xi dG(\xi)}{\delta(\eta - G(\delta))} = \frac{\partial}{\partial \eta} \frac{m}{\delta\left(1 - \frac{G(\delta)}{\eta}\right)}$, where $m = \frac{c}{p_a}$. Now, we first check the monotonicity of $\frac{G(\delta)}{\eta}$ on η. Taking derivative, we have:

$$\begin{aligned} \frac{\partial}{\partial \eta} \frac{G(\delta)}{\eta} &= \frac{\eta g(\delta) \frac{\partial \delta}{\partial \eta} - G(\delta)}{\eta^2} = \frac{\int_l^\delta \xi dG(\xi) - \delta G(\delta)}{\delta \eta^2} \\ &= \frac{-\int_l^\delta G(\xi) d\xi}{\delta \eta^2} < 0 \end{aligned}$$

where the last equality comes from integration by parts. Since $\frac{G(\delta)}{\eta}$ strictly decreases in η and δ strictly increases in η, $\delta\left(1 - \frac{G(\delta)}{\eta}\right)$ strictly increases in η, given that $G(\delta) \le \eta$. Thus, $\frac{m}{\delta\left(1 - \frac{G(\delta)}{\eta}\right)}$ strictly decreases in η and $A(p_a)$ strictly decreases in η for any feasible p_a, which implies p_a^* strictly decreases in η. Next, we show that for any $\eta \ge \eta_a^*(p_0, c)$, $\Pi_a(p_a^*) = p_a^* d(p_a^*)(1 - \frac{G(\delta)}{\eta})$ increases in η. Taking derivative, we have $\frac{d\Pi_a(p_a^*)}{d\eta} = \frac{\partial \Pi_a(p_a^*)}{\partial \eta} + \frac{\partial \Pi_a(p_a^*)}{\partial p_a} \frac{\partial p_a^*}{\partial \eta} = \frac{\partial \Pi_a(p_a^*)}{\partial \eta}$ due to the Envelope Theorem. Straightforward calculation shows that $\frac{\partial \Pi_a(p_a^*)}{\partial \eta} > 0$, which concludes the proof. □

Proof of Corollary 2: To simplify the expression, we normalize $b = 1$ without loss of generality. **Part (i):** The optimal production quantity under the linear demand and uniform yield is $q_a^*(\eta) = (a - p_a^*)\sqrt{\frac{p_a^*}{2c\eta}}$, where p_a^* is the largest root of polynomial equation $8\eta p^3 + p^2(-8a\eta - 9c) + p(2a^2\eta + 6ac) - a^2 c = 0$. Taking derivative with the Implicit Function Theorem and after some necessary simplifications, we have q_a^* decreases in η if and only if $c < \hat{c}_a$ and $\eta > \hat{\eta}_a$, where the threshold $\hat{c}_a$ is the smallest real root of polynomial $27c^3 + 9a^2 c - 2a^3 = 0$ and the threshold $\hat{\eta}_a$ is the smallest real root of polynomial equation $2a^3\eta^3 - 9a^2 c\eta^2 - 27c^3 = 0$. **Part (ii):** Simple calculation yields that $p_a^d = \frac{a+2c}{2}$ and $q_a^d = \frac{d(p_a^d)}{\mu} = a - 2c$, which decreases in c and is independent from η. On the other hand, given c, $q_a^*(\eta)$ is non-monotone in η when c is small and increases otherwise. Comparing $q_a^*(\eta)$ with q_a^d, it is immediate the verify that there exist threshold values $\bar{c}_a$ and $\bar{\eta}_a^1 < \bar{\eta}_a^2$ such that if $c < \bar{c}_a$ and $\eta \in [\bar{\eta}_a^1, \min\{\bar{\eta}_a^2, 1\}]$, $q_a^*(\eta) \ge q_a^d$; otherwise, $q_a^*(\eta) < q_a^d$, where the threshold $\bar{c}_a$ is the smallest root of polynomial $a^6 - 18a^5 c + 117a^4 c^2 - 540a^3 c^3 + 1620a^2 c^4 - 2592ac^5 + 1728c^6 = 0$, and the thresholds $\bar{\eta}_a^1 < \bar{\eta}_a^2$ are the two real roots of polynomial $-2a^3 c + \eta(a^4 + 27a^2 c^2 - 108ac^3 + 108c^4) + \eta^3(32a^3 c - 192a^2 c^2 + 384ac^3 - 256c^4) + \eta^2(-2a^4 - 8a^3 c + 48a^2 c^2 - 48ac^3) = 0$, respectively. □

Proof of Lemma 1: Lemma 1 holds due to the concavity of $p(d)d$. □

Proof of Proposition 3: We prove **part (i)** and **part (ii)** together. For any given production quantity q_r, define $F(v) = v + \frac{1}{\eta}E_\xi \min\{\pi_r(q_r) - v, 0\}$, where $\pi_r(q_r) = \max_{p_r\in[0,p_0]}\{p_r \min\{d(p_r), q_r\xi\}\} - cq_r$. There are three cases to consider.

Case 1: If $q_r < \frac{d^*}{u}$, then $\forall \xi \in [l,u]$, $q_r\xi \in [\frac{l}{u}d^*, d^*) < d^*$. In this case, define $f(\xi) = \pi_r(q_r) - v = p(q_r\xi)q_r\xi - cq - v$, which is concave in ξ due to the concavity of $p(d)d$. Let $\xi^* = \frac{d^*}{q_r}$ such that $f'(\xi^*) = 0$. Since $f'(u) = p(q_r u) + q_r u p'(q_r u) > p(d^*) + d^* p'(d^*) = 0$, we have $\xi^* > u$. Thus, $f(\xi)$ increases in $[l,u]$. There are three sub-cases to consider. **Sub-case 1:** If $v \geq p(q_r u)q_r u - cq_r$, then $f(u) \leq 0$. In this case, $F(v) = v + \frac{1}{\eta}E_\xi(p(q_r\xi)q_r\xi - cq_r - v)$. **Sub-case 2:** If $v \leq p(q_r l)q_r l - cq_r$, then $f(l) \geq 0$. In this case, $F(v) = v$. **Sub-case 3:** If $v \in [p(q_r l)q_r l - cq_r, p(q_r u)q_r u - cq_r]$, then $f(l) \leq 0 \leq f(v)$. Let $\hat{\xi}$ such that $f(\hat{\xi}) = 0$, i.e., $p(q_r\hat{\xi})q_r\hat{\xi} - cq_r = v$. In this case, $F(v) = v + \frac{1}{\eta}\int_l^{\hat{\xi}}(p(q_r\xi)q_r\xi - cq_r - v)dG(\xi)$. Putting all three sub-cases together, we have the following:

$$F(v) = \begin{cases} v, & \text{if } v \in (-\infty, p(q_r l)q_r l - cq_r], \\ v + \frac{1}{\eta}\int_l^{\hat{\xi}}[p(q_r\xi)q_r\xi - cq_r - v]dG(\xi), & \text{if } v \in (p(q_r l)q_r l - cq_r, p(q_r u)q_r u - cq_r], \\ v + \frac{1}{\eta}\int_l^{u}[p(q_r\xi)q_r\xi - cq_r - v]dG(\xi), & \text{if } v \in (p(q_r u)q_r u - cq_r, \infty]. \end{cases}$$

Easy to check that $F(v)$ is continuously differentiable in v since $\hat{\xi}|_{v=p(q_r l)q_r l - cq_r} = l$ and $\hat{\xi}|_{v=p(q_r u)q_r u - cq_r} = u$. Taking derivative, $\frac{\partial F(v)}{\partial v}|_{v=p(q_r l)q_r l - cq_r} = 1 > 0$ and $\frac{\partial F(v)}{\partial v}|_{v=p(q_r u)q_r u - cq_r} = 1 - \frac{1}{\eta} < 0$ for $\eta \in (0,1)$. Thus, there must exist a $v^* \in (p(q_r l)q_r l - cq_r, p(q_r u)q_r u - cq_r)$ satisfying the first order condition, which pins down to $1 - \frac{1}{\eta}\int_l^{\hat{\xi}} dG(\xi) = 0$ or equivalently $\hat{\xi} = G^{-1}(\eta)$. Consequently, $F(v^*) = \frac{1}{\eta}\int_l^{G^{-1}(\eta)}(p(q_r\xi)q_r\xi - cq_r)dG(\xi)$.

Case 2: If $q_r > \frac{d^*}{l}$, then $\forall \xi \in [l,u]$, $q_r\xi > d^*$. In this case, $\pi_r(q_r) = p(d^*)d^* - cq_r - v \leq 0$ if and only if $v \geq p(d^*)d^* - cq_r$. Therefore, we have the following:

$$F(v) = \begin{cases} v, & \text{if } v \in (-\infty, p(d^*)d^* - cq_r], \\ v + \frac{1}{\eta}\int_l^{u}[p(q_r\xi)q_r\xi - cq_r - v]dG(\xi), & \text{otherwise.} \end{cases}$$

Easy to check that $F(v)$ is continuous but not differentiable with $v^* = p(d^*)d^* - cq_r$ and $F(v^*) = p(d^*)d^* - cq_r$.

Case 3: If $q_r \in [\frac{d^*}{u}, \frac{d^*}{l}]$, then $\pi_r(q_r) = p(q_r\xi_r)q_r\xi_r - cq_r - v$, if $\xi \leq \frac{d^*}{q_r}$ and $\pi_r(q_r) = p(d^*)d^* - cq_r - v$, otherwise. Let $f(\xi) = p(q_r\xi_r)q_r\xi_r - cq_r - v$ and $\xi^* = \frac{d^*}{q_r}$ defined in **Case 1** belongs to $[l,u]$. There are three sub-cases to consider. **Sub-case 1:** If $v \leq p(q_r l)q_r l - cq_r$, then $f(l) \geq 0$. In this case, $F(v) = v$. **Sub-case 2:** if $v \in [p(q_r l)q_r l - cq_r, p(d^*)d^* - cq_r]$, then $f(l) \leq 0$ and $f(\xi^*) \geq 0$. Let $\hat{\xi}$ such that $f(\hat{\xi}) = 0$. In this case, $F(v) = v + \frac{1}{\eta}\int_l^{\hat{\xi}}(p(q_r\xi)q_r\xi - cq_r - v)dG(\xi)$. **Sub-case 3:** If $v > p(d^*)d^* - cq_r$, then $f(\xi^*) < 0$. In this case, $F(v) = v + \frac{1}{\eta}\int_l^{\frac{d^*}{q_r}}(p(q_r\xi)q_r\xi - cq_r - v)dG(\xi) + \frac{1}{\eta}\int_{\frac{d^*}{q_r}}^{u}(p(d^*)d^* - cq_r - v)dG(\xi)$. To sum up, we have the following:

$$F(v) = \begin{cases} v, & \text{if } v \in (-\infty, p(q_r l)q_r l - cq_r], \\ F_1^1(v) := v + \frac{1}{\eta}\int_l^{\hat{\xi}}[p(q_r\xi)q_r\xi - cq_r - v]dG(\xi), & \text{if } v \in (p(q_r l)q_r l - cq_r, p(d^*)d^* - cq_r], \\ F_1^2(v) := v + \frac{1}{\eta}\int_l^{\frac{d^*}{q_r}}(p(q_r\xi)q_r\xi - cq_r - v)dG(\xi) + \frac{1}{\eta}\int_{\frac{d^*}{q_r}}^{u}(p(d^*)d^* - cq_r - v)dG(\xi), & \text{otherwise.} \end{cases}$$

Easy to check that $F(v)$ is continuous since when $v = p(d^*)d^* - cq_r$, $\hat{\xi} = \frac{d^*}{q_r}$. However, $\frac{\partial F(v)}{\partial v}|_{v\to(p(d^*)d^*-cq_r)^-} = 1 - \frac{1}{\eta}G(\hat{\xi}) > 1 - \frac{1}{\eta} = \frac{\partial F(v)}{\partial v}|_{v\to(p(d^*)d^*-cq_r)^+}$. Therefore, $F(v)$ is unimodal with $v^* \in [p(q_r l)q_r l - cq_r, p(d^*)d^* - cq_r]$. Let $K(q_r) = \frac{\partial F(v)}{\partial v}|_{v\to(p(d^*)d^*-cq_r)^-} = 1 - \frac{1}{\eta}G(\hat{\xi}) = 1 - \frac{1}{\eta}G(\frac{d^*}{q_r})$, since when $v = p(d^*)d^* - cq_r$, $\hat{\xi} = \frac{d^*}{q_r}$. Then $K(\frac{d^*}{l}) = 1 > 0$ and $K(\frac{d^*}{u}) = 1 - \frac{1}{\eta} < 0$. Thus, there exists a unique $\bar{q}_r \in (\frac{d^*}{u}, \frac{d^*}{l})$ s.t., $K(\bar{q}_r) = 0$, which is equivalent to $\bar{q}_r = \frac{d^*}{G^{-1}(\eta)}$. Therefore,

$$v^* = \begin{cases} s.t, G(\hat{\xi}) = \eta, & \text{if } q_r < \bar{q}_r, \\ p(d^*)d^* - cq_r, & \text{otherwise.} \end{cases}$$

And the corresponding value function is:

$$F(v^*) = \begin{cases} \frac{1}{\eta}\int_l^{G^{-1}(\eta)} (p(q_r\xi)q_r\xi - cq_r)dG(\xi), & \text{if } q_r \in [\frac{d^*}{u}, \bar{q}_r), \\ p(d^*)d^* - cq_r + \frac{1}{\eta}\int_l^{\frac{d^*}{q_r}} (p(q_r\xi)q_r\xi - p(d^*)d^*)dG(\xi), & \text{otherwise.} \end{cases}$$

Combining all the three cases together, we have the following value function:

$$\Pi_r(q_r) = F(v^*) = \begin{cases} \frac{1}{\eta}\int_l^{G^{-1}(\eta)} (p(q_r\xi)q_r\xi - cq_r)dG(\xi), & \text{if } q_r \in [0, \bar{q}_r), \\ p(d^*)d^* - cq_r + \frac{1}{\eta}\int_l^{\frac{d^*}{q_r}} (p(q_r\xi)q_r\xi - p(d^*)d^*)dG(\xi), & \text{if } q_r \in [\bar{q}_r, \frac{d^*}{l}], \\ p(d^*)d^* - cq_r & \text{otherwise.} \end{cases} \quad \text{(EC.1)}$$

Easy to check that $\Pi_r(q_r)$ is continuously differentiable since when $q_r = \bar{q}_r$, $\frac{d^*}{q_r} = G^{-1}(\eta)$. Checking the second derivative, it is immediate that $\frac{\partial^2\Pi_r(q_r)}{\partial q_r^2} \leq 0$ due to the concavity of $p(d)d$, and $\frac{\partial^2\Pi_r(q_r)}{\partial q_r^2} < 0$ if $p(d)d$ is strictly concave. Thus, $\Pi_r(q_r)$ is continuously differentiable and concave. Checking the boundary condition, $\frac{\partial\Pi_r(q_r)}{\partial q_r}|_{q_r=0} = \frac{p_0}{\eta}\int_l^{G^{-1}(\eta)} \xi dG(\xi) - c$. Let $g(\eta) := \frac{\int_l^{G^{-1}(\eta)} \xi dG(\xi)}{\eta}$. We have $g'(\eta) = \frac{\eta G^{-1}(\eta) - \int_l^{G^{-1}(\eta)} \xi dG(\xi)}{\eta^2} > 0$. Moreover, $g(1) = \mu \geq \frac{c}{p_0}$ and $g(0) = G^{-1}(0) = l$ by applying the L' Hospital's rule. If $l \leq \frac{c}{p_0}$, there exists a unique $\eta_r^*(p_0, c) > 0$ such that $\frac{\partial\Pi_r(q_r)}{\partial q_r}|_{q_r=0} > 0$ holds iff $\eta > \eta_r^*(p_0, c)$. And if $l > \frac{c}{p_0}$, $\forall\eta > 0 := \eta^*(p_0, c)$, $\frac{\partial\Pi_r(q_r)}{\partial q_r}|_{q_r=0} > 0$ holds.

Now, we focus on the case when $\eta \geq \eta_r^*(p_0, c)$, which indicates that $\frac{\partial\Pi_r(q_r)}{\partial q_r}|_{q_r=0} \geq 0$. Together with the facts that $\frac{\partial\Pi_r(q_r)}{\partial q_r}|_{q_r=\frac{d^*}{l}} = -c < 0$ and $\Pi_r(q_r)$ is continuously differentiable and concavity in the entire feasible region of q_r, there exists an optimal $q_r^* \geq 0$ satisfying the first order condition. Let $\bar{c}_r := \frac{1}{\eta}\int_l^{G^{-1}(\eta)} \left[p\left(\frac{\xi d^*}{G^{-1}(\eta)}\right) + \left(\frac{\xi d^*}{G^{-1}(\eta)}\right)p'\left(\frac{\xi d^*}{G^{-1}(\eta)}\right)\right]\xi dG(\xi)$, under which $q_r^* = \bar{q}_r = \frac{d^*}{G^{-1}(\eta)}$. Then, the concavity of the objective function together with the boundary derivative conditions immediately indicate that the optimal q_r^* must satisfy the following first order condition:

$$q_r^* \text{ solves } \begin{cases} \frac{1}{\eta}\int_l^{G^{-1}(\eta)} [p'(q_r^*\xi)(q_r^*\xi) + p(q_r^*\xi)]\,\xi dG(\xi) = c & c \in [\bar{c}_r, \max\{\bar{c}_r, p_0\mu\}] \\ \frac{1}{\eta}\int_l^{\frac{d^*}{q_r^*}} [p'(q_r^*\xi)(q_r^*\xi) + p(q_r^*\xi)]\,\xi dG(\xi) = c & c \in [0, \bar{c}_r]. \end{cases}$$

This completes the proof. $\square$

Proof of Corollary 3: Part (i): when $c = 0$, we have $q_r^* = \frac{d^*}{l} > \frac{d^*}{\mu} = q_r^d$. Since both q_r^* and

q_r^d are continuous in c, there must exist a $\delta > 0$ such that $q_r^* > q_r^d$, $\forall c \in [0, \delta)$. On the other hand, when $c = p_0\mu$, $q_r^d = 0 = q_r^*$. Moreover, $\eta_r^*(p_0, p_0\mu) = 1$. Next, taking derivative and we have $\frac{\partial q_r^d}{\partial c}|_{c=p_0\mu} = \frac{1}{2p'(0)\mu^2}$ and $\frac{\partial q_r^*}{\partial c}|_{c=p_0\mu} = \frac{1}{2p'(0)E\xi^2}$. Since $E\xi^2 > \mu^2$, $|\frac{\partial q_r^d}{\partial c}|_{c=p_0\mu}| > |\frac{\partial q_r^*}{\partial c}|_{c=p_0\mu}|$. Thus, there exists a $\delta > 0$ such that $q_r^* < q_r^d$, $\forall c \in (p_0\mu - \delta, p_0\mu)$. In particular, if $d(p) = a - bp$ with b normalized to 1 and $\xi \sim \text{Uniform}[0, 1]$, then there exists a unique threshold $\hat{c}(\eta)$ such that $q_r^* > q_r^d$ holds if and only if $c \in [0, \hat{c})$, where $\hat{c} = \frac{4a\eta^2 - 3a\eta}{8\eta^2 - 6}$ if $\eta < \frac{1}{2}\left(3 - \sqrt{3}\right)$ and $\hat{c}$ is the first root of polynomial $96\text{c}^3\eta - 96\text{c}^2 a\eta + 24\text{c}a^2\eta - a^3 = 0$ otherwise.

Next, we show $\Pi_r(q_r^*) < \Pi_r^d(q_r^d)$. By part (iii), since $\Pi_r(q_r^*)$ increases in η, it is clear that $\Pi_r(q_r^*)|_{\eta<1} < \Pi_r(q_r^*)|_{\eta=1}$. When $\eta = 1$, the firm is risk neutral with expected profit concave in ξ. Thus, due to convex order, $\Pi_r(q_r^*)|_{\eta=1} < \Pi_r^d(q_r^d)$, which concludes the proof.

Part (ii): When $c \le \bar{c}_r$, q_r^* is defined by $\frac{1}{\eta}\int_l^{\frac{d^*}{q_r^*}} [p'(q_r^*\xi)(q_r^*\xi) + p(q_r^*\xi)]\,\xi dG(\xi) = c$. It is immediate that q_r^* decreases in η due to the concavity of $p(d)d$. When $c > \bar{c}_r$, Figure 2 and the proof of Corollary 4 provide illustrations that the q_r^* may not be monotonic in η.

Part (iii): We show that for any $\eta \ge \eta_r^*(p_0, c)$, $\Pi_r(q_r^*)$ increases in η. Taking derivative, we have $\frac{d\Pi_r(q_r^*)}{d\eta} = \frac{\partial \Pi_r(q_r^*)}{\partial \eta} + \frac{\partial \Pi_r(q_r^*)}{\partial q_r}\frac{\partial q_r^*}{\partial \eta} = \frac{\partial \Pi_r(q_r^*)}{\partial \eta}$ due to the Envelope Theorem. Straightforward calculation shows that $\frac{\partial \Pi_r(q_r^*)}{\partial \eta} > 0$, where $\Pi_r(q_r)$ is given in Equation (EC.1). □

Proof of Corollary 4: To simplify the expression, we normalize $b = 1$ without loss of generality. The optimal production quantity under the linear demand and uniform yield is $q_r^*(\eta) = \frac{a}{2\sqrt{6}}\sqrt{\frac{a}{c\eta}}$ if $0 < c < \frac{a\eta}{6}$ and $q_r^*(\eta) = \frac{3(a\eta - 2c)}{4\eta^2}$ otherwise. Straightforward calculation yields that $q_r^*(\eta)$ decreases in η if and only if $c < \frac{a}{4} := \hat{c}_r$ and $\eta \in (\frac{4c}{a}, 1]$. □

Proof of Lemma 2: By Yao et al. (2006), demand functions in parts (i)-(v) all satisfy the IPE property. Next, we show that they also satisfy Assumption 2. For part (i), $p(d) = (\frac{a-d}{b})^{\frac{1}{k}}$. Taking derivative, we have $(p(d)d)'' = \frac{((a-d)/b)^{\frac{1}{k}}(d + dk - 2ak)}{(a-d)^2k^2} \le 0$, since $d \le a$ and $d \le dk \le ak$. For part (ii), $p(d) = -d^{-\frac{1}{a}}(-1 + d^{\frac{1}{a}})$. Taking derivative, we have $(p(d)d)'' = \frac{(1-a)d^{-\frac{1+a}{a}}}{a^2} < 0$. For part (iii), $p(d) = (\frac{d}{a})^{-\frac{1}{b}}$. Taking derivative, we have $(p(d)d)'' = \frac{(1-b)(\frac{d}{a})^{-\frac{1}{b}}}{b^2 d} < 0$. For part (iv), $(p(d)d)'' = 2p'(d) + dp''(d) < 0$ since $p'(d) = \frac{1}{d'(p(d))} < 0$ and $p''(d) = \frac{-d''(p(d))p'(d)}{(d'(p(d)))^2} < 0$. □

Proof of Proposition 4: Part (i): From the proofs of Propositions 2 and 3, it can be easily seen that $\eta_a^*(p_0, c) = \eta_r^*(p_0, c) = 0$, if $p_0 l > c$. Otherwise, $\eta_a^*(p_0, c) = \eta_r^*(p_0, c)$ is the unique solution of $g(\eta) = \frac{c}{p_0}$, where $g(\eta) = \frac{1}{\eta}\int_l^{G^{-1}(\eta)} \xi dG(\xi)$. **Part (ii):** For any given l and p_0, if $c \le p_0 l$, then $\eta^* = 0$. If $c > p_0 l$, then $\eta^* > 0$ is defined by $g(\eta) = \frac{c}{p_0}$. It is immediate that η^* increases in c and decreases in p_0, since $g(\eta)$ increases in η. Moreover, as η^* is continuous in c, it increases in c for all feasible c. Similar argument can be made to show η^* decreases in p_0.

Part (iii): Suppose $\xi_1 \geq_{st} \xi_2$ in stochastic order. Then, for any ξ, $G_1(\xi) \leq G_2(\xi)$ or equivalently for any η, $G_1^{-1}(\eta) \geq G_2^{-1}(\eta)$. Now, suppose $\eta_1^* > \eta_2^*$. By definition, $g_1(\eta_1^*) = g_2(\eta_2^*) = \frac{c}{p_0}$, where $g_i(\eta) = \frac{1}{\eta}\int_l^{G_i^{-1}(\eta)} \xi dG_i(\xi)$, $i = 1, 2$. Since $g_i(\eta)$ increases in η, we have $g_1(\eta_2^*) < g_1(\eta_1^*) = g_2(\eta_2^*)$, which is equivalent to

$$\int_l^{G_1^{-1}(\eta_2^*)} \xi dG_1(\xi) < \int_l^{G_2^{-1}(\eta_2^*)} \xi dG_2(\xi). \tag{EC.2}$$

Utilizing integration by parts, we have (EC.2) holds $\longleftrightarrow \eta_2^*(G_1^{-1}(\eta_2^*) - G_2^{-1}(\eta_2)) < \int_l^{G_2^{-1}(\eta_2^*)}[G_1(\xi) - G_2(\xi)]d\xi + \int_{G_2^{-1}(\eta_2^*)}^{G_1^{-1}(\eta_2^*)} G_1(\xi)d\xi$. Since $\int_{G_2^{-1}(\eta_2^*)}^{G_1^{-1}(\eta_2^*)} G_1(\xi)d\xi < \eta_2^*(G_1^{-1}(\eta_2^*) - G_2^{-1}(\eta_2^*))$ and $G_1(\xi) \leq G_2(\xi)$, (EC.2) leads to a contradiction, which implies that $\eta_1^* \leq \eta_2^*$. **Part (iv):** Let $k = \frac{c}{p_0}$. Simple calculation yields that $\eta^* = 1 - \frac{\mu - k}{\sigma}$, which increases in σ, since $k \leq \mu$. □

Proof of Proposition 5: Part (i): To simplify the expression, we normalize $b = 1$ without loss of generality. Adopting the formulations in Propositions 2 and 3, we can explicitly calculate q_r^* and q_a^* in the following expressions.

$$q_r^* = \begin{cases} \frac{a}{2}\sqrt{\frac{a}{6c\eta}}, & \text{if } c \in [0, \frac{a\eta}{6}], \\ \frac{3(a\eta - 2c)}{4\eta^2}, & \text{if } c \in [\frac{a\eta}{6}, \frac{a}{2}], \end{cases}$$

and $q_a^* = (a - p_a^*)\sqrt{\frac{p_a^*}{2c\eta}}$, where p_a^* is the third (largest) root of equation $8\eta p_a^3 - (8a\eta + qc)p_a^2 + (ac + 2a^2\eta)p_a - a^2c = 0$. Comparing q_a^* and q_r^* and after some simplifications, there exists a unique threshold $\hat{c}$ such that $q_r^* \leq q_a^*$ iff $c \leq \hat{c}$ and $\hat{c}$ is the first root of equation $1323c^3 - 1512a\eta c^2 + 432a^2\eta^2 c - 32a^3\eta^3 = 0$. In addition, it is straightforward to calculate the $\hat{c}$ increase in η via the Implicit Function Theory. **Part (ii)**: To illustrate that $q_r^* - q_a^*$ may not be monotonic, we further normalize $a = 1$ for expositional convenience. Straightforward calculation yields that there exists a threshold hold risk aversion parameter $\hat{\eta} \in (\eta^*, 1]$, which is the 2nd root of polynomial $163296c^6 - 101088c^5\eta + 60534c^4\eta^2 - 36639c^3\eta^3 + 9594c^2\eta^4 - 840c\eta^5 + 16\eta^6 = 0$. For $c \in [0, 0.3182]$, $q_r^* - q_a^*$ first increases in $\eta \in (\eta^*, \hat{\eta}]$ and then decreases $\eta \in (\hat{\eta}, 1]$. For $c \in (0.3182, 0.5]$, $q_r^* - q_a^*$ always increases in η. □

Proof of Proposition 6: To simplify the expression, we normalize $b = 1$ without loss of generality. Adopting the formulations in Propositions 2 and 3, we can explicitly calculate q_r^* and q_a^*, which are given in the proof of Proposition 5. Plugging q_r^* and q_a^*, we have Π_r^* and Π_a^* given in the following expressions.

$$\Pi_r^* = \begin{cases} \frac{a}{12}(3a - 2c\sqrt{\frac{6a}{c\eta}}), & \text{if } c \in [0, \frac{a\eta}{6}], \\ \frac{3(a\eta - 2c)^2}{16\eta^2}, & \text{if } c \in [\frac{a\eta}{6}, \frac{a}{2}], \end{cases}$$

and $\Pi_a^* = (a - p_a^*)p_a^*(1 - \frac{1}{\eta}\sqrt{\frac{2c\eta}{p_a^*}})$, where p_a^* is the third root of equation $8\eta p_a^3 - (8a\eta + qc)p_a^2 + (ac + 2a^2\eta)_a - a^2c = 0$. Comparing Π_a^* and Π_r^* and after some necessary yet cumbersome simplifications, we obtain a unique threshold $\tilde{c}$, which is the first root of polynomial $1323c^3 - 1512ac^2 + 432a^2c - 32a^3 = 0$. If $c < \tilde{c}$, then $\Pi_r^* - \Pi_a^*$ increases (decreases) in η if $\eta < \hat{\eta}$ ($\eta > \hat{\eta}$), where $\hat{\eta}$ is the third root of polynomial $32a^3\eta^3 - 432a^2c\eta^2 + 1512ac^2\eta - 1323c^3 = 0$. Otherwise, $\Pi_r^* - \Pi_a^*$ increases in η. □

Proof of Proposition 7: To simplify the expression, we normalize $b = 1$ without loss of generality. **Part (i):** Adopting the formulations in Propositions 2 and 3, we can explicitly calculate q_r^*, q_a^* and p_a^*, which are given in the proof of Proposition 5. Plugging q_r^*, q_a^* and p_a^*, we have CS_r^* and CS_a^* given in the following expressions:

$$CS_r^* = \begin{cases} \frac{1}{24}a\left(3a - 2c\eta\sqrt{\frac{6a}{c\eta}}\right), & \text{if } c \in [0, \frac{a\eta}{6}), \\ \frac{a^2(a(9-4\eta)\eta - 18c)}{72(a\eta - 2c)}, & \text{if } c \in [\frac{a\eta}{6}, \frac{(3a\eta - 2a\eta^2)}{6}), \\ \frac{3(a\eta - 2c)^2}{32\eta^4}, & \text{if } c \in [\frac{(3a\eta - 2a\eta^2)}{6}, \frac{a}{2}], \end{cases}$$

and $CS_a^* = \frac{1}{6}(a - p_a^*)^2\left(3 - \sqrt{2}\sqrt{\frac{c\eta}{p_a^*}}\right)$, where p_a^* is the third root of equation $8\eta p_a^3 - (8a\eta + qc)p_a^2 + (ac + 2a^2\eta)p_a - a^2c = 0$. Taking derivatives (by using the implicit function theorem) and after some necessary simplifications, we have $\frac{\partial CS_r^*}{\partial \eta} < 0$ iff $\eta \in [\frac{4c}{a}, \max\{1, \frac{4c}{a}\}]$, and $\frac{\partial CS_a^*}{\partial \eta} > 0$ always holds. **Part (ii)**: since $p_a^* \le a$, it is immediate that $CS_a^* \le \frac{1}{6}(a - p_a^*)^2\left(3 - \sqrt{2}\sqrt{\frac{c\eta}{a}}\right) := \hat{CS}_a$. It remains sufficient to compare CS_r^* with $\hat{CS}_a$. When $c \in [\frac{a\eta}{6}, \frac{(3a\eta - 2a\eta^2)}{6})$ (respectively, $c \in [\frac{(3a\eta - 2a\eta^2)}{6}, \frac{a}{2}]$), $CS_r^* = \frac{a^2(a(9-4\eta)\eta - 18c)}{72(a\eta - 2c)}$ (respectively, $CS_r^* = \frac{3(a\eta - 2c)^2}{32\eta^4}$), necessary simplifications and comparisons yield that $CS_r^* > \hat{CS}_a \ge CS_a^*$ in this case. For the remaining case of $c \in [0, \frac{a\eta}{6}]$ or equivalently $\eta \in [\frac{6c}{a}, \max\{1, \frac{6c}{a}\}]$, $CS_r^* = \frac{1}{24}a\left(3a - 2c\eta\sqrt{\frac{6a}{c\eta}}\right)$, and it is clear that CS_r^* decreases in η, whereas CS_a^* increases in η by part (i). Checking boundary condition, we have $CS_r^* - CS_a^*|_{\eta=1} > 0$ for all $c \in [0, \frac{a}{6}]$. Consequently, $CS_r^* > CS_a^*$ in this case. Next, we illustrate that $CS_r^* - CS_a^*$ may not be monotonic in η. Recall from part (i) that when $c < \frac{a}{4b}$, CS_r^* increases in $\eta \in (\hat{\eta}, \frac{4bc}{a}]$ and decreases otherwise, whereas CS_a^* always increases in η. This implies that $CS_r^* - CS_a^*$ decreases in η at least when $\eta \in [\frac{4bc}{a}, 1]$. Together with the fact that $CS_r^* - CS_a^*|_{\eta \to \hat{\eta}} = 0 < CS_r^* - CS_a^*|_{\eta=1}$, $CS_r^* - CS_a^*$ must increase in η for some $\eta \in (\hat{\eta}, \frac{4bc}{a}]$, which implies that $CS_r^* - CS_a^*$ is non-monotonic in η in this case. Combining the above arguments, part (ii) is proved. □

Proof of Lemma 3: Let $\Pi_r(q_r) = EU(\pi_r(q_r))$. There are two cases to consider. If $q_r \le d^*$, then $\pi_r(q_r) = p(q_r\xi)q_r\xi - cq_r$ and $\Pi_r(q_r) = EU(p(q_r\xi)q_r\xi - cq_r)$. If $q_r > d^*$, then $\Pi_r(q_r) = \int_0^{\frac{d^*}{q_r}} U(p(q_r\xi)q_r\xi - cq_r)dG(\xi) + \int_{\frac{d^*}{q_r}}^1 U(p(d^*)d^* - cq_r)dG(\xi)$. It is easy to check that $\Pi_r(q_r)$ is continuously differentiable and concave in q_r with $\frac{\partial \Pi_r(q_r)}{\partial q_r}|_{q_r=0} = U'(0)(p_0\mu - c)$ and $\frac{\partial \Pi_r(q_r)}{\partial q_r}|_{q_r\to+\infty} = -cU'(p(d^*)d^* - cq_r)|_{q_r\to+\infty} < 0$. Thus, $q_r^* \geq 0$ if and only if $p_0\mu \geq c$. Next, define the riskless quantity as $q_r^d = \arg\max p(q_r^d\mu)q_r^d\mu - cq_r^d$. Applying the same argument as used in the proof of Corollary 3, we have $q_r^* \to +\infty > \frac{d^*}{\mu} = q_r^d$ when $c = 0$. Due to continuity, there must exist a $\delta > 0$ such that $q_r^* > q_r^d$ for $c \in [0, \delta)$. On the other hand, when $c = p_0\mu$, we have $q_r^* = 0 = q_r^d$. In addition, straightforward calculation through L' Hospital rule yields that $\frac{\partial q_r^d}{\partial c}|_{c=p_0\mu} = \frac{1}{2p'(0)\mu^2}$ and $\frac{\partial q_r^*}{\partial c}|_{c=p_0\mu} = \frac{U'(0)}{2U'(0)p'(0)E\xi^2} = \frac{1}{2p'(0)E\xi^2}$. Since $E\xi^2 > \mu^2$, $|\frac{\partial q_r^d}{\partial c}|_{c=p_0\mu}| > |\frac{\partial q_r^*}{\partial c}|_{c=p_0\mu}|$. Thus, there exists a $\delta > 0$ such that $q_r^* < q_r^d$, $\forall c \in (p_0\mu - \delta, p_0\mu)$. Thus, Corollary 3(i) continues to hold under the expected utility criterion. □

Proof of Lemma 4: Let $U(q) = E\pi(q) - \lambda Var(\pi(q))$, where $\pi(q) = p\min\{d(p), q\xi\} - cq$. If $q < d(p)$, then we have $U(q) = (p\mu - c)q - \lambda p^2\sigma^2 q^2$, which is concave in q. If $q > d(p)$, we have $E\pi(q) = p(\int_0^{\frac{d(p)}{q}} q\xi dG(\xi) + \int_{\frac{d(p)}{q}}^1 d(p)dG(\xi)) - cq$ and $Var(\pi(q)) = p^2 Var(min\{d(p), q\xi\}) = p^2[E\min\{d(p), q\xi\}^2 - (E\min\{d(p), q\xi\})^2]$. Taking derivative, we have $\frac{\partial E\pi(q)}{\partial q} = p\int_0^{\frac{d(p)}{q}} \xi dG(\xi) - c$ and $\frac{\partial Var(\pi(q))}{\partial q} = 2p^2[\int_0^{\frac{d(p)}{q}} q\xi^2 dG(\xi) - (\int_0^{\frac{d(p)}{q}} \xi dG(\xi))(\int_0^{\frac{d(p)}{q}} q\xi dG(\xi) + \int_{\frac{d(p)}{q}}^1 dG(\xi))]$. It can be easily verified that $\frac{\partial U(q)}{\partial q}|_{q=d(p)^-} = \frac{\partial U(q)}{\partial q}|_{q=d(p)^+} = (p\mu - c) - 2\lambda d(p)p^2\sigma^2$. In addition, $\frac{\partial U(q)}{\partial q}|_{q\to+\infty} = -c$ and $\frac{\partial U(q)}{\partial q}|_{q=0} = p\mu - c \geq 0$ if and only if $p \geq \frac{c}{\mu}$. In this case, there must exist at least one maximal q^* satisfying the FOC. In addition, if $\frac{\partial U(q)}{\partial q}|_{q=d(p)} = (p\mu - c) - 2\lambda d(p)p^2\sigma^2 < 0$ for some p, then it could be possible that $q^* < d(p)$. Finally, we discuss the variance term. It is immediate that $Var(\pi(q))$ increases in q for $q \in [0, d(p)]$ with $Var(\pi(q))|_{q=0} = 0 = \frac{\partial Var(\pi(q))}{\partial q}|_{q=0}$, $\frac{\partial Var(\pi(q))}{\partial q}|_{q=d(p)} = 2d(p)p^2\sigma^2 > 0$ and $Var(\pi(q))|_{q\to+\infty} = 0 = \frac{\partial Var(\pi(q))}{\partial q}|_{q\to+\infty}$, which indicates that the $Var(\pi(q))$ is non-monotonic in q. Such non-monotonicity, together with the fact that $E\pi(q)$ is concave in q, may lead to the fact that $U(q)$ is not unimodal in q. For example, Figure EC.1 provides a numerical example in which $U(\pi(q))$ is bi-modal in q under Uniform yield distribution with optimal $q^* < d(p) = 10 - 7 = 3$. □

Proof of Lemma 5: The proof and additional discussions on the analysis of general cost structure are given in Appendix EC.5. □

Proof of Lemma 6: For any given price p and production quantity q, define $F(v) = v + \frac{1}{\eta}E_K \min\{\pi(q) - v, 0\}$, where $\pi(q) = p\min\{q, d(p), K\} - cq$. There are two cases to consider.

Case 1: If $q < d(p)$, then $\pi(q) = p\min\{q, K\} - cq$. Simple comparison yields that:

$$F(v) = \begin{cases} v, & \text{if } v \in (-\infty, -cq], \\ F_2^1(v) := v + \frac{1}{\eta}\int_l^{\frac{cq+v}{p}} [pK - cq - v]dF(K), & \text{if } v \in (-cq, (p-c)q], \\ F_2^2(v) := v + \frac{1}{\eta}(\int_l^q [pK - cq - v]dF(K) + \int_q^{+\infty} [(p-c)q - v]dF(K)), & \text{otherwise .} \end{cases}$$

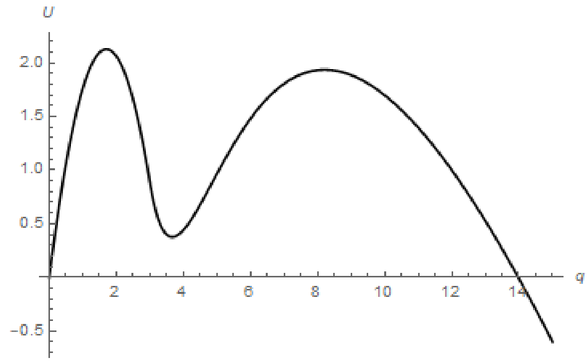


Figure EC.1 $U(\pi(q))$ **with** $\xi\sim$ **Uniform**$[0,1]$**,** $d(p)=10-p$**,** $p=7$**,** $c=1$ **and** $\lambda=0.18$

It is clear that $F(v)$ is continuous and concave in v. Moreover, we have $\frac{\partial F(v)}{\partial v}|_{v=-cq^-}=\frac{\partial F(v)}{\partial v}|_{v=-cq^+}=1$ and $\frac{\partial F(v)}{\partial v}|_{v=-(p-c)q^-}=1-\frac{1}{\eta}\int_0^q dF(K)>1-\frac{1}{\eta}=\frac{\partial F(v)}{\partial v}|_{v=-(p-c)q^+}$. Then, it is immediate that $v^*=(p-c)q$ if $q<F^{-1}(\eta)$ and $v^*=pF^{-1}(\eta)-cq$ otherwise. Plugging back, given $q<d(p)$, we have

$$F(v^*)=\begin{cases}(p-c)q+\frac{1}{\eta}\int_0^q p(K-q)dF(K), & \text{if } q<F^{-1}(v),\\ \frac{1}{\eta}\int_0^{F^{-1}(\eta)}(pK-cq)dF(K), & \text{otherwise .}\end{cases}$$

Case 2: If $q\geq d(p)$, then $\pi(q)=p\min\{d(p),K\}-cq$. Applying the same analysis as used in Case 1, we can obtain that $v^*=pd(p)-cq$ if $d<F^{-1}(\eta)$ and $v^*=pF^{-1}(\eta)-cq$ otherwise. Plugging back, given $q\geq d(p)$, we have

$$F(v^*)=\begin{cases}pd(p)-cq+\frac{1}{\eta}\int_0^q p(K-d)dF(K), & \text{if } d<F^{-1}(v),\\ \frac{1}{\eta}\int_0^{F^{-1}(\eta)}(pK-cq)dF(K), & \text{otherwise .}\end{cases}$$

Combining the above cases, there are two scenarios that could happen depending on the magnitude of $d(p)$. On the one hand, if $d(p)<F^{-1}(\eta)$, we have

$$\pi(q)=F(v^*)=\begin{cases}(p-c)q+\frac{1}{\eta}\int_0^q p(K-q)dF(K), & \text{if } q<d(p),\\ pd(p)-cq+\frac{1}{\eta}\int_0^q p(K-d)dF(K), & \text{otherwise.}\end{cases}$$

In this case, straightforward analysis yields the optimal production quantity as:

$$q^*=\begin{cases}d(p), & \text{if } d(p)<F^{-1}(\frac{\eta(p-c)}{p}),\\ F^{-1}(\frac{\eta(p-c)}{p}), & \text{if } F^{-1}(\frac{\eta(p-c)}{p})\leq d(p)<F^{-1}(\eta).\end{cases}$$

On the other hand, if $d(p)\geq F^{-1}(\eta)$, we have

$$\pi(q)=F(v^*)=\begin{cases}(p-c)q+\frac{1}{\eta}\int_0^q p(K-q)dF(K), & \text{if } q<F^{-1}(\eta),\\ \frac{1}{\eta}\int_0^{F^{-1}(\eta)}(pK-cq)dF(K), & \text{otherwise.}\end{cases}$$

In this case, straightforward analysis yields the optimal production quantity $q^*=F^{-1}(\frac{\eta(p-c)}{p})$.

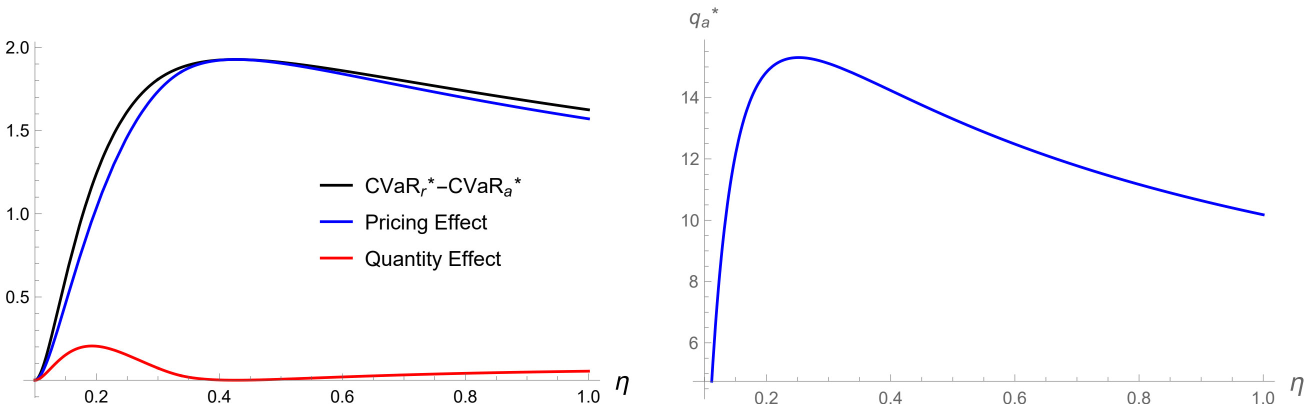
To sum up, combining the above two scenarios, the final optimal production quantity is given as:

$$q^*=\begin{cases}d(p), & \text{if } d(p)<F^{-1}(\frac{\eta(p-c)}{p}),\\ F^{-1}(\frac{\eta(p-c)}{p}), & \text{if } F^{-1}(\frac{\eta(p-c)}{p})\leq d(p).\end{cases}$$

In addition, it is immediate that $q^*\leq d(p)$ and weakly increases in η. $\square$

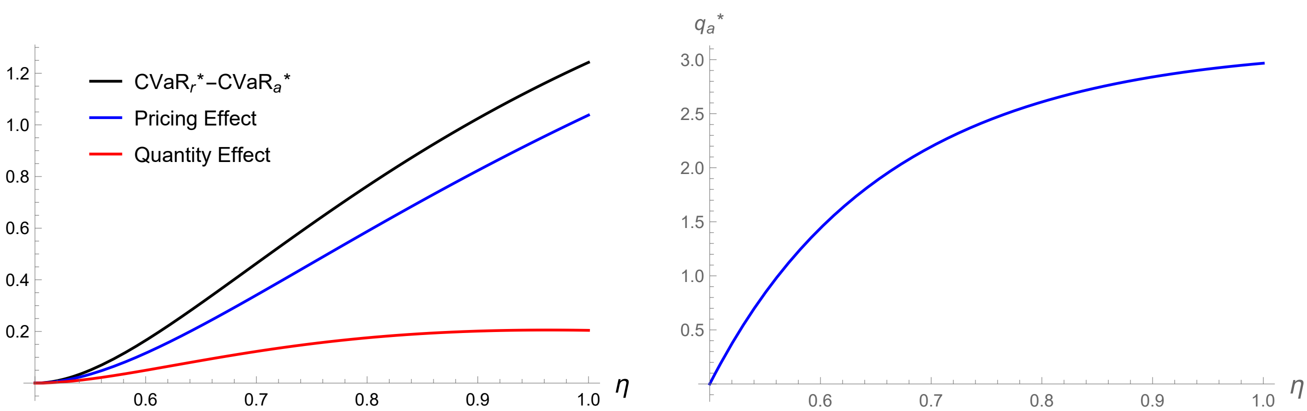
EC.2. Quantity and Pricing Effects

In this appendix, we demonstrate the quantity and the pricing effects discussed in Section 5.3 under the linear demand function $d(p) = 10 - p$ and Uniform $[0,1]$ yield distribution. Figure EC.2 [Figure EC.3] depicts both effects and the optimal order quantity under ex ante pricing with $c = 0.5$ [$c = 2.5$].



(a) Quantity and Pricing Effects (b) Optimal Quantity under Ex Ante Pricing

Figure EC.2 Impact of Risk Aversion on the Decomposed Effects and Optimal Quantity: $c = 0.5$.



(a) Quantity and Pricing Effects (b) Optimal Quantity under Ex Ante Pricing

Figure EC.3 Impact of Risk Aversion on the Decomposed Effects and Optimal Quantity: $c = 2.5$.

EC.3. CVaR Properties

In this appendix, we present some fundamental properties of the CVaR objective, which are used in our analysis.

Property 1 *For a random profit* π*, the definition* $CVaR_\eta(\pi) = E[\pi | \pi \le VaR_\eta(\pi)]$ *is equivalent to that of* $CVaR_\eta(\pi) = \max_{v \in R}\{v + \frac{1}{\eta} E \min\{\pi - v, 0\}\}$.

Proof of Property 1: Let ξ be the random factor in the profit function π with c.d.f. $G(\xi)$. Given $\mathrm{VaR}_\eta(\pi) = \inf\{z|\mathrm{P}(\pi \le z) \ge \eta\}$, then by definition $\mathrm{CVaR}_\eta(\pi) = E[\pi|\pi \le \mathrm{VaR}_\eta(\pi)] = \frac{E\left(\pi \& 1_{\{\pi \le VaR_\eta(\pi)\}}\right)}{E\left(1_{\{\pi \le VaR_\eta(\pi)\}}\right)} = \frac{1}{\eta}\int_{\pi \le VaR_\eta(\pi)} \pi dG(\xi)$. On the other hand, let $F(v) = v + \frac{1}{\eta}E\min\{\pi - v, 0\} = v + \frac{1}{\eta}\int_{\pi \le v}(\pi - v)dG(\xi)$, which is clearly concave in v. Taking derivative, we have $\frac{\partial F(v)}{\partial v} = 1 - \frac{1}{\eta}\int_{\pi \le v} dG(\xi) = 0$, impling that $v^* = \mathrm{VaR}_\eta(\pi)$. Plugging v^* back, we have $F(v^*) = v^* + \frac{1}{\eta}\int_{\pi \le v^*}(\pi - v^*)dG(\xi) = \frac{1}{\eta}\int_{\pi \le VaR_\eta(\pi)} \pi dG(\xi) = \mathrm{CVaR}_\eta(\pi)$, which completes the proof. □

Property 2 *If $\pi(q|\xi)$ is concave in q, then $CVaR(\pi(q)) = \max_{v \in R}\{v + \frac{1}{\eta}E_\xi \min\{\pi(q|\xi) - v, 0\}\}$ is also concave in q.*

Proof of Property 2: Since $\pi(q|\xi)$ is concave in q, then $\pi(q|\xi) - v$ is jointly concave in (q, v) and $\min\{\pi(q|\xi) - v, 0\}$ is also jointly concave in (q, v). As concavity is preserved under expectation, then $E_\xi \min\{\pi(q|\xi) - v, 0\}$ and $v + \frac{1}{\eta}E_\xi \min\{\pi(q|\xi) - v, 0\}$ are both jointly concave in (q, v). Finally, since concavity is preserved under maximization, $\mathrm{CVaR}(\pi(q)) = \max_{v \in \mathrm{R}}\{v + \frac{1}{\eta}\mathrm{E}_\xi \min\{\pi(q|\xi) - v, 0\}\}$ is concave in q. □

EC.4. Discussions on the Expected Utility Criterion

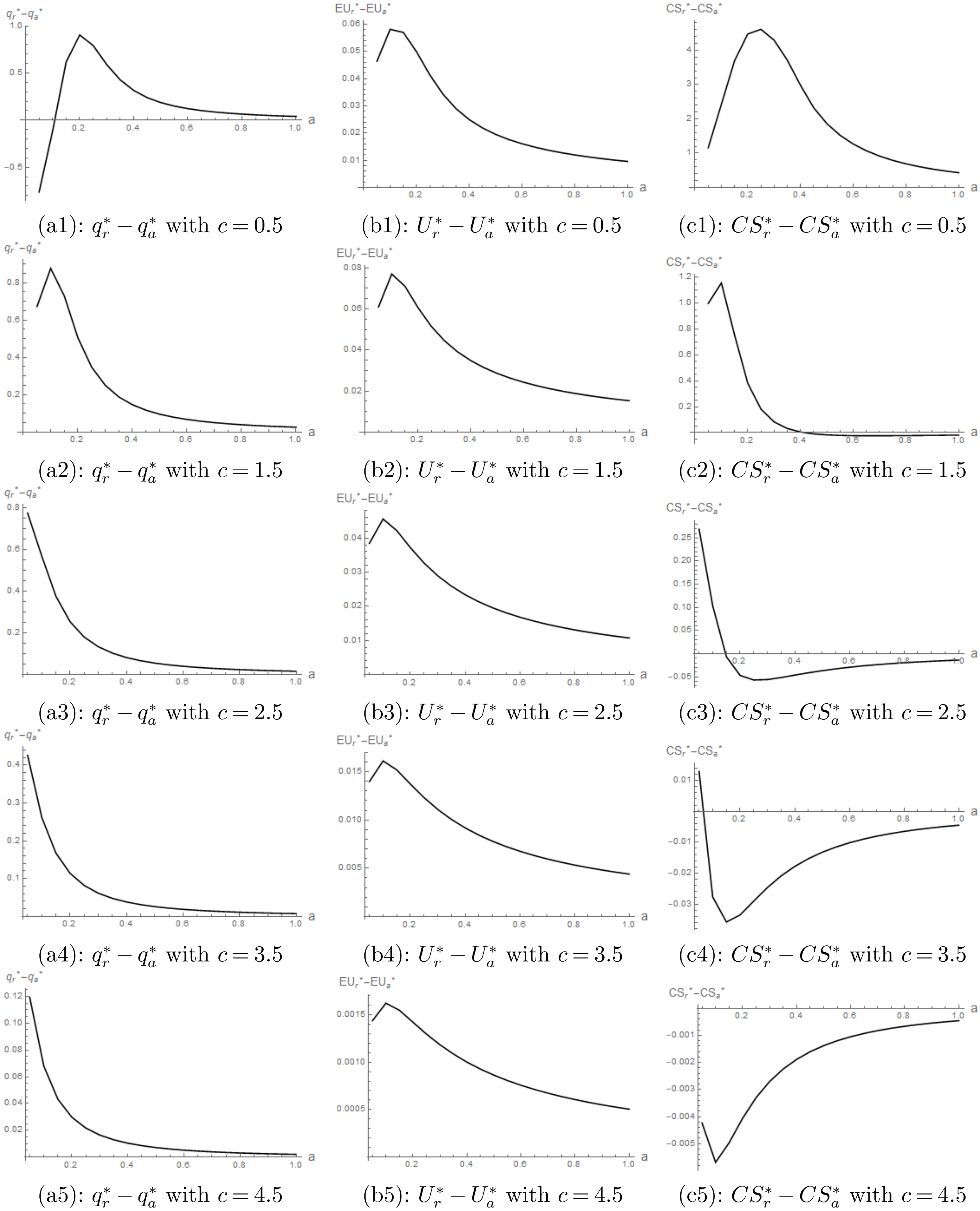
In this appendix, we list the numerical results obtained under the expected utility criterion in Figure EC.4, which depicts the impact of absolute risk aversion level (i.e., a) on the differences of production quantity $q_r^* - q_a^*$ (column a), optimal utility $U_r^* - U_a^*$ (column b), and consumer surplus $CS_r^* - CS_a^*$ (column c), respectively. The parameter combinations used in this numerical experiment are given in Section 7.1.

EC.5. Discussions on General Cost Structure

In this appendix, we analyze the firm's optimal decisions under the general payment structure discussed in Section 7.2 and prove Lemma 5. We first analyze the ex ante pricing scheme and then the responsive pricing scheme. For expositional brevity, we assume that $\xi \in [0, 1]$.

EC.5.1. The Ex ante Pricing Scheme

Let q_a and p_a be the production quantity and sales price, respectively. Define $F(v) = v + \frac{1}{\eta}E_\xi \min\{\pi_a(p_a, q_a) - v, 0\}$, where $\pi_a(p_a, q_a) = p\min\{d(p_a), q_a\xi\} - (c + \bar{c}\xi)q_a$. The firm needs to decide both p_a and q_a to maximize its $\mathrm{CVaR}(p_a, q_a) = \max_{v \in \mathbb{R}} F(v)$. Similar to the analysis in the main model, we adopt the sequential optimization approach by solving q_a as a function of p_a and the discuss the analysis of p_a. For any given p_a, there are two cases to consider.



(a1): $q_r^* - q_a^*$ with $c = 0.5$ (b1): $U_r^* - U_a^*$ with $c = 0.5$ (c1): $CS_r^* - CS_a^*$ with $c = 0.5$

(a2): $q_r^* - q_a^*$ with $c = 1.5$ (b2): $U_r^* - U_a^*$ with $c = 1.5$ (c2): $CS_r^* - CS_a^*$ with $c = 1.5$

(a3): $q_r^* - q_a^*$ with $c = 2.5$ (b3): $U_r^* - U_a^*$ with $c = 2.5$ (c3): $CS_r^* - CS_a^*$ with $c = 2.5$

(a4): $q_r^* - q_a^*$ with $c = 3.5$ (b4): $U_r^* - U_a^*$ with $c = 3.5$ (c4): $CS_r^* - CS_a^*$ with $c = 3.5$

(a5): $q_r^* - q_a^*$ with $c = 4.5$ (b5): $U_r^* - U_a^*$ with $c = 4.5$ (c5): $CS_r^* - CS_a^*$ with $c = 4.5$

Figure EC.4 **Impact of Constant Absolute Risk Aversion Level** a **on** $q_r^* - q_a^*$ **(Column a),** $U_r^* - U_a^*$ **(Column b) and** $CS_r^* - CS_a^*$ **(Column c) under Expected Utility Criterion:** c **increases in** $\{0.5, 1.5, 2.5, 3.5, 4.5\}$ **from top to bottom.**

Case 1: If $q_a \le d(p_a)$, then $\pi_a(p_a,q_a) = [p\xi - (c+\bar{c}\xi)q_a]$. In this case, $\pi_a(p_a,q_a) - v \le 0$ if and only if $\xi \le \frac{v+cq_a}{(p-\bar{c})q_a}$. Comparing $\frac{v+cq_a}{(p-\bar{c})q_a}$ with the interval $[0,1]$, we have:

$$F(v) = \begin{cases} v, & \text{if } v \in (-\infty, -cq_a], \\ v + \frac{1}{\eta}\int_0^{\frac{v+cq_a}{(p-\bar{c})q_a}} [p_a\xi - (c+\bar{c}\xi)q_a] - v dG(\xi), & \text{if } v \in (-cq_a, (p_a-c)q_a], \\ v + \frac{1}{\eta}\int_0^1 [p_a\xi - (c+\bar{c}\xi)q_a] - v dG(\xi), & \text{if } v \in ((p_a-c)q_a, \infty]. \end{cases}$$

Applying the same analysis as conducted in the proof of Proposition 1, it is immediate that $F(v)$ is continuouly differentiable and concave in v with the v^* satisfying $\frac{v+cq_a}{(p_a-\bar{c})q_a} = G^{-1}(q_a)$. Plugging v^* back, we have $F(v^*) = \frac{1}{\eta}\int_0^{G^{-1}(\eta)} [p_a\xi - (c+\bar{c}\xi)]q_a dG(\xi)$.

Case 2: If $q_a > d(p_a)$, then $\pi_a(p_a,q_a) = p\min\{q_a\xi, d(p_a)\} - (c+\bar{c}\xi)q_a = [p\xi - (c+\bar{c}\xi)]q_a := \pi_a^1$ if $\xi \le \frac{d(p_a)}{q_a}$ and $\pi_a(p_a,q_a) = p_a d(p_a) - (c+\bar{c}\xi)q_a := \pi_a^2$, otherwise. When $\pi_a = \pi_a^1$, we have $\pi_a^1 - v \le 0$ iff $\xi \le \frac{v+cq_a}{(p-\bar{c})q_a}$. When $\pi_a = \pi_a^2$, we have $\pi_a^2 - v \le 0$ iff $\xi \ge \frac{p_a d(p_a) - cq_a - v}{\bar{c}q_a}$. Comparing $\frac{v+cq_a}{(p-\bar{c})q_a}$ and $\frac{p_a d(p_a) - cq_a - v}{\bar{c}q_a}$ with respective threshold values $\frac{d(p_a)}{q_a}$, 0, and 1, we can obtain the following:

$$F(v) = \begin{cases} v, & \text{if } v \in (-\infty, -cq_a], \\ v + \frac{1}{\eta}\int_0^{\frac{v+cq_a}{(p-\bar{c})q_a}} [p_a\xi q_a - (c+\bar{c}\xi)q_a - v] dG(\xi), & \text{if } v \in (-cq_a, p_a d(p_a) - (c+\bar{c})q_a], \\ v + \frac{1}{\eta}\int_0^{\frac{v+cq_a}{(p-\bar{c})q_a}} [p_a\xi q_a - (c+\bar{c}\xi)q_a - v] dG(\xi) + \frac{1}{\eta}\int_{\frac{p_a d(p_a) - cq_a - v}{\bar{c}q_a}}^1 [p_a d(p_a) - (c+\bar{c}\xi)q_a - v] dG(\xi), & \\ & \text{if } v \in (p_a d(p_a) - (c+\bar{c})q_a, p_a d(p_a) - cq_a - \bar{c}d(p_a)], \\ v + \frac{1}{\eta}\int_0^{\frac{d(p_a)}{q_a}} [p_a\xi q_a - (c+\bar{c}\xi)q_a - v] dG(\xi) + \frac{1}{\eta}\int_{\frac{d(p_a)}{q_a}}^1 [p_a d(p_a) - (c+\bar{c}\xi)q_a - v] dG(\xi), & \text{otherwise.} \end{cases}$$

It can be verified that $F(v)$ is continuously differentiable and concave in v. Since $\frac{\partial F(v)}{\partial v}|_{v=-cq_a} = 1 > 0$ and $\frac{\partial F(v)}{\partial v}|_{v=p_a d(p_a) - cq_a - \bar{c}d(p_a)} = 1 - \frac{1}{\eta} < 0$, v^* must be in the interval $(-cq_a, p_a d(p_a) - cq_a - \bar{c}d(p_a))$. Define $\bar{q}_a$ such that $\frac{\partial F(v)}{\partial v} = 0$, which is equivalent to $\frac{p_a d(p_a) - \bar{c}\bar{q}_a}{(p-\bar{c})\bar{q}_a} = G^{-1}(\eta)$. On one hand, when $q_a \le \bar{q}_a$, $v^* \in (-cq_a, p_a d(p_a) - (c+\bar{c})q_a]$ and satisfies $\frac{v^* + cq - a}{(p_a - \bar{c})q_a} = G^{-1}(\eta)$, which implies to $F(v^*) = \frac{1}{\eta}\int_0^{G^{-1}(\eta)} [p_a\xi - (c+\bar{c}\xi)]q_a dG(\xi)$. On the other hand, when $q_a > \bar{q}_a$, $v^* \in (p_a d(p_a) - (c+\bar{c})q_a, p_a d(p_a) - cq_a - \bar{c}d(p_a))$ and satisfies that $\int_0^{\frac{v+cq_a}{(p-\bar{c})q_a}} dG(\xi) + \int_{\frac{p_a d(p_a) - cq_a - v}{\bar{c}q_a}}^1 dG(\xi) = \eta$, which implies that $F(v^*) = \frac{1}{\eta}\int_0^{\frac{v^*+cq_a}{(p-\bar{c})q_a}} [p_a\xi - (c+\bar{c}\xi)]q_a dG(\xi) + \frac{1}{\eta}\int_{\frac{p_a d(p_a) - cq_a - v^*}{\bar{c}q_a}}^1 [p_a d(p_a) - (c+\bar{c}\xi)q_a] dG(\xi)$.

Combining the above two cases, for any given sales price p_a, we can express the firm's CVaR as a function of q_a as follow:

$$\Pi_a(q_a|p_a) := F(v^*) = \begin{cases} \frac{1}{\eta}\int_0^{G^{-1}(\eta)} (p_a\xi - c - \bar{c}\xi)q_a dG(\xi), & \text{if } q_a \in (-\infty, \bar{q}_a], \\ \frac{1}{\eta}\int_0^{\frac{v^*+cq_a}{(p-\bar{c})q_a}} [p_a\xi - (c+\bar{c}\xi)]q_a dG(\xi) + \frac{1}{\eta}\int_{\frac{p_a d(p_a) - cq_a - v^*}{\bar{c}q_a}}^1 [p_a d(p_a) - (c+\bar{c}\xi)q_a] dG(\xi), & \text{if } q_a \in (\bar{q}_a, \infty). \end{cases}$$

It can be easily verify that $\Pi_a(q_a|p_a)$ is continuously differentiable in q_a, since $\Pi_a(\bar{q_a}^-|p_a) = \Pi_a(\bar{q_a}^+|p_a)$ and $\frac{\partial \Pi_a(q_a|p_a)}{\partial q_a}|_{q_a=\bar{q}_a^-} = \frac{\partial \Pi_a(q_a|p_a)}{\partial q_a}|_{q_a=\bar{q}_a^+} = \frac{1}{\eta}\int_0^{G^{-1}(\eta)} (p_a\xi - c - \bar{c}\xi) dG(\xi)$. In addition, by the Property 2 in Appendix EC.3, $\Pi_a(q_a|p_a)$ is concave in q_a since $\pi_a(p_a,q_a)$ is concave in q_a. Thus, $\Pi_a(q_a|p_a)$ is continuously differentiable and concave in q_a, and it is optimal for the firm to produce if and only if $\frac{\partial \Pi_a(q_a|p_a)}{\partial q_a}|_{q_a=0} = \frac{1}{\eta}\int_0^{G^{-1}(\eta)} (p_a\xi - c - \bar{c}\xi) dG(\xi) \ge 0$, which is equivalent to $p_a \ge$

$\frac{c\eta}{\int_0^{G^{-1}(\eta)}\xi dG(\xi)}+\bar{c}:=\tilde{p}_a$. When $p_a\geq\tilde{p}_a$, $\Pi_a(q_a|p_a)$ increases in $q_a\in(-\infty,\bar{q}_a)$ and the optimal $q_a^*\geq\bar{q}_a$ satisfying the corresponding first order condition: $\int_0^{\frac{v^*+cq_a}{(p-\bar{c})q_a}}[p_a\xi-(c+\bar{c}\xi)]dG(\xi)-\int_{\frac{p_a d(p_a)-cq_a-v^*}{\bar{c}q_a}}^1(c+\bar{c}\xi)dG(\xi)=0$, where v^* satisfies $\int_0^{\frac{v+cq_a}{(p-\bar{c})q_a}}dG(\xi)+\int_{\frac{p_a d(p_a)-cq_a-v}{\bar{c}q_a}}^1 dG(\xi)=\eta$.

Plugging q_a^* back, after some simplifications, we have $\Pi_a(p_a)=p_a d(p_a)(1-\frac{1}{\eta}G(\frac{p_a d(p_a)-cq_a^*-v^*}{\bar{c}q_a^*}))$, given that $p_a\geq\tilde{p}_a$. Clearly, we have $\Pi_a(\tilde{p}_a)=\Pi_a(p_0)=0$. Since $\Pi(p_a)$ is a continuous function defined on a compact set $p_a\in[\min\{\tilde{p}_a,p_0\},p_0]$, there must exists at least a global maximal point. Moreover, taking derivative and after some cumbersome calculation, we have $\frac{\partial\Pi_a(p_a)}{\partial p_a}|_{p_a=\tilde{p}_a}>0$ and $\frac{\partial\Pi_a(p_a)}{\partial p_a}|_{p_a=p_0}<0$, then p_a^* is not a corner point and must satisfy the first order condition, as long as the set of feasible sales price $[\min\{\tilde{p}_a,p_0\},p_0]$ is non-empty.

Next, we discuss the threshold risk aversion level. To ensure that it is optimal for the firm to produce, we need to ensure the feasible region for the pricing decision is non-empty, because the firm's optimal (p_a^*,q_a^*) always exists given the non-empty feasible price set, which pins down to require $\tilde{p}_a:=\frac{c\eta}{\int_0^{G^{-1}(\eta)}\xi dG(\xi)}+\bar{c}\leq p_0$. Let $f(\eta):=\frac{c\eta}{\int_0^{G^{-1}(\eta)}\xi dG(\xi)}+\bar{c}$. It is clear that $f(\eta)$ decreases in η with $f(0)\to+\infty$ and $f(1)=\frac{c}{\mu}+\bar{c}\leq p_0$ (i.e., the effective per unit total cost is no larger than p_0). Thus, there must exists a unique $\eta_a(p_0,c,\bar{c})$ such that $f(\eta)\leq p_0$ if and only if $\eta\geq\eta_a(p_0,c,\bar{c})$. That is, it is optimal for the firm to produce if and only if $\eta\geq\eta_a(p_0,c,\bar{c})$.

To conclude, we remark that q_a^* is no longer a linear inflation of $d(p_a)$ based on the above analysis unless $\bar{c}=0$. When $\bar{c}=0$, the region $[p_a d(p_a)-(c+\bar{c})q_a, p_a d(p_a)-cq_a-\bar{c}d(p_a)]$ clusters into a single point $p_a d(p_a)-cq_a$. In this case, function $F(v)$ defined in Case 2 only has 3 pieces. Similar analysis yields that: when $q_a>\bar{q}_a$, $v^*=\arg\max_{v\in\mathbb{R}}F(v)=p_a d(p_a)-cq_a$ and $F(v^*)=\frac{1}{\eta}\int_0^{\frac{d(p_a)}{q_a}}[p_a\xi-c]q_a dG(\xi)+\frac{1}{\eta}\int_{\frac{d(p_a)}{q_a}}^1[p_a d(p_a)-cq_a]dG(\xi)$. Then, all the subsequent analysis is the same as that in the proof of Proposition 1. In addition, when plugging q_a^* back, we have $\Pi_a(p_a)=p_a d(p_a)(1-\frac{1}{\eta}G(\frac{d(p_a)}{q_a^*(p_a)}))$. All the analysis in the proof of Proposition 2 continues to hold.

EC.5.2. The Responsive Pricing Scheme

For any given production quantity q_r, define $F(v)=v+\frac{1}{\eta}E_\xi\min\{\pi_r(q_r)-v,0\}$, where $\pi_r(q_r)=\max_{p_r\in[0,p_0]}\{p_r\min\{d(p_r),q_r\xi\}\}-(c+\bar{c}\xi)q_r$ is concave in q_r. Then, based on Property 2 in Appendix EC.3, $\Pi_r(q_r)=\text{CVaR}(\pi_r(q_r))$ is concave in q_r. Define $\bar{q}$ such that $p'(\bar{q})\bar{q}+p(\bar{q})=\bar{c}$. Clearly, $\bar{q}<d^*$ due to the concavity of $p(d)d$. There are three cases to consider.

Case 1: If $q_r\leq\bar{q}$, then let $f(\xi)=\pi_r(q_r)=p(q_r\xi)q_r\xi-(c+\bar{c}\xi)q_r$, which is concave in ξ. It can be shown that $f'(0)=(p_0-\bar{c})q>0$ and $f'(1)=p'(q)q+p(q)-\bar{c}>p'(\bar{q})\bar{q}+p(\bar{q})-\bar{c}=0$, which implies that $f(\xi)$ increases in $\xi\in[0,1]$. Then, let $\hat{\xi}$ satisfies $f(\xi)=v$, we have the following:

$$F(v)=\begin{cases}v, & \text{if } v\in(-\infty,-cq_r],\\ v+\frac{1}{\eta}\int_0^{\hat{\xi}}[p(q_r\xi)q_r\xi-cq_r-\bar{c}\xi q_r-v]dG(\xi), & \text{if } v\in(-cq_r,p(q_r)q_r-(c+\bar{c})q_r],\\ v+\frac{1}{\eta}\int_0^1[p(q_r\xi)q_r\xi-cq_r-\bar{c}\xi q_r-v]dG(\xi), & \text{otherwise .}\end{cases}$$

It is clear that $F(v)$ is continuously differentiable and concave in v. We have v^* such that $\hat{\xi} = G^{-1}(\xi)$ and $F(v^*) = \frac{1}{\eta}\int_0^{G^{-1}(\eta)}(p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r)dG(\xi)$.

Case 2: If $q_r \in [\bar{q}, d^*]$, let $f(\xi) = \pi_r(q_r) = p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r$ which is concave in ξ with $f'(0) > 0$ and $f'(1) = p'(q_r)q_r + p(q_r) - \bar{c} < p'(\bar{q})\bar{q} + p(\bar{q}) - \bar{c} = 0$. Let $\xi^* \in (0,1)$ satisfying $f'(\xi) = 0$. If $v \le p(q_r)q_r - (c+\bar{c})q_r$, then there exists a $\hat{\xi}_1$ such that $f(\xi) = v$. If $p(q_r)q_r - (c+\bar{c})q_r < v \le f(\xi^*)$, there exist $\hat{\xi}_1 \le \hat{\xi}_2$ such that $f(\xi) = v$. Otherwise, $v > f(\xi)$ always holds. We have:

$$F(v) = \begin{cases} v, & \text{if } v \in (-\infty, -cq_r], \\ v + \frac{1}{\eta}\int_0^{\hat{\xi}_1}[p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r - v]dG(\xi), & \text{if } v \in (-cq_r, p(q_r)q_r - (c+\bar{c})q_r], \\ v + \frac{1}{\eta}\int_0^{\hat{\xi}_1}[p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r - v]dG(\xi) + \frac{1}{\eta}\int_{\hat{\xi}_2}^1[p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r - v]dG(\xi), & \text{if } v \in (p(q_r)q_r - (c+\bar{c})q_r, f(\xi^*)], \\ v + \frac{1}{\eta}\int_0^1[p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r - v]dG(\xi), & \text{otherwise}. \end{cases}$$

It is immediate that $F(v)$ is continuously differentiable and concave in v. Let $\bar{q}_r^1$ such that $\frac{\partial F(v)}{\partial v}|_{v=p(q_r)q_r-(c+\bar{c})q_r} = 1 - \frac{1}{\eta}\int_0^{\hat{\xi}_1} dG(\xi) = 0$. On one hand, if $q_r < \bar{q}_r^1$, then $v^* \in (-cq_r, p(q_r)q_r - (c+\bar{c})q_r)$ and $F(v^*) = \frac{1}{\eta}\int_0^{G^{-1}(\eta)}(p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r)dG(\xi)$. On the other hand, if $q_r > \bar{q}_r^1$, then $v^* \in (p(q_r)q_r - (c+\bar{c})q_r, f(\xi^*))$ satisfying $(\int_0^{\hat{\xi}_1} + \int_{\hat{\xi}_2}^1)dG(\xi) = \eta$, and $F(v^*) = \frac{1}{\eta}(\int_0^{\hat{\xi}_1} + \int_{\hat{\xi}_2}^1)(p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r)dG(\xi)$.

Case 3: If $q \ge d^*$, then let $f(\xi) = \pi_r^1(q_r) = p(q_r\xi)q_r\xi - (c+\bar{c}\xi)q_r$ if $\xi \le \frac{d^*}{q_r}$ and $f(\xi) = \pi_r^2(q_r) = p(d^*)d^* - (c+\bar{c}\xi)q_r$ otherwise. Clearly, $f(\xi)$ is concave in ξ with $f'(0) = (p_0 - \bar{c})q_r > 0$ and $f'(\xi)|_{\xi \ge \frac{d^*}{q_r}} = -\bar{c}q_r$. Then, ξ^* satisfying $f'(\xi) = 0$ must belong to $[0, \frac{d^*}{q_r})$. If $v \le p(q_r)q_r - (c+\bar{c})q_r$, then let $\hat{\xi}_1$ be the unique solution such that $f(\hat{\xi}_1) = \pi_r^1(q_r) = v$. If $v^* \in (p(q_r)q_r - (c+\bar{c})q_r, p(d^*)d^* - (c+\bar{c})q_r]$, then let $\hat{\xi}_1 < \frac{d^*}{q_r}$ such that $f(\xi) = \pi_r^1(q_r) = v$ and let $\hat{\xi}_2 \ge \frac{d^*}{q_r}$ such that $f(\xi) = \pi_r^2(q_r) = v$. If $v \in [p(d^*)d^* - (c+\bar{c})q_r, f(\xi^*)]$, then let $\hat{\xi}_1 < \hat{\xi}_2$ be the two solutions of $f(\xi) = \pi_r^1(q_r) = v$. Combining all the cases, we have:

$$F(v) = \begin{cases} v, & \text{if } v \in (-\infty, -cq_r], \\ v + \frac{1}{\eta}\int_0^{\hat{\xi}_1}[p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r - v]dG(\xi), & \text{if } v \in (-cq_r, p(q_r)q_r - (c+\bar{c})q_r], \\ v + \frac{1}{\eta}\int_0^{\hat{\xi}_1}[p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r - v]dG(\xi) + \frac{1}{\eta}\int_{\hat{\xi}_2}^1[p(d^*)d^* - cq_r - \bar{c}\xi q_r - v]dG(\xi), & \text{if } v \in (p(q_r)q_r - (c+\bar{c})q_r, p(d^*)d^* - (c+\bar{c})q_r], \\ v + \frac{1}{\eta}\int_0^{\hat{\xi}_1}[p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r - v]dG(\xi) + \frac{1}{\eta}\int_{\hat{\xi}_2}^1[p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r - v]dG(\xi), & \text{if } v \in (p(d^*)d^* - (c+\bar{c})q_r, f(\xi^*)], \\ v + \frac{1}{\eta}\int_0^{\frac{d^*}{q_r}}[p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r - v]dG(\xi) + \frac{1}{\eta}\int_{\frac{d^*}{q_r}}^1[p(d^*)d^* - cq_r - \bar{c}\xi q_r - v]dG(\xi), & \text{otherwise}. \end{cases}$$

It is immediate to verify that $F(v)$ is continuously differentiable and concave in v. Moreover, since $q_r > d^* > \bar{q}_r^1$ defined in Case 2, then $\frac{\partial F(v)}{\partial v}|_{v=p(q_r)q_r-(c+\bar{c})q_r} > 0$. And $\frac{\partial F(v)}{\partial v}|_{v=f(\xi^*)} = 1 - \frac{1}{\eta} < 0$. Then v^* must be in $(p(q_r)q_r - (c+\bar{c})q_r, f(\xi^*))$. Let $\bar{q}_r^2$ satisfy $\frac{\partial F(v)}{\partial v}|_{v=p(d^*)d^*-cq_r-\bar{c}q_r} = 1 - \frac{1}{\eta}(\int_0^{\hat{\xi}_1} + \int_{\frac{d^*}{q_r}}^1)dG(\xi) = 0$. It is clear that $\bar{q}_r^2 > \bar{q}_r^1$. On one hand, if $q < \bar{q}_r^2$, then $\frac{\partial F(v)}{\partial v}|_{v=p(d^*)d^*-cq_r-\bar{c}q_r} > 0$ and $v^* \in (p(d^*)d^* - (c+\bar{c})q_r, f(\xi^*))$ satisfying $(\int_0^{\hat{\xi}_1} + \int_{\hat{\xi}_2}^1)dG(\xi) = \eta$, and $F(v^*) = \frac{1}{\eta}(\int_0^{\hat{\xi}_1} + \int_{\hat{\xi}_2}^1)(p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r)dG(\xi)$. On the other hand, if $q_r > \bar{q}_r^2$, then $\frac{\partial F(v)}{\partial v}|_{v=p(d^*)d^*-cq_r-\bar{c}q_r} < 0$ and $v^* \in (p(q_r)q_r - (c+\bar{c})q_r, p(d^*)d^* - (c+\bar{c})q_r)$ satisfying $(\int_0^{\hat{\xi}_1} + \int_{\hat{\xi}_2}^1)dG(\xi) = \eta$, and $F(v^*) = \frac{1}{\eta}\int_0^{\hat{\xi}_1}(p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r)dG(\xi) + \frac{1}{\eta}\int_{\hat{\xi}_2}^1(p(d^*)d^* - cq_r - \bar{c}\xi q_r)dG(\xi)$.

Combining all the above three cases, we have the following value function:

$$\Pi_r(q_r) = F(v^*) = \begin{cases} \frac{1}{\eta}\int_0^{G^{-1}(\eta)}(p(q_r\xi)q_r\xi - cq_r)dG(\xi), & \text{if } q_r \in [0, \bar{q}_r^1), \\ \frac{1}{\eta}(\int_0^{\hat{\xi}_1} + \int_{\hat{\xi}_2}^1)(p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r)dG(\xi), & \text{if } q_r \in [\bar{q}_r^1, \bar{q}_r^2], \\ \frac{1}{\eta}\int_0^{\hat{\xi}_1}(p(q_r\xi)q_r\xi - cq_r - \bar{c}\xi q_r)dG(\xi) + \frac{1}{\eta}\int_{\hat{\xi}_2}^1(p(d^*)d^* - cq_r - \bar{c}\xi q_r)dG(\xi) & \text{otherwise.} \end{cases}$$

It is immediate to verify that $\Pi_r(q_r)$ is continuously differentiable. And by Property 2 in Appendix EC.3, $\Pi_r(q_r)$ is concave in q_r. Next, we discuss whether it is optimal for the firm to produce by checking the sign of $\frac{\partial \Pi_r(q_r)}{\partial q_r}|_{q_r=0}$. Taking derivative, we have $\frac{\partial \Pi_r(q_r)}{\partial q_r}|_{q_r=0} = \frac{1}{\eta}\int_0^{G^{-1}(\eta)}(p_0\xi - c\xi - c)dG(\xi) \geq 0$, which is equivalent to require that $p_0 \geq \bar{c} + \frac{c\eta}{\int_0^{G^{-1}(\xi)}\xi dG(\xi)}$. Let $f(\eta) := \frac{c\eta}{\int_0^{G^{-1}(\eta)}\xi dG(\xi)} + \bar{c}$. It is clear that $f(\eta)$ decreases in η with $f(0) \to +\infty$ and $f(1) = \frac{c}{\mu} + \bar{c} \leq p_0$. Thus, there must exists a unique $\eta_r(p_0, c, \bar{c})$ such that $f(\eta) \leq p_0$ if and only if $\eta \geq \eta_r(p_0, c, \bar{c})$. That is, it is optimal for the firm to produce if and only if $\eta \geq \eta_r(p_0, c, \bar{c})$.

To conclude, we remark that when $\bar{c} = 0$, $\bar{q}$ defined before Case 1 always equals to d^*. Therefore, Case 2 disappears. In addition, for Case 3, $\xi^* = \frac{d^*}{q_r}$ and $f(\xi)$ is a weakly increasing function. As such, $F(v)$ in Case 3 reduces to have only three pieces within the intervals $(-\infty, -cq_r]$ (i.e., piece 1), $(-cq_r, p(d^*)d^* - cq_r]$ (i.e., piece 2), and $[p(d^*)d^* - cq_r, \infty)$ (i.e., piece 5), respectively. Then, all the subsequent analysis is the same as that in the proof of Proposition 3.

EC.5.3. Comparison of the Risk Aversion Thresholds

Based on the analysis conducted in Appendices EC.5.1 and EC.5.2, it is immediate that $\eta_a(p_0, c, \bar{c}) = \eta_r(p_0, c, \bar{c})$ as they are both satisfying the equation of $p_0 = \bar{c} + \frac{c\eta}{\int_0^{G^{-1}(\xi)}\xi dG(\xi)}$, which is shown to admit only a unique solution. That is, price postponement does not affect the firm's risk aversion threshold, which proves Lemma 5.