### Bayesian updates for indoor thermal comfort models

K.W. Mui<sup>a</sup>, T.W. Tsang<sup>a</sup>, L.T. Wong<sup>a,\*</sup>

<sup>a</sup> Department of Building Services Engineering, Faculty of Construction and Environment, The

Hong Kong Polytechnic University, Hung Hom, Hong Kong, China.

\*Corresponding author. Tel.: +852 2766 7783; fax: +852 2765 7198;

E-mail address: beltw@polyu.edu.hk

# Abstract

Achieving thermal comfort through sustainable indoor design is an increasing concern. Thermal comfort modelling is crucial for achieving building energy saving. This study reviews and categorizes major developments and trends in the field of thermal comfort research in recent years. Discrepancies between actual and predicted results of thermal sensation and thermal satisfaction suggests a performance gap in Fanger's model. Based on the current research gaps identified, a practical solution is proposed to improve the reliability of thermal comfort predictions. Two Bayesian updating protocols, namely individual updating and global updating, are put forward and the use of Bayesian approach to systemically update current thermal comfort beliefs with openly available field data is demonstrated. Besides being a practical tool for modelling thermal comfort using the best information available (i.e. existing models and field survey data), the proposed Bayesian updating provides an achievable solution to the present challenges in establishing a reliable thermal comfort prediction model.

## Keywords

Thermal comfort; Acceptance; Prediction; Bayesian updating

## 1. Introduction

Maintaining high standards of thermal comfort ensures excellent indoor environmental quality; however, huge amounts of energy will be consumed by the Heating, Ventilation and Air Conditioning (HVAC) systems to achieve this [1]. Over the past 40 years, despite the immense research efforts put into evaluating building thermal performance, only a number of thermal comfort models have been proposed for predicting occupant thermal satisfaction based on human psychological and physiological responses towards the thermal environment [2].

The most highly cited model for the evaluation of indoor thermal environment is Predicted Mean Vote (PMV) model developed by Fanger in 1972. The model considers the heat balance between human subjects and the thermal environment in a controlled climate chamber experiment. It is based on the assumption that responses driven by thermal stimulation are purely physiological regardless of the potential influencing factors like ventilation mode and climatic variation, which was confirmed by Fanger's experiment in which subjects with different climatic experience produced similar thermal preferences. With four indoor parameters (air temperature ( $T_a$ ), mean radiant temperature ( $T_r$ ), relative humidity (RH) and air velocity ( $V_a$ )) and two occupant's criteria (metabolic rate ( $M_e$ ) and clothing value ( $C_L$ )), the predicted mean vote (PMV) and predicted percentage dissatisfied (PPD) under certain environmental conditions are determined for thermal assessment [3]. PMV model has been the basis and reference of thermal comfort modelling and standards including ANSI/ASHRAE 55-1992 and ISO 1994.

The universality of the PMV model has been questioned due to the discrepancies between the model and field surveys. Humphrey found a strong relation between indoor comfort temperature and outdoor climate, suggesting strong climatic influence could happen on thermal

comfort in building with natural ventilation [4]. Evidence also shows that thermal sensation is influenced not only by physical and physiological interactions but also by cultural, sociological and climactic factors [5]. An adaptive thermal comfort model was therefore developed by de Dear and Brager to take occupant adaptive behaviour into account. They concluded that thermal adaptation can be obtained from field data instead of collecting from experiment. Based on the RP-884 database which contained approximately 21,000 thermal comfort field data from a wide range of climatic zone, the adaptive model proposes that contextual factors and past thermal history (thermal adaptation) affect one's thermal preferences and thus thermal satisfaction. There are three categories of thermal adaptation identified in this model: behavioural adjustment, physiological and psychological. For premises that are naturally ventilated (NV), where occupants have higher degree of indoor climate control than those in office buildings, the adaptive model is more suitable. Adaptive models have been implemented in ANSI/ASHRAE 55-2004 and 55-2010 for NV buildings, EN 15251 for mixed-mode buildings under natural ventilation and ISO 7730.

In 2001, Huizenga et al. [7] developed the Berkeley Comfort Model (CBE model) based on Stolwik's 25-node thermoregulation model. Unlike the PMV model that requires a steady state for accurate thermal comfort prediction, this model is able to simulate transient and spatially asymmetric environmental conditions. Sequential phases with respect to duration,  $M_e$ , physiological constants, clothing,  $T_a$ ,  $T_r$ ,  $V_a$ , RH and contact surface thermal properties are simulated for sixteen body segments and the corresponding equivalent homogenous temperatures (EHT) are used for comfort predictions. The CBE model can identify local thermal discomfort and is applicable to transient non-uniform thermal environments [8,9]. Among the above-mentioned thermal comfort models, Fanger's PMV model remains the most generally accepted. In his review, Van Hoof [10] summarized the discrepancies between actual field data and predictions by the PMV model, and other criticisms regarding the PMV model and its input parameters. Considering the inadequacy of the PMV model, some adjustments and modifications have been proposed in early 2000 to improve the accuracy, reliability and applicability of the model. Two distinct examples are: 1) using ePMV to include the expectancy of occupants in the calculation [11]; and 2) employing PMV<sub>new</sub> to reduce bias and extend the applicability of the PMV model [12]. Nevertheless, all attempts seem to be unable to generalize the original PMV model and make it applicable to all types of environment and all kinds of people. PMV model is still the most cited one and widely adopted in building research and design reference. In order that smart, green buildings of today and tomorrow can be fully realized, an accurate and reliable thermal comfort model is essential.

The present study aims at reviewing the development of thermal comfort research especially in the area of thermal comfort sensation and acceptance modelling and identifying any solution to improve the model accuracy. It is important that this study does not intent to quarrel with any existing model or propose a new model, but rather provide analytical solution to improve a thermal comfort model. This paper first reviews and categorizes major developments and research trends in the field of thermal comfort in recent 10 years. Current gap in thermal comfort research is then identified and the impacts of inaccurate thermal comfort models on other thermal comfort research is discussed. Using Fanger's PMV–PPD model as the basis, Bayesian updating framework is proposed to provide an analytical solution to improve the existing models using field survey data. It is believe that with limited resources, this Bayesian approach for PMV–PPD (or any) model updating can be a solution to improve the prediction accuracy.

# 2. Literature review

Focusing on thermal comfort model development and improvement, this review mainly aims to identify recent research trends in the field of thermal comfort and continue the work by Van Hoof [9]. No attempt will be made to review all thermal comfort topics but mainly concentrate on those related to thermal comfort model in steady thermal environment. To enhance the coverage, indoor thermal comfort reviews published recently are also discussed. The literature search tasks are listed as follows:

- 1) Discuss on recently published review paper related to indoor thermal comfort;
- 2) Categorize and analyze recent research trends related to thermal comfort;
- Identify developments and improvements in modelling thermal comfort sensation and acceptance; and
- 4) Provide suggestions for future development of thermal comfort modelling.

Databases on the Web of Science were searched for relevant articles. Figure 1 shows the results of SCI publications on indoor thermal comfort in the past 10 years (2008–2018) with the search words "CFD or numerical or simulate", "system control", "field study", "thermal manikin", "model development", "climate change" and "energy efficiency". It can be seen that there is a spike in 2013 and the number of publications has quadrupled over the past 6 years. Table 1 exhibits the contribution of each category to the total number of indoor thermal comfort publications from 2008 to 2018. The search results may be attributed to the increasing concerns related to climate change (*p*-value<0.01, *t*-test) and building energy efficiency (*p*-value<0.01, *t*-test). While research efforts on simulation, system control and thermal comfort sensation have stayed the same, the focus on model development was slightly greater in 2018. It can be concluded that thermal comfort research is still very active after nearly 45 years.

### 2.1. Review paper related to indoor thermal comfort modeling

Indoor thermal comfort model review paper in recent 10 years is searched using Web of Science and looked into. Some researcher focused on the thermal comfort parameters and discussed about their impacts on thermal comfort sensation and satisfaction. Karjalainen [13] investigated the differences in thermal comfort between genders and found out that females are usually more sensitive and easily dissatisfied especially in cool environment. Mishra and Ramgopal [14] reviewed about thermal comfort field studies based on climatic zones and discussed the effects of relevant environmental, physiological and contextual factors on thermal comfort. The adaptive opportunities in terms of the use of air-conditioners, selection of building materials, occupant's behavior, etc. were also examined. Vesely and Zeiler presented the effect of personalized conditioning systems on thermal comfort and suggested an energy reduction up to 60% can be achieved with the use of personalized conditioning system. Halawa et al. [15] investigated the impacts of thermal radiation field on thermal comfort and the ways to minimize it. Authors concluded that the existing thermal comfort standards have not adequately addressed the influence of radiation on thermal comfort, that is to say, existing thermal comfort needs to be improved. Djamila [16] reviewed and identified the methodological problem of thermal comfort data analysis using the ASHRAE RP-884 database. Indoor thermal comfort parameters and collected data were investigated and analysed. New classifications for temperature and relative humidity were proposed to describe indoor climate. The study presented a new procedure to find out natural temperature, which gives similar results by existing least squares linear regression analysis, but provides more insight and understanding to the database.

Others reviewed about the existing thermal comfort model, standards and indices. Djongyang et al. [17] comprehensively reviewed existing thermal comfort models, both heat-balanced approach and adaptive approach. Human body thermoregulatory system and the mathematical model for heat exchanged between human body and the environment were also presented. Halawa and van Hoof [18] summarized the foundation and principle of adaptive thermal comfort approach and emphasized that future research should look into the improvement of validity of Fanger's PMV model and extension of its application. Cheng et al. [2] reviewed and compared thermal comfort models for non-uniform (transient) thermal environment. Human thermal physiological and psychological models were reviewed. By comparing the CBE model and ISO 14505, the author suggested that ISO 14505 is more sensitive to warm environment than cold environment, which makes it suitable only for thermal neutral situation. Carlucci and Pagliano [19] discussed about the indices for evaluating thermal comfort and concluded the necessity of having a new index that more comprehensive. Taleghani et al. [20] described the development of the concept of thermal comfort and reviewed existing adaptive thermal comfort standards. It was concluded that the standards were fundamentally different in terms of the equations since the database used to generate the standards were different. Holopainen et al. [21] presented existing thermal comfort assessment approaches and compared different thermal comfort indices. The potential applications of these indices for enhancing building sustainability were also discussed. Croitoru et al. [22] reviewed some thermal comfort models and methods for thermal comfort assessment in indoor environment, suggested that most of them are not comprehensive enough or limited in application. Fu et al. [23] reviewed on the human heat transfer and thermoregulatory responses model. A manikin-human thermal model coupling simulator was developed to improve the performance and validity of the human thermoregulatory model. Katic et al. [24] conducted a review of existing thermos-physiological models for whole body and isolated body segments, and the applications of the models were

discussed. Enescu [25] presented an overview of thermal comfort parameter and indices and illustrated the application of thermal comfort model in system control strategies. Author proposed to include adaptive comfort model into the control system for the consideration of occupant's preference.

#### 2.2. Simulation

Refined searches suggest that the majority of thermal comfort research is related to numerical simulation of indoor thermal environment. Simulation software programs, including computational fluid dynamic (CFD) and the building energy simulation (BES) tool EnergyPlus<sup>TM</sup>, are commonly used to study the thermal environment and evaluate the performance of building ventilation system or design. Chiang et al. [26] used CFD to simulate the indoor air temperature distribution in an office environment with a radiant cooling ceiling system and validated the outcome experimentally. The PMV model was then applied to evaluate the original and improved thermal conditions. In some passive houses in Sweden, Rohdin et al. [27] employed CFD to simulate indoor air flow and air temperature and used BES to predict energy consumption. For evaluating the passive house thermal performance, both PMV and PPD were utilized to relate the simulated physical parameters and thermal comfort sensation. Du et al. [28] used DesignBuilder, a user interface to EnergyPlus<sup>TM</sup>, and CFD to simulate indoor temperature and indoor air velocity distribution respectively. Their simulated results were analysed using adaptive thermal comfort model. In short, numerical analysis and physical building structures are used to analyze the distribution of thermal comfort parameters (e.g. air temperature, air velocity, radiation). In order that thermal sensation and acceptance can be assessed based on simulated results, a reliable thermal sensation model and a corresponding thermal acceptance model (i.e. PMV-PPD model) are required.

### 2.3. System Control

System control research related to thermal comfort usually focuses on optimizing the performance of building HVAC system to achieve energy efficiency as well as acceptable thermal comfort. Freire et al. [29] proposed two model based predictive control strategies for promoting thermal comfort and reducing energy consumption. Based on ASHRAE comfort zone and PMV, their control algorithm optimizes the temperature set-point and humidity control to maintain indoor hygrothermal conditions within the comfort zone or the PMV comfort boundary, while balancing thermal comfort and energy consumption. Mossolly et al. [30] proposed two control strategies for a multi-zone air conditioning system: 1) to maintain temperature set-point and indoor air quality (IAQ) by varying supply air temperature and fresh air flow rate; and 2) to maintain PMV and IAQ by varying supply air temperature, fresh air flow rate and the amount of fresh air. Results showed that maintaining the PMV instead of the temperature set-point saved about 20% more energy, indicating that multi-variable control strategies perform better than single-variable ones. In short, system control research related to thermal comfort requires a reliable thermal comfort model as the reference for developing the system control algorithm. To attain energy savings, occupant thermal comfort may have to be maintained at acceptable level rather than at optimal level.

#### 2.4. Field Study

Thermal comfort field studies contribute some 20% to the total number of publications on thermal comfort. A field study, which is able to capture various contextual factors and occupant adaptive behaviours, is undoubtedly the most accurate way to assess thermal comfort in an indoor environment. However, it is not desirable because of the huge amount of resources involved. It is also not feasible for buildings in design stages. There are usually two parts in a thermal comfort field study: physical measurements of thermal parameters, and questionnaires for collecting occupant responses to the thermal environment. Some studies will compare their field results with existing models and relate them using regression analysis. Lu et al. [31], Cheng et al. [32], Yu et al. [33], Ning et al. [34], Yang et al. [35] and Jiao et al. [36] investigated thermal comfort in living environments. Rupp and Ghisi [37], Thapa et al. [38], Kajtar et al. [39], Gallardo et al. [40], Manu et al. [41] and Luo et al. [42] looked into occupant's thermal comfort in office environment. Hamzah et al. [43], Fang et al. [44], Liu et al. [45], Wang et al. [46], Calis and Kuru [47] and Hamza et al. [48] carried out thermal comfort survey in schools and classrooms. Cardoso et al. [49] conducted a thermal comfort survey in a bus terminal in Portugal. Wang et al. [50] investigated and analysed the thermal environment and thermal adaption of worker in a rubber factory. Sattayakorn et al. [51] carried out survey to identify the thermal comfort of healthcare occupants in Thailand. Liu et al. [52] studied the thermal comfort, vibration and noise in a ship cabin during winter time. Yang et al. [53] investigated thermal comfort in a cotton textile workshop. Yang et al. [54] conducted an adaptive thermal comfort study in an environmental chamber. Hussin et al. [55] compared actual field data with PMV prediction in an air-conditioned mosque in Malaysia.

## 2.5. Thermal manikin

Research related to thermal manikin focuses on the development of thermal manikin which is capable of mimicking the human thermos-physiological responses toward different thermal conditions. Koelblen et al [56] identified in their study that precise tools like human simulator (thermal manikin) can provide reliable response data for thermal model to predict thermal sensation and acceptability, which save us from time-consuming and resource-demanding tasks like conducting survey and field data collection. A methodology that combines thermal manikin, thermoregulation model and thermal sensation model was therefore proposed improve the prediction performance. Apart from thermal sensation prediction, human manikin is also used to reproduce the air flow, thermal environment and particle concentration around the breathing zone in order to validate the CFD simulation related to building ventilation system and heat transfer between human and the surroundings. For example, Alsaad and Voelker used thermal manikin to validate a CFD model in order to evaluate the performance of ductless personalized ventilation system [57]. Assaad et al. investigated the performance of an intermittent periodic personalized ventilation coupled with mixing ventilation with a transient 3-D CFD model which was validated by experiment conduced in a climatic chamber using thermal manikin [58,59]. Mustakallio et al. used thermal manikin to determine the manikin-based equivalent temperature in order to compare the thermal environment in an office with different cooling system [60]. Mao et al. conducted a numerical study on the convective heat transfer between a sleeping individual and the surrounding environment in bedroom equipped with a task/ambient air conditioning system using a thermal manikin [61].

### 2.6. Model development: Thermal sensation model

Research related to improving or developing thermal comfort prediction models is limited. Yao et al. [62] developed a theoretical adaptive thermal comfort model based on PMV and the "Black Box" theory. The model takes cultural, climatic and social factors into account and incorporates an adaptive coefficient into the PMV model. Adaptive behaviour can thus be related to the experimental results by Fanger, and differences between measured and predicted mean votes shall be minimized. Langevin et al. [63] used the Bayesian parameter estimation approach to extend the PMV model to field use. They developed Bayesian thermal sensation, acceptability and preference distributions to formulate a new relationship between PMV and PPD. Wong et al. [64] presented a Bayesian approach to refine Fanger's model with the use of field survey data. The approach allows systematic updates on our current beliefs about thermal

dissatisfaction. Based on the best information available (i.e. existing models and field survey data), it evaluates the statistical importance of field data with a chosen target sample size and an acceptable error value. By integrating the PMV model with the adaptive approach, Marino et al. [65] developed a subjective-adaptive thermal comfort model for predicting thermal sensation. This approach, which uses a multi-agent system (MAS) to survey user thermal preferences and adapts itself to user choices, is able to achieve personalized thermal comfort controls.

Alternatively, thermal comfort can be assessed individually. In fact, the number of personal comfort models is on the rise. Personal thermal comfort model is a data-driven approach to assess thermal comfort by predicting individuals' responses instead of averaging the thermal comfort of a group of occupants. Individuals' thermal comfort data are directly feedback to the system with the help of Internet of Things (IoT), and with the additional personal data, machine learning algorithms, such as logistic regression techniques [66], support vector regression [67] and Bayesian network [68] are employed to train a personal comfort model [43]. With six different machine learning algorithms (Classification Tree, Gaussian Process Classification, Gradient Boosting Method, Kernel Support Vector Machine, Random Forest, Regularized Logistic Regression), Kim et al. [69] showed that personal comfort models gave much better prediction performance than conventional PMV and adaptive thermal comfort models. Although a personal comfort model has its data-driven flexibility, its machine learning approach requires an expensive feedback and sensing system for identifying actual individuals' preferences. Besides, it is not feasible for buildings in design stages. As a result, personal comfort model is excluded from the discussion in this study.

#### 2.7. Model development: Thermal acceptance model

While considerable research has been devoted to developing or improving thermal sensation models, far too little effort has been directed towards assessing thermal acceptance. Despite the fact that new Bayesian approaches have been developed for the improvement of PMV–PPD representation (e.g. [63,64]), the conventional PMV–PPD model is still the primary tool for assessing the thermal acceptance of occupants in most thermal comfort research studies.

#### 2.8. Summary of literature review

Figure 2 illustrates the relationship of different research areas, connected by thermal sensation and acceptance models, in the field of thermal comfort. Simulation, system control and thermal manikin are the modules for analysing building performance with respect to thermal comfort; and field study is the module for investigating the relationship between predicted and actual thermal sensations as well as the relationship between predicted and actual thermal acceptances. In model development, efforts are currently put into generating refined models. While thermal sensation modelling is the main focus of thermal comfort research, research efforts in thermal acceptance are lacking.

## 3. Current gap in thermal comfort research

Some studies found discrepancies between actual and predicted results of thermal sensation (thermal sensation vote (TSV) and PMV) and thermal satisfaction (actual percentage dissatisfied (APD) and PPD) of occupants. The correlation between TSV and PMV can be expressed by Eq. (1).

$$TSV = C_1 \times PMV + C_0 \tag{1}$$

Research has shown that this correlation depends on the following: ventilation system type [49,70], thermal perception, tolerance and adaptation of occupants [31,44,49,50,70–80], occupant characteristics (gender and age) [76,81], climatic or seasonal variation [49,82–86], and the state of environmental characteristics (i.e. steady or transient) [49]. Table 2 summarizes some on-site thermal comfort assessment results over the past five years. C<sub>1</sub> and C0 shown were either acquired from the data reported in the study, or estimated from graph provided. The differences found between TSV and PMV suggest that PMV model adjustment is required for actual field use.

As buildings are designed to provide an acceptable environment for the occupants, extreme TSV values (i.e. +/-3, representing hot and cold) are rare in field data. According to Table 2, +/-3 votes contribute an average of 5.08% to the total number of thermal votes. Depending on the analysis method adopted, such a small sample size (e.g. less than 5 extreme votes in some assessments) will make the regression output either statistically insensitive or biased. As a result, the reliability of the extrapolated PMV–TSV regression is questionable [64].

Similarly, the thermal acceptance of occupants was found different when compared to Fanger's PPD model. Some field study results over the past five years are summarized in Table 3. A field study conducted in a tropical island region (Hainan, China) reported that the APD at an extreme value of TSV (-3: 8.7% or +3: 40.91%) was much lower than the corresponding PPD (99%). In that study, there were 59.7% and 43.5% of occupants expecting no changes in indoor temperature at TSV = -2 (cool) and TSV = -3 (cold) respectively [31]. Another study carried out in Bangkok hospitals showed that while the medical staff were satisfied with the predicted thermal neutrality, patients and visitors preferred a warmer environment [37,43,51,87–89]. In

fact, many studies of thermal preferences revealed a broader thermal acceptance range among building occupants [37,43,51,52,87–89], which can be due to the thermal tolerance and adaption [90–93]. This finding suggests that there is a certain degree of disagreement between field outcome and the PPD by Fanger's model.

While thermal sensation is related to thermal environmental parameters, thermal acceptance examines whether the thermal environment is acceptable to building occupants. From a practical point of view, discussing the sensation may not be useful if the correlation between sensation and acceptance is inconsistent most of the time. According to the field study results, a cold (-3)/ cool (-2)/ warm (+2)/ hot (+3) sensation does not necessarily mean an unacceptable thermal environment, and a neutral sensation (0) does not imply an acceptable thermal environment.

## 3.1. Effects and implications of the performance gap of PMV–PPD model

The development of thermal comfort models has not made much progress since 2008 due to the complex relationships between physical parameters and choice-making aspects. Although Fanger tried hard to make his model as objective as possible, subjective psychological effects have increasingly been proved to exert great influences on thermal sensation and acceptance. The discrepancies between predicted and measured results suggest a performance gap in the PMV–PPD model, and that may induce research errors.

A number of studies applied PMV control to improve energy performance together with thermal comfort. For instance, a study using PMV as the reference parameter for controlling ground-source heat pump system (GSHP) to maintain thermal comfort showed that a 20% of energy could be saved without jeopardizing thermal comfort [94]. Another study employing PMV control rather than dry-bulb air temperature control reported 7.3% less annual energy consumed by gas boilers and 28.8% less annual electricity used for cooling [95]. Yet, regardless of how impressive these findings look, their implications would not be valid or useful if the model basis itself is inaccurate.

According to the field survey, PMV = 0 does not necessarily give TSV = 0. According to the field results shown in Table 2, the corresponding range of PMV to TSV = -1, 0 and 1 by Eq. (1), and the corresponding PPD are illustrated in Table 4. It can be seen that TSV of -1 to 1 gives a range of PMV from -3.59 to 5.64 (mean: -1.79 to 1.51), which basically covers to whole range of PPD (mean: 66.7% to 51.7%). If PMV is assumed to be equal to TSV, i.e. as presumed in most thermal comfort studies, the PPD values for the votes TSV = -1, 0 and 1 shall be 26.1%, 5% and 26.1% respectively, indicating a PPD difference of up to 73.9%.

The use of Fanger's model as the basis of thermal comfort research also results in differences between PPD and APD. Currently, maintaining a minimum value of 5% thermally dissatisfied persons for PMV = 0 is adopted in thermal comfort management practices and research related to system control and simulation. However, the field study outcome in Table 3 revealed that occupants were actually satisfied with a wider PMV range when PMV = TSV. Examples include a study by Lu et al. [40] that demonstrated a TSV range from -2 to 0 corresponded to ADP < 2.8%, and an assessment by Pereira et al. [96] that reported a minimum percentage dissatisfied when TSV  $\neq$  0.

If the discrepancies between PMV and TSV as well as those between PMV and PPD are taken into consideration, the PMV–PPD model may be unfit for thermal comfort analysis. This can be shown using the GSHP study by Fang et al. [94] as an example. In that study, a non-linear relationship between PMV = -0.05-0.4 and power consumption = 1.4-2.5 kW (power consumption = 1.77 kW at PMV = 0) was described. The study also reported that a 20% of energy could be saved by maintaining the PMV at a level of -0.07, corresponding to a PPD of 5.1%. According to Table 5, which presents the corresponding TSV values at PMV = 0 and -0.07 determined from the assessment results in Table 2, however, thermal comfort (PPD < 5%) can neither be maintained at PMV = 0 nor -0.07. On the other hand, thermal comfort can be achieved at PMV = -0.14 (corresponding to a mean value of TSV = 0), while energy reduction can be attained at PMV = -0.24 (corresponding to a mean value of TSV = -0.07). The difference between PMV and TSV can be easily noticed.

Figure 3 shows the power consumption for the PMV data extracted from the GSHP study, with the assumption PMV = TSV. It should be noted that a linear relationship was assumed to simplify the calculation. Based on the field data collected from the literature search, the actual PMV values, which are calculated using Equation (1) and mean C<sub>1</sub> and C<sub>0</sub> from all studies (shown in Table 2), are plotted in the figure for comparison. The uncertainty range resulted from the difference between PMV and TSV was from 31.5% to 3.0%, with an average of 14.8%. This range is extremely significant when compared to the 20% energy savings claimed in the study.

Another uncertainty can be found in the range of PMV/TSV that represents the 5% dissatisfied. Figure 4 exhibits the relationship between PMV and thermal dissatisfaction. It can be seen that the APD is generally lower than the PPD, resulting in a wider PMV range (i.e. PMV = -0.64 - 0.000 (0.58) for maintaining the thermal comfort level with less than 5% dissatisfied while achieving higher energy efficiency. Since the GSHP study did not discuss about the power consumption below PMV = -0.07, the effect of energy savings with a wider range of acceptable PMV values cannot be quantified when no actual energy data is available. Nevertheless, a wider acceptable PMV range offers greater energy savings potential for both heating and cooling systems.

In spite of the fact that the PMV–PPD model may not be able to accurately evaluate thermal comfort, it is still being used as the basis of most thermal comfort research, especially for research related to indoor environment simulation and system control. Before a model that can truly represent thermal comfort sensation and acceptance is available, the PMV–PPD representation can be updated accordingly using the field data gathered from worldwide research efforts to minimize the performance gap of the PMV–PPD model.

Based on the efforts by Wong et al. on Bayesian numerical representation [64], this study presents a novel analytical solution for target sample size selection and demonstrates the use of Bayesian approach to systemically update the PMV–PPD model with openly available field data.

### 4. Bayesian estimates and parameters

Bayes' theorem, which relates the conditional and marginal probabilities of stochastic events A and B (where B has a non-vanishing probability), asserts that the probability of event A given event B depends not only on the relation between events A and B but also on the marginal probability of occurrence of each event. This theory can be applied to a sample size not large enough for decision-making purposes, yet relevant enough for statistical analysis. Its general formulation and various applications are available in the literature [97].

The proposed approach predicts collective acceptance of an environmental condition using the readily available information (event A) and the new measurements from an indoor environment (event B) [64].

If a measured acceptance value  $\rho$  is significantly different from a prior belief of the acceptance  $\rho_0$ , then  $|\rho_0 - \rho| > \varepsilon$ , where  $\varepsilon$  is the cut-off value of an acceptable error.

Given a measured acceptance value  $\rho$  of an environment with attributes *j* approximated by a normal distribution,  $\rho_{j,m} \sim N(\mu, \sigma^2)$ , the posterior estimate of the acceptance  $\rho_{j,1} \sim N(\mu_1, \sigma_1^2)$  is expressed by the following Bayesian rules [98], where  $\rho_{j,0} \sim N(\mu_0, \sigma_0^2)$  is the prior estimate of the acceptance towards environmental attributes *j*, *p* is the probability,  $\mu$  and  $\sigma^2$  are the mean and variance of the normal distribution function, and  $\mu$ ,  $\mu_0$ , and  $\mu_1$  are the best estimates of the measured, prior and posterior acceptance values respectively,

$$p(\rho_{j,1}|\rho_{j,m}) = p(\rho_{j,0})p(\rho_{j,m}|\rho_{j,0})$$
(2)

$$\sigma^{2} = \frac{1}{\sigma_{0}^{-2} + \sigma^{-2}}; \, \mu_{1} = \frac{\mu_{0}\sigma_{0}^{-2}}{\sigma_{0}^{-2} + \sigma^{-2}} + \frac{\mu\sigma^{-2}}{\sigma_{0}^{-2} + \sigma^{-2}} \tag{3}$$

In these rules, the weightings are proportional to their respective variances, and the posterior mean is a weighted average of the prior mean and the measured value given. This posterior mean can be characterized by the ratio of standard deviations and expressed as a parameter  $\beta^2$ .

$$\beta^2 = \frac{\sigma^2}{\sigma_0^2} \tag{4}$$

Suppose repeated measurements will deliver the measured acceptance  $\rho$  and denote  $X = \frac{\sigma_0^{-2}}{\sigma_0^{-2} + \sigma^{-2}} = \frac{\beta^2}{1 + \beta^2}$  and  $Y = \frac{\mu \sigma^{-2}}{\sigma_0^{-2} + \sigma^{-2}} = \frac{\mu}{1 + \beta^2}$ , then the posterior estimates  $\mu_1, \mu_2, \dots, \mu_n$  are given by,

$$\mu_{1} = \mu_{0}X + Y;$$

$$\mu_{2} = \mu_{0}X^{2} + XY + Y;$$

$$\vdots$$

$$\mu_{n} = \mu_{0}X^{n} + Y(X^{n-1} + X^{n-2} + \dots + X + 1)$$

$$= \mu_{0}X^{n} + \frac{Y(1-X^{n})}{1-X}$$
(5)

In Eq. (6),  $\mu_n \rightarrow \mu$  when  $n \rightarrow \infty$ . Taking *n* as a finite number of the repeated observations such that the n-th estimate shows no significant difference from the measured acceptance, i.e.  $|\mu_n - \mu| \le \varepsilon$ , then  $\beta^2$  can be determined by,

$$\mu_n = \mu_0 X^n + \frac{Y(1-X^n)}{1-X} = \mu_0 \left(\frac{\beta^2}{1+\beta^2}\right)^n + \frac{\mu}{1+\beta^2} \times \frac{1-\left(\frac{\beta^2}{1+\beta^2}\right)^n}{1-\left(\frac{\beta^2}{1+\beta^2}\right)} = \mu + \varepsilon$$
(6)

$$\beta^{2} = \frac{c_{r}^{\frac{1}{n}}}{1 - c_{r}^{\frac{1}{n}}}; c_{r} = \frac{\varepsilon}{|\mu_{0} - \mu|}$$
(7)

Constant  $c_r$  is the ratio of the acceptable error to the difference between the prior PD value  $\mu_0$ and the measured PD value  $\mu$ .

With a sample size m < n and  $\beta^2$  as given in Eq. (7), the Bayesian estimate for the PD value  $\mu_p$  is expressed by,

$$\mu_p = \mu_0 X^m + \frac{Y(1 - X^m)}{1 - X}; \quad X = \frac{\beta^2}{1 + \beta^2}; \quad Y = \frac{\mu}{1 + \beta^2}$$
(8)

### 4.1. Thermal comfort database

Thermal comfort database selection aims to demonstrate the percentage effects of field data sample size (*m*) on target sample size (*n*) under the Bayesian approach. A total of 4 thermal comfort datasets, outlined in Table 6, were selected for the demonstration: 1) residential buildings in Hainan, China (m = 1944) [31]; 2) hospitals in Bangkok, Thailand (m = 928) [51]; 3) elderly homes in Shanghai, China (m = 672) [36]; and 4) residential buildings in Hong Kong, China (m = 177) [99,100]. The first three datasets, obtained from the literature search conducted in this study, contain the necessary parameters for Bayesian thermal comfort analysis and cover samples of very small apartments [101]. The fourth is a published dataset created by this research team. Showing typical field survey results, all datasets have votes heavily concentrated (about 78%) in the range from -1 to +1; and their percentages of extreme votes (i.e. -3 and +3) are all below 10 % except for the +3 votes in Dataset 4. In Table 6, the PMV values (corresponding to each TSV) were calculated using the correlation coefficients C<sub>1</sub> and C<sub>0</sub>, while APD (µ) and the sample size of each TSV (m), with PPD (corresponding to each PMV) as the prior acceptance (µ<sub>0</sub>), were used to compute the posterior acceptance (µ<sub>1</sub>).

### 4.2. Bayesian updating procedures, results and practical implications

Two updating protocols, namely individual and global, are proposed in this paper to update the current PMV–PPD belief. Since individual updating uses one single dataset to update the prior belief, the sample size of each TSV is required (Datasets 2–4). This kind of updating, which is based on both prior information (PMV–PPD relationship) and new information (survey data), generates a unique relationship between PMV and percentage dissatisfied (PD) of a particular

environmental setting. Figure 5 shows the posterior PD estimated by the Bayesian thermal comfort model. With a selected acceptable error  $\varepsilon = 0.001$  (i.e. 0.1%) and a target sample size n = 1000, posterior estimation of PD can be computed using Equations. (5)–(8). An exemplary calculation demonstration is presented in Appendix 1 for reference. Results show that the posterior PD estimated is always closer to the measured PD than PPD. If the sample size m of each vote is significant comparing to the target sample size n, the posterior estimate will be closer to the measured PD. This can be observed generally at vote = 0 because most of the environments are designed to provide comfort for occupants. On the other hand, the sample size of an extreme vote (i.e. -3 or +3) is usually small, therefore the posterior PD is closer to PPD instead. As Bayesian estimation can evaluate the significance of a small dataset (as small as a one-sample dataset) and update the prior belief (the PPD in this case), the reliability concerns in regression analysis when the extreme vote sample size is too small are eliminated [64]. This individual updating protocol gives a thermal comfort model that incorporates the adaptive and contextual parameters from occupants in a specific type of environment (or even as specific as from a particular environment). After updating with available field, the posterior PD can act as an updated tailor-made model that can be used as basis for further thermal comfort study.

Global updating treats each dataset as one sample size and updates the PPD belief for a general indoor environment rather than a particular environmental setting. Presently, PMV–PPD based comfort standard is widely used regardless of the type of environment. Although contextual factors and adaptive behaviours strongly influence thermal comfort acceptability, modelling thermal comfort for each unique environment is resource demanding as field data collection is inevitable. By adopting the PMV–PPD concept, global updating can update the PPD belief using field data from different environments to generate a model that incorporates the influence

of field settings on thermal comfort. Figure 6 graphs the posterior PD estimated by the Bayesian thermal comfort model with acceptable error  $\varepsilon = 0.05$  and different target sample size n = 5, 10 and 20 to demonstrate the effects of target sample size difference. It can be seen that since one vote is regarded as one sample, when sample size is considered small and less significant compared to a preset target number (in case of n = 20), the posterior estimates are closer to the prior PPD belief (i.e. Fanger's as demonstrated) than the actual field data. With a smaller target sample size (in case of n = 5), Bayesian estimate will give an updated PMV–PPD model that makes prediction closer to actual data than the original model. The figure demonstrates that Bayesian updating can significantly improve prediction quality. An exemplary calculation demonstration can be found in Appendix 1.

To further illustrate the practical implications of using Bayesian updating, the proposed global protocol was applied to the GSHP study by Fang et al. [90], with error  $\varepsilon = 0.05$  and target sample size n = 10. Showing a PMV range from -0.062 to +0.062 for having 5% thermally dissatisfied people, the updated PMV–PPD relationship was found slightly narrower than the original PMV–PPD. As a result, the minimum power consumption would be approximately 1.46kW at a PMV of -0.062.

# 5. Conclusion

This study reviewed thermal comfort research in recent years and found that indoor simulation, system control and field survey are the three most discussed categories in the field of thermal comfort. While a developed thermal comfort prediction model is required as the basis of reference in these categories, the existing models are not yet comprehensive enough to give accurate thermal acceptance prediction. Some efforts has been done to improve Fanger's model by introducing adaptive parameters and expanding the applicability, still the modifications

could not achieve model generalization. This study identified the current research gaps in thermal comfort modelling include: the need for improving the predicted mean vote (PMV) model for actual field use, lack of adequate field data from extreme votes, and disagreement between actual percentage of dissatisfied (APD) and Fanger's predicted percentage of dissatisfied (PPD) model. The performance gap between actual field data and model prediction can lead to substantial error in research that based on an inaccurate model. In order to overcome these research gaps, this study proposed a novel Bayesian approach to update existing thermal comfort model. Two Bayesian updating protocols, namely individual and global, were presented to demonstrate the analytical solution for target sample size selection to systemically update current thermal comfort beliefs with openly available field data. This method allows the incorporation of field settings into any existing model even with a small sample size. Results showed that with Besides being a practical tool for modelling thermal comfort using the best information available (i.e. existing models and field survey data), the proposed Bayesian updating provides an achievable solution to the present challenges in establishing a reliable thermal comfort prediction model. While existing model can be updated using Bayesian approach, a comprehensive data-driven thermal comfort model shall be developed in the future.

## Acknowledgement

The work described in this paper was partially supported by grants from the Research Grants Council of the Hong Kong Special Administrative Region (HKSAR), China (PolyU 152088/17E, 15203019).

Nome	nclature	0	prior
$T_a$	air temperature	1/ p	Posterior estimate/ (with measured sample size m)
$T_r$	mean radiant temperature	ρ	measured acceptance
RH	relative humidity	З	acceptable error
$V_a$	air velocity	j	environmental attributes
$M_e$	metabolic rate	Ν	normal distribution
$C_L$	clothing value	μ	mean of measured acceptance
PMV	predicted mean vote	$\sigma^2$	variance
PPD	predicted percentage dissatisfied	р	probability
TSV	thermal sensation vote	β	ratio of standard deviations ( $\sigma / \sigma_0$ )
APD	actual percentage dissatisfied	n	target sample size
$C_{l}$	slope of PMV–TSV plot	т	measured sample size
$C_0$	y-intercept of PMV-TSV plot	$C_r$	Ratio of $\varepsilon$ to difference between prior and
<i>A</i> , <i>B</i>	events		measured acceptance

Table 1. Contribution of different categories to the total number of publications. To identify significant differences between contributions before and after 2013, paired *t*-tests were done and the *p*-values are shown in the table.

			,					r				
	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	р
Simulation	52%	48%	40%	42%	51%	46%	53%	55%	55%	51%	43%	0.250
System control	37%	31%	41%	34%	29%	32%	30%	35%	34%	33%	35%	0.656
Field study	11%	17%	23%	20%	19%	20%	18%	20%	21%	20%	19%	0.438
Thermal manikin	0%	1%	5%	2%	1%	4%	2%	2%	3%	2%	1%	0.852
Model development	0%	1%	3%	1%	1%	3%	2%	1%	1%	1%	1%	0.547
Climate change	2%	11%	11%	10%	10%	14%	12%	12%	14%	15%	12%	0.053
Energy efficiency	13%	10%	14%	19%	17%	17%	21%	23%	31%	25%	26%	0.004

Remark: The total percentage in each year does not add up to 100% as some of the publications belong to more than one category.



Figure 1. Number of publications related to indoor thermal comfort from 2008 to 2018 on the Web of Science. Shown in the figure are the results with the search words "CFD or numerical or simulate", "system control", "field study", "thermal manikin", "model development", "climate change" and "energy efficiency". Overlapping of categories may appear during refined searching.



Figure 2. Involvement of different research areas in the field of thermal comfort. APD: actual percentage dissatisfied; TSV: thermal sensation vote.

Def	Lessier	Desilding	Vantilation	Köppen–Geiger	Saaaa		C	C			TSV	(no. of	vote)		
Ref.	Location	Building	ventilation	climate	Season	Sample size	$C_1$	$C_0$	-3	-2	-1	0	1	2	3
[31]	Hainan, China	Residential building	FR	Dry-winter humid subtropical	Transitional season	1944	0.9426	-0.3076	_	-	_	_	_	-	_
[32]	Tibet, China	Stone dwellings	NV	Cold semi-arid	Winter	327	1.371	0.979	27	41	154	95	11	0	0
[33]	Tibet, China	Residential building	NV	Cold semi-arid	Summer Winter	609 573	0.6883 0.7573	0.382 0.3883	8 13	26 18	129 51	351 202	74 173	17 79	4 37
[34]	Harbin China	Residential	н	Monsoon-influenced	Cool exposure	304	1.1346	0.7558	4	6	62	187	24	19	2
	Harbin, China	building	11	continental	Warm exposure	321	0.7345	0.0213	0	2	28	209	40	20	22
		Flderly		Hot-summer humid	Cooling	114	0.32	0.15	0	0	4	68	40	2	0
[35]	Korea	centre	NV/AC/H	continental	Mid-season	182	1.16	0.44	8	22	50	80	22	0	0
				continental	Heating	102	0.84	0.15	2	26	42	20	10	2	0
[36]	Shanghai, China	Elderly	FR	Humid subtropical	Winter	342	0.598	0.394	1	52	33	212	43	1	0
	C A	home		1	Summer	330	0.373	0.038	0	0	11	188	82	46	3
[27]	Drozil	Office	AC	Tropical savanna/ Humid subtropical	Spring to early	1236 (A)	0.51	0.15	2	48	328 190	/13	132	10	0
[37]	Drazii	building	AC/NV		winter	823 (Б) 530 (С)	1.08	0.22	3 0	24 6	100	401 266	106	15	5 10
[38]	India	Office	AC	Hot semi-arid/ Tropical savanna	All year	444	0.963	0.00	1	33	166	165	71	8	0
[39]*	Hungary	Office	AC	Warm humid continental	Winter	278	1	0.275	NA	50	106	72	31	19	NA
[40]	Quito, Ecuador	NV office	NV	Temperate oceanic	Summer	441	0.3203	0.0698	7	12	88	246	81	7	0
			NV	Hot somi arid/		2005	0.8	0.664	-	-	_	_	_	-	_
[41]	India	Office	NV/AC	Tropical savanna	All year	2470	0.7647	-0.4924	-	-	—	—	—	-	—
			AC	110pical savallia	-	1849	0.65	-0.5275	-	-	-	-	-	-	_
[42]	Shenzhen,	Office	AC	Monsoon-influenced	Summor	321	0.5702	0.1428	3	9	16	174	102	11	6
[+2]	China	Office	NV	humid subtropical	Summer	513	0.4550	0.0603	21	45	17	241	183	4	2

Table 2. Occupants' thermal sensation votes (TSV) in various studies over the past 5 years (2018–2014)

MVS: mechanical ventilation system; AC: air-conditioned; FR: Free-running; HVAC: Heating, ventilation, and air conditioning; NV: natural ventilation; H/NH: Heating/ no heating;

'-' indicates that the TSV values are not available in the corresponding studies;

\*a 5-point scale was used for thermal sensation evaluation.

Pof	Location	Building	Vontilation	Köppen–Geiger	Sasson	Sampla siza	C.	C			TSV	(no. of	vote)		
Kei.	Location	Building	Ventilation	climate	Season	Sample Size	Cl	$C_0$	-3	-2	-1	0	1	2	3
[43]	Makassar, Indonesia	Secondary school	NV	Tropical monsoon	Summer	1594	0.676	-1.052	0	21	317	588	493	167	8
[44]	Hong Kong, China	University classroom (Chamber)	HVAC	Monsoon-influenced humid subtropical	Summer	946	0.667	0.382	_	_	_	_	_	_	_
[45]	Weinan and Wuwei, China	Rural school	NV	Cold semi-arid	Winter	763	0.4184	-0.1044	11	58	230	362	82	17	3
	Shaanxi, China			Cold semi-arid		345	0.45	0.1175	14	45	110	131	36	8	1
[46]	Gansu, China	School	H/NH	Cold semi-arid	All year	360	0.3514	0.1292	6	16	70	213	40	11	4
	Qinghai, China			Cold semi-arid		421	0.3902	-0.5112	3	14	68	126	126	69	15
[47]	Aegean Greek	Classroom	HVAC	Hot-summer	Heating	449	0.9702	0.2942	0	14	36	139	139	85	36
['']	Regean, Greek	Clussioolii	nivite	Mediterranean	Cooling	345	1.2906	0.0288	14	24	42	62	69	55	79
[48]	Indonesia	University classroom	NV	Tropical rainforest	Autumn	118	0.4624	0.4306	0	0	19	26	50	20	3
[49]	Porto, Portugal	Bus station	MVS	Warm-summer Mediterranean	Summer	240	0.603	1.065	0	1	17	105	71	38	8
[50]	Shandong, China	Rubber factory	NV	Hot humid continental	Summer	40	0.888	-1.21	0	0	2	10	16	10	2
	Bangkok					451 (Patient)	0.5187	0.0035	5	45	74	255	41	25	6
[51]	Thailand	Hospital	AC	Tropical savanna	Summer	146 (Staff)	1.2372	-0.9764	8	27	45	25	20	14	7
	Thuhuhu					331 (Visitor)	0.6278	0.0518	8	36	61	182	26	18	0
	China			Monsoon-influenced		100 (Seated)	0.971	0.444	-	-	—	-	-	-	—
[52]	subtropical monsoon area	Ship cabin	AC	humid subtropical	Winter	100 (Light working)	1.24	1.133	_	_	_	-	-	-	_
		Cotton				123 (worker)	0.5869	0.3402	0	0	0	6	42	48	27
[53]	Henan, China	textile workshop	AC	Humid subtropical	Summer	69 (student)	0.9068	0.7629	0	0	0	0	16	29	24
[54]	Chongqing, China	Environ- mental chamber	Controlled	Humid subtropical climate	All year	440	0.45	-0.1	_	_	_	_	_	_	_
[55]	Penang, Malaysia	Mosque	AC	Tropical rainforest	Cooler and hotter seasons	330	0.2462	-0.3888	1	5	39	108	105	69	3

Table 2 (cont.'). Occupants' thermal sensation votes (TSV) in various studies over the past 5 years (2018–2014)

Daf	Location	Duilding	Types of	Köppen–Geiger	Saacan	Total sample					TSV			
Kel.	Location	Building	ventilation	climate	Season	size, $\sum n$		-3	-2	-1	0	1	2	3
[31]	Hainan, China	Residential building	FR	Dry-winter humid subtropical	Transitional season	1944		8.7	2.3	2.8	2.8	19.3	23.2	40.9
[51]	Bangkok, Thailand	Hospital	AC	Tropical savanna	Summer	451 (Patient) 146 (Staff) 331 (Visitor)		66.2 91.5 71.5	31.5 62.3 34.6	8.5 26.2 8.5	0 7.7 0	3.1 11.5 2.3	9.2 23.1 6.2	22.3 38.5 16.2
[56]	New South	Primary school	NV/AC	Humid subtropical	Summer	3545	APD (%)	85	49	16	8	17	38	65
W	Wales, Australia	Secondary school		climate		1321		60	23	8	9	20	43	72
[61]	Beja, Portugal	Classroom A Classroom B	HVAC	Hot-summer Mediterranean	Spring to summer	26 19		NA NA	NA NA	17 NA	0 1	0 0	NA 0	NA NA
[36]	Shanghai, China	Elderly home	FR	Humid subtropical	Winter Summer	342 330		100 NA	94 NA	79 27	0 0	7 84	100 87	NA 100
				•			Min	8.7	2.3	2.8	0	0	0	16.2
							Max	100	94	79	9	84	100	100
							Mean	69.0	42.4	21.4	2.9	16.4	36.6	50.7
											PMV			
								-3	-2	-1	0	1	2	3
				Predicted percen	tage dissatisfied % (1	PPD) in Fanger's	s model	99	75	25	5	25	75	99

Table 3. Review of actual percentage dissatisfied (APD; %) in various studies over the past 5 years (2018–2014)

FR: Free-running; NV: natural ventilation; AC: air-conditioned; HVAC: Heating, ventilation, and air conditioning; 'NA' due to 0 sample size under the vote.

Table 4. Corresponding PMV and PPD for TSV = -1, 0 and 1

TSV	transforming TSV to PMV by Eq. (1)	PPD (Assume $TSV = PMV$ )	PPD (transforming TSV to PMV by Eq. (1))
-1	-3.59 - 0.24 (mean = $-1.79$ )	26.1%	100% - 6.2% (mean = 66.7%)
0	-1.77 - 1.58 (mean = $-0.14$ )	5%	65.3% - 55.2% (mean = 5.4%)
1	-0.11 - 5.64 (mean = 1.51)	26.1%	5.2% - 100% (mean = $51.7%$ )

Table 5. Corresponding TSV and PPD for PMV = 0 and -0.07

	Minin	num	Maxin	num	Mean			
$\mathbf{D}\mathbf{M}\mathbf{V}=0$	TSV	-1.21	TSV	1.13	TSV	0.14		
$P   \mathbf{v}   \mathbf{v} = 0$	PPD (%)	35.7	PPD (%)	32.0	PPD (%)	5.4		
$\mathbf{D}\mathbf{M}\mathbf{V} = 0.07$	TSV	-1.27	TSV	1.05	TSV	0.086		
$P_{1V1} v = -0.07$	PPD (%)	38.8	PPD (%)	28.1	PPD (%)	5.2		

Table 6. Selected databases for Bayesian thermal comfort model demonstration

D	Total sample size,	C	C					TSV			
Ref.	$\sum m$	$C_1$	$C_0$	-	-3	-2	-1	0	1	2	3
				т	—	_	_	-	_	_	_
[31]	1944	0.9426	-0.3076	PMV	-2.86	-1.80	-0.73	0.33	1.39	2.45	3.51
[31] [51]				APD (%)	8.7	2.3	2.8	2.8	19.3	23.2	40.9
				m	5	45	74	255	41	25	6
	451 (Patient)	0.5187	0.0035	PMV	-5.79	-3.86	-1.93	-0.01	1.92	3.85	5.78
				APD (%)	66.2	31.5	8.5	0	3.1	9.2	22.3
				m	8	27	45	25	20	14	7
[51]	146 (Staff)	1.2372	-0.9764	PMV	-1.64	-0.83	-0.02	0.79	1.60	2.41	3.21
[]				APD (%)	91.5	62.3	26.2	7.7	11.5	23.1	38.5
				m	8	36	61	182	26	18	0
	331 (Visitor)	0.6278	0.0518	PMV	-4.86	-3.27	-1.68	-0.08	1.51	3.10	4.70
	· · · · ·			APD (%)	71.5	34.6	8.5	0	2.3	6.2	16.2
				m	1	52	33	212	43	1	0
	342	0.598	0.394	PMV	-5.68	-4.00	-2.33	-0.66	1.01	2.69	4.36
10.01				APD (%)	100	94	79	0	7	100	NA
[36]				m	0	0	11	188	82	46	3
	330	0.373	0.038	PMV	-8.14	-5.46	-2.78	-0.10	2.58	5.26	7.94
				APD (%)	NA	NA	27	0	84	87	100
				m	0	2	15	76	47	12	25
[99,100]	177	2.49	-0.02	PMV	-1.20	-0.80	-0.39	0.01	0.41	0.81	1.21
. / .				APD (%)	NA	50	0	0	8.51	66.7	100

Remark: PMV values (corresponding to each TSV) were calculated using the correlation coefficients C<sub>1</sub> and C<sub>0</sub>;

'-' indicates that TSV values are not available;

'NA' due to 0 sample size under the vote.

![](_page_30_Figure_0.jpeg)

Figure 3. Plot of thermal sensation vote against power consumption. PMV data was extracted from Fang et al. [94]. Actual votes were calculated using Bayesian using coefficients gathered from field studies.

![](_page_30_Figure_2.jpeg)

Figure 4. Plot of PMV against thermal dissatisfaction.

![](_page_31_Figure_0.jpeg)

Figure 5. Posterior PD by Bayesian thermal comfort model using individual updating method with  $\varepsilon = 0.001$  and n = 1000; (a) Patient, Sattayakorn et al. [51]; (b) Staff, Sattayakorn et al. [51]; (c) Visitor, Sattayakorn et al. [51]; (d) Winter, Jiao et al. [36]; (e) Summer, Jiao et al. [36]; (f) Residential, Lai et al. and Mui et al. [99,100].

![](_page_32_Figure_0.jpeg)

## Predicted dissatisfaction

Figure 6. Posterior PD by Bayesian thermal comfort model with  $\varepsilon = 0.05$  and n = 5, 10 and 20; a) Lu et al. [31]; b) Patient, Sattayakorn et al. [51]; c) Staff, Sattayakorn et al. [51]; d) Visitor, Sattayakorn et al. [51]; e) Jiao et al. [36]; g) Residential, , Lai et al. and Mui et al. [99,100].

## References

- Pérez-Lombard L, Ortiz J, Pout C. A review on buildings energy consumption information. Energy and Buildings 2008;40(3):394–398.
- [2] Cheng Y, Niu JL, Gao NP. Thermal comfort models: A review and numerical investigation. Building and Environment 2012;47:13–22.
- [3] Fanger PO. Thermal comfort. New York, USA: McGraw-Hill; 1972.
- [4] Humphreys MA. Outdoor temperatures and comfort indoors. Building Research and Practice 1978;6(2): 92–105.
- [5] Brager GS, de Dear R. Thermal adaptation in the built environment: A literature review. Energy and Building 1998;27(1):83–96.
- [6] de Dear R, Brager GS. Developing an adaptive model of thermal comfort and preference. ASHRAE Transactions 1998;104(1):145–167.
- [7] Huizenga C, Zhang H, Arens E, Duan T. A model of human physiology and comfort for assessing complex thermal environments. Building and Environment 2001;36(6):691–699.
- [8] Zhang H, Huizenga C, Arens E, Wang D. Thermal sensation and comfort in transient non-uniform thermal environment. European Journal of Applied Physiology 2004;92:729–733.
- [9] Zhang H, Huizenga C, Arens E, Yu T. Modelling Thermal Comfort in Stratified Environments. Proceedings. Indoor Air 2005: 10th International Conference on Indoor Air Quality and Climate, Beijing, China, September.
- [10] Van Hoof J. Forty years of Fanger's model of thermal comfort: comfort for all? Indoor Air 2008;18(3):182–201.

- [11] Fanger PO, Toftum J. Extension of the PMV model to non-air-conditioned buildings in warm climates. Energy and Buildings 2002;34(6):533–536.
- [12] Nicol JF, Humphreys MA. Adaptive thermal comfort and sustainable thermal standards for buildings. Energy and buildings 2002;34(6):563–572.
- [13] Karjalainen S. Thermal comfort and gender: a literature review. Indoor Air 2012;22(2):96–109.
- [14] Mishra AK, Ramgopal M. Field studies on human thermal comfort–An overview.Building Environment 2013;64:94–106.
- [15] Halawa E, van Hoof J, Soebarto V. The impacts of the thermal radiation field on thermal comfort, energy consumption and control–A critical overview. Renewable and Sustainable Energy Reviews 2014;37:907–918.
- [16] Djamila H. Indoor thermal comfort predictions: Selected issues and trends.Renewable and Sustainable Energy Reviews 2017;74:669–680.
- [17] Djongyang N, Tchinda R, Njomo D. Thermal comfort: a review paper. Renewable and Sustainable Energy Reviews 2010;14(9):2626–2640.
- [18] Halawa E, van Hoof J. The adaptive approach to thermal comfort: a critical overview.Energy and Building 2012;51:101–110.
- [19] Carlucci S, Pagliano L. A review of indices for the long-term evaluation of the general thermal comfort conditions in buildings. Energy and Building 2012;53:194– 205.
- [20] Taleghani M, Tenpierik M, Kurvers S, van den Dobbelsteen A. A review into thermal comfort in buildings. Renewable and Sustainable Energy Reviews 2013;26:201–215.

- [21] Holopainen R, Tuomaala P, Hernandez P, Häkkinen T, Piira K, Piippo J. Comfort assessment in the context of sustainable buildings: comparison of simplified and detailed human thermal sensation methods. Building and Environment 2014;71:60– 70.
- [22] Croitoru C, Nastase T, Bode F, Meslem A, Dogeanu A. Thermal comfort models for indoor spaces and vehicles–Current capabilities and future perspectives. Renewable and Sustainable Energy Reviews 2015;44:304–318.
- [23] Fu M, Weng W, Chen W, Luo N. Review on modeling heat transfer and thermoregulatory responses in human body. Journal of Thermal Biology 2016;62(B):189–200.
- [24] Katić K, Li R, Zeiler W. Thermophysiological models and their applications: a review. Building and Environment 2016;106:286–300
- [25] Enescu D. A review of thermal comfort models and indicators for indoor environments. Renewable and Sustainable Energy Reviews 2017;79:1353–1379.
- [26] Chiang WH, Wang CY, Huang JS. Evaluation of cooling ceiling and mechanical ventilation systems on thermal comfort using CFD study in an office for subtropical region. Building and Environment 2012;48:112–127.
- [27] Rohdin P, Andreas M, Bahram M. Experiences from nine passive houses in Sweden -Indoor thermal environment and energy use. Building and Environment 2014;71:176– 185.
- [28] Du CY, Bokel R, van den Dobbelsteen A. Building microclimate and summer thermal comfort in free-running buildings with diverse spaces: A Chinese vernacular house case. Building and Environment 2014;82:215–227.

- [29] Freire RZ, Oliveira GHC, Mendes N. Predictive controllers for thermal comfort optimization and energy savings. Energy and Building 2008;40(7):1353–1365.
- [30] Mossolly M, Ghali K, Ghaddar N. Optimal control strategy for a multi-zone air conditioning system using a genetic algorithm. Energy 2009;34(1):58–66.
- [31] Lu SL, Pang B, Qi YF, Fang K. Field study of thermal comfort in non-air-conditioned buildings in a tropical island climate. Applied Ergonomics 2018;66:89–97.
- [32] Cheng B, Fu YL, Khoshbakht M, Duan LB, Zhang J, Rashidian S. Characteristics of thermal comfort conditions in cold rural areas of China: a case study of stone dwellings in a Tibetan Village. Buildings 2018;8(4):49.
- [33] Yu W, Li BZ, Yao RM, Wang D, Li KT. A study of thermal comfort in residential buildings on the Tibetan Plateau, China. Building and Environment 2017;119:71–86.
- [34] Ning HR, Wang ZJ, Ji YC. Thermal history and adaptation: does a long-term indoor thermal exposure impact human thermal adaptability? Applied Energy 2016;183:22– 30.
- [35] Yang JH, Nam I, Sohn JR. The influence of seasonal characteristics in elderly thermal comfort in Korea. Energy and Buildings 2016;128:583–591.
- [36] Jiao Y, Wang T, An YS, Yu YF. Thermal comfort and adaptation of the elderly in free-running environments in Shanghai, China. Building and Environment 2017;118:259–272.
- [37] Rupp RF, Ghisi E. Predicting thermal comfort in office buildings in a Brazilian temperate and humid climate. Energy and Buildings 2017;144:152–166.

- [38] Thapa S, Bansal AK, Panda GK. Thermal comfort in naturally ventilated office buildings in cold and cloudy climate of Darjeeling, India – An adaptive approach. Energy and Buildings 2018;160:44–60.
- [39] Kajtar L, Nyers J, Szabo J, Ketskemety L, Herczeg L, Leitner A, Bokor B. Objective and subjective thermal comfort evaluation in Hungary. Thermal Science 2017;21(3):1409–1418.
- [40] Gallardo A, Palme M, Lobato-Cordero A, Beltran RD, Gaona G. Evaluating thermal comfort in naturally conditioned office in a temperate climate zone. Buildings 2016;6(3):27.
- [41] Manu S, Shukla Y, Rawal R, Thomas LE, de Dear R. Field studies of thermal comfort across multiple climate zones for the subcontinental India model for adaptive comfort (IMAC). Building and Environment 2016;98:55–70.
- [42] Luo MH, Cao B, Damiens J, Lin BR, Zhu YX. Evaluating thermal comfort in mixedmode buildings: afield study in a subtropical climate. Building and Environment 2015;88:46–54.
- [43] Hamzah B, Gou ZH. Mulyadi R, Amin S. Thermal comfort analyses of secondary school students in the tropics. Buildings 2018;8(4):56.
- [44] Fang ZS, Zhang S, Cheng Y, Fong AML, Oladokun MO, Lin Z, Wu HJ. Field study on adaptive thermal comfort in typical air conditioned classrooms. Building and Environment 2018;133:73–82.
- [45] Liu YF, Jiang J, Wang DJ, Liu JP. The indoor thermal environment of rural school classrooms in Northwestern China. Indoor and Built Environment 2017;26(5):662–679.

- [46] Wang DJ, Jiang J, Liu YF, Wang YY, Xu YC, Liu JP. Student responses to classroom thermal environments in rural primary and secondary schools in winter. Building and Environment 2017;115:104–117.
- [47] Calis G, Kuru M. Assessing user thermal sensation in the Aegean region against standards. Sustainable Cities and Society 2017;29:77–85.
- [48] Hamzah B, Lshak MT, Beddu S, Osman MY. Thermal comfort analyses of naturally ventilated university classrooms. Structural survey 2016;34(4/5):427–445.
- [49] Cardoso VE, Ramos NM, Almeida RM, Barreira E, Martins JP, Simões ML, Sanhudo
   L. A discussion about thermal comfort evaluation in a bus terminal. Energy and
   Buildings 2018;168: 86–96.
- [50] Wang HY, Sun L, Guan HY, Hu ST. Thermal environment investigation and analysis on thermal adaption of workers in a rubber factory. Energy and Buildings 2018;158:1625–1631.
- [51] Sattayakorn S, Ichinose M, Sasaki R. Clarifying thermal comfort of healthcare occupants in tropical region: A case of indoor environment in Thai hospitals. Energy and Buildings 2017;149:45–57.
- [52] Liu HM, Lian ZW, Gong ZH, Wang YC, Yu GJ. Thermal comfort, vibration, and noise in Chinese ship cabin environment in winter time. Building and Environment 2018;135:104–111.
- [53] Yang RL, Liu L, Ren Y. Thermal environment in the cotton textile workshop. Energy and Buildings 2015;102:432–441.
- [54] Yang Yu, Li BZ, Liu H, Tan ML, Tai RM. A study of adaptive thermal comfort in a well-controlled climate chamber. Applied Thermal Engineering 2015;76:283–291.

- [55] Hussin A, Salleh E, Chan HY, Mat S. The reliability of Predicted Mean Vote model predictions in an air-conditioned mosque during daily prayer times in Malaysia.
   Architectural Science Review 2015;58(1):67–76.
- [56] Koelblen B, Psikuta A, Bogdan A, Annaheim S, Rossi RM. Human simulator A tool for predicting thermal sensation in the built environment. Building and Environment 2018;143:632–644.
- [57] Alsaad H, Voelker C. Performance assessment of a ductless personalized ventilation system using a validated CFD model. Journal of Building Performance Simulation 2018; 11(6):689–704.
- [58] Al Assaad D, Ghali K, Ghaddar N, Habchi C. Mixing ventilation coupled with personalized sinusoidal ventilation: Optimal frequency and flow rate for acceptable air quality. Energy and Buildings 2017;154:569–580.
- [59] Al Assaad D, Habchi C, Ghali K, Ghaddar N. Effectiveness of intermittent personalized ventilation in protecting occupant from indoor particles. Building and Environment 2018;128:22–32.
- [60] Mustakallio P, Bolashikov Z, Rezgals L, Lipczynska A, Melikov A, Kosonen R. Thermal environment in a simulated double office room with convective and radiant cooling systems. Building and Environment 2017;123:88–100.
- [61] Mao N, Song MJ, Pan DM, Deng SM. Computational fluid dynamics analysis of convective heat transfer coefficients for a sleeping human body. Applied Thermal Engineering 2017;117:385–396.
- [62] Yao RM, Li BZ, Liu J. A theoretical adaptive model of thermal comfort AdaptivePredicted Mean Vote (aPMV). Building and Environment 2009;44(10):2089–2096.

- [63] Langevin J, Wen J, Gurian PL. Modeling thermal comfort holistically: Bayesian estimation of thermal sensation, acceptability, and preference distributions for office building occupants. Building and Environment 2013;69:206–226.
- [64] Wong LT, Mui KW, Cheung CT. Bayesian thermal comfort model. Building and Environment 2014;82:171–179.
- [65] Marino C, Nucara A, Peri G, Pietrafesa M, Pudano A, Rizzo G. An MAS-based subjective model for indoor adaptive thermal comfort. Science and Technology for the Built Environment 2015;21(2):114–125.
- [66] Daum D, Haldi F, Morel N. A personalized measure of thermal comfort for building controls. Building and Environment 2011;46:3–11
- [67] Rana R, Kusy B, Jurdak R, Wall J, Hu W. Feasibility analysis of suing humidex as an indoor thermal comfort predictor. Energy and Buildings 2013;64:17–25.
- [68] Ghahramani A, Tang C, Becerik-Gerber B. An online learning approach for quantifying personalized thermal comfort via adaptive stochastic modeling. Building and Environment 2015;92:86–96.
- [69] Kim J, Schiavon S, Brager G. Personal comfort models A new paradigm in thermal comfort for occupant-centric environmental control. Building and Environment 2018;135:114–124.
- [70] Rupp RF, de Dear R, Ghisi E. Field study of mixed-mode office buildings in Southern Brazil using an adaptive thermal comfort framework, Energy and Building 2018;158:1475–1486.
- [71] Daniel L. 'We like to live in the weather': Cooling practices in naturally ventilated dwellings in Darwin, Australia. Energy and Building 2018;158:549–557.

- [72] de Dear R, Kim JS, Parkinson T. Residential adaptive comfort in a humid subtropical climate – Sydney Australia. Energy and Buildings 2018;158:1296–1305.
- [73] Indraganti M, Boussaa D. An adaptive relationship of thermal comfort for the Gulf
   Cooperation Council (GCC) Countries: The case of offices in Qatar. Energy and
   Buildings 2018;159:201–212.
- [74] Kleber M, Wagner A. Investigation of indoor thermal comfort in warm-humid conditions at a German climate test facility. Building and Environment 2018;128:216–224.
- [75] Kotopouleas A, Nikolopoulou M. Evaluation of comfort conditions in airport terminal buildings. Building and Environment 2018;130:162–178.
- [76] Liu H, Wu YX, Lei DN, Li BZ. Gender differences in physiological and psychological responses to the thermal environment with varying clothing ensembles.
   Building and Environment 2018;141:45–54.
- [77] Tan Z, Roberts AC, Christopoulos GI, Kwok KW, Car J, Li XZ, Soh CK. Working in underground spaces: Architectural parameters, perceptions and thermal comfort measurements. Tunneling and Underground Space Technology 2018;71:428–439.
- [78] Tartarini F, Cooper P, Fleming R. Thermal perceptions, preferences and adaptive behaviors of occupants of nursing homes. Building and Environment 2018;132:57– 69.
- [79] Wu TL, Cao B, Zhu YX. A field study on thermal comfort and air-conditioning energy use in an office building in Guangzhou. Energy and Buildings 2018;168:428– 437.

- [80] Yang B, Olofsson T, Wang F, Lu WZ. Thermal comfort in primary school classrooms: A case under subarctic climate area of Sweden. Building and Environment 2018;135:237–245.
- [81] Wang Z, Yu H, Jiao Y, Wei Q, Chu XY. A field study of thermal sensation and neutrality in free-running aged-care homes in Shanghai. Energy and Buildings 2018;158:1523–1532.
- [82] Li BZ, Du CQ, Yao RM, Yu W, Costanzo V. Indoor thermal environments in Chinese residential buildings responding to the diversity of climates. Applied Thermal Engineering 2018;129:693–708.
- [83] Luo MH, Wang Z, Brager G, Cao B, Zhu YX. Indoor climate experience, migration, and thermal comfort expectation in buildings. Building and Environment 2018;141:262–272.
- [84] Martinez-Molina A, Boarin P, Tort-Ausina I, Vivancos JL. Assessing visitors' thermal comfort in historic museum buildings: Results from a Post-Occupancy Evaluation on a case study. Building and Environment 2018;132:291–302.
- [85] Wang ZJ, Ji YC, Su XW. Influence of outdoor and indoor microclimate on human thermal adaptation in winter in the severe cold area, China. Building and Environment 2018;133:91–102.
- [86] Zhang ZJ, Zhang YF, Jin L. Thermal comfort in interior and semi-open spaces of rural folk houses in hot-humid areas. Building and Environment 2018;128:336–347.
- [87] Ioannou T, Laure I, Tushar A. In-situ real time measurements of thermal comfort and comparison with the adaptive comfort theory in Dutch residential dwellings. Energy and Buildings 2018;170:229–241.

- [88] Kim JS, de Dear R. Thermal comfort expectations and adaptive behavioral characteristics of primary and secondary school students. Building and Environment 2018;127:13–22.
- [89] Cardoso V, Ramos NMM, Almedia RMSF, Barreira E, Martins JP, Simões ML, Sandudo L, Ribeiro B. Thermal comfort evaluation in cruise terminals. Building and Environment 2017;126:276–287.
- [90] Jabbari SG, Maleki A, Kaynezhad MA, Olesen BW. Inter-personal factors affecting building occupants' thermal tolerance at cold outdoor condition during an autumnwinter period. Indoor and Built Environment 2019.
- [91] Mui KW, Tsang TW, Wong LT, Yu YP. Evaluation of an indoor environmental quality model for very small residential units. Indoor and Built Environment 2018; 28(4):470–478.
- [92] Jindal A. Thermal comfort study in naturally ventilated school classrooms in composite climate of India. Building and Environment 2018;142:34–46.
- [93] Perez-Fargallo A, Pulido-Arcas JA, Rubio-Bellido C, Trebilcock M, Piderit B, Attia
   S. Development of a new adaptive comfort model for low income housing in the central-south of Chile. Energy and Building 2018;178:94–106.
- [94] Fang J, Feng ZB, Cao SJ, Deng YL. The impact of ventilation parameters on thermal comfort and energy-efficient control of the ground-source heat pump system. Energy and Buildings 2018;179:324–332.
- [95] Hong SH, Lee JM, Moon JW, Lee KH. Thermal comfort, energy and cost impacts of PMV control considering individual metabolic rate variations in residential building Energies 2018;11(7):1767.

- [96] Pereira LD, Raimondo D, Corgnati SP, da Silva MG. Assessment of indoor air quality and thermal comfort in Portuguese secondary classrooms: methodology and results.
   Building and Environment 2014;81:69–80.
- [97] Vick SG. Degrees of belief: subjective probability and engineering judgment.Virginia, USA: ASCE; 2002.
- [98] Lee PM. Bayesian statistics, 3rd Ed., London, UK: Hodder Arnold; 2004.
- [99] Lai ACK, Mui KW, Wong LT, Law LY. An evaluation model for indoor environmental quality (IEQ) acceptance in residential buildings. Energy and Buildings 2009;41:930–936.
- [100] Mui KW, Tsang TW, Wong LT, Yu YPW. Evaluation of an indoor environmental quality model for very small residential units. Indoor and Built Environment 2018;28(4): 470–478.
- [101] Wong LT. Tiny affordable housing in Hong Kong. Indoor and Built Environment 2018;27(9):1159–1161.

# Appendix

Exemplary calculation steps

Sample 1: Individual updating – Dataset 2, TSV vote = 0; PMV vote = -0.01

Given: Target sample size n = 1000

Acceptable error  $\varepsilon = 0.001$ Sample size m = 255Prior acceptance  $\rho_{1,0}/\mu_0 = 0.05$ Measured acceptance  $\rho_{1,m}/\mu = 0$  By Eq. (7)

$$c_r = \varepsilon (\mu_0 - \mu)^{-1} = 0.001 \times (0.05 - 0)^{-1} = 0.02$$
  
 $\beta^2 = c_r^{1/n} / (1 - c_r^{1/n}) = 0.02^{1/1000} / (1 - 0.02^{1/1000}) = 255.12$ 

By 
$$X = \sigma_0^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \beta^2 / (1 + \beta^2) = 255.12 / (1 + 255.12) = 0.9961$$
  
 $Y = \mu \sigma^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \mu / (1 + \beta^2) = 0 / (1 + 255.12) = 0$ 

By Eq. (8)  

$$\mu_p = \mu_0 X^m + Y (1 - X^m) / (1 - X) = 0.05 \times 0.9961^{255} + 0 \times (1 - 0.9961^{255}) / (1 - 0.9961)$$

$$= 0.01846$$

Sample 2: Global updating – Dataset 1, TSV vote = -2; PMV vote = -1.80

Given: Target sample size n = 10

Acceptable error  $\varepsilon = 0.05$ Sample size m = 1 (one study is treated as 1 sample) Prior acceptance  $\rho_{1,0}/\mu_0 = 0.67$ Measured acceptance  $\rho_{1,m}/\mu = 0.023$ 

By Eq. (7)  

$$c_r = \varepsilon (\mu_0 - \mu)^{-1} = 0.05 \times (0.67 - 0.023)^{-1} = 0.0775$$

$$\beta^2 = c_r^{1/n} / (1 - c_r^{1/n}) = 0.0775^{1/10} / (1 - 0.0775^{1/10}) = 3.432$$

By 
$$X = \sigma_0^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \beta^2 / (1 + \beta^2) = 3.432 / (1 + 3.432) = 0.774$$
  
 $Y = \mu \sigma^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \mu / (1 + \beta^2) = 0.023 / (1 + 3.432) = 0.00519$ 

By Eq. (8)  

$$\mu_p = \mu_0 X^m + Y (1 - X^m) / (1 - X) = 0.67 \times 0.774^1 + 0.00519 \times (1 - 0.774^1) / (1 - 0.774)$$

$$= 0.522$$