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Bayesian updates for indoor environmental quality (IEQ) acceptance model for residential buildings

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An accurate indoor environmental quality (IEQ) model is essential to design and maintain a comfortable indoor environment. Due to the complexity of IEQ modelling and subjective nature of IEQ responses, there is a need to update the subjective–objective relationship of IEQ model when new information is available. In this study, a Bayesian approach for IEQ model updating is proposed to systematically relate new subjective IEQ responses towards the environment to existing beliefs. With a selected target sample size and an acceptable error, the statistical significance of data is evaluated and incorporated into the updated IEQ model. Bayesian updating framework is able to enhance the accuracy of IEQ prediction and shall be a useful tool for managerial decision making in maintaining a comfortable indoor environment.

1. Introduction

Indoor environmental quality (IEQ) acceptance is a significant topic in built environment as it affects occupant's comfort, productivity and health. A longitudinal study in offices showed that inadequate IEQ reduced self–reported work performance, measured cognitive performance and well–being by indirectly lowering motivation and enhancing tiredness and distractibility (Lamb and Kwok 2016). Poor IEQ can also lead to Sick building syndrome (SBS), mental health problems and a number of long–term illnesses (Bluyssen 2009, 2014; Al Horr et al. 2016a).

Current IEQ research mainly focuses on working environment, since the cost to presenteeism, i.e. working under reduced productivity (Johns 2010), and absenteeism can be detrimental to the business. An estimated 2% decrease in productivity due to Sick Building Syndrome (SBS) in US would impose an annual nationwide cost of $60 billion (Fisk 2000). In another recent cross–sectional study conducted in Japan, based on human capital approach, the monetary value due to absenteeism and presenteeism was \$520 and \$3055 per person per year (Nagata et al. 2018).

Less emphasis has been put onto IEQ in living space, despite that people generally spend more time at home than in work (BLS 2019). IEQ research at home mostly focuses on energy performance and building sustainability, for example Chen et al. (2016), Aydin and Mihlayanlar (2017). Besides conducting field studies, some researchers attempted to develop IEQ models that relate objective environmental conditions with subjective occupant's satisfaction. Several studies use collective physical and subjective measurement data to develop multivariate logistic regression models for assessing IEQ acceptance in public buildings (Mui and Chan 2005; Cao et al. 2012) and residential building (Lai et al. 2009). However, IEQ regression model is found to be unable to give accurate prediction if the occupants have their own perception and/ or adaption towards the environment (Tsang et al. 2019; Mui et al. 2018). The selection of regression model also significantly affects the prediction results (Majcen et al. 2013).

In view of the limitations, an open probabilistic acceptance model using frequency distribution function is recently developed to handle diverse range of descriptive IEQ parameters in addition to the four major numerical factors forming the argument of environmental quality (i.e. thermal comfort, indoor air quality (IAQ), visual and aural comfort). It makes model updating easier and is more robust in reflecting occupant's environmental perception (Wong et al. 2018). Nevertheless, the characteristics of data used strongly affect the accuracy, relevance and applicability of any model (Heinzerling et al. 2013). If the model is created from a database with small sample size, bias may exist, therefore creating a performance gap between the predicted IEQ acceptance and the actual one, which shall be minimized by calibration.

Humphreys (2005) pointed out that due to cross–parameter effects and prioritization of IEQ parameters, it is impossible to develop an internationally valid index to evaluate IEQ. Wong et al. (2014) addressed the problem of discrepancies between model and survey outcome and recognized that as a "fundamental problem of judgmental decision making based on the best information available". They therefore proposed a Bayesian approach to contemporize field survey data and thermal comfort model. This approach enables the updating of existing model based on best information available (Vick 2002).

Hong Kong as one of the most populated places in the world, has been facing a challenge of meeting the housing demand due to population expansion and limited land supply. Recently some very small living spaces with median floor area of $10m^2$ have emerged in the housing market as an affordable alternative choice of accommodation for the underprivileged (Transport and Housing Bureau 2013; Lai et al. 2016). "Nano flats" with less than $20m^2$ are also becoming popular in the private housing market as the housing price continuously rising (Legislative Council Secretariat 2018). It can be foreseen that the development of housing in Hong Kong is tending to be smaller and smaller as the demand is ever expanding (Wong 2018), our understanding on IEQ responses to residential environments needs be expanded and updated. IEQ modelling shall be able to accommodate the fast changing housing situation with minimum research effort. Following the idea, in this study, we propose and demonstrate a Bayesian IEQ acceptance model framework based on a developed open acceptance model (Wong et al. 2018) and available survey data (Tsang et al. 2019; Mui et al. 2018).

1.1 Research questions and Objectives

The research questions are: Are occupant's responses towards IEQ in very small flat units significantly different from those in average residential buildings? If so, how the different responses improve the existing understandings of occupant's responses towards IEQ in residential buildings?

This paper first explains the complexity of IEQ and the need for having a subjective–objective IEQ model. The development of Bayesian IEQ acceptance model is then introduced. IEQ responses collected by this research team from very small flats units in Hong Kong that have been published previously (Tsang et al. 2019; Mui et al. 2018) are presented, which, together with the probability of acceptance of each environmental case predicted by an existing IEQ model, are used to demonstrate the Bayesian updating procedure. The characteristics of Bayesian model proposed in this study and the future development of IEQ modelling are discussed in the end. The novelty of this study is to quantify the parameter of Bayesian rules for this special case of discrete measurement that the variance can be related to target sample size and difference between prior and measured acceptance of a case. The results shall provide an analytical solution to building owners/operators regarding the choice of IEQ parameters in environmental design and management.

2. Literature review: Understanding the complexity of IEQ modelling

2.1 Factors affecting IEQ satisfaction

Many factors can influence one's perception towards IEQ. Environmental–related constituents include thermal comfort, IAQ, visual comfort, aural comfort, layout, etc. (Kang et al. 2017; Al Horr et al. 2016a, 2016b). Occupant's socioeconomic status and gender also have impact on comfort level, for example Indraganti and Rao (2010) found that people with lower income had higher tolerance to temperature than higher income group, Hansen et al. (2019) found that women and older people considered IEQ at home to be more important than their counterparts. Differences in IEQ satisfaction in workplace were also found to be dependent on occupants' demographics (Bae et al. 2019).

Researchers have taken two different approaches to investigate IEQ. Single–factor studies mainly aim at investigating the effect of a particular aspect and establishing the acceptable environmental condition range, with thermal comfort and indoor air quality (IAQ) as more popular ones (Andargiea et al. 2019; ASHRAE 2019, 2017; ISO 2005). Multi–factor approach instead assesses more than one IEQ aspects concurrently, either independently as separated factors, i.e. assuming the factors are unrelated, or inter–relatedly as if the factors have influences on each other (Andargiea et al. 2019; CEN 2019; ISO 2012).

A number of studies have found that indoor environmental factors have significant interactions and effects on each other. In a controlled experiment designed to investigate the impact of temperature and humidity on perceived IAQ, it was found that levels of indoor air enthalpy (lower than 50kJ/kg) had significant (p–value<0.05) negative effects on acceptability of air quality, suggesting a strong correlation of enthalpy and acceptability of air (Fang et al. 1999). In case of high air enthalpy, perceived air quality appeared to be worse (Lan et al. 2011). This result was also confirmed by a study in Swedish hospitals which found that higher thermal comfort sensation was associated with better perception of air quality (Fransson et al. 2007). In a controlled field survey, Huang et al. (2012) identified that when one IEQ parameter reached the highest satisfaction level, occupants tended to have higher tolerance to another parameter. For example, when temperature was at the optimal level, subjects found a higher noise level acceptable. To conclude, under the same IEQ, different factors may offset each other. Due to the complexity of interaction between IEQ parameters, most of the existing IEQ models only focus on a limited number of four physical factors, namely thermal comfort, IAQ, visual and aural environment (Heinzerling et al. 2013).

In addition to cross–parameter effects, a prioritization of IEQ factors is observed to be space and occupant specific (Sakhare and Ralegaonkar 2014). Field studies investigating the impacts of IEQ on occupant's satisfaction have found that thermal comfort is usually the most important among other IEQ factors, and visual environment is the least concerned (Cao et al. 2012; Frontczak et al. 2012; Huang et al. 2012). Occupants sometimes would have a specific IEQ preference for building with different usage, for instant a quite aural condition was more important than other aspects in learning environment (Lee et al. 2012). Alternatively, even for building with the same usage, relative importance of IEQ factors would also be different deemed by occupants with different demographics. For example, Lai and Yik (2009) concluded that noise was more important than IAQ for lower income group residing in public housing, but the situation was reverse in private housing where the high income group lives in.

From the above, we can see that IEQ is an intricate, inter–related and subjective matter that cannot be entirely explained by physical equations alone. Subjective IEQ responses collected by survey are therefore important input for predicting occupant's IEQ satisfaction.

2.2 IEQ model

IEQ research in relation to occupants looks for the deterministic causal connections between environmental quantities and occupant's comfort. This approach views these relationships purely physical which can be expressed in a mathematical equation or model (Baggs and Chemero 2018; Wellems et al. 2020). Therefore, IEQ models that relate one or multiple objective IEQ parameters to occupant's overall IEQ comfort response were developed to explain this relationship. Heinzerling et al. (2013) categorized IEQ models into two basic types: subjective–objective and objective–criteria. The former one gives single–variable, linear or multivariate regression equations to predict overall IEQ satisfaction (an index) that defines the level of IEQ of an environment, the latter compares objective measurements with a fixed set of comfort IEQ criteria that are derived from previous subjective–objective studies.

While most buildings are designed and operated according to comfort objective–criteria, it has been found that even if comfort requirements were met, occupants still felt unsatisfied (Burge 2004). Heinzerling et al. (2013) summarized in their review on IEQ assessment models that none of the existing models accounted for inter–category relationship between IEQ parameters which could be space–specific. The assessment classes of objective–criteria IEQ models also lack justification and are not always aligned with occupant's actual satisfaction. It can be seen that the causal relationship sometimes cannot explain people's conscious experience to an environment that is ever changing (Stanton 1983). As a result, IEQ research cannot fully adopt a reductive physicalism in exploring the environmental quantities–occupant's comfort relationship. Conducting subjective questionnaire can therefore address the phenomenal characteristics of mental state, for example perception, feelings and emotions. Yet, we cannot rely solely on field questionnaires to evaluate building IEQ performance due to its subjective nature and the lack of universal judgement (Asaid et al. 2017). Heinzerling et al. (2013) also pointed out that occupant's satisfaction is the major concern of building operators, but using only subjective survey for assessing IEQ may not be able to capture IEQ–related energy issues. As a result, there is a need to have accurate subjective–objective IEQ model for predicting IEQ satisfaction.

3. Materials and Methods

In the following section, field data collected and published previously, which are used for the demonstration of model updating is first introduced. The open acceptance model for IEQ developed previously (Wong et al. 2018), shown below in Eq. (1)–(6), is then described. The open acceptance model was based on frequency distribution function of occupant's responses towards IEQ parameters, and the overall IEQ acceptance is defined by the logistic function of the probability of acceptance of individual parameter. The fundamental of Bayesian rules, described in Eq. (7) and (8), are explained. Finally, the Bayesian framework proposed in this study, introduced in Eq. (9)–(13), is incorporated into open acceptance model by updating the probability of acceptance of the environmental cases using newly available data.

3.1 Database for model updating

Objective and subjective IEQ data were collected in very small flat units through on–site field measurement and interviews from October to December 2016 previously by this team in very small flat units. These data have been published in Tsang et al. (2019) and Mui et al. (2018).

Basic IEQ parameters including indoor air temperature (T_a), radiant temperature (T_r), air velocity (V_a), relative humidity (*RH*), were measured by Lutron Heat Index WBGT Meter (WBGT–2009) and Lutron Hot Wire Anemometer (AM–4204HA), carbon dioxide (CO_2) by TSI Q–Trak IAQ Monitor (TSI–8551), horizontal illuminance level by Lutron Digital Lux Meter (LX–1108) and equivalent noise level by Lutron Digital Sound Level Meter (SL–4001) for evaluating the thermal, air quality, visual and aural environment. These environmental parameters allow us to objectively evaluate the indoor environmental conditions of very small flats units and compare the environment with average residential buildings.

Subjective IEQ responses were collected through individual interviews with occupants. Their thermal sensation and acceptance to air quality were evaluated by ASHRAE seven–point thermal sensation scale and a five–point scale (very good, good, neutral, bad and very bad). Aural and visual environments were assessed by a maximum of 100 score. Besides, to determine the overall IEQ acceptance, occupants were asked a total of five direct polar acceptable/ unacceptable questions regarding the above–mentioned four IEQ aspects and the overall IEQ. These particular information are necessary to evaluate occupant's subjective responses toward the perceived environmental conditions. Comparison of subjective responses by occupants from very small flats unit and average residential buildings will be made to identify the difference in their subjective–objective IEQ relationship. These results are later used as input for Bayesian updating described in the next section.

3.2 Proposed Bayesian IEQ acceptance model

This work proposes a Bayesian updating framework for IEQ model in order to update the subjective–objective relationship of the model to improve the accuracy and model applicability. In this particular example of updating IEQ acceptance model for residential building in Hong Kong, open acceptance model for IEQ developed by Wong et al. (2018) is adopted. This probabilistic acceptance model assesses the overall IEQ performance using frequency distribution function of occupant's responses towards IEQ parameters. This model is selected due to its robustness and the flexibility about the range/ type of IEQ factors and the addition of new observed data for model updating.

3.2.1 Open acceptance model for IEQ

The collective overall IEQ acceptance Φ is given in Eq. (1) by the overall individual's acceptance to the exposed environmental conditions of the respective environmental parameters $\delta_i = \delta_i\,(x_i)$, where i is the number of the environmental parameters resulted in a total of $j = 1, 2, \ldots, i^2{-}1, i^2$ environmental case. The occurrence of case j can be expressed by Eq. (2), while $\rho_j = \rho_1, \rho_2, \ldots, \rho_{i^2}$ is the acceptance with respect to the environmental conditions φ_j.

$$\Phi = \sum_{j=1}^{i^2} \varphi_j \rho_j \; ; \; \varphi_j = \varphi_j\,(\delta_i) \tag{1}$$

$$\begin{aligned} \varphi_j &= \varphi_1, \varphi_2, \ldots, \varphi_{i^2} \\ &= (1-\delta_1)(1-\delta_2)\ldots(1-\delta_{i-1})(1-\delta_i), (1-\delta_1)(1-\delta_2)\ldots(1-\delta_{i-1})(\delta_i), \\ &\quad (1-\delta_1)(1-\delta_2)\ldots(\delta_{i-1})(1-\delta_i), \ldots\ldots, (\delta_1)(\delta_2)\ldots(\delta_{i-1})(\delta_i) \end{aligned} \tag{2}$$

The acceptance function $\delta(x)$ of an environmental parameter $x \in [a, b]$ is expressed in Eq. (3).

$$\delta = \begin{cases} 1-\int_a^x \tilde{x}_{su}dx & \delta(a) > \delta(b) \\ \quad ; \\ \int_a^x \tilde{x}_{su}dx & \delta(a) < \delta(b) \end{cases} \tag{3}$$

The probability density function of normalized occupant votes $\tilde{x}_{su}$ for the environmental acceptance θ_{su} is given by Eq. (4), where θ_s and θ_u, shown in Eq. (5), are percentage votes for acceptance and unacceptance with sample sizes n_s and n_u, y_s and y_u in Eq. (6) are the cumulative frequency distributions for the mass density functions of parameters $\tilde{x}_s$ and $\tilde{x}_u$,

$$\tilde{x}_{su} = \frac{\theta_{su}}{\int_a^b \tilde{\theta}_{su}dx}; \ \theta_{su}(x) = 1 - \left|\theta_s - \theta_u\right| \tag{4}$$

$$\theta_s = \frac{n_s y_s}{n_s y_s + n_u y_u}; \quad \theta_u = \frac{n_u y_u}{n_s y_s + n_u y_u} \tag{5}$$

$$y_s = 1 - \int_a^x \tilde{x}_s dx; \quad y_u = \int_a^x \tilde{x}_u dx \tag{6}$$

$\tilde{x}_s \sim \tilde{x}_s$ (μ_s, σ_s), $\tilde{x}_u \sim \tilde{x}_u$ (μ_u, σ_u) are the collective occupant responses to the environment obtained from site survey studies and can be approximated by parametric distribution functions, where μ and σ are the means and standards deviation of parameters x_s and x_u.

3.2.2 Development of Bayesian updating framework

Bayes' theorem, which relates the conditional and marginal probabilities of stochastic events A and B (where B has a non–vanishing probability), asserts that the probability of an event A given by event B depends not only on the relation between events A and B but also on the marginal probability of occurrence of each event. This theory can be applied to a sample size not large enough for decision–making purposes, yet relevant enough for statistical analysis. Its general formulation and various applications are available in the literature (Vick 2002).

The proposed approach predicts collective acceptance to an environmental condition using the readily available information (event A) and the new measurements from an indoor environment (event B) (Wong et al. 2014). Given a measurement acceptance value ρ (event B) is significantly different from a prior belief of the acceptance ρ_0 (event A) that $|\rho_0 - \rho| > \varepsilon$, where ε is a cut–off value of the acceptable error.

Assuming the measured acceptance value ρ of an environment at attributes j can be approximated by a normal distribution, $\rho_{j,\mathrm{m}} \sim N(\mu,\sigma^2)$, the posterior estimate of the acceptance $\rho_{j,1} \sim N(\mu_1,\sigma_1^2)$ is expressed by the following Bayesian rules in Eq. (7) and (8) (Lee 2004), where $\rho_{j,0} \sim N(\mu_0,\sigma_0^2)$ is the prior estimate of the acceptance towards environmental attributes j, p is the probability, μ and σ^2 are the mean and variance of a normal distribution function, μ, μ_0, and μ_1 are the best estimates of the measured, prior and posterior acceptance value respectively,

$$p(\rho_{j,1}|\rho_{j,m}) = p(\rho_{j,0})\, p(\rho_{j,m}|\rho_{j,0}) \tag{7}$$

$$\sigma_1^2 = (\sigma_0^{-2} + \sigma^{-2})^{-1};\ \mu_1 = \mu_0\, \sigma_0^{-2} / (\sigma_0^{-2} + \sigma^{-2}) + \mu\, \sigma^{-2} / (\sigma_0^{-2} + \sigma^{-2}) \tag{8}$$

In these rules, the weightings are proportional to their respective variances, and the posterior mean is a weighted average of the prior mean and the measured value given in Eq. (9). This posterior mean can be characterized by the ratio of standard deviations and expressed as a parameter β.

$$\beta^2 = \sigma^2 / \sigma_0^2 \tag{9}$$

Suppose repeatedly measurements give the measurement acceptance ρ and denote $X = \sigma_0^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \beta^2 / (1 + \beta^2)$ and $Y = \mu\, \sigma^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \mu / (1 + \beta^2)$, posterior estimates $\mu_1, \mu_2, \ldots, \mu_n$ are given below in Eq. (10).

$$\mu_1 = \mu_0 X + Y,$$

$$\mu_2 = \mu_0 X^2 + XY + Y,$$

$$\vdots$$

$$\mu_n = \mu_0 X^n + Y (X^{n-1} + X^{n-2} + \ldots + X + 1) = \mu_0 X^n + Y (1 - X^n) / (1 - X) \tag{10}$$

It is noted for Eq. (10) $\mu_n \rightarrow \mu$ when $n \rightarrow \infty$. Taking n is a finite number of the repeated observations such that the N–th estimate shows no significant difference from measured acceptance, i.e. $|\mu_n - \mu| \leq \varepsilon$, and β^2 can be determined by Eq. (11)–(13),

$$\mu_0 X^n + Y (1 - X^n) / (1 - X) = \mu + \varepsilon \tag{11}$$

$$\mu_0 \left(\frac{\beta^2}{1+\beta^2}\right)^n + \left(\frac{\mu}{1+\beta^2}\right)\left(\frac{1-\left(\frac{\beta^2}{1+\beta^2}\right)^n}{1-\left(\frac{\beta^2}{1+\beta^2}\right)}\right) = \mu + \varepsilon \tag{12}$$

$$\beta^2 = c_r^{1/N} / (1 - c_r^{1/N});\ c_r = \varepsilon\, (\mu_0 - \mu)^{-1} \tag{13}$$

The constant c_r is the ratio of acceptable error to the difference between the prior acceptance μ_0 and the measured acceptance μ, while N is the target number of estimate which is large enough for superseding the prior belief.

In order to present this Bayesian approach for IEQ model updating, target sample sizes N of 5 (choice A) and 10 (choice B), and an acceptable error ε of 0.01 are chosen as example managerial decisions. The flow of Bayesian approach is presented in Figure 1 for easy understanding.

4. Results and Discussions

4.1 IEQ data from very small flat units

Table 1 exhibits the measurement results of selected IEQ parameters in very small flat units (Tsang et al. 2019; Mui et al. 2018) with comparison to another IEQ study conducted in average residential buildings (Lai et al. 2009). For thermal environment, as though no significant differences were recorded between all temperatures in average residential buildings and very small flat units, the overall predicted mean vote (PMV) in very small flat units was significantly higher due to higher metabolic rate. This may indicate that small unit occupants have adapted to a warmer environment as a “factual reality”. No significant differences were found between satisfied (and unsatisfied) groups in the two studies, suggesting that human thermal sensation on thermal satisfaction (and dissatisfaction) are somewhat compatible. It is noteworthy that air velocity (V_a) (overall and both satisfied and unsatisfied group) in small units was much lower than that in average residential buildings, however its contribution to thermal comfort determined by the operative temperature (T_o) is not significant. Overall speaking, the thermal environments between average residential buildings and very small units were objectively the same.

Subjective thermal sensation vote (TSV) revealed a narrower thermal acceptability range of occupants from small units than PMV model, given by a slope of 2.79 shown in Eq. (14). They also preferred a slightly cool environment than thermal neutral with a PMV = –0.12 at TSV = 0. Occupants from average residential buildings also gave similar responses with a slope of 2.2 and a PMV=–0.15 at TSV = 0. The results suggested that occupants of very small units were more sensitive and more easily dissatisfied with hot environment than residents in average residential buildings, although the thermal conditions were found to be comparable.

$$TSV = 2.79PMV + 0.12;\ 0 \leq TSV \leq 3 \tag{14}$$

Investigation of thermal acceptance showed that as though small unit residents were more sensitive to warmth with a preference to slightly cool environment, some of them still accepted a thermal environment with PMV ≥ 2, compare to a zero acceptance at PMV = 1.5 for occupants from average residential buildings. It suggested that small unit residents might have developed some degree of tolerance or psychological resistance to heat with a wider range of acceptable thermal condition. A greater sensitivity to operative temperature was also discovered for small unit residents with as low as 9% acceptance at 32°C, while 74% of occupants from average residential buildings accepted the thermal environment at that temperature.

For IAQ, visual and aural aspect, small unit occupants in general preferred low CO_2 level, high horizontal illuminance level and low equivalent noise level. However, variabilities of acceptance against these parameters were very small within the measurable range, meaning that the acceptance towards these aspects were not depending much on the changing environmental conditions, but rather influenced mainly by their own perceptions to the environment. On the other hand, occupants from average residential buildings were sensitive to changing CO_2 level, horizontal illuminance level and equivalent noise level. Zero acceptance can be observed within the measurable boundary for these three aspects. It suggested that unlike the occupants from average residential buildings who considered thermal and aural environments as more important contributors to IEQ, small unit residents were more concerned about the thermal environment, while the remaining aspects were less important to them.

In summary, basic IEQ parameters in very small flat units were found compatible to the residential buildings, but occupant's subjective responses to the environmental conditions were different. Bayesian approach is therefore appropriate to be adopted for IEQ model updating.

4.2 Bayesian updating procedure

To demonstrate the Bayesian updating procedure for IEQ model, two prior beliefs are adopted. First, a uniform prior $\rho_{j,0}$ which environment contributors weigh equally in the overall IEQ acceptance (i.e. thermal comfort, IAQ, visual and aural condition affect occupant's IEQ acceptance in equal manner) is assumed to represent a situation when we do not have any previous IEQ understandings of a new environment. The predicted probability of acceptance of 16 environmental cases generated by logistic regression model for average residential buildings by Lai et al. (2009) are also adopted. This prior belief represents an example where some degree of understandings of a certain environment are known, and newly acquired information are available to improve the accuracy of existing model.

Table 2 shows the prior IEQ acceptance under different cases of environmental conditions (total number of cases $j = 2^4 = 16$ cases). IEQ contributors with binary notation 0 = unsatisfied and 1 = satisfied for thermal comfort, IAQ, visual and aural acceptance are presented. In average residential buildings, most of the occupants voted for case $j = 16$, which indicated that they were mostly satisfied with the environment conditions. It is assumed that people have more control over the living environments and therefore they adjusted to those that fit them. It is also noteworthy that only 11 out of 16 cases were recorded with vote, and only 4 cases with $n \geq 5$. In regression analysis, survey data with small sample size are not included, making the model less sensitive to poor conditions. On the other hand, for residents of very small flat units, substantial of them voted for case $j = 1$ to 4, indicating that the majority of them were not satisfied with the environmental conditions. Only 2 out of 16 cases did not record any vote, showing that the occupant's opinions towards the environmental conditions were more diverse.

Bayesian approach has the power to evaluate the statistical significance of field measurement data based on its sample size and relate it to existing model with a choice of target sample size N and acceptable error ε (Wong et al. 2014). Different target sample size N would result in different posterior probability $\rho_{j,1}$. Calculation steps of two selected cases are demonstrated in Appendix 1 for reference.

Table 3 exhibits the posterior acceptance ($\rho_{j,1}$) with (a) uniform prior and (b) probability of acceptance by regression model under managerial decisions choice A (N = 5, ε = 0.01) and choice B (N = 10, ε = 0.01). Figure 2 is the graphical presentation of the Bayesian estimation. It is noteworthy that in some cases no sample were recorded (i.e. n = 0, annotated with '$^{\sigma}$'), prior acceptance becomes the sole and the best information available for prediction, therefore the posterior acceptance is the same as prior acceptance (i.e. $\rho_{j,0} = \rho_{j,1}$).

When the sample size is small comparing to target sample size, e.g. case j = 5, 7, 8 and 14 of choice B, by Bayesian approach, survey data has small influence on the prior acceptance, resulting a posterior acceptance that is closer to prior than measured acceptance. On the other hand, for cases with larger sample sizes, e.g. case j = 15 of choice A and case j = 4 of choice B, influence of survey data on prior belief is larger and therefore posterior estimation is closer to measured acceptance. For cases which sample size is larger or equal to target sample size (annotated with '$^{\#}$'), i.e. $n \geq N$, e.g. case j = 1, 2, 4 and 16 of choice A and case j = 16 of choice B, the posterior estimate is equal to measured acceptance plus acceptable error (i.e. $\mu_1 = \mu \pm \varepsilon$, where $\rho_{j,1} \sim N(\mu_1, \sigma_1^2)$). From the above, it can be seen that the target sample size significantly affects the resulting posterior estimation by Bayesian approach.

Some cases with measured acceptance equal to the prior belief (annotated with '*'), e.g. case j = 15 and 16 of uniform prior, case j = 12 and 16 of regression model, posterior acceptance is the same as the measured and the prior because the predicted and actual data agree with each other (i.e. if $\rho_{j,0} = \rho_{j,\mathrm{m}}$, then $\rho_{j,0} = \rho_{j,\mathrm{m}} = \rho_{j,1}$). When the difference between measured acceptance and prior acceptance is equal or smaller than acceptable error (annotated with '$^\tau$'), i.e. $|\mu_n - \mu| \leq \varepsilon$ but $\neq 0$, e.g. case j = 5, 7, 13, and 14 of regression model, no significant difference between measured data and prior belief is considered, therefore posterior estimate is equal to prior belief. It is also recognizable that the selection of acceptable error greatly influences the estimation. For a large error, accuracy of the model is lower because a large difference between survey data and prior belief is accepted as measurement error, and therefore failing to update the prior with actual occupant's response.

4.3 Future work

Bayesian approach benefits IEQ modeling by allowing easy updating with newly acquired data, which handles the limitations of existing IEQ models. In addition, this approach is not limit to continuous IEQ parameters, discrete parameters that can be used to anticipate IEQ acceptance can also be processed by Bayesian approach if field data is available.

We agree with Willems et al. (2020) that considering occupant's perception towards an environment as a causal, reducible relationship may be easier for setting up guidelines and comfort requirements, but it may not truly reflect the actual experience. It is supported by studies that showed people being dissatisfied with an environment that met with comfort requirements suggested by the guidelines. The fundamental problem is that the criteria are derived from previous subjective–objective studies, and the relationship between subjective vote and objective physical measurement may change with different group of occupants. The perception toward environmental conditions and the above–mentioned relationship can change over time and with lived experience even with the same group of people. As a result, IEQ modelling shall be constantly updated with newly available subjective data. This Bayesian IEQ acceptance model can therefore be a useful tools for improving the IEQ model accuracy before any holistic prediction model that can resolve the epistemic nature of occupant's perception is developed.

5. Conclusion

Assessing IEQ cannot solely rely on objective tools or subjective survey. An accurate subjective–objective IEQ model is therefore crucial for building engineers to predict occupant's satisfaction. Field study in very small flat units revealed that occupant's IEQ response to a similar environment can be different due to their own perception and/ or adaption. In view of the fast changing housing situation in Hong Kong, it is essential to update our understanding on residential IEQ and to expend the applicability of residential IEQ model. In this study, Bayesian IEQ acceptance model is proposed based on an existing open acceptance model. Expressions for overall IEQ acceptance given by discrete binary responses of IEQ parameters that are ready solved by Bayesian rules are devised analytically. This method provides a systematic approach to related additional survey data to current belief. With selected target sample size and acceptable error, statistical significances of data are considered and incorporated into Bayesian analysis. Bayesian updating of previous residential IEQ model is demonstrated by using subjective IEQ responses from very small flat units as inputs. It shows that the posterior acceptance is close to prior belief when the sample size is small. With large sample size, the posterior is instead close to the measured acceptance. For sample size that meets with the target sample number, posterior is equal to measured acceptance plus acceptable error. Updating of IEQ prediction model can therefore be achieved even with a small quantity of field data from a similar environment. This study presents a significant step forward from a numerical solution in limited cases to a general analytical solution for IEQ with the Bayesian rules applied. The findings suggest that the Bayesian IEQ acceptance model can be a useful tool for indoor environmental design with a selection of target sample size and acceptable error based on managerial decision.

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Nomenclature

<u>Environmental parameters</u>

T_a	Indoor air temperature
T_r	Radiant temperature
RH	Relative humidity
V_a	Air velocity

<u>Bayesian IEQ acceptance model</u>

δ_i	Acceptance to environmental parameter
Φ	Overall IEQ acceptance
x_i	Level of environmental parameter
i	Number of environmental parameter
j	Environmental conditions correspond to environmental parameter i
ρ_j	Acceptance to environmental condition j
φ_j	Occurrence of environmental condition j

<u>Occurrence of environmental conditions</u>

$[a, b]$	Range of level of environmental parameter
θ_{su}	Environmental acceptance
θ_s	Acceptance
θ_u	Unacceptance
x_{su}	Occupant votes
x_{su}	Probability density function of normalized occupant votes
x_{su}^*	Level of environmental parameter when acceptance and unacceptance are equal
n_s	Acceptance sample size
n_u	Unacceptance sample size
$\tilde{x}_s$	Collective acceptance occupant responses to the environment
$\tilde{x}_u$	Collective unacceptance occupant responses to the environment
y_s	Cumulative frequency distributions for the mass density functions of parameters $\tilde{x}_s$
y_u	Cumulative frequency distributions for the mass density functions of parameters $\tilde{x}_u$
μ_s / μ_u	Mean of x_s / x_u
σ_s / σ_u	Standard deviation of x_s / x_u

<u>Bayesian acceptance of environmental conditions</u>

A, B	Events
$\rho_{j,0}$	Prior acceptance
$\rho_{j,m}$	Measured acceptance
$\rho_{j,1}$	Posterior acceptance
ε	Acceptable error
j	Environmental attributes
μ	Mean of measured acceptance
σ^2	Variance
p	Probability

<u>Bayesian updating framework</u>

β	Ratio of standard deviations (σ/σ_0)
N	Target sample size
n	Measured sample size
c_r	Ratio of ε to difference between prior and measured acceptance

Table 1. Measurement results of IEQ parameters

Parameter	**Residential buildings (Lai et al. 2009)**	**Very small flat units (Tsang et al. 2019; Mui et al. 2018)**	***p*-value, *t*-test**
Per capita area (m^2)	13.1	5.7 (3.4)	<0.0001
Predicted mean vote *PMV*	0.27 (0.88)	0.56 (0.82)**	<0.05
Unsatisfied	0.65 (0.95)	0.94 (0.43)	0.43
Satisfied	0.24 (0.86)	0.32 (0.92)	0.65
Air temperature T_a (°C)	27.3 (2.2)	27.4 (2.2)**	0.81
Unsatisfied	28.1 (2.3)	28.3 (1.2)	0.86
Satisfied	27.3 (2.2)	26.9 (2.5)	0.43
Radiant temperature T_r (°C)	27.5 (2.0)	27.3 (1.8)**	0.63
Unsatisfied	28.1 (2.4)	28.2 (1.2)	0.94
Satisfied	27.4 (1.9)	26.8 (2.0)	0.12
Air velocity V_a (ms^{-1})	0.37 (0.2)	0.2 (0.19)	<0.05
Unsatisfied	0.49 (0.3)	0.18 (0.2)	<0.05
Satisfied	0.36 (0.2)	0.21 (0.2)	<0.05
Operative temperature T_o (°C)	27.4 (2.0)	27.3 (2.0)**	0.93
Unsatisfied	28.1 (2.4)	28.2 (1.2)	0.91
Satisfied	27.3 (2.0)	26.9 (2.2)	0.25
Relative humidity *RH* (%)	83.9 (10.5)	73.5 (12.3)	<0.05
Unsatisfied	84.1 (10.3)	76.1 (10.3)	0.09
Satisfied	83.9 (10.4)	71.8 (13.2)	<0.05
Metabolic rate M_e (Met)	1.06 (0.11)	1.13 (0.10)	<0.05
Unsatisfied	1.11 (0.13)	1.15 (0.09)	0.45
Satisfied	1.05 (0.10)	1.12 (0.10)	<0.05
Clothing value I_{cl} (clo)	0.48 (0.11)	0.40 (0.11)	<0.05
Unsatisfied	0.48 (0.11)	0.39 (0.10)	<0.05
Satisfied	0.48 (0.11)	0.41 (0.12)	<0.05
Carbon dioxide ζ_2 (ppm)	675 (328)	1046 (500)	<0.05
Unsatisfied	497 (345)	1240 (609)	<0.05
Satisfied	689 (327)	925 (369)	<0.05
Horizontal illuminance level ζ_3 (lux)	187 (273)	191 (127)	0.88
Unsatisfied	307 (435)	156 (112)	0.36
Satisfied	178 (252)	213 (131)	0.29
Equivalent noise level ζ_4 (dBA)	67.3 (6.2)	62.6 (4.8)	<0.05
Unsatisfied	70.6 (7.9)	62.4 (5.0)	<0.05
Satisfied	67.1 (6.0)	62.8 (4.7)	<0.05

Remarks: Standard deviation in brackets; *t*-test between satisfied and unsatisfied groups for each indoor environmental parameter, where **: *p*-value ≤ 0.05

Table 2. The prior IEQ acceptance ($\rho_{j,0}$) in case j = 1, 2, 3, …, 16 in (a): an IEQ model with uniform prior acceptance such that each of the four IEQ contributor contributes equally to the overall IEQ acceptance; (b): a multivariate logistic regression model for IEQ in average residential buildings by Lai et al. (2009); and the measured environmental acceptance $\rho_{j,\mathrm{m}}$ in very small flat units.

IEQ Contributor					Uniform Prior	Regression model		Very small flat units	
j	Thermal	IAQ	Visual	Aural	$\rho_{j,0}$	n	$\rho_{j,0}$	n	$\rho_{j,\mathrm{m}}$
1	0	0	0	0	0	1	2×10^{-15}	6	0.167
2	0	0	0	1	0.25	0	8×10^{-6}	5	0.2
3	0	0	1	0	0.25	1	3×10^{-10}	3	0.333
4	0	0	1	1	0.5	2	0.5	8	0.875
5	0	1	0	0	0.25	0	1×10^{-14}	1	0
6	0	1	0	1	0.5	1	4×10^{-5}	0	–
7	0	1	1	0	0.5	2	2×10^{-9}	1	0
8	0	1	1	1	0.75	6	0.83	1	1
9	1	0	0	0	0.25	1	9×10^{-6}	0	–
10	1	0	0	1	0.5	0	0.9999	2	0
11	1	0	1	0	0.5	0	0.55	2	1
12	1	0	1	1	0.75	2	1	2	1
13	1	1	0	0	0.5	0	5×10^{-5}	3	0
14	1	1	0	1	0.75	7	0.9999	1	1
15	1	1	1	0	0.75	7	0.86	4	0.75
16	1	1	1	1	1	95	1	13	1
				Total	–	125	–	52	–

Table 3. Posterior acceptance with (a) uniform prior and (b) regression model under managerial decisions choice A (target sample size N = 5, acceptable error ε = 0.01) and choice B (N = 10, ε = 0.01). Column "$\rho_{j,1}$" shows the posterior acceptance updated by Bayesian approach based on prior estimate ($\rho_{j,0}$) and measured acceptance ($\rho_{j,\mathrm{m}}$) collected.

Case	Very small flat units		Uniform Prior			Regression model		
				Posterior ($\rho_{j,1}$)			Posterior ($\rho_{j,1}$)	
j	n	Measured ($\rho_{j,\mathrm{m}}$)	Prior ($\rho_{j,0}$)	A	B	Prior ($\rho_{j,0}$)	A	B
1	6	0.167	0	$0.167^{\#}$	0.136	2×10^{-15}	$0.167^{\#}$	0.136
2	5	0.2	0.25	$0.2^{\#}$	0.222	8×10^{-6}	$0.2^{\#}$	0.155
3	3	0.333	0.25	0.310	0.289	3×10^{-10}	0.292	0.217
4	8	0.875	0.5	$0.875^{\#}$	0.854	0.5	$0.875^{\#}$	0.854
5	1	0	0.25	0.132	0.181	$1\times10^{-14\tau}$	1×10^{-14}	1×10^{-14}
6^{σ}	0	–	0.5	0.5	0.5	4×10^{-5}	4×10^{-5}	4×10^{-5}
7	1	0	0.5	0.229	0.339	$2\times10^{-9\tau}$	2×10^{-9}	2×10^{-9}
8	1	1	0.75	0.869	0.819	0.83	0.904	0.872
9^{σ}	0	–	0.25	0.25	0.25	9×10^{-6}	9×10^{-6}	9×10^{-6}
10	2	0	0.5	0.105	0.229	0.9999	0.158	0.398
11	2	1	0.5	0.895	0.771	0.55	0.902	0.790
12	2	1	0.75	0.931	0.869	1^{*}	1	1
13	3	0	0.5	0.048	0.155	$5\times10^{-5\tau}$	5×10^{-5}	5×10^{-5}
14	1	1	0.75	0.869	0.819	0.9999^{τ}	0.9999	0.9999
15	4	0.75	0.75^{*}	0.75	0.75	0.86	0.766	0.792
16	13	1	1^{*}	$1^{\#}$	$1^{\#}$	1^{*}	$1^{\#}$	$1^{\#}$

Measured acceptance of cases with no sample (i.e. n = 0) is marked as "–". These cases are annotated with '$^{\sigma}$'. '$^{\tau}$' indicates difference between prior acceptance and measured acceptance is smaller than the acceptable error; '*' indicates the prior acceptance is the same as measured acceptance; '$^{\#}$' indicates that the sample size meets with the target sample size and therefore the posterior acceptance is equal to the measured acceptance.

Appendix: Example calculation steps

Sample 1 – Case 1, with uniform prior

Given: Target sample size $N = 10$

Acceptable error $\varepsilon = 0.01$

Sample size $n = 6$

Prior acceptance $\rho_{1,0} / \mu_0 = 0$

Measured acceptance $\rho_{1,\mathrm{m}} / \mu = 0.167$

By Eq. (14)

$c_r = \varepsilon\, (\mu_0 - \mu)^{-1} = 0.01 \times (0.167 - 0)^{-1} = 0.060$

$\beta^2 = c_r^{1/N} / (1 - c_r^{1/N}) = 0.0599^{1/10} / (1 - 0.0599^{1/10}) = 3.076$

By $X = \sigma_0^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \beta^2 / (1 + \beta^2) = 3.076 / (1 + 3.076) = 0.755$

$Y = \mu\, \sigma^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \mu / (1 + \beta^2) = 0.167 / (1 + 3.076) = 0.041$

By Eq. (11)

$\mu_6 = \mu_0\, X^n + Y\, (1 - X^n) / (1 - X) = 0 \times 0.755^6 + 0.041 \times (1 - 0.755^6) / (1 - 0.755)$

$= 0.136$

Sample 2 – Case 10, with residential model as prior

Given: Target sample size $N = 5$

Acceptable error $\varepsilon = 0.01$

Sample size $n = 2$

Prior acceptance $\rho_{1,0} / \mu_0 = 0.9999$

Measured acceptance $\rho_{1,\mathrm{m}} / \mu = 0$

By Eq. (14)

c_r = ε (μ_0 − μ)$^{-1}$ = 0.01 × (0.9999 – 0)$^{-1}$ = 0.010

$\beta^2 = c_r^{1/N} / (1 - c_r^{1/N}) = 0.0100^{1/5} / (1 - 0.0100^{1/5}) = 0.661$

By $X = \sigma_0^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \beta^2 / (1 + \beta^2) = 0.661 / (1 + 0.661) = 0.398$

$Y = \mu \sigma^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \mu / (1 + \beta^2) = 0 / (1 + 0.661) = 0$

By Eq. (11)

$\mu_6 = \mu_0 X^n + Y (1 - X^n) / (1 - X) = 0.9999 \times 0.398^2 + 0 \times (1 - 0.398^2) / (1 - 0.398)$

$= 0.158$

Figure 1. Schematic diagram of Bayesian approach on IEQ acceptance model.

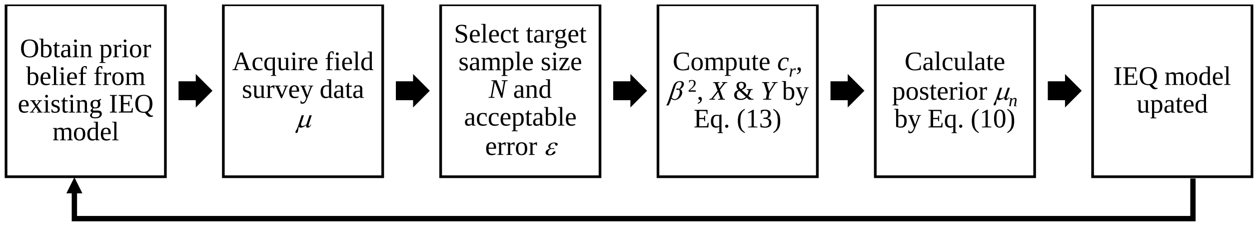
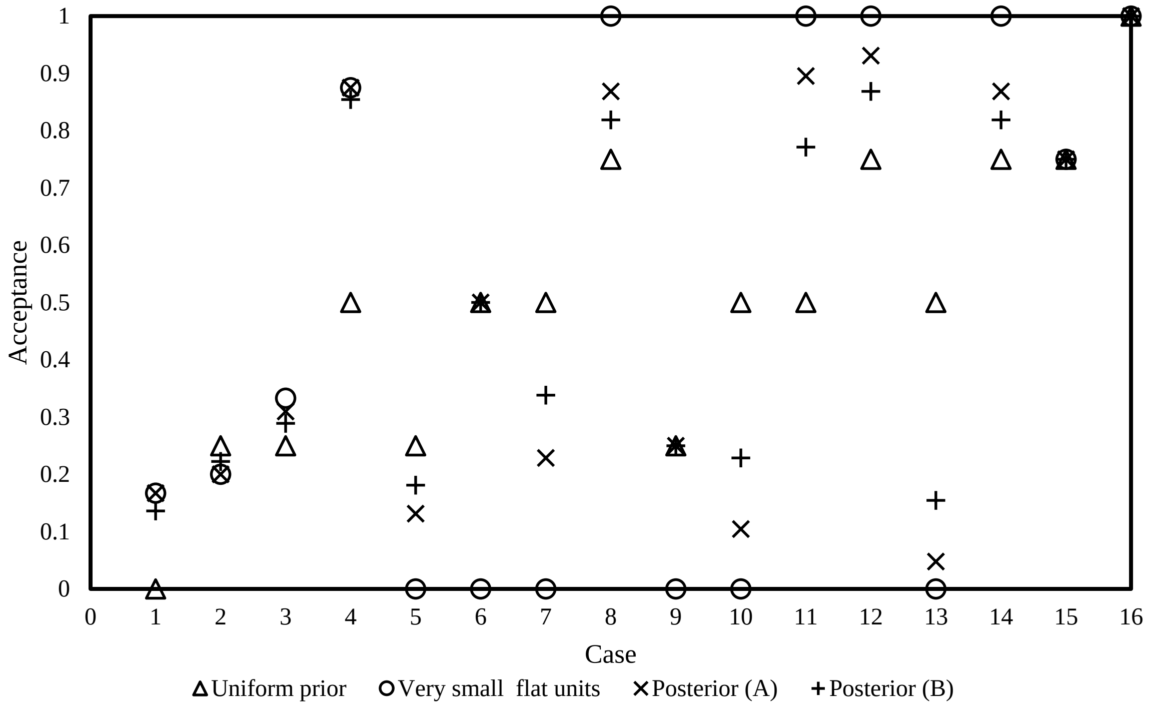
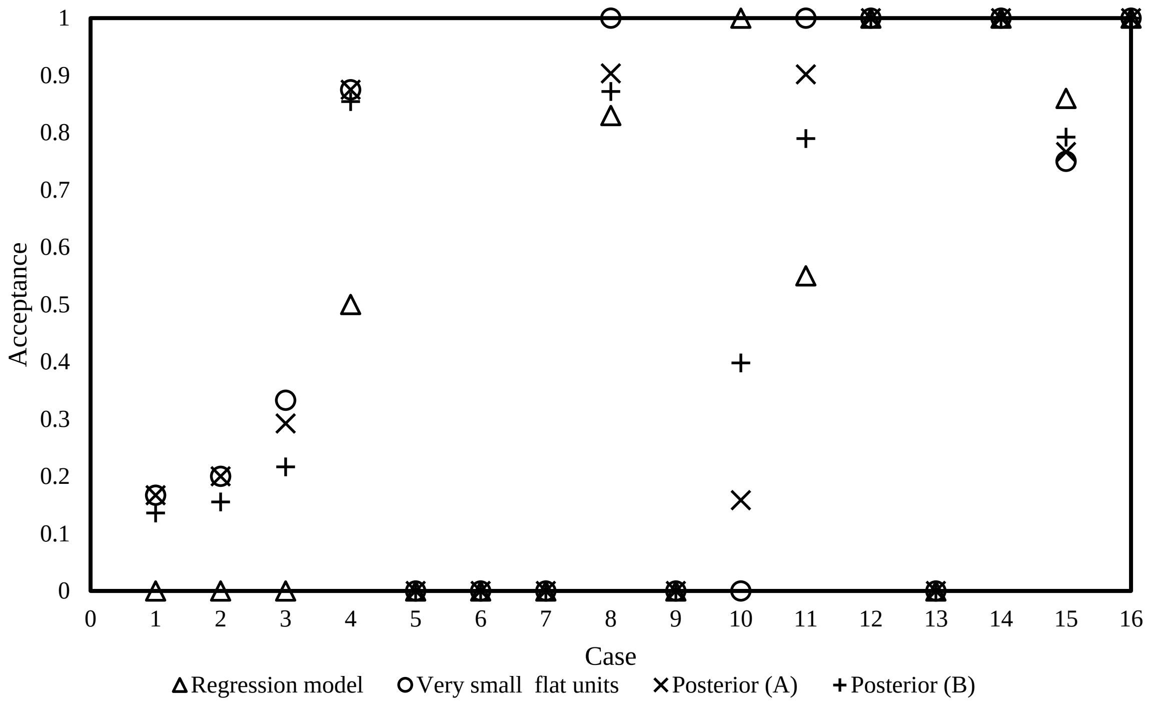


Figure 2. Graphical presentation of the Bayesian estimation.



(a) IEQ model with uniform prior acceptance



(b) Multivariate logistic regression model for IEQ in average residential buildings by Lai et al. (2009)