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Development of bi-stretch auxetic woven fabrics based on re-entrant hexagonal geometry

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Abstract: This paper reports a development of bi-stretch auxetic woven fabrics by using nonauxetic yarns and weaving technology. The fabric structure was first designed based on a reentrant hexagonal geometry by combination of loose weaves and tight weave in a woven fabric structure, and then fabricated on a rapier weaving machine by using both non-elastic and elastic yarns. Two variations of the fabric were produced by using different elastic yarn arrangement in weft direction. The designed auxetic geometry was finally achieved after a washing process due to creation of non-uniform contraction or shrinkage profiles within the fabric structural unit cell. The testing results showed that the developed fabrics exhibited negative Poisson ratio effect in both weft and warp directions in a large range of tensile strain.

Keywords: auxetic, bi-stretch, woven, non-uniform contraction profile, negative Poisson's ratio

1. Introduction

In this era of technological advancement as the demand for new technologies is growing day by day, the demand for new smart materials with improved performance cannot be underrated. Keeping in view the importance of such smart materials, the development in smart textile fabrics for performance-based clothing applications is the need of the day. Auxetic fabric is one of the options for such clothing applications that have come to the forefront in recent times. Unlike conventional fabrics, auxetic fabrics have negative Poisson's ratio (NPR), which means that, if they are stretched longitudinally, they will endure expansion in lateral

direction as well.^[1, 2] The word auxetic was derived by Evans K from the Greek word (auxetos) which means "that something which tends to increase".^[3] The exceptional properties associated with auxetic behavior or NPR effect of auxetic fabrics which make them superior than conventional fabrics include synclastic behavior for better formability^[4], improved comfort and form fitting at joint parts^[5] and increased porosity under stress,^[6] etc.

Most commonly, the shortest possible route adopted to fabricate the auxetic fabrics is to use weaving technology and the auxetic yarns either in warp or weft. Up till today, two types of auxetic woven fabrics by using helix auxetic yarn (HAY) in weft were produced. The first fabric exhibited an out of plane NPR, and an in-plane NPR up to -0.1 is obtained only when the fabric is tested under thickness constraints.^[7] In the second type, the basic weaves including plain, 2/2 twill and 3/5(3) satin, were used. It was reported that the plain and twill fabrics exhibited most auxeticity and the satin woven fabric was found less auxetic.^[6] A 2-ply plain narrow woven fabric by using HAY in warp was also produced. The fabric showed an in-plane NPR value of - 0.1 at 32% strain.^[8] The demerit of the above mentioned woven fabrics is that there are very few HAYs available and the NPR effect of HAYs cannot be fully utilized due to the constraints of woven structure.^[9] In addition, the NPR effect of these fabrics is achieved in the orthogonal direction within the same plane. However, the NPR effect is smaller and only achieved when the fabrics are stretched along the direction in which HAYs are used. That is why such fabrics have very limited applications.

The second method of fabrication involves the use of conventional yarns and weaving or knitting technology. Since the auxetic behavior is purely linked to the geometrical shape of the fabric structural units, the realization of auxetic geometry capable of inducing auxetic behavior into a fabric structure is the key technique for the development of auxetic fabrics by using this method.^[10, 11] In past few years, various auxetic geometries have been developed and tested to realize their mechanical properties. The geometries mostly used to produce

textile and polymer materials include foldable structures,^[12] re-entrant structures,^[13] rotating rigid structures^[14] and nodule and fibril structures.^[15] The auxetic geometries used to produce weft knitted fabrics include foldable geometries,^[12, 13] rotating rectangle geometry,^[13] re-entrant hexagonal geometry and double arrowhead auxetic geometry.^[16] Whereas, the auxetic warp knitted fabrics developed up till today are based on rotational hexagonal loops geometry,^[17] double arrowhead geometry,^[10] spacer structure^[18] and re-entrant hexagonal knit structures.^[19-21] Nevertheless, most of the developed auxetic knitted fabrics have certain limitations such as low structural stability, because the geometrical shape can be easily deformed under loading and yarn slippage effect increases the axial deformation, resulting in reduced NPR effect. In addition, certain parts of geometrical unit cell become inactive during loading, which also lead towards lower NPR effect. Correspondingly, the opening of foldable geometries is less at the beginning of loading. Therefore, less opening of folds and thickness constraint of the fabric resulted in smaller NPR effect.

Most recently, the design and development of uni-stretch auxetic woven fabrics based on second method have also been described.^[22] The development of such fabrics aimed to address the problems related to auxetic knitted fabrics, especially structural stability and thickness. It was reported that it is possible to realize auxetic geometries into stable woven structure having reduced thickness.^[22] The rotating rectangle geometry, foldable geometries and re-entrant hexagonal geometry were realized through creation of the phenomenon of differential shrinkage or non-uniform contraction profile within the fabric structural unit cell.^[22] This phenomenon is created by using elastic and non-elastic yarns together with combinations of loose and tight weaves within the unit cell of interlacement pattern of the fabric.^[22] It was reported that the foldable geometry and re-entrant hexagonal geometry were realized successfully and produced an NPR value of -0.1 when stretched along weft direction. It was also reported that the fabric based on re-entrant hexagonal geometry exhibited NPR

effect over a large tensile strain range from 3% to 52%. This shows that re-entrant hexagonal geometry is more suitable auxetic geometry to be realized into woven structure. However, the major drawbacks associated with these fabrics are extensibility and smaller NPR effect only in one direction, because the phenomenon of non-uniform contraction profile or differential shrinkage is created only in weft direction due to the absence of elastic yarn in warp direction. In view of the limitations related to auxetic fabrics developed up till today, it is implicit that such limitations can be overcome by fabricating auxetic woven fabrics having extensibility in both directions (bi-stretch), with larger NPR effect and made of conventional yarns. This paper reports the work performed towards the achievement of this goal based on re-entrant hexagonal auxetic geometry. The developed fabrics were tested and exhibited NPR effect when stretched along warp and weft direction respectively.

2. Design and Fabrication

2.1. Realization of re-entrant hexagonal geometry into woven fabric

The re-entrant hexagonal geometry used in this study is illustrated in **Figure 1**(a). Its unit cell is shown in Figure 1(b). When the structure is stretched in either direction, the diagonal ribs 1-5, 5-2, 3-6 and 6-4 will move to the vertical direction, which leads to an increase of the distance between point 5 and 6 in horizontal direction. As a result, the dimensions of the whole structure increase and the NPR effect is achieved as shown in Figure 1(c).

The re-entrant hexagonal geometry is a hollow structure which consists of rib segments. Since the woven fabric is not a hollow structure, therefore it is not possible to realize the true rib segments of re-entrant hexagonal geometry into woven fabric. However, an approximation of this geometry can possibly be realized by creating the phenomenon of non-uniform contraction profile or differential shrinkage within the fabric structural unit cell. This phenomenon enables different sections of fabric structural unit cell to endure different levels of shrinkage upon relaxation. In this study, this technique is adopted to fabricate bi-stretch

auxetic woven fabrics. In a woven fabric structure, the differential shrinkage phenomenon can be created in either direction based on an interlacements pattern with combination of loose and tight weaves having different contraction properties and the use of elastic yarns and nonelastic yarns. While the elastic yarns induce elasticity into the fabric structure and act as a return spring, the non-elastic yarns impart stability to the fabric structure. The sections of interlacement pattern with tight weave undergo less shrinkage and loose weave sections undergo higher shrinkage upon relaxation. When the fabric is stretched in either direction, the shrinkage is transposed to increase the transverse dimensions of the fabric resulting in the auxetic effect.^[22]

To realize the approximation of re-entrant hexagonal geometry into woven fabric, the loose and tight weaves within the unit cell of interlacements pattern must be arranged in a suitable way. To achieve this, two possible arrangements were fabricated in the preliminary study. In the first arrangement, the fabric section at ribs 1-3 and 2-4 of the unit cell as shown in Figure 1(b) were tightly woven and the tightness of weave is decreased progressively towards center of the fabric unit cell at point 5 and 6. It was observed that upon relaxation, this arrangement did not realize the shape as assumed. Because of higher shrinkage of the loose weave sections, the tight weave sections were deformed and lost their shape. The second arrangement which produced good results and realized the approximation of re-entrant hexagonal geometry is shown in Figure 2(d). In a unit cell of this arrangement as highlighted with thicker red lined box, there are three sections in terms of weaving tightness namely A, B and C. Firstly, the sections A, at ribs 1–3 and 2–4, are loosely woven by using (4/1) weave as shown in Figure 2(a). Secondly, the sections B which are next to the ribs 1–3 and 2–4, are tightly woven by using (1/1) weave as shown in Figure 2(b). Thirdly, the central section designated as section C which is loosely woven such that each alternate warp yarn is raised above all the weft yarns within the unit cell. This weave can also be described as a weave in which the warp yarns are

free of interlacement and raised above weft yarns alternately while all the weft yarns within the section have exactly same interlacement pattern as shown in Figure 2(c). Therefore, the order of weaving tightness of three sections is B > A > C. It was assumed that by using strong loose weave at the center of the unit cell as compared to other sections, the fabric unit cell will undergo higher shrinkage at points 5 and 6 upon relaxation and acquire the shape of re-entrant hexagon as highlighted by dashed lines in Figure 2(d).

Based on the above designed interlacement pattern, two variations of the fabric were developed, as listed in Table 1. The first variation used elastic and non-elastic yarns in alternate fashion and is designated as (1R,1L) arrangement in weft direction. In the second variation, all elastic yarns were used in weft direction and is designated as (L) arrangement. The aim of these variations is to study the effect of elastic yarn arrangement in weft directions on the realization of re-entrant hexagon geometry and ultimately on the NPR effect. In the warp direction, alternate elastic and non-elastic yarns arrangement is used for both variations of the fabric.

2.2. Fabrication of bi-stretch auxetic woven fabrics and post fabrication treatment

As the concept of developing bi-stretch auxetic woven fabric in this study involves the use of elastic and non-elastic yarn within the same fabric, the tension during warping as well as weaving for both types of yarns cannot be kept the same if they are wound and supplied from the same warp beam. To avoid this problem, a weaving machine with second beam attachment having separate control of let-off motion and tension must be used. The rapier weaving machine (Model: SL8900S) manufactured by CCI Intech Taiwan as shown in Figure 3(a) was used to weave the fabrics because it can meet the requirements. This machine has the options of eight weft supplies, second beam assembly attachment with separate controls as shown in Figure 3(b) and dobby shedding mechanism. All the fabrics were fabricated by using Ne 40/1 cotton spun yarn as non-elastic yarn and Ne 40(40D) core spun cotton spandex yarn as elastic

yarn. The weft and warp densities used are of 31.50/cm and 25.20/cm respectively. The warp yarns are sized by using water soluble Polyvinyl Alcohol (PVA). The specifications of PVA are PH: 5-7; Hydrolysis: 86.5-89; Viscosity: 20.5 to 24.5 CP's and Ash contents: 0.7%. The sizing can be removed just by washing and no extra de-sizing process is required. Application of PVA makes the elastic yarns rigid due to binding of fibers on the surface of yarn. As a result, the elastic warp yarns are in rigid form during weaving and not truly elastic. It is also important to mention that while using all elastic yarns in the weft, separate heald frames for selvedge must be used. The separate frames for selvedge make strong grip of weft yarn on picking and receiving side of machine. Due to strong grip on both sides of the machine, the elastic yarn retract problem which can cause miss picks or weft loose at receiving side is avoided.

After weaving, all the fabrics were washed for about 45 minutes with Luke warm water (40-45°C) and followed by drying and relaxation at room temperature for 24 hours. This process removes the sizing material of the warp yarns and makes the elastic warp yarns to regain their elastic nature again. The elastic warp and weft yarns then shrink and due to different shrinkage levels of elastic and non-elastic yarns together with different contraction properties of loose and tight weaves. Therefore, the non-uniform contraction profile/differential shrinkage effect is created into fabric structure.

3. Tensile test

The developed auxetic woven fabrics have extensibility in both directions. Therefore, tensile tests were carried out along warp as well as along weft direction on an Instron 5566 tensile testing machine. The capacity of the load cell used was 500N. The gauge length and tensile speed were set at 150mm and 50mm/min, respectively. The testing setup is schematically shown in Figure 4. A set of three fabric strips of dimension (50mm×200mm) were cut for each sample with the length along warp and along weft, respectively. The central point of the fabric

strip was first located and then four points were marked with the central point at 20mm to facilitate recording the information of fabric deformation during the tensile test. The tensile test was video recorded by using a camera (Canon power shot G10) until the sample breaks. The photographs were then extracted from the video after every 1-2% of tensile strain for each sample. Then, the distances of the marks in the photographs were measured via a screen ruler for both the un-stretched state and stretched state. The engineering strains of the fabric structure in both tensile direction and transversal direction were then calculated based on the measured distances. The process was repeated for all three strips. Finally, Poisson's ratio \boldsymbol{v} was calculated using Eq.1 and 2 [1].

$$v_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} \tag{1}$$

$$v_{yx} = -\frac{\varepsilon_x}{\varepsilon_y} \tag{2}$$

Where v_{xy} is the Poisson's ratio, ε_x is the tensile strain and ε_y is the transverse strain when the fabric is stretched in the x-direction. Whereas, v_{yx} is the Poisson's ratio, ε_y is the tensile strain and ε_x is the transverse strain when the fabric is stretched in the y-direction. To confirm the auxetic behavior, the data obtained for tensile strain, transverse strain and Poisson's ratio when stretched along the warp and weft directions were used to generate tensile strain versus transverse strain and Poisson's ratio versus tensile strain curves.

4. **Results and discussion**

4.1. Re-entrant hexagonal geometry realization and NPR effect

Figure 5 shows the photos of a typical fabric sample (REH-B) in the relaxed state. The three sections of fabric unit cell are indicated in Figure 5(a). As assumed, upon relaxation, the three sections of fabric unit cell having different tightness of weave endure different levels of shrinkage and a non-uniform contraction/shrinkage profile is created within the unit cell. Section C endures the highest shrinkage which decreases the horizontal distance between

points 5 and 6. Correspondingly, due to shrinkage at sections A, the length 1-3 and 2-4 are decreased. Section B undergoes the least shrinkage and because of differential shrinkage of sections C and A, the fabric at the edges of section B is rucked up in the diagonal form on both sides of section C and the fabric at the center of section B remains flat. In this way, the edges of section B form an imitation of diagonal segments 1–5, 5–2, 3–6 and 6–4 and an approximation of re-entrant hexagonal geometry is realized, which is clearly shown in Figure 5(b). Moreover, at section C, the long floats of warp yarns are prominent. At section A the long floats of warp and weft yarns are also prominent on the face and back of the fabric, respectively. In addition, at sections C and A, the fabric is flat and thicker because of yarn swelling coming from shortening of yarn length which is instigated by the shrinkage. It was also observed that the warp yarns do not reside in the same positions at all three sections. Because of higher shrinkage at sections C and A, the warp yarns depart from the positions which they held at section B and become narrower.

When the fabric is stretched, the yarns tend to get straight and the shrinkage at sections A and C is transposed. This transposition of shrinkage makes the imitations of diagonal segments 1– 5, 5–2, 3–6 and 6–4 at section B to move towards the horizontal disposition and translate to the straight form. Hence, the distance between point 5 and 6 increases in transverse direction which increases the dimensions of the whole structure and the NPR effect is achieved. After that, the straightening of yarns in the tensile direction happens until slippage point is reached. Subsequently, the yarns in the stretch direction tend to come closer and the width of the fabric decreases, which results in a decrease of NPR behavior.

Figure 6(a) and 6(b) show the transverse strain versus tensile strain curves when stretched along warp and weft direction for fabric REH-A and fabric REH-B respectively. It can be seen that the transverse strain increases with increasing tensile strain for both fabrics. In both fabrics, the transverse strain increases upto 50% of tensile strain when stretched in weft

direction and the increase is sustained upto 100% of tensile strain. When stretched in warp direction, in case of fabric REH-A, the transverse strain increases upto 50% of tensile strain and the increase is sustained upto 100% of tensile strain whereas, in the case of fabric REH-B the transverse strain increases upto 80% of tensile strain then started decreasing. The Poisson's ratio versus tensile strain curves of two fabrics when stretched along warp and weft direction are presented in Figure 7(a) and 7(b), respectively. It can be seen that both the fabrics produce NPR effect when stretched in either weft or warp direction.

4.2. Effect of stretch direction and weft yarn arrangement on NPR

From Figure 7, it can be also found that the NPR effect first increases and reaches its highest level at smaller tensile strains and then reduces with the increase in tensile strain. This is because the transposition of the shrinkage at sections C and A starts at smaller tensile strain and increases with increase in tensile strain. This results in higher initial NPR which decreases with increasing tensile strain. Moreover, the NPR effect is higher and achieved at smaller tensile strains when fabric is stretched in warp direction. While when stretched in weft direction, the NPR effect is smaller and achieved at higher tensile strains. This is because in case of woven fabrics, the weft direction is always more extensible than warp direction and inherently there is always higher shrinkage along weft direction as compared to warp direction. In addition, the weft density for both fabrics is higher than warp density and the warp yarns acquire more interlacements with weft yarns, making the fabric stiffness increased. Therefore, the fabrics are stiffer and less extensible along warp direction as compared to weft direction. This is also evident from the tensile stress vs. tensile strain curves for the fabrics with two weft yarn arrangements as presented in Figure 8. Consequently, when the fabrics are stretched along warp direction, the transpose of shrinkage at sections C and A along transverse direction is achieved at smaller tensile strains, resulting in higher NPR. On the other hand, when the fabric is stretched along weft direction due to higher shrinkage and

extensibility along this direction, the transposition of shrinkage at sections C and A starts at larger tensile strain, resulting in smaller NPR effect.

Figure 9 compares the NPR effect produced by fabrics with two weft yarn arrangements. It is found that, between two types of weft yarn arrangements, the NPR effect with all elastic weft yarn arrangement is higher than that of alternate elastic and non-elastic weft yarn arrangement. This is because by using only elastic yarn arrangement, the shrinkage at sections C and A is increased upon relaxation. Therefore, higher decrease in the horizontal distance between points 5, 6 and in lengths 1-3 and 2-4 is achieved. When the fabric is stretched, the transposition of shrinkage at sections C and A along transverse direction is also higher which results in higher NPR effect.

It was also found that the shape realization of re-entrant hexagonal unit cell is prominently influenced by weft yarn arrangement. If alternate elastic and non-elastic yarns are used, the long floats of elastic yarn undergo higher shrinkage while the non-elastic yarns form loops at section C on the face of the fabric, resulting in unclear shape of unit cell. This problem is even more along weft direction due to higher shrinkage. Whereas, if only elastic yarns are used then because of higher shrinkage, the surface will appear more even and smoother and there will be no yarn loops on the fabric surface and the shape is very clear as shown in Figure 5 for fabric REH-B. In addition, the stick-slip effect or jumps are observed in the Poisson's ratio versus tensile strain curves as shown in Figure 7 and Figure 9. This is because the transpose of shrinkage and increase in transverse strain occurs in steps with an increase in tensile strain. The decrease in NPR is sustained up to the tensile strain value, where there is an increase in the transverse strain again, at which point the NPR increases again. This behavior is continued until the slippage point.

5. Conclusions

In this study, a new class of bi-stretch auxetic woven fabrics are designed and fabricated. An approximation of re-entrant hexagonal auxetic geometry is realized into bi-stretch woven fabric structure by employing an interlacement pattern with combination of tight and loose weaves and by using elastic and non-elastic yarns. The developed fabrics exhibit the NPR effect over a wide range of tensile strain, when stretched along either warp or weft direction. The following conclusions can be drawn from this study.

- The re-entrant hexagonal auxetic geometry can be realized into bi-stretch woven fabrics by creating non-uniform contraction or shrinkage profiles within the fabric structural unit cell. This phenomenon can be created by using combinations of elastic and non-elastic yarns and by placing loose weave and tight weave precisely across the unit cell of interlacement pattern.
- 2. By realizing re-entrant hexagonal geometry into bi-stretch woven fabrics, the NPR effect can be produced in both weft and warp directions. However, the NPR effect is higher when fabric is stretched in warp direction than that when stretched in weft direction.
- 3. The NPR effect can be enhanced by using all elastic yarns in weft direction.

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Figure 1. Re-entrant hexagonal geometry: (a) un-stretched state; (b) unit cell; (c) stretched

state



Figure 2. Re-entrant hexagonal geometry formation: (a) weave of section A; (b) weave of section B; (c) weave of section C; (d) arrangement of loose and tight weave within the unit cell of interlacement pattern



Figure 3. Weaving machine: (a) CCI Rapier weaving machine; (b) second beam assembly



Figure 4. Schematic illustration of testing setup for the auxetic fabric





hexagonal geometry



Figure 6. Transverse strain vs. tensile strain curve: (a) fabric REH-A; (b) fabric REH-B



Figure 7. Poisson's ratio vs. tensile strain curve: (a) fabric REH-A; (b) fabric REH-B



Figure 8. Tensile stress vs. tensile strain curve: (a) fabric REH-A; (b) fabric REH-B



Figure 9. Effect of weft yarn arrangement on NPR: (a) when stretched along warp direction;

(b) when stretched along weft direction

Fabric ID	REH-A	REH-B
Weft yarn arrangement	(1R,1L)	(L)
Warp yarn arrangement	1R,1L	

Table 1. Variation of fabric based on weft yarn arrangement

R = rigid yarn, L= elastic yarn, REH = Re-entrant hexagon