

The Shore Power Deployment Problem for Maritime Transportation

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Abstract

In this paper, we study a shore power deployment problem in a container shipping network. The aim of the problem is to develop a subsidy program for a government that achieves the utmost reduction of at-berth emissions from ships in the network. We formulate the problem as a mathematical model that captures the involved relationships among the government, container ports, and shipping lines. The model is hard to solve because it involves a multi-phase process that does not have a closed-form solution. To solve the problem, we develop a tailored labeling algorithm. Extensive numerical experiments are conducted, and the results demonstrate the applicability and efficiency of the solution method for solving practical instances. The results also demonstrate that the solutions delivered by our algorithm to the problem can significantly reduce the at-berth emissions from ships in the shipping network.

Keywords: Container Shipping, Green Shipping, Port Operations, Shore Power

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1. Introduction

Ports have long been the main gateways for global trade and are critical to economies around the world (Qu and Meng, 2012). However, they are major sources of ship pollution, cargo handling equipment emissions, and noise (McArthur and Osland, 2013; Wang et al., 2019). When berthing at ports, ships use their diesel auxiliary engines to generate electricity for hoteling, unloading, and loading activities, and they emit huge amounts of greenhouse gases (GHGs), sulfur dioxide (SO₂), nitrogen oxides (NO_x), particulate matter (PM), and other harmful pollutants (Merk, 2014). These emissions can cause adverse impacts on the environment (e.g., climate change, acid rain, water contamination) and contribute to significant health problems for local communities including premature mortality, cardiovascular and respiratory diseases (Sharma, 2006).

One effective measure to reduce at-berth emissions is to provide electricity to the ships from the shore-side national grid while turning off the ships' auxiliary diesel engines at ports, which can significantly reduce the emissions of air pollutants and GHGs. U.S. Environmental Protection Agency (2017) summarized 13 studies on the benefits of shore power (SP) at different ports in America, Europe, and Asia, and found that the reductions in emissions of CO₂ and pollutants (e.g., SO₂, NO_x, and PM) from ships at most ports were between 60% and 80%. To date, shore power facilities have been set up at more than 30 ports, mostly in Europe and North America. Many Asian ports are planning to deploy SP to curb their increasingly serious port emissions.

Several issues must be tackled before SP can be adopted. A major issue is the expensive retrofitting process—shore-side electricity supply infrastructure (SPI) is required at ports, and ships should be retrofitted with equipment that enables the connection to SP (SPE). In particular, it typically takes around 2 million US dollars to build up SPI at a port, and retrofitting a container ship with SPE can require 0.3 to 0.5 million US dollars' investment. Another fundamental and inherent issue is the “chicken and egg” dilemma (Winkel et al., 2016): ports do not install SPI until ships are SPE ready, while ships wait for SPI to be set up at ports prior to being retrofitted. This dilemma also exists in the investment in electric vehicle infrastructure (Zhang et al., 2019; Qu et al., 2020). Therefore, governmental subsidies play a critical role in promoting SP. In addition, the network effect in SP deployment further complicates the problem: as more ports build SPI, ships with SPE will have more opportunities to use SP and this will reduce the cost for shipping companies (for ships with SPE, using SP electricity when berthing at ports with SPI is cheaper

than generating electricity using the auxiliary engines); as a result, more ships will be retrofitted and then even more ports will install SPI (providing SP electricity to ships brings in extra revenues for a port) (Wang et al., 2015). This procedure will be repeated between ports and shipping lines, and considering the network effect is essential for the promotion of SP.

In practice, many governments provide subsidies to shipping companies and ports that are under their administration (including both state-owned and private shipping companies and ports). These subsidies relate to support for national flags, seafarer employment, the competitiveness of maritime clusters, promoting high quality standards and maintaining maritime connectivity (International Transport Forum, 2019). The types of maritime subsidies include direct subsidies, tax expenditures (e.g., tax exemption), and transfer of financial risk to governments (International Transport Forum, 2019). As estimated by International Transport Forum (2019), at least 3 billion EUR per year is spent on maritime subsidies in 36 countries included in the report. For example, in many large ports, government acts as owners or shareholders and governmental investments and subsidies play a key role in their operations and developments. Besides, in China, South Korea, and the U.S., governmental subsidies are provided to ships under the domestic flags or to state-owned shipping companies (International Transport Forum, 2019). Some European countries (including Italy, the United Kingdom, Sweden, and Norway) offered subsidies to shipping companies in order to reduce congestion and greenhouse gas emissions (International Transport Forum, 2019). In particular, the European Union (EU) has provided financial incentives to attract ships to be equipped with SPE. It also offers subsidies to ports to install SPI under the Marco Polo and Trans-European Transport Network programs (European Commission, 2019). In addition, the United Kingdom’s recently released Maritime 2050 strategy clearly states that the government is considering granting subsidies and investments to ports and ships to increase the uptake of SP (Department for Transport, 2019).

Container ships are the most polluting ships among all types of ships (Smith et al., 2014). This paper studies a Shore Power Deployment Problem (SPDP) in a container shipping network. The objective of the problem is to develop a subsidy program for a government that achieves the utmost reduction of at-berth emissions in the network. We formulate the problem as a mathematical model that captures the interaction between government and stakeholders (container ports and shipping lines) and the network effect in SP deployment. The model is difficult to solve because it involves a multi-phase process that does not have a closed-form solution. We prove that the problem is

NP-hard. To solve the problem, we develop a tailored labeling algorithm that takes advantage of the network effect in the problem. The great effectiveness of the algorithm is demonstrated through a series of numerical experiments.

Literature on container shipping studies can be divided into the stream of port operations (e.g., [Kim and Park, 2004](#); [Du et al., 2011](#); [Song et al., 2016](#)) and the stream of shipping operations (e.g., [Dong and Song, 2009](#); [Wang and Meng, 2012b](#); [Ng and Lin, 2018](#)). [Bierwirth and Meisel \(2015\)](#) and [Meng et al. \(2013\)](#) have provided excellent reviews of studies in port operations and shipping operations, respectively. Most SP-related studies have focused on the cost-benefit analysis of whether a port should install SPI (the health benefit from the reduction of emissions versus the installation cost of SPI) by assuming a fixed percentage of ships that visit the port will use SP. [Ballini and Bozzo \(2015\)](#) assumed 60% of all cruise ships visiting Copenhagen used SP and calculated that the total capital cost of establishing SPI in Copenhagen would be recovered by the health benefits in 12-13 years. [Zis et al. \(2016\)](#) analyzed the payback period for a ship to be retrofitted with SPE and found that the payback time depended on the price of fuel, the electricity price, and the time spent at ports. [Wang et al. \(2015\)](#) assumed that 40% of ships visiting the Port of Shenzhen (China) came from ports in Europe and North America and were already equipped with SPE, and they evaluated the potential emissions reduction if the port were to install SP infrastructure. [Vaishnav et al. \(2016\)](#) calculated ships and ports in the U.S. that should be switched to SP to maximize the social benefit. They assumed that port operators and ship owners act in a socially optimal manner. To the best of our knowledge, although governmental subsidization is key to the promotion of SP, there are no existing studies that aim at generating an SP-related subsidization plan for a government. In addition, no studies have considered the network effect in SP deployment in a quantitative manner. Our paper is the first study that considers the SPDP in a container shipping network. By solving this problem, we aim to formulate a subsidization plan for a government whose goal is to minimize the at-berth emissions from ships in the container shipping network. The network effect is also considered in the problem to ensure that the delivered result is applicable to real cases.

For better readability, we summarize notations used in this paper in the Table 1.

Table 1: Notations.

Indices:	
p	Index of ports.
r	Index of routes.
n	Index of phases in the network effect.
Sets:	
P :	Set of ports in the shipping network.
R :	Set of routes in the shipping network.
P_r :	Set of ports visited in route r .
R_p :	Set of routes that visit port p .
Parameters:	
h_{rp} :	The annual fuel cost of the auxiliary engine of ships on route r when at berth of port p , without using SP.
e_{rp} :	The annual cost of using shore electricity of ships on route r when at berth of port p if using SP.
u_{rp} :	The annual profit for port p for providing shore power to ships on route r .
B :	Budget of the government for subsidization.
C_p^1 :	Cost to set up SPI at port p .
C_r^2 :	Cost of retrofitting the ships on route r with SPE.
a :	The coefficient that converts the cost to set up SPI at a port into the annualized cost (aC_p^1 equals the annualized cost of setting up SPI at port p).
b :	The coefficient that converts the cost to retrofit a ship with SPE into the annualized cost (bC_r^2 equals the annualized cost of retrofitting the ships on route r with SPE).
N :	Number of phases after which the network effect reaches its equilibrium.
Variables:	
x_p :	= 1, if port p is subsidized for building up SPI; =0, otherwise.
y_r :	= 1, if ships on route r are subsidized for retrofitting with SPE; =0, otherwise.

- \hat{P}_n : Set of ports that have set up SPI in any of the phases 0, 1, ..., n of the network effect, where $n = 0, 1, \dots, N$. In particular, \hat{P}_0 represents the initial set of ports that have set up SPI (all are subsidized by the government).
- \hat{R}_n : Set of routes on which the ships are retrofitted with SPE in any of the phases 0, 1, ..., n of the network effect, where $n = 0, 1, \dots, N$. In particular, \hat{R}_0 represents the initial set of routes on which the ships have been equipped with SPE (all are subsidized by the government).

Auxiliary Variables:

- $\alpha_p(\hat{R})$: = 1, if port p finds setting up SPI using its own funding is profitable and thus does so; =0, otherwise, given a set of routes (\hat{R}) on which the ships are SPE-ready.
- $$\alpha_p(\hat{R}) = \begin{cases} 1 & \text{if } \sum_{r \in \hat{R}} u_{rp} \geq aC_p^1 \\ 0 & \text{if } \sum_{r \in \hat{R}} u_{rp} < aC_p^1. \end{cases}$$
- $\beta_r(\hat{P})$: = 1, if the shipping line operating route r finds that retrofitting ships on this route using its own funding reduces its operating cost and thus does so; =0, otherwise, given a set of ports (\hat{P}) that have set up SPI.
- $$\beta_r(\hat{P}) = \begin{cases} 1 & \text{if } \sum_{p \in P_r} h_{rp} \geq bC_r^2 + \sum_{p \in P_r \cap \hat{P}} e_{rp} + \sum_{p \in P_r \setminus \hat{P}} h_{rp} \\ 0 & \text{if } \sum_{p \in P_r} h_{rp} < bC_r^2 + \sum_{p \in P_r \cap \hat{P}} e_{rp} + \sum_{p \in P_r \setminus \hat{P}} h_{rp}. \end{cases}$$

94 In the following, we formally describe the considered problem in Section 2. We formulate
 95 the problem as a mathematical model in Section 3. The complexity of the SPDP is discussed
 96 in Section 4. The labeling algorithm for solving the problem is introduced in Section 5. The
 97 computational results are reported in Section 6. Finally, we conclude our main findings in Section 7.

98 2. Problem description

99 Consider a container shipping network. In the network, there are a set of container ports P
 100 that are managed by a government and a set of shipping routes R (operated by shipping lines

101 under the administration of the government) that call at the ports. The Shore Power Deployment
 102 Problem (SPDP) involves three types of players: the government, container port operators (ports),
 103 and shipping lines (ships), as shown in Figure 1. In order to reduce the at-berth emissions from
 104 the ships when they are calling at the ports, the government considers to subsidize a set $P' \subseteq P$
 105 of ports to set up SPI and subsidize ships that sail on a set $R' \subseteq R$ of routes to be retrofitted
 106 with SPE. Given an SP subsidization plan from the government, ports and shipping lines affect
 107 each other. In particular, given a set of ports that newly set up SPI in a route, shipping lines will
 108 retrofit their ships on this route with SPE if the cost of retrofitting is no larger than the benefits
 109 brought by it. Meanwhile, given a set of routes on which ships are newly retrofitted with SPE,
 110 ports contained in these routes will build up SPI if the revenue brought by selling SP electricity to
 111 ships with SPE outweighs the relevant cost.

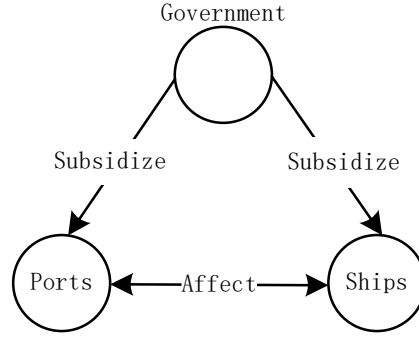


Figure 1: Three players in the SPDP.

112 Therefore, the SPDP is a two-stage optimization problem, in which the government decides
 113 its SP subsidization plan in the first stage and in the second stage, the ports and the shipping
 114 lines make decisions in reaction to (i) the subsidization plan from the government in the first
 115 stage and (ii) the network effect among ports and shipping lines in the second stage. Note that
 116 since the government aims at minimizing the at-berth emissions in the long run, when making the
 117 subsidization plan, it should also consider the second-stage decisions made by the ports and the
 118 shipping lines.

119 2.1. The government

120 The aim of the government is to reduce at-berth emissions from the ships in the shipping
 121 network by minimizing the amount of fuel consumption of the auxiliary engines when the ships

are berthing at the ports in the network. To this end, it decides to subsidize a set of ports to set up SPI and ships on a set of routes to be retrofitted with SPE. The budget of the government for subsidizing ports and ships is denoted by B .

In this study, there is only one government (e.g., China, the U.S., or EU) that provides subsidies to the ports and shipping lines in the shipping network. The ports and the shipping lines considered in the network are all managed by this government. Therefore, the subsidization plan can be generated in a coordinated fashion among all ports and shipping lines.

Our study can be directly applied in the scenarios where the routes connect ports that are all managed by one government. Such scenarios include the deployment of SP on the ships and ports on the shipping routes in the Yangtze River, China, the deployment of SP on the ships and ports on the shipping routes along China's or the U.S.'s coastline, and the deployment of SP on the ships and ports on the shipping routes within EU.

In addition, by carefully setting the problem parameters, this study can also be applied in scenarios where a shipping route visits ports managed by different governments. For instances, in a shipping route connecting China and the U.S., some ports are managed by China and the others are managed by the U.S. In this sense, the network considered in our problem does not necessarily contain all ports that are visited by the routes in practice, i.e., we only consider ports in the routes that are managed by one particular government. Also note that a shipping route may visit multiple ports in one country (e.g., China or the U.S.). We illustrate the method to handle such routes in the SPDP in Section 2.5.

2.2. Ports

The shipping network contains a set P of container ports. For each port $p \in P$, let R_p denote the set of routes that include it. Let C_p^1 where $p \in P$ be the one-time set-up cost of building up SPI at port p . Note that if p is subsidized by the government, then the set-up cost C_p^1 is afforded by the government, otherwise, the cost is paid by the port operators. The annualized cost of setting up shore power is aC_p^1 , where a is the coefficient that converts the set-up cost C_p^1 to the annualized cost. Note that $0 < a < 1$, because the SPI at a port lasts more than one year.

Ports with SPI obtain electricity from the national grid and provide electricity to berthing ships with SPE. Suppose that the SP electricity is provided to all ports with SPI at an identical unit price from the national grid, and that SP electricity is also sold at an identical unit price to

ships with SPE from these ports. Then a port makes profits due to the difference between the unit prices of purchasing and selling electricity. Given a route r , suppose ships sailing on it have been equipped with SPE. Then, the annual profit of the port p from providing SP electricity to ships on route r is denoted by u_{rp} ($u_{rp} = 0$ if the route does not include port p).

Let \hat{R} be the set of routes on which ships are retrofitted with SPE. Given \hat{R} , let $\alpha_p(\hat{R}) \in \{0, 1\}$ be equal to 1 if port p finds setting up SPI using its own funding is profitable and thus does so and zero otherwise. Then, $\alpha_p(\hat{R}) = 1$ if $\sum_{r \in \hat{R}} u_{rp} \geq aC_p^1$ and zero otherwise.

2.3. Shipping routes

A set R of routes is included in the container shipping network. Consider a shipping route $r \in R$ on which ships visit a set $P_r \in P$ of different ports $p_{r,1}, p_{r,2}, \dots, p_{r,|P_r|}$ and then return to $p_{r,1}$. The cost of retrofitting the ships on the route with SPE is C_r^2 . The retrofitting cost of ships on a route is paid by the government if the government decides to subsidize these ships. In comparison, the shipping line that operates a route pays the retrofitting cost of ships on the route, if the ships are not subsidized by the government. Note that the cost of retrofitting a ship is mainly decided by the characteristics of the ship itself (e.g., capacity of the ship). In this paper, we assume that the ships deployed on each shipping route are known and fixed. As a result, the cost of retrofitting all ships on a route is also known and fixed. Further, let bC_r^2 denote the annualized retrofitting cost, where b is the coefficient that converts the set-up cost C_r^2 to the annualized cost. Note that since the SPE on a ship can be used in more than a year, we set $0 < b < 1$.

To simplify the analysis, we assume that there is a fixed ratio between (1) the amount of SP electricity a ship with SPE uses when berthing at a port with SPI and (2) the fuel consumption of the same ship's auxiliary engine when it berths at the same port without using SP. It is also assumed that the ratios are identical for all ships visiting all ports in the network. In other words, we assume that the amount of electricity a ship consumes when at a port keeps unchanged no matter whether the electricity is provided by SP or by its auxiliary engine. This is reasonable because the berthing time of a ship at a port will not be affected by the adoption of SP. In addition, we assume that the fuel-to-electricity conversion rate is a constant for auxiliary engines in all ships. This is also a reasonable assumption since most ships use the same fuel (i.e., Marine Gas Oil, MGO) in their auxiliary engines when berthing at ports (Zis et al., 2016).

Supposing no ships on r are equipped with SPE, then the annual fuel cost of the auxiliary

engines of all ships on r when berthing at port $p \in P_r$ is h_{rp} . Alternatively, if all ships on r are equipped with SPE and port $p \in P_r$ has set up SPI, the annual cost of all ships on r using SP electricity when berthing at port p is e_{rp} . Suppose that $e_{rp} < h_{rp}$, $\forall r \in R, \forall p \in P$. Several aspects regarding the parameter settings here are worth mentioning. First, in this paper, we only consider two cases regarding the ships on a route: all ships on the route are equipped with SPE or none of the ships are equipped with SPE. This is because ships deployed on the same route are typically of the same type (Wang and Meng, 2012a; Ng, 2017). Hence, from the perspective of shipping lines, if it is preferable (or cost-effective due to the lower costs of using SP at ports) to retrofit one ship on a particular route, retrofitting other ships on the route will also be preferable. Therefore, treating all ships on a route as a whole will not lead to sub-optimal retrofitting decisions for shipping lines. Second, given a fixed fleet of ships on a shipping route r and the fixed amount of electricity each ship on r consumes at a port p , e_{rp} is decided by the SP electricity price, and h_{rp} is decided by the fuel price. In practice, $e_{rp} < h_{rp}$ holds in many ports, including the ports in many European countries (Transport Malta, 2014; Kanellakis, 2016; Gutierrez Saenz, 2019), ports in the U.S. (Vaishnav et al., 2016), and Port of Shenzhen in China (Peng, 2016). In addition, European Commission (2017) is also considering offering lower taxation rates on shore-supplied power.

Let \hat{P} be the set of ports with SPI. Given \hat{P} , let $\beta_r(\hat{P}) \in \{0, 1\}$ be equal to 1 if the shipping line operating route r finds that retrofitting ships on this route using its own funding reduces its operating cost and thus does so and zero otherwise. Then, $\beta_r(\hat{P}) = 1$ if $\sum_{p \in P_r} h_{rp} \geq bC_r^2 + \sum_{p \in P_r \cap \hat{P}} e_{rp} + \sum_{p \in P_r \setminus \hat{P}} h_{rp}$ and zero otherwise.

2.4. Network effect and its long-term equilibrium

Given a subsidization plan from the government, let \hat{P}_0 and \hat{R}_0 be the initial set of ports deployed with SPI and the initial set of routes on which the ships are retrofitted with SPE after the subsidization. The network effect can be described as follows. To begin with, because ships sailing on routes in \hat{R}_0 are equipped with SPE, some ports may find it profitable to set up SPI and providing electricity to ships that are SPE-ready (we only need to consider ports $p \in P \setminus \hat{P}_0$, since ports $p \in \hat{P}_0$ have already set up SPI and will not take any action). In particular, for each $p \in P \setminus \hat{P}_0$ if $\alpha_p(\hat{R}_0) = 1$, then port p will set up SPI (through its own investment). Define $\hat{P}_1 = \hat{P}_0 \cup \{p | p \in P \setminus \hat{P}_0, \alpha_p(\hat{R}_0) = 1\}$. Similarly, consider a route $r \in R \setminus \hat{R}_0$. The shipping line that operates the route may find that it is favorable to invest on retrofitting ships on r with SPE

(i.e., $\beta_r(\hat{P}_0) = 1$). Define $\hat{R}_1 = \hat{R}_0 \cup \{r | r \in R \setminus \hat{R}_0, \beta_r(\hat{P}_0) = 1\}$. To describe the evolvement of the network effect, we introduce the following definitions for *phases* and the *long-term equilibrium* in the network effect.

Definition 1. Phases. Given the initial SP deployment (i.e., the initial set of ports deployed with SPI and the initial set of routes on which the ships are retrofitted with SPE), then (1) all ports in the network without SPI will make their decisions regarding SPI establishment based on the initial set of routes on which the ships are retrofitted with SPE and (2) the operators (i.e., shipping lines) of all routes on which ships are not retrofitted with SPE will decide whether or not to retrofit their ships with SPE based on the initial set of ports with SPI. Given such decisions made by the ports and the shipping lines, a phase in the network effect refers to a status in which (1) the SPI has been established on all the ports that decide to set up SPI and (2) the SPE has been set up on the ships that are decided to be retrofitted. Note that the SP deployment in the current phase becomes the initial SP deployment for the next phase.

Definition 2. The long-term equilibrium. The long-term equilibrium of SP deployment in the network effect is a state in which no more ports have incentives to invest in setting up SPI and no more routes on which the shipping lines will be better off by retrofitting their ships with SPE. Note that given a subsidization plan from the government, its utmost emission reduction in the network is achieved at the equilibrium.

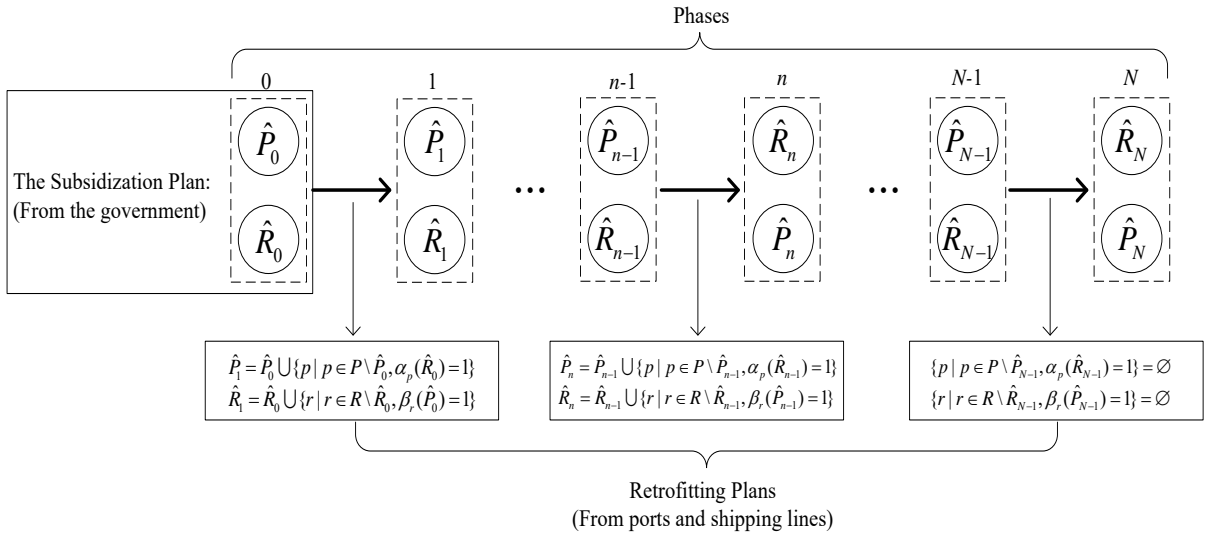


Figure 2: The evolvement of the network effect.

230 The evolvement process of the network effect is demonstrated in Figure 2. In particular, we
 231 define \hat{P}_0 and \hat{R}_0 as the result of the initial phase (Phase 0) which are decided by the government's
 232 subsidization plan. \hat{P}_1 and \hat{R}_1 as the result of the first phase (Phase 1) of the network effect. Now
 233 consider the following two cases. First, if $\hat{P}_1 = \hat{P}_0$ and $\hat{R}_1 = \hat{R}_0$, then no ships on routes $r \in R \setminus \hat{R}_1$
 234 will be retrofitted and no ports $p \in P \setminus \hat{P}_1$ will invest on SPI. Otherwise, we can generate \hat{P}_2
 235 and \hat{R}_2 , which are the result of the second phase (Phase 2) of the network effect. In particular,
 236 $\hat{P}_2 = \hat{P}_1 \cup \{p | p \in P \setminus \hat{P}_1, \alpha_p(\hat{R}_1) = 1\}$, and $\hat{R}_2 = \hat{R}_1 \cup \{r | r \in R \setminus \hat{R}_1, \beta_r(\hat{P}_1) = 1\}$.

237 This procedure repeats until the N -th ($N \geq 1$) phase (or Phase N) of the network effect such
 238 that:

$$\hat{P}_N = \hat{P}_{N-1}, \quad (1)$$

$$\hat{R}_N = \hat{R}_{N-1}, \quad (2)$$

240 where \hat{P}_N and \hat{R}_N are the sets of ports with SPI and routes on which ships are retrofitted with
 241 SPE at the N -th phase. We refer to the state in which these conditions hold as the long-term
 242 equilibrium of SP deployment in the network effect, and \hat{P}_N and \hat{R}_N are the sets of ports with SPI
 243 and routes on which ships are retrofitted with SPE in the equilibrium, respectively. Note that in
 244 the long-term equilibrium, SP devices are installed in the ports and on the ships if (1) they are
 245 subsidized to set up SP devices by the government or (2) they find that setting up SP devices is
 246 profitable (or cost-effective) due to the network effect.

247 **Lemma 1.** $\alpha_p(\hat{R}') \leq \alpha_p(\hat{R}'')$ if $\hat{R}' \subset \hat{R}''$.

248 *Proof.* Considering $\hat{R}' \subset \hat{R}''$, we have $\sum_{r \in \hat{R}'} u_{rp} \leq \sum_{r \in \hat{R}''} u_{rp}$. It can be readily seen that $\alpha_p(\hat{R}') \leq$
 249 $\alpha_p(\hat{R}'')$. \square

250 **Lemma 2.** $\beta_r(\hat{P}') \leq \beta_r(\hat{P}'')$ if $\hat{P}' \subset \hat{P}''$.

251 *Proof.* Let $D(\hat{P}) = \sum_{p \in P_r \cap \hat{P}} e_{rp} + \sum_{p \in P_r \setminus \hat{P}} h_{rp}$. It is sufficient to prove that $D(\hat{P}') \geq D(\hat{P}'')$. To
 252 see this, consider $D(\hat{P}') - D(\hat{P}'') = \sum_{p \in P_r \cap (\hat{P}' \setminus \hat{P}'')} e_{rp} - \sum_{p \in P_r \cap (\hat{P}'' \setminus \hat{P}')} e_{rp} + \sum_{p \in P_r \cap (\hat{P}'' \setminus \hat{P}')} h_{rp} -$
 253 $\sum_{p \in P_r \cap (\hat{P}' \setminus \hat{P}'')} h_{rp}$. Since $\hat{P}' \subset \hat{P}''$, we have $D(\hat{P}') - D(\hat{P}'') = \sum_{p \in P_r \cap (\hat{P}'' \setminus \hat{P}')} (h_{rp} - e_{rp}) \geq 0$. \square

254 **Proposition 1.** *Given any subsidization plan from the government, the equilibrium of the network*
 255 *can be obtained after at most $\min\{2|P|, 2|R|\}$ phases.*

256 *Proof.* Suppose the equilibrium of the network effect with $P \neq \emptyset$ and $R \neq \emptyset$ is reached after
 257 N phases. Let \hat{P}_0 and \hat{R}_0 denote the sets of ports with SPI and routes on which the ships are
 258 retrofitted, respectively, at 0-th (initial) phase. If at the first phase, we have $\hat{P}_1 = \hat{P}_0$ and $\hat{R}_1 = \hat{R}_0$,
 259 then the network effect has achieved its equilibrium (i.e., $N = 1$). Otherwise, suppose that at stage
 260 $\hat{n} \geq 2$, we have $\hat{P}_{\hat{n}} = \hat{P}_{\hat{n}-2}$. It is easy to infer that $\hat{P}_{\hat{n}} = \hat{P}_{\hat{n}-1} = \hat{P}_{\hat{n}-2}$. Then, from Lemma 2, we
 261 have $\hat{R}_{\hat{n}-1} = \hat{R}_{\hat{n}}$. Therefore, we have $\hat{P}_{\hat{n}} = \hat{P}_{\hat{n}-1}$ and $\hat{R}_{\hat{n}} = \hat{R}_{\hat{n}-1}$, which indicate $N = \hat{n}$. Similarly,
 262 suppose that at stage $\hat{n} \geq 2$, we have $\hat{R}_{\hat{n}} = \hat{R}_{\hat{n}-2}$. Then by following the same logic, we have
 263 $\hat{P}_{\hat{n}} = \hat{P}_{\hat{n}-1}$ and $\hat{R}_{\hat{n}} = \hat{R}_{\hat{n}-1}$, which indicate $N = \hat{n}$.

264 Consider the following cases:

- 265 **(I)** Suppose we have $\hat{P}_1 = \hat{P}_0$ and $\hat{R}_1 = \hat{R}_0$. Then, $N = 1$, and $N \leq \min\{2|P|, 2|R|\}$.
- 266 **(II)** Suppose we have $\hat{P}_2 = \hat{P}_1$ and $\hat{R}_2 = \hat{R}_1$. Then, $N = 2$, and $N \leq \min\{2|P|, 2|R|\}$.
- 267 **(III)** Let k be any positive integer such that $k \geq 2$, and suppose $N = 2k$ or $2k - 1$. Then, it
 268 is necessary to have (1) $\hat{P}_3 \supset \hat{P}_1$, $\hat{P}_5 \supset \hat{P}_3, \dots$, $\hat{P}_{2k-1} \supset \hat{P}_{2k-3}$, and (2) $\hat{R}_3 \supset \hat{R}_1$, $\hat{R}_5 \supset$
 269 \hat{R}_3, \dots , $\hat{R}_{2k-1} \supset \hat{R}_{2k-3}$, which are equivalent to (a) $|\hat{P}_3| - |\hat{P}_1| \geq 1$, $|\hat{P}_5| - |\hat{P}_3| \geq 1, \dots$,
 270 $|\hat{P}_{2k-1}| - |\hat{P}_{2k-3}| \geq 1$, and (b) $|\hat{R}_3| - |\hat{R}_1| \geq 1$, $|\hat{R}_5| - |\hat{R}_3| \geq 1, \dots$, $|\hat{R}_{2k-1}| - |\hat{R}_{2k-3}| \geq 1$.
 271 If $|\hat{P}_1| = 0$ or $|\hat{R}_1| = 0$, it is easy to infer that $N = 1$, therefore in this case, we have
 272 $|\hat{P}_1| \geq 1$ and $|\hat{R}_1| \geq 1$. Following this, we have $|\hat{P}_3| \geq 2$, $|\hat{R}_3| \geq 2, \dots$, $|\hat{P}_{2k-1}| \geq k$, and
 273 $|\hat{R}_{2k-1}| \geq k$. Considering $|\hat{P}_{2k-1}| \leq |P|$ and $|\hat{R}_{2k-1}| \leq |R|$, we have $k \leq \min\{|P|, |R|\}$.
 274 Therefore, $N \leq 2k \leq \min\{2|P|, 2|R|\}$.

275 Summarizing the above results gives us $N \leq \min\{2|P|, 2|R|\}$. □

276 2.5. Extensions

277 In this section, we show how to handle routes that visit ports managed by multiple governments
 278 in the SPDP. Suppose a government is considering subsidizing a set of ports P and ships on a set
 279 of shipping routes R to be retrofitted with SP devices. We refer to this government as the *target*
 280 *government*. Now consider a route $r \in R$ that visits a set of ports denoted by P_r^+ . Assume that
 281 P_r^+ is composed of ports managed by different governments (including this target government).
 282 Let \hat{P}_r^+ denote the set of ports in P_r^+ that have deployed SPI.

283 Our aim is to construct an artificial route denoted by r' to replace r in the SPDP. To this end,
 284 we first partition P_r^+ into two subsets: P_{r1}^+ and P_{r2}^+ . Particularly, $P_{r1}^+ = P_r^+ \cap P$ and $P_{r2}^+ = P_r^+ \setminus P$.

285 The parameters associated with r' are set as follows. First, let $P_{r'} = P_{r1}^+$. Second, let $e_{r'p} = e_{rp}$,
 286 $h_{r'p} = h_{rp}$, and $u_{r'p} = u_{rp}$ for each $p \in P_{r'}$ and $C_{r'}^2 = C_r^2$. Finally, given the set of ports in P that
 287 have deployed SPI (\hat{P}), we redefine the auxiliary variable $\beta_{r'}(\hat{P})$ associated with r' to be

$$\beta_{r'}(\hat{P}) = \begin{cases} 1 & \text{if } \sum_{p \in P_r^+} h_{rp} \geq bC_r^2 + \sum_{p \in P_r^+ \cap (\hat{P} \cup \hat{P}_r^+)} e_{rp} + \sum_{p \in P_r^+ \setminus (\hat{P} \cup \hat{P}_r^+)} h_{rp} \\ 0 & \text{if } \sum_{p \in P_r^+} h_{rp} < bC_r^2 + \sum_{p \in P_r \cap (\hat{P} \cup \hat{P}_r^+)} e_{rp} + \sum_{p \in P_r \setminus (\hat{P} \cup \hat{P}_r^+)} h_{rp}. \end{cases}$$

288 The parameters of r' satisfy three properties that allow the correct incorporation of r into the
 289 SPDP (which is replaced by r' in the problem). First, the target government manages all ports on
 290 route r' . Second, the cost to retrofit ships on r' and once they are retrofitted, the revenue generated
 291 to each port $p \in P_r^+ \cap P$ remain unchanged when compared with the “real” scenario. Third, the
 292 new auxiliary variable $\beta_{r'}(\hat{P})$ ensures that the ships on r' will be retrofitted by a shipping line
 293 (with or without governmental subsidies) if and only if the shipping line that operates the “real”
 294 route r finds that retrofitting ships on route r using its own funding reduces its operating cost.

295 3. Model formulation

296 In this section, we formulate the SPDP as a mathematical model. The model is difficult to
 297 solve. This is because, in this model, we have to depict the network effect in SP deployment, which
 298 is, in essence, a multi-phase process without a closed-form solution.

299 In the problem, we assume that there are no ports that have SPI and no routes on which the
 300 ships are retrofitted with SPE, before the government makes its subsidization decision. Note that
 301 our model and the solution method proposed in the following section can also be used to solve the
 302 SPDP in which this assumption does not hold, after small adaptations.

303 Let x_p ($p \in P$) be the decision variable which is equal to 1, if port p is subsidized by the
 304 government. Let y_r ($r \in R$) be the decision variable which is equal to 1, if ships on route r
 305 are subsidized by the government. Let $N = \min\{2|P|, 2|R|\}$. According to Proposition 1, the
 306 equilibrium of the network effect can be obtained after at most N phases (i.e., at N -th phase). Let
 307 \hat{P}_n , $n = 0, 1, \dots, N$ be the decision variable that represents the set of ports that set up SPI in any
 308 of the phases $0, 1, \dots, n$ of the network effect, and let \hat{R}_n , $n = 0, 1, \dots, N$ be the decision variable

that represents the set of routes on which the ships are retrofitted with SPE in any of the phases
0, 1, ..., n of the network effect. The SPDP can be formulated as the following model.

$$(M1) \max Z = \sum_{r \in \hat{R}_N} \sum_{p \in P_r \cap \hat{P}_N} e_{rp}, \quad (3)$$

subject to:

$$\sum_{p \in P} C_p^1 x_p + \sum_{r \in R} C_r^2 y_r \leq B, \quad (4)$$

$$x_p \in \{0, 1\} \quad \forall p \in P, \quad (5)$$

$$y_r \in \{0, 1\} \quad \forall r \in R, \quad (6)$$

$$\hat{P}_0 = \{p | x_p = 1, p \in P\}, \quad (7)$$

$$\hat{R}_0 = \{r | y_r = 1, r \in R\}, \quad (8)$$

$$\hat{P}_{n+1} = \hat{P}_n \cup \{p | \alpha_p(\hat{R}_n) = 1, p \in P \setminus \hat{P}_n\}, \quad \forall n \in \{0, 1, \dots, N-1\}, \quad (9)$$

$$\hat{R}_{n+1} = \hat{R}_n \cup \{r | \beta_r(\hat{P}_n) = 1, r \in R \setminus \hat{R}_n\}, \quad \forall n \in \{0, 1, \dots, N-1\}, \quad (10)$$

$$\hat{P}_N = \hat{P}_{N-1}, \quad (11)$$

$$\hat{R}_N = \hat{R}_{N-1}. \quad (12)$$

The objective function (3) maximizes the total cost of ships for using SP electricity in the shipping network in a year. As described in Section 2, since the SP electricity is provided to all ships at an identical unit price at all ports, maximizing the cost of using SP electricity is equivalent to maximizing the usage amount of SP electricity. Further, there is a fixed ratio between the usage

amount of SP electricity and the amount of fuel consumption of the auxiliary engines for all ship
berthing at all ports in the network. Therefore, maximizing the usage amount of SP electricity is
equivalent to minimizing the at-berth fuel consumption from the ships. Constraint (4) ensures the
total expenditure of the subsidization does not exceed the budget. Constraints (5) and (6) require
the decision variables x_p 's and y_r 's to be binary. Constraints (7)–(12) formulate the network effect
of SP deployment. In particular, Constraints (7) and (8) calculate the initial set of ports with
SPI and the initial set of routes on which the ships are retrofitted with SPE, respectively. The
relationship between two consecutive phases is depicted in Constraints (9) and (10). Finally, the
equilibrium result of the network effect is given in Constraints (11) and (12).

4. Complexity of the problem

In this section, we show that the SPDP is NP-hard. To do so, we show the decision version of
the SPDP is NP-hard. That is, given a set of ports and a set of shipping routes and all parameters
 $B, u_{rp}, h_{rp}, e_{rp}, C_p^1, C_r^2, a$, and b , it cannot be determined in polynomial time whether the objective
value Z of the problem is no smaller than a given constant Γ unless P=NP.

We prove the NP-hardness of the SPDP by reducing a well-known NP-hard problem—the
Knapsack Problem—to a decision version of the SPDP.

Theorem 1. *The SPDP is NP-hard.*

Proof. We transform the Knapsack Problem to the decision version of the SPDP. The Knapsack
Problem can be stated as follows. There is a set I of given items to be packed in a knapsack of
capacity K . Each item p has a profit f_p and a weight w_p . The problem asks whether there exists
a packing method such that a subset of items whose total weight does not exceed K and whose
total profit is no less than a constant \underline{F} are packed in the knapsack.

Given an arbitrary instance of the Knapsack Problem, we construct a corresponding instance
of the SPDP. In the instance, there is only one route r ($|R| = 1$) that contains all ports in P (i.e.,
 $P_r = P$). Specifically, we set other parameters in the problem as follows.

$$B = K, \tag{13}$$

350

$$P = I, \quad (14)$$

351

$$C_p^1 = w_p, \quad \forall p \in P, \quad (15)$$

352

$$C_r^2 = 0, \quad (16)$$

353

$$e_{rp} = f_p, \quad \forall p \in P, \quad (17)$$

354

$$aC_p^1 > u_{rp}, \quad \forall p \in P, \quad (18)$$

355

$$\sum_{p \in P_r} h_{rp} < bC_r^2 + \sum_{p \in P_r} e_{rp}, \quad (19)$$

356

$$\Gamma = \underline{F}. \quad (20)$$

357 Clearly, this transformation can be completed in polynomial time. We will show that there
 358 exists a feasible solution to the constructed instance of SPDP if and only if the answer to the
 359 Knapsack Problem is “yes”.

360 Suppose the answer to the Knapsack Problem is “yes”. Let I^* denote the items selected to
 361 be packed in the knapsack. Clearly, we have (i) $\sum_{p \in I^*} w_p \leq K$, and (ii) $\sum_{p \in I^*} f_p \geq \underline{F}$. Then
 362 consider the following solution (\mathbb{S}) to the constructed instance of the SPDP. In \mathbb{S} , the government
 363 subsidizes ships on route r . Then, corresponding to each $p \in I^*$, the government subsidizes port
 364 p to set up SPI. The feasibility of \mathbb{S} to the SPDP instance can be verified as follows. First,
 365 given Equation (16), the total cost of this subsidization plan equals $\sum_{p \in I^*} C_p^1 = \sum_{p \in I^*} w_p \leq K$.
 366 Considering $B = K$, the total cost does not exceed the budget. Second, let $\hat{P}_0 = I^*$ and $\hat{R}_0 = R$ be
 367 the initial set of ports with SPI and routes on which the ships are retrofitted with SPE. Considering
 368 the network effect among ports and routes, suppose that the equilibrium of the network effect is
 369 obtained after N phases. Let \hat{P}_N and \hat{R}_N be the set of ports with SPI and the set of routes on

370 which the ships are retrofitted in the equilibrium. We have $I^* \subseteq \hat{P}_N$ and $R = \hat{R}_N$. Further,
 371 $Z = \sum_{r \in \hat{R}_N} \sum_{p \in P_r \cap \hat{P}_N} e_{rp}$. Hence, we have $Z \geq \sum_{p \in P_r \cap I^*} e_{rp}$. Since $P_r = P$ and $e_{rp} = f_p$, we
 372 have $Z \geq \sum_{p \in I^*} f_p = \underline{F}$. Therefore, \mathbb{S} is feasible to the constructed instance of the SPDP.

373 Conversely, suppose that there exists a feasible solution to the constructed instance of the
 374 SPDP such that $Z \geq \Gamma$. Let P^* denote the set of ports that are subsidized by the government to
 375 set up SPI. Since $C_r^2 = 0$, we have $\sum_{p \in P^*} C_p^1 \leq B$, which is equivalent to:

$$\sum_{p \in P^*} w_p \leq K. \quad (21)$$

376 Let $\hat{P}_0 = P^*$ and \hat{R}_0 be the set of ports subsidized by the government to set up SPI and the set of
 377 routes on which the ships are subsidized to be retrofitted with SPE, respectively. Suppose that the
 378 equilibrium of the network effect is obtained after N phases. Let \hat{P}_N and \hat{R}_N be the set of ports
 379 with SPI and the set of routes on which the ships are retrofitted in the equilibrium. Considering
 380 $Z \geq \Gamma$, we have $\sum_{r \in \hat{R}_N} \sum_{p \in \hat{P}_N \cap P_r} e_{rp} \geq \underline{F}$. Supposing $\underline{F} > 0$ (the case such that $\underline{F} \leq 0$ is trivial),
 381 it is easy to infer that $\hat{R}_N = R$. Because $P_r = P$, and $f_p = e_{rp}$, we have:

$$\sum_{p \in \hat{P}_N} f_p \geq \underline{F}, \quad (22)$$

382 Further, given (18) and (19), it is easy to infer that $\hat{R}_N = \hat{R}_0 = R$ and that $\hat{P}_N = \hat{P}_0 = P^*$, which
 383 imply that (22) is equivalent to:

$$\sum_{p \in P^*} f_p \geq \underline{F}. \quad (23)$$

384 Therefore, given (21) and (23), we can construct a feasible solution to the Knapsack Problem by
 385 packing items $p \in P^*$ into the knapsack. This completes the proof. \square

386 **Remark 1.** *In the proof of Theorem 1, the constructed instance of the SPDP has only one shipping*
 387 *route and only the route and the ports that are included in the subsidization plan will set up SPI.*
 388 *Therefore, the SPDP is NP-hard even if there is only one shipping route and the network effect*
 389 *between the ports and routes is not considered.*

5. Solution method

In this section, we propose the solution method for the SPDP. We solve the problem by using a labeling algorithm, in which all feasible subsidization plans from the government are considered implicitly and the optimal deployment plan is generated dynamically. For the ease of exposition, we introduce the following notations. We define $\Psi := P \cup R$, and define $\bar{C}_i = \begin{cases} C_i^1, & \text{if } i \in P, \\ C_i^2, & \text{if } i \in R, \end{cases}$ for each $i \in \Psi$. In what follows, we first introduce the method to derive the long-term equilibrium of the network effect given a set of ports with SPI and a set of routes on which the ships are retrofitted with SPE in Section 5.1. The procedures of the labeling algorithm are presented in Section 5.2. We propose several dominance rules for the algorithm in Section 5.3.

5.1. Deriving the long-term equilibrium

Let \hat{P}_0 and \hat{R}_0 denote the initial set of ports with SPI and the initial set of routes on which the ships are retrofitted, respectively. For example, in a subsidization plan, \hat{P}_0 is the set of ports that are subsidized to set up SPI and \hat{R}_0 is the set of routes on which the ships are retrofitted with SPE. Note that $\sum_{p \in \hat{P}_0} C_p^1 + \sum_{r \in \hat{R}_0} C_r^2 \leq B$, if \hat{P}_0 and \hat{R}_0 are the set of ports subsidized by the government and the set of routes on which ships are subsidized by the government, respectively. Also, note that subsidizations from the government are only provided to the ports and ships in the initial phase (Phase 0) of the entire network effect. Let $\tilde{\Omega} = \hat{P}_0 \cup \hat{R}_0$. Then, procedure $\mathbf{F}(\tilde{\Omega})$ which is shown in Algorithm 1 finds the long-term equilibrium SP deployment denoted by $\hat{\Omega}$. Here, $\hat{\Omega} = \hat{P} \cup \hat{R}$, where \hat{P} is the set of ports with SPI and \hat{R} is the set of routes on which the ships are retrofitted with SPE in the long-term equilibrium.

5.2. Procedures of the labeling algorithm

To begin with, to define a label, we introduce some notations related to a partial subsidization plan. Given a partial deployment plan, let \hat{P}_0 and \hat{R}_0 respectively denote the set of ports that are subsidized and the set of routes on which the ships are subsidized in the plan, and let $\bar{\Psi} := \hat{P}_0 \cup \hat{R}_0$. In addition, let \bar{B} denote the remaining budget, i.e., $\bar{B} = B - \sum_{i \in \bar{\Psi}} \bar{C}_i$. Further, let \hat{P} and \hat{R} denote the set of ports with SPI and the set of routes on which the ships are retrofitted in the long-term equilibrium, respectively. It is easy to infer that $\hat{P}_0 \subseteq \hat{P}$, and $\hat{R}_0 \subseteq \hat{R}$. Finally, define $\hat{\Omega} = \hat{P} \cup \hat{R}$. In the algorithm, a label $L = (\bar{\Psi}, \bar{B}, \hat{\Omega})$ is associated with a partial deployment plan such that (1)

Algorithm 1 The long-term SP deployment equilibrium calculation procedure ($\mathbf{F}(\tilde{\Omega})$).

Input: $\tilde{\Omega}$;

Output: $\hat{\Omega}$;

```

1:  $\hat{P} = P \cap \tilde{\Omega}$ ,  $\hat{R} = R \cap \tilde{\Omega}$ ;
2: while True do
3:    $\hat{P}' = \hat{P}$ ,  $\hat{R}' = \hat{R}$ ;
4:   for  $r \in R \setminus \hat{R}$  do
5:     if  $\beta_r(\hat{P}) = 1$  then
6:        $\hat{R} = \hat{R} \cup \{r\}$ ;
7:     end if
8:   end for
9:   for  $p \in P \setminus \hat{P}$  do
10:    if  $\alpha_p(\hat{R}) = 1$  then
11:       $\hat{P} = \hat{P} \cup \{p\}$ ;
12:    end if
13:   end for
14:   if  $\hat{R} = \hat{R}'$  &  $\hat{P} = \hat{P}'$  then
15:      $\hat{\Omega} = \hat{P} \cup \hat{R}$ ;
16:     Return.
17:   end if
18: end while

```

the ports and routes in $\bar{\Psi}$ are subsidized to deploy SPI or the ships on which are retrofitted, (2) the remaining budget is \bar{B} , and (3) the set $\hat{\Omega}$ of ports and routes set up SPI or the ships on which are retrofitted in the long-term equilibrium.

The algorithm starts from an initial label $L_0 = (\emptyset, B, \emptyset)$. The extension of a label $L = (\bar{\Psi}, \bar{B}, \hat{\Omega})$ is as follows. First, we define a candidate pool denoted by Φ for extending L as $\Phi = \{i | \bar{C}_i \leq \bar{B}, i \in \Psi \setminus \hat{\Omega}\}$. Then, for each $i \in \Phi$, we extend L to a new label $L' = (\bar{\Psi}', \bar{B}', \hat{\Omega}')$, where $\bar{\Psi}' = \bar{\Psi} \cup \{i\}$, $\bar{B}' = \bar{B} - \bar{C}_i$, and $\hat{\Omega}' = \mathbf{F}(\hat{\Omega} \cup \{i\})$.

A label $L = (\bar{\Psi}, \bar{B}, \hat{\Omega})$ is terminated if its candidate pool $\Phi = \emptyset$. For a terminated label, we calculate $Z(L)$ which is the usage amount of SP electricity generated by the subsidization plan ($\bar{\Psi}$) using the following equation:

$$Z(L) = \sum_{r \in \hat{R}} \sum_{p \in \hat{P} \cap P_r} e_{rp}, \quad (24)$$

where $\hat{R} = R \cap \hat{\Omega}$, and $\hat{P} = P \cap \hat{\Omega}$.

Let $\bar{\Psi}^*$ denote the incumbent optimal subsidization plan found by the algorithm and let Z^* be the usage amount of SP electricity generated by $\bar{\Psi}^*$. Then, if $Z(L) > Z^*$, we update $\bar{\Psi}^* = \bar{\Psi}$, and $Z^* = Z(L)$.

5.3. Dominance rules

The performance of a labeling algorithm heavily relies on the efficiency of the dominance rules, which allow one to discard a significant number of labels. For the labeling algorithm, we propose the following dominance rules.

Proposition 2. *Label $L_1 = (\bar{\Psi}_1, \bar{B}_1, \hat{\Omega}_1)$ dominates Label $L_2 = (\bar{\Psi}_2, \bar{B}_2, \hat{\Omega}_2)$ if (1) $\hat{\Omega}_1 \supseteq \hat{\Omega}_2$, and (2) $\bar{B}_1 \geq \bar{B}_2$.*

Proof. Considering (1) and (2), it is easy to infer that for any extension $L'_2 = (\bar{\Psi}'_2, \bar{B}'_2, \hat{\Omega}'_2)$ from L_2 , there exists an extension of L_1 , denoted by $L'_1 = (\bar{\Psi}'_1, \bar{B}'_1, \hat{\Omega}'_1)$ such that $\hat{\Omega}'_1 \supseteq \hat{\Omega}'_2$. Therefore, we have $Z(L'_1) \geq Z(L'_2)$. This indicates that for any subsidization plan $\bar{\Psi}'_2$ generated from extensions of L_2 , there exist some subsidization plans generated from extensions of L_1 that are no worse than $\bar{\Psi}'_2$. This completes the proof. \square

Proposition 3. *Label $L_1 = (\bar{\Psi}_1, \bar{B}_1, \hat{\Omega}_1)$ dominates Label $L_2 = (\bar{\Psi}_2, \bar{B}_2, \hat{\Omega}_2)$ if (1) $\hat{\Omega}_1 \supseteq \hat{\Omega}_2$ and (2) $\Phi_2 \setminus \hat{\Omega}_1 = \emptyset$, where Φ_2 is the candidate pool for extending L_2 .*

Proof. Given (1) and (2), we have that for any extension $L'_2 = (\bar{\Psi}'_2, \bar{B}'_2, \hat{\Omega}'_2)$ from L_2 , $\hat{\Omega}'_2 \subseteq \hat{\Omega}_1$ (i.e., $\bar{\Psi}_2$ cannot be extended to include any $i \notin \hat{\Omega}_1$). It follows that the $Z(L_1) \geq Z(L'_2)$. Therefore, subsidization plan $\bar{\Psi}_1$ is no worse than the subsidization plan generated from any extension from L_2 . \square

Proposition 4. *Label $L_1 = (\bar{\Psi}_1, \bar{B}_1, \hat{\Omega}_1)$ dominates Label $L_2 = (\bar{\Psi}_2, \bar{B}_2, \hat{\Omega}_2)$ if (1) $\hat{\Omega}_1 \supseteq \hat{\Omega}_2$, and (2) $\bar{B}_1 \geq \max_{i \in \Phi_2 \setminus \hat{\Omega}_1} \bar{C}_i$ hold, and (4) or (5) holds:*

$$(4) |\Phi_2 \setminus \hat{\Omega}_1| \geq 2, \text{ and } \bar{B}_2 < \min_{i \in \Phi_2 \setminus \hat{\Omega}_1} \bar{C}_i + \min[2]_{i \in \Phi_2 \setminus \hat{\Omega}_1} \bar{C}_i;$$

$$(5) |\Phi_2 \setminus \hat{\Omega}_1| = 1.$$

Here, Φ_2 is the candidate pool for extending L_2 , and $\min[2]_{i \in \Phi_2 \setminus \hat{\Omega}_1} \bar{C}_i$ returns the second smallest \bar{C}_i from $\Phi_2 \setminus \hat{\Omega}_1$.

Proof. Suppose $L'_2 = (\bar{\Psi}'_2, \bar{B}'_2, \hat{\Omega}'_2)$ is extended from L_2 . Since (4) or (5) holds, for any L'_2 , we have $|\bar{\Psi}'_2 \setminus \hat{\Omega}_1| \leq 1$.

First consider the case $|\bar{\Psi}'_2 \setminus \hat{\Omega}_1| < 1$, that is $\bar{\Psi}'_2 \setminus \hat{\Omega}_1 = \emptyset$. Then according to Proposition 3, L'_2 is dominated by L_1 .

Then, consider the case $|\bar{\Psi}'_2 \setminus \hat{\Omega}_1| = 1$. Let $\{I\} = \bar{\Psi}'_2 \setminus \hat{\Omega}_1$. Considering (2), we have $\bar{C}_I \leq \bar{B}_1$. This indicates that there exists a label $L'_1 = (\bar{\Psi}'_1, \bar{B}'_1, \hat{\Omega}'_1)$ that is extended from L_1 such that $\bar{\Psi}'_1 = \bar{\Psi}_1 \cup \{I\}$. From (1), it is easy to infer that $\hat{\Omega}'_1 \supseteq \hat{\Omega}'_2$. Therefore, for any subsidization plan $\bar{\Psi}'_2$ generated from extensions of L_2 , there exist some subsidization plans generated from extensions of L_1 that are no worse than $\bar{\Psi}'_2$. This completes the proof. \square

6. Numerical experiments

In this section, we perform a series of computational experiments to verify the effectiveness of our proposed model and solution method. All the experiments are coded in C++ and are conducted on an Intel Core i7 2.20 GHz PC with 32 GB RAM.

To test the performance of our algorithm, we first generate 12 instances in terms of different input parameters. In particular, the number of ports $|P|$ is selected from $\{20, 30, 40\}$ and the number of shipping routes $|R|$ is selected from $\{20, 30, 40, 50\}$. We set the other parameters as follows. The parameters are set using the data provided by Papoutsoglou (2012) and Wang et al. (2015). First, we randomly generate C_p^1 from the uniform distribution $U(1.5, 2.5)$ (million dollars). Second, for each shipping route r , the cost for retrofitting a ship on this route with SPE (denoted by \dot{c}_r) is generated from $U(0.3, 0.5)$ (million dollars). We assume that each $r \in R$ provides a weekly service (i.e., ships on the route call at each port $p \in P_r$ once a week). It is also assumed that all ships on r are identical. The number of ships deployed in a route (denoted by η_r which is an integer) is randomly generated from $[1, 8]$. Hence, for each route r , C_r^2 is set equal to $\dot{c}_r \eta_r$. In addition, we randomly generate $|P_r|$ (an integer) from $[2, 6]$, and the ports in P_r are randomly selected from P . Third, we set $a = 0.025$ and $b = 0.03$. Fourth, considering the weekly service routine, the h_{rp} is set to be $52\lambda_{rp}$, where 52 represent the number of weeks in a year and λ_{rp} (which is the cost of at-berth fuel consumption of one ship on route r when berthing at port p) is randomly generated using $U(3000, 5000)$ (dollars). Then we randomly generate $e_{rp} = \alpha h_{rp}$ and $u_{rp} = \beta h_{rp}$, where α and β are randomly generated from $U(0.7, 0.8)$ and $U(0.1, 0.15)$, respectively. Finally, B is randomly generated using $U(0.05D, 0.1D)$, where $D = \sum_{p \in P} C_p^1 + \sum_{r \in R} C_r^2$.

The computational results are demonstrated in Table 2. Columns 1 and 2 report the number of ports and the number of routes in an instance, respectively. We report the computational time for solving an instance in Column 3. Column 4 presents the number of ports subsidized by the

government to set up SPI ($|\hat{P}_0^*|$). Column 5 presents the number of routes on which the ships are subsidized to be retrofitted with SPE ($|\hat{R}_0^*|$). The number of ports with SPI and the number of routes on which the ships are retrofitted with SPE in the long-term equilibrium ($|\hat{P}_N^*|$ and $|\hat{R}_N^*|$) are reported in Columns 6 and 7, respectively. In Column 8, we report the amount of at-berth fuel consumption at all ports from all ships sailing on routes in R without using SP in a year, which is calculated by $OBK = \sum_{r \in R} \sum_{p \in P_r} h_{rp}/q$, where q is the unit cost (dollars per ton) of bunkers. Column 9 reports the similar value after the SP deployment among ports and ships has achieved its equilibrium, which is calculated by $EBK = OBK - \sum_{r \in \hat{R}_N^*} \sum_{p \in P_r \cap \hat{P}_N^*} h_{rp}/q$, where \hat{P}_N^* is the set of ports that have set up SPI and \hat{R}_N^* is the set of routes on which the ships have equipped with SPE in the equilibrium obtained from the optimal subsidization plan delivered by the algorithm. In the experiments, we set $q = 700$ (dollars per ton). Finally, the last column reports fuel consumption reduction (in percentage) between OBK and EBK, which equals $\frac{OBK-EBK}{OBK} \cdot 100$.

Table 2: Computational results.

$ P $	$ R $	Time ¹	$ \hat{P}_0^* $	$ \hat{R}_0^* $	$ \hat{P}_N^* $	$ \hat{R}_N^* $	OBK ²	EBK ²	Reduction (%)
20	20	< 1	3	1	19	19	23900	872	96.35
20	30	< 1	1	0	20	30	35882	0	100.00
20	40	< 1	1	0	20	40	48381	0	100.00
20	50	< 1	1	0	20	50	57457	0	100.00
30	20	< 1	2	3	18	19	20593	3589	82.57
30	30	< 1	4	1	28	30	36665	1218	96.68
30	40	< 1	3	0	30	40	49530	0	100.00
30	50	< 1	1	0	30	50	70074	0	100.00
40	20	2016	4	1	17	15	24468	9077	62.90
40	30	809	4	2	30	29	36823	2980	91.91
40	40	< 1	5	1	37	40	48403	583	98.79
40	50	2	2	2	35	49	53777	2708	94.96

Note¹. In seconds.

Note². In tons.

We can see from Table 2 that the labeling algorithm efficiently solves all instances (with practical sizes). Besides, the results indicate that promoting SP usage among ports and ships generates tremendous environmental and health benefits. In particular, after SP is adopted, the average amount of at-berth fuel consumption in the instances reduces by 93.86%, and in some instances, the reduction percentages reach 100%. The results also verify the network effect among ports and shipping routes. In particular, the average ratio of between $|\hat{P}_N^*| + |\hat{R}_N^*|$ (i.e., the total number of

ports with SPI and routes on which ships are equipped with SPE in the equilibrium) and $|\hat{P}_0^*| + |\hat{R}_0^*|$ (i.e., the total number of subsidized ports and routes on which ships are subsidized) is 30.16. The network effect also explains why the solution times of the two largest instances (i.e., the instances with 40 ports and 40 or 50 routes) are shorter than those of the instances with 40 ports and 20 or 30 routes. As a matter of fact, when the container shipping network is denser (i.e., ports are linked by more routes), setting up SPI at particular ports or retrofitting ships on particular routes have higher chances to drive more ports to build up SPI and ships on more routes to deploy SPE, and this reduces the searching space in the labeling algorithm.

7. Conclusion

In this paper, we analyzed a Shore Power Deployment Problem that aims at generating a subsidization plan for a government whose goal is to minimize at-berth emissions from ships in a container shipping network. The problem was formulated in a framework that captures the complex relationship between the government, ports, and ships. We showed that the problem is NP-hard. For solving the problem, a tailored labeling algorithm was proposed. We conducted extensive numerical experiments to test the performance of the algorithm. The results demonstrated that the proposed algorithm can efficiently solve problems with practical sizes and that the delivered subsidization plans generate great environmental and health benefits.

Ports are the key nodes in a global supply chain. However, they are also a major source of various pollutants. SP provides a potential cure to the adverse environmental impacts of ports. Due to the huge infrastructure set-up costs, governmental subsidization plays a vital role in SP deployment. We have shown both in theory and by the numerical experience that by utilizing the network effect in the SP deployment, the subsidization plan of a government can lead to a significant reduction in at-berth fuel consumption in a shipping network.

Both port-based and ship-based SP devices are expensive and the budget for subsidizing SP deployment from a government is limited. Therefore, identifying the optimal subsidization plan is critical for a government. However, such a problem is very difficult to solve. In this work, we provide a solution method for this important yet challenging problem. In particular, by subsidizing a small proportion of ports and ships to be retrofitted with SP devices, many more ports and ships will be voluntary to set up SPI and SPE in the long-term equilibrium. Considering the huge

environmental benefits brought by SP, our study provides theoretical support for governments to devise effective subsidization plans for promoting SP.

The model and algorithm proposed in this work may also provide references to other infrastructure deployment problems in which governmental subsidization is required and the network effect should be considered. Examples include the deployment of charging stations for electronic vehicles in a national road traffic network and the deployment of base stations in a communication network.

There are some potential directions for extending the current study. First, in this study, we only consider the subsidization policy in which the government should decide whether all ships on a shipping route should be subsidized to get retrofitted with SPE or not. However, a better subsidization plan may be possible if more flexibilities are allowed (e.g., the government is allowed to subsidize only part of ships on a route). Similarly, we assume that a shipping line that operates a route can only choose to either get all ships on the route equipped with SPE or let none of the ships equipped with SPE. In practice, a shipping line may also choose to partially retrofit its fleet deployed on a route (a possible reason is the lack of sufficient funds to retrofit the entire fleet). Future studies should consider how to generate subsidization plans that allow more flexibilities in SP deployment. Second, in this study, all parameters are considered to be deterministic and constant. In practice, the price of SP electricity and the price of fuel for auxiliary engines of ships may change over time and can also be uncertain. Future studies should consider how to generate subsidization plans that are robust against the volatilities and uncertainties in these prices. Finally, the current study only considers the situation that there is only one government that invests in SP deployment in a shipping network. Future studies can extend this study by considering the situation where multiple governments are making their subsidization plans for SP deployment in a network and each of them manages different ports and shipping routes (shipping lines). Hence, an interesting topic is how to coordinate these governments and design a subsidization plan for each of them.

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