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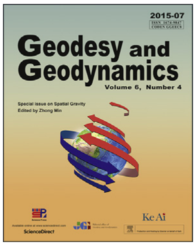

Monthly gravity field solution from GRACE range measurements using modified short arc approach

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ABSTRACT

In this paper we present a series of monthly gravity field solutions from Gravity Recovery and Climate Experiment (GRACE) range measurements using modified short arc approach, in which the ambiguity of range measurements is eliminated via differentiating two adjacent range measurements. The data used for developing our monthly gravity field model are same as Tongji-GRACE01 model except that the range measurements are used to replace the range rate measurements, and our model is truncated to degree and order 60, spanning Jan. 2004 to Dec. 2010 also same as Tongji-GRACE01 model. Based on the comparison results of the $C_{2,0}$, $C_{2,1}$, $S_{2,1}$, and $C_{15,15}$, $S_{15,15}$, time series and the global mass change signals as well as the mass change time series in Amazon area of our model with those of Tongji-GRACE01 model, we can conclude that our monthly gravity field model is comparable with Tongji-GRACE01 monthly model.

1. Introduction

The main approaches for the Earth's gravity field solutions from the satellite to satellite tracking data, such as Gravity Recovery and Climate Experiment (GRACE) [1,2], are the dynamic approach, the short arc approach and the acceleration approach. The dynamic approach was used by the Centre for Space Research (CSR), the GeoForschungsZentrum (GFZ) and the Jet Propulsion Laboratory (JPL) to develop the official gravity field models, namely GSM_UTCSR RL05 [3], GSM_Eigen

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RL05a [4] and GSM_JPLEM RL05 [5]. Xu [6] expressed the position and velocity observations as the perturbations with respect to the kinematic orbits, which can theoretically handle long arc of orbit with small linearization error. The mean acceleration approach was used by the Delft Institute of Earth Observation and Space Systems to develop their gravity field models [7–9]. Recently the mean acceleration approach was modified by considering the corrections of the orbit positions in computing gravitational forces and used to develop monthly gravity field models [10].

The short arc approach, which was first presented by Schneider [11], was used by the Bonn University to develop ITG series of gravity field models [12–14]. The short arc approach was modified by Shen et al. [15] and the modified short arc approach was used to develop monthly gravity field models from the range rate measurements [16] and static gravity field models from GRACE orbits and range or range rate measurements [17,18]. This paper will use the range measurements to develop monthly gravity field model spanning Jan. 2004 to Dec. 2010 using modified short arc approach and evaluate the developed model, since the present monthly gravity field models developed by using short arc approach are all using range rate measurements. At first, we will briefly introduce the short arc approach and its modified version; then we describe data processing method and present our monthly gravity field solution. At last, we try to evaluate the developed monthly model and compare it with the Tongji-GRACE01 monthly model.

2. Short arc approach for gravity field solution and its modified version

In the short arc approach of gravity field solution, the satellite's position and velocity vectors are formulated as a boundary value problem of an integral equation [12,13],

$$\boldsymbol{r}(\tau) = (1-\tau)\boldsymbol{r}_0 + \tau\boldsymbol{r}_N - \mathrm{T}^2\int_0^1 \mathrm{K}(\tau,\tau')\boldsymbol{a}(\boldsymbol{r}(\tau'),\boldsymbol{u},\boldsymbol{p},\tau')\mathrm{d}\tau' \qquad (1)$$

$$\dot{\boldsymbol{r}}(\tau) = (\boldsymbol{r}_N-\boldsymbol{r}_0)/\mathrm{T} + \mathrm{T}\int_0^1 \frac{\partial \mathrm{K}(\tau,\tau')}{\partial\tau}\boldsymbol{a}(\boldsymbol{r}(\tau'),\boldsymbol{u},\boldsymbol{p},\tau')\mathrm{d}\tau' \qquad (2)$$

where, $\boldsymbol{r}(\tau)$ and $\dot{\boldsymbol{r}}(\tau)$ denote satellite's position and velocity vectors, τ is the normalized time of the arc of time interval T, $\boldsymbol{r}_0$ and $\boldsymbol{r}_N$ are satellite's position vectors at two boundaries of integral arc, $\boldsymbol{a}(\boldsymbol{r}(\tau),\boldsymbol{u},\boldsymbol{p},\tau)$ is the force acting on the unit mass of a satellite, in which $\boldsymbol{u}$ is the geopotential unknowns and $\boldsymbol{p}$ is arc-specific unknowns such as the biases of accelerometer measurements, the integral kernel K is described as,

$$\mathrm{K}(\tau,\tau') = \begin{cases} \tau(1-\tau'), \tau\le\tau' \\ \tau'(1-\tau), \tau\ge\tau' \end{cases} \qquad (3)$$

The inter-satellite range rate measurement can be expressed as the projection of the velocity differences of two satellites as

$$\dot\rho(\tau) + v_{\dot\rho}(\tau) = \boldsymbol{e}_{\mathrm{AB}}^{\mathrm{T}}(\tau)\cdot(\dot{\boldsymbol{r}}_{\mathrm{B}}(\tau) - \dot{\boldsymbol{r}}_{\mathrm{A}}(\tau)) \qquad (4)$$

where, $\dot\rho(\tau)$ denotes range rate measurement, $v_{\dot\rho}(\tau)$ represents its correction, $\mathbf{e}_{\mathrm{AB}}(\tau) = (\mathbf{r}_{\mathrm{B}}(\tau) - \mathbf{r}_{\mathrm{A}}(\tau))/\rho(\tau)$, and $\rho(\tau)$ is the range between the GRACE satellite A and B, the symbol "·" denotes the inner product of two vectors.

The equation (1) was used by Mayer-Gürr et al. [12] to solve ITG-CHAMP01 model, in which the impact of kinematic orbit error in computing $\boldsymbol{a}(\boldsymbol{r}(\tau),\boldsymbol{u},\boldsymbol{p},\tau)$ was ignored. Since this impact cannot be ignored for the GRACE range or range rate observations, Mayer-Gürr [13] solved GRACE Earth's gravity field after using gradient corrections to correct kinematic orbit errors beforehand. However, the gradient corrections are computed with equation (1) by using a priori gravity field model. Moreover correcting the satellite orbit errors and resolving the gravity field model in two steps is not rigorous theoretically. Therefore, Shen et al. [15] presented a modified short arc approach in GRACE science team meeting 2013, where all positions in equations (1) and (2) are expressed as,

$$\boldsymbol{r}(\tau) = \boldsymbol{r}_k(\tau) + \boldsymbol{v}_r(\tau) \qquad (5)$$

where, $\boldsymbol{r}_k(\tau)$ denotes the position vector of kinematic orbit, $\boldsymbol{v}_r(\tau)$ is its correction vector. Substituting equation (5) into equations (1) and (2), we have,

$$\boldsymbol{r}_k(\tau) + \boldsymbol{v}_r(\tau) = (1-\tau)\left(\boldsymbol{r}_0 + \boldsymbol{v}_{r_0}\right) + \tau\left(\boldsymbol{r}_{\mathrm{N}} + \boldsymbol{v}_{r_{\mathrm{N}}}\right) - \mathrm{T}^2\int_0^1 \mathrm{K}(\tau,\tau')\boldsymbol{a}(\boldsymbol{r}_k(\tau') + \boldsymbol{v}_r(\tau'),\boldsymbol{u},\boldsymbol{p},\tau')d\tau' \qquad (6)$$

$$\dot{\boldsymbol{r}}(\tau) = \left(\boldsymbol{r}_{\mathrm{N}} + \boldsymbol{v}_{r_{\mathrm{N}}} - \boldsymbol{r}_0 - \boldsymbol{v}_{r_0}\right)/\mathrm{T} + \mathrm{T}\int_0^1 \frac{\partial\mathrm{K}(\tau,\tau')}{\partial\tau}\boldsymbol{a}(\boldsymbol{r}_k(\tau') + \boldsymbol{v}_r(\tau'),\boldsymbol{u},\boldsymbol{p},\tau')d\tau' \qquad (7)$$

Handling linearization of equations (6) and (7) with respect to the kinematic orbit and numerical integration with Adams–Moulton method, then substituting them into equation (4), we can derive the observation equations for position and range rate measurements. The detailed descriptions of the observation equation can be referred to Chen et al. [16].

If we use the range rate measurements for the gravity field solution, it is obvious that both equations (6) and (7) need to be integrated. If using range measurements to replace range rate measurements, we only need to integrate equation (6) since the range measurements are expressed as follows,

$$\rho(\tau) + \Delta\rho + v_\rho(\tau) = \boldsymbol{e}_{\mathrm{AB}}^{\mathrm{T}}(\tau)\cdot(\boldsymbol{r}_{\mathrm{B}}(\tau) - \boldsymbol{r}_{\mathrm{A}}(\tau)) \qquad (8)$$

in which, $v_\rho(\tau)$ is the correction of range measurement $\rho(\tau)$, the bias $\Delta\rho$ can be eliminated by differentiating two range measurements. After linearization, the discrete observation equations (6) and (8) can be expressed as follows,

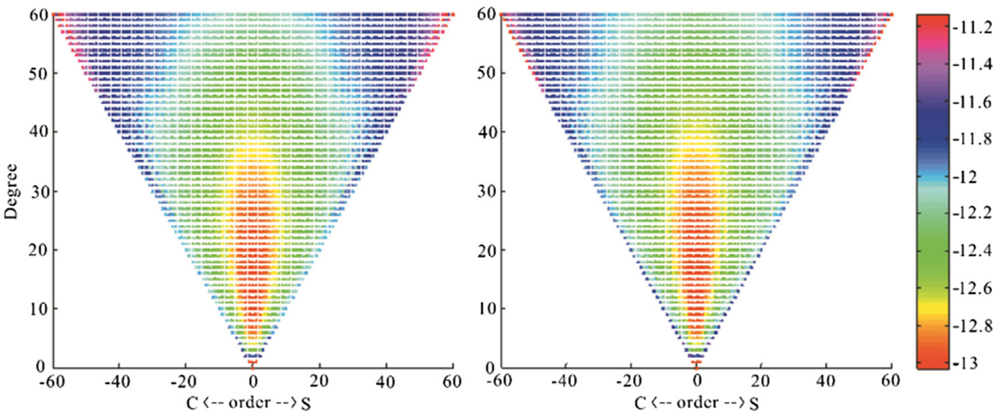
$$\boldsymbol{Ax} + \boldsymbol{Bv} = \boldsymbol{w} \qquad (9)$$

where, $\boldsymbol{x}$ is the vector of unknown parameters including the geopotential coefficients $\boldsymbol{u}$ and the arc-specific parameters $\boldsymbol{p}$, $\boldsymbol{v}$ is the vector of corrections of satellite orbits and range measurements, $\boldsymbol{A}$ and $\boldsymbol{B}$ are the coefficient matrices, $\boldsymbol{w}$ is misclosure vector. By using weighted least squares adjustment, we first eliminate the arc-specific parameters with the normal equations of each arc, then merge the normal equations of all arcs to solve the geopotential coefficients. One can also refer to Chen et al. [16] for the details of solving the observation equation (9).

3. Monthly gravity field solution from range measurements

Gravity field solution using range measurements is implemented in the Satellite Gravimetry Analysis Software (SAGAS) developed by Tongji University. The GRACE Level-1B measurements used in our solution are same as those in Chen et al. [16] for computing Tongji-GRACE01 model except that the range measurements are used to replace the range rate measurements. The background force models used in our solution were described in detail in Chen et al. [16]. The arc length is two hours, the accelerometer biases are estimated every hour and the accelerometer scales are not estimated, which are the same as that in Chen et al. [16]. The weights of orbit and range measurements are determined based on the priori accuracies of about 2 cm for orbit [19] and of about 10 μm for range [20]. Since our solution is complete to degree and order 60, the gravitational effects beyond degree and order 60 are removed with the ITG-GRACE2010s model. In summary, the methods used in this paper are also same as those in Chen et al. [16]. Nevertheless, two GRACE satellite's velocities needn't be integrated using equation (7) since we use range measurements. Thereby, using range measurements is more efficient in computation than using range rate measurements.

The formal errors and the degree geoid heights of Tongji-GRACE01 and our model with respect to EIGEN6C2 are presented in Fig. 1 and Fig. 2, respectively, which demonstrate that both the patterns and values of the two models are very similar. Since the $C_{2,0}$, $C_{2,1}$, $S_{2,1}$ coefficients and the resonance coefficients $C_{15,15}$, $S_{15,15}$ are with lower accuracies in GRACE solution, we present the time series of these coefficients in Figs. 3–5, where the time series of two models are consistent well. According to the above results, we can conclude that our monthly gravity field model is comparable with Tongji-GRACE01 monthly model.



a – Tongji-GRACE01 model b – Our model

Fig. 1 – The formal errors of models in Jan. 2010.

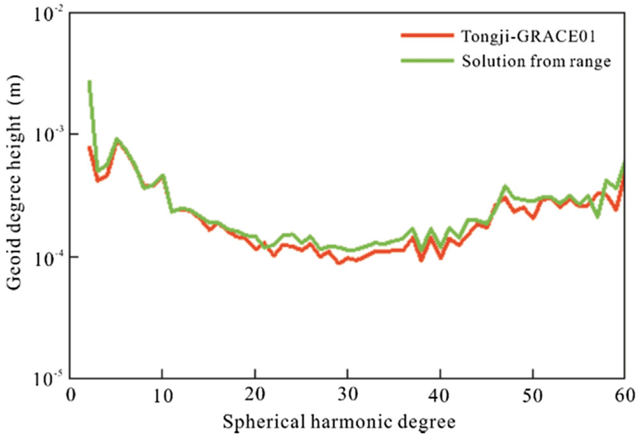


Fig. 2 – Tongji-GRACE01 and our model in Jan. 2010 (w.r.t EIGEN6C2 in terms of geoid height).

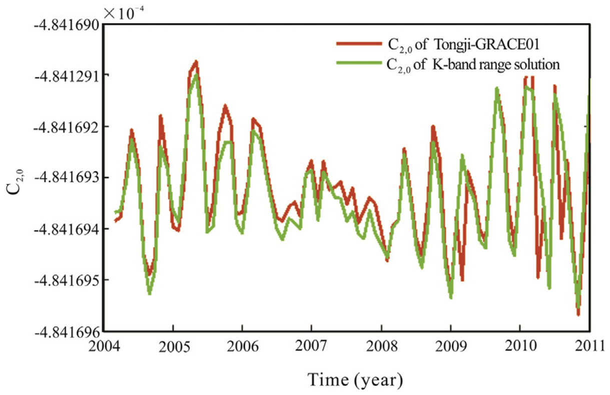


Fig. 3 – $C_{2,0}$ time series of Tongji-GRACE01 and our model.

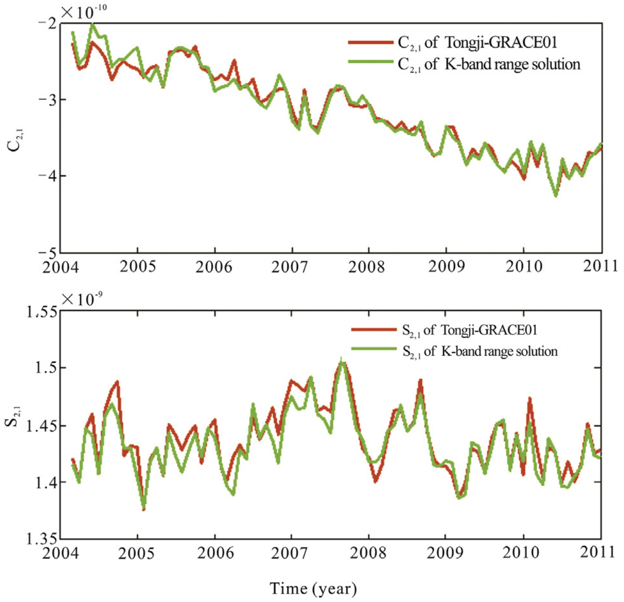


Fig. 4 – **$C_{2,1}$ and $S_{2,1}$ time series of Tongji-GRACE01 and our model.**

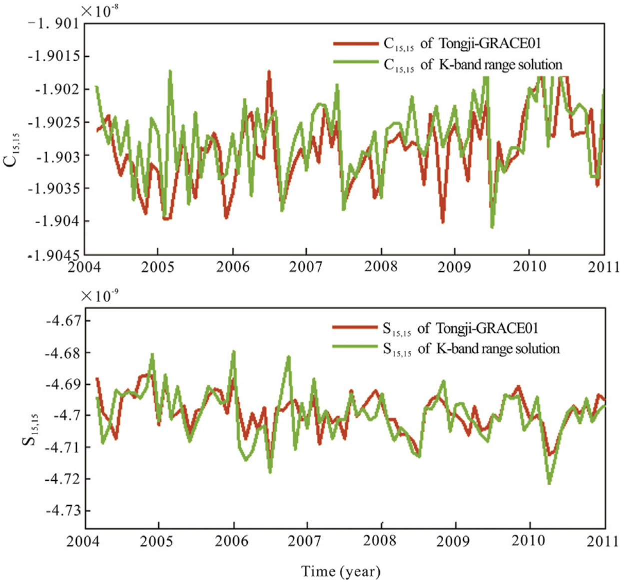


Fig. 5 – **$C_{15,15}$ and $S_{15,15}$ time series of Tongji-GRACE01 and our model.**

4. Mass change analysis

The mass change derived from monthly gravity field model is as follows [21],

$$\Delta h(\theta,\lambda) = \frac{a\rho_{ave}}{3\rho_w}\sum_{l=0}^{N_{max}}\sum_{m=0}^{l}\frac{2l+1}{1+k_l}\bar{P}_{lm}(\cos\theta)W_{l,m}(\Delta C_{lm}\cos(m\lambda) + \Delta S_{lm}\sin(m\lambda)) \qquad (10)$$

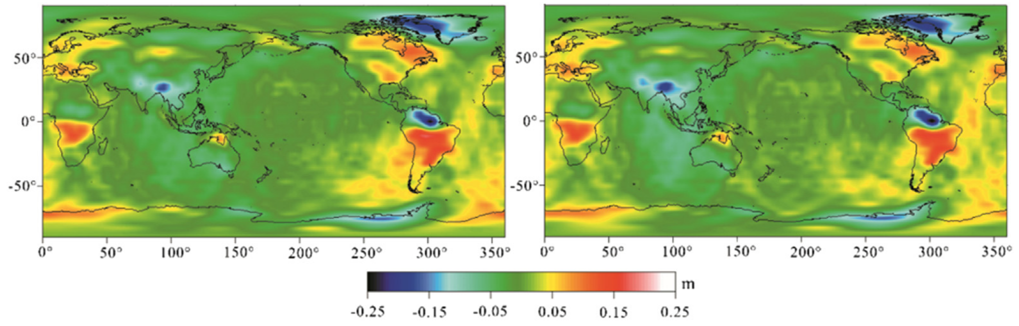
where, $\Delta h(\theta,\lambda)$ denotes the mass change in equivalent water height, θ and λ are the co-latitude and longitude, ρ_{ave} and ρ_w are the Earth's average density (5517 kg/m^3) and the density of water, a denotes the semi-major radius of the Earth, N_{max} is the maximum degree of the monthly gravity field model, $\bar{P}_{lm}(\cos\theta)$ is normalized associated Legendre function, k_l is l-th Love number, ΔC_{lm} and ΔS_{lm} are the geopotential coefficients at degree l and order m, $W_{l,m}$ is Gaussian filtering function.

The global mass change signals of our solution in Jan. 2010 are presented in Fig. 6 together with those of Tongji-GRACE01 in which a Fan smoothing [22] with a radius of 300 km is applied to remove high frequency noise in the GRACE estimates and no decorrelation filtering is used. The $C_{2,0}$ term is replaced with that from satellite laser ranging [23] and the degree one coefficients are substituted with those from Swenson et al. [24].

Fig. 6 shows that the mass change patterns of the two models are very consistent to each other. The mass change time series of the two models in Amazon River Basin are illustrated in Fig. 7 for the period Feb. 2004 to Dec. 2010, in which the two time series are very close to each other.

5. Conclusions

This paper presents the monthly gravity field model computed with the range measurements using modified short arc approach, and other GRACE data are same as those for computing Tongji-GRACE01 model. Our model is complete to degree and order 60 and spanning Jan. 2004 to Dec. 2010. The main advantage of recovering gravity field model from range measurements is that the equation (7) needs not to be integrated, which can improve the computational



a– Tongji-GRACE01 b–Our model

Fig. 6 – **Global mass change signals in Jan. 2010.**

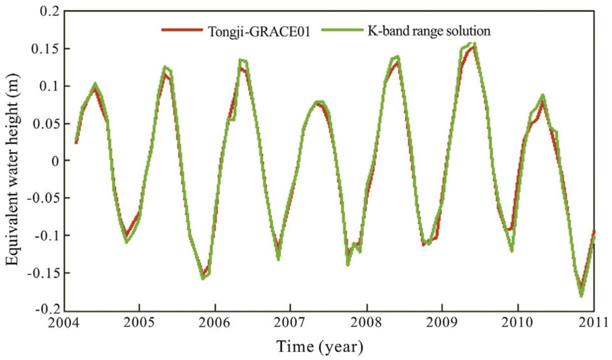


Fig. 7 – Mass change time series in terms of equivalent water height in the Amazon River Basin for the period Feb. 2004 to Dec. 2010 from Tongji-GRACE01 and our model.

efficiency. However the ambiguity of range measurements must be determined or eliminated. This ambiguity is successfully eliminated in this paper by differentiating of adjacent range measurements.

The formal errors of our model are very similar to those of Tongji-GRACE01 model, both in values and patterns. The $C_{2,0}$, $C_{2,1}$, $S_{2,1}$, and $C_{15,15}$, $S_{15,15}$ time series of our model are also very consistent with those of Tongji-GRACE01 model. The global mass change signals and the mass change time series in Amazon area derived from our model are also closer to those from Tongji-GRACE01 model. Based on these results, we can confidently conclude that our monthly gravity field model is comparable with Tongji-GRACE01 monthly model. Therefore, the gravity field model derived from the range measurements has comparable accuracy compared to that derived from range rate measurements.

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