An Adaptive Interpolating Moving Least Squares Response Surface Model Applied to the Design Optimizations of Electromagnetic Devices

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Abstract—A Response Surface Model (RSM) based on a combination of Interpolating Moving Least Squares (IMLS) and multi-step method is proposed. The proposed RSM can automatically adjust the supports of its weight functions according to the distribution of the sampling points when it is used to reconstruct a computationally heavy design problem. The TEAM Workshop Problem 22 is solved to demonstrate the feasibility of the proposed method for solving inverse problems.

I. A Multi-step IMLS Interpolation Scheme

Nowadays, it is very common to use RSMs to enhance the solution speed of a stochastic optimal method for multi-modal functions of inverse problems [1],[2]. The most popular RSMs are those based on globally supported Radial Basis Functions (RBF) because of their interpolating power in dealing with both grid and scattered data. However, global RBFs have the inherent drawbacks of having the need to manipulate the full interpolation matrices. Consequently, the sample points of available RSMs cannot exceed an upper limit [2]. In order to address such drawback, a compactly supported RSM based on a combination of IMLS and the multi-step method is proposed.

The reconstruction of a function $f(x): D \rightarrow R$ on the basis of its values f_i at a set of sample points $x_i \in$D ($i=1,2,\ldots,N$) in terms of basis function $b=\{b^{(i)}\}_{i=1}^{n}$ $(n \le N)$ using IMLS is:

$$f(x)=\sum_{i=1}^{n} a_i(x) b^{(i)}(x) \tag{1}$$

where, $a(x)=A^{-1}(x)B(x)f$, $f=[f_1\ f_2\ \cdots\ f_N]^T$,

$$A(x)=\sum_{i=1}^{N} w^{(i)}(x) b(x_i) b^T(x_i) ,$$

$$B(x)=[w^{(1)}(x)b(x_1)\quad w^{(2)}(x)b(x_2)\quad \cdots\quad w^{(N)}(x)b(x_N)] ,$$

where $w^{(i)}(x)$ is the weight function, and its general form is:

$$w_f^i(\|x-x_i\|)/\|x-x_i\|^{\alpha} \tag{2}$$

where, α is a positive even integer, $\|\bullet\|$ is the Euclidean norm, $w_f^i(\|x-x_i\|)$ is a compactly supported function.

Ideally, the sample points of an objective function should be distributed irregularly such that the point densities are comparatively high in regions where the local optima are likely to exist. Thus, every weight function should be able to adjust its support according to the point density around it. Correspondingly, the weight function $w^{(i)}(x)$ is of the form $w^{(i)}(\cdot/\beta)$ for β>0. For this purpose, the multi-step method as proposed in [3] is used and the set of sample points, X, is decomposed into a nested sequence as follows

$$X^1 \subset X^2 \subset \cdots X^{M-1} \subset X^M = X , \tag{3}$$

of the subset X^k of X,

$$X^k=\{x_1^{(k)}, x_2^{(k)}, \cdots, x_{N_k}^{(k)}\} \quad (1 \le k \le M) , \tag{4}$$

and the interpolation problem is also decomposed into M steps as described below.

Starting with k=1, one will match the error function at the k^{th} step as

$$f-(s^1+s^2+\cdots s^{k-1}) , \tag{5}$$

on X^k by computing the coefficients of the k^{th} interpolant

$$s^k(x)=\sum_{i=1}^{n} a_i(x) b^{(i)}(x) \tag{6}$$

with $a(x)=A^{-1}(x)B(x)f$, $f=[f_1\quad f_2\quad \cdots\quad f_{N_k}]^T$, and

$$A(x)=\sum_{i=1}^{N_k} w^{(i)}(x/\beta_k) b(x_i^{(k)}) b^T(x_i^{(k)}) ,$$

$$B(x)=[w^{(1)}(x/\beta_k)b(x_1^{(k)})\quad w^{(2)}(x)b(x_2^{(k)})\quad \cdots\quad w^{(N_k)}(x)b(x_{N_k}^{(k)})] ,$$

after the value of β_k of the weight function has been chosen.

It follows naturally that

$$f|_x=(s^1+s^2+\cdots+s^M)|_X \tag{7}$$

II. Numerical Example

The TEAM Workshop problem 22 is selected as the case study of the proposed method. In the numerical implementation, 495 sampling points are firstly generated and the objective/constraint function values are obtained through Finite Element (FE) simulations. The optimal problem is then reconstructed from these sampling points and their function values using the proposed method in which a liner basis {$1,x,y,z$} and a cubic spline function are adopted. It is then solved using a tabu search method. Table I compares the performance of the proposed method with that of the traditional solution approach for which the tabu search is run directly on the original problem. It can be seen that the proposed technique can virtually reach the same optimal solution as the traditional optimization algorithm, even though the former uses only about one quarter of the FE analysis computations of the latter.

TABLE I
Performance Comparison of the Proposed and a Tabu Method

Algorithm	R_2	$h_2/2$	d_2	f_{opt}	No. of FEM Computations
Proposed	3.09	0.242	0.389	8.20×10^{-2}	495
Tabu	3.10	0.240	0.388	8.19×10^{-2}	1842

III. References

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