## Article

# Modeling Study of Design Flow Rates for Cascade Water Supply Systems in Residential Skyscrapers 

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#### Abstract

Skyscrapers are common nowadays around the world, especially in cities with limited development area. In order to pump water up to the higher level of a skyscraper, a cascade water supply system has to be installed. Currently, cascade water supply systems are mainly designed based on practical experiences or requirements of existing standards/guidelines that, in fact, are not specifically for skyscrapers. However, thorough studies on cascade water supply system designs are still limited in the literature. This study proposes mathematical models and uses Monte Carlo simulations to evaluate the design flow rate of a typical cascade water supply system that feeds various appliances in a residential skyscraper in Hong Kong. Graphs that showed the correlations between the inflow rate in the supply pipe and water volume in the tank are obtained. While tank storage volume is confirmed, the design flow rate of the cascade water supply system can be determined from these graphs. The proposed mathematical models can also be applied to evaluate the design flow rate of cascade water supply systems in other types of skyscrapers (e.g., office, commercial building) as well as with the changes in water demand patterns in the models.


Keywords: cascade water supply system; design flow rate; Monte Carlo simulation; skyscrapers

## 1. Introduction

Skyscrapers are common nowadays around the world, especially in cities with limited development area. In Hong Kong, about 7.5 million people (total population in Hong Kong) [1] are living in less than $25 \%$ developed area of around 1100 square km (population density of 6659 people per square km ) [2], and skyscrapers are a trend of the city development. This challenges the design of water supply systems inside skyscrapers. The design of the water supply system not only affects the water demand inside buildings but also influences the pumping energy consumption. It was reported that $40 \%$ of the total pumping energy in some developed cities is consumed inside buildings [3,4], and this percentage is quite bigger in skyscrapers as the pumping energy consumption is proportional to building height [5]. The proper design of the water supply system in skyscrapers would benefit the sustainable development of buildings nowadays. Existing projects of water supply systems in skyscrapers can be found around the world; however, these water supply systems are designed based on practical experiences or requirements of existing codes/ordinances that, in fact, are not specifically for skyscrapers. A thorough investigation of water supply system design in skyscrapers is still limited.

Furthermore, in practices of exiting codes/ordinances, maximum simultaneous water demand can be taken as the design flow rate of the water supply system [6,7], it may be not an optimal solution as the maximum simultaneous water demand usually lasts only for a short time over the system lifetime [8].

In fact, several studies revealed that the calculated maximum flow rates by current standards/guidelines are overestimated [9,10].

The application of current standards/guidelines to skyscrapers needs to be justified. Previous studies have recommended several restoration actions for some common problems in water pipe networks [11]; however, the design flow rate of water supply systems in skyscrapers is still not covered. Therefore, it is necessary to thoroughly evaluate the design flow rate in skyscrapers.

A cascade water supply system is commonly used in skyscrapers (Figure 1). This system divides a skyscraper into several zones, with a water tank on the top of each zone feeding demand points below it, which is similar to traditional roof tank water supply systems [5]. However, the intermediate tanks (e.g., Tank 1 and Tank 2 in Figure 1) also supply water to the adjacent tank on the upper zone through a transfer pump. In practice, the application of a cascade water supply system avoids the reliance of using water pumps with a superhigh pump head for pumping all of the water to a tank at the top level of a skyscraper. The pumps with superhigh pump heads are currently still limited. This study investigates the cascade water supply system for skyscrapers.

Tank 3


Pump 1
Figure 1. A cascade water supply system in a skyscraper.
In order to save fresh water resources, sea water has been used for toilet flushing since the 1950s in Hong Kong, and the current sea water supply coverage is for about $80 \%$ of the total population [12] while the remainder is supplied by freshwater. Sea water is abstracted at the seafront pumping stations, and then firstly screened by strainers to remove debris and large-sized particles [13]. After that, sea water is disinfected by sodium hypochlorite produced on-site by electro-chlorination before being
distributed to consumers or pumped to service reservoirs [13]. Figure 2 shows the schematic diagram of a typical sea water supply system in Hong Kong [13].

Sea water and fresh water are being separately supplied to buildings in Hong Kong. This study focuses on the supply of fresh water to appliances (not include water closet (WC)) in buildings. In addition, as water demand profiles vary with building types (e.g., residential building, office building), examining the design flow rate in specific types of buildings is necessary. Therefore, this paper thoroughly studies the design flow rate for a typical cascade water supply system that feeds various appliances, including showerheads, wash basins, kitchen sinks, and washing machines in a residential skyscraper in Hong Kong.


Figure 2. The schematic diagram of a typical sea water supply system in Hong Kong [13].

## 2. Water Demand Models, Occupant Load, and Water Demand Pattern

The estimation of instantaneous flow rates is necessary for sizing pipes and other components of water systems in buildings [14]; thus, demand models have been developed to determine the design flow rate (i.e., probable maximum simultaneous demands). One major type of demand model was developed based on a simulation and time series approach, and this approach has been studied by a number of studies [15-17]. This current study also uses the simulation and time series approach to investigate the design flow rate of a cascade water supply system. Design flow rates can be determined from the instantaneous demand time series via Monte Carlo sampling techniques [18].

The simulation accuracy of water demand by the Monte Carlo method is highly dependent on the input parameter values (i.e., occupant load, water demand pattern) resulting from field surveys. The occupant load and water demand pattern in high-rise buildings were studied by Wong and Mui via two surveys in residential households in Hong Kong [19,20]. The surveys reported that the average number of occupants per household is 4.2 in Hong Kong [19,20]. The occupant load variation factor was defined as the occupant load at a time as a percentage of the maximum occupant load [19]. Figure 3 presents the occupant load variation factor obtained from the surveys [5,8]. The occupant load in the morning and at night is high, while in the daytime, the occupant load is low. That is because most occupants are out to work during the day on weekdays, while in holidays, some occupants like to choose outdoor activities in the daytime.


Figure 3. Occupant load variation factor $\gamma(t)$ : (a) weekdays; (b) holidays [5,18].
Figure 4 shows the hourly demand patterns of each type of appliance in an apartment throughout a day, including showers, water taps for wash basins, kitchen sinks, and washing machines. The data of occupant load in Figure 4 are obtained by the self-reports of occupants who stayed at home for the longest time every day $[19,20]$. Figure 4 presents that the night demand peaks for all types of appliances are obvious, and, comparatively, morning demand peaks are unobvious for some types of appliances, such as showerheads. These demand patterns of appliances are correlated with occupant load variation in an apartment and behaviors of users. At night, most occupants come back home after work or study (namely, the occupant load is high at night), and this is one factor that causes the night demand peak at around 18:00 to 21:00. In addition, most occupants have the habit of cleaning themselves (e.g., using showerheads and water taps) and washing clothes at night and cooking dinner at home (e.g., using kitchen sink). The morning demand peak at round 7:00-8:00 is because some occupants clean themselves (e.g., showering, wash the face, and rinse the mouth) for work or study and prepare breakfast.


Figure 4. Hourly demand of each type of appliances in an apartment: (a) showerhead; (b) wash basin; (c) kitchen sink; (d) washing machine [16,17].

## 3. Methodology

Monte Carlo simulations of the design flow rates for a cascade water supply system in a residential skyscraper that feeds various appliances including showers, water taps for wash basins, kitchen sinks, and washing machines were performed. It was assumed that the vertical height of each zone was around 50 m , and the number of appliances of each appliance type in each zone was 600 in this study.

The design flow rate in each zone was simulated sequentially from the uppermost zone to the lowest one, for example, $q_{23}$ in Zone 3 was simulated first, and $q_{01}$ in Zone 1 was simulated lastly.

The Monte Carlo simulations were carried out in two steps: the first step was to simulate the simultaneous water demands generated by all appliance types in the time series; the second step was to integrate the water demand time series, with the considered tank volume, to determine the design flow rate. For the simulation of the simultaneous water demands by all appliance types, a water demand time series was simulated for each appliance type first, and a total water demand time series was then generated by aggregating the individual water demand time series data points. The problem of discharge overlapping [21-23] among different types of the appliance can be avoided during the aggregation process because the simulation of water consumed by each appliance type is based on the field-surveyed water demand pattern.

## Models of Design Flow Rate

Mass balance equations were proposed at each tank of the cascade water supply system as shown in Figure 1, to decide the inflow rate of the supply pipes, as Equations (1) to (3), where $q_{w, 3}\left(\mathrm{~L} \cdot \mathrm{~s}^{-1}\right), q_{w, 2}$ $\left(\mathrm{L} \cdot \mathrm{s}^{-1}\right)$, and $q_{w, 1}\left(\mathrm{~L} \cdot \mathrm{~s}^{-1}\right)$ are time-variant water demands in Zone 3, Zone 2, and Zone 1, respectively; $q_{0,3}\left(\mathrm{~L} \cdot \mathrm{~s}^{-1}\right), q_{0,2}\left(\mathrm{~L} \cdot \mathrm{~s}^{-1}\right)$, and $q_{0,1}\left(\mathrm{~L} \cdot \mathrm{~s}^{-1}\right)$ are inflow rates of supply pipes to Tank 3, Tank 2, and Tank 1, respectively; $\tau_{\infty, 3}, \tau_{\infty, 2}, \tau_{\infty, 1}$ are demand time periods in Zone 3, Zone 2, and Zone 1, respectively; $V_{3}$ (L), $V_{2}(\mathrm{~L}), V_{1}(\mathrm{~L})$ are the water volumes in Tank 3, Tank 2, and Tank 1, respectively. The minimum and maximum values of $V_{3}, V_{2}$, and $V_{1}$ are used to determine the start and stop of the pumps, respectively In practical application, the tank volumes of Tank 3, Tank 2, and Tank 1 should be larger than the maximum values of $V_{3}, V_{2}$, and $V_{1}$, respectively, in order to ensure the water supply security.

$$
\begin{gather*}
\int_{\tau_{\infty, 3}} q_{w, 3} d_{t} \leq q_{0,3} \tau_{\infty, 3}+V_{3},  \tag{1}\\
\int_{\tau_{\infty, 2}} q_{w, 2} d_{t}+q_{o, 3} \tau_{\infty, 2} \leq q_{0,2} \tau_{\infty, 2}+V_{2}  \tag{2}\\
\int_{\tau_{\infty, 1}} q_{w, 1} d_{t}+q_{o, 2} \tau_{\infty, 1} \leq q_{0,1} \tau_{\infty, 1}+V_{1}, \tag{3}
\end{gather*}
$$

Assuming the time-variant water demands and the demand time periods in the three zones are the same in this study, namely,

$$
\begin{align*}
q_{w, 3} & =q_{w, 2}=q_{w, 1}=q_{w}  \tag{4}\\
\tau_{\infty, 3} & =\tau_{\infty, 2}=\tau_{\infty, 1}=\tau_{\infty} . \tag{5}
\end{align*}
$$

Therefore, Equations (1) to (3) can be rewritten as,

$$
\begin{gather*}
\int_{\tau_{\infty}} q_{w} d_{t} \leq q_{o, 3} \tau_{\infty}+V_{3}  \tag{6}\\
\int_{\tau_{\infty}} q_{w} d_{t}+q_{0,3} \tau_{\infty} \leq q_{0,2} \tau_{\infty}+V_{2}  \tag{7}\\
\int_{\tau_{\infty}} q_{w} d_{t}+q_{0,2} \tau_{\infty} \leq q_{0,1} \tau_{\infty}+V_{1} \tag{8}
\end{gather*}
$$

For Equations (6) to (8), there are solution pairs $\left(V_{3}, q_{0,3}\right),\left(V_{2}, q_{0,2}\right)$, and $\left(V_{1}, q_{0,1}\right)$, respectively, at any time period within the time period of demand.

The water demand $q_{w}\left(\mathrm{~L} \cdot \mathrm{~s}^{-1}\right)$ is defined by a number of water appliances (i.e., $1,2, \ldots, k$ ) operating at any time $t \in \tau_{\infty}$,

$$
\begin{equation*}
q_{w}=\sum_{k} q_{c, k}(t) \tag{9}
\end{equation*}
$$

The operation of an appliance is random within a time period $\tau_{w}(\mathrm{~s})$, which starts at time $t_{w, 0}(\mathrm{~s})$ and ends at time $t_{w, \infty}(\mathrm{~s})$; it equals to the sum of time periods of non-zero demands $\tau_{w, l}(\mathrm{~s})$ and zero demands $\tau_{0, l}(\mathrm{~s})$ for $l=1,2, \ldots, N_{a}$, where $N_{a}\left(\mathrm{~h}^{-1}\right)$ is the hourly demand of an appliance within the time period, and the time periods are represented by the appliance demand start time $t_{w 1, l}(\mathrm{~s})$ and the appliance demand end time $t_{w 2, l}(\mathrm{~s})$ [24],

$$
\begin{gather*}
\tau_{w}=t_{w, \infty}-t_{w, 0} \\
=\left\{\begin{array}{c}
\tau_{0,1}+\tau_{w, 1}+\tau_{0,2}+\tau_{w, 2}+\cdots+\tau_{0, l}+\tau_{w, l}+\cdots+\tau_{0, N_{a}}+\tau_{w, N_{a}}+\tau_{0, N_{a}+1} \\
\tau_{w, 0}+\tau_{0,1}+\tau_{w, 1}+\tau_{0,2}+\tau_{w, 2}+\cdots+\tau_{0, l}+\tau_{w, l}+\cdots+\tau_{0, N_{a}}+\tau_{w, N_{a}}-\tau_{0, N_{a}+1}
\end{array} ; \begin{array}{l}
t_{w 2, N_{a}} \leq t_{w, \infty} \\
t_{w 2, N_{a}}>t_{w, \infty}
\end{array}\right.  \tag{10}\\
\tau_{0, l}=t_{w 1,1}-t_{w 2, l-1} ; \tau_{w, l}=t_{w 2, l}-t_{w 1, l}  \tag{11}\\
\tau_{0,1}=\left\{\begin{array}{c}
t_{w 1,1}-t_{w, 0} \\
t_{w 1,1}-t_{w 2,0}
\end{array} ; \tau_{0, N_{a}+1}=\left\{\begin{array}{c}
t_{w, \infty}-t_{w 2, N_{a}} \\
t_{w 2, N_{a}}-t_{w, \infty}=t_{w 2,0}-t_{w, 0}=\tau_{w, 0}
\end{array} ; \begin{array}{l}
t_{w 2, N_{a}} \leq t_{w, \infty} \\
t_{w 2, N_{a}}>t_{w, \infty}
\end{array}\right.\right. \tag{12}
\end{gather*}
$$

The demand start time $t_{w 1, l}(\mathrm{~s})$ is given by a randomly distributed fractional demand start time $\hat{t}_{w 1, l}(\mathrm{~s})$, which can be determined via Monte Carlo simulations using a uniformly distributed fractional demand start time $U(\mathrm{~s})$, where $\vartheta \in[0,1]$ is a random number between 0 and 1 [24].

$$
\begin{gather*}
t_{w 1, l}=\hat{t}_{w 1, l} \tau_{w},  \tag{13}\\
\vartheta=\int_{-\infty}^{\hat{t}_{w 1, l}} U d_{t} . \tag{14}
\end{gather*}
$$

The Monte Carlo model that embodied in the model of design flow rate has been used to predict the hourly flushing water consumption in two typical residential buildings in Hong Kong and has been validated with hourly measurement [25]. Good agreement between the predicted and the measured flushing water consumption was found [25]. The validated Monte Carlo model ensures the accuracy of the predicted design flow rate in this study.

Previous studies have pointed out that residential water consumption is regional dependent, as the community attitudes and behaviors, water stock efficiency profiles, environmental conditions, water pricing structures, government water restriction regimes, and conservation message intensity vary in different countries or regions [26]. All these factors influence the residential water consumption. In this study, the values of input parameters for the Monte Carlo simulation are identified from local public references and summarized in Table 1. As the data of input parameters are limited in public references, and the statistical methods for the data in different public references are different, which causes the inconsistency of the parameter units in Table 1, for example, some parameters are with Max, Min, and Mean, while other parameters are with arithmetic mean (AM) and geometric mean (GM).

Table 1. Values of input parameters.

| Appliance | Parameter |  | Value | Reference |
| :---: | :---: | :---: | :---: | :---: |
| showerhead | Flow rate (L/s) | Max | 0.20 | [27] |
|  |  | Min | 0.10 | [27] |
|  |  | Mean | 0.16 | [27] |
|  | Discharge time (s) | Max | 359 | [27] |
|  |  | Min | 240 | [27] |
|  |  | Mean | 310.2 | [27] |
| Washbasin | Flow rate (L/s) | Max | 0.23 | [28] |
|  |  | Min | 0.03 | [29] |
|  |  | $\mathrm{AM}^{1}$ | 0.13 | [30] |
|  | Discharge time (s) | GM ${ }^{2}$ | 23.2 | [30] |
| Kitchen sink | Flow rate (L/s) | Max | 0.26 | [28] |
|  |  | Min | 0.03 | [29] |
|  |  | AM ${ }^{1}$ | 0.15 | [30] |
|  | Discharge time (s) | $\mathrm{GM}^{2}$ | 257 | [30] |
| Washing machine | Flow rate (L/s) | AM ${ }^{1}$ | 0.2 | [30] |
|  | Discharge time (s) | $\mathrm{GM}^{2}$ | 150 | [30] |

${ }^{1} \mathrm{AM}$ : arithmetic mean; ${ }^{2}$ GM: geometric mean.

## 4. Results and Discussion

### 4.1. Simulated Water Demand Time Series

Figure 5 shows the simulation results of the time series of total demand flow rates $q_{w}(t)$ for all types of appliances in terms of maximum and minimum daily volumetric consumption, on the condition that the water supply system is in use for 100 years. The use of " 100 years" was based on the findings that there was no significant difference in the simulation results with an increase in years of operation after 100 . Some design guides suggest a $1 \%$ failure rate for the design demand flow rate [31]; therefore, 1 out of 100 years was taken as a reference calculation in this study. The time step of the daily demand time series is 1 s . As the simulation is conducted with inputs of the local-scale field-surveyed data, it is considered as an accurate representation of the water usage pattern in Hong Kong.

The daily water consumption was acquired by summing the demand flow rates across the time series shown in Figure 5. Results in Figure 5 indicated that the simulated daily consumption range was from $534.8 \mathrm{~m}^{3} \cdot \mathrm{~d}^{-1}$ to $590.0 \mathrm{~m}^{3} \cdot \mathrm{~d}^{-1}$, with an average of $562.4 \mathrm{~m}^{3} \cdot \mathrm{~d}^{-1}$.


Figure 5. Total demand flow rates: (a) maximum daily consumption ( $590.0 \mathrm{~m}^{3} \cdot \mathrm{~d}^{-1}$ ); (b) minimum daily consumption ( $534.8 \mathrm{~m}^{3} \cdot \mathrm{~d}^{-1}$ ).

### 4.2. Simulated Design Flow Rates for Cascade Water Supply Systems

Monte Carlo simulations are performed by a coded program and output a series of solution pairs of inflow rate and water volume in the tank. Figure 6 demonstrates the solution pairs $\left(V_{3}\right.$, $q_{0,3}$ ) for Equation (6) regarding integration time periods $\tau_{o}=10,60$, and 300 s for the demand flow rates in the time series shown in Figure 5. Since the simulated solution pairs with an integration time period $\tau_{o}=1 \mathrm{~s}$ for the WC demand in a previous study showed no significant difference from those with $\tau_{0}=10 \mathrm{~s}$ [8], $\tau_{0}=10 \mathrm{~s}$ was chosen as the minimum integration time period in this study. As demonstrated in Figure 6, a great discrepancy occurred with a rough integration time period $\tau_{o}$ (e.g., 300 s ), i.e., the simulated inflow rate with $\tau_{0}=300 \mathrm{~s}$, was greatly lower than that with $\tau_{o}=10 \mathrm{~s}$ or 60 s ; however, no significant difference was found between the solutions for large storage volumes.

With the specific value of water volume $V_{0,3}$, inflow rate $q_{0,3}$ can be determined from Figure 6 . Figure 6 also shows that the inflow rate $q_{0,3}$ decreases with the water volume $V_{0,3}$. In practical application, the volume of Tank 3 should be larger than the water volume $V_{o, 3}$, and te corresponding design flow rate is less than the $q_{0,3}$ determined by Figure 6 a , but should be larger than the $q_{0,3}$ determined by Figure $6 \mathbf{b}$. The inflow rate $q_{0,3}$ determined from Figure $6 \mathbf{a}, \mathrm{~b}$ are the maximum and minimum values of the design flow, respectively. In practical application, while the design flow rate is in the range of inflow rate determined by Figure 6a,b, water supply security can be ensured. Comparatively, a security factor is usually used to ensure the water supply in traditional design methods [6]. For Hong Kong practice, the total volume of water tanks (including sump and roof tanks, and the proportion of capacity of sump tank to roof tank is $1: 3$ ) shall be on the basis of 135 L for each of the first 10 flats and 90 L thereafter for each additional flat [32]. Each zone of the cascade water supply system can be taken as a roof tank water supply system and assuming the 600 simulated appliances of each type are installed in 600 flats in that zone. Taking the water volume $V_{0,3}$ as $41,000 \mathrm{~L}(\approx 40,838 \mathrm{~L}$ $(=3 / 4(135 \times 10+90 \times 590))$. With the water volume $V_{0,3}$ of 4100 L , inflow rate $q_{0,3}$ of $17.9 \mathrm{~L} \cdot \mathrm{~s}^{-1}$, and
$16.4 \mathrm{~L} \cdot \mathrm{~s}^{-1}$ for the maximum and minimum demand time series, respectively, can be determined from Figure 6. In practical application, the design flow rate can be in the range of $17.9 \mathrm{~L} \cdot \mathrm{~s}^{-1}$ and $16.4 \mathrm{~L} \cdot \mathrm{~s}^{-1}$.


Figure 6. Solutions of inflow rate and storage volume at Zone 3 (a) for the maximum demand time series in Figure 5a and (b) for the minimum demand time series in Figure 5b.

For the engineering application, results in Figure 6 can be presented in the dimensionless form, as shown in Figure 7. The $x$-axis is normalized by the daily consumption of the installation $V_{d}$, $\left(V_{d}\right.$ is $590,000 \mathrm{~L}$ and 534,800 L, shown in Figure 5, for Figure 7a,b, respectively); the y-axis is normalized by the maximum flow rate $q_{0, \max }$, theoretically determined by $V_{3}=0 \mathrm{~L} / \mathrm{s}\left(q_{0, \max }\right.$ are $28.3 \mathrm{~L} / \mathrm{s}$ and $27.3 \mathrm{~L} / \mathrm{s}$, shown in Figure 5, for Figure 7a,b, respectively).


Figure 7. Solutions of normalized inflow rate and storage volume at Zone 3 (a) for the maximum demand time series in Figure 5a and(b) for the minimum demand time series in Figure 5b.

As shown by Equation (7), inflow rate $q_{0,2}$ is influenced by three parameters, for example, the demand flow rate at Zone $2\left(q_{w}\right)$, the water volume in Tank $2\left(V_{2}\right)$, and the inflow rate to Tank $3\left(q_{0,3}\right)$, in which $q_{0,3}$ is correlated with the water volume in Tank $3\left(V_{3}\right)$. With one parameter, for example, $V_{0,3}$, is confirmed, the two-dimensional correlation of $q_{0,2}$ to $V_{2}$ can be determined by Equation (7). The solution pairs ( $q_{0,2}, V_{0,2}$ ) for Equation (7) regarding integration time periods $\tau_{0}=10$ under different values of $V_{0,3}\left(q_{0,3}\right)$ are plotted in Figure 8.


Figure 8. Solutions of inflow rate and storage volume at Zone 2 (a) for the maximum demand time series in Figure 5a and (b) for the minimum demand time series in Figure 5b.

Equation (8) shows that inflow rate $q_{o, 1}$ is influenced by the demand flow rate at Zone $1\left(q_{w}\right)$, the water volume in Tank $1\left(V_{1}\right)$, and the inflow rate to Tank $2\left(q_{0,2}\right)$, in which $q_{0,2}$ is correlated with the water volumes in Tank 2 and Tank 3 (shown in Figure 8). With $V_{0,3}=41,000 \mathrm{~L}$, the solution pairs ( $q_{0,1}$, $V_{1}$ ) for Equation (8) regarding integration time periods $\tau_{o}=10$ under different values of $V_{o, 2}\left(q_{0,2}\right)$ are plotted in Figure 9.


Figure 9. Solutions of inflow rate and storage volume at Zone 1 (a) for the maximum demand time series in Figure 5a and (b) for the minimum demand time series in Figure 5b.

In practical application, the tank size is determined based on the balance of water demand, fire safety requirements, and space available in the plant room. As shown in Figures 6-9, since tank storage volume is known, the inflow rate (maximum value of the design flow rate) in each zone of the cascade water supply system can be obtained from these graphs. The dimensionless form of the results, as Figure 7, can be generally used for other projects. The proposed model in this study will be useful for the design of a cascade water supply system, urban planning in water facilities, and water treatment for cities of highly dense environments similar to Hong Kong.

Previous studies have validated that the Monte Carlo model can reasonably predict the water consumption in residential buildings [25]. Therefore, it can be believed that the simulated water consumption and the design flow rate of a cascade water supply system in this study is closer to the measured values in a real situation. Furthermore, previous studies have revealed that existing standards/guidelines overestimate the design flow rate by 2 to 3 times [33]. Comparing with the design flow rate determined by existing standards/guidelines, the simulated design flow rate in this study is smaller, and therefore, lower friction head loss in the pipelines will be caused due to the lower water velocity in the supply pipe. Friction head loss is expressed by Equation $H_{f}=\lambda \frac{u^{2}}{2 g d} L_{e}$, where $H_{f}$ is friction head loss, $\lambda$ is the $d^{\prime}$ Arcy friction coefficient, $u$ is the flow velocity, $d$ is the hydraulic diameter, $g$ is the gravitational acceleration, $L_{e}$ is the pipe equivalent length taking all pipe fittings into account [6]. In this equation, the value of $\lambda$ is in the range of $0.008-0.1$ [34]. The low value of $\lambda$ would weaken the influence of flow velocity on friction head loss and the system energy efficiency. It can be deduced that the energy efficiency of the cascade water supply system would not improve greatly when with the simulated design flow in this study; however, it benefits the proper selection of water pump. The proper selection of pump will reduce the energy consumption of the pump by $30 \%$ [35].

## 5. Conclusions

The rapid growth in the construction of skyscrapers around the world raises concerns about the design of the water supply system within them. Cascade water supply systems are commonly used in skyscrapers nowadays. However, thorough studies about the design of cascade water supply systems are limited, and there is currently no standard or guide specifically for the design of cascade water supply systems in skyscrapers.

This study proposed mathematical models and used Monte Carlo simulations to evaluate the design flow rate for a typical cascade water supply system that feeds various appliances, including showerheads, wash basins, kitchen sinks, and washing machines in a residential skyscraper in Hong Kong. The water volume in the tank is included as a study parameter in the mathematical models to avoid/alleviate the overestimation of the design flow rate, which is limited in classical methods for the determination of the design flow rate. In addition, the problem of discharge overlapping among different types of appliances is avoided/alleviated by the Monte Carlo models in this study. As residential water consumption is regionally dependent, parameters in the Monte Carlo model were quantified by local public references. Water demand patterns were obtained first by the Monte Carlo simulations with input data from field surveys in Hong Kong. Corresponding to the water demand pattern, correlations between the inflow rate in the supply pipe and tank storage volume were acquired and plotted in graphs. It also shows that the inflow rate in the upper zone of a cascade water supply system influences the inflow rate in the lower zone. In practical application, while the tank storage volume was confirmed (balancing the water demand, fire safety requirement, and space available in plant room), the design flow rate of cascade water supply systems can be determined from these graphs. For future application with different demand patterns, the correlations between inflow rate and water volume in the tank can be obtained based on the Monte Carlo model proposed in this study, and further design flow rates of cascade water supply systems can be determined. The solid data of water demand patterns and proposed mathematical models in this study can be a reference for further study of the design of a cascade water supply system, as well as for the practical application.

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