Research Article

ENERGY EXPLORATION & EXPLOITATION

Energy Exploration & Exploitation
0(0) 1–17

DOI: 10.1177/0144598720942852
journals.sagepub.com/home/eea
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Comparisons of the accuracy of different wake models in wind farm layout optimization

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Abstract

Accurate wake model in wind farm layout optimization can help extracting maximum power generation, minimizing cost of energy and prolonging wind turbines' lifetime as well. With the development of different wake models, the wind farm layout optimization results based on the models should be updated. This paper investigates the performances of four wake models in wind farm layout optimization using multi-population genetic algorithm (MPGA) with the wind farm power generation, COST/AEP and wind farm efficiency been reported. Comparison of results between typical wake models' performance shows that Jensen's wake model reported a higher wind farm power generation and efficiency because it underestimates the velocity deficit in the wake, and to the contrary, in the Frandsen wake model, the velocity in the wake is underestimated, resulting in a deceased power generation. The expression of 2D_k model shall be out of work in complicated wind condition. The 2D Jensen–Gaussian wake model performed better in the wind farm layout optimization using the MPGA program which can be promoted in real-world wind farm micrositing.

Keywords

Wake model, wind turbine layout optimization, total power, wind farm efficiency, cost of energy

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Introduction

Wind power is already the most competitive renewable technology as well as the most significant to utilities and independent power producers (IPPs): efficient, reliable, sustainable, predictable and cost competitive energy which can meet the current and future electricity demand (REN21, 2019; Sahu, 2018; Sun et al., 2012). Increased installed capacity in 2017 was 52,573 MW, bringing the global cumulative capacity to 539,581 MW (Dupont et al., 2018).

However, most operating wind farms cannot produce adequate energy as it was predicted because the energy harvested will be definitely reduced due to the loss of available kinetic energy in the wake flow. The discrepancy ranges from 10% in Middelgrunden wind farm to 23% in the Lillgrund wind farm (loss in annual energy production) (Barthelmie et al., 2009; Wu and Porte-Agel, 2012), which can be attributed to the loss in the availability of energy due to wake effects – the shading effect of a wind turbine on other wind turbines downstream from it (Christiansen and Hasager, 2005). The significance of wind farm layout optimization which can extract maximum power generation, minimizing cost of energy (COE) and prolonging wind turbines' lifetime as well has been widely accepted.

The problem of optimal micrositing of wind turbines in onshore/offshore wind farms has been widely studied in the existing literature (Sun et al., 2019a, 2019b). It is a highly complex optimization problem that was first presented by Mosetti et al. (1994), whose research revealed the evolutionary computational techniques and introduced the earliest method named Genetic Algorithm (GA). In their study, a $2\,\text{km}\times 2$ km wind farm is divided into a square grid with three typical wind cases considered and numerically tested. These typical cases are used to assess the optimization algorithms in many subsequent papers. Grady et al. (2005) improved the results by using larger population and more generation of evolution in GA and also included some improvements in the economic model regarding the work of Mosetti et al. Kusiak and Song (2010) and Kusiak and Zheng (2010) introduced the Weibull distribution to describe the incoming wind characteristics instead of individual separated wind cases, based on which GA with special mutation and selection operators is applied to optimize the turbine layout within a circular wind farm area. Emami and Noghreh (2010) introduced a novel coding method that maps the locations within a square wind farm to a matrix of zeros and ones. Lee and Lam (2008) proposed a Hybrid Distributed Genetic Algorithm, which is a Distributed Genetic Algorithm (DGA) followed by a heuristic Hill-Climbing (HC) approach. The DGA divides the population into small demes, which improves the performance of GA by preserving diversity, and the HC approach is used to improve the solution of DGA even further. Gao et al. (2016) used a similar variation of GA, the multi-population GA, to optimize the layout under typical cases. The method was also applied to a real offshore wind farm in Hong Kong, demonstrating its effectiveness in handling realistic conditions. Dupont et al. (2018)'s work was devoted to search for the optimum wind farm layout using binary real-coded genetic algorithm based local search, gathering robust single wake model with suitable wake interaction modeling. The binary part of GA was used to represent the location of turbines. Results were compared with earlier studies using GA and also random search algorithm, and it was shown that the proposed approach was found to be superior in finding the optimal solution, in which better configurations with higher power productivity are attained. The attempt for changing a different wake model in wind farm layout optimization program was conducted by Gao et al. in 2016, in which Jensen wake model is replaced by 2D Jensen–Gaussian wake model. Results in the study are more practical than those in previous studies.

By careful observation, we can find that most of the mentioned researches concentrated on the algorithm promotion and neglected the accurate wake model selection, and in which the most widely used wake model is Jensen wake model. However, detailed knowledge of the flows and turbulence structures within wakes is crucial for the wind farm layout optimum, and wake model is the theoretical basis in wake numbering checking and wind velocity calculation for individual wind turbine in wind farm. In recent years, numbers of wake model were proposed to describe the velocity deficits and turbulence intensity in the turbines wake with the objective to describe the wake characteristics accurately.

The wake model proposed by Jensen (Bekele and Palm, 2009), also known as the Park wake model, has been used in the vast majority of studies addressing the wind farm layout optimal (WFLO) problem and proved to predict the energy loss acceptable within wind farm. However, this model assumes a linearly expanding wake with a velocity deficit that is only dependent on the distance behind the rotor and keeps constant in the cross-wind direction. As mentioned by Katic and Jensen (1986), the purpose of the Jensen model was not to accurately describe the wind speed in wake area, but to predict the energy content of the wind farm so as to better estimate the wind farm power generation. However, the relevant wind tunnel experimental data and the measured data of the wind farm indicate that the Jensen wake model underestimates the velocity deficit in the far wake region. The other major limitation of Jensen wake model is an improper constant wake decay parameter expressed by k through the entire wake flow field. In Jensen's wake model, the value of k is 0.1; nevertheless, the suggested values of k in the literature are 0.075 for onshore turbines and 0.05 for the offshore ones (Sanderse, 2009). In fact, the growth of the wake is governed by many more factors such as shear-generated turbulence and the turbulence created by the turbine other than the ambient turbulence, which leads to the conclusion that the wake decay rate should not be a constant but a variable parameter taking the effective wake turbulence (Politis et al., 2012; Sorensen, 2011; Sumner et al., 2010; Troldborg et al., 2011).

Later, based on the law of earth's rotation and drag, the Frandsen model was proposed which adopted the momentum conservation to derive an additional turbulence formula for the center of wind turbine's hub height (Frandsen et al., 2006). In this model, the wake expansion was assumed to be nonlinear, and the velocity distribution was uniform in radial direction. Frandsen claimed that the model was not for a single wake, but focused on the wake of the entire wind farm. However, the wind speed in the wake region predicted by this model is also higher than the measured value. Bastankhah and Porte-Agel (2014) pointed out that the main reason for this model to underestimate the velocity deficit in the wake region was that the assumption of nonlinear expansion for the wake region was inconsistent with the wake expansion curve obtained by large eddy simulation (LES), and the assumption of the "top-hat" shape for the velocity distribution in the wake region was unreasonable.

Ainslie (1988) firstly announced a two-dimensional (2D) field model which analyzed the effect of wake meandering on wake deficits by relating wake meandering to the variability in wind direction, when 2D field models assume axial symmetry in the wakes. Inspired by Ainslie and the Jensen's wake model, our team used the Gaussian function to describe the velocity distribution curve of the wake region, and the wake model was proposed based on the conservation of mass and the linear expansion of wake (Gao et al., 2016, 2019, 2020). Compared with the one-dimensional wake model, the newly proposed 2D Jensen–Gaussian wake model greatly improves the accuracy of the velocity deficit prediction in the wake region, and the predicted value is in good agreement with the measured value. However, the wake expansion coefficient in this model is still determined by the empirical formula proposed by Katic and Jensen (1986). As mentioned above, Jensen assumed that the wake region was in the turbulent flow zone, at the same time, the local atmospheric turbulence intensity was not considered and the blade tip vortex generated by the wind turbine was neglected. The result of the above simplification is to underestimate the influence of turbulence intensity on the wake recovery rate, so the wake model would overestimate the velocity deficit to some extent. In addition to the 2D Jensen–Gaussian wake model, Tian et al. (2015) brought out the 2D Jensen wake model with the assumption that the velocity distribution in the wake region exhibited a cosine function shape. There is no essential difference between the cosine function and the Gaussian distribution in describing the velocity distribution curve of the wake region. The most direct difference between the 2D_k Jensen–Gaussian model and the 2D Jensen–Gaussian model is reflected in the correction of the wake expansion coefficient, which can better predict the turbulence intensity in the wake. Based on the work of Crespo and Hernández (1996) and Frandsen et al. (2006), a simple empirical engineering model of turbulence intensity in wake area was proposed by Gao et al. (2016), which made the value of wake expansion coefficient take into account not only the influence of turbulence intensity of incoming flow, but also the influence of the additional mechanical turbulence intensity in the wake area. It means that the corrected wake expansion coefficient K is larger than the value in the 2D wake model. It is well known that strong turbulence intensity can accelerate the convective diffusion of the wake and the surrounding free flow and accelerate the recovery of the wake. Therefore, compared with the 2D wake model, the velocity recovery rate predicted by the 2D_k wake model is faster in the wake region, especially in the far-wake region.

To validate the performance of different wake models in wind farm layout optimization program, in this paper, four typical wake models, i.e. the Jensen wake model, the Frandsen wake model, the 2D_k Jensen wake model, and the 2D Jensen–Gaussian wake model, are used for the wind turbine layout optimization combined with the multi-population genetic algorithm (MPGA) optimization method, respectively. The performance of different wake models are studied and compared. The wake model by which the wind farm layout optimization program can harvest more power with less COE will be chosen and recommended. Thus, the second part emphasized on introducing the abovementioned for wake models and the methodologies, followed by the results and discussion in the third part. Conclusion and recommendations are given in the last section of this paper.

Materials and methods

Wake model

The mentioned four typical wake models used in the wind farm layout optimization program are shown in Table 1. The Jensen and Frandsen wake models are both regarded as linear models in the description of the velocity deficits in the downwind region of turbines , while the Jensen-k and Jensen-Gaussian wake models are two-dimensional models. As it was reported that the Jensen and Frandsen wake models underestimate the velocity deficits in the wake, which causes the wind velocity downwind a turbine to be overestimate. The 2D Jensen-k and Jensen–Gaussian wake models considered the turbulence intensity both inside

Table 1. Wake models used in the wind farm layout optimization program.

Wake model	*Wake radius and velocity distribution in the wake*
Jensen (1983)[a]	$r_x = k_{wake}x + r_a,\ \ r_a = r_d\sqrt{(1-2a)\frac{1-a}{}},\ \ a = \frac{1}{2}\left(1-\sqrt{1-C_T}\right)$ $u = u_0\left[1 - 2a/(1+k_{wake}x/r_a)^2\right]$
Frandsen et al. (2006)[b]	$\beta = \frac{1}{2}\frac{1+\sqrt{1-C_T}}{\sqrt{1-C_T}},\ \ r_x = r_d\left(\beta^{k/2} + \alpha\cdot s\right)^{1/k},\ \ s = \frac{x}{2r_d}$ $a = 1-\sqrt{1-C_T},\quad C_T < 1;\ \ \alpha = \beta^{K/2}\left[\left(1+2\alpha_{(noj)}s\right)^k - 1\right]s^{-1},\ \ \alpha_{(noj)}\approx 0.05$ $\begin{cases} u = \frac{u_0}{2}\left(1+\sqrt{1-2\frac{A_0}{A}C_T}\right) & a \le 0.5 \\ u = \frac{u_0}{2}\left(1-\sqrt{1-2\frac{A_0}{A}C_T}\right) & a > 0.5 \end{cases}$ $A_0 = \pi r_d^2,\ \ A = \pi r_x^2$
2D Jensen-k (Tian et al., 2015)[c]	$u^* = u_0\left[1 - \frac{1-\sqrt{1-C_T}}{(1+k_{wake}x/r_d)^2}\right],\ \ r_x = k_{wake}x + r_d$ $u = (u_0 - u^*)\cos\left(\frac{\pi}{r_x}\times r + \pi\right) + u^*$
2D Jensen–Gaussian (Gao et al., 2016)[c]	$r_x = k_{wake}x + r_a, r_a = r_d\sqrt{(1-a)/(1-2a)}, a = \frac{1}{2}\left(1-\sqrt{1-C_T}\right)$ $\begin{cases} u^* = u_0\left[1-2a/(1+k_{wake}x/r_a)^2\right] \\ u = u_0 - (u_0 - u^*)\frac{5.16}{\sqrt{2\pi}}\cdot e^{-r^2/2\cdot(r_x/2.58)^2} \end{cases}$

[a] r_a is the immediately downstream rotor radius of the turbine, a is the axial induction factor and C_T is the thrust coefficient.
[b] For Frandsen model, assuming monotonic expansion of the wake for increasing x, the equation has solutions only for $a \le 0.5$. And $k = 2$ is chosen.
[c] The 2D Jensen and 2D Jensen–Gaussian models assume that the wake center is at the location $r = 0$.

Table 2. Wind turbine properties.

Parameters	*Values*
Hub height (Z)	60 m
Rotor radius (r_d)	40 m
Thrust coefficient (C_T)	0.88

and outside the wake which perform better in the wake characteristics prediction. However, their performance on wind farm layout optimization should be further discussed.

Models of a wind farm

Wind turbine characteristics and power generation. The same wind turbine with the same properties parameters will be used in this paper which is exactly the same as those used in the previous studies (Emami and Noghreh, 2010; Gao et al., 2016; Gonzalez et al., 2010; Grady et al., 2005; Marmidis et al., 2008; Mosetti et al., 1994; Pookpunt and Ongsakul, 2013;

Tenguria et al., 2010), and are shown in Table 2. The power generation of the turbine can be calculated using the following equation

$$P = C_p(\lambda, \beta)\rho A u^3/2 \quad (1)$$

The power coefficient $C_p(\lambda, \beta)$ of WTs is defined as 40% (Christiansen and Hasager, 2005; Grady et al., 2005; Tenguria et al., 2010; Zhang et al., 2011), and the above equation will be converted to the following equation

$$P = \frac{1}{2} C_p(\lambda, \beta)\rho \pi R^2 u^3 = 40\% \times 1.2 \times \pi \times 20^2 \times u^3/2 = 0.3u^3 \quad (2)$$

Cost model. Taking COE into consideration is essential for the purpose of maximizing the wind energy capture and minimizing the investment. Mosetti et al. (1994) assumed that the non-dimensionalized cost/year of a single turbine is assumed as "1" and a maximum cost reduction of 1/3 for each additional turbine. In order to make a comparison of the results between present and previous researches, the cost model for the entire wind park is selected the same as previous studies, which can be expressed as the following equation

$$COST = N \times \left[\frac{2}{3} + \frac{1}{2} e^{-0.00174} N^2\right] \quad (3)$$

Meanwhile, a non-dimensional value which represents the COE is introduced in this paper, which is expressed by the following division formula

$$\zeta = COST/AEP \quad (4)$$

AEP represents annual power generation. A minimum of ζ is one of the objectives of the optimization program.

Wind farm efficiency. To analyze the fluctuation of power generation caused by wake effect quantitatively, wind farm efficiency is introduced in this study, which is a percentage of the total power generation for the entire wind farm taking the wake effect into account and the total power generation of all the wind turbines at free stream without wake effect between wind turbines. The efficiency can be given by the equation

$$\eta_{WF} = \sum_{1}^{N} 0.3 \times {u_i}^3/N \times (0.3 \times {u_0}^3) \quad (5)$$

Wind scenarios. The wind scenarios are also the same as those in the past studies, i.e. (a): uniform wind direction with a wind speed of 12 m/s; (b): uniform wind speed of 12 m/s and variable wind directions from 0° to 360°. The wind direction is divided into 36 angles with equal fraction of occurrence and (c): variable wind speed of 8, 12 and 17 m/s and variable wind directions. The fraction of occurrence for each angle at each wind speed is shown in Figure 1.

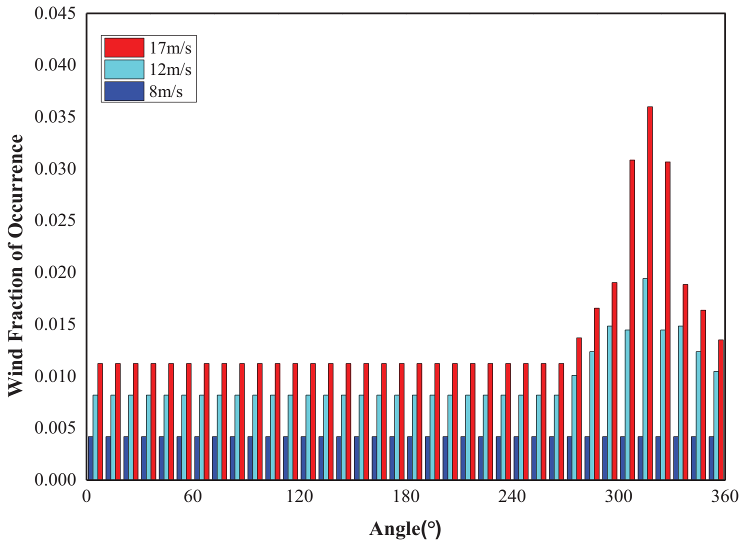

Figure 1. Variable wind direction, variable wind speed of case (c).

Table 3. Initial parameters setting for MPGA.

Parameters	*Value*
Population number	10
Probability of crossover	0.7–0.9
Probability of mutation	0.001–0.05
Number of individual	Set by cases
The least keeping generations	500
Binary digits of variable	20

Optimization methodology

The multi-population genetic algorithm. GA is one of the artificial intelligence algorithms that are performed to optimize wind farm layout problems inspired from the nature genetic and evolution mechanisms. Details can be checked in Larsen et al. (1997) and Gao et al. (2014).

Program design. The initial wind farm condition and wind turbine characters are the same as those in previous studies. The 2 km $\times$ 2 km wind farm is subdivided into a 10 $\times$ 10 grid, with a cell size of 100 m $\times$ 100 m (Grady et al., 2005; MirHassani and Yarahmadi, 2017; Pookpunt and Ongsakul, 2013; Turner et al., 2014). The turbine can only be installed in the center of the cell due to the binary coding method of the GAs. However, in Wan et al. (2010) and Gao et al. (2014), the positions of turbines in a wind farm can be adjusted freely to reduce wake effects and capture more wind energy. To ensure the operation safety distance between each turbine, a minimum of 5D distance is required.

Table 4. Comparison of results of case (a).

Studies	*Wake models*	*Turbine numbers*	*Total power (kW)*	*ζ(COST/AEP) (×10^−3^)*	*Efficiency (%)*
Turner et al.'s (2014)	Jensen's model	30	14,800	–	95.24
Grady et al.'s (2005)			14,310	1.544	92.09
Gonzalez et al.'s (2010)			14,310	1.544	92.09
Tenguria et al.'s (2010)			14,336	1.541	92.18
Pookpunt and Ongsakul's (2013)			14,310	1.544	92.02
Wan's (2010)			15,220	–	97.87
Zhang et al.'s (2011)			14,310	1.544	92.02
Gao et al.'s (2014) previous study			15,346	1.440	98.67
Present study	Frandsen's model		13,475	1.639	86.65
	2D_k model		15,542	1.421	99.94
	2D Jensen–Gaussian model		15,325	1.442	98.55

MATLAB is used for the MPGA program writing. The number of optimization variable is 2 N(X,Y) (Ilinca et al., 2002). Some important parameters for the MPGA program are shown in Table 3.

Results and discussions

The optimization results of the three cases based on the four referred wake models (Jensen's, Frandsen's, 2D Jensen-k wake model and 2D Jensen–Gaussian wake model) using the MPGA program are presented and compared to the results of previous studies (MirHassani and Yarahmadi, 2017; Turner et al., 2014). The performance of these wake models on wind farm layout optimization can be observed.

Case (a): Constant wind speed of 12 m/s with fixed wind direction

For comparison, the micrositing of same numbers of turbine ($N = 30$) using different wake models is optimized in this study, and the results are compared with those of previous studies.

Table 4 presents the optimization results of total power generation, wind farm efficiency as well as the fitness value of turbines' micrositing in different studies. It can be checked that Jensen's wake model is widely used in previous studies. In this study, four wake models are used in the MPGA optimization process which can provide comprehensive results for the analysis of the models' performance. The micrositing of the same numbers of wind turbine using the same Jensen's wake model is more effective in our previous studies, which indicates the superiority of the proposed MPGA program. The wind farm efficiency can reach to 98.67% for 30 turbines' location been optimized which is higher than those in previous studies. This has been proved and analyzed in our previous studies (Gao et al., 2014). This paper is with emphasis on the performance of different wake models in wind farm optimization.

After the superiority of the proposed MPGA is validated, the performance comparisons are conducted. It can be checked from Table 4 that the performances of the referred four wake models are almost the same. With 30 turbines been optimized, the wind farm efficiencies are 98.67%, 86.65%, 99.94% and 98.55%, respectively. The optimized wind farm efficiency based on Frandsen's wake model is a litter lower than the other three, which is caused by differences of the velocity calculation method of this model. In general, the power curve of specific turbine is calculated using the velocity at the turbine's hub height without consideration of the rotor's swept area. However, in Frandsen's model, the velocity in the wake is calculated using the proportion of the downwind turbine's rotor area in the wake caused by the upstream turbines which underestimate the velocity at the hub height and thus, result in a deceased power generation. On the whole, for the simple wind condition in case (a), with a constant wind direction and incoming wind speed, the optimization results are excellent using those wake models, and no great differences are observed between the 1D and 2D models.

The optimization results of wind turbines' layout for case (a) of previous and the present study are shown in Figure 2. In the optimization process, the turbine's downstream distance should be large enough to avoid the wake effect between each other when the wind blows at a fixed direction. As shown in Figure 2, the optimized turbines are installed with more freedom in the present study using MPGA compared to the layout of the previous studies. More free distribution means better performance of the wind farm.

Case (b): Constant wind speed of 12 m/s with variable wind directions

For the more complicated case (b) with multi-directional wind, different scenarios with different numbers of wind turbines ($N = 19$, $N = 30$ and $N = 40$) are attempted in the present study to compare the wake models' performance as well as with the results of previous studies. Figure 3 shows some typical optimized configuration results compared with previous studies. In this case, there is no prevailing wind direction where each angle has an equal probability of wind fraction of occurrence. It is obvious to observe that the wind turbines under this specific wind condition prefer to be installed around the outer perimeter of the wind farm, while few in the center both in previous and present studies. What is more, wind turbines can be installed anywhere within the wind farm using more dense grids only in the present study, which can provide more flexibility for the wind turbine installation. More flexibility means more wind energy capture.

Table 5 shows a comparison of total power generation and wind farm efficiency for each configuration in different studies. Irrespective of the wake model, in general, the more wind turbines been installed and optimized within the wind farm, the more power can be generated.

The optimization results using the 2D_k and 2D Jensen–Gaussian wake model have lower wind farm power generations and wind farm efficiency compared with that using Jensen wake model. The reason is that Jensen's wake model underestimates velocity deficits in the wake.

Comparing the results of these two 2D wake models, i.e. the 2D_k and 2D Jensen–Gaussian models, the wind farm efficiencies for $N = 19$, 38, 39 and 40 are 96.14%, 93.78%, 91.94%, 89.20% and 97.42%, 77.83%, 78.47%, 81.88%, respectively. Superficially, the performance of 2D_k wake model is better than that of 2D Jensen–Gaussian wake model because more power with higher wind farm efficiency can be

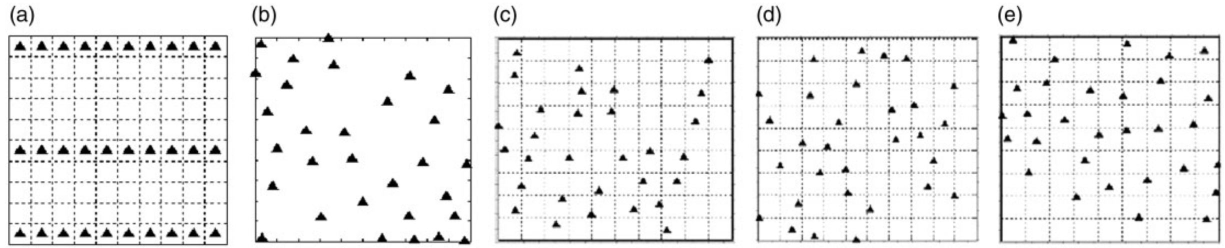

Figure 2. Optimal layout configurations of case (a) when $N = 30$: (a) under Jensen model in Grady et al.'s (2005) and Zhang et al.'s (2011) study; (b) under Jensen model in Gao et al.'s (2014) previous study; (c) under Frandsen's model in the present study; (d) under 2D_k Jensen wake model in the present study; (e) under 2D Jensen–Gaussian wake model in the present study (▲ represents wind turbine).

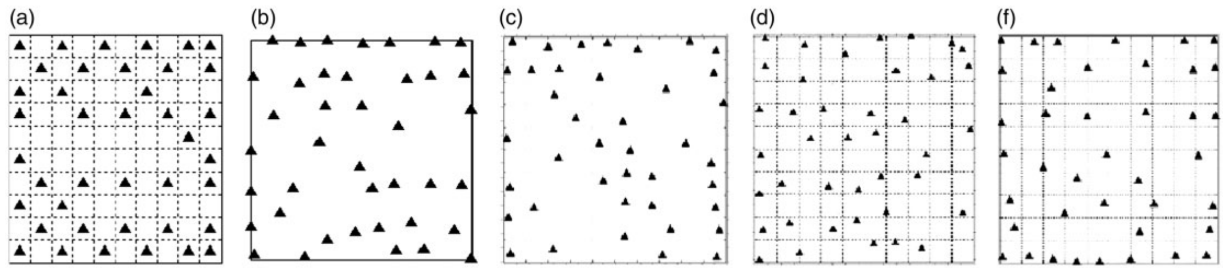

Figure 3. Optimal layout configurations of case (b) when $N = 40$: (a) under Jensen model in Zhang et al.'s (2011) study; (b) under Jensen model in Gao et al.'s (2014) previous study; (c) under Frandsen's model in the present study; (d) under 2D_k Jensen wake model in present study; (e) under 2D Jensen–Gaussian wake model in present study (▲ represents wind turbine).

Table 5. Comparison of results of case (b).

Studies		Turbine numbers	Total power (kW)	ζ(COST/AEP) ($\times 10^{-3}$)	Efficiency (%)
Mosetti et al.'s (1994)	Jensen's model	19	8,711	–	88.44
Turner et al.'s (2014)		19	9,549	–	96.94
		39	18,336	–	90.69
Grady et al.'s (2005)		39	17,220	1.567	85.17
Gonzalez et al.'s (2010)		39	18,065	1.490	89.35
Tenguria et al.'s (2010)		38	17,259	1.527	87.61
Pookpunt and Ongsakul's (2013)		40	18,632	1.476	89.81
Wan's (2010)		39	17,953	–	88.80
Zhang et al.'s (2011)		39	17,611	1.532	87.11
		40	17,991	1.528	86.76
Gao et al.'s (2014) previous study		38	19,075	1.382	96.83
		39	19,478	1.382	96.34
		40	19,964	1.377	96.23
Present study	Frandsen's model	19	8,953	1.701	90.89
		38	16,466	1.601	83.58
		39	16,866	1.596	83.42
		40	17,382	1.579	83.82
	2D_k model	19	9,470	1.683	96.14
		38	18,475	1.427	93.78
		39	18,588	1.448	91.94
		40	18,497	1.486	89.20
	2D Jensen–Gaussian wake model	19	9,596	1.672	97.42
		38	15,333	1.756	77.83
		39	15,866	1.661	78.47
		40	16,979	1.619	81.88

generated after the layout been optimized using 2D_k wake model. However, by extending the formulas of the two models shown in Table 1, it is not difficult to find that in the 2D_k model, the speed of the coda wave is described by a cosine function, which is periodic. It means that, if the wake expansion in the radial direction is less than the half cycle of the function, the velocity deficit can be described. Once the wake expansion exceeds the half cycle along with the downwind distance, the expression shall be out of work. On the contrary, no such problem exists in the 2D Jensen–Gaussian wake model. And the optimized power generation and wind farm efficiency are in accordance with the report that the power losses caused by wind turbine wakes are of the order of 10%–20% of the total power output of large wind farms (Barthelmie et al., 2009; Sanderse, 2009).

Case (c): Variable wind speed of 8, 12, 17 m/s respectively, with variable wind directions

Case (c) is the most sophisticated wind regime with three different wind speeds and multi-directional wind which is close to the real situation. To further examine the performance of the referred four models, different numbers of turbine's layout optimization are attempted.

As shown in Figure 1, the prevailing wind direction is between the angles of 270° and 360°, from north to south. In order to harvest as much electricity as possible, it is better to

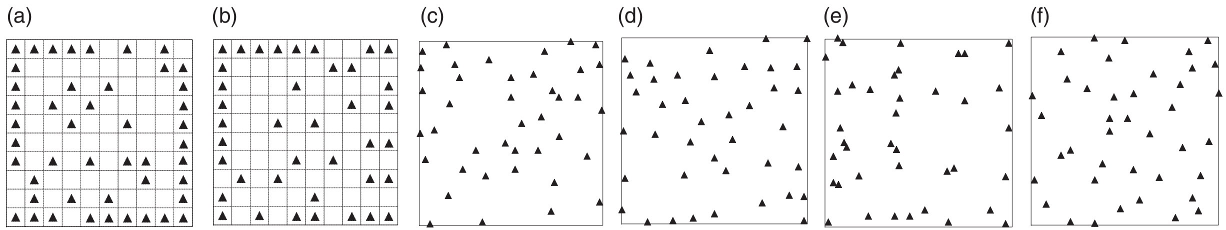

Figure 4. Optimal layout configurations of case (c): (a) under Jensen model in Pookpunt and Ongsakul's (2013) study of 46 turbines; (b) under Jensen model in Gonzalez et al.'s (2010) study of 39 turbines; (c) under Jensen's model in Gao et al.'s previous (2014) study of 40 turbines; (d) under Frandsen's model in the present study of 40 turbines; (e) under 2D_k Jensen wake model in the present study of 40 turbines; (f) under 2D Jensen–Gaussian wake model in the present study of 40 turbines (▲ represents wind turbine).

Table 6. Comparison of results of case (c).

Studies		*Turbine numbers*	*Total power (kW)*	*ζ(COST/AEP) ($\times 10^{-4}$)*	*Efficiency (%)*
Mosetti et al.'s (1994)	Jensen's model	15	13,343	–	93.65
Turner et al.'s (2014)		15	13,494	–	94.71
		39	32,453	–	87.61
Grady et al.'s (2005)		39	32,086	8.031	86.62
Gonzalez et al.'s (2010)		39	32,793	8.210	89.38
Tenguria et al.'s (2010)		41	33,262	8.438	86.73
Pookpunt and Ongsakul's (2013)		46	39,359	7.894	83.83
Wan's (2010)		39	32,921	–	88.98
Zhang et al.'s (2011)		39	33,553	–	91.45
Gao et al.'s (2014) previous study	Jensen's model	39	34,210	7.959	93.24
		41	36,281	7.736	94.60
		46	39,838	7.795	84.85
Present study	Frandsen's model	15	12,459	10.738	87.45
		39	27,635	9.699	74.60
		40	28,343	9.700	77.25
	2D_k model	39	–	–	–
		41	–	–	–
		46	–	–	–
	2D Jensen–Gaussian model	15	13,641	9.809	95.74
		39	25,970	10.365	70.11
		41	30,092	9.327	77.27

install the wind turbines along the prevailing wind direction and minimize the wind deficit of each turbine from upstream turbines. Figure 4 shows some typical optimized layout configurations using different models. Also, small cell size provides more freedom for the wind turbine installation and improves the performance of the wind farm.

Table 6 shows a comparison of the optimization results in different studies. It can be observed that more power is captured in the present study using 2D Jensen–Gaussian wake model by optimizing fewer number of wind turbines ($N = 15$) in which the wake effect can be avoided by the optimal layout configuration. For example, the total energy harvested is 13,641 kW with 15 WTs in the present study while the value of that is 13,343 kW in Mosetti's study and is 13,641 kW in our previous study. It can be concluded that if there are fewer turbine positions to be optimized, the more accurate the model, the better the optimization results.

However, when the turbines' number increases, situation changes. The wake effect is inevitable, however, an accurate wake model can well predict the lack of wake velocity. Thus, especially for complex wind condition, optimization based on accurate wake model is of great significance. With $N = 39$ and 41, the optimized wind farm efficiencies are 70.11% and 77.27% using 2D Jensen–Gaussian wake model. The 2D_k model which should consider the wake expansion to accommodate with its initial functions' periodicity is out of one's element. No optimal layout configuration has been generated no matter how many times the program runs.

Compared with case (a) and case (b), the performance of Frandsen's wake model in this complicated wind condition is not so bad. The underestimation of the power generation still persists. The optimal wind farm efficiencies are 87.45%, 74.60% and 77.25% for $N = 15$, 39 and 40, respectively. In conclusion, with complicated and changeable wind condition, the 2D Jensen–Gaussian wake model is recommended.

Conclusions

With the development of different wake models, the wind farm layout optimization results based on the models should be updated. In this paper, four wake models including Jensen wake model, Frandsen wake model, 2D_k Jensen wake model and 2D Jensen–Gaussian wake model are applied in the MPGA program to examine their performance in the WFLO problem by extracting the maximum total power with a minimum COE. The performance of different wake in wind farm layout optimization in this paper in three different wind conditions are reported and compared with the previous research results.

- In the simple wind condition in case (a), with a constant wind direction and incoming wind speed, the optimization results are excellent using those wake models, and no great differences were observed between the 1D and 2D models.
- In general, for the three cases, Jensen's wake model reported a higher wind farm power generation and efficiency because it underestimates the velocity deficit in the wake as it assumed a linearly expanding wake with a velocity deficit that is only dependent on the distance behind the rotor and keeps constant in the cross-wind direction.
- Instead, the performances of Frandsen wake models in the three cases are opposite to that of Jensen's wake model—lower wind farm power generations and wind farm efficiencies are reported. It is caused by differences of the velocity calculation method of this model. In general, the power curve of specific turbine is calculated using the velocity at the turbine's hub height without consideration of the rotor's swept area. However, in Frandsen's model, the velocity in the wake is calculated using the proportion of the downwind turbine's rotor area in the wake caused by the upstream turbines which underestimate the velocity at the hub height and thus result in a deceased power generation.
- The two 2D wake models (2D_k Jensen and 2D Jensen–Gaussian wake model) can predict the velocity profile in the wake more accurately than Jensen's and Frandsen wake model. Therefore, the wind turbine layout optimization based on these two models can report more realistic power generation and wind farm efficiencies with optimal layout configuration. For example, in case (b), the wind farm efficiencies for $N = 19$, 38, 39 and 40 under 2D_k and 2D Jensen–Gaussian models, are 96.14%, 93.78%, 91.94%, 89.20% and 97.42%, 77.83%, 78.47%, 81.88%, respectively, which is in accordance with the report that the power losses caused by wind turbine wakes are of the order of 10%–20% of the total power output of large wind farms (Barthelmie et al., 2009; Sanderse, 2009). However, in a more complicated wind condition in case (c), the 2D_k model results in chaos and the reason is the velocity in the wake is described by the cosine function, which is a cyclical one. It means that, if the wake expansion in the radial direction is less than the half cycle of the function, the velocity deficit can be described. Once the wake expansion exceeds the half cycle along with the downwind distance, the expression shall be out of work.

According to the comparisons in this paper, the 2D Jensen–Gaussian wake model performed better in the wind farm layout optimization using the MPGA program. The accuracy of the 2D Jensen–Gaussian wake model in the real-world wind farm should be demonstrated.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was funded by Natural Science Foundation of Hebei Province, grant number E2019502072 and The Foundation Research Funds for the Central Universities, grant number 2020MS107.

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