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Linkages Between Chinese Stock Price Index and Exchange Rates-An Evidence From the Belt and Road Initiative

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ABSTRACT This paper selects the daily data of the exchange rates of Chinese Yuan (CNY) over the currencies of 14 countries along the Belt and Road, Shanghai composite index and Shenzhen composite index to study the influence of the Belt and Road Initiative on the linkages between exchange rates and Chinese stock index based on the flow-oriented model and the stock-oriented model. To reflect the fluctuations in daily data and reduce the central bank's interference with the exchange rate, two fuzzy techniques are used to process data, that is, the centroid based measure and the integral based measure. Then we judge the relationship between exchange rate and stock index through the Pearson correlation coefficient and the Granger causality test. Besides, we further compare the results and their differences by the classic crisp method and our two fuzzy techniques, which enable us to judge their correlation more accurately, and provide a reference for a wider application of the proposed fuzzy methods. We find that there is a correlation between exchange rate and stock index under certain conditions, and the Belt and Road initiative strengthens the relationship between the Chinese foreign exchange market and the stock market, more importantly, the fuzzy techniques are effective to judge this relation.

INDEX TERMS Stock price index, exchange rate, fuzzy techniques, the Belt and Road initiative.

I. INTRODUCTION

The exchange rate is the international price of the domestic currency, which reflects m economic fundamental change of the domestic currency's international purchasing power. The stock price index can quickly reflect the subtle changes in the real economy. Equity markets and currency markets both play a facilitating role in economic, and are expected to interact with each other. With the acceleration of the global economic integration, financial markets will be more closely related. Specifically, exchange rate fluctuations and stock price fluctuations will also have greater interaction.

Most existing studies on the relationship between exchange rate and stock index focus on developed countries [3], [10], [6] or individual developing countries [26], [31]. In addition, the exchange rate selected by their research is largely influenced by the monetary policies of various countries, resulting in some information loss [26], [6], [31]. Different from previous studies, we focus on small and medium-sized countries. In view of the highly acclaimed Belt and Road (B&R) initiative, this article studies its impact on the exchange rates of some small and medium-sized countries that implement the floating exchange rate regime and the Chinese stock indexes. This work is an in-depth understanding of the interaction between the stock market and the foreign exchange market and has a great significance in further substantiating the B&R initiative.

Considering that both the exchange rates and stock price indexes are real-time and continuous in each trading day, it is doubtful whether the closing price can really reflect the

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changes in data within each trading day. Also, studies have shown the monetary policy of the central bank will affect the effectiveness of the exchange rate [35]. Motivated by these questions, we used the two fuzzy methods to convert the highest, lowest and closing prices into fuzzy data in our research. The effectiveness of fuzzy numbers in processing imprecise data has been verified by comparing the findings with the previous research results [18], [21]. Our findings reveal that the fuzzy techniques can be applied to ascertain the relationship between the two financial markets and are expected to play a major role in future research.

In summary, the most important contribution of our research is that we used the fuzzy methods to process exchange rate data to study the linkages between exchange rates and stock indexes, which expands the application of fuzzy theory. On the theoretical side, we analyzed the influence of the B&R initiative on the linkages between exchange rates and stock indexes and our empirical research also proved it.

The rest paper is organized as follows. Section II presents a brief review of the related research. Section III presents the theoretical framework of this paper. Section IV describes the data used in the work and presents fuzzy techniques. Section V outlines the results and findings by using closing price data and fuzzy techniques. Section VI discusses the empirical results and some future insights.

II. LITERATURE REVIEW

A. STUDIES ON THE RELATIONSHIP BETWEEN STOCK MARKET AND EXCHANGE RATE MARKET

The interplay of exchange rate and stock index has become an increasingly important research topic in the context of globalization, and many researchers have studied it from different research perspectives. However, most of the researchers have no mutual consensus on the relationship between the two.

Through empirical research, many researchers advocated that there was causality between the exchange market and the stock market [11], [17], [20], [23], [28]. Hsueh studied the correlation between China's exchange rate and the Shanghai A-share stock price index [7]. And in this work, data from 1996 to 2016 were analyzed to prove that there was a significant correlation between the two. Zhang studied the monthly exchange rate of the US dollar against RMB exchange rate and the Shanghai composite index from 2005 to 2017, and demonstrated that there was a two-way influence mechanism between stock market and exchange market, indicating that the appreciation of RMB will bring the stock index down [29]. Jacob used the Autoregressive Distributed Lagged (ARDL) model to analyze the impact of exchange rate on the stock market return in India, finding that they had a negative correlation [26]. Wong found there was a link between the exchange rate market and the stock market in Malaysia but not every real stock price return was significantly linked with the real exchange rate [6].

Literatures also indicate that some authors consider that there is no influence of the exchange rate over the stock price. Nieh and Lee found there was no long-run significant relationship between stock prices and exchange rates in the G-7 countries, which was consistent with Bahmani-Oskooee and Sohrabian's finding [3]. In addition, many studies showed that the interaction between exchange rate and stock index was always realized through various transmission channels [19], [30]. Tang found that in emerging markets, the inner-financial structure played an important role in the link between exchange rates and stock prices [32]. Li applied wavelet analysis to investigate the role of short-run international capital movements in the interaction mechanism between the exchange rate and stock prices and proved that it played an important role during the process of Renminbi internationalization and capital account liberalization [31].

Previous findings are mostly based on relatively perfect capital markets of the developed countries or economies and there were no consistent conclusions on the relationship between the two variables [9], [22]. Additionally, few studies choose an economy or the relevant countries within a certain region for research. An attempt made in this article is selecting some small countries along the B&R route to study the causality directions between the exchange rate and stock index, which also provides a new thought to predict the future stock price.

B. STUDIES ON METHODS OF MEASURING FUZZY NUMBERS

In actual research, sometimes the variables we measured are not precise values but uncertain values, and it is necessary to find a way to measure the correlation between the fuzzy data. Gerstenkorn and Manko defined a function, measuring the interrelation of intuitionistic fuzzy sets, the so-called correlation of these sets, and introduced a correlation coefficient to examine its properties [25]. Chiang and Lin, adopted a method from statistics to calculate the correlation coefficient for fuzzy data [4]. Hadi-Vencheh and Mokhtarian proposed a new fuzzy Multi-Criterion Decision Making (MCDM) approach based on the centroid of fuzzy numbers [1]. Liu and Kao argued that the fuzzy number could be used to calculate the correlation coefficient and proposed a-cut technique [24]. Ruan *et al.* presented a centroid-based method and proved that it was superior to Liu and Kao's method [10]. The fuzzy methods are usually used to solve various decision problems. For example, Xu *et al.* incorporated a loss aversion function to study the optimal option purchase decision of a loss-averse retailer [34]. Xu and Meng introduced a coordination model for a supplier and a retailer in a two-stage supply chain to maximize the profit of the supplier and the retailer [33]. Jiang *et al.* introduced an approach for stock index forecasting based on fuzzy time series and a fuzzy logical relationship map [8]. Besides, these fuzzy methods can also be used to process data, such as the Internet of things (IoT) finance, big data finance and real-time data from agriculture sensors [12], [13].

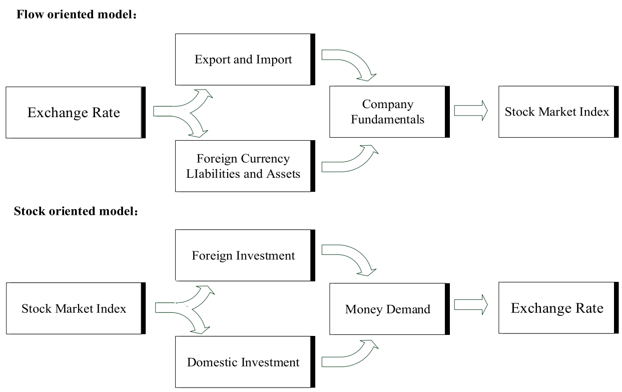

FIGURE 1. Flow oriented model and Stock oriented model.

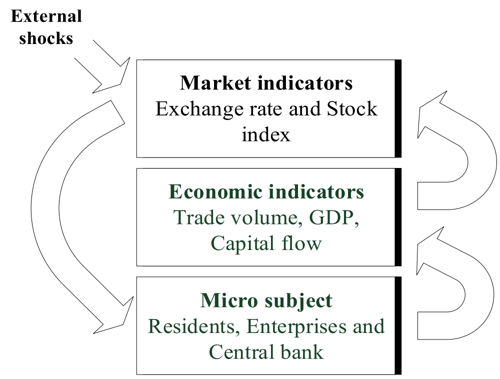

FIGURE 2. Three-level economic system.

The previous studies of exchange rate and stock index mostly used the closing price data, but it is doubtful whether the closing price can really reflect the changes in data in each intraday trading [15]. Thus, in this work, we use the centroid based measure and the integral based measure to examine the relationship between stock price index and exchange rate.

III. THEORETICAL ANALYSIS

A. THEORETICAL FRAMEWORK

There are two main theoretical foundations in the extant studies on the relationship between exchange rate and stock price. One is the ''flow-oriented'' model proposed by Dornbusch [19], and the other is the ''stock-oriented'' model proposed by Branson and Frankel [14].

The flow-oriented model is based on the perspective that the exchange rate is affected by the country's current account balance or trade balance. According to Fig. 1, the appreciation of a country's currency will lead to a decrease in the cost of import enterprises and a rise of the cost of export enterprises, which will affect the operation of enterprises and ultimately reflect on the stock price. On the other hand, the change of the exchange rate will affect the value of overseas assets and liabilities, resulting in a decrease of both, and then affect the basic situation and stock price of the company. Based on the flow-oriented model, the impact of exchange rate on stock price is uncertain, and the final direction of the impact is determined by the degree of positive and negative impact. Although the flow-oriented model emphasizes the influence of trade and capital account, it only considers the impact of exchange rate on stock index, while ignore the opposite direction. Therefore, only when the current account is convertible can the flow-oriented model explain the impact of exchange rate on the stock index. In addition, if both the current account and the capital account are convertible, the flow-oriented model can fully explain the impact of the exchange rate on the stock index.

On the contrary, according to the stock-oriented approach, the causality runs from the stock market to exchange rate. It claims that the change of stock index will affect the behavior of domestic and overseas investors, which will cause the fluctuation of money demand, and finally affect the exchange rate. A consistent increase in stock price will attract an inflow of foreign capital. This leads to an excess demand for home currency thereby leading to the appreciation of the home currency. Also, an increase in stock price will increase the wealth of the investors, which ultimately led to an increase in demand for the home currency and the appreciation of home currency. The stock-oriented model emphasizes that capital and financial accounts are the main factors that cause the exchange rate changes. It only considers the impact of stock index on exchange rate and believes that a stronger stock index will bring a stronger domestic currency. When a country's capital account can be freely convertible, the stock index will have a very significant impact on the exchange rate.

From the above analysis, we conclude that microeconomic entities will obtain information according to various macro market indicators and take rational actions to face the fluctuation of economic indicators such as the exchange rate. This will lead to changes in other economic variables such as trade and investment. Therefore, we divide the whole economic system into three levels, as shown in Fig. 2. The first is the layer of the micro subjects, including residents, enterprises, central banks and so on. The second level is economic indicators, such as trade volume, GDP and capital flow. The third level refers to macro market indicators, including exchange rate, stock index, and interest rate. Exchange rate, stock index, and other market indicators are essentially determined by supply and demand. When there is no external shock, the exchange rate and stock index are stable and cannot show the relevant relationship. However, if there is an external shock and the economic system is no longer stable, the micro subject will change its behavior, which will lead to the change of macro indicators, and then convey the shock information to other micro subjects, thus affecting the whole economic system. And then, we can observe the correlation between exchange rate and stock index. Therefore, the change between exchange rate and stock index is the transmission of external shocks and internal unstable information between the two.

Based on the three-level economic system, the stock-oriented model and the flow- oriented model, we got the theoretical framework of the interaction between exchange

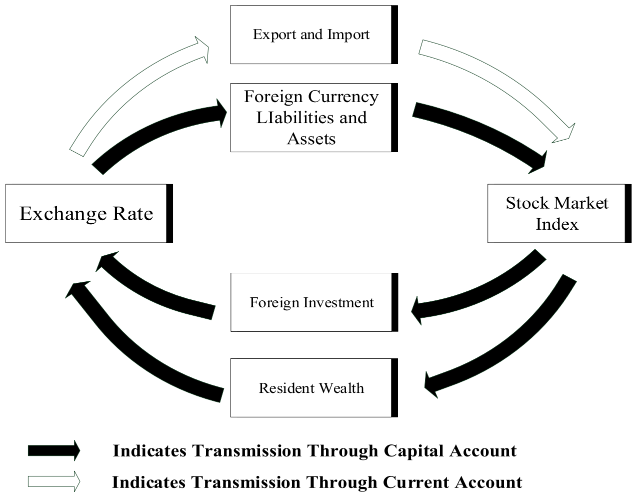

FIGURE 3. The transmission channel of interaction between exchange rate and stock index.

rate and stock index. The exchange rate influences the stock index through trade and foreign assets, and the stock index influences the exchange rate through investment, as shown in Fig 3. Therefore, we conclude that when the current account and capital account can flow freely, the exchange rate and the stock index have a significant influence on each other. This relationship only exists when the exchange rate and the stock index market are completely floating and the amount of trade and investment is large enough.

The B&R initiative is helpful to strengthen the links between trade and investment of China and some countries along the B&R initiative, so that more Chinese capital will flow into these countries, and their imports and exports will also increase. According to our analysis and related literature [14], [19], trade and investment are the key factors influencing the interaction between exchange rate and stock index of two countries. When trade or investment rises, the interaction between exchange rate and stock index will be more obvious, so we can analyze the impacts of the B&R initiative through the changes of their relationship before and after the B&R initiative. Furthermore, according to the causality of exchange rate and stock index, we can separately infer whether the trade and investment between the two countries have changed significantly after the B&R initiative was proposed. It is worth emphasizing that we can observe these links only when the exchange rates are close to free floating.

IV. MATERIALS AND METHODS

A. DATA DESCRIPTION

The B&R initiative includes 64 countries. Among these countries, some experienced a long-term war or their domestic political and economic environments were unstable. Some countries' exchange rate regimes have been adjusted between fixed and floating. Whereas, the exchange rates of some countries have long been anchored by other developed countries' currencies. For example, Fig. 4 shows the real exchange rates of Bulgaria (BGN), Czech Republic (CZK), Croatia (KHR), and Macedonia (MKD) anchored by the euro. As we can see, their exchange rates are moving in a similar direction. Thus, we collected daily trading data of countries with the floating exchange rate regimes from 16 September 2008 to 16 October 2018 and used indirect quotations for research. The above-mentioned cases, however, are excluded. Table 1 shows the 14 selected countries with their corresponding symbols.

TABLE 1. Countries and corresponding symbols.

Countries	Exchange rate of CNY over the currency of the country	Countries	Exchange rate of CNY over the currency of the country
Albania	ALL	Kuwait	KWD
Armenia	AMD	Romania	RON
Kazakhstan	KZT	Moldova	MDL
Indonesia	IDR	Malaysia	MYR
Kyrgyzstan	KGS	Philippines	PHP
Hungary	HUF	India	INR
Israel	ILS	Singapore	SGD

B. THE FUZZY TECHNIQUE

1) THE PEARSON CORRELATION COEFFICIENT

The Pearson correlation coefficient also referred to the Pearson product-moment correlation coefficient. It is a measure to describe the strength of the linear relationship of two random variables, which is bounded between −1 and 1.

Let $X = \{x_1, x_2, \ldots x_n\}$ and $Y = \{y_1, y_2, \ldots y_n\}$ be two random samples of size n, then the sample means and the sample standard deviations can be written as:

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \bar{Y} = \frac{\sum_{i=1}^{n} y_i}{n} \tag{1}$$

$$S_x = \sqrt{\frac{\sum_{i=1}^{n} \left(x_i - \bar{X}\right)^2}{n-1}} \quad S_y = \sqrt{\frac{\sum_{i=1}^{n} \left(y_i - \bar{Y}\right)^2}{n-1}} \tag{2}$$

and the Pearson correlation coefficient r_{XY} between X and Y can be defined as:

$$r_{XY} = \frac{\sum_{i=1}^{n} \left(x_i - \bar{X}\right)\left(y_i - \bar{Y}\right) / (n-1)}{S_x \cdot S_y} \tag{3}$$

If $r_{XY} = 0$, X and Y are linearly independent, if $r_{XY} > 0$, X and Y are linear positive correlation, if $r_{XY} < 0$, X and Y are linear negative correlation. It is one of the classic indexes for measuring the correlation of two crisp datasets.

2) A CENTROID BASED MEASURE

In the above analysis, we conclude that only when the exchange rates can reflect the changes of supply and demand, the linkages between exchange rates and stock indexes are

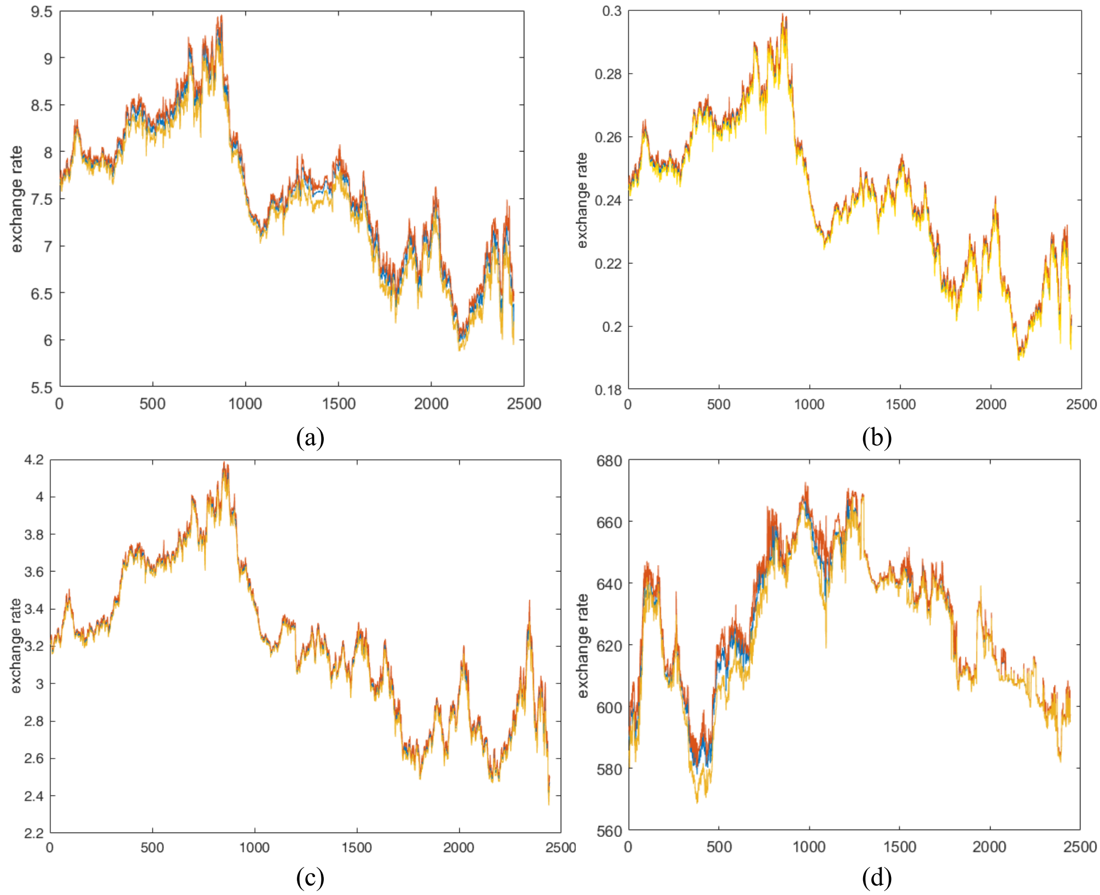

FIGURE 4. This figure shows the exchange rates of CNY over the currencies of some countries: (a) Description of the exchange rate of CNY-BGN; (b) Description of the exchange rate of CNY-CZK; (c) Description of the exchange rate of CNY-KHR; (d) Description of the exchange rate of CNY-MKD.

true and meaningful. Hinkle and Nsengiyumva believed that the closing price of the exchange rate could not fully reflect the real exchange rate changes, and it is easily disturbed by various factors such as the operation of the central bank [15].

As compared to the previous studies on the exchange markets and the stock markets, which used the closing price data, the present work uses the centroid based method and the integral based method to convert the data into fuzzy numbers. We use the similar approach of Chiang and Nancy' work [4], which analyzed the correlation between the two fuzzy sets by simply calculating the mean value of fuzzy data. First, we regard the real exchange rates of a day as triangular fuzzy sets, and construct their membership functions through the closing price, the highest price and the lowest price of the exchange rates. Then, we get the crisp number which can represent the whole fuzzy sets by the centroid based measure and the integral based measure to study the linkages between exchange rates and stock indexes.

In our work, the universe of discourse X is the set of all possible values of real exchange rate. Given a map,

$$u : X \rightarrow [0, 1] \tag{4}$$

u determines a fuzzy subset s on X. We think that the price lower than the lowest price or higher than the highest price cannot be the real exchange rate. Since most studies directly used the closing price [2], [4], [5], we believe that the closing price is often the closest to the real exchange rate. Meanwhile, all data of each trading day are between the lowest price and the highest price which may be relatively far from the real exchange rate. Therefore, we can determine the membership function of the real exchange rate according to the triangular fuzzy set, as defined in Eqs. (5). We assume that the fuzzy subset is a set of the highest price and the lowest price of exchange rate. For all $x \in X$, the value $u(x)$ is the membership degree of x to s, and the function $u(x)$ is the membership function of fuzzy set s. The range of values for $u(x)$ is from 0 to 1, implying that the range of values for the probability that x belongs to s is from 0 to 1, that is, the exchange rate between the highest price and the lowest price has a certain probability of being the real exchange rate. And if $u(x) = 0$, x does not belong to s at all, if $u(x) = 1$, x most likely belongs to s.

Hadi-Vencheh and Mokhtarian proposed a fuzzy centroid based method [1]. which is simple and effective in the extant measures for comparing fuzzy numbers. For a triangular fuzzy number $\tilde{A}=(a_1, a_2, a_3; 1)$, a_1, a_2 and a_3 represent the left threshold, the middle threshold and the right threshold of $\tilde{A}$ respectively. Its membership function $u_{\tilde{A}}(x)$ is:

$$u_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \frac{x-a_2}{a_3-a_2}, & a_2 < x \le a_3 \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

and its centroid respectively denoted by $\bar{\tilde{A}}_{cent}$ is:

$$\bar{\tilde{A}}_{cent} = \frac{1}{3}(a_1 + a_2 + a_3) \quad (6)$$

For two fuzzy datasets $\tilde{X}$, $\tilde{Y}$ which represent the exchange rates and the stock price indexes, their fuzzy datasets are calculated by the centroid based measure respectively and are denoted by $\bar{\tilde{X}}_{cent}$ and $\bar{\tilde{Y}}_{cent}$, the results are calculated by the integral based measure denoted by $\bar{\tilde{x}}_i^{cent}$ and $\bar{\tilde{y}}_i^{cent}$:

$$\tilde{X} = \{\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n\} = \left\{\left(x_1^1, x_1^2, x_1^3\right), \ldots, \left(x_n^1, x_n^2, x_n^3\right)\right\} \quad (7)$$

$$\tilde{Y} = \{\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_n\} = \left\{\left(y_1^1, y_1^2, y_1^3\right), \ldots, \left(y_n^1, y_n^2, y_n^3\right)\right\} \quad (8)$$

$$\bar{\tilde{X}}_{cent} = \left\{\bar{\tilde{x}}_1^{cent}, \bar{\tilde{x}}_2^{cent}, \ldots, \bar{\tilde{x}}_n^{cent}\right\}$$

$$\bar{\tilde{Y}}_{cent} = \{\bar{\tilde{y}}_1^{cent}, \bar{\tilde{y}}_2^{cent}, \ldots, \bar{\tilde{y}}_n^{cent}\} \quad (9)$$

$$\bar{\tilde{x}}_i^{cent} = \frac{1}{3}(x_i^1 + x_i^2 + x_i^3) \quad \forall i = 1, 2, \ldots, n \quad (10)$$

$$\bar{\tilde{y}}_i^{cent} = \frac{1}{3}(y_i^1 + y_i^2 + y_i^3) \quad \forall i = 1, 2, \ldots, n \quad (11)$$

Then, the means and the standard deviations of two fuzzy datasets based on the centroid method can be written as:

$$\bar{\bar{\tilde{X}}}^{cent} = \frac{1}{n}\sum_{i=1}^{n} \bar{\tilde{x}}_i^{cent} \quad \bar{\bar{\tilde{Y}}}^{cent} = \frac{1}{n}\sum_{i=1}^{n} \bar{\tilde{y}}_i^{cent} \quad (12)$$

$$S_{\bar{\tilde{X}}_{cent}} = \sqrt{\frac{\sum_{i=1}^{n}\left(\bar{\tilde{x}}_i^{cent} - \bar{\bar{\tilde{X}}}^{cent}\right)^2}{n-1}}$$

$$S_{\bar{\tilde{Y}}_{cent}} = \sqrt{\frac{\sum_{i=1}^{n}\left(\bar{\tilde{y}}_i^{cent} - \bar{\bar{\tilde{Y}}}^{cent}\right)^2}{n-1}} \quad (13)$$

and the Pearson correlation coefficient of the centroid based measure is:

$$r_{\bar{\tilde{X}}_{cent}\bar{\tilde{Y}}_{cent}} = \frac{\sum_{i=1}^{n}\left(\bar{\tilde{x}}_i^{cent} - \bar{\bar{\tilde{X}}}^{cent}\right)\left(\bar{\tilde{y}}_i^{cent} - \bar{\bar{\tilde{Y}}}^{cent}\right)/(n-1)}{S_{\bar{\tilde{X}}_{cent}} \cdot S_{\bar{\tilde{Y}}_{cent}}} \quad (14)$$

Compared with the traditional studies using the closing price, the centroid based measure gives the highest, lowest and closing prices a same weight, indicating that each price is equally important for fuzzy numbers.

3) AN INTEGRAL BASED MEASURE

Liou and Wang proposed an integral method which considered the optimism level [27]. For a triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3; 1)$, γ represents the optimism level, $\gamma \in [0,1]$, and can be used to change the weight of each fuzzy number. When γ changes from 0 to 1, the weight of a_1 changes from 0 to 1/2, which means that the weight of the lowest price in calculating the fuzzy number of the exchange rates and stock price indexes changes from 0 to 1/2. With its membership function $u_{\tilde{A}}(x)$ defined in formulas Eqs. (5), the formula for the integral based measure is expressed as:

$$\bar{\tilde{A}}_{intel} = \frac{1}{2}\left(\gamma a_1 + a_2 + (1-\gamma)\, a_3\right) \quad (15)$$

For two fuzzy datasets $\tilde{X}$, $\tilde{Y}$ defined in formulas Eqs. (7) and (8), their fuzzy datasets are calculated by the integral based measure denoted by $\bar{\tilde{X}}_{intel}$ and $\bar{\tilde{Y}}_{intel}$, the results are calculated by the integral based measure denoted by $\bar{\tilde{x}}_i^{intel}$ and $\bar{\tilde{y}}_i^{intel}$:

$$\bar{\tilde{X}}_{intel} = \left\{\bar{\tilde{x}}_1^{intel}, \bar{\tilde{x}}_2^{intel}, \ldots, \bar{\tilde{x}}_n^{intel}\right\}$$

$$\bar{\tilde{Y}}_{intel} = \{\bar{\tilde{y}}_1^{intel}, \bar{\tilde{y}}_2^{intel}, \ldots, \bar{\tilde{y}}_n^{intel}\} \quad (16)$$

$$\bar{\tilde{x}}_i^{intel} = \frac{1}{2}\left(\gamma x_i^1 + x_i^2 + (1-\gamma)x_i^3\right) \quad \forall i = 1, 2, \ldots, n \quad (17)$$

$$\bar{\tilde{y}}_i^{intel} = \frac{1}{2}\left(\gamma y_i^1 + y_i^2 + (1-\gamma)y_i^3\right) \quad \forall i = 1, 2, \ldots, n \quad (18)$$

Then, the means and the standard deviations of two fuzzy datasets based on the integral method can be written as:

$$\bar{\bar{\tilde{X}}}^{intel} = \frac{1}{n}\sum_{i=1}^{n} \bar{\tilde{x}}_i^{intel} \quad \bar{\bar{\tilde{Y}}}^{intel} = \frac{1}{n}\sum_{i=1}^{n} \bar{\tilde{y}}_i^{intel} \quad (19)$$

$$S_{\bar{\tilde{X}}_{intel}} = \sqrt{\frac{\sum_{i=1}^{n}\left(\bar{\tilde{x}}_i^{intel} - \bar{\tilde{X}}^{intel}\right)^2}{n-1}}$$

$$S_{\bar{\tilde{Y}}_{intel}} = \sqrt{\frac{\sum_{i=1}^{n}\left(\bar{\tilde{y}}_i^{intel} - \bar{\tilde{Y}}^{intel}\right)^2}{n-1}} \quad (20)$$

and the Pearson correlation coefficient of the integral based measure is:

$$r_{\bar{\tilde{X}}_{intel}\bar{\tilde{Y}}_{intel}} = \frac{\sum_{i=1}^{n}\left(\bar{\tilde{x}}_i^{intel} - \bar{\bar{\tilde{X}}}^{intel}\right)\left(\bar{\tilde{y}}_i^{intel} - \bar{\bar{\tilde{Y}}}^{intel}\right)/(n-1)}{S_{\bar{\tilde{X}}_{intel}} \cdot S_{\bar{\tilde{y}}_{intel}}} \quad (21)$$

Compared with the centroid based method, the integral based method gives the highest and the lowest price changing weights and it adds the parameter γ, which can be considered as the level of investors' expectations for future markets in the financial markets. As the parameter γ approaches 0, the investors have higher expectations for the future. It is valuable to further compare the two methods to find out whether there are other differences between the two methods.

4) COMPARISON BETWEEN FUZZY NUMBER AND CLOSING PRICE

According to above theoretical analysis, an important condition for the significant interaction between exchange rate and stock index is that the two markets are completely floating. At present, since most countries do not regard the stability of the stock market as a policy objective, we think that the stock market is completely floating. However, in the exchange rate market, most central banks take maintaining a stable currency as an important policy goal and adopt various methods such as open market operations to keep the exchange rate stable. As a participant in the foreign exchange market, the central bank has an impact on the change of exchange rate. Therefore, considering the unpredictability of the real exchange rate, we regard the exchange rate as a fuzzy set which can reflect more features of continuous data to use the fuzzy data rather than the closing price to study the linkages between exchange rates and stock indexes.

We try to show that fuzzy data can better reflect the real exchange rate information and get more accurate research results than closing price. As for the comparison between the fuzzy data and the closing price, we should determine the standard of comparison first. Here we take the amount of information contained in the fuzzy data and the closing price as the standard.

We take a group of values as an example: in a trading day, the lowest price, closing price and the highest price of the exchange rate can be written as a triangular fuzzy number $\widetilde{A}_1 = (60, 100, 160)$, and the data on the other day are written as $\widetilde{A}_2 = (80, 100, 160)$. Obviously, we can find that for these two triangular fuzzy numbers, their most likely values are the same. However, their minimum values are different, leading to different membership degrees, so the uncertainty of the two fuzzy sets is different, which is also called fuzzy entropy.

We assume that S represents the closing price, H represents the highest price, and L represents the lowest price, then the fuzzy entropy of triangular fuzzy data of exchange rate can be expressed as:

$$\begin{aligned} H\left(\tilde{A}\right) = -k \int_L^S [&\left(\frac{x-L}{S-L}\right) Ln \left(\frac{x-L}{S-L}\right) \\ &+ \left(1-\frac{x-L}{S-L}\right) Ln \left(1-\frac{x-L}{S-L}\right)]dx \\ &+ \int_S^H \left[\left(\frac{x-S}{H-S}\right) Ln \left(\frac{x-S}{H-S}\right)\right. \\ &\left. + \left(1-\frac{x-S}{H-S}\right) Ln \left(1-\frac{x-S}{H-S}\right)\right] dx \\ &(k \text{is a constant}) \end{aligned} \tag{22}$$

Then, the fuzzy entropies of the above two fuzzy data are $50k$ and $40k$, implying that the fuzzy degree of $\widetilde{A}_1$ is higher than that of $\widetilde{A}_2$. Only considering the closing price of exchange rates cannot reflect this difference. However, based on the fuzzy theory, we can calculate their fuzzy mean values for the two triangular fuzzy numbers, which are 107 and 113, respectively. The difference of fuzzy mean is due to the different membership function, which can reflect the difference of fuzzy entropy to some extent. We can find that the lowest price in the first trading day is smaller than that in the second trading day, and we think that the operation of the central bank whose goal is to keep the exchange rate stable has less impact on the first day. Compared with the same closing price data in two trading days, the fuzzy representative value which reflects the fuzzy entropy difference of fuzzy sets can show this difference. Additionally, we use the boundary values of triangular fuzzy sets to calculate the fuzzy data. Compared with the most probable value of fuzzy sets, that is, the closing price in our work, the fuzzy degree of the fuzzy data considering the boundary of fuzzy sets is the fuzzy entropy. Therefore, we conclude that using the fuzzy number can reflect more real information.

C. RELATIONSHIP MEASUREMENT METHODS

Firstly, we used the closing price to make a preliminary judgment on the relationship between exchange rate and stock index using the Pearson correlation coefficient. Pearson correlation coefficient can only show whether there is a linear correlation between exchange rate and stock index, but not whether there is a causal relationship between them. Then we processed the exchange rate data during the period of 2008.09.16-2018.10.16 by two fuzzy methods, that is, the centroid based measure and the integral based measure, and collected the data of SSE and SZSE composite indexes during the same period. We used the Granger causality test to study the relationship between the exchange rates and SSE composite index (SHI) as well as SZSE composite index (SZI) to find what are the linkages between the two variables and whether the proposal of the B&R initiative impacts the linkages. More importantly, we found that the empirical results of using fuzzy data were better than the results of using crisp closing price data.

V. RESULTS AND ANALYSIS

A. ANALYSIS OF THE CORRELATION BETWEEN EXCHANGE RATE AND STOCK INDEX

The Pearson correlation coefficient r is used to reflect the degree of linear correlation between two variables. The absolute value of r is related to the strength of the correlation. We analyzed the correlation between exchange rates and stock indexes during the study period in this paper.

Fig. 5 shows the correlation coefficients between the exchange rates of CNY over the currencies of the countries along the B&R route SHI during the period of 2008.09.16-2018.10.16. Among the 14 selected countries, only the correlation coefficient between the exchange rate of CNY over Israel shekel and the SHI is negative. It can be understood that with the rise of the exchange rate of CNY over Israel shekel, the SHI showed a downward trend. Whereas, the correlation coefficients between exchange rates of CNY over the currencies of Kyrgyzstan, Moldova, Malaysia, and the SHI are positive and greater than 0.5.

TABLE 2. ADF test results of the exchange rates and the stock price indexes before and after the B&R initiative.

Variables	C (Closing price)		Centroid (The centroid based measure)		The integral based measure						Results	
					Intel0(γ=0)		Intel0.5(γ=0.5)		Intel1(γ=1)			
	before	after	before	after	before	after	before	after	before	after	before	after
SHI	-0.3585	-0.5735	—	—	—	—	—	—	—	—	unstable	unstable
SZI	-0.4843	-0.2589	—	—	—	—	—	—	—	—	unstable	unstable
ALL	-1.1611	0.2486	-1.1128	0.2531	-1.0986	0.2778	-1.1122	0.2650	-1.1318	0.2571	unstable	unstable
AMD	-2.0732	-0.4539	-2.1746	-0.4696	-2.0218	-0.4844	-2.1486	-0.4701	-2.1807	-0.4613	stable	unstable
HUF	-0.9078	-0.5536	-0.8906	-0.5568	-0.8798	-0.5540	-0.8861	-0.5581	-0.8871	-0.5686	unstable	unstable
IDR	-1.3002	-1.0842	-1.3518	-1.0653	-1.2739	-1.0920	-1.2597	-1.0654	-1.2662	-1.1208	unstable	unstable
ILS	-0.6645	0.6601	-0.5831	0.6560	-0.5859	0.6485	-0.6016	0.6609	-0.6248	0.6550	unstable	unstable
INR	-2.0751	-0.4790	-1.9673	-0.5985	-2.2796	-0.5477	-2.2768	-0.5422	-2.2678	-0.5330	stable	unstable
KGS	-2.7513	-1.1797	-2.9366	-1.1645	-3.0003	-1.2041	-2.8326	-1.1746	-2.7852	-1.2165	stable	unstable
KWD	-2.8708	0.6348	-2.8730	0.6252	-2.5610	0.6466	-2.8638	0.6361	-2.9029	0.6244	stable	unstable
KZT	-1.9161	-1.8123	-1.8739	-1.8674	-1.7327	-1.7983	-1.9956	-1.8867	-1.9195	-1.8955	unstable	unstable
MDL	-1.8966	-0.7283	-2.0108	-0.6664	-1.9595	-0.7305	-1.1611	-0.6669	-2.0800	-0.6996	unstable	unstable
MYR	-0.2541	-0.7812	-0.2479	-0.8405	-0.2473	-0.8162	-0.2465	-0.8319	-0.2476	-0.8338	unstable	unstable
PHP	-2.8729	-0.8876	-0.2831	-0.8936	-2.7662	-0.9018	-2.6791	-0.9018	-2.6794	-0.9086	stable	unstable
RON	-1.0808	-0.4413	-1.1343	-0.4755	-1.0974	-0.4476	-1.1261	-0.4501	-1.1469	-0.4572	unstable	unstable
SGD	0.1248	0.1604	0.1366	0.1864	0.1514	0.1800	0.1459	0.1824	0.1420	0.1806	unstable	unstable

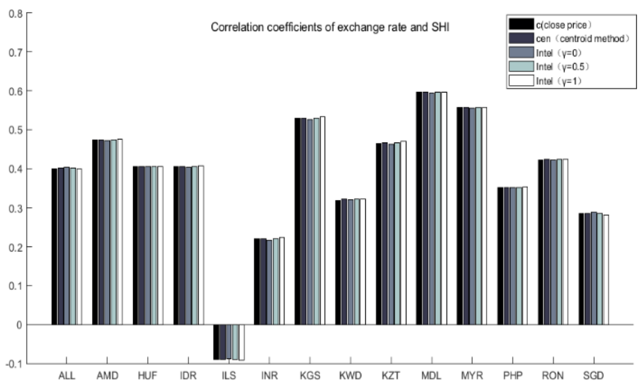

FIGURE 5. Correlation coefficient between exchange rate and the Shanghai stock price index.

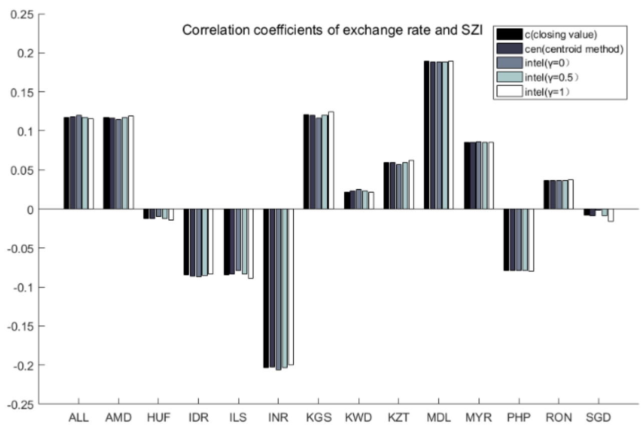

FIGURE 6. Correlation coefficient between exchange rate and the Shenzhen stock price index.

Fig. 6 shows the correlation coefficients between the exchange rates and SZI. Obviously, the correlation coefficients between the exchange rates and the SZI are also mostly positive. However, correlation coefficients between the same exchange rate and SHI as well as SZI are different. In specific, exchange rates of CNY over Hungary, Indonesia, India, Philippines and Singapore are positively correlated with SHI, but are negatively correlated with SZI. And the correlation coefficients between the SZI and the exchange rates of CNY over the currencies of Hungary and Singapore are rather weak. Similarly, with the rise of the exchange rate of CNY over Indian rupee, the SZI shows a weaker downward trend. The remaining exchange rates are positively correlated with the SZI and the correlation coefficients are small.

The correlations between those two variables are quite different, like the correlations between exchange rates and the SHI are generally higher than SZI. The result is not only influenced by the large volume of sample data, but also is related to the composition of the stock indexes. The SHI is a composite index, which comprehensively reflects the share trend of all A-share and B-share listed stocks in the Shanghai stock market, covering a more comprehensive range. The SZI refers to the component stock index, which is used to extract the stocks of 40 listed companies with market representativeness from all the listed stocks as the calculation object. The data source is representative of influential companies in all the listed companies and has certain one-sidedness.

TABLE 3. ADF test results of first order difference before and after the B&R initiative proposed.

Variables	C (Closing price)		Centroid (The centroid based measure)		The integral based measure						Results	
					Intel0(γ=0)		Intel0.5(γ=0.5)		Intel1(γ=1)			
	before	after	before	after	before	after	before	after	before	after	before	after
SHI	-14.987	-14.987	—	—	—	—	—	—	—	—	stable	stable
SZI	-31.576	-31.576	—	—	—	—	—	—	—	—	stable	stable
ALL	-37.091	-37.091	-27.7166	-27.7166	-31.210	-31.210	-29.3413	-29.3413	-30.307	0.2571	stable	stable
HUF	-38.166	-38.166	-26.7911	-26.7911	-30.637	-30.637	-29.0496	-29.0496	-30.382	-0.5686	stable	stable
IDR	-30.477	-30.477	-27.1120	-27.1120	-28.402	-28.402	-27.3115	-27.3115	-23.695	-1.1208	stable	stable
ILS	-27.683	-27.683	-24.7698	-24.7698	-25.118	-25.118	-25.1662	-25.1662	-25.338	0.6550	stable	stable
KZT	-26.870	-26.870	-17.7396	-17.7396	-33.477	-33.477	-18.1415	-18.1415	-17.380	-1.8955	stable	stable
MDL	-38.296	-38.296	-21.0525	-21.0525	-22.742	-22.742	-21.5320	-21.5320	-31.145	-0.6996	stable	stable
MYR	-31.063	-31.063	-24.0019	-24.0019	-23.407	-23.407	-23.9102	-23.9102	-24.506	-0.8338	stable	stable
RON	-37.098	-37.098	-23.1254	-23.1254	-29.554	-29.554	-28.4159	-28.4159	-30.042	-0.4572	stable	stable
SGD	-38.979	-38.979	-25.0066	-25.0066	-25.244	-25.244	-25.0883	-25.0883	-25.354	0.1806	stable	stable

TABLE 4. Granger test results of the exchange rates and the SHI before the B&R initiative proposed.

Null Hypothesis:	Prob				
	C	Centroid	The integral based measure		
			Intel(γ=0)	Intel(γ=0.5)	Intel(γ=1)
ALL does not Granger Cause SHI	0.4182	0.2505	0.0031 ***	0.1220	0.2194
prrrrv	0.0000 ***	0.0001 ***	0.0000 ***	0.0002 ***	0.0001 ***
HUF does not Granger Cause SHI	0.0013 ***	0.0000 ***	0.0002 ***	0.0004 ***	0.0126 **
SHI does not Granger Cause HUF	0.0000 ***	0.0000 ***	0.0000 ***	0.0000 ***	0.0000 ***
IDR does not Granger Cause SHI	0.1364	0.2854	0.4100	0.2174	0.1094
SHI does not Granger Cause IDR	0.1705	0.1639	0.0507 *	0.0356 **	0.1926
ILS does not Granger Cause SHI	0.3466	0.0064 ***	0.0084 ***	0.0131 **	0.0922 *
SHI does not Granger Cause ILS	0.0004 ***	0.0057 ***	0.0002 ***	0.0021 ***	0.0061 ***
KZT does not Granger Cause SHI	0.6395	0.4382	0.4471	0.5042	0.3668
SHI does not Granger Cause KZT	0.0983 *	0.1383	0.8679	0.1252	0.1725
MDL does not Granger Cause SHI	0.4387	0.6404	0.3937	0.5462	0.6984
SHI does not Granger Cause MDL	0.3347	0.5974	0.7904	0.4432	0.2124
MYR does not Granger Cause SHI	0.3940	0.4059	0.1353	0.2387	0.8234
SHI does not Granger Cause MYR	0.8491	0.2311	0.7060	0.2880	0.3065
RON does not Granger Cause SHI	0.0001 ***	0.0006 ***	0.0073 ***	0.0030 ***	0.0207 **
SHI does not Granger Cause RON	0.0000 ***	0.0000 ***	0.0000 ***	0.0000 ***	0.0000 ***
SGD does not Granger Cause SHI	0.0787 *	0.0022 ***	0.0016 ***	0.0028 ***	0.0642 *
SHI does not Granger Cause SGD	0.0000 ***	0.0002 ***	0.0000 ***	0.0000 ***	0.0000 ***

B. THE UNIT ROOT TEST BEFORE AND AFTER THE B&R INITIATIVE

In our work, we regard exchange rate and stock index as two groups of random time series data. If their statistical characteristics do not change with time, they can be called as stationary time series, otherwise they are non-stationary time series. The stationary time series can be used to make the Granger test and build econometric models, but the non-stationary time series data can neither be used for Granger test, nor be used to predict the future changes of time series.

In order to check the stationary to avoid the pseudo-regression problem caused by the unstable time series data,

TABLE 5. Granger test results of the exchange rates and the SHI after the B&R initiative proposed.

Null Hypothesis:	Prob.				
	C	Centroid	The integral based measure		
			Intel(γ=0)	Intel(γ=0.5)	Intel(γ=1)
ALL does not Granger Cause SHI	0.0438**	0.0065***	0.0475**	0.0144**	0.0043***
SHI does not Granger Cause ALL	0.0079***	0.0041***	0.0022***	0.0018***	0.0092***
HUF does not Granger Cause SHI	0.0908*	0.1404	0.239	0.1701	0.1041
SHI does not Granger Cause HUF	0.1407	0.5694	0.1456	0.2658	0.5967
IDR does not Granger Cause SHI	0.9566	0.3395	0.6794	0.7182	0.7256
SHI does not Granger Cause IDR	0.1503	0.4111	0.7141	0.4719	0.1368
ILS does not Granger Cause SHI	0.7306	0.6950	0.4422	0.5034	0.6306
SHI does not Granger Cause ILS	0.8299	0.7723	0.3964	0.0195**	0.9722
KZT does not Granger Cause SHI	0.1565	0.2265	0.2565	0.2519	0.0508*
SHI does not Granger Cause KZT	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
MDL does not Granger Cause SHI	0.1482	0.0035***	0.0022***	0.0087***	0.044**
SHI does not Granger Cause MDL	0.024**	0.2746	0.122	0.1583	0.3663
MYR does not Granger Cause SHI	0.049**	0.0373**	0.0034***	0.0194**	0.0481**
SHI does not Granger Cause MYR	0.0592*	0.0357**	0.0223**	0.0631*	0.0823*
RON does not Granger Cause SHI	0.0019***	0.0064***	0.0364**	0.0084***	0.0014***
SHI does not Granger Cause RON	0.0016***	0.0148**	0.0008***	0.0063***	0.0223**
SGD does not Granger Cause SHI	0.0025***	0.0067***	0.0041***	0.0081***	0.0313**
SHI does not Granger Cause SGD	0.0185**	0.0026***	0.0135**	0.0065***	0.0534*

we need to perform ADF (the unit root test) on the data before the Granger test. When the ADF test results show that the data are stable, it means we can use the stationary time series to make the Granger test and predict the actual situation by establishing an econometric model.

Table 2 shows the unit root test results of the exchange rates and stock price indexes before the B&R initiative (2008.09.16-2013.10.31) and after the B&R initiative (2013.11.01-2018.10.16). The exchange rates of CNY over currencies of the five countries, Armenia, India, Kyrgyzstan, Kuwait, and Philippines, were steadily time-series data at a 5% confidence level (−1.9411) before the B&R initiative was proposed, and all became unstable time series after the B&R initiative was proposed. The rest exchange rates were unstable time series data at the 5% confidence level both before and after the B&R initiative.

The non-stationary time series can be transformed into stable time series by differential processing. Thus, after the first-order difference, we performed the unit root test again and the results are shown in Table 3. The ADF test results show that all exchange rates and stock indexes have passed the unit root test at the 1% confidence level whether it is before or after the raise of the B&R initiative, implying we can make the Granger test for these stationary time series.

C. THE GRANGER CAUSALITY TEST BEFORE AND AFTER THE B&R INITIATIVE

The Granger causality test is usually used to analyze the causality between variables. For time series data, the Granger causality between two economic variables X and Y is defined as: if variable X is useful to explain the future change of variable Y, then variable X is considered as the Granger cause of variable Y, which is also expressed as "X does Granger cause Y" in Granger test. If X represents the exchange rate, and Y represents the stock index, the "flow-oriented" model argues that X is the Granger cause of Y, on the contrary, the "stock-oriented" model argues that Y is the Granger cause of X.

In order to further study the relationship between the exchange rates and the stock indexes, we use the two stock indexes and the exchange rates of CNY over nine countries' currencies, to make the Granger test, all of which are first-order single-integrated data. The results of the exchange rates and the Stock market index are shown in Tables 4 to 7.

By analyzing the Granger test results, we find that the linkages between exchange rates and SHI as well as SZI changes in different degrees before and after the B&R initiative was proposed. (1) Whether the closing price or fuzzy data are used, the SHI and the exchange rates of CNY

TABLE 6. Granger test results of the exchange rates and the SZI before the B&R initiative proposed.

Null Hypothesis:	Prob.				
	C	Centroid	The integral based measure		
			Intel(γ=0)	Intel(γ=0.5)	Intel(γ=1)
ALL does not Granger Cause SZI	0.4053	0.3876	0.0370**	0.2151	0.2987
SZI does not Granger Cause ALL	0.0002***	0.0001***	0.0004***	0.0008***	0.0005***
HUF does not Granger Cause SZI	0.2271	0.0168**	0.0267**	0.0509*	0.0227**
SZI does not Granger Cause HUF	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
IDR does not Granger Cause SZI	0.1456	0.3652	0.1846	0.2584	0.0611*
SZI does not Granger Cause IDR	0.1240	0.2028	0.4479	0.0659	0.1814
ILS does not Granger Cause SZI	0.4491	0.2943	0.1395	0.2419	0.1345
SZI does not Granger Cause ILS	0.0016***	0.0049***	0.0004***	0.0004***	0.0094***
KZT does not Granger Cause SZI	0.8727	0.6673	0.5579	0.7455	0.7068
SZI does not Granger Cause KZT	0.1495	0.1557	0.9617	0.1492	0.2141
MDL does not Granger Cause SZI	0.4186	0.7490	0.3780	0.5707	0.7441
SZI does not Granger Cause MDL	0.0735*	0.3785	0.3938	0.2359	0.1233
MYR does not Granger Cause SZI	0.2893	0.5163	0.1873	0.3165	0.7468
SZI does not Granger Cause MYR	0.8243	0.2100	0.4146	0.2553	0.4115
RON does not Granger Cause SZI	0.0018***	0.0057***	0.0036***	0.0038***	0.0021***
SZI does not Granger Cause RON	0.0002***	0.0000***	0.0000***	0.0000***	0.0001***
SGD does not Granger Cause SZI	0.1991	0.0180**	0.0080***	0.0237**	0.0268**
SZI does not Granger Cause SGD	0.0000***	0.0005***	0.0001***	0.0001***	0.0008***

over the currencies of Romania, Hungary, and Singapore have bidirectional causality before the B&R initiative was proposed. (2) There is no Granger causality between the exchange rate of CNY over the currencies of Hungary, while SHI and the exchange rates of CNY over the currencies of Albania and Malaysia have bidirectional causality after the B&R initiative. (3) For SZI, there is bidirectional causality between it and the exchange rate of CNY over the currencies of Romania, both before and after the B&R initiative. (4) SZI and the exchange rate of CNY over the currencies of Albania show a bidirectional causality after the B&R initiative. The three yellow circles in Fig. 9 represent there are consistent Granger test results between only three exchange rates and SHI or the SZI before and after the B&R initiative, and the six red circles represent the different results of the Granger test between six exchange rates and SHI or the SZI. Therefore, we can conclude that the proposal of the B&R initiative had an impact on the Granger test results of these variables. In the context of economic globalization and financial liberalization, the B&R initiative promoted the trade between China, South Asia and other countries more rapidly, and had a certain impact on the exchange rates, and stock indexes.

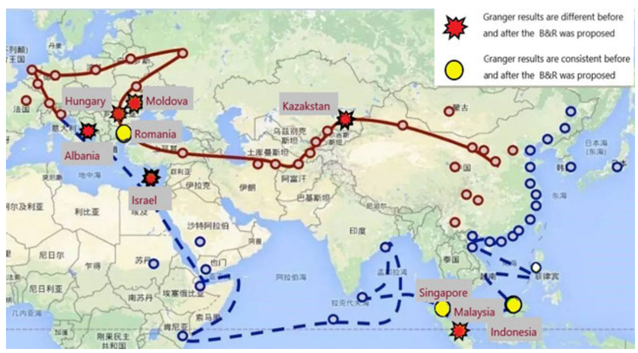

FIGURE 7. Granger test results before and after the B&R initiative.

Whether using the closing price or the fuzzy number, we can find that except the bidirectional causality, the most results of the Granger test have proved that stock price index is the Granger cause of exchange rate, implying the fluctuation of stock prices will cause the fluctuation of one country's exchange rate, which is consistent with the "stock-oriented" model.

Additionally, because of the influence of the central bank's monetary policy, the exchange rate data may not fully reflect the information that affects the exchange rate, which can

TABLE 7. Granger test results of the exchange rates and the SZI after the B&R initiative proposed.

Null Hypothesis:	Prob.				
	C	Centroid	The integral based measure		
			Intel(γ=0)	Intel(γ=0.5)	Intel(γ=1)
ALL does not Granger Cause SZI	0.0101**	0.0021***	0.0002***	0.0014***	0.0027***
SZI does not Granger Cause ALL	0.0218**	0.0190**	0.0051***	0.0043***	0.0073***
HUF does not Granger Cause SZI	0.0243**	0.0268**	0.0489**	0.0260**	0.0053***
SZI does not Granger Cause HUF	0.2477	0.1060	0.5529	0.1795	0.1328
IDR does not Granger Cause SZI	0.9256	0.1599	0.8177	0.4812	0.2963
SZI does not Granger Cause IDR	0.2471	0.6036	0.8331	0.6884	0.2410
ILS does not Granger Cause SZI	0.5728	0.3680	0.0761*	0.4137	0.4086
SZI does not Granger Cause ILS	0.9000	0.6880	0.2554	0.5280	0.8279
KZT does not Granger Cause SZI	0.1006	0.3246	0.2531	0.3665	0.0615*
SZI does not Granger Cause KZT	0.0001***	0.0013***	0.0002**	0.0008***	0.0053***
MDL does not Granger Cause SZI	0.0422**	0.0059***	0.0010***	0.0060***	0.0268**
SZI does not Granger Cause MDL	0.1397	0.1671	0.1489	0.1490	0.2109
MYR does not Granger Cause SZI	0.0102**	0.0076***	0.0016***	0.5311	0.9208
SZI does not Granger Cause MYR	0.2850	0.1368	0.0568*	0.7101	0.6661
RON does not Granger Cause SZI	0.0064***	0.0039***	0.0008***	0.0058***	0.0017***
SZI does not Granger Cause RON	0.0065***	0.0970*	0.0034***	0.0400**	0.0352**
SGD does not Granger Cause SZI	0.3483	0.1446	0.2356	0.1836	0.0748*
SZI does not Granger Cause SGD	0.0399**	0.0281**	0.0571*	0.0854*	0.0496**

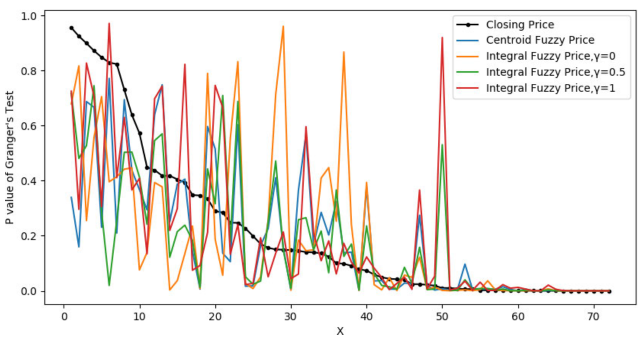

FIGURE 8. Comparison of Granger's results between the fuzzy number and the closing price.

lead to false results. Therefore, we compare the Granger test results of the closing price and the fuzzy numbers, which are shown in Fig. 8. We can see that when the p-value of the Granger test using the closing price is greater than 0.1, that is, at the 10% confidence level, the results of fuzzy numbers show no obvious consistency. But when the p-value of the Granger test using the closing price is less than 0.1, most of their results show special consistency.

If there is a causal relationship between the exchange rate and the stock index, it means that the fuzzy treatment of the exchange rate can offset part of the impact of the central bank's monetary policy, making their mutual influence more significant, which is shown in the figure as the consistency of the results of various fuzzy methods. However, when there is no real causal relationship between them, since the highest and lowest exchange rates have nothing to do with the change of stock index, the results of various fuzzy methods are quite different and not credible. Only when the p-value of the Granger test of the closing price and the fuzzy numbers are less than 0.1, we can believe that there is a causal relationship between exchange rate and stock index.

According to the results of fuzzy numbers, we can see that it is not credible to test the results of "SHI does not Granger cause MDL" and "MYR does not Granger cause SZI". We underlined it in Table 5 and Table 7. So, we can get the relationship between SHI and the exchange rate from Table 4 - 5. Before the B&R initiative, the exchange rate of the CNY over Albania was the Granger cause of the SHI but after the B&R initiative was proposed, the two have bidirectional causality. There is no Granger causality before the B&R initiative between the exchange rates of CNY over the currencies of the Kazakhstan, Moldova, Malaysia and the SHI. However, they show significant Granger causality after the B&R initiative.

VI. DISCUSSION AND INSIGHTS

This paper makes an empirical analysis of the linkages between the exchange rates and stock price indexes during the period from 2008.09.16 to 2018.10.16. The results of both the ADF and the Granger causality test provide evidence that the B&R initiative can influence the foreign exchange market and the Chinese stock market. After the B&R initiative, the Granger causal relationship between those variables has been strengthened. Specifically, we have found the two fuzzy methods, that is, the centroid based method and the integral based method, which have some advantages over the crisp method using the closing price. These can compress the data and reflect the volatility of the financial markets in each trading day. In addition, the integral based method adds the parameter of optimism level which can make the research results more accurate and comprehensive.

There are also some limitations of this work that need to be addressed: (1) This paper only analyzes the possible transmission mechanism between exchange rate and stock index on the theoretical level. Considering the difficulty of data acquisition, it is difficult to add trade and investment into the model for empirical research. Therefore, this paper only uses the data of exchange rate and stock index for empirical tests. When we can obtain the high-frequency data of trade and investment, the fuzzy techniques and some mixed frequency data processing methods like Mixed Data Sampling (MIDAS) will be used to make our empirical research more perfect. (2) In this paper, fuzzy technology is only used to analyze the causal relationship between exchange rate and stock index. Based on the empirical results, we preliminarily prove the effectiveness of fuzzy technology, but there is no basis for further analysis of fuzzy technology.

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