A Self-Training Numerical Method to Calculate the Magnetic Characteristics for Switched Reluctance Motor Drives

X. D. Xue, K. W. E. Cheng, *Member, IEEE*, and S. L. Ho

***Abstract*—Based on the two-dimensional (2-D) least squares method, this paper presents a novel numerical method to calculate the magnetic characteristics for switched reluctance motor drives. In this method, the 2-D orthogonal polynomials are used to model the magnetic characteristics. The coefficients in these polynomials are determined by the 2-D least squares method. These coefficients can be computed off line and can also be trained on line. The computed results agree well with the experimental results. In addition, the effect of the order number of the polynomials on the computation errors is discussed. The proposed method is very helpful in torque prediction, simulation studies and development of sensorless control of switched reluctance motor drives.**

***Index Terms*—Least squares methods, reluctance motors, reluctance motor drives, variable speed drives.**

I. Introduction

PRECISE computation of the nonlinear magnetic characteristics at an arbitrary rotor position and a current is crucial when performance predictions, simulations, computer-aided designs, torque control, as well as sensorless control of the switched reluctance motor (SRM) drives are carried out. However, the previous methods [1]–[3] are executed off line. In [1], Torrey and Lang present an analytical expression for the flux linkage/current/position data. While their equation can provide all the magnetic information, the resultant expression is a complicated function that includes cosine function and exponent function. Thus the coefficients have to be computed by a Fourier cosine series. On the basis of [1], Torrey, Niu, and Unkauf develop the piece-wise linear model [2]. In [3], the nonlinear model of the SRM uses the equivalent magnetic circuit of the motor as a set of reluctances linked in series and in parallel. However, those methods cannot describe accurately the dynamics of the SRM drives.

The nonlinear magnetic characteristics in the SRM are the functions of both the rotor position and the current. To implement accurate simulation and real-time control, the designers have to develop novel techniques to calculate precisely the nonlinear magnetic characteristics of the SRM both on line and off line. Reference [4] presents a method based on artificial neutral networks (ANN) which is suitable for both off line and on line. However, the ANN method needs a large number of given data to train the ANN model.

In this study, a novel method based on two-dimensional (2-D) least squares algorithm with orthogonal polynomials is being proposed to compute the motor's magnetic characteristics, which is applicable both off line and on line. Furthermore, the proposed method only needs a limited amount of data for training the coefficients in the proposed model. It can be employed to compute the flux linkage at any arbitrary rotor position and current. The results from the experiment and the computation based on the proposed method show that the errors are fairly small and that the proposed method is effective and accurate.

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X. D. Xue is with the Department of Electrical Engineering, Hong Kong Polytechnic University, Hong Kong, on leave from Tianjin University, Tianjin, China (e-mail: xdxue.ee@polyu.edu.hk).

K. W. E. Cheng and S. L. Ho are with the Department of Electrical Engineering, the Hong Kong Polytechnic University, Hong Kong (e-mail: eekwcheng@polyu.edu.hk; eeslho@polyu.edu.hk).

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II. Computation Modeling

A. Modeling of Nonlinear Magnetic Characteristics

In general, nonlinear magnetic characteristics in the SRM drives are obtained from measurements on existing motor or from numerical computations such as finite-element (FE) analysis. Assuming there are $n \times m$ flux linkage values ψ_{kj} with respect to both the rotor position θ_k and the currents i_j being known $(k = 0, 1, \ldots, n-1; j = 0, 1, \ldots, m-1)$, the proposed modeling of the nonlinear magnetic characteristics in the SRM drives is given by

$$\psi(\theta, i) = \sum_{k=0}^{p-1}\sum_{j=0}^{q-1} a_{kj}(\theta - \bar{\theta})^k (i - \bar{i})^j \tag{1}$$

where ψ denotes the flux linkage, θ denotes the rotor position, i denotes the current, a_{kj} are the coefficients computed from the derivation in the next section, $p \le n, q \le m$, and

$$\bar{\theta} = \sum_{k=0}^{n-1} \theta_k / n, \quad \bar{i} = \sum_{j=0}^{m-1} i_j / m. \tag{2}$$

B. Establishing the Modeling

First, the m polynomials with respect to the rotor position θ are constructed [5]:

$$g_j(\theta) = \sum_{u=0}^{p-1} \lambda_{uj}\gamma_u(\theta), \quad j = 0, 1, \cdots, m-1 \tag{3}$$

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where $\gamma_u(\theta)(u=0,1,\ldots,p-1)$ are the orthogonal polynomials to θ and are determined by the following recursive expression

$$\begin{aligned}\gamma_0(\theta)&=1\\ \gamma_1(\theta)&=\theta-\alpha_0\\ \gamma_{u+1}(\theta)&=(\theta-\alpha_u)\gamma_u(\theta)-\beta_u\gamma_{u-1}(\theta),\\ &\qquad u=1,2,\ldots,p-1.\end{aligned} \quad (4)$$

If setting

$$d_u=\sum_{k=0}^{n-1}\gamma_u^2(\theta_k),\quad u=0,1,\ldots,p-1 \quad (5)$$

then

$$\begin{aligned}\alpha_u&=\sum_{k=0}^{n-1}\theta_k\gamma_u^2(\theta_k)/d_u,\quad u=0,1,\ldots,p-1\\ \beta_u&=d_u/d_{u-1},\quad u=1,2,\ldots,p-1\end{aligned} \quad (6)$$

can be derived.

According to the least squares principle

$$\lambda_{uj}=\sum_{k=0}^{n-1}\psi_{kj}\gamma_u(\theta_k)/d_u,\quad \begin{array}{l}j=0,1,\ldots,m-1\\ u=0,1,\ldots,p-1\end{array} \quad (7)$$

can be derived from (3)–(6). Next, the p polynomials with respect to the current i are constructed, similarly to (3) [5]:

$$h_u(i)=\sum_{v=0}^{q-1}\mu_{uv}\eta_v(i),\quad u=0,1,\ldots,p-1 \quad (8)$$

where $\eta_v(i)(v=0,1,\ldots,q-1)$ are the orthogonal polynomials to i and are determined by the following recursive expression:

$$\begin{aligned}\eta_0(i)&=1\\ \eta_1(i)&=i-\alpha_0'\\ \eta_{v+1}(i)&=(i-\alpha_v')\eta_v(i)-\beta_v'\eta_{v-1}(i),\\ &\qquad v=1,2,\ldots,q-1.\end{aligned} \quad (9)$$

Using the substitution of

$$\delta_v=\sum_{j=0}^{m-1}\eta_v^2(i_j),\quad v=0,1,\ldots,q-1 \quad (10)$$

one can obtain

$$\begin{aligned}\alpha_v'&=\sum_{j=0}^{m-1}i_j\eta_v^2(i_j)/\delta_v,\quad v=0,1,\ldots,q-1\\ \beta_v'&=\delta_v/\delta_{v-1},\quad v=1,2,\ldots,q-1.\end{aligned} \quad (11)$$

According to the least squares principle

$$\mu_{uv}=\sum_{j=0}^{m-1}\lambda_{uj}\eta_v(i_j)/\delta_v,\quad \begin{array}{l}u=0,1,\ldots,p-1\\ v=0,1,\ldots,q-1\end{array} \quad (12)$$

can be obtained from (8)–(11).

Finally, the 2-D polynomials can be obtained as

$$\psi(\theta,i)=\sum_{u=0}^{p-1}\sum_{v=0}^{q-1}\mu_{uv}\gamma_u(\theta)\eta_v(i) \quad (13)$$

which is employed to describe the magnetic characteristics of the SRM drives.

The expression (13) can be changed into the standard polynomials, which is given by

$$\psi(\theta,i)=\sum_{k=0}^{p-1}\sum_{j=0}^{q-1}a_{kj}\theta^k i^j \quad (14)$$

where the coefficients a_{kj} have to be computed from (13) and (14) using the recursive algorithm. The detailed description is seen in [5].

To prevent computation overflow, θ and i should be replaced by $(\theta-\bar{\theta})$ and $(i-\bar{i})$, respectively, to give the modeling as described by (1).

C. Training the Coefficients

1) Off-Line: From the above derivation, it is seen that the coefficients in the model (1) are only dependent on the given flux linkage values, rotor position angles, and phase currents. Thus, the coefficients can be computed off line if the $n\times m$ flux linkage values with respect to the n rotor positions and m phase currents are obtained through either measurement on existing motor or numerical computation.

2) On-Line: The coefficients are determined generally off line using the static magnetization data. Due to the static nature of the solution, these coefficients could be unsatisfactory in the dynamic operating regime of the SRM drives. However, for the proposed method in this study, the above coefficients can be trained and corrected on line for the dynamic range if the voltage applied to phase winding, rotor position, and current are measured. The flux linkage can be computed from the measured voltage and current using the trapezoidal method as given in

$$\begin{aligned}\psi(l+1)&=\psi(l)+\frac{1}{2}T_s[V(l+1)+V(l)\\ &\qquad -ri(l+1)-ri(l)]\\ \psi(0)&=0\end{aligned} \quad (15)$$

where $\psi(l+1)$ and $\psi(l)$ are, respectively, the flux linkage values at the sampling instants $(l+1)$ and (l); $V(l+1)$ and $V(l)$ are, respectively, the voltage values applied to the phase winding at the sampling instants $(l+1)$ and (l); $i(l+1)$ and $i(l)$ are the phase current values at the sampling instants $(l+1)$ and (l), respectively; r is the resistance value of the phase winding and T_s is the sampling time.

It is clear from the above derivation that the proposed method is different from the ANN method. However, it is similar to the ANN method with respect to the self-training features of the algorithms.

III. Applications

In this study, 13 rotor position data, seven current data, and 13×7 linkage data (i.e., $n=13$ and $m=7$) are obtained experimentally on a four-phase SRM drive prototype. The main data of this prototype are listed as follows: the number of the phases $=4$, the number of the stator poles $=8$, the number of the rotor poles $=6$, the phase resistance $=0.687\ \Omega$, the phase flux linkage when the stator pole is aligned with the rotor pole $=0.4164$ Wb at the phase current $=12$ A, the phase flux

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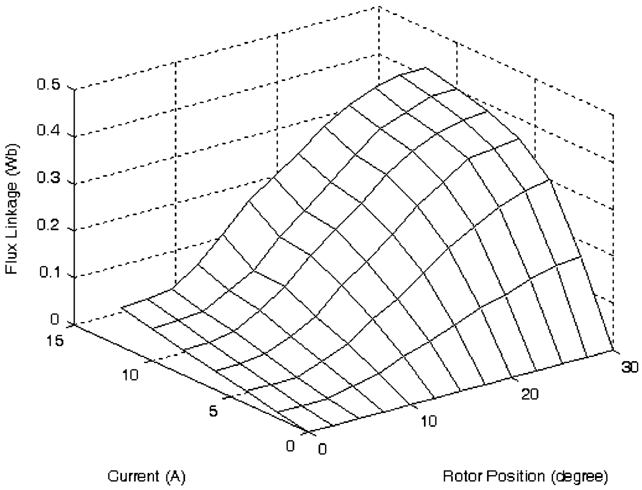

Fig. 1. Flux linkage with respect to the rotor position and the current obtained experimentally within half a period.

TABLE I
COEFFICIENTS a_{kj} IN THE PRESENTED MODELLING

j \ k	0	1	2	3
0	0.185506E+0	0.172323E-1	0.495901E-4	-0.138488E-5
1	0.212722E-1	0.464162E-3	-0.580348E-4	0.821745E-5
2	-0.681549E-3	-0.266467E-3	-0.231760E-4	-0.316703E-5
3	-0.100343E-3	0.662989E-4	0.220016E-5	-0.659221E-6
4	-0.892657E-4	-0.811327E-6	0.666872E-6	0.236683E-6
5	0.403938E-5	-0.120180E-5	-0.465463E-7	0.157412E-7
6	0.197376E-5	0.251275E-7	-0.673947E-8	-0.347280E-8

j \ k	4	5	6	7
0	0.599126E-6	-0.319811E-6	-0.244672E-8	0.864888E-9
1	-0.401359E-7	-0.799320E-7	0.531650E-9	0.182501E-9
2	0.400549E-7	0.418121E-7	0.230158E-9	-0.108192E-9
3	0.195339E-7	0.410335E-8	-0.786641E-10	-0.817040E-11
4	0.578146E-8	-0.262243E-8	-0.392510E-10	0.658349E-11
5	-0.329661E-9	-0.100879E-9	0.142795E-11	0.202721E-12
6	-0.174014E-9	0.393419E-10	0.907334E-12	-0.984998E-13

linkage when the stator pole is unaligned with the rotor pole $=$ 0.0839 Wb at the phase current $= 12$ A, the phase flux linkage when the stator pole is aligned with the rotor pole $= 0.1676$ Wb at the phase current $= 2$ A, and the phase flux linkage when the stator pole is unaligned with the rotor pole $= 0.01264$ Wb at the phase current $= 2$ A. The given magnetic characteristics from the experiment are illustrated by Fig. 1.

The maximum order of the polynomials with respect to rotor position is selected to be seven and the maximum order of the polynomial with respect to current is selected to be six (i.e., $p = 8$ and $q = 7$). From (2), $\bar{\theta}$ and $\bar{i}$ in (1) are equal to 15° and 6.0 A, respectively. The computed coefficients in the proposed modeling are shown in Table I.

Fig. 2 illustrates the phase flux linkage versus the phase current from both the experiments and the proposed method. The

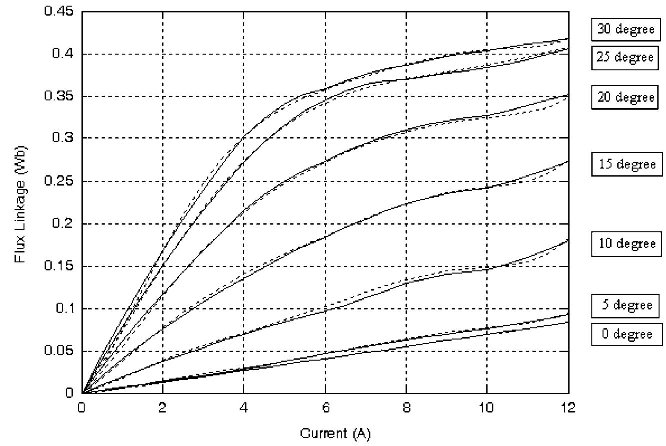

Fig. 2. Comparisons between the experiment and the computation with respect to the current.

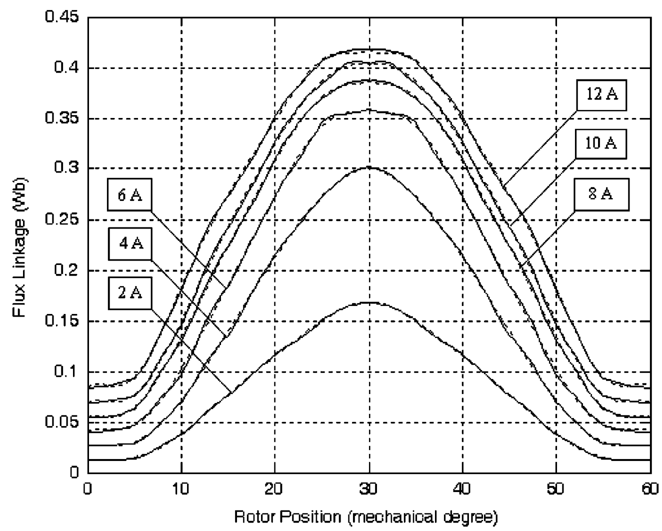

Fig. 3. Comparisons between the experiment and the computation with respect to the rotor position.

phase flux linkage versus the rotor position obtained both experimentally and from the proposed method is shown in Fig. 3. In Fig. 2 and Fig. 3, the solid curves are obtained experimentally and the dotted curves are obtained from the proposed method. It is clear from Fig. 2 and Fig. 3 that the experimental curves agree well with the computed curves based on the proposed method. This indicates that the proposed method can describe the magnetic characteristics in the SRM drive accurately.

IV. ERROR ANALYSIS

In general, the accuracy of the proposed method is dependent on the maximum order of the polynomials in the model. However, that does not mean that a high order in the model would always result in a high accuracy [5]. The accuracy of the proposed analytical model can be analyzed using the following four error functions, which are computed from (16), (17), (18), and (19), respectively.

$$\mathrm{SSE} = \sum_{k=0}^{n-1} \sum_{j=0}^{m-1} [\psi(\theta_k, i_j) - \psi_{kj}]^2 \tag{16}$$

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TABLE II
ERRORS RESULTS

Group number	1	2	3	4
p	3	4	5	6
q	3	4	5	6
SSE	0.0337	0.00278	0.00143	0.000812
SAVE	1.35	0.382	0.275	0.198
MAVE	0.0537	0.0176	0.0124	0.00868
MRE	2.94	0.496	0.403	0.577
Group number	5	6	7	8
p	7	8	9	10
q	7	7	7	7
SSE	0.000595	0.000377	0.000324	0.000298
SAVE	0.160	0.131	0.113	0.106
MAVE	0.00764	0.00583	0.00582	0.00558
MRE	0.404	0.0993	0.0991	0.126

$$\mathrm{SAVE} = \sum_{k=0}^{n-1}\sum_{j=0}^{m-1} |\psi(\theta_k, i_j) - \psi_{kj}| \tag{17}$$

$$\mathrm{MAVE} = \max_{\substack{0 \le k \le n-1 \\ 0 \le j \le m-1}} |\psi(\theta_k, i_j) - \psi_{kj}| \tag{18}$$

$$\mathrm{MRE} = \max_{\substack{0 \le k \le n-1 \\ 0 \le j \le m-1}} \left(\frac{|\psi(\theta_k, i_j) - \psi_{kj}|}{\psi_{kj}}\right) \tag{19}$$

where SSE denotes the sum of the squares errors, SAVE denotes the sum of the absolute values of the errors, MAVE denotes the maximum value of the absolute errors, MRE denotes the maximum relative error, $\psi(\theta_k, i_j)$ denotes the computed flux linkage value from the proposed method, and ψ_{kj} denotes the given flux linkage value from the experiment, with respect to the rotor position θ_k and the current i_j.

Table II shows the variations of the four errors in eight different groups at various p and q values.

It can be observed that the maximum order numbers of the polynomials in the model have a crucial effect on the accuracy of the proposed method. If a small value is selected as the order number, such as the order numbers of Group 1 ($p = 3$ and $q = 3$), the four error values are fairly large and, thus, the accuracy of the model is low. When the order numbers go from small to large, the four error values decrease. However, the four errors are hardly changed if the order numbers being selected are already large, such as the values in Group 6 ($p = 8$ and $q = 7$). In the meantime, the large order numbers result in slow computation. Hence, the determination of the maximum order numbers of both the rotor position and the current should take into account of the number of the given rotor position angles, the number of given currents, the errors, and the corresponding computation speed. Actually, SSE, SAVE, and MAVE indicate the errors between the fitted values and the given values, but they do not contain the amplitudes of the given values. However, MRE can consider both the errors and the amplitudes of the given values. Hence, in this study, MRE is used to determine the maximum order numbers, and MRE should not be larger than 10%. Under this constraint, it is found that $p = 8$ and $q = 7$. It is seen from Table II that SSE, SAVE, MAVE, and MRE are all fairly small. This also validates the effectiveness and accuracy of the proposed method.

V. CONCLUSION

This paper presents a novel numerical method to compute precisely the nonlinear magnetic characteristics of the SRM drives. The salient advantages of the proposed method are that the modeling can be trained both off line and on line by using a limited set of given data that are much less than that required by the ANN method. The nonlinear flux linkage at arbitrary rotor positions and currents can also be computed precisely by using the proposed method. The experimental results are used to validate the effectiveness and accuracy of the proposed method as reported in this paper. Furthermore, it is concluded that the proposed method can be applicable to performance prediction, torque control, and sensorless control of the SRM drives, regardless of whether the machines are operating in a static or dynamic state.

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