An Improved Ant Colony Optimization Algorithm and Its Application to Electromagnetic Devices Designs

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Based on the success in the design of a new global search procedure on the development of a novel trail updating mechanism and the introduction of an elitist strategy to available ant colony optimization (ACO) methods, an improved ACO algorithm is proposed. In order to facilitate the implementation of the search procedure, the available local search phase is simplified also. The algorithm is tested on a mathematical function and an inverse problem, and its performances are compared with those of other well designed methods.

***Index Terms*—Ant colony optimization (ACO) method, heuristic algorithm, inverse problem, optimal design.**

I. Introduction

FOR GLOBAL optimization of multimodal continuous objective functions of electromagnetic devices, it is difficult to use deterministic methods to obtain global solutions. Hence, increasing efforts are devoted to the development of heuristic and meta-heuristic algorithms in computational electromagnetics. The most notable progress in this respect is the development of evolutionary methods. Recently, a new entrant to the family of evolutionary algorithms, the ant colony optimization (ACO) method, has been proposed [1]. The ACO metaphor is inspired by researches to try understanding how almost blind ants establish the shortest path from their colony to their feeding sources and back. Essentially, it is found that ants use pheromone as the means of communication. As an ant moves, it lays varying amounts of pheromone, which are detectable by other ants, along its path, thereby marking the path by a trail of such substances. As more ants pass by, more pheromone is deposited on the path. Since ants chase after pheromone, the richer the trail of pheromone in a path, the more likely it would be followed by other ants. Therefore, the ants can establish the shortest way from their colony to the feeding sources and back.

An intuitive description of the foraging behavior of ants is shown in Fig. 1. Initially, three ants leave their nest in random directions to search for food. As they wander around, they deposit certain amount of pheromone trails, which will evaporate slowly but are detectable by other ants. Now assume *Ant* 1 finds a food source. It will pick up some food and return to the nest by following its own pheromone trail, laying additional pheromone on the same path while *Ant* 2 and *Ant* 3 are still wandering randomly. When the next group of ants leave their nest to search for food, they detect twice as much pheromone on *Path* 1 than on *Path* 2 and *Path* 3, assuming the evaporation of pheromone is negligible. Since the probability for a path to be followed is proportional to its pheromone value, more ants will follow *Path* 1 in this second round of search for food. In this way, the ants can establish the shortest path from their colony to the feeding sources. Obviously, even if an isolated ant moves randomly, it can, by means of communication using pheromones, follow the collective behavior of ant colonies and exhibits the "autocatalytic" characteristics. Due to their power to learn and search for an autocatalytic self-organization such as an ant colony, ACO based algorithms are becoming increasingly popular for solving difficult combinatorial optimal problems [2]–[5]. However, there are few applications of ACOs to continuous variable problems [6]–[8]. Bilchev and Parmee first propose an ACO method for continuous variable problems with the ACO embedded in a genetic algorithm (GA) for fine (local) searching so as to improve the quality of the final solution of the GA [6]. The algorithm is subsequently extended by Wodrich and Bilchev to become a general global searching procedure of continuous variable problems [7]. It is further improved by Mathur *et al.* for large-scale continuous optimal ones [8]. The common features of the available continuous ACO algorithms are that the search process is divided into bi-level search procedures, having a local and a global search one; and two GAs are used in the global search procedure.

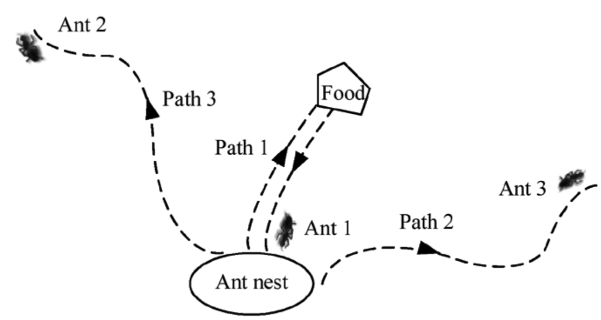

Fig. 1. Intuitive description of ants foraging for food.

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However, such global search technique has no metaphoric link to the behaviors of an ant colony. Moreover, since coded and arithmetic crossovers as well as mutation operations are performed, the corresponding search procedure is very awkward in numerical implementation and parameter tuning. Consequently, an improved continuous ACO algorithm is proposed to address the aforementioned shortcomings.

II. Improved Ant Colony Optimization Algorithm

As similar to available continuous ACO algorithms, the iterative procedure of the proposed algorithm is divided into bi-level search procedures, comprising a local search and a global search, with the procedures switching alternatively between the two. However, the proposed one differs from the others mainly in the following aspects: 1) it uses dynamically updated regions rather than pre-discretized regions in the search process; 2) a simple, novel but robust global search mechanism is designed; 3) an elitist strategy is incorporated; and 4) a new trail assignment schedule is proposed. To facilitate the understanding and explanation of the proposed ACO algorithm, its procedural steps are summarized as follows.

- Step 1) *Initialization*: Create initial feasible solutions (ants) and determine the corresponding objective function values; Set initial values of pheromones and other variables.
- Step 2) *Local Search*: Activate the local search procedure to send some local ants to regions of the global ants.
- Step 3) *Global Search*: Start the global search to generate some new ants.
- Step 4) *Elitist:* Implement an elitist strategy to reinforce the trails of the best solution found in the present cycle.
- Step 5) *Termination Test*: If the test is passed, stop; Otherwise, go to Step 2).

A. Global Search

The objective of this phase is to create N_G new global ants as feasible regions. For this purpose, one first chooses a predator x^p from the G global ants using a Roulette wheel selection scheme according to the following fitness values of the ants:

$$f_i(k) = \frac{\tau_i(k)}{\sum_{j=1}^{G} \tau_j(k)} \tag{1}$$

where $\tau_i(k)$ is the pheromone value of the ant i at step k.

One then selects another N_h ants, referred as preys, $x^i (i = 1, \ldots, N_h)$, from the G global ants using the same Roulette wheel selection scheme according to their fitness values defined in (1). An increment vector $v^p(t+1)$ is computed using

$$v_d^p(k+1) = r v_d^p(k) + (1-r)\left[\sum_{i=1}^{N_h} \frac{\tau_i(k)}{\sum_{j=1}^{N_h} \tau_j(k)} \left(x_d^i - x_d^p(k)\right)\right] \tag{2}$$

where r is a random parameter uniformly chosen from the interval [0, 1], d is the dimension variable, $v_d^p(k)$ is the step increment of predator x^p in the dth dimension at step k.

Finally, a new global ant is created from

$$x_d^p(k+1) = x_d^p(k) + v_d^p(k+1). \tag{3}$$

This process will repeat until N_G new global ants are generated. Instead of choosing the N_G best ones [8], a new population of G global ants is chosen from the total group of $G + N_G$ ants using the Roulette wheel selection scheme based on the ants' pheromone values in order to maintain the diversities of the ants in the parameter space.

It should be emphasized that the communication between different ants in the proposed search procedure is set up in a heterarchical rather than a hierarchical manner. Thus, a fast convergent speed is expected [9].

B. Local Search

The local search of the proposed algorithm is to send some local ants to the neighborhood of the selected global ants for intensification searches. The search mechanism proposed is similar to that of [8] but simplified. The local search starts by setting the initial trail values of the ants to 1 s. To select L regions (ants) from the G global ones, one uses the same Roulette wheel selection scheme according to their trail values. Every local ant being sent will move a short distance around the selected global one. The direction of the movement will be the same as that of the previous one if there is an improvement in the fitness. If there is no improvement, it will search in a random direction. If an improvement in objective function values is observed, the ant position is updated to become the current one. The corresponding pheromone deposited on this solution is adjusted in accordance to the increase in fitness value. This process will be repeated until a predefined number of successive iterations with no improvement in fitness values has been reached.

C. Elitist Strategy

At the end of every search cycle, the pheromone trails laid on the solution with the best objective function value found in this cycle is reinforced to facilitate the search around the specified point for the best solutions.

D. Trail Updating Schedule

Once a new global ant is created in the global search phase, it is mandatory to assign a trail value to it. Mathur *et al.* suggest to assign the trail of the new ant to a value between those of its parent ants, irrespective the objective function value of the new ant. However, such treatment may slow down the convergence speed of the algorithm since the information about the objective function of the new ant is not fully used to guide the generation of other new ants. To address this problem, a new trail updating schedule is designed in the proposed ACO algorithm. Thus, the trail value of a newly generated global ant is assigned the weighted sum of two different parts. The first part is the contribution of the trails from its parents. Since more than two parents are generally used in the proposed algorithm to generate a new ant, it is necessary to use some interpolation technique to determine the trail value of the child ant from those of its parents. For this purpose, the moving least squares approximation based response surface model as proposed by the authors [10] is used. The second part is the influence of the objective function which is proportional to the inverse of the function (for minimization problems) value of the new ant. The advantage of the proposed trail updating schedule is that it favors the selection of ants with better fitness values, other things being equal, for both exploration and exploitation.

E. Approaches to Mitigate Stagnation

As is well known, a major weakness of ACO algorithms is the stagnation in which all ants are taking the same position. If this occurs, the algorithm may be trapped in a local optimal point. To alleviate the stagnation problem of ACO algorithms, two different approaches are used in the proposed method to guarantee the diversity of the ants to avoid stagnations.

1) Evaporation: To reduce the influence of the previous experiences to avoid the algorithm from becoming stagnant, the evaporation operator which models ants foraging for food is also used in the proposed algorithm. At the end of each global search cycle, the trail of an ant is reduced using the following equation:

$$\tau_i(k) = \rho\tau_i(k) \tag{4}$$

where ρ is a coefficient referred as the evaporation parameter, $(1-\rho)$ presents the evaporation amount of the pheromone.

2) Aging: To increase the diversity of the ants, an age variable is also assigned to each ant, and it will increase with the number of iterations if its position is not changed. After the evaporation process, the trail value of an ant is penalized further according to its age variable such that the larger the value of the age variable, the larger is the reduction of the trail value for an ant.

III. Numerical Examples

The proposed improved ACO algorithm is applied to both mathematical benchmark problems and electromagnetic design problems with promising numerical results. Here, only some typical results are reported.

A. Testing on Mathematical Functions

To validate the proposed algorithm and to compare its performances with those of an available ACO algorithm, it is firstly used to solve a well designed mathematical function, which is selected from [8]. The function is highly multimodal and has many similar peaks near the origin which makes it a difficult one to handle by an optimal algorithm. Mathematically, it is formulated as

$$\max \frac{1}{0.1+\left[\sum_{i=1}^{10}\frac{x_i^2}{4000}-\prod_{i=1}^{10}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1\right]} \times [-512 \le x_i \le 512\ (i=1,2,\ldots,10)]. \tag{5}$$

The global optimal solution of this function is at $x_{\rm opt} = (0,0,0,0,0,0,0,0,0,0)$ with $f_{\rm opt} = 10$.

The parameters used by the proposed algorithm for the test function are the same as those used by the continuous ACO algorithm (CACO) of [8] when applicable. The value of the parameter N_h of the proposed algorithm is set to 20% of the number of the total global ants using a trial and error approach. For this test function, the proposed algorithm is independently run 100 times starting from randomly generated initial points, and the numerical results are summarized in Table I. It can be seen from Table I that the average number of function evaluations for the proposed algorithm to converge to a solution for the 100 independent runs is 3146, which is nearly 40% less than that required by the CACO algorithm. However, the success rate for finding the global optimal solution of the proposed method is slightly less than that of the CACO. However, considering the fact that, in general, the objective function of an engineering design problem is not as complex as that of the test function examined and the success rate of the proposed algorithm, even for such a complex problem, is still 92%, the robustness of the proposed improved ACO algorithm for finding global solutions of engineering problems is considered as satisfactory. It should be noted that the proposed algorithm is very simple both in parameter tuning and numerical implementation, as compared with the original CACO algorithm. Also, from the performance comparison results of the CACO and that of the GA as reported in [8], it can be seen that the CACO is slightly better than GA.

TABLE I
PERFORMANCE COMPARISON OF DIFFERENT OPTIMAL METHODS ON TEST FUNCTION FOR 100 INDEPENDENT RUNS

Algorithms	No. of averaged iterations	No. of runs for finding the global solutions
Proposed ACO	3146	92
CACO	5000	100

B. Application

To evaluate the performances of the proposed algorithm for electromagnetic design problems, it is used to minimizing the torque ripple of a cage induction motor fed from pulsewidth modulated (PWM) inverter operating at rated conditions as reported in [11]. For this purpose, the rms value of the harmonic electromagnetic torque should be as small as possible when compared to that of the fundamental torque. Consequently, this problem is formulated as

$$\begin{aligned} \min \quad & f = \frac{1}{T_0}\sqrt{\sum_{i=0}^{N_f} T_i^2(\alpha)} \\ \text{s.t.} \quad & \alpha_{i-1}+\Delta_{\min} \le \alpha_i \le \alpha_{i+1}-\Delta_{\min}(i=2,\ldots,N) \\ & \alpha_1 \ge \alpha_{\min}, \alpha_N \le \alpha_{\max} \end{aligned} \tag{6}$$

where N is the number of switching angles within one quarter period of the PWM inverter; α_i is the ith switching angle; T_i is the ith component of the electromagnetic torque of the machine ($i=0$ represents the direct component); $\Delta_{\min}$, $\alpha_{\min}$, and $\alpha_{\max}$ are dependent on the switching frequency of the power electronic elements and the operating conditions.

To determine the steady-state performances of the entire drive system, the iterative procedure based on a time-stepping finite element method coupled with the external circuit model and a complex finite element method of [11] is used. Therefore, both the nonlinear property of the magnetic materials and the relative movement between the stator and the rotor are properly

TABLE II
PERFORMANCE COMPARISON OF DIFFERENT OPTIMAL ALGORITHMS

	α_1(deg)	α_2(deg)	α_3(deg)	α_4(deg)	α_5(deg)	f_{opt}	No. of Iterations
Tabu	5.17	18.04	19.92	33.73	34.99	1.01	7862
Proposed	5.18	18.02	20.04	33.68	35.14	1.009	7482
SA	5.18	18.00	20.03	33.70	35.04	1.009	16986

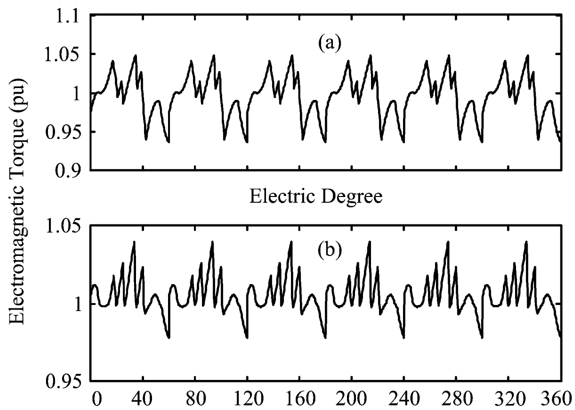

Fig. 2. Steady-state electromagnetic torques. (a) and (b) are, respectively, the torque before and after optimization.

modeled. In the numerical implementation being reported, the algorithm parameters used are the same as those used for the mathematical test function. For the purpose of performance comparisons, a tabu search [12] and a simulated annealing (SA) algorithm [13] are also used to solve the same problem. The final solutions obtained by different optimal methods and the corresponding iteration numbers required to converge to the final solution are given in Table II. The profiles of the electromagnetic torques before and after the optimizations exercise using the proposed ACO algorithm are given in Fig. 2. It can be seen that the electromagnetic torque ripple with optimized switching angles is smaller than that for the unoptimized ones.

From these numerical results, it is obvious that the following hold.

1) All the three optimal algorithms converge to nearly identical solutions.
2) The proposed algorithm is the most efficient one among the three methods, although the iteration number required by it is only slightly less than that required by the tabu search.
3) However, the proposed algorithm is very complex in terms of mathematical content and numerical implementation when compared with the other two algorithms; moreover, the parameters of the proposed algorithm have to be tuned carefully.

IV. CONCLUSION

An ACO-based algorithm for optimizations of functions with continuous variables is proposed. It is tested on a mathematical function and an optimal design problem of electromagnetic devices as reported in this paper. The numerical results show that the performances of the proposed algorithm are comparable to that of its ancestors, and is a strong competitor to other well developed heuristic algorithms, i.e., the simulated annealing, the genetic and the tabu search algorithms, at least for the tested problems reported in this paper. Our further work will focus on a comprehensive study of the algorithm parameters and the simplification of the search process. However, to the best of our knowledge, there are only a few applications of ACO algorithms including the proposed one in computational electromagnetics. Therefore, the authors hope that this paper will promote the study of ACOs among fellow researchers in the development of ACO based algorithms with excellent robustness and convergence.

REFERENCES

[1] A. Colorni, M. Dorigo, and V. Maniezzo, “Distributed optimization by ant colonies,” in *Proc. Eur. Conf. Artificial Life*, 1991, pp. 134–142.
[2] M. Dorigo, V. Maniezzo, and A. Colorni, “Ant system: Optimization by a colony of cooperative agents,” *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 26, no. 1, pp. 29–41, Feb. 1996.
[3] C. Gagne, W. L. Price, and M. Gravel, “Comparing an ACO algorithm with other heuristics for the single machine scheduling problem with sequence-dependent setup times,” *J. Oper. Res. Soc.*, vol. 53, pp. 895–906, 2002.
[4] I. K. Yu, C. S. Chou, and Y. H. Song, “Application of the ant colony search algorithm to short-term generation scheduling problem of thermal units,” in *Proc. IEEE Power System Technology Conf. (POWERCON’98)*, vol. 1, Aug. 1998, pp. 552–556.
[5] N. Karaboga, K. Guney, and A. Akdagli, “Null steering of linear antenna arrays with use of modified touring ant colony optimization algorithm,” *Int. J. RF Microw. CAE*, vol. 12, pp. 375–383, 2002.
[6] G. Bilchev and I. C. Parmee, “The ant colony metaphor for searching continuous design spaces,” *Lect. Notes Comp. Sci.*, vol. 993, pp. 25–39, 1995.
[7] M. Wodrich and G. Bilchev, “Cooperative distributed search: The ants’ way,” *Contr. Cybern.*, vol. 26, pp. 413–446, 1997.
[8] M. Mathur, S. B. Karale, S. Priyee, V. K. Jayaraman, and B. D. Kulkarni, “Ant colony approach to continuous function optimization,” *Ind. Eng. Chem. Res.*, vol. 39, pp. 3814–3822, 2000.
[9] J. Dreo and P. Siarry, “A new ant colony algorithm using the heterarchical concept aimed at optimization of multiminima continuous functions,” *Lect. Notes Comp. Sci.*, vol. 2463, pp. 216–221, 2002.
[10] S. L. Ho, Y. Shiyou, N. Peihong, and H. C. Wong, “Development of an efficient global optimal design technique-a combined approach of MLS and SA algorithm,” *Compel*, vol. 21, pp. 604–614, 2002.
[11] S. L. Ho, S. Yang, P. Zhou, H. C. Wong, and M. A. Rahman, “A combined finite element-domain elimination method for minimizing torque ripples in inverter-fed AC drive systems,” *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 1817–1821, Jul. 2000.
[12] J. M. Machado, S. Yang, S. L. Ho, and G. Ni, “A common tabu search algorithm for the global optimization of engineering problems,” *Comput. Methods Appl. Mech. Eng.*, vol. 190, pp. 3501–3510, 2001.
[13] S. Yang, J. M. Machado, G. Ni, S. L. Ho, and P. Zhou, “A self-learning simulated annealing algorithm for global optimizations of electromagnetic devices,” *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 1004–1008, Jul. 2000.

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