Incorporating *A Priori* Preferences in a Vector PSO Algorithm to Find Arbitrary Fractions of the Pareto Front of Multiobjective Design Problems

S. L. Ho[1], Shiyou Yang[2], and Guangzheng Ni[2]

[1]Department of Electrical Engineering, the Hong Kong Polytechnic University, Hong Kong
[2]College of Electrical Engineering, Zhejiang University, Hangzhou 310027, China

To incorporate the knowledge or preference of a decision maker or domain expert into a vector optimizer in the search for a series of subsets of the entire Pareto optimal solutions, a vector particle swarm optimization (PSO) algorithm that implements the reference point-based approach together with a desirability function is proposed. The fitness assignment strategy and the neighborhood relationship of the PSO algorithm are redefined to facilitate the realization of the aforementioned objective. To validate and demonstrate the advantages of the proposed algorithm, its applications on two different multiobjective problems are reported.

***Index Terms*—Desirability function, multiobjective design, particle swarm optimization (PSO) algorithm, reference point.**

I. Introduction

IN MULTIOBJECTIVE design studies in engineering, the usual practice is to find the complete set of nondominated or Pareto solutions so as to allow the decision maker to have the flexibility to choose the best tradeoffs among conflicting criteria. Whilst most decision makers do have some expert knowledge about what are the best tradeoffs, it is extremely difficult for common multiobjective optimizers to find the entire Pareto front for multiobjective design problems having a large number of design objectives [1]. In order to enhance the convergence speed of the algorithm, it is therefore essential if one could use the knowledge of experts to guide the iterative search process so as to minimize trivial or nonproductive explorations of the parameter and objective spaces. Consequently, it is desirable to develop a vector optimizer that incorporates the knowledge or preference of domain experts so as to find, efficiently yet accurately, a conflicting fraction of the entire Pareto front in multiobjective design problems.

In the context of including the preference or knowledge of decision makers or domain experts into vector optimizers for finding some preferable fractions of nondominated or Pareto optimal solutions, as opposed to the finding of the entire Pareto solutions, some case studies have been reported [1]–[3].

Generally, these studies can be classified into three categories according to the strategies which are used to integrate the *a priori knowledge* of the experts. In the first category, the strategy that integrates the preference of the decision maker can be described as an aggregation approach. More specially, the essence of these approaches is to focus on a study of the weightings of the objectives rather than on the individual preferences themselves [2]. To guide the search toward individual preference domains, Deb *et al.* propose to extend the classical reference point method [1], and this approach constitutes the second group of the studies. However, in addition to the disadvantages as will be explained below, the nondominated sorting algorithm used in this strategy is not strictly based on a Pareto optimal sense. The final solutions of these optimizers are only rough approximations of the exact Pareto ones. The third strategy, along the same direction that integrates the domain experts' preferences in vector optimizers, is the use of desirability functions [3]. Upon transferring the original multiobjective problem into an equivalent multiobjective one which comprises of the corresponding desirability functions of the original objectives, a classical vector optimizer can be used simply to find a fraction of the Pareto solutions which are close to the reference point. However, such approach works well with only one reference point at a time. In other words, it cannot be used to find points corresponding to the multiple preference conditions simultaneously. Furthermore, the transformed problem is not always equivalent to the original one in the Pareto optimal sense. As a result, the parameters of the desirability function must be specified with care. Otherwise, the approach could produce untrustworthy or even false Pareto optimal solutions in the preference region [4].

It is desirable to combine the merits of both the reference point and the desirability function approaches into a vector optimizer to include the preference of a decision maker or the knowledge of a domain expert in an optimizing process. In this regard, the reference point-based strategy is improved and combined with a desirability function into a PSO vector algorithm for finding arbitrary multiple fractions of the entire Pareto front for a multiobjective design problem.

II. Reference Point-Based Vector PSO Algorithm

To incorporate the preference of the decision maker into a vector optimizer, a reference point which provides the necessary information about the subregion of the objective space being studied is commonly being used. However, most of available preference point-based approaches cannot be used for finding solutions corresponding to multiple reference points simultaneously. To address this problem, the approach proposed in [1] is improved by using the desirability function which is then implemented in a vector PSO algorithm.

A. Grouping of Populations Using a Clustering Algorithm

Since the proposed algorithm will tackle multiple preference points in a single run, the desirability function of an individual

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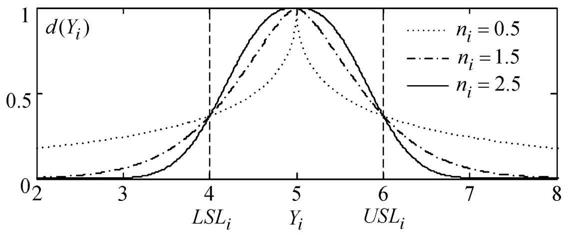

Fig. 1. Schematic diagram of Harrington's two-sided desirability function.

should be defined in a way which is different from those in available works. In this regard, the population of the PSO algorithm is firstly divided into different groups with respect to different predefined preference points. The desirability function for each member in a group is then determined by considering only members of the same group. For this purpose, a clustering algorithm [5] is used to classify the grouping of an individual based on its Euclidean distances to the predefined reference points. During the search, the grouping of a particle is dynamically changed according to its position. Also, there will be rare cases with no particles being near to a reference point. If such case happens, one shall generate new particles in the neighborhood of the specified reference points and then substitute the particles in the groups with the highest density of particles among all current groups.

B. Introduction of Desirability Function

To guide the search toward a reference point, the fitness of an individual should be assigned in such a way that the closer an individual to the reference point, the larger is its fitness value. To fulfill this goal, Deb *et al.* proposed a ranking approach [1]. However, such scheme only considers whether one solution is closer than another solution, and not how close the point is to the reference point. Therefore, it offers a coarse guidance toward the specific preference point only. The rank values in such approach are discrete integers and hence are rough estimations. To address such issue, the desirability function is introduced to measure more accurately the "quality" of the closeness of an individual to the preference point. The desirability function is proposed by Harrington to transform a multicriteria optimization into an overall quality value (desirability index) [6] which are, indeed, being used extensively as the quality index in many optimization studies in different disciplines. One of the most commonly used desirability function in the studies of the multiobjective designs is the following Harrington's two-sided function:

$$d_i(Y_i') = \exp\left(-|Y_i'|^{n_i}\right) \quad (1)$$

$$Y_i' = 2Y_i - (\mathrm{USL}_i + \mathrm{LSL}_i)/|\mathrm{USL}_i - \mathrm{LSL}_i| \quad (2)$$

where USL_i and LSL_i are, respectively, the upper and lower specification levels characterizing a symmetric desirability around a target value midway between the two limits of the ith objective or criterion; $n_i > 0$ controls the shape of the two-sided desirability function as shown in Fig. 1.

However, the so-defined desirability function cannot be used directly to measure the "closeness" of a solution to a specified preference point in the present study. Therefore, it needs changing. Moreover, to enable the desirability function to have the ability to adjust its function values according to the characteristics of the individuals in a group, another parameter is also introduced in the proposed algorithm. The modified Harrington's two-sided desirability function for an individual which belongs to the ith group is then proposed as

$$d_i\left(Y_{ij}'\right) = \exp\left(-\left|Y_{ij}'\right|^{n_i}\right) \quad (3)$$

$$Y_{ij}' = Y_{ij} - (Y_i)_{\min}/\alpha\left((Y_i)_{\max} - (Y_i)_{\min}\right) \quad (4)$$

where Y_{ij} is the distance of individual j to the ith preference point; $(Y_i)_{\min}$ and $(Y_i)_{\max}$ are, respectively, the maximum and minimum distances of the individuals in the ith group to the ith preference point; $n_i > 0$ and $0 < \alpha < 1$ are two predefined constants which control the kurtosis of the desirability function to adequately meet the expert's preference.

Obviously, for each of these modified desirability functions, an equal emphasis is given to the solutions in each group which are close to the corresponding specific preference points, thereby allowing several regions of interests to be explored simultaneously in a single run.

C. Fitness Assignment

As discussed before, solutions of a multiobjective design problem are sets of optimals (Pareto optimals). Therefore, the final solutions searched by the proposed algorithm are not simply those near the predefined preference points, they are also optimal ones in the Pareto optimal sense. Consequently, to guide the search toward fractions of Pareto front which are in the vicinity of the predefined preference points, it is suggested that the fitness value of an individual in the proposed algorithm be the weighted sum of its commonly defined fitness, i.e., the fitness is defined in a Pareto optimal sense, and its desirability function values. For example, if individual x_k belongs to the group of the ith preference point, its fitness value is determined from

$$f_{\mathrm{fit}}(x_k) = w_1 f_{\mathrm{fit}}^{\mathrm{nor}}(x_k) + (1 - w_1) d_i\left(Y_{ik}'\right) \quad (5)$$

where $f_{\mathrm{fit}}^{\mathrm{nor}}(x_k)$ is the commonly defined fitness of x_k; w_1 is a weighting constant.

Moreover, to guide the search to favor the search for Pareto solutions in the early stage and to concentrate the search around those sub-Pareto fractions which are close to the predefined preference points, the weighting constant w_1 is decreased from its maximum value to its minimum value during the iterating process of the proposed algorithm.

D. Definition of Neighborhood Relations

As the emphasis of the present work is to find fractions of the Pareto front which are close to some predefined preference points of a multiobjective problem, the neighborhood definition of the corresponding vector PSO algorithm should be different from that of a classical PSO one that aims to reproduce the entire Pareto front. Consequently, the neighborhood of the particles being studied in the proposed algorithm is defined according to groupings rather than the topology structures of the whole populations. For example, if a particle belongs to the group of the ith preference point, all the particles in the same group are the neighbors of this specific particle. As a result, the neighborhood relation of the proposed algorithm is dynamically changed in the searching process. The convergence speed of the algorithm is enhanced by the proposed neighborhood definition because the best solutions so far searched around the preference point are being used by all particles in the vicinity of the reference point.

E. Updating of the External Pareto Set

To report the so-far searched nondominated (Pareto optimal) solutions, a Pareto set called the External Pareto Set is introduced. To further improve the finding of some fractions of the entire Pareto optimal solutions corresponding to the preferences points, the members of this external Pareto set are updated at the end of the iterative process in every generation by considering all individuals in the current generation and the external Pareto set according to the fitness definition of (5). Accordingly, members in this external Pareto set are also grouped using the same clustering algorithm.

III. Numerical Validation and Example

A. Mathematical Validation

To validate the proposed algorithm, it is firstly used to solve a deliberately designed mathematical function, and its performance is compared with those of other vector optimizers. This function is given as

$$\min F = [f_1(x,y) f_2(x,y) f_3(x,y)]^T \tag{6}$$

$$\begin{aligned} f_1(x,y) &= 0.5(x^2+y^2)+\sin(x^2+y^2) \\ f_2(x,y) &= (3x-2y+4)^2/8+(x-y+1)^2/27+15 \\ f_3(x,y) &= 1/(x^2+y_2+1)-1.1e^{(-x^2-y^2)} \\ &\quad (-3 \le x,y \le 3). \end{aligned} \tag{7}$$

The Pareto front of this test function is a 3-D curve following a convoluted path in the objective space. In the numerical experiments, three reference points are predefined in the implementation of the proposed algorithm. This mathematical function is then solved by using the proposed and a common purpose PSO [7] algorithm. In the study, the parameters for the proposed algorithm are Population Size $= 20$, $\alpha = 0.5$, $n_i = 1.5$, the initial value of w_1 is 0.6, and w_1 will decrease during the iterative process according to

$$w_1 = \frac{w_1^{\text{int}}}{(1+\beta)^i} \tag{8}$$

where w_1^{int} is the initial value of w_1; β is a positive constant, and is set to 0.1 for this case study; i is the generation number.

The other parameters used by the two algorithms are the same as those of [7]. The two algorithms will stop their iterative processes when either the density of the searched Pareto solutions (in the specific regions) exceeds a threshold value or the number of the searched Pareto solutions (in the specific regions) is unchanged after 100 successive iterations. To demonstrate the "average" performances of the two algorithms, they are run independently ten times on this test problem. Moreover, to elucidate the robustness of the proposed algorithm to possible changes in parameter values, the aforementioned parameters for the proposed algorithm are randomly perturbed by some small values of about 5% from their base values. The averaged iterative numbers used by the common purpose and the proposed ones are, respectively, 1026 and 635. The searched Pareto fronts using the common purpose PSO and the proposed algorithms in a typical run are depicted in Fig. 2. From these numerical results the following can be seen.

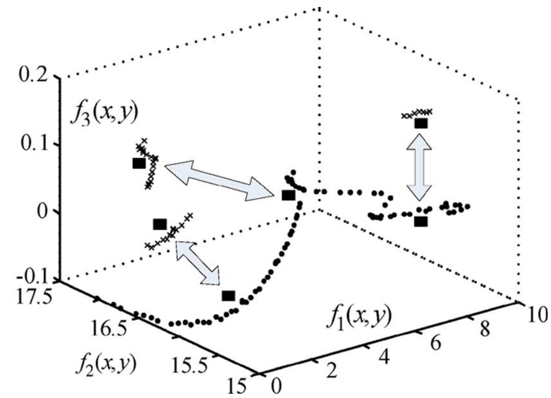

Fig. 2. Searched Pareto fronts: × by using a common purpose PSO algorithm, • by using the proposed algorithm (■ the reference points).

1) The proposed algorithm can successfully find a number of the exact Pareto fronts which are nearest to the predefined reference points.
2) With respect to a special part (surface) of the Pareto solutions searched by using the proposed algorithm, the smaller the distance of a position on the surface to the reference point, the large is the number of Pareto solutions found. This observation allows the decision maker to make better and more reliable decisions conveniently and readily.
3) Moreover, small perturbations on the parameters of the proposed algorithm have virtually no effect on the finally searched solutions.

B. Application

As an application on inverse multiobjective problems, the proposed algorithm is used to find the optimal solutions of the geometrical design of the multisectional pole arcs of large hydrogenerators [7]. This problem is formulated as

$$\begin{aligned} &\max \quad B_{f1}(X) \\ &\min \quad (e_v, \text{THF}) \\ &\text{s.t} \quad \text{SCR} - \text{SCR}_0 \ge 0 \\ &\qquad X'_d - X'_{d0} \le 0 \end{aligned} \tag{9}$$

where B_{f1} is the amplitude of the fundamental component of the flux density in the air gap; e_v is the distortion factor of a sinusoidal voltage of the machine at no-load conditions; THF is the abbreviation of the Telephone Harmonic Factor; X'_d is the direct axis transient reactance of the generator; SCR is the abbreviation of the short circuit ratio.

The details including the schematic diagram of the decision parameters about this case study are given in [8]. For performance comparison purpose, this problem is also solved using a

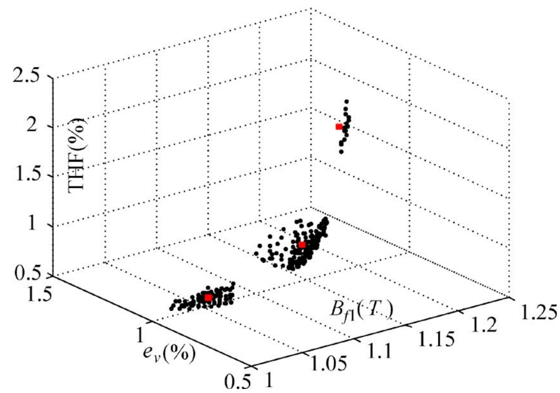

Fig. 3. Searched Pareto solutions of a 300-MW hydrogenerator by using the proposed algorithm (■ the reference points).

common purpose PSO [7] and the proposed method. In this case study, three reference points are employed for the proposed algorithm to work. Again, all the parameter values for the two algorithms are the same as those used in the previous section, except that the rigorous termination criteria used in the previous section is relaxed to some extent because computational heavy finite element analysis are involved in deciding the objective function. To show the robustness of the two algorithms on different initial conditions, they are independently and randomly run five times. The average numbers of the function evaluations for the original PSO and the proposed ones are 1678 and 796, respectively. The searched Pareto solutions of a 300-MW, 44-pole hydrogenerator using the proposed algorithm in a typical run are given in Fig. 3. Compared with the corresponding results of the original PSO algorithm [7, Fig. 3], the merits of the proposed ones can be described as follows.

1) It does not only have the ability to find a small set of the exact Pareto fronts for a reference point to reflect the decision-makers' interest, it also has the power to find multiple fractions of the Pareto front that corresponds to different preference conditions in a single run.
2) The average iterative number used by the proposed algorithm is less than half of that used by common PSO algorithms which are the conventional vector optimizers.

When compared with available vector optimizers, the numerical results for the aforementioned two examples have both demonstrated that the preference or knowledge of a decision maker can be used effectively in the proposed algorithm to guide the search toward specific tradeoff solutions of the entire Pareto front with superior searching efficiency.

IV. Conclusion

This paper strives to address the need to equip an available vector optimizer for multiobjective design problems to have the ability to find some fractions of the Pareto solutions which are nearest to multiple preference regions predefined by a decision maker. Compared with similar techniques that integrate a decision maker's preference into vector optimizers in related works, the salient advantage of the proposed techniques is that the Pareto optimality and the nearest desirability to the preference points are compromised in an optimized manner. To make the proposed algorithm to become a brand vector optimizer with guiding ability, the future work of the authors is to continue our effort to search for the optimal parameter values for the algorithms. Also, the sensitivity of the desirability function on the algorithm's performance is currently under investigation.

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References

[1] K. Deb and J. Sundar, "Reference point based multi-objective optimization using evolutionary algorithms," in *Proc. 8th Annu. Conf. Genetic and Evolutionary Computation*, 2006, pp. 635–642.
[2] J. Branke, T. Kaugler, and H. Schmeck, "Guidance in evolutionary multi-objective optimization," *Adv. Eng. Softw.*, vol. 32, pp. 499–507, 2001.
[3] J. Mehnen and H. Trautmann, "Integration of expert's preferences in Pareto optimization by desirability function techniques," in *Proc. 5th CIRP Inte. Seminar on Intelligent Computation in Manufacturing Engineering (CIRP ICME'06)*, R. Teti, Ed., Ischia, Italy, Jul. 2006, pp. 293–298.
[4] H. Trautmann and J. Mehnen, A method for including a-priori-preferences in multicriteria optimization Univ. Dortmund, Tech. Rep. 49/05, SFB 475, 2005.
[5] M. S. Floater and A. Iske, "Multistep scattered data interpolation using compactly supported radial functions," *J. Comput. Appl. Math.*, vol. 73, pp. 65–78, 1996.
[6] J. Harrington, "The desirability function," *Ind. Quality Control*, vol. 21, pp. 494–498, 1965.
[7] S. L. Ho, S. Y. Yang, G. Z. Ni, E. W. C. Lo, and H. C. Wong, "A particle swarm optimization-based method for multiobjective design optimizations," *IEEE Trans. Magn.*, vol. 41, pp. 1756–1759, 2005.
[8] R. Y. Tang, S. Y. Yang, Y. Li, G. Wen, and T. M. Mei, "Combined strategy of improved simulated annealing and genetic algorithm for inverse problems," *IEEE Trans. Magn*, vol. 32, pp. 1326–1329, 1996.

Manuscript received June 24, 2007. Corresponding author: S. L. Ho (e-mail: eeslho@polyu.edu.hk).