Hysteresis Modeling of Magnetic Devices Using Dipole Distribution

Ka Wai Eric Cheng, *Member, IEEE*, Y. Lu, and S. L. Ho

Department of Electrical Engineering, the Hong Kong Polytechnic University, Hung Hom, Hong Kong

A new modeling method based on the normal distribution of dipoles is used to represent the B–H loop of magnetic materials. The dipole representation can be expressed either in 2-orientation or 4-orientation. Depending on the accuracy required, the modeling method typically requires only magnetic saturation parameters, the critical H-field, and a few control parameters. Experimental results show that the method can be used to represent the B–H loop of materials very accurately.

***Index Terms*—B–H loop, dipole, hysteresis, modeling, normal distribution.**

I. Introduction

MAGNETIC materials are important elements for many power conversion systems. Ferrites, iron powder, and molydbenum Permalloy powder (MPP) are commonly used for the fabrication of transformer and inductors for high-power and high-frequency applications. For today's power converter application, the magnetic component related losses account for more than 30% of the total system losses. An accurate design of the magnetic device reduces loss and optimizes the operation. Therefore, a proper modeling of the magnetic materials is an important step in the design of a power system. The modeling of B–H loop using finite element [1] and Preisach model [2], [3] has been proposed by many researchers with success. A Hammerstein-based model [4] which is an extension of the Preisach model has also found promising results. Recently, modeling using scalar product [5], neural network [6] and Jiles–Atherton mean model [7], and statistic method [8] have also been reported. These models are particularly suitable for highly nonlinear hysteresis loop characterization. On the other hand, the basic magnetic characteristics of the field orientation can be considered using the dipole theory of the magnetics which facilitates the consideration of the magnetic dipoles in two or more orientations that can be computed using the accumulation effect. This accumulation can be easily described effectively by the normal distribution of a probability density function.

In the absence of magnetization, the domains are randomly distributed and the total number of dipoles in either orientation is equal. When magnetization occurs, the dipole distributions are then aligned toward one orientation. As the number of dipoles in a material is large, their density function can be assumed to be normally distributed and it will be shown that the computation using the proposed method will reduce the complexity of the model to result in a reduction in computation time.

Digital Object Identifier 10.1109/TMAG.2005.845069

II. Dipole Orientation Theory

Assume N_0 and N_{180} are the total number of dipoles in the two orientations, the total number N_T which is a constant is

$$N_T = N_0 + N_{180}. \tag{1}$$

The normally distributed domains tend to align along the same direction as that of the applied magnetization. Therefore, the B-field for a given H is

$$B(H) = k\left(\int_{-\infty}^{H} p(x)dx - \int_{-\infty}^{0} p(x)dx\right) \tag{2}$$

where k is a scaling distribution constant and p is the normal distribution function, which is

$$p(x) = e^{-\frac{(x-m)^2}{2\sigma^2}} \tag{3}$$

where m is the mean value of $p(x)$ and σ is the standard deviation. The first integral of (2) is to describe the distribution of the dipoles whereas the second integral is to ensure the B–H hysteresis is balanced between the positive and negative values. Its value can be approximated by

$$\int_{-\infty}^{0} p(x)dx = 1.25\sigma. \tag{4}$$

A B–H loop is formed by an upward trajectory and a downward trajectory. Equation (2) is a function of four independent variables. Therefore, the trajectory is rewritten as $B(H, k, m, \sigma)$. The upward trajectory can then be represented by

$$B_u = B\left(H, k, -\frac{H_c}{2}, \frac{B_{\mathrm{sat}}}{(1.25k)}\right) \tag{5}$$

and the downward trajectory is similarly represented by

$$B_d = B\left(H, k, \frac{H_c}{2}, \frac{B_{\mathrm{sat}}}{(1.25k)}\right) \tag{6}$$

where $B_{\rm sat}$ is the saturated field and the domains are either aligned to 0° or 180° of the applied field; H_c is the critical H-field when $B = 0$. It should be noted that

$$B_{\rm sat} = 1.25k\sigma \tag{7}$$

$$H_C = m. \tag{8}$$

The B_u and B_d form a limiting loop. The above equations form a simple and realistic mechanism for the modeling of the B–H loop of a magnetic material. The method can be used to calculate the minor loop under dc and ac operations also. Once the B–H loop is determined, the hysteresis loss P_h can be determined easily by calculating the area enclosed by the B–H loop.

III. Higher Order Orientation

Higher order orientations such as 4, 8, etc., can also be used. The number of domains N_T in one core is constant. That is

$$N_0 + N_{90} + N_{180} + N_{270} = N_T. \tag{9}$$

The normal distributed domains are oriented along the same direction during the magnetizing process and they vary in a normal distribution manner. Therefore, the B-field at H is given by

$$B(H) = k_1\left(\int_{-\infty}^{H} p_1(x)dx - \int_{-\infty}^{0} p_1(x)dx\right) + k_2\left(\int_{-\infty}^{H} p_2(x)dx - \int_{-\infty}^{0} p_2(x)dx\right) \tag{10}$$

where k_1 *and* k_2 are the distribution constants and p_1 and p_2 are the normal distribution functions for 0° or 180° and 90° or 270°, respectively. That is

$$p_1(x) = e^{-\frac{(x-m_1)^2}{2\sigma_1^2}} \tag{11}$$

and

$$p_2(x) = e^{-\frac{(x-m_2)^2}{2\sigma_2^2}} \tag{12}$$

where m_1 and m_2 are the mean values of p_1 and p_2, and σ is the standard deviation. The second and fourth integrals of (10) are to ensure the B–H hysteresis is balanced between the positive and negative values. The value of the integral of p_1 and p_2 can be approximated by

$$\int_{-\infty}^{0} p_1(x)dx = 1.25\sigma_1 \tag{13}$$

$$\int_{-\infty}^{0} p_2(x)dx = 1.25\sigma_2. \tag{14}$$

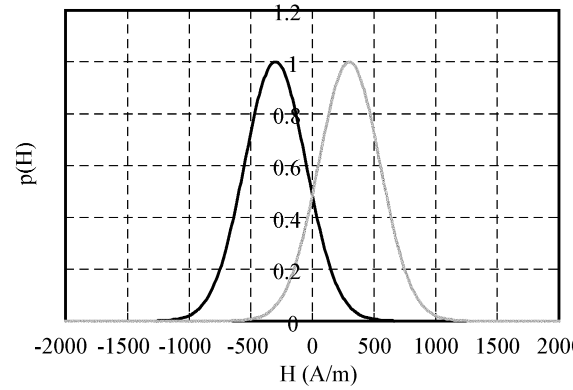

Fig. 1. Probability distribution function of the domain.

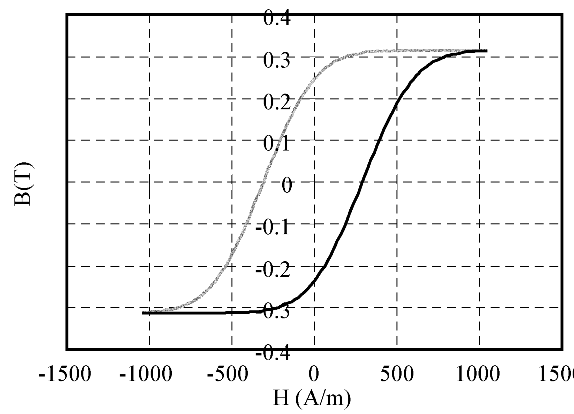

Fig. 2. B–H loop formed by the probability distribution function.

A B–H loop is formed by an upward trajectory and a downward trajectory. Equation (10) is in fact a function of seven independent variables. Therefore, the trajectory is rewritten as $B(H, k_1, k_2, m_1, m_2, \sigma_1, \sigma_2)$. The upward and downward trajectories are dependent on the positions of m_1 and m_2. In particular, when $k_1 = k_2$, and $\sigma_1 = \sigma_2$, the critical field is

$$\left|\frac{m_1 + m_2}{2}\right| = H_C. \tag{15}$$

It should also be noted that

$$B_{\rm sat} = 1.25(k_1\sigma_1 + k_2\sigma_2). \tag{16}$$

The above equations form a higher order method to simulate the B–H loop of a material by using a limiting loop.

IV. Computational Result

A. Two Orientations

Fig. 1 shows the normal distribution of the domains for the upward magnetization of $m = -300$ A/m and downward magnetization of $m = 300$ A/m for a typical ferrite. The corresponding B–H loop is shown in Fig. 2 with the following describing parameters: $B_{\rm sat} = 0.31$ T, $H_c = 300$ A/m that are based on $k = 0.001$ H/m, $\sigma = 250$ A/m.

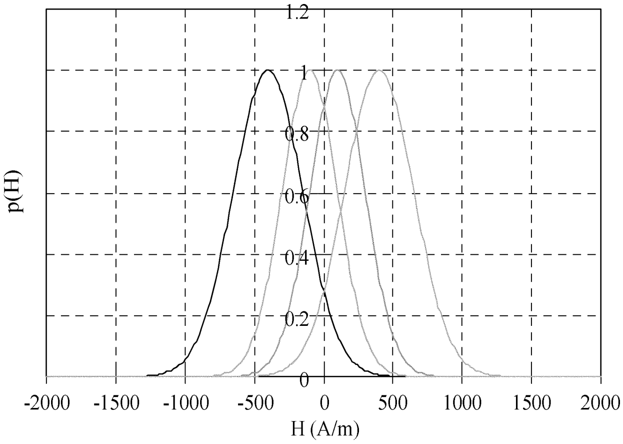

Fig. 3. Probability distribution function of domain using four orientations.

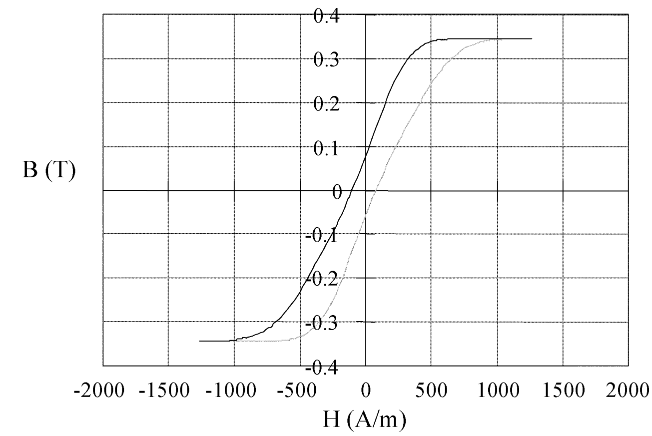

Fig. 4. B–H loop formed by the probability distribution function using four orientations.

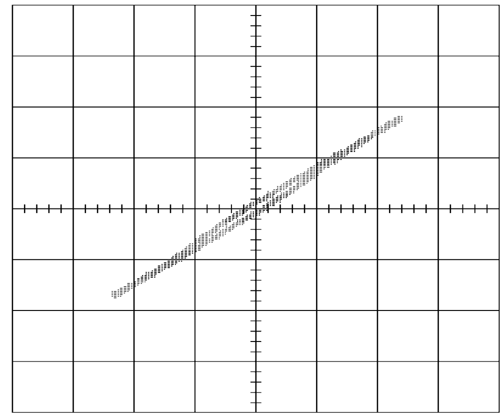

Fig. 5. Measured B–H loop of the new material poly7 (y: 0.06 T/div, x: 13.3 kA $\cdot$ m^{-1}/div).

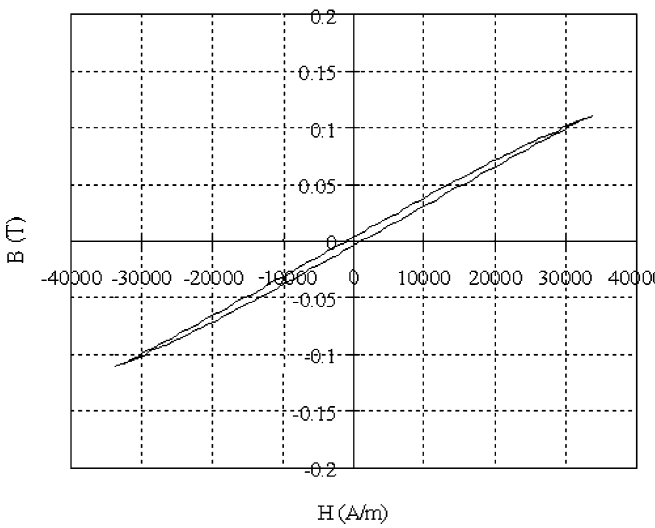

Fig. 6. Simulation of the B–H loop of material poly10.

B. Higher Order Orientations

Fig. 3 shows the normal distribution of the domains for upward and downward magnetizations using the parameters of: $m_1 = 400$ A/m, $m_2 = 100$ A/m, $\sigma_1 = 250$ A/m, $\sigma_2 = 200$ A/m. It can be seen that the location of the mean value m_1 and m_2 of the normal distributions can be used to describe the critical field. The standard deviation σ is to indicate the deviation of the dipole from the mean value and, hence, it can be used to describe the upward and downward trajectories accurately. The distribution constants k_1 and k_2 can be different when describing subtle movements of the trajectory. Fig. 4 shows the corresponding B–H loop based on the distributions. The parameters for k_1 and k_2 are 0.0032 and 0.0004.

It is clear than the 4-orientation arrangement can give more details of the subtle B–H loop variation as H is changing. When a higher order is used, very detailed trajectory B–H loops can be modeled. For common application, two or four orientations are sufficient.

Because of the basic theory of the dipole movement, the number of orientations is 2^{n}. The number of describing variables is twice that of the orientations. Higher order is only necessary for complicated B–H characteristics. One of the advantages of the proposed method is that it can model imbalance B–H characteristics because the dc offset can be described by different values of k, m, and σ.

V. Measurements and Simulations

A. Two Orientations

Fig. 5 shows the B–H curve of the low-permeability materials poly10 with the composition of Co and Ni in the ratio of 20 : 10. The H-field is increased to 33 kA/m in order to produce a B-field of 0.11 T. It can be seen that the materials have a very low relative permeability of only 2.7. The corresponding hysteresis loss is 480 J/m^3.

The above modeling techniques can be used to model new materials. Fig. 6 shows the computed B–H loop. The parameters used in the simulations are

$$k = 0.0035 \text{ H/m}, \quad m = 1000 \text{ A/m}, \quad \sigma = 45\,000 \text{ A/m}.$$

The accuracy of the measured B–H has been compared with that derived from the model and the error is less than 2% on each measured point. It can be seen that the proposed method represents a simple method to model the B–H loops of magnetic materials. The 2-orientation is more suitable for simple

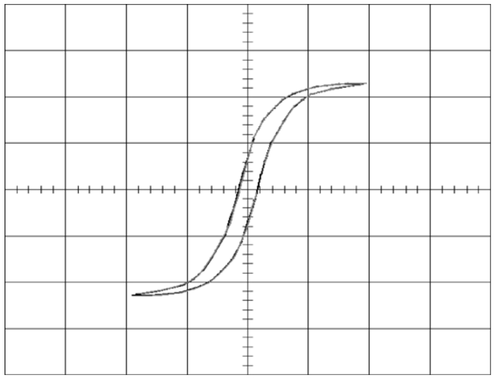

Fig. 7. Measurement of the B–H of the material 3C85 (y: 0.1 T/div, x: 100 $\mathrm{A \cdot m^{-1}/div}$).

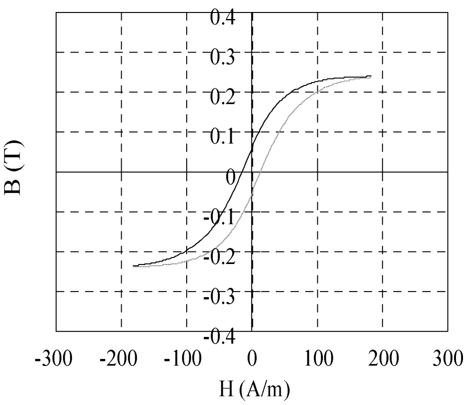

Fig. 8. Simulation of the B–H loop of 3C85.

B–H relationships. It can be seen that only three variables are required to model the characteristics in good agreement.

B. Higher Order Orientations

Higher order orientations can be used to describe more complicated B–H loops. Fig. 7 shows the B–H loop of the typical ferrite material 3C85 measured at 10 kHz. The H-field in this case is increased to 200 A/m in order to drive the material to near saturation at 0.238 T. The measurement was performed by using a high power amplifier to energize the material to high frequency.

In view of the relatively complexity of B–H characteristics of 3C85, its modeling is based on a 4-orientation dipole movement. Fig. 8 shows the B–H modeling results of the materials.

The describing parameters for the B–H of the material are

$$k_1 = 0.0018 \text{ H/m}, \quad m_1 = 20 \text{ A/m}, \quad \sigma_1 = 70 \text{ A/m}$$
$$k_2 = 0.00216 \text{ H/m}, \quad m_2 = 10 \text{ A/m}, \quad \sigma_2 = 30 \text{ A/m}.$$

By using a 4-orientation model, a more detailed curvature of the B-loop can be modeled with very good agreement with the actual characteristics. For more complicated B–H loops, higher order can also be used to give better representations.

VI. Conclusion and Discussions

The proposed modeling method gives a simple but powerful representation of the characteristics of magnetic materials. This new method is based on the dipole orientation and is a natural modeling methodology. The distribution of the dipole orientation is first described by a normal distribution and each distribution is controlled by the mean value m, standard deviation σ, and distribution constant k. In this paper, the basic theory of two and four orientations has been described in detail. Higher order representations can also be derived similarly. It can be shown that the higher the number of orientations being used, the more accurate model becomes. For a simple B–H loop, usually a 2-orientation is sufficient. In this case, only the values of k, m, and σ are necessary. For a 4-orientation model, the describing parameters consist of six variables. For higher order of orientations, the number of describing variables is twice as many as the orientation order. The external field attached to the materials can be decomposed into different vectors indicating the dipole orientation. Each orientation gives the component of the external field that in turn gives certain alignment of the dipoles. As the shape of the magnetic materials may be nonuniform, the number of orientations may need to be increased in order to give an accurate modeling.

Computation of the model is fast and no complicated algorithm is needed. The method has been tested for more than 20 models and only two are shown in the paper. Comparison between the computation and experiment confirms good agreement, and thus it can be concluded that the proposed method offers an alternative way to model the B–H characteristics of magnetic materials.

Acknowledgment

This work was supported by the Research Grants Committee of Hong Kong under Grant PolyU 5103/01E and by the Research Committee of the Hong Kong Polytechnic University under Project BQ474.

References

[1] H. L. Toms, R. G. Colclaser, and M. P. Krefta, "Two-dimensional finite element magnetic modeling for scalar hysteresis effects," *IEEE Trans. Magn.*, vol. 37, no. 2, pp. 982–988, Mar. 2001.
[2] S. R. Naidu, "Time domain model for the ferromagnetic core including the effects of hysteresis and eddy currents," *Inst. Elect. Eng. Proc.—A*, vol. 138, no. 1, pp. 44–50, Jan 1991.
[3] S. Y. R. Hui, J. G. Zhu, and V. S. Tamsden, "A generalized dynamic circuit model of magnetic cores for low- and high-frequency applications," *IEEE Trans Power Electron.*, vol. 11, no. 2, pp. 251–259, Mar. 1996.
[4] J. T. Hsu and K. D. T. Ngo, "A Hammerstein-based dynamic model for hysteresis phenomenon," *IEEE Trans. Power Electron.*, vol. 12, no. 3, pp. 406–413, May 1997.
[5] O. Henze, W. M. Rucker, M. Jesenik, V. Gorican, M. Trlep, A. Hamler, and B. Stumberger, "Hysteresis model and examples of ferromagnetic material," *IEEE Trans. Magn.*, vol. 39, no. 3, pp. 1151–1154, May 2003.
[6] H. H. Saliah, D. A. Lowther, and B. Forghani, "Modeling magnetic materials using artificial neural networks," *IEEE Trans. Magn.*, vol. 34, no. 5, pp. 3056–3059, Sep. 1998.
[7] F. T. Calkins, R. C. Smith, and A. B. Flatau, "Energy-based hysteresis model for magnetostrictive transducers," *IEEE Trans. Magn.*, vol. 36, no. 2, pp. 429–439, Mar. 2000.
[8] P. R. Bissell, M. Cerchez, R. W. Chantrell, and A. Stancu, "Evaluation of statistical and mean field magnetostatic interactions using generalized $\Delta \mathrm{M}$ plots: Experiments and Preisach modeling," *IEEE Trans. Magn.*, vol. 36, no. 5, pp. 2438–2440, Sep. 2000.

Manuscript received June 8, 2004.