Investigation on Rotation Response of Spin-Exchange Relaxation-Free Atomic Spin Gyroscope

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ABSTRACT Spin-exchange relaxation-free atomic spin gyroscope (SERF gyro) has ultrahigh theoretical sensitivity and is regarded as one of the most potential atomic gyroscopes. To investigate its rotation response, a high linear atomic spin precession detection module was developed firstly by modifying the fiber reflective Sagnac interferometer for atomic spin precession detection and operating it in closed-loop mode. The SERF gyro based on K-Rb-21Ne comagnetometer was built and tested and appeared obvious nonlinear response feature. The nonlinear rotation response model was deduced by assuming the rotation-induced nuclei spin procession detecting as an in-situ magnetometer inside the SERF gyro and the fitting curves are in good agreement with the experimental results. The optimized pump power for maximum sensitivity and measurable dynamic range were analyzed theoretically and demonstrated experimentally. The investigation results show that the nonlinear rotation response is an intrinsic property of SERF gyro and its measurable dynamic range is limited due to the saturation characteristic of the sum of pumping and relaxation rate. This indicates that SERF gyro is propitious to a position gyroscope with high sensitivity.

INDEX TERMS Atomic spin gyroscope, spin-exchange relaxation-free, nonlinear rotation response model, atomic spin precession detection.

I. INTRODUCTION Gyroscopes are considered as core components of the inertial system which play an important role in the inertial navigation system [1]–[3]. In recent years, atomic gyroscopes have attracted extensive attention due to their ultrahigh sensitivity, high precision and miniaturized structure [4], [5]. Spin-exchange relaxation-free atomic spin gyroscope (SERF gyro) which was first demonstrated in 2005 by Kornack et. al [6] has ultrahigh theoretical sensitivity and is regarded as one of the most potential atomic gyroscopes now. A bias instability of 0.009 deg/h was achieved by using a train of 87Rb π pulses and a +/− optical pumping technology [7] and a dual-axis SERF gyro using the parametric modulation technique was demonstrated and a rotation sensitivity of 1.093 × 10⁻⁶ rad/s/Hz¹/² was reported [8]. Up to now, most studies on SERF gyro have focused on the bias instability and rotation sensitivity based on the linear model demonstrated in [6]. However, as key performance parameters, its dynamic range and scale factor linearity have not attracted sufficient attention and the rotation response of SERF gyro has not been specially investigated. In this paper, we experimentally and theoretically investigated the rotation response of SERF gyro.

In SERF gyro, it is significant to detect precisely and linearly the atomic spin precession due to rotation. The conventional detection method is based on the classical Malus’s Law by using an off-resonance laser with a well-defined linear polarization and analyzing its polarization state after passing through the vapor [9]–[11]. And the high precision has been obtained based on the differential polarization detection.
and modulation technology, etc. [12]–[15]. However, there are some drawbacks, such as limited linear measurement range, bulky optical configuration, sensitive to thermal and vibrations fluctuations, etc. Recently, our group proposed and experimentally demonstrated a single-fiber Sagnac-like interferometer for atomic spin precession detection with two oppositely circularly polarized beams instead of linear polarized light. The open-loop detection system has been first constructed and successfully demonstrated in a SERF magnetometer [16]. In this paper, we improved the detection system further and operated it in closed-loop mode with excellent linearity in large measure range. Based on this technology, the rotation response of a SERF gyro was investigated successfully. In section II, the linear atomic spin precession detection module was proposed and developed by modifying the closed-loop fiber reflective Sagnac interferometer and let it operate in closed-loop mode. Its construction and operation principle were described and demonstrated briefly. In section III, the experiment setup of a SERF gyro based on K-Rb-²¹Ne comagnetometer was built and its rotation response was tested under different pump powers. The experimental results revealed its nonlinear response feature. In section IV, the cooperation condition of the self-compensating K-Rb-²¹Ne comagnetometer was analyzed in detail and it was found that the nuclei spin precession detection can be equivalent to an in-situ magnetometer inside the SERF gyro. The nonlinear rotation response model was established based on this assumption and the model fitting results agree well with the experimental data. Furthermore, the optimized pump power for maximum sensitivity and measurable dynamic range were analyzed theoretically and demonstrated experimentally. The investigation shows the nonlinear rotation response is an intrinsic property of SERF gyro and it is propitious to a position gyroscope with high sensitivity.

II. LINEAR ATOMIC SPIN PRECESSION DETECTION

Figure 1 shows the linear atomic spin precession detection scheme based on the modified closed-loop fiber optic spin precession detection scheme. It is composed of a fiber reflective Sagnac interferometer, a probe module, and an all-digital closed-loop detection circuit. The fiber reflective Sagnac interferometer is similar with the initial structure proposed in [16], but a broadband superluminescent diode (SLD) is used instead of the narrow linewidth diode laser to reduce the interference noise in the reciprocal optical path. The probe module consists of a collimating lens, an optical spectrum filter, a quarter-wave (λ/4) plate, a vapor cell, and a mirror. Where two circularly polarized probe lights of opposite helicity are taken as the probe lights instead of traditional linearly polarized light. The first three devices are assembled in a lens tube and the mirror is mounted on the opposite side of the vapor cell.

The fiber reflective Sagnac interferometer is a typical polarization mode interferometer and the evolution of polarization modes in the interferometer has been described in detail in [16]. Two polarization modes formed at 45° splice point have traveled along both axes of the polarization-maintaining fiber (PMF) delay-line as right and left circularly polarized lights in the vapor cell only in reverse order. This ensures the interferometer excellent reciprocity and insensitivity to environmental disturbances. The only source of phase shift between the two polarization modes is caused in the polarized vapor cell. The interference light intensity detected by the photodetector can be given by,

\[ I_{out}(t) = \frac{1}{2} I_s \{ 1 + \cos[\varphi_m(t) - \varphi_m(t - \tau) + \Delta \varphi] \} \]  

where \( I_{s} \) is the source intensity of SLD, \( \eta \) is the optical loss coefficient, \( \tau \) is the propagation delay time of the two waves passing through the modulator forward and backward. \( \varphi_m(t) \) and \( \varphi_m(t - \tau) \) represent the modulation phase produced by modulator at time of \( t \) and \( t - \tau \), respectively. \( \varphi \) is the phase difference between the two circularly polarized probe lights, it is four times the optical rotation angle when the linearly polarized lights were used as probe lights.

Based on the closed-loop fiber optic gyroscope (FOG) technique [17], this interferometer takes a Ti-diffused LiNbO₃ integrated optical straight waveguide as birefringence modulation and is operated in closed-loop mode with the all-digital closed-loop detection circuit. The \( \pi/2 \) bias modulation waves (a) and the phase step waves (b) were added on the modulator simultaneously to provide bias modulation and a nonreciprocal phase shift equal and opposite to the phase difference, the first harmonic of the periodic waveform (c) was thereby nulled and the nonreciprocal phase shift was then a measure for \( \varphi \). The output digital quantity of the closed-loop detection circuit can be written as [9], [17],

\[ D = K_D \cdot \Delta \varphi = K_D \cdot K_V \cdot I \cdot P_x^e \]  

\[ K_V = \frac{\pi}{3} n r e c [\text{Im}[V(v - v_{D2})] - \text{Im}[V(v - v_{D1})]] \]  

where \( K_D \) is the conversion coefficient from the phase difference \( \varphi \) to the output digital quantity. \( K_V \) is the equivalent Wilder coefficient of the polarized vapor cell. \( I \) is the propagation distance of light in the vapor cell. \( P_x^e \) is the alkali-metal atoms spin polarization component along the x axis.
as shown in Fig. 1. $n$ is the density of alkali-metal atoms, $r_e$ is the electron radius of the alkali-metal atom, $c$ is the velocity of light in vacuum, $\nu$ is the light frequency, $\nu_{D1}$ and $\nu_{D2}$ are the resonance frequency of D1 and D2 line of alkali-metal atoms, respectively. Im[$V(\nu-\nu_{D1})$] and Im[$V(\nu-\nu_{D2})$] represent the imaginary part of the Voigt profile of resonance frequency $\nu_{D1}$ and $\nu_{D2}$, respectively. As described in (2) and (3), the $P_e$ can be measured linearly and accurately within a large phase difference range without any approximation. Thanks to mature closed-loop FOG technology, the excellent linearity of less than $10^{-6}$ and high phase resolution of less than $10^{-8}$ rad can be achieved [17] in this linear atomic spin precession detection system.

### III. EXPERIMENTAL SETUP AND RESULTS

Figure 2(a) shows the schematic illustration of the experimental setup of the SERF gyro based on K-Rb-21Ne comagnetometer. The core unit was a 15 mm diameter spherical cell made from GE180 aluminosilicate glass. The cell contained a mixture of K, Rb (72.2% 85Rb and 27.8% 87Rb) alkali-metals, 2.7 atm of 21Ne gas and 50 torr of N2 gas for quenching, the density ratio of K to Rb atoms was about 1:100. The cell was mounted in a boron nitride ceramic shell frame and heated to $\sim$185 °C by a non-magnetic electric heater driven by a 50 kHz AC source. A set of three orthogonal Helmholtz coils was driven independently by three high current function generators (Keithley Model 6221) to zero the magnetic field at the location of the vapor cell. All the components above were mounted inside the three-layer $\mu$-metal magnetic shields with a shielding factor of 105. The pump laser module contained a 770.1 nm (K atom D1 line) ECDL laser, a tapered amplifier, and their driver. The lens tube 2 contained a collimator and a $\lambda/4$ plate. The amplified pump laser was led through PMF to the lens tube 2 and circularly polarized by the $\lambda/4$ plate. In the K-Rb-21Ne comagnetometer, the function of optically pumped K atoms is to polarize Rb atoms while the polarized Rb atoms are used not only to hyperpolarize 21Ne nuclei but also to sense the spin precession due to the applied rotation rate.

The fiber reflective Sagnac interferometer and all-digital closed-loop detection circuit shown in Fig. 1 were integrated to form the closed-loop fiber reflective Sagnac interferometer for atomic spin precession detection (abbreviated as FSI ASP detection module) and packaged in an instrument box with the size of 22 cm × 17 cm × 6 cm. The photograph of the FSI ASP detection module is shown in Fig. 2(b). The mirror, vapor cell and lens tube 1 containing a collimator, a filter and a $\lambda/4$ plate constituted the probe module. As shown in Fig. 2(a), the FSI ASP detection module, the pump laser module, and the probe module were all mounted on a rotary platform and the y-axis of the SERF gyro was parallelly aligned to the rotation axis of the rotary platform. The K atoms in vapor cell were polarized along the z-axis and the Rb atoms electron spin polarization component $P_e$ along the x-axis was detected by the FSI ASP detection module and the output was recorded by a computer.

To reduce the interference noise due to stray reflection light of optical components and parasitic interference in fibers, the broadband SLD with the mean wavelength of 787 nm and bandwidth of 43 nm was used as the light source and its spectrum is shown in Fig. 3. The normalized $K_V$ was simulated based on (3) with the typical parameters listed in [9] and shown in Fig. 3. It can be seen the spectrum width of SLD is larger than the space between Rb D1 and D2 lines and the wavelength dependence $K_V$ has reverse values at the left and right of the Rb D1 (and D2) line. This results in smaller output because the output is proportional to the integral value over the source spectrum bandwidth. To guarantee high sensitivity, a specially designed band-pass optical filter was inserted before the $\lambda/4$ plate. The filtered spectrum is shown in Fig. 3 and covers almost the spectrum space between Rb D1 and D2 lines. Therefore, high sensitivity and low interference noise were satisfied simultaneously.

During the experiment, the residual magnetic fields in the volume of the K-Rb-21Ne vapor cell was compensated firstly with the in-situ triaxial magnetic field compensation technique by analyzing the pump light intensity transmitted through the cell [18], and the residual magnetic field of less
than 2 nT was achieved. The SERF gyro is operated in the SERF regime and the special cooperation condition, i.e. $B_c = -(B^e + B^n)$ must be satisfied, as shown in Fig. 5 (a), where $B_c$ is the applied bias magnetic field parallel to the pump beam ($z$-axis), $B^e$ and $B^n$ are created by the nuclear and electron magnetization respectively along the $z$-axis. For this purpose, the procedure of magnetic field $B_c$ compensating was carried out and the zero compensating point was identified based on the response to a $B_c$ modulation square wave at various $B_c$ values and the compensation value of $B_c$ was set accurately [19]. Under the SERF regime and self-compensation condition, we tested the gyroscopic rotation response from $-10.0$ deg/s to $+10.0$ deg/s with an interval of 1 deg/s under different pump powers of 51 mW and 155 mW preliminarily. The tested data were shown in Fig. 4 and appeared typically Lorentzian dispersion feature, i.e. obvious nonlinearity within the applied rotation rate range. Besides, the measurable monotone range is smaller and closely related to the pump power. In our previous work, this similar phenomenon has been observed in the parametrically modulated dual-axis atomic spin gyroscope [8] and the investigation on the real-time closed-loop control of the compensation point in the K-Rb-Ne comagnetometer [20] and can confirm these tested results further.

At the rotation rate of $\pm 10.0$ deg/s, the corresponding phase difference $\Delta \phi$ between the two circularly polarized lights of opposite helicity is far less than $2\pi$ rad and the linearity of the FSI ASP detection module can be well guaranteed because its measurable phase difference range is beyond $2\pi$. Therefore, the observed nonlinear response must be attributed to the intrinsic property of the SERF gyro itself.

### IV. ROTATION RESPONSE MODEL

As mentioned above, the SERF gyro is operated in the SERF regime while the special cooperation condition for the self-compensating comagnetometer is satisfied, as shown in Fig. 5(a). Under this cooperation condition, the nuclear magnetization will adiabatically cancel to any first-order fluctuations in the external field along transverse axes and all net magnetic field $B$ and light shift $L$ can be regarded as zero. The rotation response equation was simplified as following in the early literature [6], [13]:

$$P_x^e = \frac{P_x^{e_{tot}}}{\gamma_e R_{tot} + \gamma_n R_{rel}}$$

(4)

$$R_{tot} = R_p + R_{rel}$$

(5)

where $P_x^e = R_{rel}/R_{tot}$ is the equilibrium polarization of the electron along the $z$-axis, $R_p$ is the pumping rate of the pump beam, $R_{tot}$ is sum of the pumping rate $R_p$ and the relaxation rate $R_{rel}$. $\Omega_y$ is the applied rotation rate along the $y$-axis, $\gamma_e$ and $\gamma_n$ are the gyromagnetic ratio of electron and nuclear spin, respectively. Up to now, most studies on SERF gyro have focused on the bias instability and rotation sensitivity based on this linear model. However, referring to the experimental results shown in Fig. 4, it is found that this linear model may be valid only within a very small rotation rate range and further refinement for large rotation rate range is needed.

In SERF gyro based on K-Rb$^{21}$Ne comagnetometer, for example, the polarized Rb atoms are taken to polarize $^{21}$Ne atoms and sense the spin procession due to the applied rotation rate. When a rotation rate $\Omega_y$ was applied along the $y$-axis, only the equivalent magnetic field $B_y$ will be experienced by the Rb atoms and the component of electron spin polarization $P_x^e$ will be detected with the FSI ASP detection module, as shown in Fig. 5(b). This procedure can be equivalent to an in-situ magnetometer inside the SERF gyro and described with the classical Bloch equation of the electron spins [21]:

$$\frac{\partial P^e}{\partial t} = \frac{1}{q} \left[ \gamma_e B \times P^e + R_p \left( \hat{z} - P^e \right) - R_{rel} P^e \right]$$

(6)

where the pump rate of the probe light is neglected, $P^e$ is the electron spin polarization vector, $B$ is the magnetic field experienced by the electrons. When the cooperation condition mentioned above was satisfied and the applied rotation rate is $\Omega_y$, $B_z = 0$, $B_y = \Omega_y/\gamma_n$, and $B_z = 0$, the component of electron spin polarization $P_x^e$ can be solved out as following,

$$P_x^e = \frac{P_x^{e_{tot}} \gamma_e \Omega_y}{R_{tot} \gamma_n \left( 1 + \left( \frac{\gamma_e}{R_{rel} \gamma_n} \right)^2 \Omega_y^2 \right)}$$

(7)
Equation (7) is a typical Lorentzian dispersion response function and the maximum values will be reached at $\pm \Omega_{ym}$ described as following:

$$\Omega_{ym} = \gamma_n R_{tot} / \gamma_e$$  \hspace{1cm} (8)

When $\Omega_y << \Omega_{ym}$, the term containing $\Omega_y^2$ can be ignored and (4) can be obtained from (7). This indicates that the linear response model (4) is the special linear form of (7) at a small rotation rate by approximation.

Combining (2) and (7), the rotation rate $\Omega_y$ can be expressed in terms of output numerical quantities $D$ which can be rewritten to fit the Lorentzian dispersion function:

$$D = \frac{K_1 \Omega_y}{1 + K_2 \Omega_y^2}$$  \hspace{1cm} (9)

$$K_1 = \left( K_D K_V R_p \gamma_e \right) / \left( R_{tot}^2 \gamma_n \right)$$  \hspace{1cm} (10)

$$K_2 = \left( \gamma_e / R_{tot} \gamma_n \right)^2$$  \hspace{1cm} (11)

where $K_1$ is a comprehensive coefficient related to the polarized vapor cell and the FSI ASP detection module. The coefficient $K_2$ is inversely proportional to the square of $R_{tot}$.

The tested data under different pump powers are fitted according to (9) and the fitting curves are plotted in Fig. 4. The R-square values (the goodness of the fit) of the nonlinear fitting are all superior to 0.99, this shows a good agreement between the tested data and the nonlinear rotation response model and proves further that it is reasonable to regard the rotation rate sensing behavior in SERF gyro as an in-situ magnetometer which is sensitive only to the equivalent magnetic field produced by inertial rotation. The fitted model parameters are all listed in Table 1. In the SERF gyro based on K-Rb-$^{21}$Ne comagnetometer, $\gamma_e = 2.80 \times 10^{10}$ Hz/T and $\gamma_n = 3.37 \times 10^6$Hz/T, the $R_{tot}$ and the rotation rate $\Omega_{ym}$ at the maximum response were calculated out according to (11) and (8) and listed in Table 1 too. The experimental and theoretical investigation results both reveal the nonlinear rotation response characteristic of the SERF gyro and its rotation sensitivity and dynamic range are both related closely to $R_{tot}$.

Substituting (5) to (10), when $R_p = R_{rel}$, we can get the maximum rotation sensitivity as following:

$$K_1 = \left( K_D K_V / \gamma_e \right) / \left( 4 R_p \gamma_n \right)$$  \hspace{1cm} (12)

This indicates there must be an optimal pump power to achieve maximum rotation sensitivity for the SERF gyro.

From the equation (8), the SERF gyro measurable maximum rotation rate $\Omega_{ym}$ is determined by the $R_{tot}$. In our previous works [22], we have modeled and demonstrated that the spin-exchange relaxation in K-Rb-$^{21}$Ne comagnetometer is related closely to spin-exchange interaction between Rb and $^{21}$Ne spins and effected closely by the spin polarization of Rb. This will let $R_{tot}$ saturated when the pump power was big enough.

Increasing the pump power, we investigated the evolution characteristic of $R_{tot}$ in the SERF gyro by testing the rotation response at each pump power. The values of $R_{tot}$ were calculated out based on the fitting results according to (9)-(11). The similar results were obtained and shown in Fig. 6. The fitting curve was obtained by nonlinear fitting in Origin software. The saturation value of $R_{tot}$ is $\sim 3059$ s$^{-1}$ when the pump power is bigger than $\sim 250$ mW. Based on the tested data, the sensitivity coefficient $K_1$ and the maximum rotation rate $\Omega_{ym}$ under each pump powers were carried out and plotted in Fig. 6. The experimental results agree well with the theoretical prediction. The maximum sensitivity of 3818 (deg/s)$^{-1}$ is achieved under the optimized pump power of $\sim 82$ mW. The tested rotation response and the fitting curve under 82 mW were added in Fig. 4 and Table 1. As shown in Fig.7, the maximum rotation rate of $\sim 21$ deg/s is reached when pump power is bigger than 250 mW.
V. CONCLUSION

In summary, we developed a high linear atomic spin precession detection technique by modifying the fiber reflective Sagnac interferometer and operating it in closed-loop mode. The experimental setup of SERF gyro based on K-Rb-Ne comagnetometer with this detection module was built and its rotation response under different pump powers was tested experimentally. The tested results show that the gyro rotation response is nonlinear and the measurable dynamic range is limited. The cooperation condition of the self-compensating K-Rb-Ne co-magnetometer was analyzed and the nonlinear rotation response model was established by assuming the nuclei spin procession detecting as an in-situ magnetometer inside the SERF gyro. The model fitting curves agree well with the experimental results and confirm the assumption rationality. Furthermore, the optimized pump power for maximum sensitivity and measurable dynamic range of the SERF gyro were analyzed theoretically and demonstrated experimentally. And the curve of sensitivity and maximum measurable rotation rate were obtained with the proposed model. The investigation results indicate that the SERF gyro is propitious to a position gyroscope with high sensitivity.

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