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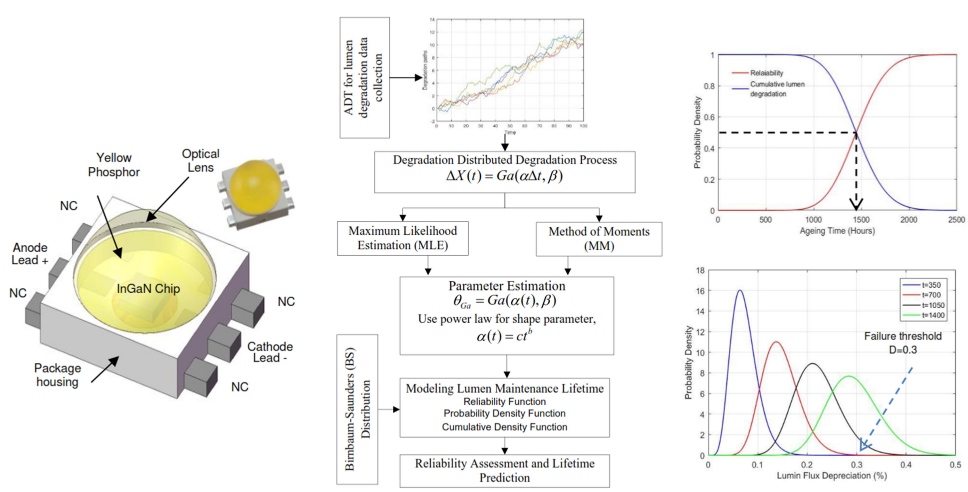

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Lumen Degradation Lifetime Prediction for High-Power White LEDs Based on the Gamma Process Model

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Abstract: Nowadays, due to the advancement of design and manufacturing technology, there are many consumer products with high reliability. Similarly, the competition in the business sector influences the product development time to become shorter and that makes it difficult for manufacturers to evaluate the reliability of current products before new products are released to the market. This phenomenon is manifested in the lighting industry, especially for the high power white light-emitting diodes (LEDs) as these products have a long lifetime and high reliability. Currently, the standard to predict the lifetime of LEDs is based on a deterministic nonlinear least squares method which has low prediction accuracy. To overcome this, degradation models are being used to study the reliability of such products, considering the uncertainties and the quality characteristics whose degradation over a period of time can be related to the product lifetime. A stochastic approach based on gamma distributed degradation (GDD) is proposed in this study to estimate the long-term lumen degradation lifetime of phosphor-converted white LEDs. An accelerated thermal degradation test was designed to gather luminous flux degradation data which was analyzed based on maximum likelihood estimation (MLE) and the method of moments (MM) to estimate the parameters for the GDD model. The MLE method has shown superiority over MM in terms of the estimation of the model parameters due to its iterative algorithm being likely to find the optimal estimation. The lifetime prediction results show that the accuracy of the proposed method is much better than the TM-21 nonlinear least squares (NLS) approach which makes it promising for future industrial applications.

Index Terms: Light-emitting diodes (LEDs), luminous flux degradation, gamma distributed degradation (GDD), maximum likelihood estimation, method of moments.

1. Introduction

In the past few years, it has been challenging to evaluate the reliability of highly reliable products such as light-emitting diodes (LEDs), aircraft components, lithium-ion batteries, etc., based on classical approaches including censoring and/or accelerated lifetime-based methods that record time-to-failure. This is due to the long lifetime of the products that contributes to the recording of few or no failures during the reliability testing and little information can be extracted from such life data [1], [2]. It was later believed that degradation data is more convenient to obtain when compared with failure data (life data), so that degradation modeling methods can be implemented to estimate the lifetime, make inferences about failure times and important reliability information [3], [4]. The quality characteristics that provide important reliability information depend on product properties such as the light output and chromaticity shift for LEDs [5], [6], mechanical vibration for bearings [7], capacitance for capacitors [8], charging capacity in batteries [9], wheels of a train [10] and so on. These quality characteristics are often manifested in dynamic and time-dependent or temporal variability properties. Based on this, the relationship between the amount of degradation data and failure times enabled the development of degradation path models, two-step models and similar statistical methods to assess the reliability and lifetime of products, such as fatigue crack growth, tire wear and so on [4], [11], [12]. The gradual degradation of such products can also be modeled based on discrete-time Markov processes as well as continuous-time Markov and stochastic processes such as the compound Poisson process, Wiener processes with drift and Gamma processes, due to the independent incremental properties in these methods [13]. The non-linear as well as random and dynamic variation multiple performance characteristics of products can be explained with the implantation of stochastic processes [14].

In general electronic systems, the failure mechanisms occur in the form of either catastrophes (overstress) or degradation (wear out). The catastrophic failures are due to a single stress (load) condition (electrical, mechanical, chemical, etc.) that exceeds a certain failure threshold while the degradation failures occur over a certain period of time due to cumulative damage caused by stresses or loads [15]. Characterized as being sudden and complete, catastrophic failures (hard failures) result in the termination of service (operation), while degradation failures (also known as soft failures) are characterized by partial and gradual performance degradation [16]. Irrespective of the high reliability and long lifetime properties, the high power white LEDs are known to experience both catastrophic failures and deterioration of lumen flux, as well as chromaticity shifting over time during their service lifetime [6], [17]. However, there is lower probability of catastrophic failure (no light output) due to open circuits for InGaN based LEDs in which the dominant failure mode is performance degradation in terms of luminous flux and chromaticity shift [6], [18]. The lumen degradation and chromaticity shift (including CCT shift, Duv) phenomenon occur simultaneously, however the degradation pattern and the rate of degradation of luminous flux and color shift is different. Studies show that the degradation dependency between the two performance indicators lumen maintenance and CCT is not strong, thus both could be simply considered as independent variables. In most cases the color shift degrades faster compared to the luminous depreciation [19], [20].

Among stochastic approaches, the Wiener process has been widely studied for degradation analysis due to its important mathematical properties [21]. One of the main advantages of a conventional Wiener process is that the distribution of the failure times can be formulated analytically in terms of a transformed inverse Gaussian distribution. In addition, the Wiener process is appropriate for degradation modeling where there are bi-directionally varying degradation processes with Gaussian noise [22], [23]. Therefore, if the degradation overtime is linear, the Wiener process with linear drift is appropriate and can be demonstrated on laser data with unit to unit variability [22], and lamps in contact image scanner [23]. On the other hand, nonlinear degradation can also be linearized by time-scale transformation [24] or log-transformation [25]–[27] to make it suitable for Wiener process analysis. However, not all nonlinear degradations can be transformed by these techniques. Doksum [28] employed the Wiener process to model the reliability of products based on the process of degradation, so that failure occurred when the degradation crosses a critical threshold. Si *et al.* [29] developed a general Wiener process with nonlinear drift and formulated an analytical approximation for failure time distributions and residual life under mild assumptions. Huang *et al.* [30] used a modified Wiener process to estimate the lifetime for a mid-power white LED. Ibrahim *et al.* [31] applied the Wiener process for lumen maintenance lifetime prediction of phosphor-converted white LEDs. There have been a number of reviews on the Wiener process method. A comprehensive review on stochastic modeling, three variants of the Wiener process with measurement error, with random effects as well as with covariates, was presented by Ye and Xie [32]. Recently Zhang *et al.* [33] presented a comprehensive systematic review on Wiener processes and its variants. Although the Wiener process has many advantages, it also has a major weakness in that it ignores the information given by the entire sequence of observations and it only make uses of information in the current degradation data.

The Gamma process is also considered to be well suited for modeling the temporal variability of degradation [34]. Since the introduction of the Gamma process to assess the reliability and model random degradation of wear processes in the 1970's by Abdel-Hameed [35], it has been increasingly used to model stochastic degradation. It is believed that the Gamma process is most appropriate method for the stochastic modeling of monotonic and gradual deterioration or degradation. The gamma process is one of the widely used types of stochastic process suitable for analyzing and modeling monotonic degradation processes, that take place in a sequence of tiny increments over time [34]. The Gamma process is a natural model for degradation processes that vary one-directionally such as fatigue crack propagation and wear process. The other applications of the Gamma process include corrosion, fatigue, erosion, consumption, swell, creep, degrading health index, etc [13]. For example, Lawless and Crowder [36] developed a tractable gamma process that incorporates random effects to model the crack propagation of metallic particles. Freitas *et al.* [10] used train wheel degradation data to model the reliability of the wheels based on the Gamma process and Brownian motion, and Gamma process based methods have been used to model the degradation of products mainly involving multiple accelerating variables [37]. Liu *et al.* [38] presented inverse Gaussian and Gamma process methods for monotonic degradation data and used Bayesian model averaging for reliability parameter estimation. Recently, the Gamma process has been used to predict the lifetime of LEDs based on the CCT shift [39], however, in this study the lumen maintenance life was unfortunately not discussed as this considered to be the dominant failure mode for phosphor-converted white LEDs in general applications. The particle filter-based approach has also been proposed which considers measurement dynamics and uncertainties of lumen degradation [40], [41]. However, this method requires a robust approach for parameter initialization or requires prior knowledge on the initial parameters estimates. Currently, many LED manufacturers use the IES-TM-21-11 standard to predict the lifetime of LED light sources [42]. This standard is based on the nonlinear least squares (NLS) regression method approved by the Illuminating Engineering Society of North America (IESNA), and the lumen maintenance data is also collected according to the LM-80-08 standard [43]. Later, the IES-TM-28 standard was release to project the long-term lumen maintenance lifetime for LED lamps and luminaires [44]. Similarly, the luminous flux and color coordinates data can be obtained according to the approved IES-LM-84-14 standard [45].

This study is aimed at assessing the reliability and predicting the lifetime of phosphor-converted white LEDs from the luminous flux degradation data based on the Gamma distributed degradation (GDD) approach, which can handle temporal variability independent and non-negative degradation increments. The maximum likelihood estimation approach is used for parameter estimation which gives the estimated results after automatic iterations that do not require a sophisticated approach. The Moment Method has also been applied to validate the results from the MLE method. The estimated values were used for the prediction of the lumen maintenance lifetime, assessing the lifetime distribution and the related reliability information.

The remainder of this paper is organized as follows: Section 2 presents the degradation test to obtain performance characteristics data and lifetime data analysis. The description of the gamma process approach along with the parameter estimation methods is presented in detail in Section 3. In Section 4, the simulation results for the luminous flux degradation obtained based on the described

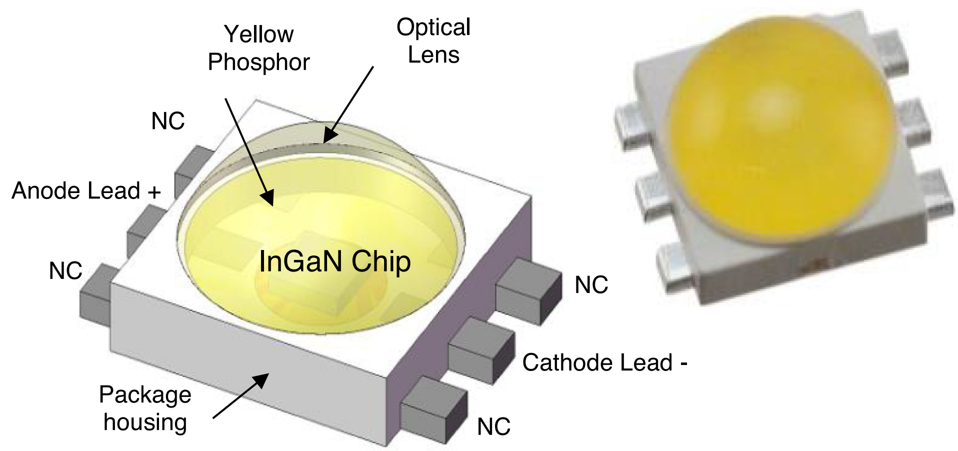

Fig. 1. Test sample architecture schematic view (left) and pictorial view (right).

methodology and comparison with the traditional nonlinear least square approach are described. Finally, the concluding remarks are stated in Section 5.

2. Degradation Test and Lifetime Data Analysis

2.1 Test Samples

The state of the art production of white light from LEDs for general lighting is typically based on three approaches: the phosphor-converted LEDs, color-mixed LEDs and hybrid-method LEDs. Nowadays, the phosphor-converted white LEDs based approach is the most dominant architecture to produce white light for LEDs. The test samples used in this study are phosphor-converted white LEDs which are composed of yellow phosphor and GaN based blue LEDs with an electrical power capacity of 3W, with high-brightness. The samples used were manufactured by Avago with part number ASTM-JN-31-NTV01. The phosphor-converted white LED package detailed architectural, as well as schematic and pictorial views, are presented in Fig. 1. The white light is generated based on the principle of electroluminescence from the package when photons from the blue LED pass through yellow phosphor once an electric current is applied. Previously, Fan *et al.* [20] investigated the color shift failure mechanisms and predicted the residual color lifetime of these test samples [20]. In this study, the features of spectral power distribution are extracted using Gaussian and Lorentzian model and then the spectral power distribution feature trajectories are modeled by a nonlinear filtering approach to finally estimate the CRI and CCT and quantify the color shift failures.

In addition, a lumen maintenance lifetime prediction study was conducted on these samples [40]. In the study, the particle filter-based prognostic approach was implemented, robustness of the particle filter-based method investigated and resulting a better prediction accuracy than the NLS method.

2.2 Experimental Setup and Test Condition

The phosphor-converted white LEDs have high reliability and long lifetime characteristics in normal working conditions. Thus the accelerated degradation test based on electrical and thermal stress was conducted to obtain photometric and colorimetric degradation data. The thermal stress was conducted in an electro-thermal chamber with a constant ageing temperature of 90 °C whereas the electrical stress was provided by a DC power source (Device model: E3611A Agilent) at a constant current of 0.2A. A total of 16 samples were tested and the test conditions in this study are shown in Table 1. The degradation experiment setup in our lab is demonstrated in Fig. 2.

TABLE 1
Test Condition for the Experiment

Terms		Test Conditions
Testing duration		1663 hrs (71 test cycles)
Interval of optical data collection	Luminous flux	every 23 hrs
	Chromaticity coordinates	every 23 hrs
Input current (DC powered)		200 mA
Temperature	Test	90 °C
	Ambient	25 °C
Relative humidity		18%RH

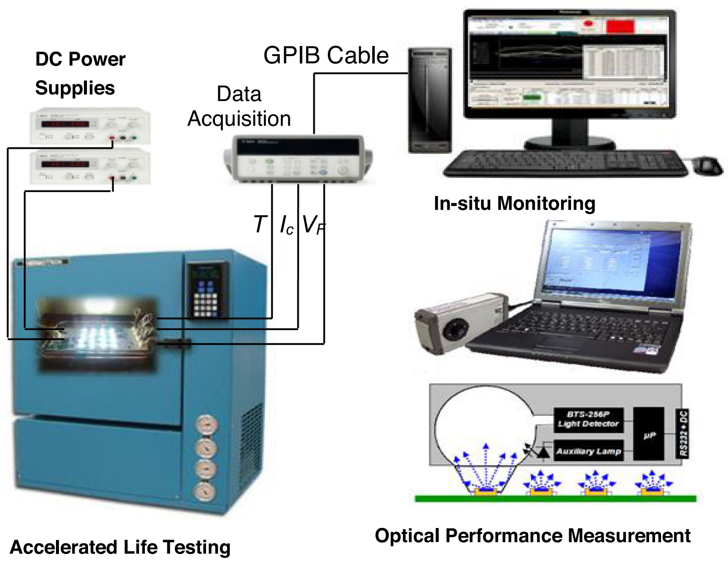

Fig. 2. Experimental setup for data collection.

The luminous flux of the phosphor-converted white LED depreciates as the operation time increases from the start to 1633 hrs. The lumen degradation data for the 16 test samples is presented in Fig. 3. The photometric and colorimetric characteristics were measured every 23 hrs and the test was terminated after 1633 hrs, over a total of 71 test cycles.

The cumulative degradation X_n of the relative luminous flux is the sum of the deterioration on light output after n measurements due to the stress condition i.e $X_n = X_0 + X_1 + \cdots + X_{n-1}$ where $X_0 = 0$ is as shown in Fig. 4. It can be noted from the plot that the cumulative degradation data increases as the ageing time increases, and finally a few components crossed the failure threshold.

3. Theory and Methodology

In this section, the methodologies used in this study are described respectively as the Nonlinear Least Square (NLS) regression approach and the Gamma process based approaches, along with the two parameter estimation methods – maximum likelihood and method of moments.

3.1 Nonlinear Least Square (NLS) Method

The IES-TM-21 is the standard method approved by IESNA to project the lumen maintenance lifetime of LEDs light sources and is used by many LED manufacturers [42]. The LED lifetime is

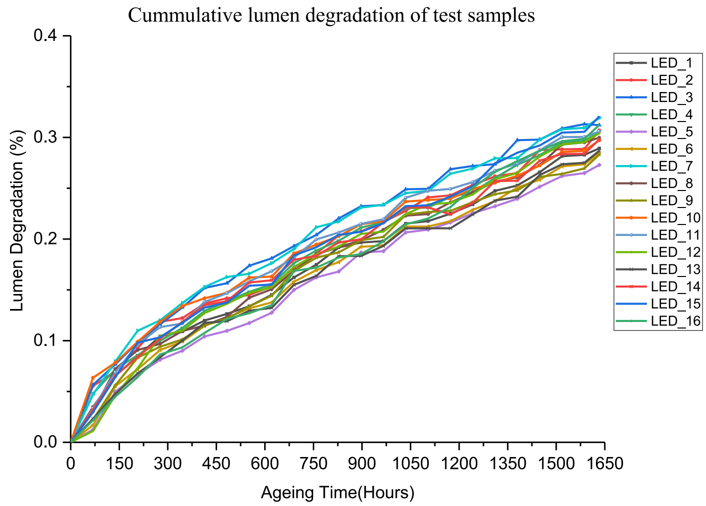

Fig. 3. Lumen flux degradation with time (all samples).

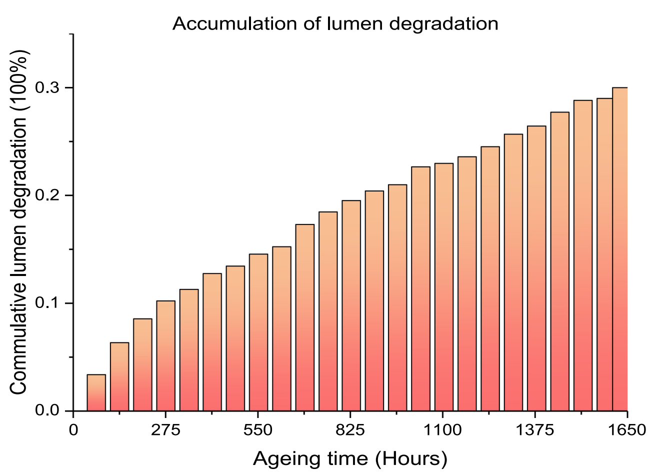

Fig. 4. Relative lumen flux cumulative degradation with time.

projected according to the data collected in the IES-LM-80-08 standard [43]. The lumen maintenance lifetime for general applications is defined as the operating time that the lumen flux will maintain 70% (L70) of the initial light output, according to the Alliance for Solid-State Illumination Systems and Technologies (ASSIST) [46]. This method is implemented according to the following procedures (i) the lumen flux data should be collected according to the IES-LM-80-08 standard, and all the data gathered should be normalized to a value of 1 in order to determine the lumen maintenance life from the initial light output. $LM(t) = [\Phi(t)/\Phi(0)] * 100\%$ (ii) perform a curve-fit based on an exponential least-square regression method using the equation $LM(t) = \beta * \exp(-\alpha * t)$ where β is the projected initial constant, α is the decay rate constant and t is the operating time, (iii) Project the lumen maintenance lifetime using $L_{70} = \ln(\beta/0.7)/\alpha$ to estimate the L_{70} lumen maintenance lifetime of LED light sources.

3.2 Gamma Process: Definition and Reliability Modeling

A continuous time stochastic degradation process which can be described as a Gamma process satisfies the following properties [47].

i) The initial degradation $X(0) = 0$ with a probability one,
ii) $X(t)$ has steady and independent increments $X(t_4) - X(t_3)$ and $X(t_2) - X(t_1)$ with respect to time $t \geq 0$ only if $t_4 > t_3 > t_2 > t_1$
iii) The increments $\Delta X(t) = X(t + \Delta t) - X(t)$ follow the Gamma distribution, and the degradation model M_G is represented as:

$$M_G : \Delta X(t) \sim Ga(\alpha \Delta t, \beta) \tag{1}$$

where $\alpha(t) > 0$ is the shape parameter and $\beta > 0$ is the scale parameter. In the Gamma process, the shape parameter α(t) has a positively increasing value with degradation time while the scale parameter β has a fixed constant value. When the performance degradation parameters are modeled based on the Gamma process, the influence of stress (due to temperature, current, etc) on the products' performance is described by the shape parameter, while the effect of random factors such as human and environmental factors, and material differences are described by the scale parameter. Thus, it is generally assumed that the scale parameter is not related to stress and the shape parameter is affected by stress [48].

According to the definition of the gamma process, the probability density function (PDF) of X(t), can be given as in Eq. (2).

$$f_{X(t)}(x|\alpha(t), \beta) = \begin{cases} \frac{1}{\Gamma(\alpha(t))\beta^{\alpha(t)}} x^{\alpha(t)-1} \exp(-x/\beta)\, I_{(0,\infty)}(x), & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{2}$$

where $I_{(0,\infty)}(x)$ is expressed as:

$$I_{(0,\infty)}(x) = \begin{cases} 1, & x \in (0, \infty) \\ 0, & x \notin (0, \infty) \end{cases} \tag{3}$$

and $\Gamma(\alpha)$ is the complete Gamma distribution function expressed as:

$$\Gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha-1} dx \tag{4}$$

Statistically, it is known that for a Gamma process where $X = \{X(t), t \geq 0\}$, the expectation (mean) and variance are given as:

$$E(X(t)) = \frac{\alpha(t)}{\beta} \text{ and } Var(X(t)) = \frac{\alpha(t)}{\beta^2} \tag{5}$$

Empirical studies showed that the shape parameter α for an expected degradation over time based on the Gamma process is proportional following a Power law:

$$\alpha(t) = ct^b \tag{6}$$

where $c > 0$ and $b > 0$ are physical parameters. The value of the constant b depends on the condition of the degradation process and is determined based on available engineering knowledge. If the increment of degradation is only dependent on the increment of time i.e $(t + \Delta t) - X(t)$, the degradation model is a stationary gamma process and the value of the coefficient b for such stationary model is often 1.0. However, it is worth noting that the Gamma process is not limited by the use of the power law to model the expected degradation of products [49].

3.3 Gamma Process Based Lumen Flux Degradation Model

The luminous flux degradation and chromaticity shift are the main failure modes for LED products and particularly the luminous flux degradation is considered as the dominant reliability concern for general applications [50], [51]. The lifetime of an LED is defined as the number of operating hours for the luminous flux to reach 70% of the initial light output level. Suppose a random variable T represents the lifetime of an LED, while $\{X(t), t \geq 0\}$ denotes the time-dependent stochastic degradation process, thus the LED will fail, when the $X(t)$ reaches its failure threshold denoted as D. This can be described mathematically as: $T = \inf\{t | X(t) \geq D, t \geq 0\}$.

The cumulative distribution function (CDF) of the lifetime T for an LED can be expressed as:

$$\begin{aligned} F_T(t) &= P(T \le t) = P(X(t) \ge D = 1 - P\,(X(t) < D) = 1 - Ga(D\,|\alpha,\beta) \\ &= \int_D^{\infty} f_X(x;\alpha(t),\beta)dx \\ &= \int_D^{\infty} \frac{\beta^{\alpha(t)}}{\Gamma\,(\alpha\,(t))} x^{\alpha(t)-1} \exp\,(-\beta\chi)\,dx \\ &= \frac{1}{\Gamma\,(\alpha\,(\mathrm{t}))} \int_{\frac{D}{\beta}}^{\infty} \xi^{\alpha(t)-1} \exp^{\xi} d\xi = \frac{\Gamma\left(\alpha t, \frac{D}{\beta}\right)}{\Gamma\,(\alpha t)} \end{aligned} \tag{7}$$

Here, it can be noted that, $\Gamma(\alpha t, D/\beta)$ could be stated as $\Gamma(a, z) = \int_z^{\infty} \xi^{\alpha(t)-1} \mathrm{e}^{\xi} d\xi$ and is an incomplete gamma function.

Based on the CDF definition, the reliability function of an LED is presented as:

$$R(t) = 1 - F_T(t) = 1 - P(X(t) \ge D) = 1 - F_T(t) = \frac{\Gamma\left(\alpha t, \frac{D}{\beta}\right)}{\Gamma\,(\alpha t)} \tag{8}$$

The PDF of the lifetime T for an LED can also be derived from the CDF as follows:

$$f_T(t) = \frac{dF_T(t)}{dt} = \frac{\alpha}{\Gamma\,(\alpha t)} \int_0^{\frac{D}{\beta}} \left[\ln\,(\xi) - \frac{\Gamma'\,(\alpha t)}{\Gamma\,(\alpha t)}\right] \xi^{\alpha(t)-1} \mathrm{e}^{\xi} d\xi \tag{9}$$

Here the logarithmic derivative of the gamma function (also known as digamma function) and the generalized hyper geometric functions are included in the above results. Due to his, further differentiation on PDF of the lifetime T is complicated and analytically intractable for engineering applications [52]. Thus, the Birnbaum-Saunders (BS) distribution is implemented to overcome this challenge.

The BS distribution [53] is a reasonable alternative for modeling the lifetime distribution model, where it applies standard normal distribution to approximate the distribution function for lifetime T. Thus the result is obtained as shown below:

$$F_T(t) \approx \Phi\left[\frac{1}{h}\left(\sqrt{\frac{t}{k}} - \sqrt{\frac{k}{t}}\right)\right], t > 0 \tag{10}$$

In this expression, Φ(h, k) is standard normal distribution, with h $= \sqrt{\beta/D}$, and k $= D/\alpha\beta$

The corresponding probability distribution function is also expressed as:

$$f_T(t) = \frac{1}{2\sqrt{2\pi}hk}\left[\left(\frac{k}{t}\right)^{\frac{1}{2}} + \left(\frac{k}{t}\right)^{\frac{3}{2}}\right].exp^{\left[-\frac{1}{2h^2}\left(\frac{t}{k}-2+\frac{k}{t}\right)\right]}, t > 0 \tag{11}$$

It can be further simplified and presented as:

$$f_T(t) = \frac{\left[\left(\frac{k}{t}\right)^{\frac{1}{2}} + \left(\frac{k}{t}\right)^{\frac{3}{2}}\right]}{2\sqrt{2\pi}hk}.exp^{\left[\frac{(t-k)^2}{2tkh^2}\right]}, t > 0$$

Based on Eq. (7) and (10), the reliability function can be represented as:

$$R = 1 - F_T(t) = 1 - \Phi\left[\frac{1}{h}\left(\sqrt{\frac{t}{k}} - \sqrt{\frac{k}{t}}\right)\right], t > 0 \tag{12}$$

The mean time to failure (MTTF) and the variance of LED luminous flux maintenance under this model $M_{G,}$ can be approximated as [54]:

$$E(t) = MTTF_G \cong k\left(1+\frac{h^2}{2}\right) = \frac{D}{\alpha\beta}+\frac{1}{2\alpha},$$
$$VAR(t) = \sqrt{(hk)^2\left(1+\frac{5h^2}{4}\right)} \tag{13}$$

3.4 Parameter Estimation for the Gamma Process Model

The estimation of the shape parameter α and the scale parameter β is an important part of the Gamma process model. Two methods widely used for parameter estimation are maximum the likelihood estimation (MLE) and the method of moments (MM) [49]. In this study, both the moment method and the maximum likelihood estimation are implemented to estimate the shape and scale parameters for the Gamma process model. Based on the stationary gamma process assumptions mentioned above, the moment method helps to estimate the value of the physical parameter c in the shape parameter α(t) and the value of scale parameter β based on Eqs (14) and (15). The moment method is found to be easier to compute compared to MLE based method which requires the use of special functions such as fminsearch/fmincon/fsolve in MATLAB.

Suppose m samples are tested, n is the number of measurements taken (total data) based on a uniform interval of time for each sample, then x_{ij} denotes the degradation readout at the *i*-th time for the *j*-th test sample. The degradation observation vector and matrix for each tested sample can be denoted respectively as:

$$X_m = \left[x_{j1}, x_{j2}, \ldots, x_{jm}\right], \qquad Xnm = \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1n} \\ x_{21} & x_{22} & \ldots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \ldots & x_{nm} \end{bmatrix}$$

Thus, for any arbitrary sample, x_i denotes the *i*-th cumulative degradation, t_i is *i*-th time for measuring, $\delta_i = x_i - x_{i-1}$ is a degradation difference between (*i-1*)th and *i*-th time, and $\omega_i = t_i^b - t_{i-1}^b$.

$$\bar{\delta} = \frac{x_n}{t_n^b} = \frac{\sum_{i=1}^n \delta_i}{\sum_1^n \omega_i} = \frac{\hat{c}}{\hat{\beta}} \tag{14}$$

$$\frac{x_n}{\hat{\beta}}\left(1-\frac{\sum_{i=1}^n \omega_i^2}{\left[\sum_{i=1}^n \omega_i\right]^2}\right) = \sum_{i=1}^n \left(\delta_i - \bar{\delta}\omega_i\right)^2 \tag{15}$$

On the other hand, the parameters α and β can also be determined from the available cumulative degradation data based on the MLE as follows:

$$\hat{\beta} = \frac{\hat{c}t_n^b}{x_n}, \qquad \sum_{i=1}^n \left[t_i^b - t_{i-1}^b\right]\left\{\psi\left(\hat{c}\left[t_i^b - t_{i-1}^b\right]\right) - \log\delta_i\right\} = t_n^b \log\left(\frac{\hat{c}t_n^b}{x_n}\right) \tag{16}$$

In general, the Gamma process model implementation flow chart for luminous flux degradation is shown in Fig. 5. Once the cumulative degradation is observed, the gamma process model analysis is carried out based on the parameter estimation methods to evaluate the shape parameter and scale parameters. In summary, the gamma process has been found to be effective for lifetime prediction due to its numerous advantages over the other approaches. One of the benefits of the Gamma process for RUL estimation is that the physical meaning contained is easy to understand and the required mathematical calculations are relatively straightforward [13]. Nevertheless, it is worth noting that the nature of the degradation process will affect the choice for gamma process as

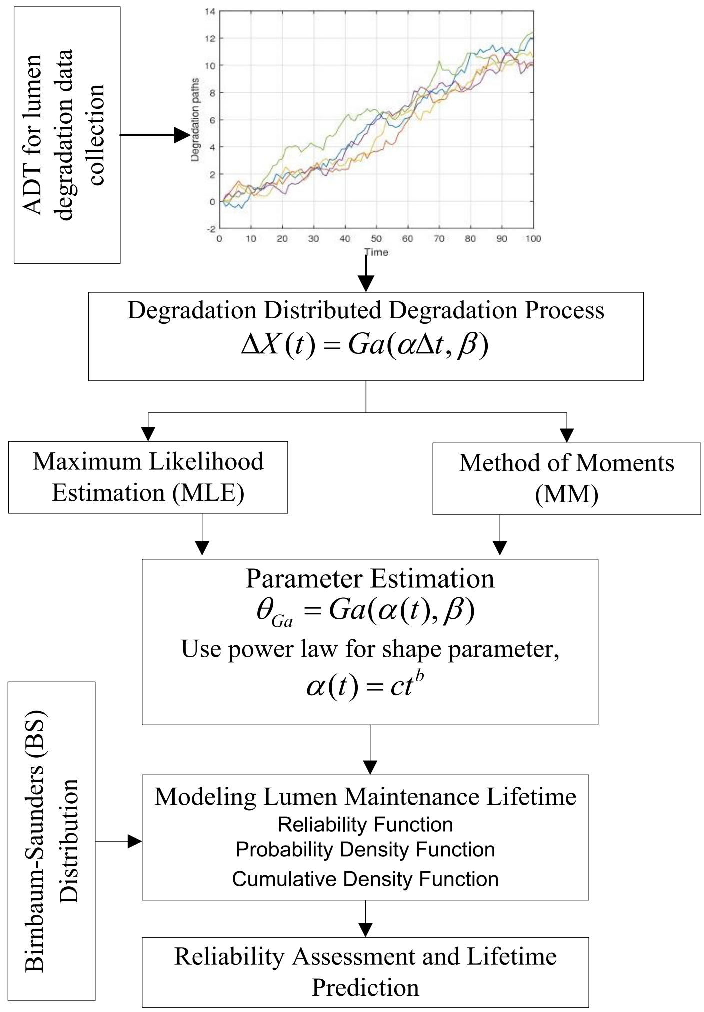

Fig. 5. Schematic description of degradation modelling processes.

this method seems appropriate for monotonic performance degradation over a period of time. This might affect the robustness of the method for random modeling of particular degradation processes.

4. Results and Discussion

4.1 Lumen Degradation of Test Samples

The suitability of the GDD model for the lumen degradation data is demonstrated based on a cumulative degradation over 690 hrs ageing time (45% of the full lifetime) and 966 hrs ageing time (63% of the full lifetime) from the degradation data. The probability plot can be used to assess as to whether the cumulative lumen degradation increments can follow the gamma distribution or exponential distribution, as shown in Fig. 6. The gamma distribution plot shows that the data fits to the distribution path, demonstrating that the gamma process is better suited for the cumulative lumen degradation data.

In addition, the implementation of the Gamma distributed degradation (GDD) model also requires the availability of non-decreasing cumulative (monotonic) luminous degradation. The luminous flux degradation shown in Fig. 7 is taken to demonstrate the GDD model by considering data points every 69 hrs. Based on this customization, the luminous degradation data shows a very good monotonic degradation trend which makes it suitable for use based on the Gamma process method.

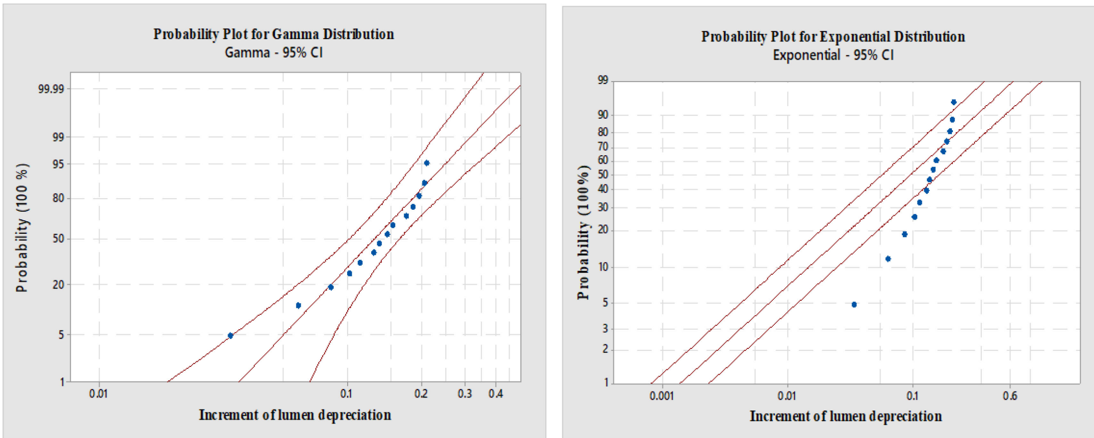

Fig. 6. Probability distribution plots with gamma distribution (Left) and exponential distribution (Right).

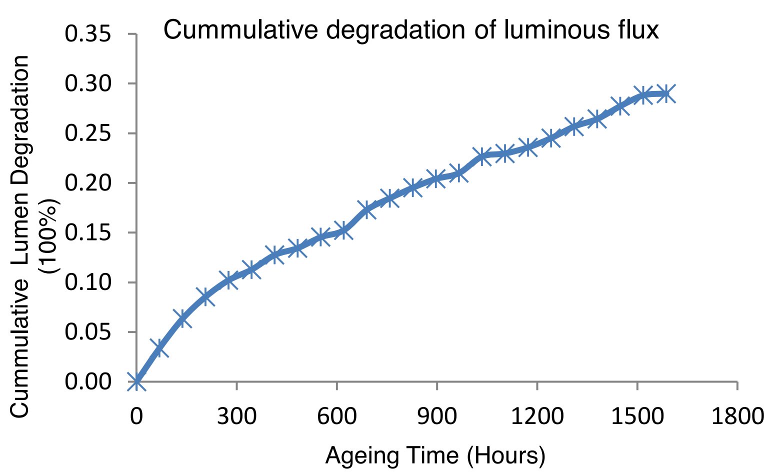

Fig. 7. Averaged cumulative luminous flux degradation.

4.2 Estimation of Parameters for GDD Model

As described in the methodology part, the IESNA recommended method for LED lumen maintenance lifetime prediction is the TM-21 nonlinear least squares (NLS) method. According to the IES-TM-21 standard, the parameter estimation as well as the long term lifetime prediction was conducted based on the luminous data from 0 to 690 hrs (about 11 test cycles considering 69 hrs for each test, i.e 45% of full lifetime) and 0 to 966 hrs (15 test cycles, i.e 63% of full lifetime). From this degradation data the parameters for the NLS method and the prediction results for the test samples are shown in Table 2. It is worth noting that the prediction accuracy is lower with less data (690 hrs) compared with the accuracy of using relatively more data (966 hrs).

Similarly, as mentioned in Section 3.4, parameter estimation is in the primary phase in the Gamma process model implementation. If the shape parameter $\alpha = 1$, then the gamma distribution will transform to an exponential distribution, on the other hand if $\beta = 1$, it becomes a standard gamma distribution. The parameter estimation results based on cumulative lumen degradation for the test samples and their average value are shown in Fig. 8. The parameter estimation is based on the maximum likelihood estimation as well as the method of moments as indicated in Eqs. (14)–(16) from the lumen degradation data at two prediction times, 690 hrs (45% of full degradation test) and 966 hrs (63% of the full degradation test). For the averaged 690 hrs lumen

TABLE 2
Prediction Results Comparison: TM-21 Method and GDD Process

Test Samples	Prediction Time	Lumen maintenance lifetime prediction error %		
		IES-TM-21	GDD Model	
			Based on MLE	Based on MM
LED_3	690	24.87	12.87	19.20
	966	17.59	2.90	7.57
LED_7	690	23.66	12.08	17.93
	966	17.22	3.20	7.67
LED_11	690	19.47	8.37	13.83
	966	11.80	-0.58	1.29
LED_12	690	16.43	4.00	8.92
	966	7.65	-10.44	-4.86
Avg. Lumen	690	18.11	10.88	15.43
	966	11.93	4.90	3.77

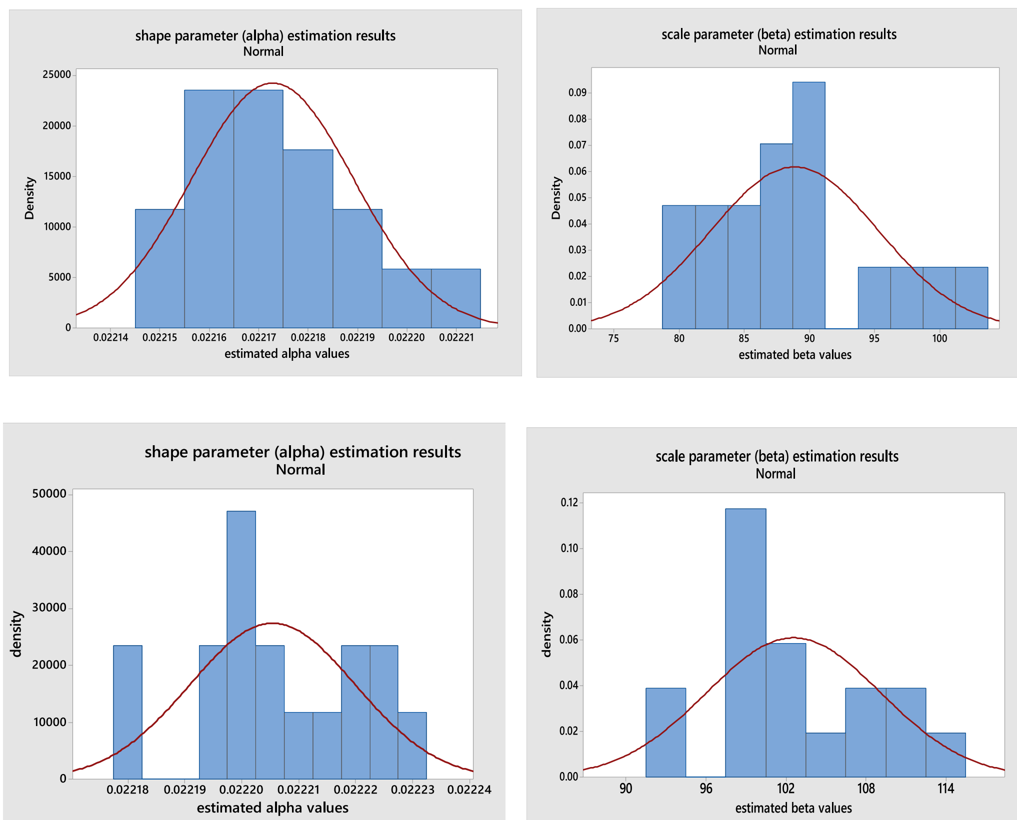

Fig. 8. Parameter estimation results based on 45% (upper plots) and 63% (lower plots) data with normal distribution fit.

degradation data, the MLE method was implemented to estimate the shape and scale parameters respectively as $\hat{c} = 0.02217258$ and $\beta = 88.454425$. For the purpose of validation and examination of the parameter estimation accuracy, the method of moments was employed and the shape parameter was estimated as $\hat{c} = 0.0416141$ and the scale parameter $\beta = 166.01373$. The method of moments is a commonly used method in statistics to estimate model parameters by evaluating sample moments against unobservable population moments. As the use of iterations to find the

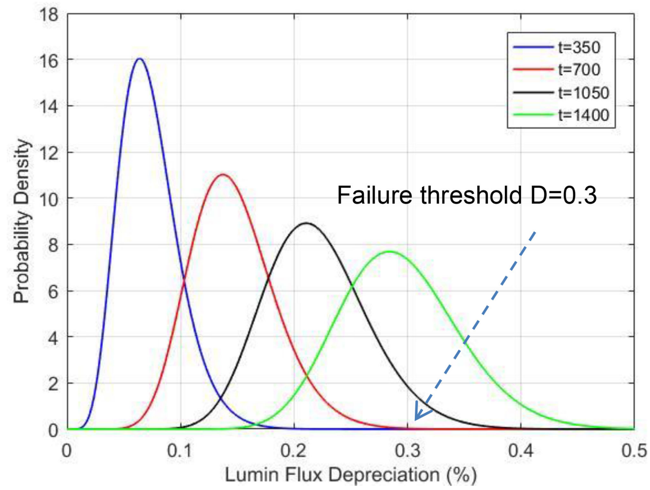

Fig. 9. PDF Plots for cumulative lumen degradation for average data.

unknown parameters is not required in the process of parameter estimation using the method of moments, and this makes the method of moments easier for implementation. However, it is also found to be less accurate compared with the MLE method, which is widely applicable and computationally demanding. The estimated parameters for all the independent test samples and different prediction times, following a normal distribution, are shown in Fig. 8.

4.3 GDD Model Based Analysis of Results

The PDF for the cumulative lumen degradation of phosphor-converted white LEDs at different measurement time points is shown in Fig. 9. It can be observed from the plots that the maximum height of the probability density decreases while the width of the PDF plots increases as the ageing time increases from left to right. This demonstrates that the uncertainty in the lifetime prediction increases as the prediction time is much greater than the test times. Similarly, the prediction uncertainty also varies among different samples for such longer prediction times. The complexity of the LED-based lights and their failure mechanisms could be the reason for such variability in the prognostics of white light pc-LEDs.

The skewness is a measure of the degree of symmetry for statistical distribution. If the value of the skewness is less than 0.5, then the statistical distribution is considered as a normal distribution and on the other hand if the skewness is greater than 1.0, it is highly skewed. The equation for the Gamma process skewness is $S = 2/\sqrt{\alpha(t)}$ and for stationary gamma process with constant time increment, the skewness is expressed as $S = 2/\sqrt{ct}$. Based on this, the skewness of the average luminous flux degradation is estimated to be $S = 0.949$ at $t = 200$ hrs, $S = 0.718$ at $t = 350$ hrs, $S = 0.508$ at $t = 700$ hrs, $S = 0.415$ at $t = 1050$ hrs and $S = 0.359$ at operating time of 1400 hrs. The skewness trend shows a decreasing trend towards zero as the operating time increases, and this shows that the lumen degradation becomes more normally distributed as the ageing time of the test samples increases.

The reliability and cumulative lumen degradation function plots are shown in Fig. 10. The 50% probability density value on this plot corresponds to the median value of the lifetime distribution curve in Fig. 11. Mathematically, integration of the CDF of the lifetime gives the results for the PDF curve and the PDF curve of time to failure for the lumen flux as shown in Fig. 11. As it can be seen from the plot, the skewness is less than 0.5 which indicates a normal distribution with a mean time to failure of 1455.3 hrs. The L70 time to failure for the average lumen degradation according to LM-21 is about 1337.2 hrs while the actual failure time is about 1633 hrs. This confirms that the lifetime

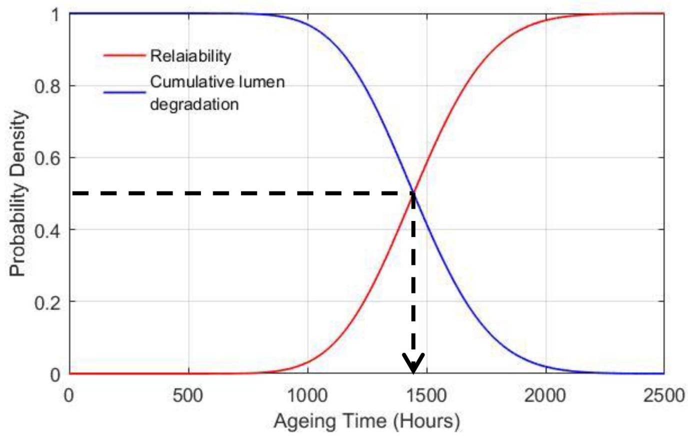

Fig. 10. CDF and Reliability Plots.

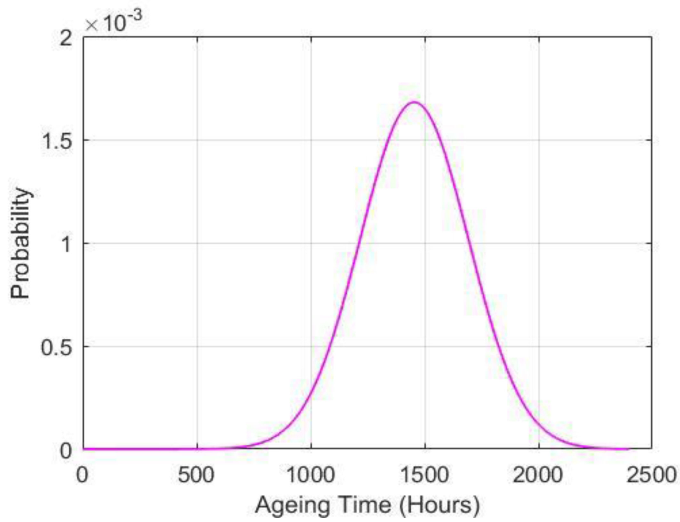

Fig. 11. Plot for PDF at mean failure time.

estimation based on the Gamma process is much better than the traditional NLS approach recommended by IESNA. The appropriateness of the GDD method for monotonic luminous degradation is also demonstrated on randomly selected test samples as shown in Table 2. It is worth noting however, that the results depend on the accuracy of the parameter estimation method. Here the MLE based parameter estimation shows superiority over the method of moments, as the prediction accuracy is better.

In general, a thermal degradation test designed to collect the performance characteristics of white LEDs in a relatively shorter time than normal operating conditions. The gamma distributed degradation approach is suitable to model the luminous flux degradation of phosphor-converted white LEDs. The maximum likelihood estimation based method for parameter estimation shows a better accuracy compared to the method of moments to model the lifetime distribution of white LEDs degradation. The GDD model lifetime prediction result based on the MLE estimated parameters showed superiority over the MM parameter results based on randomly selected four test samples (LED_3, LED_7, LED_11 and LED_12) and the averaged lumen degradation prediction. The average error in the GDD model was about 10% with a confidence interval of 95% while the NLS-based IES-TM-21 has a prediction error of about 20% based on 45% of the full lifetime data. Thus, it can be concluded that the GDD estimation results shown better accuracy compared to the IES-TM-21 approach.

5. Conclusion

In this study, the lifetime prediction of phosphor converted white LEDs is demonstrated based on a stationary gamma process approach that considers the degradation at constantly increasing operating times. As one of the dominant quality characteristics of white LEDs, lumen maintenance prediction is based on the nonlinear least squares method which introduced prediction inaccuracy. Thus, the gamma distributed degradation (GDD) modeling approach is proposed to handle temporal variability of lumen maintenance. The advantages of the GDD approaches over other methods are that the physical meaning contained is easy to understand and the required mathematical calculations are relatively straightforward. To demonstrate the GDD model lumen maintenance data from an accelerated thermal ageing lifetime test was collected while the maximum likelihood estimation and method of moments are used for parameter estimation. The GDD model lifetime prediction result based on MLE estimated parameters showed superiority over the MM parameter results based on four randomly selected samples (LED_3, LED_7, LED_11 and LED_12) and the averaged lumen degradation prediction. The average error in the GDD model was about 10% with a confidence interval of 95%, while the NLS-based IES-TM-21 showed a prediction error of about 20% based on 45% of the full lifetime data. Thus, the GDD estimation results show better accuracy compared to the IES-TM-21 NLS approach.

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