mathematics

Article

The Loss-Averse Retailer's Order Decisions Under Risk Management

Felix T. S. Chan [1] and Xinsheng Xu [2,*]

1 Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong, China
2 College of Science, Binzhou University, Binzhou 256600, China
* Correspondence: xxs0905@163.com

Received: 27 May 2019; Accepted: 28 June 2019; Published: 2 July 2019

Abstract: This paper characterizes the retailer's loss aversion by introducing a loss aversion coefficient and proposes a new loss aversion utility function for the retailer. To hedge against the risk arising from the uncertain market demand, we use the Conditional Value-at-Risk (CVaR) measure to quantify the potential risks and obtain the optimal order quantity for the retailer to maximize the CVaR objective of loss aversion utility. It is shown that that the optimal order quantity for a retailer to maximize the expected loss aversion utility is smaller than expected profit maximizing (EPM) order quantity in the classical newsvendor model, which can help to explain decision bias in the newsvendor model. This study shows that the optimal order quantity with the CVaR objective can decrease in retail price under certain conditions, which has never occurred in the newsvendor literature. With the optimal order quantity under the CVaR objective, it is proved that the retailer's expected loss aversion utility is decreasing in the confidence level. This confirms the fact that high return means high risk, while low risk comes with low return. Based on the results, several management insights are suggested for the loss-averse newsvendor model.

Keywords: newsvendor model; conditional value-at-risk; loss-averse; optimal order quantity

1. Introduction

The newsvendor model has been well applied to several fields including supply chain contracts ([1–3]). For example, Khouja [1] showed that the newsvendor model has been used to aid decision making in fashion and sporting industries, and service industries such as airlines and hotels. For a perishable product with a short selling season and a stochastic market demand, a retailer needs to maximize his/her expected profit by selecting an optimal order quantity. If the retailer's order quantity is bigger than the realized market demand, there is a loss from the excess orders. Otherwise, if the retailer's order quantity is smaller than the realized market demand, there is a shortage cost for the lost sales. Thus, the retailer needs to give a balance between ordering too many and too few.

In the traditional research about the newsvendor model, the expected profit maximizing (EPM) order quantity has been well documented ([4–6]). For example, Qin et al. [6] showed that the EPM order quantity has caused extensive concern in the study on the newsvendor model. However, some recent studies found that maximizing the expected profit can not always reflects the reality. The realized order quantity of the sales managers in practice always deviates from the EPM order quantity, which is referred to as "decision bias" in the newsvendor model (Schweitzer and Cachon [7]). To study such a deviation, researchers introduced several extensions of the newsvendor model by relaxing some original premises ([8–13]). Furthermore, researchers also introduced risk preferences to study the newsvendor model ([14–22]).

In the newsvendor model, there often exists some loss that comes from the over-ordering items or the excess demands that can not be satisfied. In practice, this kind of loss often has a severer impact on the benefit of the retailer than the profit obtained in the selling season. For example, Bayle [23] found that the loss caused by stockouts is significant and can drive the customers away from the retailer permanently. The fact makes the retailers in reality are more averse to such a loss than they are attracted to the same amount of profit, which can be characterized as the retailer's loss aversion preference in reality. Loss aversion is a branch of the prospect theory proposed by Kahneman and Tversky [24], which is well supported in the economics and decision theory. For example, Mohammed and Wehurng [25] showed that loss aversion can help to explain why people always deviate from the expected utility. In real world applications, loss aversion has been well applied to varieties of settings (e.g., economics and management). The empirical studies by Shapira [26] showed that managers' decision-making behaviors in the real world are always consistent with loss aversion. Up to now, the loss aversion theory has been well applied in various areas, such as financial markets (Putler [27]), organizational behavior (Maor [28]) and labor supply (Farber [29]). In view of this issue, it is necessary and meaningful to study the effects of loss aversion on the optimal order decisions in the newsvendor model.

By integrating a loss aversion coefficient, this paper proposes a loss aversion utility function which characterizes the retailer's loss aversion preference. We obtain the optimal order quantity for the retailer to maximize the expected loss aversion utility. The result shows that this optimal order quantity is no more than the EPM order quantity of the loss-neutral newsvendor. To reduce the potential risks, we use the CVaR measure to quantify the risks and obtain the optimal order quantity for a retailer to maximize CVaR objective of utility. We showed that under certain conditions, the optimal order quantity with CVaR objective is decreasing in the retail price. Under the optimal order quantity with CVaR objective, it is shown that the retailer's expected loss aversion utility decreases in the confidence level. This further confirms that low risk means low return while high return comes with high risk. This paper thus contributes to the newsvendor literature in two main aspects. First, we show how loss aversion influences the optimal order decisions in the newsvendor model and find that loss aversion provides an alternative to explain the decision bias in the newsvendor model. Second, we introduce CVaR measure into this study and show how to select the optimal order quantity to hedge against potential risks for the loss-averse retailers.

The rest of this paper is organized as follows. In Section 2, we give a detailed description on the proposed model. Section 3 analyzes the optimal order quantities for the retailer to maximize the expected loss aversion utility and CVaR objective of utility. Section 4 gives a numerical example to show the results and presents several management insights, with conclusions given in Section 5.

2. Materials and Methods

Consider the newsvendor model. It is supposed that ξ is the stochastic market demand, and let $f(\cdot)$ be its probability density function and $F(\cdot)$ be its cumulative distribution function. Moreover, let q denote the order quantity of the retailer and D denotes the realized market demand. On the one hand, the retailer can obtain the following profit

$$P(q) = (p-c)\min\{q, D\} \tag{1}$$

in the selling season. Here, p is the unit retail price and c is the unit wholesale price. On the other hand, when the selling season is due, there is the following loss

$$L(q) = (c-r)(q-D)^+ + s(D-q)^+ \tag{2}$$

from the excess orders or the lost sales. Here, $X^+ = \max\{X, 0\}$, r is the salvage price of unit product and s is the shortage penalty price for each lost sale. Indeed the shortage cost ranging from profit loss to some unspecified loss of goodwill of the customers has an important influence on the benefit of the newsvendor. Therefore we exert a shortage penalty price s on each lost sale. In the right hand of Equation (2), the first item $(c-r)(q-D)^+$ represents the loss from excess orders and the second $s(D-q)^+$ the shortage cost of the lost sales. In general, it is assumed that $p \geq c \geq r \geq 0$. Based on the analysis, we have the following loss aversion utility function

$$U(q) = P(q) - \lambda L(q) = (p-c)\min\{q, D\} - \lambda[(c-r)(q-D)^+ + s(D-q)^+], \tag{3}$$

where $\lambda \geq 1$ reflects the the retailer's degree of loss aversion. The bigger the value of λ is, the more loss-averse the retailer becomes.

The utility function $U(q)$ implies that the loss from the excess orders or the lost sales has a more important impact on the retailer than the same amount of profit. In the following, we introduce different objectives on the loss aversion utility function $U(q)$ for the retailer, and study the optimal order quantities to optimize the objectives, respectively.

3. Results

This section studies the optimal order quantities for the retailer to maximize two objectives on the loss aversion utility. In Section 3.1, we investigated the optimal order quantity for a retailer to maximize the expected utility $E[U(q)]$(E is the expectation operator). In Section 3.2, we introduce the CVaR measure for the retailer to hedge against the potential risks. Under the CVaR measure, we obtained an optimal order quantity for the retailer to maximize the expectation of the utility above a given target level.

3.1. Optimal Order Quantity to Maximize Expected Utility

As studied before, a conventional approach to the newsvendor model is to select an order quantity to maximize the retailer's expected utility. Following this idea, we first analyzed the optimal order quantity for the retailer to maximize the expected loss aversion utility $E[U(q)]$.

Theorem 1. *To maximize the expected loss aversion utility $E[U(q)]$, the optimal order quantity of the retailer is given as*

$$q^* = F^{-1}\left[\frac{p-c+\lambda s}{p-c+\lambda(c-r+s)}\right]. \tag{4}$$

Proof. For the loss aversion utility, we have

$$U(q) = (p-c)\min\{q, D\} - \lambda[(c-r)(q-D)^+ + s(D-q)^+]. \tag{5}$$

It follows from $\min\{q, D\} = q - (q-D)^+$ and $(D-q)^+ = (D-q) + (q-D)^+$ that

$$U(q) = (p-c+\lambda s)q - \lambda sD - [p-c+\lambda(c-r+s)](q-D)^+. \tag{6}$$

Thus the expected value of $U(q)$ can be given as

$$E[U(q)] = (p-c+\lambda s)q - \lambda sE(\xi) - [p-c+\lambda(c-r+s)]\int_0^q (q-t)dF(t), \tag{7}$$

which implies

$$\frac{\partial E[U(q)]}{\partial q} = (p-c+\lambda s) - [p-c+\lambda(c-r+s)]F(q). \tag{8}$$

Therefore we have

$$\frac{\partial^2 E[U(q)]}{\partial q^2} = -[p-c+\lambda(c-r+s)]f(q) < 0, \tag{9}$$

which proves that $E[U(q)]$ is concave in q. Then it follows from the first order condition and (7) that $E[U(q)]$ achieved its maximum value at

$$q^* = F^{-1}\left[\frac{p-c+\lambda s}{p-c+\lambda(c-r+s)}\right]. \tag{10}$$

Theorem 1 gives the optimal order quantity to maximize the expected loss aversion utility $E[U(q)]$ for a loss-averse retailer. When $\lambda = 1$, the retailer becomes to be loss-neutral. It follows with $\lambda = 1$ in Theorem 1 that q^* reduces to

$$q_1^* = F^{-1}\left[\frac{p-c+\lambda s}{p-c+\lambda(c-r+s)}\right] = F^{-1}\left[\frac{p-c+s}{p-r+s}\right]. \tag{11}$$

It is same to the EPM order quantity in the classical newsvendor model. Moreover, we have

$$\frac{p-c+s}{p-r+s} - \frac{p-c+\lambda s}{p-c+\lambda(c-r+s)} = \frac{(\lambda-1)(p-c)(c-r)}{(p-r+s)(p-c+\lambda(c-r+s))} \geq 0,$$

which implies

$$\frac{p-c+s}{p-r+s} \geq \frac{p-c+\lambda s}{p-c+\lambda(c-r+s)}. \tag{12}$$

Then it follows that

$$F^{-1}\left[\frac{p-c+s}{p-r+s}\right] \geq F^{-1}\left[\frac{p-c+\lambda s}{p-c+\lambda(c-r+s)}\right] \tag{13}$$

and

$$q^* \leq q_1^*. \tag{14}$$

The result reveals that the optimal order quantity to maximize the expected loss aversion utility $E[U(q)]$ was no more than the EPM order quantity. By Theorem 1, we can obtain the following results. □

Corollary 1. *To maximize the expected loss aversion utility $E[U(q)]$, the optimal order quantity q^* increases in the retail price p, the salvage price r and the shortage penalty price s, and decreases in the wholesale price c.*

It follows that $q^* = F^{-1}[\frac{p-c+\lambda s}{p-c+\lambda(c-r+s)}] = F^{-1}(1) = +\infty$ when $c = r$. This implies that the retailer will order as many as possible to avoid the understock loss when the salvage price is equal to the wholesale price.

Corollary 2. *For the newsvendor model, the optimal order quantity q^* for a loss-averse retailer to maximize the expected utility $E[U(q)]$ is decreasing in the loss aversion coefficient λ.*

Proof. By Theorem 1, the optimal order quantity q^* is given as

$$q^* = F^{-1}\left[\frac{p-c+\lambda s}{p-c+\lambda(c-r+s)}\right]. \tag{15}$$

It follows that

$$\frac{\partial q^*}{\partial \lambda} = -\frac{(p-c)(c-r)}{f[F^{-1}(\frac{p-c+\lambda s}{p-c+\lambda(c-r+s)})](p-c+\lambda(c-r+s))^2} \leq 0, \tag{16}$$

which implies that q^* decreases in the loss aversion coefficient λ. □

As stated above, the loss aversion coefficient λ indicates the retailer's degree of loss aversion. This result implies that the retailer will order fewer items when he/she becomes more loss-averse.

In this subsection, we study the optimal order quantity to maximize the expected loss aversion utility. However, this expected utility maximizing order quantity is insufficient, since it is not possible to hedge against potential risks and thus can not be accepted by some risk-averse retailers. To control the potential risks, we use the CVaR measure for the loss-averse newsvendor in the next subsection.

3.2. Optimal Order Quantity to Maximize CVaR of Utility

In the above subsection, we derive the optimal order quantity for the retailer to maximize the expected loss aversion utility. In recent years, some unpredictable disasters often bring great risks and losses to the retailers. Therefore, several risk control measures have been introduced into the decision framework of the newsvendor model, such as the CVaR measure ([30–36]). The CVaR measure is known as a famous risk measure which is coherent and consistent with the stochastic dominance (Gotoh and Takano [31]). It has been widely used in the financial risk management. This measure is a type of downside risk measure which pays more attention to the utility going down to some target level, while the utility above the target level is ignored. Then it is more appealing than some other risk control measures.

For the utility $U(q)$ of the retailer, the Value-at-Risk (VaR) of $U(q)$ is defined by

$$VaR_\alpha[U(q)] = \sup\{y \in R|\Pr\{U(q) \geq y\} \geq \alpha\}, \tag{17}$$

where $\Pr\{U(q) \geq y\}$ represents the probability of the utility $U(q)$ above the value y. $VaR_\alpha[U(q)]$ indicates the maximum utility that the retailer can obtain under the confidence level α. Taking $VaR_\alpha[U(q)]$ as the target utility, the following CVaR objective of utility $U(q)$ is given as

$$CVaR_\alpha[U(q)] = E[U(q)|U(q) \leq VaR_\alpha[U(q)]]. \tag{18}$$

It gives the expectation of the utility below the given target level $VaR_\alpha[U(q)]$. To maximize the CVaR objective, we can obtain an optimal order quantity for the retailer to maximize the expected value of utility below the target level $VaR_\alpha[U(q)]$. We have the following result to address the issue.

Theorem 2. *To maximize CVaR objective of loss aversion utility, the optimal order quantity q^α is given by*

$$q^\alpha = \frac{[p-c+\lambda(c-r)]F^{-1}\left[\frac{(1-\alpha)(p-c+\lambda s)}{p-c+\lambda(c-r+s)}\right] + \lambda s F^{-1}\left[\frac{(1-\alpha)(p-c+\lambda s)}{p-c+\lambda(c-r+s)} + \alpha\right]}{p-c+\lambda(c-r+s)}. \tag{19}$$

Proof. For an order quantity q of the retailer and a realized market demand D, it follows with (5) that

$$U(q) = (p-c+\lambda s)q - \lambda s D - [p-c+\lambda(c-r+s)](q-D)^+. \tag{20}$$

We give an auxiliary function

$$\begin{aligned} h(q,v) &= v - \frac{1}{1-\alpha}E[v-U(q)]^+ \\ &= v - \frac{1}{1-\alpha}\int_0^{+\infty}[v-(p-c+\lambda s)q + \lambda s t + (p-c+\lambda(c-r+s))(q-t)^+]^+ dF(t) \\ &= v - \frac{1}{1-\alpha}\int_0^{q}[v+\lambda(c-r)q - (p-c+\lambda(c-r))t]^+ dF(t) \\ &\quad - \frac{1}{1-\alpha}\int_q^{+\infty}[v-(p-c+\lambda s)q + \lambda s t]^+ dF(t). \end{aligned} \tag{21}$$

By the result in Rockafellar and Uryasev [37], $h(q,v)$ is concave in (q,v) since $U(q)$ is concave in q.

It follows with the result in Rockafellar and Uryasev [38] that, the optimal order quantity for a retailer to maximize the CVaR objective equals to the optimal solution to the problem

$$\max_{q\geq 0}[\max_{v\in R} h(q,v)]. \tag{22}$$

Then, for any fixed q, we distinguish the following cases:

Case 1. $v \leq -\lambda(c-r)q$.

In this case, it follows with (21) that

$$h(q,v) = v - \frac{1}{1-\alpha}\int_{\frac{(p-c+\lambda s)q-v}{\lambda s}}^{+\infty}[v-(p-c+\lambda s)q+\lambda st]dF(t) \tag{23}$$

and

$$\frac{\partial h(q,v)}{\partial v} = 1 - \frac{1}{1-\alpha}\Big[1 - F\Big(\frac{(p-c+\lambda s)q - v}{\lambda s}\Big)\Big]. \tag{24}$$

There exists a sufficiently small v ($F(\frac{(p-c+\lambda s)q-v}{\lambda s}) \geq \alpha$) such that $\frac{\partial h(q,v)}{\partial v} \geq 0$.

If it satisfies

$$\frac{\partial h(q,v)}{\partial v}|_{v=-\lambda(c-r)q} = 1 - \frac{1}{1-\alpha}\Big[1 - F\Big(\frac{(p-c+\lambda(c-r+s))q}{\lambda s}\Big)\Big] \leq 0, \tag{25}$$

that is $q \leq \frac{\lambda s F^{-1}(\alpha)}{p-c+\lambda(c-r+s)}$, it follows from (24) that the optimal solution v^* to problem $\max_{v\in R} h(q,v)$ solves

$$1 - \frac{1}{1-\alpha}\Big[1 - F\Big(\frac{(p-c+\lambda s)q - v^*}{\lambda s}\Big)\Big] = 0. \tag{26}$$

It implies that

$$v^* = (p-c+\lambda s)q - \lambda s F^{-1}(\alpha). \tag{27}$$

Case 2. $-\lambda(c-r)q \leq v \leq (p-c)q$.

In this case, it follows with (21) that

$$\begin{aligned} h(q,v) = v &- \frac{1}{1-\alpha}\int_0^{\frac{v+\lambda(c-r)q}{p-c+\lambda(c-r)}}[v+\lambda(c-r)q-(p-c+\lambda(c-r))t]dF(t) \\ &- \frac{1}{1-\alpha}\int_{\frac{(p-c+\lambda s)q-v}{\lambda s}}^{+\infty}[v-(p-c+\lambda s)q+\lambda st]dF(t) \end{aligned} \tag{28}$$

and

$$\frac{\partial h(q,v)}{\partial v} = 1 - \frac{1}{1-\alpha}\Big[1 + F\Big(\frac{v+\lambda(c-r)q}{p-c+\lambda(c-r)}\Big) - F\Big(\frac{(p-c+\lambda s)q-v}{\lambda s}\Big)\Big]. \tag{29}$$

It satisfies

$$\frac{\partial h(q,v)}{\partial v}|_{v=(p-c)q} = 1 - \frac{1}{1-\alpha} \leq 0. \tag{30}$$

If it satisfies

$$\frac{\partial h(q,v)}{\partial v}|_{v=-\lambda(c-r)q} = 1 - \frac{1}{1-\alpha}\Big[1 - F\Big(\frac{(p-c+\lambda(c-r+s))q}{\lambda s}\Big)\Big] \geq 0, \tag{31}$$

that is $q \geq \frac{\lambda s F^{-1}(\alpha)}{p-c+\lambda(c-r+s)}$, it follows from (29) that the optimal solution v^* to problem $\max_{v\in R} h(q,v)$ solves

$$1-\frac{1}{1-\alpha}\Big[1+F\Big(\frac{v^*+\lambda(c-r)q}{p-c+\lambda(c-r)}\Big)-F\Big(\frac{(p-c+\lambda s)q-v^*}{\lambda s}\Big)\Big]=0. \tag{32}$$

Case 3. $v \geq (p-c)q$.

In this case, it follows with (21) that

$$h(q,v)=v-\frac{1}{1-\alpha}\int_0^q[v+\lambda(c-r)q-(p-c+\lambda(c-r))t]dF(t)$$
$$-\frac{1}{1-\alpha}\int_q^{+\infty}[v-(p-c+\lambda s)q+\lambda st]dF(t) \tag{33}$$

and

$$\frac{\partial h(q,v)}{\partial v}=1-\frac{1}{1-\alpha}\leq 0. \tag{34}$$

Based on the analysis above, the optimal solution v^* to problem $\max_{v\in R} h(q,v)$ for any fixed q is given by

$$v^*=\begin{cases}(p-c+\lambda s)q-\lambda sF^{-1}(\alpha) & q\leq \frac{\lambda sF^{-1}(\alpha)}{p-c+\lambda(c-r+s)},\\ v^1 & q>\frac{\lambda sF^{-1}(\alpha)}{p-c+\lambda(c-r+s)},\end{cases} \tag{35}$$

where v^1 solves (32).

In the following, to solve the problem $\max_{q\geq 0}[\max_{v\in R} h(q,v)]=\max_{q\geq 0} h(q,v^*)$, we distinguish between two different cases:

(i) $q \leq \frac{\lambda sF^{-1}(\alpha)}{p-c+\lambda(c-r+s)}$.

In this case, it follows from (35) that

$$v^*=(p-c+\lambda s)q-\lambda sF^{-1}(\alpha). \tag{36}$$

Then it follows with (21) that

$$h(q,v^*)=(p-c+\lambda s)q-\lambda sF^{-1}(\alpha)-\frac{1}{1-\alpha}\int_{F^{-1}(\alpha)}^{+\infty}[\lambda s(t-F^{-1}(\alpha))]dF(t) \tag{37}$$

and

$$\frac{\partial h(q,v^*)}{\partial q}=p-c+\lambda s>0. \tag{38}$$

(ii) $q > \frac{\lambda sF^{-1}(\alpha)}{p-c+\lambda(c-r+s)}$.

In this case, it follows from (32) and (35) that $v^*=v^1$ satisfies

$$F\left[\frac{(p-c+\lambda s)q-v^1}{\lambda s}\right]-F\left[\frac{v^1+\lambda(c-r)q}{p-c+\lambda(c-r)}\right]=\alpha. \tag{39}$$

Then it follows with (21) that

$$h(q,v^1)=v^1-\frac{1}{1-\alpha}\int_0^{\frac{v^1+\lambda(c-r)q}{p-c+\lambda(c-r)}}[v^1+\lambda(c-r)q-(p-c+\lambda(c-r))t]dF(t)$$
$$-\frac{1}{1-\alpha}\int_{\frac{(p-c+\lambda s)q-v^1}{\lambda s}}^{+\infty}[v^1-(p-c+\lambda s)q+\lambda st]dF(t) \tag{40}$$

and

$$\frac{\partial h(q,v^1)}{\partial q} = -\frac{1}{1-\alpha}\Big[\lambda(c-r)F\Big(\frac{v^1+\lambda(c-r)q}{p-c+\lambda(c-r)}\Big) + (p-c+\lambda s)\Big(F\Big(\frac{(p-c+\lambda s)q-v^1}{\lambda s}\Big)-1\Big)\Big]. \tag{41}$$

Then it follows from (41) that the optimal solution q^α to problem $\max_{q\geq 0} h(q,v^*)$ solves

$$\lambda(c-r)F\Big[\frac{v^1+\lambda(c-r)q^\alpha}{p-c+\lambda(c-r)}\Big] + (p-c+\lambda s)\Big[F\Big(\frac{(p-c+\lambda s)q^\alpha-v^1}{\lambda s}\Big)-1\Big] = 0. \tag{42}$$

Then it follows from (39) and (42) that the optimal solution q^α to problem $\max_{q\geq 0} h(q,v^*)$ is given as

$$q^\alpha = \frac{[p-c+\lambda(c-r)]F^{-1}\Big[\frac{(1-\alpha)(p-c+\lambda s)}{p-c+\lambda(c-r+s)}\Big] + \lambda s F^{-1}\Big[\frac{(1-\alpha)(p-c+\lambda s)}{p-c+\lambda(c-r+s)}+\alpha\Big]}{p-c+\lambda(c-r+s)}. \tag{43}$$

If it satisfies $\alpha = 0$, the risk-averse retailer becomes risk-neutral, it follows from the above result that q^α reduces to q^* in Theorem 1. Similar to Corollary 1, we have the following result about the optimal order quantity q^α. □

Corollary 3. *For the newsvendor model, the optimal order quantity q^α for a retailer to maximize the CVaR objective is increasing in the salvage price r and the shortage penalty price s, and decreasing in the wholesale price c.*

Proof. By Theorem 2, we have

$$q^\alpha = \frac{[p-c+\lambda(c-r)]F^{-1}\Big[\frac{(1-\alpha)(p-c+\lambda s)}{p-c+\lambda(c-r+s)}\Big] + \lambda s F^{-1}\Big[\frac{(1-\alpha)(p-c+\lambda s)}{p-c+\lambda(c-r+s)}+\alpha\Big]}{p-c+\lambda(c-r+s)}. \tag{44}$$

Let

$$F^{-1}\left[\frac{(1-\alpha)(p-c+\lambda s)}{p-c+\lambda(c-r+s)}\right] = M, \quad F^{-1}\left[\frac{(1-\alpha)(p-c+\lambda s)}{p-c+\lambda(c-r+s)}+\alpha\right] = N. \tag{45}$$

It follows

$$N \geq M \tag{46}$$

and

$$q^\alpha = \frac{[p-c+\lambda(c-r)]M+\lambda s N}{p-c+\lambda(c-r+s)}. \tag{47}$$

Then we have

$$\frac{\partial q^\alpha}{\partial c} = -\left[\frac{\lambda(1-\alpha)(p-c+\lambda s)}{[p-c+\lambda(c-r+s)]^3}\Big(\frac{p-c+\lambda(c-r)}{f(M)}+\frac{\lambda s}{f(N)}\Big) + \frac{\lambda(\lambda-1)s(N-M)}{[p-c+\lambda(c-r+s)]^2}\right] \leq 0, \tag{48}$$

$$\frac{\partial q^\alpha}{\partial r} = \frac{\lambda(1-\alpha)(p-c+\lambda s)}{[p-c+\lambda(c-r+s)]^3}\left[\frac{p-c+\lambda(c-r)}{f(M)}+\frac{\lambda s}{f(N)}\right] + \frac{\lambda^2 s(N-M)}{[p-c+\lambda(c-r+s)]^2} \geq 0, \tag{49}$$

$$\frac{\partial q^\alpha}{\partial s} = \frac{\lambda^2(1-\alpha)(c-r)}{[p-c+\lambda(c-r+s)]^3}\left[\frac{p-c+\lambda(c-r)}{f(M)}+\frac{\lambda s}{f(N)}\right] + \frac{\lambda(p-c+\lambda(c-r))(N-M)}{[p-c+\lambda(c-r+s)]^2} \geq 0, \tag{50}$$

which implies that q^α increases in the salvage price r and the shortage penalty price s, and decreases in the wholesale price c. □

Remark 1. *Recall that the optimal order quantity q^* to maximize the expected loss aversion utility is increasing in the retail price p. It is important to point out that this property does not hold for the optimal order quantity q^α. Indeed it follows from Corollary 3 that*

$$q^\alpha = \frac{[p-c+\lambda(c-r)]M + \lambda sN}{p-c+\lambda(c-r+s)}. \tag{51}$$

Then we have

$$\frac{\partial q^\alpha}{\partial p} = \frac{\lambda(1-\alpha)(c-r)}{[p-c+\lambda(c-r+s)]^3}\left[\frac{p-c+\lambda(c-r)}{f(M)} + \frac{\lambda s}{f(N)}\right] - \frac{\lambda s(N-M)}{[p-c+\lambda(c-r+s)]^2}, \tag{52}$$

which implies $\frac{\partial q^\alpha}{\partial p}$ may be positive or negative. Therefore the optimal order quantity q^α may increase, or decrease in the retail price p. In the traditional research about the newsvendor model, the retailer will order more units of product if the retail price increases. Remark 1 shows that the optimal order quantity q^α may violate this result.

Corollary 4. *To maximize CVaR objective of loss aversion utility, the optimal order quantity q^α may increase, or decrease in the loss aversion coefficient λ.*

Proof. By Corollary 3, we have

$$q^\alpha = \frac{[p-c+\lambda(c-r)]M + \lambda sN}{p-c+\lambda(c-r+s)}. \tag{53}$$

Then it follows that

$$\frac{\partial q^\alpha}{\partial \lambda} = -\frac{(1-\alpha)(p-c)(c-r)}{[p-c+\lambda(c-r+s)]^3}\left[\frac{p-c+\lambda(c-r)}{f(M)} + \frac{\lambda s}{f(N)}\right] + \frac{s(p-c)(N-M)}{[p-c+\lambda(c-r+s)]^2}, \tag{54}$$

which implies $\frac{\partial q^\alpha}{\partial \lambda}$ can be positive or negative. Then the optimal order quantity q^α can be increasing, or decreasing in the loss aversion coefficient λ.

This result is different from Corollary 2, where the optimal order quantity q^* for a retailer to maximize expected utility is decreasing in the loss aversion coefficient λ. It can be explained as follows. When the selling time is due, the retailer's loss comes from the excess orders or the lost sales, which can be defined as the overage loss and underage loss, respectively. Since the underage loss and overage loss may be not equal, and the dominated term decides the changing direction of the optimal order quantity q^α. If the overage loss is bigger than the underage loss, the retailer should order fewer units of product when he/she becomes more loss-averse. Otherwise, the overage loss is smaller than the underage loss, the retailer should order more units of product when he becomes more loss-averse. □

Corollary 5. *To maximize CVaR objective of loss aversion utility, the optimal order quantity q^α may increase, or decrease in the confidence level α.*

Proof. By Corollary 3, we have

$$q^\alpha = \frac{[p-c+\lambda(c-r)]M + \lambda sN}{p-c+\lambda(c-r+s)}. \tag{55}$$

It follows that

$$\frac{\partial q^\alpha}{\partial \alpha} = \frac{1}{[p-c+\lambda(c-r+s)]^2}\left[\frac{\lambda^2 s(c-r)}{f(N)} - \frac{(p-c+\lambda s)(p-c+\lambda(c-r))}{f(M)}\right], \tag{56}$$

which implies $\frac{\partial q^\alpha}{\partial \alpha}$ can be positive or negative. Then the optimal order quantity q^α may increase or decrease in the confidence level α. □

As stated above, the optimal order quantity q^α reduces to the optimal order quantity q^* when $\alpha = 0$. Therefore Corollary 5 implies that the optimal order quantity for the retailer to maximize CVaR of utility may be bigger or smaller than the expected utility maximizing order quantity q^*.

The above result shows that when the retailer becomes more risk-averse, he/she may select a bigger or smaller order quantity to hedge the potential risks. Then how does the retailer's expected utility under the optimal order quantity q^α change when the the confidence level changes? The answer to this question is given as follows.

Corollary 6. *To maximize CVaR objective of loss aversion utility, the retailer's expected utility $E[U(q^\alpha)]$ under the optimal order quantity q^α is decreasing in the confidence level α.*

Proof. It follows from (7) that

$$E[U(q)] = (p - c + \lambda s)q - \lambda s E(\xi) - [p - c + \lambda(c - r + s)] \int_0^q (q - t) dF(t). \tag{57}$$

Then it follows

$$\frac{\partial E[U(q^\alpha)]}{\partial \alpha} = [(p - c + \lambda s) - (p - c + \lambda(c - r + s))F(q)] \frac{\partial q^\alpha}{\partial \alpha}. \tag{58}$$

By Corollary 5, if the optimal order quantity q^α is increasing in the confidence level α, we have $\frac{\partial q^\alpha}{\partial \alpha} \geq 0$ and $q^\alpha \geq q^*$. Then it follows from $q^* = F^{-1}\left[\frac{p-c+\lambda s}{p-c+\lambda(c-r+s)}\right]$ that

$$p - c + \lambda s - (p - c + \lambda(c - r + s))F(q^\alpha) \leq p - c + \lambda s - (p - c + \lambda(c - r + s))F(q^*) = 0. \tag{59}$$

It follows from (58), (59) and $\frac{\partial q^\alpha}{\partial \alpha} \geq 0$ that

$$\frac{\partial E[U(q^\alpha)]}{\partial \alpha} \leq 0, \tag{60}$$

which proves that $E[U(q^\alpha)]$ is decreasing in the confidence level α. Otherwise, if the optimal order quantity q^α is decreasing in the confidence level α, we have $\frac{\partial q^\alpha}{\partial \alpha} \leq 0$ and $q^\alpha \leq q^*$. Then it follows from $q^* = F^{-1}\left[\frac{p-c+\lambda s}{p-c+\lambda(c-r+s)}\right]$ that

$$p - c + \lambda s - (p - c + \lambda(c - r + s))F(q^\alpha) \geq p - c + \lambda s - (p - c + \lambda(c - r + s))F(q^*) = 0. \tag{61}$$

It follows from (58), (61) and $\frac{\partial q^\alpha}{\partial \alpha} \leq 0$ that

$$\frac{\partial E[U(q^\alpha)]}{\partial \alpha} \leq 0,$$

which proves that $E[U(q^\alpha)]$ is decreasing in the confidence level α.

This result implies that the retailer will expect a lower utility if he/she selects an order quantity to reduce the potential risks. Therefore this result proves that low risk follows with low return while high return comes with high risk. □

4. Numerical Results and Discussions

This section presents the numerical results and suggests some management insights for the optimal order decisions of loss-averse retailers in reality.

Example 1. *Suppose that the market demand ξ is subject to the normal distribution $N(1000, 100^2)$. For the given parameters, we solve the optimal order quantities q^* and q^α for the retailer and present the sensitivity analysis.*

Let $\lambda = 2$ and $\alpha = 0.5$, we solve q^* and q^α under different retail prices p, wholesale prices c, salvage prices r and shortage penalty prices s. q^* and q^α are given in Figures 1–4, respectively. Figures 1, 3 and 4 show that both q^* and q^α increases in the retail price p, the salvage price r and the shortage penalty price s. Figure 2 shows that both q^* and q^α decreases in the wholesale price c. Besides, it satisfies $q^* > q^\alpha$ for different retail prices p, wholesale prices c, salvage prices r and shortage penalty prices s. Further, let $p = 8$, $c = 5$, $r = 2$, $s = 3$ and $\alpha = 0.5$, we compute q^* and q^α under different loss aversion coefficients λ. The results are given in Figure 5. Figure 5 shows that both the q^* and q^α decreases in the loss aversion coefficient λ. It also shows that $q^* > q^\alpha$ for different loss aversion coefficients λ. Therefore the loss-averse retailer should order fewer items than the loss-neutral retailer. The more loss-averse the retailer becomes, the fewer he/she should order. Finally, let $p = 8$, $c = 5$, $r = 2$, $s = 3$ and $\lambda = 2$, we compute the optimal order quantities q^* and q^α with different confidence levels α. The results are given in Figure 6. Figure 6 shows that q^α decreases in the confidence levels α. It also shows that $q^* > q^\alpha$ for different confidence levels α. Therefore the risk-averse retailer should order fewer items than the risk-neutral retailer. The more risk-averse the retailer becomes, the fewer he orders.

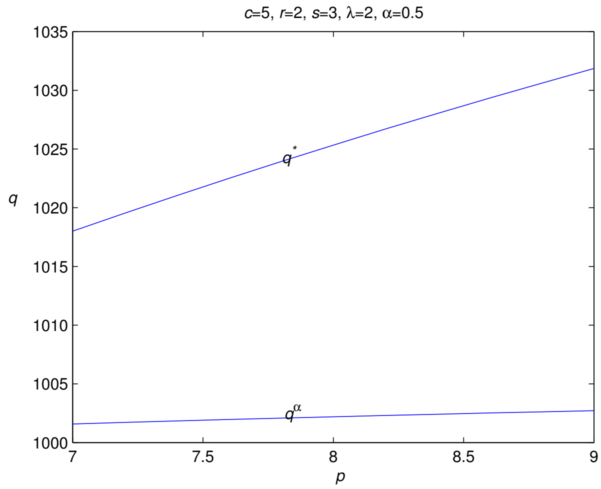

Figure 1. Optimal order quantities q^* and q^α with different retail prices p.

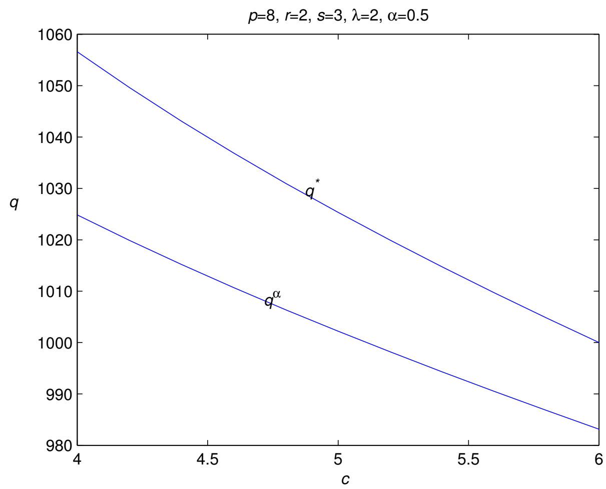

Figure 2. Optimal order quantities q^* and q^α with different wholesale prices c.

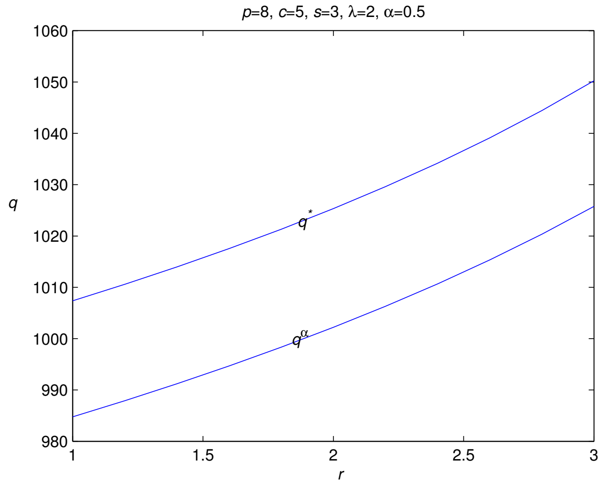

Figure 3. Optimal order quantities q^* and q^α with different salvage prices r.

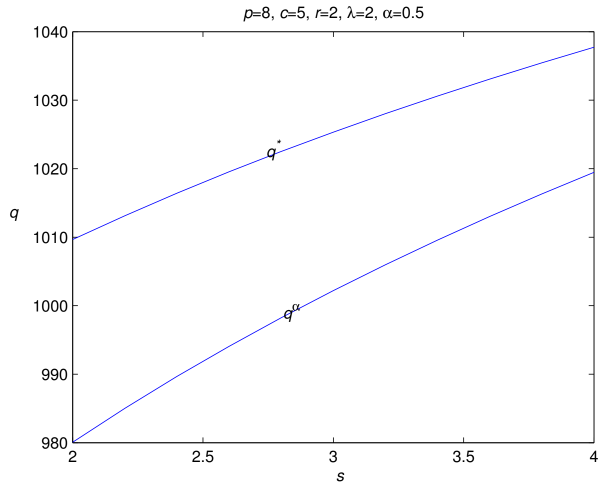

Figure 4. Optimal order quantities q^* and q^α with different shortage penalty prices s.

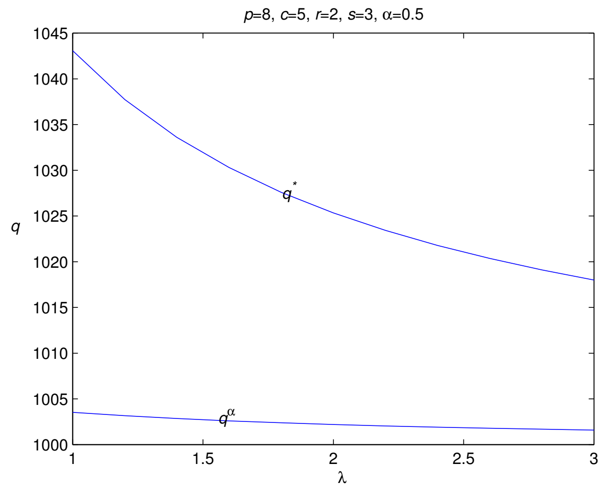

Figure 5. Optimal order quantities q^* and q^α with different loss aversion coefficients λ.

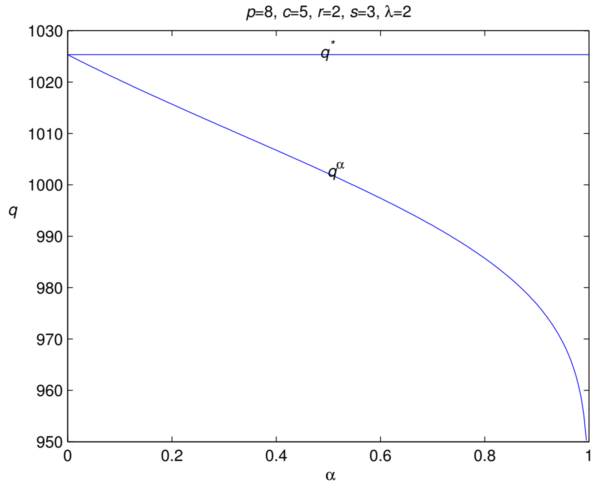

Figure 6. Optimal order quantities q^* and q^α with different confidence levels α.

Example 2. *Suppose that the market demand ξ subjects to the normal distribution $N(1000, 100^2)$. Let $p = 8$, $c = 5$, $r = 2$ and $s = 3$. For $\alpha = 0.3$, we compute the optimal order quantities q^* and q^α under different loss aversion coefficients λ, and the results are given in Figure 7. Figure 7 shows that both the optimal order quantities q^* and q^α are decreasing in the loss aversion coefficients λ. Further, for $\alpha = 0.9$, we compute the optimal order quantities q^* and q^α under different loss aversion coefficient λ, and the results are given in Figure 8. Figure 8 shows that the optimal order quantity q^* is decreasing in the loss aversion coefficients λ, while the optimal order quantity q^α is increasing in the loss aversion coefficient λ, which is different from the result in Figure 7. Indeed, the underage loss is bigger than the overage loss in this example, therefore the retailer should order more items to hedge against potential risks when he becomes more loss-averse. This result also reveals that the retailer with different risk aversion preference should select different change direction of the optimal order quantity when he becomes more loss-averse.*

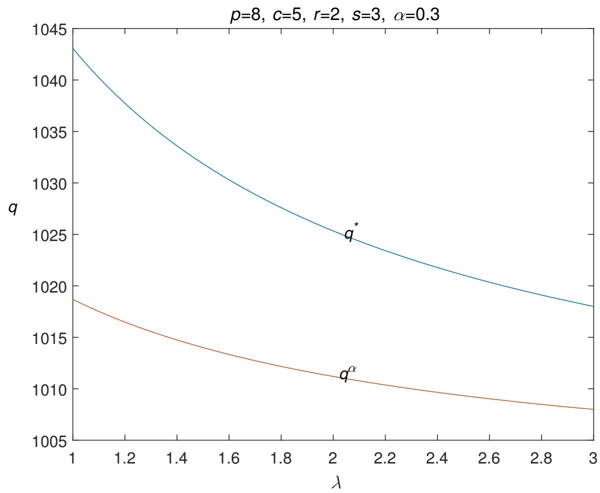

Figure 7. Optimal order quantities q^* and q^α with different loss aversion coefficients λ.

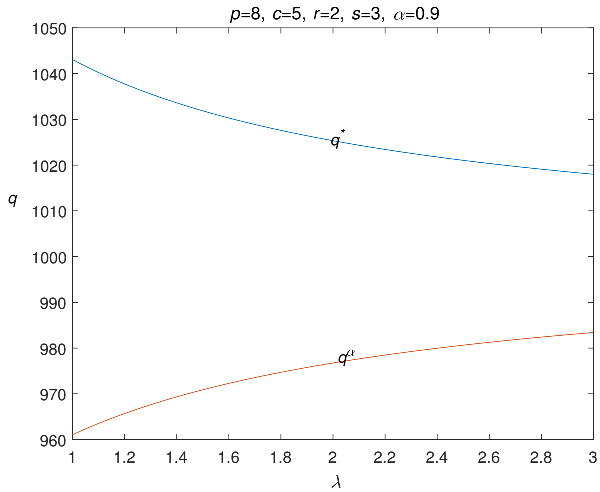

Figure 8. Optimal order quantities q^* and q^α with different loss aversion coefficients λ.

To summarize this section, the numerical results and confirmed the results in Section 3. There are following suggestions for the retailer in this example to select an optimal order quantity to maximize the expected loss aversion utility and CVaR objective about loss aversion utility: compared with the optimal order quantity to maximize expected utility, the retailer should order fewer units of product to maximize CVaR of utility.

5. Conclusions

In this paper, we study the optimal order decisions in the loss-averse newsvendor model. We propose a novel loss aversion utility function to characterize the retailer's loss aversion. We achieve the optimal order quantities for the retailer to maximize the expected loss aversion utility and CVaR objective of loss aversion utility, respectively. This study shows that the optimal order quantity for a retailer to maximize the expected loss aversion utility is smaller than EPM order quantity in the classical newsvendor model. It is also found that the optimal order quantity for a retailer to maximize CVaR objective of utility can be decreasing in the retail price, which has never occurred in the newsvendor literature. Besides, this study shows that the retailer's expected utility under the optimal order quantity with CVaR objective decreases in the confidence level, which verifies that low risk means low return while high return comes with high risk. Our paper thus provides an alternative choice to solve the loss-averse newsvendor model and also presents insights for the risk management in the newsvendor model.

Some extensions of this research are possible. For example, with the rapid development of customer service level, backlogging the excess demands of the unsatisfied customers has became more and more common in reality. Then a possible extension is to incorporate backordering into this study, which is a more interesting and practical issue.

Author Contributions: Conceptualization, F.T.S.C.; methodology, X.X.; validation, X.X.; formal analysis, F.T.S.C.; investigation, F.T.S.C.; writing-original draft preparation, X.X.; writing—review and editing, X.X.; visualization, X.X.; supervision, F.T.S.C.

Funding: This research received no external funding.

Acknowledgments: This work is supported by the National Natural Science Foundation of China with grant 71871026, the Humanities and Social Sciences Foundation of Ministry of Education of China with grant 18YJC630214 and the Natural Science Foundation of Shandong Province with grant ZR2017MG002.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Khouja, M. The single-period (news-vendor) problem: Literature review and suggestions for further research. *Omega* **1999**, *27*, 537–553. [CrossRef]
2. Gruen, T.; Corsten, D. Rising to the challenge of out-of-stocks. *ECR J.* **2002**, *2*, 45–58.
3. Zhang, B.F.; Wu, D.S.; Liang, L.; Olson, D.L. Supply chain loss averse newsboy model with capital constraint. *IEEE Trans. Syst. Man Cybern. Syst.* **2016**, *46*, 646–658. [CrossRef]
4. Fisher, M.; Ramen, A. Reducing the cost of demand uncertainty through accurate response to early sales. *Oper. Res.* **1996**, *44*, 87–99. [CrossRef]
5. Palar, M.; Weng, Z.K. Balancing desirable but conflicting objectives in the newsvendor problem. *IIE Trans.* **2003**, *35*, 131–142. [CrossRef]
6. Qin, Y.; Wang, R.X.; Vakharia, A.J.; Chen, Y.W.; Seref, M.M.H. The newsvendor problem: Review and directions for future research. *Eur. J. Oper. Res.* **2011**, *213*, 361–374. [CrossRef]
7. Schweitzer, M.E.; Cachon, G.P. Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence. *Manag. Sci.* **2000**, *46*, 404–420. [CrossRef]
8. Lau, H. The newsboy problem under alternative optimization objectives. *J. Oper. Res. Soc.* **1980**, *31*, 525–535. [CrossRef]
9. Sankarasubramanian, E.; Kumaraswamy, S. Optimal order quantity for predetermined level of profit. *Manag. Sci.* **1983**, *29*, 512–514. [CrossRef]
10. Lau, A.; Lau, H. Maximizing the probability of achieving a target profit level in a two-product newsboy problem. *Decis. Sci.* **1988**, *19*, 392–408. [CrossRef]
11. Li, J.; Lau, H.; Lau, A. A two-product newsboy problem with satisfying objective and independent exponential demands. *IIE Trans.* **1991**, *23*, 29–39. [CrossRef]
12. Ockenfels, A.; Selten, R. Impulse balance and multiple-period feedback in the newsvendor game. *Prod. Oper. Manag.* **2015**, *24*, 1901–1906. [CrossRef]
13. Niederhoff, J.; Kouvelis, P. Generous, spiteful, or profit maximizing suppliers in the wholesale price contract: A behavioral study. *Eur. J. Oper. Res.* **2016**, *253*, 372–382. [CrossRef]
14. Eeckhoudt, L.; Gollier, C.; Schlesinger, H. The risk averse (and prudent) newsboy. *Manag. Sci.* **1995**, *41*, 786–794. [CrossRef]
15. Agrawal, V.; Seshadri, S. Impact of uncertainty and risk aversion on price and order quantity in the newsvendor problem. *Manuf. Serv. Oper. Manag.* **2000**, *2*, 410–423. [CrossRef]
16. Chen, X.; Sim, M.; Simchi-Levi, D.; Sun, P. Risk aversion in inventory management. *Oper. Res.* **2007**, *55*, 828–842. [CrossRef]
17. Choi, S.; Ruszczy'skim A.; Zhao, Y. A multiproduct risk- averse newsvendor with Law-Invariant coherent measures of risk. *Oper. Res.* **2011**, *59*, 346–364. [CrossRef]
18. Tekin, M.; Ozekici, S. Mean-Variance newsvendor model with random supply and financial hedging. *IIE Trans.* **2015**, *47*, 910–928. [CrossRef]
19. Eskandarzadeh, S.; Eshghi, K.; Bahramgiri, M. Risk shaping in production planning problem with pricing under random yield. *Eur. J. Oper. Res.* **2016**, *253*, 108–120. [CrossRef]
20. Xu, X.; Wang, H.; Dang, C.; Ji, P. The loss-averse newsvendor model with backordering. *Int. J. Prod. Econ.* **2017**, *188*, 1–10. [CrossRef]
21. Xu, X.; Chan, C.K.; Langevin, A. Coping with risk management and fill rate in the loss-averse newsvendor model *Int. J. Prod. Econ.* **2018**, *195*, 296–310. [CrossRef]
22. Xu, X.; Chan, F.T.S.; Chan, C.K. Optimal option purchase decision of a loss-averse retailer under emergent replenishment. *Int. J. Prod. Res.* **2019**, 1–27. [CrossRef]
23. Bayle, M. Brand killers. *Fortune* **2003**, *8*, 51–56.
24. Kahneman, D.; Tversky, A. Prospect theory: An analysis of decision under risk. *Econometrica* **1979**, *47*, 263–291. [CrossRef]
25. MacCrimmon, K.R.; Wehrung, D.A. *Taking Risks: The Management of Uncertainty*; Free Press: New York, NY, USA, 1996.
26. Shapira, Z. *Risk in Managerial Decision Making*; Working Paper; Hebrew University: Jerusalem, Israel, 1986,
27. Putler, D. Incorporating reference price effects into a theory of household choice. *Mark. Sci.* **1992**, *11*, 287–309. [CrossRef]

28. Maor, M. Emotion-driven negative policy bubbles. *Policy Sci.* **2016**, *49*, 191–210. [CrossRef]
29. Farber, H.S. Why you can't find a taxi in the rain and other labor supply lessons from cab drivers. *Q. J. Econ.* **2015**, *130*, 1975–2026. [CrossRef]
30. Chen, X.; Sim, M.; Simichi-levi, D.; Sun, P. *Risk Aversion in Inventory Management;* MIT: Cambridge, UK, 2003
31. Gotoh, J.; Takano, Y. Newsvendor solutions via conditional value-at-risk minimization. *Eur. J. Oper. Res.* **2007**, *179*, 80–96. [CrossRef]
32. Chen, Y.; Xu, M.; Zhang, Z.G. A risk-averse newsvendor model under the CVaR criterion. *Oper. Res.* **2009**, *57*, 1040–1044. [CrossRef]
33. Wu, M.; Zhu, S.; Teunter, R. Newsvendor problem with random shortage cost under a risk criterion. *Int. J. Prod. Econ.* **2013**, *145*, 790–798. [CrossRef]
34. Xu, X.; Meng, Z.; Shen, R.; Jiang, M.; Ji, P. Optimal decisions for the loss-averse newsvendor problem under CVaR. *Int. J. Prod. Econ.* **2015**, *164*, 146–159.
35. Wang, R.; Song, S.; Wu, C. Coordination of supply chain with one supplier and two competing risk-averse retailers under an option contract. *Math. Probl. Eng.* **2016**, 1970615. [CrossRef]
36. Xu, X.; Meng, Z.; Ji, P.; Dang, C.; Wang, H. On the newsvendor model with conditional Value-at-Risk of opportunity loss. *Int. J. Prod. Res.* **2016**, *54*, 2339–2458. [CrossRef]
37. Rockafellar, R.T.; Uryasev, S. Optimization of Conditional Value-at-Risk. *J. Risk* **2000**, *2*, 21–41. [CrossRef]
38. Rockafellar, R.T.; Uryasev, S. Conditional value-at-risk for general loss distributions. *J. Bank. Financ.* **2002**, *26*, 1443–1471. [CrossRef]