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A Refinement to the Viterbi-Viterbi Carrier Phase Estimator and an Extension to the Case With a Wiener Carrier Phase Process

YAN LI[1,2], MING-WEI WU[3], (Member, IEEE), XINWEI DU[4], TIANYU SONG[4], AND POOI-YUEN KAM[2,5], (Fellow, IEEE)

[1]Department of Electronic and Information Engineering, Hong Kong Polytechnic University, Hong Kong
[2]National University of Singapore (Suzhou) Research Institute, Suzhou 215123, China
[3]School of Information and Electronic Engineering, Zhejiang University of Science and Technology, Hangzhou 310023, China
[4]Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576
[5]School of Science and Engineering, Chinese University of Hong Kong, Shenzhen 518172, China

Corresponding author: Ming-Wei Wu (mingweiwu@ieee.org)

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ABSTRACT We provide a theoretical foundation for further analysis and optimization of the Mth-power (MP) carrier phase estimator for MPSK modulation. Also known as the Viterbi-Viterbi (VV) estimator, it is commonly used in practice because it leads to low-latency receiver implementations. The MP carrier phase estimator first raises the received noisy signal samples to the Mth-power to remove the unknown phase modulation, and then extracts the unknown carrier phase of the mid-symbol using a weighted sum of these modulation-wiped-off received signal samples over a symmetrical observation window. Our starting point is the single-term, complex exponential expression for a complex sinusoid received in complex, additive, white, Gaussian noise (AWGN), which leads to a great deal of simplicity in dealing with arbitrary powers of the noisy received signal sample when compared with the conventional approach of raising the sum of signal plus noise to higher powers. The single-exponential expression enables us to first optimize the weighting coefficients of the MP carrier phase estimator with respect to the statistics of the AWGN, in a manner much simpler than previous approaches. Then, it enables us to apply the linear minimum mean square error (LMMSE) criterion to optimize the MP estimator with respect to both the statistics of the AWGN and the carrier phase noise that we model here as a Wiener process. Although the LMMSE MP estimator is computationally intensive for online implementation, a much less complex version is suggested that can be efficiently implemented in real time. Extensive simulation results are presented to demonstrate the improved performance of the LMMSE MP estimator over the conventional MP estimator. By using a sufficiently long symmetrical observation window, the LMMSE estimator does not suffer from the block length effect, which leads to much performance gain over the VV/MP estimator especially at high signal-to-noise ratio (SNR) and high phase noise. A phase unwrapping algorithm is also presented for accurate unwrapping of the estimated carrier phase before it is used in data detection. The proposed LMMSE carrier phase estimator is suitable for implementing a coherent receiver at all SNRs.

INDEX TERMS MPSK, Viterbi-Viterbi/Mth-power carrier phase estimation, Wiener carrier phase noise, AWGN, phase unwrapping, LMMSE estimation.

I. INTRODUCTION

With the ever-increasing demand for higher data rates and higher spectral efficiencies, the use of higher-order digital modulations such as M-ary quadrature-amplitude modulations (MQAM) and M-ary phase shift-keying (MPSK) is becoming imperative for future generations of data transmission systems. These linear modulations require coherent detection in order to achieve their full performance potential, and accurate carrier recovery is, thus, imperative.

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The preference for using digital signal processing (DSP) techniques over the analog phase lock loop for carrier recovery is, by now, no longer a matter of debate. Among the reasons for this is the fact that many powerful, statistically optimum, DSP techniques are available for potentially solving the carrier-phase estimation problem if we have accurate statistical models of the various noise processes involved. While much has been done on statistical estimation of carrier phase, this paper presents novel concepts, analytical techniques and simulation results that will significantly advance the state of the art in carrier recovery for an MPSK-modulated carrier that is received in the presence of the usual additive, white, Gaussian noise (AWGN) and the commonly encountered Wiener-process carrier phase noise, as we will demonstrate.

Two commonly encountered carrier phase estimators for MPSK, especially in coherent optical communications, are the Mth-power estimator [1] and the decision-aided maximum-likelihood (DA-ML) estimator [2]. Both methods rely on the assumption of quasi-static carrier phase process, i.e., a carrier phase process that is very slowly time varying such that it can be modeled as a constant over the averaging interval. Since the carrier phase is usually time-varying in practice, albeit slowly, this assumption may incur severe performance impairment due to the block length effect [3]. The DA-ML estimator is statistically optimum in the presence of AWGN and a quasi-static carrier phase process and is also computationally simpler than the Mth-power method. Despite these advantages, its application is less common in practice than that of the latter because of the latency issue associated with the need for making symbol decisions before performing the decision-aided carrier phase estimation. Since the envisioned future ultra-high speed communications systems require very low latency, the latency issue can cause a serious problem in practical receiver implementations. To the best of the authors' knowledge, more researchers in practice make use of the Mth-power estimator. Because of its practical value, our work here therefore focuses on improving the structure of the Mth-power estimator and enhancing its performance.

The Mth-power estimator is a non-decision-aided estimator because it raises the complex exponential part of the complex received signal sample to the Mth-power to remove the modulation due to the data phase. Reference [1] also suggested a nonlinear transformation of the magnitude of the complex received sample for improving the signal-to-noise ratio (SNR) of the final estimated carrier phase, and it is shown in [1] and [6] that the square-law provides the optimum transformation that maximizes the SNR. By combining and averaging the transformed complex samples over a symmetrical time interval, the phase of the resultant phasor gives the estimated carrier phase of the symbol in the middle of the interval. Note that [1] assumes that the carrier phase is quasi-static over the averaging interval, and our work in [2] uses this assumption also.

The starting point of our refinement to the Mth-power estimator is the realization in [7] that one can view the effect of the complex AWGN that perturbs the observation of a complex signal phasor as introducing an additive observation phase noise (AOPN) on the latter, where the statistics of the AOPN and those of the AWGN are related. This enables one to express the noisy complex received signal sample as a single complex exponential, instead of the sum of two terms (signal plus noise), as we have demonstrated in [7]. One can then treat the AOPN and the carrier phase noise in the received signal sample in the same manner when raising the complex exponential part of the complex received signal sample to the Mth-power to remove the data modulation. Since our approach involves only one complex exponential, it leads to tremendous analytical simplicity, when compared with the conventional approach of raising the sum of two terms (signal plus noise) to the Mth-power that leads to a complicated expression involving many signal-times-noise terms. The single-complex-exponential expression for the Mth-power of the received signal sample provides much insight into how one can recover the unknown carrier phase for a single symbol interval from signal samples observed over a symmetrical window centered at that interval concerned.

Starting with the single-exponential signal model, we provide here an explicit performance investigation into the Mth-power carrier phase estimator proposed by Viterbi and Viterbi in [1], which shows that this method is optimal only under the condition of constant carrier phase. The simplicity of the single-exponential expression also allows us to take into account explicitly the random variations in the carrier phase process that we model here as a Wiener process. This is a departure from most previous works such as [1], [2] that require the assumption that the carrier phase is quasi-static over the observation window. The use of a Wiener-process phase noise model is common in the communication theory literature, especially in optical communications [4], [8]. Using the Wiener-process model, we apply the linear minimum mean square error (LMMSE) criterion to design a new Mth-power carrier phase estimator. With this refinement, the proposed method achieves much stronger tolerance to the phase fluctuation, which greatly improves the applicability of the conventional Mth-power method in practice with time varying carrier phase. Most importantly, since the phase fluctuation is taken into account, the proposed estimator is shown to achieve strong tolerance to the block length effect, which enables an efficient implementation in practice. Through numerical simulations, we show that, instead of having to numerically search for the optimal averaging window size, one can easily approach the optimal performance by using our proposed estimator with a relatively long window. To further facilitate the implementation, a simplified version of the optimized estimator with much less computational complexity is introduced. Despite the assumption of reasonably high SNR, the simplified method is shown to approach closely to the exact version within a wide SNR range of practical interest.

We note here that the effect of the AWGN on the transmitted signal phasor is equivalent to the rotation of the latter by an angle that we call the additive observation phase noise (AOPN). This observation enables one to express the two terms (signal and noise) in the received signal model as a single exponential. We should thus emphasize the key role played by the AOPN that leads to the single-exponential received signal model. As shown in [7], the AOPN sequence is a sequence of independent, Tikhonov distributed random variables, each with a mean of zero. Their variance is a nonlinear function of three parameters, namely, the transmitted signal power, the spectral density of the AWGN and the magnitude of the noisy received signal sample. The nonlinear variance expression involves, more precisely, the modified Bessel function of the first kind of order zero, and its nonlinear nature makes the application of the Tikhonov AOPN model to the analysis and optimization of the Mth-power carrier phase estimator very complicated. To circumvent this problem, we use the fact that at reasonably high SNR, the Tikhonov distribution is accurately approximated by a Gaussian distribution [7], [9] with a simple linear expression for its variance. In this Gaussian approximation, the AOPN has a mean of zero and a variance that is a linear function of the three parameters mentioned above. The linearity of the variance expression simplifies the analytical work with the Mth-power estimator, and thus the approximate Gaussian AOPN model is adopted in our derivations here in order to arrive at explicit and practically useful results. The principle of the analytical approach is the same whether the Tikhonov model or the Gaussian approximation is used.

The AOPN model, as shown in [7], is applicable irrespective of the power of the transmitted sinusoidal signal, and therefore is applicable to MQAM signals in general. The only reason why we restrict attention here to MPSK signals is that the Mth-power estimator is originally proposed in [1] only for MPSK modulations and we are focusing on this estimator. Some ad hoc techniques have been proposed [10]–[14] to extend the Mth-power estimator to the general case of MQAM or amplitude-phase shift keying (APSK). Instead of dealing with these ad hoc extensions here, we will, moving forward, investigate into the application of our AOPN model to the optimum design of carrier phase estimators for MQAM/APSK signals in general. The results will be reported in future reports. We should caution the reader that we are not extending our results of [7] here. Rather, our goal is to demonstrate the usefulness of our ideas in [7] to the analysis and optimization of carrier phase estimators in general.

The structure of the remainder of the paper is as follows. Section II introduces the single-term exponential model of the received signal. Preview of the Mth-power phase estimator and analytical interpretation is provided in Section III. The LMMSE Mth-power phase estimator is derived and analyzed in Section IV. In Section V, we introduce several post-processing phase unwrapping methods which are crucial for phase estimation. Numerical simulations and analysis are provided in Section VI. Conclusions are drawn in Section VII.

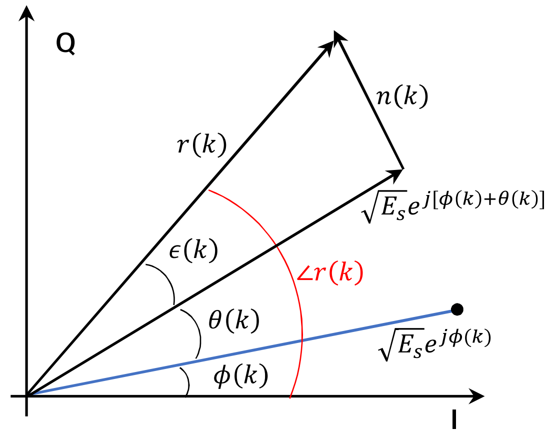

FIGURE 1. Geometric representation of the received signal $r(k)$.

II. SYSTEM MODEL

We assume perfect timing synchronization and frequency offset compensation, and the absence of other channel dispersive affects that lead to inter-symbol interference. Hence, the received is only perturbed by AWGN and carrier phase noise such as laser phase noise in an optical communication system [8], [15], [16]. In this case, a canonical model of the received symbol over the kth symbol interval is given by [8], [15], [16]

$$r(k) = m(k)e^{j\theta(k)} + n(k). \tag{1}$$

Here, $m(k)$ denotes the kth transmitted symbol. For MPSK, $m(k)$ takes on values from the signal set $\{S_i = \sqrt{E_s}e^{j\phi(i)} = \sqrt{E_s}e^{j\frac{2\pi i}{M}}\}_{i=0}^{M-1}$ with equal probability, where E_s denotes the average energy per symbol, $\phi(i)$ denotes the phase modulation and M is the number of signal points. Term $\{n(k)\}_k$ is a sequence of independent and identically distributed (i.i.d.), complex Gaussian random variables, each with mean zero and variance N_0, where N_0 is the double-sided spectral density of the AWGN.

Term $\theta(k)$ denotes the carrier phase, which is commonly modeled as a Wiener process in coherent optical communications and is given by [8]

$$\theta(k) = \theta(k-1) + \nu(k). \tag{2}$$

Term $\{\nu(k)\}_k$ is a sequence of independent, identically distributed (i.i.d.) Gaussian random variables with mean zero and variance [16]

$$\sigma_p^2 = 2\pi\Delta\nu T_s. \tag{3}$$

T_s and $\Delta\nu$ denote the symbol duration and the total 3-dB laser linewidth of the transmitter and local oscillator lasers.

From our work in [7], as shown in Fig. 1, the in-phase-and-quadrature-phase form of the received signal model in (1) can be rewritten in a single-complex-exponential form given by

$$r(k) = |r(k)|e^{j\angle r(k)} = |r(k)|e^{j[\phi(k)+\theta(k)+\epsilon(k)]}. \tag{4}$$

Here, term $\epsilon(k)$ denotes the AOPN due to the AWGN. With the knowledge of received signal $\{r(k)\}_k$, $\{\epsilon(k)\}_k$ has been shown to be a sequence of independent, Tikhonov random variables, with the probability density function (PDF) given as [7]

$$p\left(\epsilon(k)\middle||r(k)|\right)=\frac{\exp\left[\frac{|r(k)|\sqrt{E_s}}{N_0/2}\cos\epsilon(k)\right]}{2\pi I_0\left(\frac{|r(k)|\sqrt{E_s}}{N_0/2}\right)},\tag{5}$$

where $I_0(\cdot)$ denotes the modified Bessel function of the first kind of order zero. Since the PDF given in (5) is highly nonlinear, direct application of the Tikhonov AOPN model is very complicated. To simplify the theoretical work, we use the fact that assuming high SNR, the Tikhonov distribution can be accurately approximated with a Gaussian distribution with mean of zero and variance σ_ϵ^2 given as

$$\sigma_\epsilon^2(k)=\frac{N_0}{2|r(k)|\sqrt{E_s}}.\tag{6}$$

Additionally, with high SNR values, the received signal magnitude $|r(k)|$ approaches closely to $\sqrt{E_s}$ for most of the time. Hence, the AOPN variance given in (6) can be further simplified as a constant, given as

$$\sigma_\epsilon^2=\frac{N_0}{2E_s}=\frac{1}{2\gamma},\tag{7}$$

where $\gamma=E_s/N_0$ represents the SNR per symbol.

Alternatively, since the signal magnitude $\sqrt{E_s}$ may not be available in some cases in practice, we can also approximate it, especially for large γ, by the magnitude of $r(k)$ and obtain

$$\sigma_\epsilon^2(k)=\frac{N_0}{2|r(k)|^2}.\tag{8}$$

We note from the geometric representation of the received signal $r(k)$ in Fig. 1 that the single-complex-exponential signal model in (4) applies for any value of the signal power $\sqrt{E_s}$. Likewise, we note from [7] that the Tikhonov AOPN model applies also for any value of the signal power $\sqrt{E_s}$. Thus, the signal model (4) together with the Tikhonov AOPN model applies to MQAM signals in general. As we are only concerned with the estimator of [1], we will not be dealing with MQAM signals in general.

Note also that the signal model (4) together with the general Tikhonov AOPN model can be used in our subsequent work in this paper to deal with the analysis of the Mth-power carrier phase estimator. However, as can be observed from [7], the variance of the Tikhonov AOPN model involves a modified Bessel function of the first kind of order zero, whose arguments are the parameters on the right hand side of (6). This variance expression is highly nonlinear, and obviously would not lead to a tractable analysis. By assuming reasonably high SNR, the Tikhonov distribution can be approximated by a Gaussian distribution in which the variance is given by one of the simple expressions presented above. This simplification of the variance expression renders the analysis below tractable, and gives us explicit results.

III. PREVIEW OF THE *M*TH-POWER CARRIER PHASE ESTIMATOR

The Mth-power carrier phase estimator is first proposed in [1] with the consideration of MPSK only and without carrier phase noise. For each symbol, a symmetrical estimation window of size $2L+1$, including L earlier symbols, L later symbols and the current symbol, is used for the carrier recovery. With the exponential signal model given in (4), we first introduce the conventional Mth-power carrier phase estimator.

After raising to the Mth-power and taking the normalization, the phase modulation of the received symbol is removed, which gives

$$y_M(k)=\left[\frac{r(k)}{|r(k)|}\right]^M=\left[e^{j[\phi(k)+\theta(k)+\epsilon(k)]}\right]^M=e^{jM[\theta(k)+\epsilon(k)]}.\tag{9}$$

This is because, for MPSK, $M\phi(k)=2\pi i$ is a multiple of 2π, and is thus the same as a zero phase angle. Based on the observations within the symmetrical window, $\{y_M(l)\}_{l=k-L}^{k+L}$, the phase estimate $\hat{\theta}_v(k)$ using Mth-power estimator is given by [1]

$$\hat{\theta}_v(k)=\frac{1}{M}\angle\left[\frac{1}{2L+1}\sum_{l=k-L}^{k+L}F(|r(l)|)y_M(l)\right].\tag{10}$$

$F(|r(l)|)$ is a nonlinear function of $|r(l)|$, which can be chosen to optimize the estimation performance. Previously, Viterbi and Viterbi provided a group of candidates for the nonlinear function, $F(|r(l)|)=|r(l)|^N$, where $N\le M$ is an arbitrary even integer. Among them, $F(|r(l)|)=|r(l)|^2$ was suggested as the optimal choice for QPSK without frequency error [1]. The optimal nonlinearity with respect to SNR was investigated in [6] which further confirmed Viterbi and Viterbi's suggestion. In optical communication systems, for simplicity, researchers usually make use of the direct Mth-power of the received symbol, i.e., they take $F(|r(l)|)=|r(l)|^M$ [17]. For simplicity, in the rest of our paper, the suggested Mth-power estimator with the nonlinearity of $F(|r(l)|)=|r(l)|^2$ is referred as the Viterbi-Viterbi (VV) estimator, while that using $F(|r(l)|)=|r(l)|^M$ is referred as the conventional Mth-power estimator.

Here, we make a further contribution by showing that $F(|r(l)|)=|r(l)|^2$ is actually optimum for MPSK without phase noise and frequency error, in the sense that it leads to the minimum achievable estimation error variance given by the Cramér-Rao lower bound (CRLB) at relatively high SNR.

Based on the single-term exponential signal model given in (4), we will show, for the first time, that $F(|r(l)|)=|r(l)|^2$ is the optimal choice for systems with constant carrier phase, i.e., $\Delta\nu=0$. Substituting (9) into (10) and considering the suggested nonlinearity, the VV phase estimate can be rewritten as

$$\hat{\theta}_v(k)=\frac{1}{M}\angle\left[\frac{1}{2L+1}\sum_{l=k-L}^{k+L}|r(l)|^2e^{jM[\theta(l)+\epsilon(l)]}\right].\tag{11}$$

Denoting the estimated phasor by using the VV method as

$$\hat{Z}(k) = \frac{1}{2L+1}\sum_{l=k-L}^{k+L}|r(l)|^2 e^{jM[\theta(l)+\epsilon(l)]}, \tag{12}$$

assuming the constant carrier phase, i.e. $\theta(k) = \{\theta(l)\}_{l=k-L}^{k+L}$, we have

$$\hat{Z}(k) = |\hat{Z}(k)|e^{jM\hat{\theta}_v(k)} = e^{jM\theta(k)}\frac{1}{2L+1}\sum_{l=k-L}^{k+L}|r(l)|^2 e^{jM\epsilon(l)}. \tag{13}$$

Denoting the VV phase estimation error as $\theta_e(k) = \theta(k) - \hat{\theta}_v(k)$, (13) can be rewritten as

$$|\hat{Z}(k)|e^{-jM\theta_e(k)} = \frac{1}{2L+1}\sum_{l=k-L}^{k+L}|r(l)|^2 e^{jM\epsilon(l)}. \tag{14}$$

Assuming relatively small AOPN and estimation error, through applying the approximation of $e^x \approx 1+x$, (14) can be rewritten as

$$|\hat{Z}(k)|[1 - jM\theta_e(k)] = \frac{1}{2L+1}\sum_{l=k-L}^{k+L}|r(l)|^2 + \frac{jM}{2L+1}\sum_{l=k-L}^{k+L}|r(l)|^2\epsilon(l). \tag{15}$$

Equating the real and imaginary parts of (15), we have

$$|\hat{Z}(k)| = \frac{1}{2L+1}\sum_{l=k-L}^{k+L}|r(l)|^2 \tag{16a}$$

$$|\hat{Z}(k)|\theta_e(k) = -\frac{1}{2L+1}\sum_{l=k-L}^{k+L}|r(l)|^2\epsilon(l) \tag{16b}$$

Substituting (16b) into (16b), we can simplify $\theta_e(k)$ as

$$\theta_e(k) = -\frac{\sum_{l=k-L}^{k+L}|r(l)|^2\epsilon(l)}{\sum_{l=k-L}^{k+L}|r(l)|^2}. \tag{17}$$

Squaring both sides of (17) and taking the expectation, the MSE of the VV phase estimate can be reduced to

$$E[\theta_e^2(k)] = \frac{E\left[\sum_{l=k-L}^{k+L}|r(l)|^2\epsilon(l)\right]^2}{\left[\sum_{l=k-L}^{k+L}|r(l)|^2\right]^2}. \tag{18}$$

We note that in the development of the Mth-power estimator starting from (9), the received signal samples $\{r(l), l = k-L, \ldots, k+L\}$ are already available and are therefore known. Thus, in (18), the expectation is, more precisely, the conditional expectation given knowledge of the magnitudes $\{|r(l)|, l = k-L, \cdots, k+L\}$, and we need only take the conditional expectation over the set of AOPN samples $\epsilon(l)$ which remain unknown. As the AOPN samples are due to the AWGN, they are mutually independent with mean of zero. Their conditional variances given the received signal magnitudes are given in eq. (6). With this result, the MSE of the phase estimation given in (18) can be easily simplified as

$$E[\theta_e^2(k)] = \frac{\left[\sum_{l=k-L}^{k+L}(|r(l)|^2)^2 E[\epsilon^2(l)]\right]}{\left[\sum_{l=k-L}^{k+L}|r(l)|^2\right]^2}. \tag{19}$$

Assuming static AOPN variance at relatively high SNR, as shown in (8), we have

$$\begin{aligned} E[\theta_e^2(k)] &= \frac{N_0}{2}\frac{\left[\sum_{l=k-L}^{k+L}(|r(l)|^2)^2/|r(l)|^2\right]}{\left[\sum_{l=k-L}^{k+L}|r(l)|^2\right]^2} \\ &= \frac{N_0}{2}\frac{\left[\sum_{l=k-L}^{k+L}|r(l)|^2\right]}{\left[\sum_{l=k-L}^{k+L}|r(l)|^2\right]^2} \end{aligned} \tag{20}$$

For MPSK modulation with constant amplitude, we have $|r(l)| \approx \sqrt{E_s}$ at high SNR. In this case, the MSE of phase estimation, $E[\theta_e^2(k)]$, can be further simplified as

$$E[\theta_e^2(k)] = \frac{N_0}{2}\frac{\left[\sum_{l=k-L}^{k+L}E_s\right]}{\left[\sum_{l=k-L}^{k+L}E_s\right]^2} = \frac{N_0}{2(2L+1)E_s}. \tag{21}$$

From [18, eq. (30)], we have that the CRLB on the variance of any carrier phase estimate is given by

$$E[\theta_e^2] \geq \mathrm{CRLB}(\theta) = \frac{1}{2L_c}\frac{N_0}{E_s}, \tag{22}$$

where L_c is the length of estimation window. Combining (21) and (22), we can conclude that $F(|r(l)|) = |r(l)|^2$ is the optimum nonlinearity for the Mth-power carrier phase estimator particularly at high SNR.

IV. LINEAR MMSE *M*TH-POWER CARRIER PHASE ESTIMATOR

A. DEVELOPMENT OF LMMSE *M*TH-POWER ESTIMATOR

Previously, the optimal nonlinearity $F(|r(l)|)$ was discussed with AWGN only. Here, with the consideration of both AWGN and linear phase noise, the optimal $F(|r(l)|)$ is derived explicitly by using the linear minimum mean square error (LMMSE) technique.

Similar to the VV phase estimator given in (10), the linear model of the phase estimation is considered. Denoting the phasor at kth time interval after Mth-power operation as $V_M(k) = e^{jM\theta(k)}$, the estimated phasor by using LMMSE estimator is given by

$$\hat{V}_M(k) = \sum_{l=k-L}^{k+L}F(|r(l)|)y_M(l) = \mathbf{w}^H\mathbf{y_M}(k), \tag{23}$$

where $\mathbf{y_M}(k) = [y_M(k-L), \cdots, y_M(k+L)]^T$ denotes the observation vector for the kth time interval and $\mathbf{w} = [F(|r(k-L)|), \cdots, F(|r(k+L)|)]^H$ represents the complex weight coefficients. Here, superscript H denotes the Hermitian transpose and both $\mathbf{y_M}(k)$ and $\mathbf{w}$ are $(2L+1)$-dimensional vectors. Denoting the estimation error as

$e(k) = V_M(k) - \hat{V}_M(k)$, the MSE of the estimated phasor, $E[|e(k)|^2]$, can be written as

$$\begin{aligned} &E[|e(k)|^2] \\ &\quad = E[e(k)e^*(k)] \\ &\quad = E\left[\left(V_M(k) - \mathbf{w}^H \mathbf{y_M}(k)\right)\left(V_M^*(k) - \mathbf{y_M}^H(k)\mathbf{w}\right)\right] \\ &\quad = E[|V_M(k)|^2] - \mathbf{p}^H\mathbf{w} - \mathbf{w}^H\mathbf{p} + \mathbf{w}^H\mathbf{R}\mathbf{w}. \end{aligned} \tag{24}$$

Here, $\mathbf{p} = E[\mathbf{y_M}(k)V_M^*(k)]$ is the $(2L+1) \times 1$ cross-correlation vector with its lth element given as

$$p_l = E[y_M(k-L+l-1)V_M^*(k)]. \tag{25}$$

$\mathbf{R} = E[\mathbf{y_M}(k)\mathbf{y_M}^H(k)]$ is the $(2L+1) \times (2L+1)$ auto-correlation matrix with its (x, y)th element given as

$$R(x, y) = E[y_M(k-L+x-1)y_M^*(k-L+y-1)]. \tag{26}$$

Through minimizing the MSE given in (24), the optimal weight coefficient $\mathbf{w}_o$ can be obtained as

$$\mathbf{w}_o = \mathbf{R}^{-1}\mathbf{p}, \tag{27}$$

where the lth element of $\mathbf{p}$ is given as

$$p_l = e^{-\frac{1}{2}\left[M^2\left(|L-l+1|\sigma_p^2 + \sigma_\epsilon^2(k-L+l-1)\right)\right]} \tag{28}$$

and, the (x, y)th element of $\mathbf{R}$ can be calculated as

$$R(x, y) = e^{-\frac{1}{2}M^2\left[(|x-y|)\sigma_p^2 + \sigma_\epsilon^2(k-L+x-1) + \sigma_\epsilon^2(k-L+y-1)\right]}. \tag{29}$$

The detailed derivation is provided in Appendix A. Note that, since the AOPN variance $\sigma_\epsilon^2(k)$ depends on the signal amplitude $|r(k)|$, both the matrix $\mathbf{R}$ and the vector $\mathbf{p}$ are time-varying. In the next section, to simplify the computation of the matrix inverse $\mathbf{R}^{-1}$, a static approximation of the matrix $\mathbf{R}$ and vector $\mathbf{p}$ is introduced.

B. SIMPLIFIED IMPLEMENTATION

From (27), the proposed LMMSE estimator requires the inverse of the auto-correlation matrix $\mathbf{R}$ at each time interval k, which would incur high computational complexity in practical applications. Here, we introduce a simplified implementation of the LMMSE Mth-power, which is computationally more efficient.

As shown in (28) and (29), both $\mathbf{p}$ and $\mathbf{R}$ are functions of the AOPN variance $\sigma_\epsilon^2(k)$, which is shown to be time-varying due to the fluctuation of the received signal amplitude $|r(k)|$. Through applying the approximation of the static AOPN variance given in (7), the lth term of the vector $\mathbf{p}$ given in (28) can be simplified as

$$p_l = e^{-\frac{1}{2}\left[M^2\left(|L-l+1|\sigma_p^2 + \sigma_\epsilon^2\right)\right]}. \tag{30}$$

Similarly, the (x, y) term of the auto-covariance matrix $\mathbf{R}$ given in (29) can be reduced to

$$R(x, y) = e^{-\frac{1}{2}M^2\left[(|x-y|)\sigma_p^2 + 2\sigma_\epsilon^2\right]}. \tag{31}$$

Substituting (30) and (31) into (27), the optimal weight vector can be approximated as a constant, which needs to be computed only once during the entire carrier phase estimation process. Obviously, the assumption of constant σ_ϵ^2 would degrade the estimation accuracy, particularly at low SNR. To verify its feasibility, the performance impairment due to the simplified implementation is evaluated through numerical simulations in Section VI.

C. PERFORMANCE ANALYSIS

In both the VV and conventional Mth-power estimator, the carrier phase $\theta(k)$ is assumed approximately constant within the estimation window. In this case, the window length $(2L+1)$ would strongly affect the estimation performance. In general, a large window size L would lead to effective mitigation of the AOPN during the phase estimation. However, it also conflicts with the assumption of the slowly varying carrier phase within the estimation window. Known as the block length effect, it requires extensive simulations and experiments to determine the optimal window size while applying the conventional Mth-power and the VV phase estimator in practice. In contrast, with the proposed LMMSE estimator, we show here analytically that the MSE of phase estimation suffers from no block length effect at relatively high SNR.

With the optimal weight vector $\mathbf{w}_o$ given in (27), the minimum MSE by using the LMMSE estimator can be rewritten as

$$\begin{aligned} &E[|e(k)|^2]_{min} \\ &\quad = E[|V_M(k)|^2] - \mathbf{p}^H\mathbf{w}_o - \mathbf{w}_o^H\mathbf{p} + \mathbf{w}_o^H\mathbf{R}\mathbf{w}_o \\ &\quad = 1 - \mathbf{p}^T\mathbf{R}^{-1}\mathbf{p} - \mathbf{p}^T[\mathbf{R}^{-1}]^T\mathbf{p} + \mathbf{p}^T[\mathbf{R}^{-1}]^T\mathbf{R}\mathbf{R}^{-1}\mathbf{p} \end{aligned} \tag{32}$$

Based on the approximation given in (31), obviously, $\mathbf{R}$ is a symmetric matrix. Hence, we have $[\mathbf{R}^{-1}]^T = \mathbf{R}^{-1}$. In this case, $E[|e(k)|^2]_{min}$ given in (32) can be further reduced to

$$E[|e(k)|^2]_{min} = 1 - \mathbf{p}^T\mathbf{R}^{-1}\mathbf{p}. \tag{33}$$

With relatively high SNR γ and non-zero carrier phase noise, i.e., $\sigma_p^2 > 0$, we have shown in Appendix B that the $E[|e(k)|^2]_{min}$ given in (33) can be approximated as

$$E[|e(k)|^2]_{min} = e^{M^2\sigma_\epsilon^2} - 1 + \left(e^{M^2\sigma_\epsilon^2} - 1\right)^2 \frac{e^{-M^2\sigma_p^2} + 1}{e^{-M^2\sigma_p^2} - 1}, \tag{34}$$

which is independent of the estimation window length L. That is to say, with relatively high SNR, the proposed LMMSE estimator suffers from no block length effect. This observation will be further verified by the numerical simulation results. Note that in arriving at (34), we made the high SNR assumption, $e^{M^2\sigma_\epsilon^2} \approx 1$. We did this only for the purpose of deriving (34) and demonstrating that this MSE is independent of the observation window length L. For a higher modulation order M, this assumption holds only for a smaller σ_ϵ^2 or a larger SNR γ. Thus, the independence from the block length effect occurs at a higher SNR γ for an MPSK signal set with more signal points. This observation is borne out by our simulation results. Note also that this assumption is not necessary for the derivation of the LMMSE estimator in the previous two subsections.

With constant carrier phase, i.e., $\sigma_p^2 = 0$, we will show in the next subsection that the proposed LMMSE estimator can be considered as the generalization of the VV estimator, which was shown to approach the CRLB at high SNR in Section III.

D. INTUITIVE INTERPRETATION

In the LMMSE carrier phase estimation, the measured fluctuation of the received signal magnitude within the estimation window provides an insight into the AOPN fluctuation, which can be applied to mitigate the estimation error due to the AOPN. If only the current observation is applied, i.e., $2L + 1 = 1$, the resultant phasor by using the Mth-power estimator can be rewritten as

$$\hat{V}_M(k) = F(|r(l)|)y_M(k). \tag{35}$$

Since the nonlinear coefficient $F(|r(l)|)$ is commonly considered as a real number, the estimated phase after argument extraction is given by

$$\hat{\theta}(k) = \frac{1}{M}\angle\left[\hat{V}_M(k)\right] = \frac{1}{M}\angle\left[e^{jM[\theta(k)+\epsilon(k)]}\right]. \tag{36}$$

From (36), we can see that with $L = 0$, the AOPN term can not be averaged out during the estimation.

Considering a specific case, where the immediate past and current observation is used to estimate the current phase $\theta(k)$, the LMMSE Mth-power phasor estimate can be written as

$$\hat{V}_M(k) = \sum_{l=k-1}^{k} \mathbf{w}^H \mathbf{y_M}(k). \tag{37}$$

Assuming a constant carrier phase, i.e., $\Delta\nu = 0$, the optimal weight coefficient can be easily obtained as

$$\mathbf{w} = \frac{1}{2|r(k)|^2 + 2|r(k-1)|^2}\begin{bmatrix}|r(k-1)|^2\\ |r(k)|^2\end{bmatrix}. \tag{38}$$

The detailed derivation is given in Appendix C. Substituting (38) into (37), we have

$$\hat{V}_M(k) = \frac{1}{2|r(k)|^2+2|r(k-1)|^2}\sum_{l=k-1}^{k}|r(l)|^2 y_M(l) \tag{39}$$

which agrees with the suggestion of $F(|r(l)|) = |r(l)|^2$ given in [1], [6]. In other words, we can consider the VV estimator with the suggested nonlinearity, $F(|r(l)|) = |r(l)|^2$, as a special case of the LMMSE estimator when only AWGN is present. The proposed optimal Mth-power estimator, in contrast, takes both the laser phase noise and the AWGN into consideration.

V. POST-PROCESSING PHASE UNWRAPPING

The phase estimate $\hat{\theta}(k)$ obtained from the carrier phase estimator is applied to de-rotate the received signal and remove the unknown carrier phase. Referring to (4), the receiver forms the de-rotated signal

$$\begin{aligned} r'(k) &= r(k)\exp[-j\hat{\theta}(k)] \\ &= |r(k)|\exp[j(\phi(k) + (\theta(k) - \hat{\theta}(k)) + \epsilon(k))]. \end{aligned} \tag{40}$$

The argument $\angle r'(k) = \phi(k) + (\theta(k) - \hat{\theta}(k)) + \epsilon(k)$ is used to determine the information phase $\phi(k)$ by choosing the value in the set $\{i2\pi/M, i = 0, 1, \ldots, M-1\}$ that is closet to $\angle r'(k)$. Because of the Mth-power operation involved in the phase estimation, it is well known [1] that there is a $2\pi/M$ ambiguity in the phase estimate $\hat{\theta}(k)$, i.e., ambiguity as to which quadrant $\hat{\theta}(k)$ should lie in for the case of QPSK where $M = 4$, for instance. We thus have to consider the resolution of this ambiguity in relation to the issue of unwrapping the phase estimate from the Mth-power estimator. Phase unwrapping is essential because in raising the received sample $r(k)$ to the Mth power, its argument $\angle r(k)$ is multiplied M times, and is thus wrapped M times around the circle. While applying the Mth-power carrier phase estimators, the phase estimate at kth time interval $\hat{\theta}(k)$ is obtained from the principal argument of the estimated phasor. For instance, with LMMSE estimator, the principal argument of $\hat{V}_M(k)$, denoted as

$$\angle\hat{V}_M(k) = \arg[\hat{V}_M(k)], \tag{41}$$

is considered as the wrapped estimate of $M\theta(k)$. Because phase angles are measured modulo-2π, i.e., an angle α is the same as $(\alpha \pm 2m\pi)$ for any integer m, it is clear that dividing $\angle\hat{V}_M(k)$ by M to obtain the unwrapped estimation of $\theta(k)$ will result in M possible values within the principal interval $[-\pi, +\pi)$, with each value being separated from its nearest neighbors by an angle of $2\pi/M$. After argument extraction, the post-processing phase ambiguity resolution is commonly ignored in the literature. This is because, with constant carrier phase, the difference between the wrapped estimate $\angle\hat{V}_M(k)$ and the actual phase $M\theta(k)$ is relatively constant in time, based on our observations during the simulation. In this case, through applying differential data encoding and decoding, the unwrapped phase estimate can be used directly in data decoding without ambiguity resolution [1]. However, when taking the phase noise into consideration, the ambiguity resolution after the phase unwrapping of $\angle\hat{V}_M(k)$ is indispensable.

Here, we focus first on evaluating the performance improvement of the LMMSE estimator compared with the VV and the conventional Mth-power estimators. Since the potential performance impairment due to the ambiguity resolution errors might disturb the performance comparison, we first consider genie-aided phase unwrapping, where the actual phase at each time k, $\theta(k)$, is given by a genie. Assuming the phase estimation is relatively accurate, the actual phase estimate $\hat{\theta}(k)$ is chosen such that $M\hat{\theta}(k)$ lies in the 2π-interval centered at $M\theta(k)$, i.e., $M\theta(k) - \pi \leq M\hat{\theta}(k) \leq M\theta(k) + \pi$. This is done by adding multiples of $\pm 2\pi$ to

TABLE 1. Genie-aided phase unwrapping.

$$
\begin{array}{l}
\textbf{Initialize} \text{ with } \angle\hat{V}_M(k) = \arg[\hat{V}_M(k)];\\
\textbf{while } \angle\hat{V}_M(k) - M\theta(k) \geq \pi\\
\quad \angle\hat{V}_M(k) = \angle\hat{V}_M(k) - 2\pi\\
\textbf{end}\\
\textbf{while } \angle\hat{V}_M(k) - M\theta(k) \leq -\pi\\
\quad \angle\hat{V}_M(k) = \angle\hat{V}_M(k) + 2\pi\\
\textbf{end}\\
\hat{\theta}(k) = \frac{1}{M}\angle\hat{V}_M(k)
\end{array}
$$

TABLE 2. Differential phase unwrapping.

$$
\begin{array}{l}
\textbf{Initialize} \text{ with } \angle\hat{V}_M(k) = \arg[\hat{V}_M(k)];\\
\textbf{while } \angle\hat{V}_M(k) - M\hat{\theta}(k-1) \geq \pi\\
\quad \angle\hat{V}_M(k) = \angle\hat{V}_M(k) - 2\pi\\
\textbf{end}\\
\textbf{while } \angle\hat{V}_M(k) - M\hat{\theta}(k-1) \leq -\pi\\
\quad \angle\hat{V}_M(k) = \angle\hat{V}_M(k) + 2\pi\\
\textbf{end}\\
\hat{\theta}(k) = \frac{1}{M}\angle\hat{V}_M(k)
\end{array}
$$

the wrapped estimate $\angle\hat{V}_M(k)$, when the absolute difference between $M\theta(k)$ and $\angle\hat{V}_M(k)$ is greater than π as shown in Table 1. Again, this is because angles are measured modulo-2π. The ambiguity-resolved phase estimate of the kth symbol is now obtained through the unwrapping operation as

$$\hat{\theta}(k) = \frac{1}{M}\angle\hat{V}_M(k). \tag{42}$$

In practice, since the actual phase is unknown, the genie-aided phase unwrapping is unavailable. Considering the practical implementation, the LMMSE estimator with the differential phase unwrapping proposed in [7] is introduced and evaluated in this paper. With the differential method, assuming a slowly varying carrier phase, the phase unwrapping of current symbol is conducted based on the immediate past estimate. As illustrated in Table 2, similarly, the principal argument of $\hat{V}_M(k)$ is taken as the wrapped estimate. Denoting the phase estimate of the $(k-1)$th symbol after phase unwrapping as $\hat{\theta}(k-1)$, the absolute difference between $M\hat{\theta}(k-1)$ and $M\hat{\theta}(k)$ is expected to be smaller than π due to the slowly varying carrier phase. Hence, the actual phase estimate at time k, $\hat{\theta}(k)$, is chosen such that $M\hat{\theta}(k)$ is limited to within the 2π-range centered at $M\hat{\theta}(k-1)$. After phase ambiguity resolution through adding multiples of $\pm 2\pi$ to $\angle\hat{V}_M(k)$, the actual phase estimate at each time k, $\hat{\theta}(k)$, is obtained as given in (42). However, while using this differential method, an occasional phase unwrapping failure due to an error in ambiguity resolution is inevitable particularly at low SNR. To address this problem, during the simulation, the phase unwrapping is restabilized periodically based on the inserted pilots. Detailed explanation is provided later in this paper.

VI. NUMERICAL PERFORMANCE RESULTS

Monte Carlo simulations are performed to investigate the phase estimation performance of the proposed LMMSE Mth-power estimator. For comparison, the suggested VV estimator and the conventional Mth-power estimator are simulated. During the simulation, the inverse mean square error (IMSE) of the carrier phase estimate, defined as

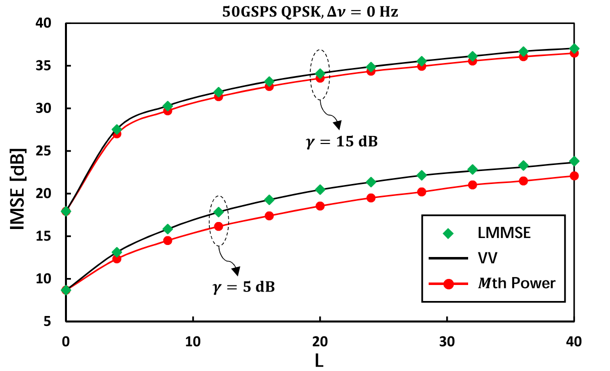

FIGURE 2. IMSE performance investigation of various phase estimators with constant carrier phase as a function of window size *L* (GSPS: Giga samples per second).

$$\text{IMSE} = -10\log_{10}\left[\frac{\sum_{k=1}^{q}[\theta(k)-\hat{\theta}(k)]^2}{q}\right], \tag{43}$$

is used as the performance metric, where q denotes the sample size for each calculation of IMSE and where all angles are measured in radians. To investigate the optimal achievable IMSE performance of the proposed LMMSE estimator, the genie-aide phase unwrapping introduced in Table 1 is first considered during the simulation.

A. BLOCK LENGTH EFFECT WITHOUT PHASE NOISE

For both the VV and the proposed LMMSE estimator, the selection of window size L would affect the estimation performance. Here, the block length effect is numerically investigated in QPSK modulated systems with constant carrier phase i.e., no phase noise. In this case, the VV estimator is shown to achieve approximately the same MSE performance compared with our LMMSE estimator. This observation further confirms our interpretation in Section IV that the VV estimator can be considered as a special case of the LMMSE estimator when only AWGN is present.

As shown in Fig. 2, with the increase of the window size L, a continuous performance improvement can be observed for all tested estimators. This is because, without laser phase noise, the AOPN becomes the only noise source during the estimation, which can be efficiently reduced through applying a sufficiently long estimation window. Both the VV and the LMMSE estimator outperform the conventional Mth-power in terms of the MSE performance. At high SNR, since the magnitude of the received symbol $r(l)$ is approximately equal to $\sqrt{E_s}$ most of the time, the performance enhancement through applying the optimal nonlinearity becomes less observable.

B. BLOCK LENGTH EFFECT WITH PHASE NOISE

With laser phase noise, the performance degradation due to the block length effect is further investigated. Since the

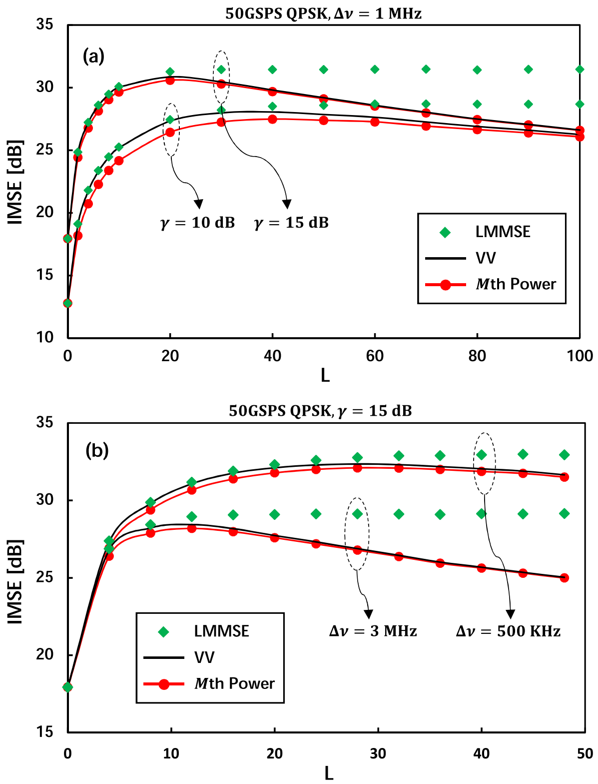

FIGURE 3. IMSE performance investigation of various estimators with different values of (a) SNR and, (b) laser linewidth.

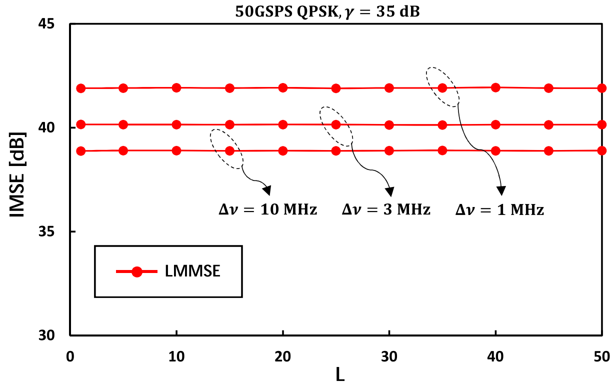

FIGURE 4. IMSE performance investigation of LMMSE estimator as a function of the window size L with different laser linewidth values.

nonlinear weight vector is optimized with the consideration of the Wiener phase noise, the proposed LMMSE estimator is shown to achieve strong tolerance to the block length effect particularly at high SNR and high phase noise.

From the simulation, the estimation window length L needs to be optimized for every value of SNR and laser linewidth while applying the VV and the conventional Mth-power phase estimators. As shown in Fig. 3 (a), due to the presence of phase noise, instead of continuous performance improvement, we can now observe an optimal window size L for both the VV and the conventional Mth-power estimator which leads to the minimum MSE for phase estimation. In addition, from Fig. 3 (a) and Fig. 3 (b), the optimal L varies for different SNR and laser linewidth values. For instance, as SNR increases from 10 to 15 dB, the observed optimal L decreases from 40 to around 20. This is because, at low SNR, the AOPN becomes the dominant noise source, where a long estimation window is preferred to effectively average out the estimation error. On the other hand, with large laser linewidth, the laser phase noise dominates. In this case, a small L is preferred to ensure that the carrier phase is approximately block-wise constant.

While applying the LMMSE phase estimator, similarly, with the increase of the window size L, a faster convergence to the optimal MSE can be observed at higher SNR and lower laser linewidth as shown in Fig. 3. However, since the phase fluctuation within the estimation window is taken into consideration in LMMSE estimation, instead of performance degradation, the measured MSE remains the same after reaching the optimal point. In this case, instead of time-consumingly searching for the optimal estimation window length, the optimal MSE performance can be easily achieved by using the LMMSE estimator with a sufficiently long estimation window.

Additionally, as we demonstrated in Section IV, with sufficiently high SNR, the proposed LMMSE phase estimator suffers from no block length effect. For instance, as shown in Fig. 4, with SNR $\gamma = 35$ dB, the measured IMSE remains constant with arbitrary values of L at a laser linewidth of up to about 10 MHz. This is because, with sufficiently high SNR, accurate phase estimation can be achieved, even if only the current observation is considered.

C. COMPARISON OF LMMSE WITH SIMPLIFIED LMMSE

While implementing the LMMSE estimator, the matrix inverse of the covariance matrix $\mathbf{R}$ as shown in (27) is required for each estimate, which would lead to extensive computational complexity. To facilitate its implementation, a simplified LMMSE estimator, which requires only one matrix inverse operation, was proposed in Section IV based on the assumption of relatively high SNR. To verify its feasibility, the performance of the simplified LMMSE method is presented and compared with that of the exact version in Fig. 5.

As shown in Fig. 5, the IMSE of phase estimation is measured as a function of SNR γ and laser linewidth $\Delta\nu$ with fixed window size of $L = 20$. As shown, with the simplified LMMSE estimator, a performance degradation of around 1 dB can be observed compared with that using the actual LMMSE estimator at SNR of less than around 6 dB and laser linewidth of up to around 10 MHz. With a larger SNR, close convergence to the actual LMMSE estimation can be achieved. Considering the SNR range of practical interest, we can conclude that, although the simplified LMMSE estimator is derived based on the assumption of high SNR, it is a close approximation to the actual LMMSE estimator. Hence, in Subsection D, the simplified LMMSE estimator is considered instead of the actual LMMSE estimator.

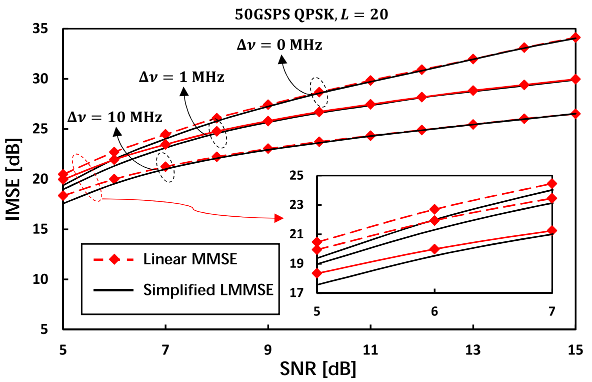

FIGURE 5. Comparison of the simplified LMMSE with the actual LMMSE estimator.

D. COMPARISON OF LMMSE WITH OPTIMIZED VV ESTIMATORS

The optimal MSE performance of the simplified LMMSE estimator, which is obtained through applying a sufficiently long estimation window, is evaluated for different SNR, laser linewidth values and modulation formats. The VV and the conventional Mth-power estimators with optimized window length for each value of SNR and laser linewidth, which is obtained through numerical search, are simulated for comparison.

Numerical results show that the simplified LMMSE estimator outperforms both the VV and the conventional Mth-power estimators in terms of the minimum achievable MSE performance particularly in systems with strong phase noise and large SNR. At constant carrier phase, the VV method achieves approximately the same performance as the simplified LMMSE method for both QPSK and 8PSK modulated systems. This observation verifies our previous conclusion that the VV estimator is a special case of the proposed LMMSE method under the condition of constant carrier phase. With laser phase noise, the simplified LMMSE estimator is shown to outperform both the VV and the conventional Mth-power method with the performance improvement increasing with the laser linewidth, particularly at high SNR. For example, compared with the VV method, approximately 3 dB performance enhancement can be obtained by using the LMMSE method within a wide range of SNR at the laser linewidth of 10 MHz. As shown in Fig. 6, compared with the phase estimation of QPSK, significant performance degradation can be observed in 8PSK modulated system at low SNR. This is because, with large M, the phase fluctuation due to the AOPN is significantly enlarged by the Mth power operation which strongly affects the phase estimation.

E. PHASE UNWRAPPING PERFORMANCE

As mentioned above, genie-aided phase unwrapping is unavailable in practice since the prior knowledge of the actual phase is required. To verify the feasibility of the proposed simplified LMMSE estimator in practice, its MSE performance with differential phase unwrapping is numerically evaluated.

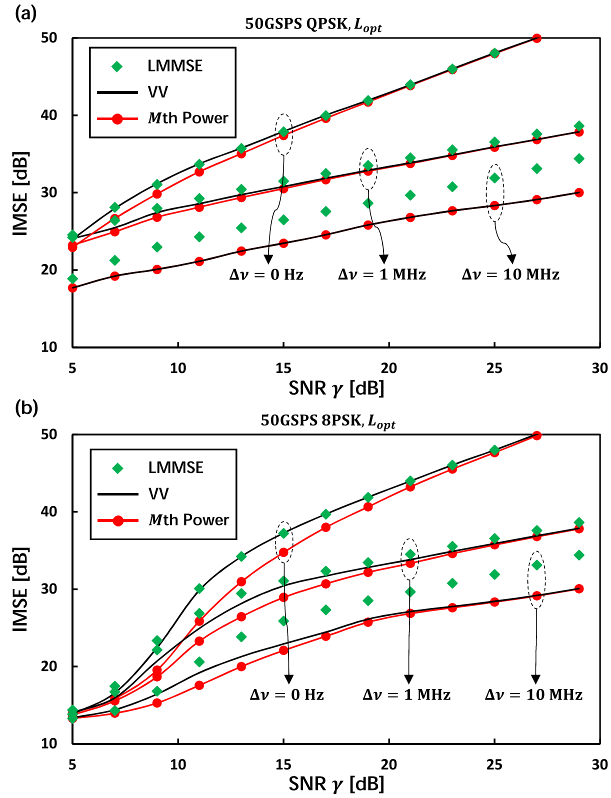

FIGURE 6. IMSE performance investigation as a function of SNR γ and laser linewidth $\Delta\nu$ for (a) QPSK, and (b) 8PSK modulated systems, with the optimized window length L_{opt} obtained by numerical search.

While using the differential method, since the phase estimate of the immediate past symbol instead of the actual carrier phase of the current symbol is used for the phase unwrapping of the current symbol, occasional phase unwrapping failure is inevitable particularly at low SNR. To overcome this problem, during the simulation, a single pilot is inserted after every D data symbols, as shown in Fig. 7 (a), to restabilize the phase unwrapping process. Suppose the kth transmitted symbol $m(k)$ as a pilot, i.e., $m(k)$ is known at the receiver side. In this case, the received noisy carrier phase of the kth symbol, denoted as $\zeta(k) = \theta(k) + \epsilon(k)$, can be easily obtained from the signal model given in (4). Due to the potential phase unwrapping error, the phase estimate at time k, $\hat{\theta}(k)$, might be unreliable. To cut off the accumulation of the phase unwrapping error, at $(k+1)$th time interval, the phase unwrapping is conducted based on the noisy carrier phase of the kth symbol $\zeta(k)$, instead of the phase estimate $\hat{\theta}(k)$.

Due to the presence of AOPN term $\epsilon(k)$ inside the noisy carrier phase $\zeta(k)$, the single-pilot insertion becomes less effective particularly at low SNR. For further improvement, instead of single-pilot insertion, a pilot window with the size of P is inserted at each refreshing interval as shown in Fig. 7 (b). In this case, the AOPN inside the noisy carrier phase can be effectively eliminated through applying the window average. For example, suppose that the ith symbol is the first data symbol after current refreshing interval. With the pilot window, the phase unwrapping at ith time interval is

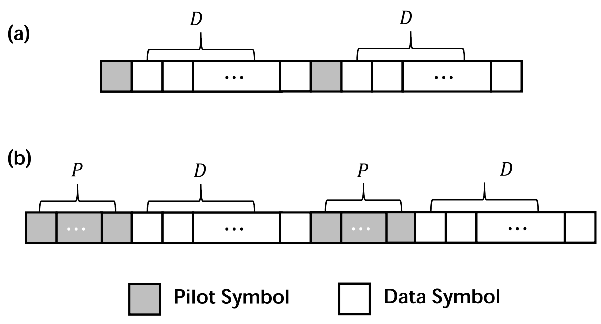

FIGURE 7. Frame structure of the pilot insertion.

conducted based on the noisy phase ξ, which is given as

$$\xi = \frac{1}{P} \sum_{l=i-P}^{i-1} \zeta(l). \tag{44}$$

From Fig. 8, taking the single pilot case as an example, performance impairment due to the use of differential phase unwrapping compared with the genie-aided case can be observed particularly in systems with low SNR and large laser linewidth. However, with the increase of SNR, gradual convergence to the genie-aided case can be observed. This is because, with low SNR and large laser linewidth, the estimated carrier phase would strongly fluctuate due to the large AOPN and phase noise after Mth-power operation, which conflicts with the assumption of slowly varying carrier phase. At high SNR, since the phase estimation becomes more reliable, differential phase unwrapping starts to converge to the genie-aided case. Additionally, with larger estimation window length, a closer convergence can be observed. This is because, the estimation error due to the AOPN can be effectively averaged out with a long estimation window.

Additionally, the performance improvement due to the use of a pilot window compared to using the single-pilot insertion is verified as shown in Fig. 8 (b). With a long pilot window of $P = 10$, the AOPN inside the noisy phase can be effectively averaged out, thereby leading to better phase unwrapping performance particularly at low SNR. However, the presence of laser linewidth would require limiting the window length to ensure that the noisy phase ξ after window averaging remains correlated with the carrier phase of the data symbols. Therefore, we would expect an optimal pilot size P for each value of SNR and laser linewidth. The detailed investigation into the optimal pilot window size will be reported later.

The frequency of the pilot insertion, i.e., the chosen value of D, would affect the phase unwrapping performance as well. Considering the single pilot insertion as an example, as shown in Fig. 8 (c), closer convergence to the genie-aided phase unwrapping can be achieved with a smaller D. This is because, with more frequent pilot insertion, the potential phase unwrapping error can be corrected more frequently, thereby generating better performance.

Similar to the LMMSE estimator, the VV method requires as well the post-processing phase unwrapping process, which

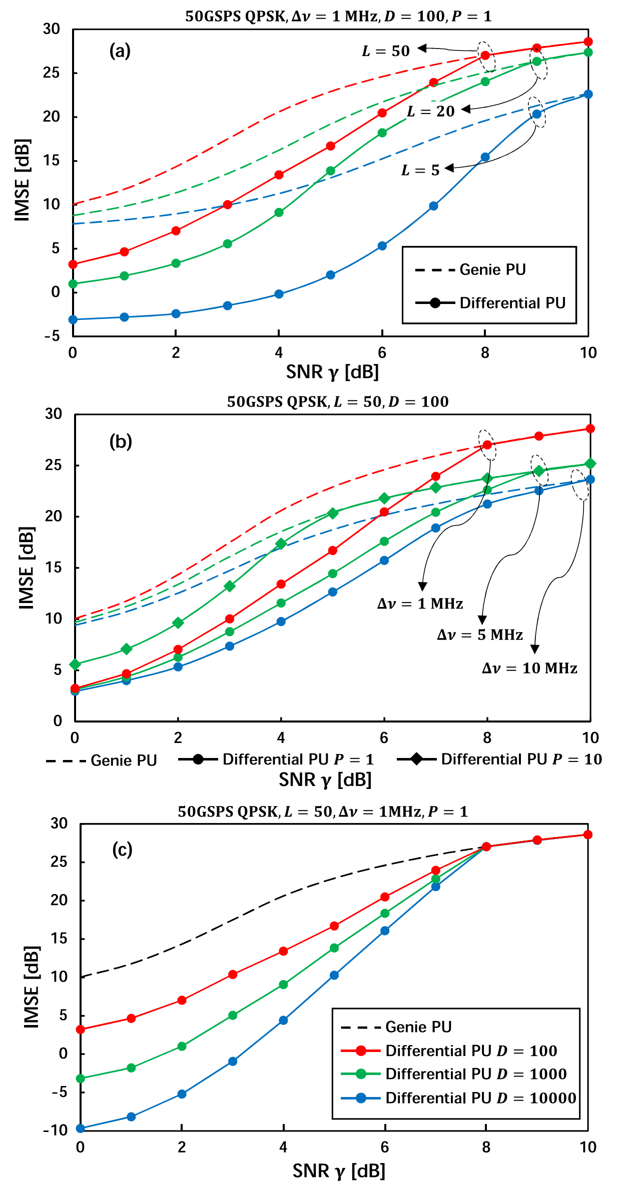

FIGURE 8. IMSE performance investigation of the simplified LMMSE estimator using various phase unwrapping methods with different values of (a) estimation window size *L*, (b) laser linewidth $\Delta\nu$, and (c) message symbols of each frame *D* (PU: phase unwrapping).

is, however, commonly ignored in the literature. Here, performance degradation due to the potential phase unwrapping error when applying the VV phase estimator is further investigated and compared with the proposed LMMSE method. From Fig. 9, similar performance impairment due to the use of differential phase unwrapping method can be observed in the VV estimator. Additionally, with both genie-aided and differential phase unwrapping, our proposed LMMSE estimator outperforms the VV method within the entire range of SNR tested, i.e., $\gamma \in [0, 10]$ dB. This observation verifies our above conclusion that despite the high-SNR assumption considered during the development, the propose LMMSE method remains feasible at low SNR. The simplified LMMSE estimator is, however, less effective compared with

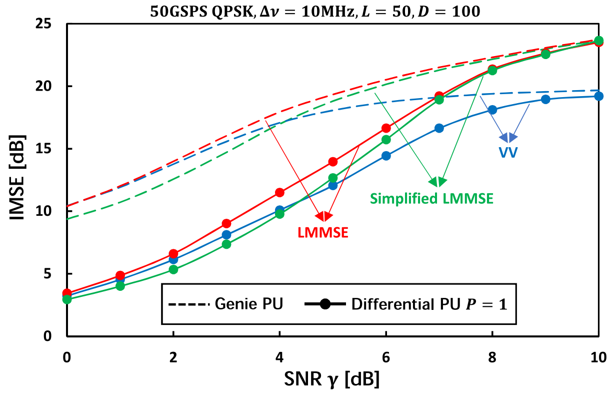

FIGURE 9. Performance comparison between different phase estimators when applying differential phase unwrapping.

the VV method at SNR of less than 4 dB in the case of both genie-aided and differential phase unwrapping. With larger SNR, since the phase fluctuation is taken into consideration, the simplified LMMSE method is shown to outperform the VV method and gradually converges to the actual LMMSE method, as we expected. Hence, considering the SNR range of practical interest, we would suggest that the simplified LMMSE method with differential phase unwrapping is a promising candidate in practice.

VII. CONCLUSIONS

The use of the Mth-power nonlinearity for non-decision-aided carrier phase estimation of MPSK signals is popular in practice, because of the low latency involved. The use of the single complex exponential expression for the complex, noisy received signal sample has proven to be effective in the design and analysis of such Mth-power carrier phase estimators. Using this approach, we have derived the LMMSE Mth-power estimator that is optimized with respect to both the channel AOPN and the carrier phase noise, and studied its performance using simulations. Our work is of significance in extending the conventional Mth-power phase estimator to include the consideration of phase fluctuation. Additionally, the single-exponential signal model provides a foundation for further research on carrier phase estimation.

We should point out that the design of a complete coherent receiver should also consider the issue of timing and frequency synchronization. However, as we are refining and extending on the work of [1], we adopt the assumption in [1] that symbol timing and carrier frequency can be tracked accurately between packets and thus these latter parameters needs only to be estimated once in the initial packet, while phase estimation on each successive packet is a requirement due to oscillator phase noise. To avoid making this paper too complicated, we will consider timing and frequency synchronization in a future report. Another relevant issue is that of amplitude estimation for automatic gain control. However, since the Mth-power carrier phase estimator is applicable only for MPSK modulations and the detection of the latter symbols requires only accurate carrier phase information and does not require amplitude information, we will not deal with the amplitude estimation problem here. Accurate amplitude estimation would be essential for detecting APSK, and therefore will become relevant in designing a carrier phase estimator for the latter case. Finally, we note that the variance of the AWGN and the carrier phase noise are assumed known in the design of the LMMSE estimator. In practice, if these variances are unknown, an adaptive version of this estimator would be necessary that can adapt its weights in response to real-time measurements from the received signal samples. In [19], we have developed such an adaptive version of our DA-ML estimator of [2]. The adaptive version of the LMMSE estimator here is currently a topic of our investigation.

APPENDIX A
DERIVATION OF THE OPTIMAL WEIGHT VECTOR

The optimal weight coefficient $\mathbf{w}_o$ can be obtained through minimizing the MSE given in (24). Note that the MSE given in (24) is a real-valued scalar function of the complex vector $\mathbf{w}$ and $\mathbf{w}^*$. Based on the theorem given in [20, Theorem 2], for a real-valued function of the complex vectors $\mathbf{b}$ and $\mathbf{b}^*$, $f(\mathbf{b}, \mathbf{b}^*)$, the vector pointing in the direction of the maximum rate of change of $f(\mathbf{b}, \mathbf{b}^*)$ is $\partial f(\mathbf{b}, \mathbf{b}^*)/\partial \mathbf{b}^*$, which is the partial derivative of $f(\mathbf{b}, \mathbf{b}^*)$ with respective of $\mathbf{b}^*$. Hence, the optimal $\mathbf{w}$ can be obtained as

$$\mathbf{w}_o = \arg\min E[|e(k)|^2] = \arg\left[\frac{\partial E[|e(k)^2|]}{\partial \mathbf{w}^*} = 0\right]. \tag{A.1}$$

Substituting (A.1) into (24), we can easily find that

$$\mathbf{w}_o = \mathbf{R}^{-1}\mathbf{p}. \tag{A.2}$$

The explicit expression of $\mathbf{p}$ and $\mathbf{R}$ is given as follow. From (25), the lth element of $\mathbf{p}$ can be expressed as

$$\begin{aligned} p_l &= E[y_M(k-L+l-1)V_M^*(k)] \\ &= E\left[e^{jM[\theta(k-L+l-1)+\epsilon(k-L+l-1)]}e^{-jM\theta(k)}\right]. \end{aligned} \tag{A.3}$$

Considering the Wiener process model of the carrier phase $\theta(k)$, we have

$$\begin{aligned} p_l &= E\left[e^{jM[\theta(k)-\sum_{i=k-L+l}^{k}\nu(i)+\epsilon(k-L+l-1)]}e^{-jM\theta(k)}\right] \\ &= E\left[e^{jM[-\sum_{i=k-L+l}^{k}\nu(i)+\epsilon(k-L+l-1)]}\right] = E[e^{j\alpha(l)}], \end{aligned} \tag{A.4}$$

where $\alpha(l) = M[-\sum_{i=k-L+l}^{k}\nu(i) + \epsilon(k-L+l-1)]$. Since the AOPN term $\epsilon(l)$ is shown to be approximately Gaussian distributed, we can conclude that $\alpha(l)$ is also Gaussian distributed with zero mean and variance of

$$\mathrm{var}[\alpha(l)] = \sigma_\alpha^2(l) = M^2(|L-l+1|\sigma_p^2 + \sigma_\epsilon^2(k-L+l-1)). \tag{A.5}$$

Considering the moment-generating function of a Gaussian random variable, $\alpha(l)$, (A.4) can be rewritten as [21]

$$\begin{aligned} p_l &= E[e^{j\alpha(l)}] = e^{1jE[\alpha(l)]+\frac{1}{2}(1j)^2\sigma_\alpha^2(l)} \\ &= e^{-\frac{1}{2}\left[M^2\left(|L-l+1|\sigma_p^2+\sigma_\epsilon^2(k-L+l-1)\right)\right]} \end{aligned} \tag{A.6}$$

The (x, y)th element of $\mathbf{R}$ can be calculated in a similar way. If $x = y$, obviously, $R(x, x) = 1$. If $x \neq y$, we have

$$\begin{aligned} R(x, y) &= E[y_M(k-L+x-1)y_M^*(k-L+y-1)] \\ &= E[e^{jM[\theta(k-L+x-1)+\epsilon(k-L+x-1)]} \\ &\quad \times e^{-jM[\theta(k-L+y-1)+\epsilon(k-L+y-1)]}] \\ &= E[e^{jM\theta(k)}e^{jM[-\sum_{i=k-L+x}^{k}\nu(i)+\epsilon(k-L+x-1)]} \\ &\quad \times e^{-jM\theta(k)}e^{-jM[-\sum_{q=k-L+y}^{k}\nu(q)+\epsilon(k-L+y-1)]}] \\ &= E[e^{jM\beta(x,y)}], \end{aligned} \tag{A.7}$$

where $\beta(x, y) = M[\sum_{q=k-L+y}^{k}\nu(q) - \sum_{i=k-L+x}^{k}\nu(i) + \epsilon(k-L+x-1) - \epsilon(k-L+y-1)]$. Similarly, $\beta(x, y)$ is Gaussian distributed with zero mean and variance

$$\begin{aligned} \sigma_\beta^2(x, y) = M^2\big[&(|x-y|)\sigma_p^2 \\ &+\sigma_\epsilon^2(k-L+x-1) + \sigma_\epsilon^2(k-L+y-1)\big]. \end{aligned} \tag{A.8}$$

Hence, based on the moment-generating function of $\beta(x, y)$, $R(x, y)$ can be reduced to

$$R(x, y) = e^{-\frac{1}{2}M^2\left[(|x-y|)\sigma_p^2+\sigma_\epsilon^2(k-L+x-1)+\sigma_\epsilon^2(k-L+y-1)\right]} \tag{A.9}$$

Substituting (A.6) and (A.9) into (A.2), we can easily calculate the optimal weight $\mathbf{w}_o$ for each time k. Obviously, $\mathbf{w}_o$ given in (A.2) is a function of the signal amplitudes within the estimation window, $\{|r(l)|\}_l$. Therefore, the components of $\mathbf{w}_o$ given in (A.2) are the optimal nonlinear functions that minimize the MSE of the Mth-power carrier phase estimation.

APPENDIX B
DERIVATION OF EQUATION (34)

As shown in (33), the minimum MSE by using the LMMSE estimator is given as

$$E[|e(k)|^2]_{min} = 1 - \mathbf{p}^T\mathbf{R}^{-1}\mathbf{p}. \tag{B.1}$$

Denoting the ith column of the simplified covariance matrix $\mathbf{R}$ given in (31) as $\mathbf{r}(i)$, from (30), we have

$$\mathbf{p} = a \times \mathbf{r}(L+1) + (1/a - a) \times \mathbf{b} \tag{B.2}$$

where $a = e^{\frac{1}{2}M^2\sigma_\epsilon^2}$ and $\mathbf{b}$ denotes the $(2L+1)$-dimensional vector $[0, \cdots, 0, 1, 0, \cdots, 0]^T$ with a "1" in the middle. Substituting (B.2) into (B.1), we have

$$\begin{aligned} E[|e(k)|^2]_{min} &= 1 - [a\mathbf{r}(L+1) + (1/a-a)\mathbf{b}]^T\,\mathbf{R}^{-1} \\ &\quad \times [a\mathbf{r}(L+1) + (1/a-a)\mathbf{b}] \\ &= a^2 - 1 - (1/a-a)^2 g(L+1, L+1), \end{aligned} \tag{B.3}$$

where $g(L+1, L+1)$ denotes the $(L+1, L+1)$th element of $\mathbf{G} = \mathbf{R}^{-1}$. Denoting $e^{-\frac{1}{2}M^2\sigma_p^2}$ by c, the simplified matrix $\mathbf{R}$ given in (31) can be rewritten as

$$\mathbf{R} = \begin{bmatrix} 1 & c/a^2 & c^2/a^2 & \cdots & c^{2L}/a^2 \\ c/a^2 & 1 & c/a^2 & \cdots & c^{2L-1}/a^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ c^{2L}/a^2 & c^{2L-1}/a^2 & c^{2L-2}/a^2 & \cdots & 1 \end{bmatrix}. \tag{B.4}$$

Assuming relatively high SNR, we have $a^2 = e^{M^2\sigma_\epsilon^2} \approx 1$. In this case, the matrix $\mathbf{R}$ given in (B.4) can be approximated as

$$\mathbf{R} \approx \frac{1}{a^2}\begin{bmatrix} 1 & c & c^2 & \cdots & c^{2L} \\ c & 1 & c & \cdots & c^{2L-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ c^{2L} & c^{2L-1} & c^{2L-2} & \cdots & 1 \end{bmatrix}. \tag{B.5}$$

Assuming non-zero carrier phase noise, i.e., $\sigma_p^2 > 0$, the inverse of the matrix $\mathbf{R}$ given in (B.5) can be easily calculated as (B.6), as shown at the top of the next page, for $L \geq 1$. Hence, the $(L+1, L+1)$th element of $\mathbf{R}^{-1}$ is

$$g(L+1, L+1) = \frac{-a^2(c^2+1)}{c^2-1}. \tag{B.7}$$

Substituting (B.7) into (B.3), the minimum MSE of the phasor estimation by using the LMMSE estimator can be obtained as

$$E[|e(k)|^2]_{min} = e^{M^2\sigma_\epsilon^2} - 1 + \left(e^{M^2\sigma_\epsilon^2} - 1\right)^2 \frac{e^{-M^2\sigma_p^2}+1}{e^{-M^2\sigma_p^2}-1}. \tag{B.8}$$

APPENDIX C
DERIVATION OF EQUATION (38)

With the immediate past and the current received signal, i.e., $\mathbf{y_M}(k) = [y_M(k-1), y_M(k)]^T$, the optimal weight coefficient for the LMMSE phase estimator is given as

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{p}, \tag{C.1}$$

where the cross-correction vector $\mathbf{p}$ can be written as

$$\begin{aligned} \mathbf{p} &= \begin{bmatrix} E[e^{jM[\theta(k-1)+\epsilon(k-1)]}e^{-jM\theta(k)}] \\ E[e^{jM[\theta(k)+\epsilon(k)]}e^{-jM\theta(k)}] \end{bmatrix} \\ &= \begin{bmatrix} e^{-\frac{M^2}{2}[\sigma_p^2+\sigma_\epsilon^2(k-1)]} \\ e^{-\frac{M^2}{2}\sigma_\epsilon^2(k)} \end{bmatrix} \end{aligned} \tag{C.2}$$

Similarly, the auto-correlation matrix $\mathbf{R}$ can be calculated as

$$\begin{aligned} \mathbf{R} &= \begin{bmatrix} 1 & E[e^{jM[\nu(k)+\epsilon(k-1)+\epsilon(k)]}] \\ E[e^{jM[\nu(k)+\epsilon(k-1)+\epsilon(k)]}] & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & e^{-\frac{M^2}{2}[\sigma_p^2+\sigma_\epsilon^2(k-1)+\sigma_\epsilon^2(k)]} \\ e^{-\frac{M^2}{2}[\sigma_p^2+\sigma_\epsilon^2(k-1)+\sigma_\epsilon^2(k)]} & 1 \end{bmatrix} \end{aligned} \tag{C.3}$$

$$\mathbf{R}^{-1} = a^2 \begin{bmatrix} \frac{-1}{c^2-1} & \frac{c}{c^2-1} & 0 & 0 & \cdots & 0 \\ \frac{c}{c^2-1} & \frac{-(c^2+1)}{c^2-1} & \frac{c}{c^2-1} & 0 & \cdots & 0 \\ 0 & \frac{c}{c^2-1} & \frac{-(c^2+1)}{c^2-1} & \frac{c}{c^2-1} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & \frac{c}{c^2-1} & \frac{-1}{c^2-1} \end{bmatrix} \tag{B.6}$$

Considering the matrix inverse of $\mathbf{R}$ given in (C.3), we have

$$\mathbf{R}^{-1} = \frac{1}{1-dgf}\begin{bmatrix} 1 & -dgf \\ -dgf & 1 \end{bmatrix} \tag{C.4}$$

where

$$d = e^{-\frac{M^2}{2}\sigma_p^2}, \quad g = e^{-\frac{M^2}{2}\sigma_\epsilon^2(k-1)}, \ f = e^{-\frac{M^2}{2}\sigma_\epsilon^2(k)}. \tag{C.5}$$

Substituting (C.4) and (C.2) into (C.1), we can easily calculate the weight vector $\mathbf{w}$ as

$$\begin{aligned}\mathbf{w} &= \frac{1}{1-dgf}\begin{bmatrix} 1 & -dgf \\ -dgf & 1 \end{bmatrix} \times \begin{bmatrix} dg \\ df \end{bmatrix} \\ &= \frac{1}{1-dgf}\begin{bmatrix} dg - d^2gf^2 \\ df - d^2g^2f \end{bmatrix}\end{aligned} \tag{C.6}$$

Through applying the approximation: $e^{jx} \approx 1+jx$, the weight vector given in (C.6) can be reduced to

$$\mathbf{w} = h\begin{bmatrix} \sigma_p^2|r(k)|^2|r(k-1)|^2 + N_0|r(k-1)|^2 \\ \sigma_p^2|r(k)|^2|r(k-1)|^2 + N_0|r(k)|^2 \end{bmatrix}, \tag{C.7}$$

where

$$h = \frac{1}{\sigma_p^2|r(k)|^2|r(k-1)|^2 + \frac{N_0}{2}|r(k-1)|^2 + \frac{N_0}{2}|r(k)|^2}. \tag{C.8}$$

Assuming constant phase, i.e., $\sigma_p^2 = 0$, $\mathbf{w}$ can be further simplified as

$$\mathbf{w} = \frac{1}{2|r(k)|^2 + 2|r(k-1)|^2}\begin{bmatrix} |r(k-1)|^2 \\ |r(k)|^2 \end{bmatrix}. \tag{C.9}$$

Substituting (C.9) into (37), the estimated phasor based on the immediate past and current observation is given as

$$\hat{V}_M(k) = \frac{1}{2|r(k)|^2 + 2|r(k-1)|^2}\sum_{l=k-1}^{k}|r(l)|^2 y_M(l). \tag{C.10}$$

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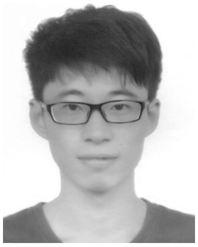

YAN LI was born in Fuyang, Anhui province, China, in 1994. He received the B.Eng. degree from Nanjing University, Nanjing, China, in 2014, and the Ph.D. degree from the National University of Singapore, Singapore, supervised by Prof. P.-Y. Kam and Prof. C. Yu. He is currently a Research Assistant with the Hong Kong Polytechnic University. His research interests include optical fiber communications, carrier recovery, and stochastic processes and algorithms.

MING-WEI WU received the B.E. (first class Hons.), M.E., and Ph.D. degrees in electrical engineering from the National University of Singapore, Singapore, in 2000, 2003, and 2011, respectively. From 2002 to 2004, she was a Research Engineer with the Institute for Infocomm Research, Singapore, and worked on Ethernet passive optical networks standardization and implementation. In 2004, she joined the School of Information and Electronic Engineering, Zhejiang University of Science and Technology, China, as a Lecturer, and is currently an Associate Professor. Her research interests include wireless communication, detection and estimation theory, and performance analysis. She is a Technical Program Committee Member for international communications conferences including the IEEE ICC, the IEEE VTC, the IEEE Globecom, and the IEEE ICCC. She received the Best Paper Award from the IEEE ICC2011, Kyoto, Japan.

XINWEI DU was born in Da'an, Jilin province, China, in 1992. She received the B.Eng. degree from the Department of Communication and Information Engineering, University of Electronic Science and Technology of China, Chengdu, China, in 2014, and the Ph.D. degree from the Department of Electrical and Computer Engineering, National University of Singapore, supervised by Prof. P.-Y. Kam, and Prof. C. Yu. She is currently a Research Assistant with the Polytechnic University of Hong Kong. Her research interests include carrier recovery for coherent optical OFDM systems, including the chromatic dispersion compensation, the timing synchronization, carrier frequency offset estimation, laser phase noise estimation, and data detection algorithms.

TIANYU SONG (S'13) was born in Wuchang, Heilongjiang province, China, in 1989. He received the B.Eng. degree from Honors School, Harbin Institute of Technology, Harbin, China, in 2011, and the Ph.D. degree from the National University of Singapore, Singapore, in 2016, supervised by Prof. P.-Y. Kam. From 2009 to 2010, he was an Exchange Student with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology, Daejon, South Korea. His research interests include free space optical communications, optimal receiver design, and stochastic processes and algorithms. He received the Best Paper Award from the IEEE/CIC ICCC2015.

POOI-YUEN KAM (F'10) was born in Ipoh, Malaysia. He received the S.B., S.M., and Ph.D. degrees in electrical engineering from the Massachusetts Institute of Technology, Cambridge, MA, USA, in 1972, 1973, and 1976, respectively. From 1976 to 1978, he was a Member of the Technical Staff with the Bell Telephone Laboratories, Holmdel, NJ, USA, where he was engaged in packet network studies. Since 1978, he had been a Professor with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore. He was the Deputy Dean of Engineering and the Vice Dean for Academic Affairs of the Faculty of Engineering, National University of Singapore, from 2000 to 2003. He joined the School of Science and Engineering, Chinese University of Hong Kong, Shenzhen, in 2019, as a Professor. His research interests include the communication sciences and information theory, and their applications to wireless and optical communications. He spent the sabbatical year 1987 to 1988 at the Tokyo Institute of Technology, Tokyo, Japan, under the sponsorship of the Hitachi Scholarship Foundation. In 2006, he was invited to the School of Engineering Science, Simon Fraser University, Burnaby, B.C., Canada, as the David Bensted Fellow. He was appointed as a Distinguished Guest Professor (Global) at the Graduate School of Science and Technology, Keio University, Tokyo, Japan, from 2015 to 2017. In 2017, he was a Visiting Professor with the Shanghai Jiaotong University, and in 2018, he held a Visiting Professorship with the University of Electronic Science and Technology of China, in Chengdu, China.

Dr. Kam is a member of Eta Kappa Nu, Tau Beta Pi, and Sigma Xi. From 2011 to 2017, he was a Senior Editor of the IEEE Wireless Communications Letters. From 1996 to 2011, he was the Editor for Modulation and Detection for Wireless Systems of the IEEE Transactions on Communications. He also served on the Editorial Board for PHYCOM, the *Journal of Physical Communications* (Elsevier), from 2007 to 2012. He was also a Co-Chair of the Communication Theory Symposium of IEEE Globecom 2014. He was elected as Fellow of the IEEE for his contributions to receiver design and performance analysis for wireless communications. He received the Best Paper Award from the IEEE VTC2004-Fall, the IEEE VTC2011-Spring, the IEEE ICC2011, and the IEEE/CIC ICCC2015. From 2016 to 2018, he served on the IEEE Fellow Committee.