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Comparative study on a newly-developed three-dimensional wind turbine wake model

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Abstract

In this paper, an analytical three-dimensional wake model is developed to study the wind turbine wake effect and the model is compared with the existing one-dimensional wake model. With the fast development of both onshore and offshore wind energy, wind turbines are upsizing rapidly and the simple assumption of one-dimensional wake model is not suitable for large wind turbines. The commonly used one-dimensional wake model is impossible to describe the spatial distribution accurately as it assumes that the wind deficit is linear distributed along the downwind distance. While the three-dimensional wake model is an effective way to solve this problem. This wake model assumes the wake deficit as Gaussian shaped and considers wind speed variation along height direction as well. In this study, the wind tunnel measurement validation of the three-dimensional wake model is demonstrated which is used to calculate the wind deficit at a downstream position. Next, the results are compared to the one-dimensional wake model's results. An error analysis and discussions are then conducted based on the comparisons. From this study, the new three-dimensional wake model contributes to the wake distribution study for researchers and a proper wake model can help developing optimized wind farm layouts as well.

Keywords: wake models; three-dimentional wake model; numerical analysis;

1. Introduction

With the fast development of both onshore and offshore wind energy, wind turbines are upsizing rapidly, reflecting in that wind turbine (WT) rotors are much larger than before and the hub heights also increase

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correspondingly. The wake effect, referring the wind speed deficit and the turbulence behind a WT, also becomes more important with it, as the wind losses are even more and the influenced area is expanded as well.

The general way to estimate the wind losses is adopting the wake models. Various analytical wake models have been presented for solving different problems. The one-dimensional (1-D) wake model is absolutely the simplest one, it uses a typical wind speed to represent the speed distribution, so it is definitely the most commonly used wake model so far. Though the rough estimation of 1-D wake model saves the calculation cost, it also decreases the precision and ignores many realistic problems. As is known to all, the wind velocity varies seriously with the height increasing, therefore, in the large and high WT swept area, using just one representative wind speed to describe the whole wind speed distribution is apparently not suitable. In a word, the 1-D wake model does not meet the requirement of today's wind energy industry.

2. One-dimensional and two-dimensional wake models

Jensen wake model (also named as Park model) is a good choice to estimate the wind farm energy losses as for its simplicity as well as the relatively high accuracy. The calculation based on Jensen wake model requires the least computation time compared with the other models [1]. Consequently, to improve wake model, further developing Jensen wake model is the most possible way.

Jensen wake model is based on momentum conservation theory, see Fig. 1 [2]. u_0 is the original wind velocity; r_0 is the radius of WT; x is the distance between upstream WT and downstream WT; and u is incoming wind velocity of downstream WT.

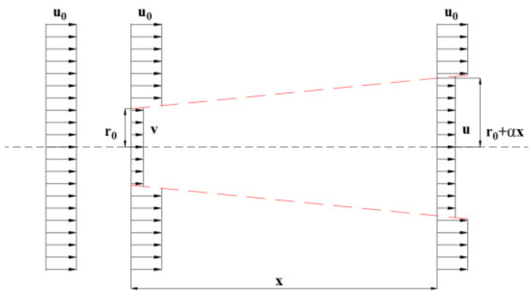

Fig. 1 Jensen wake model

Under this circumstance, a three-dimensional (3-D) analytical wake model has been established. In the 3-D wake model, apart from the downwind distance, both the radial distance and the height distance are involved in the consideration. It has been proved that the 3-D wake model has good predictions of wind deficit in all directions. In this paper, an intensive study about the characteristics of the 3-D wake model will be conducted. The distinctions between 1-D and 3-D wake models will be compared and discussed in depth.

If a WT is under one WT's wake influence, the wind velocity is equation (1), in which α is the rate of the wake expansion, with a value of 0.1 in Jensen's wake model. However, the suggested values of α in the literature are 0.075 for onshore turbines and 0.04 or 0.05 for offshore turbines [3-6]. If a WT is under several WTs' wake influence, a more complicated equation (2) should be adopted.

$$u = u_0 \left[1 - \frac{2ar_0^2}{(r_0 + \alpha x)^2} \right] \tag{1}$$

$$u_i = u_0 \left[1 - \sqrt{\sum_{i=1}^{N} \left(1 - \frac{u}{u_0} \right)^2} \right] \tag{2}$$

Except for the Jensen wake model, there some other two-dimensional (2-D) wake models that are based on it. Bastankhah and Porté-Agel [7] developed 2-D wake model that is Gaussian shaped, see formula (3).

$$\frac{\Delta U}{U_\infty} = \left(1 - \sqrt{1 - \frac{C_T}{8\left(k^* x / d_0 + 0.2\sqrt{\beta}\right)^2}}\right) \cdot \exp\left(-\frac{1}{2\left(k^* x / d_0 + 0.2\sqrt{\beta}\right)^2}\left\{\left(\frac{z - z_h}{d_0}\right)^2 + \left(\frac{y}{d_0}\right)^2\right\}\right) \quad (3)$$

Tian, et al. [8] developed a wake model with the cosine shape, the formula (4) is shown as follows.

$$\begin{cases} u^* = u_0\left[1 - 2a / \left(1 + kx / r_1\right)^2\right] \\ u = \left(u_0 - u^*\right)\cos\left(\pi / r_x \cdot r + \pi\right) + u^* \end{cases} \quad (4)$$

Gao, et al. [9] then developed the Jensen-Gaussian wake model, see formula (5).

$$\begin{cases} u^* = u_0\left[1 - 2a / \left(1 + kx / r_1\right)^2\right] \\ u = u_0 - \left(u_0 - u^*\right)\frac{5.16}{\sqrt{2\pi}} \cdot e^{-r^2/2\cdot\left(r_x/2.58\right)^2} \end{cases} \quad (5)$$

The 2-D wake models are more effective in forecasting the wind losses than 1-D wake model, however, there are still some limitations. Thus the further 3-D wake model seems significant and necessary.

3. Three-dimensional wake model

A novel 3-D wake model is then developed. The distribution of the 3-D wake model is shown in Fig. 2. This wake model is based on the Jensen wake model, so it firstly consider the effect of the downstream distance. Additionally, it also takes the wind speed variation along height into consideration and the wind deficit is uniform distribution. Above all, with this 3-D wake model, it is easy to get the wind speed and wind deficit of every point downstream a wind turbine.

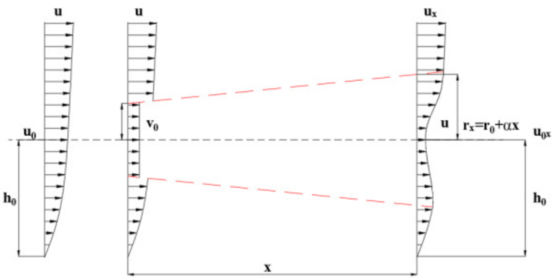

Fig. 2 the 3-D Wake Model

The equation of the wake model is shown as (6):

$$U(x,y,z) = A(x)\left(\frac{1}{2\pi\sigma(x)^2} e^{-\frac{y^2+(z-h_0)^2}{2\sigma(x)^2}}\right) + B(x) + U_0(z) \quad (6)$$

The proposed 3-D wake model has been proved to match well with the wind tunnel data [10]. A vertical profile verification has been shown in Fig. 3. This profile is at the downwind position with the 7 times of rotor diameter.

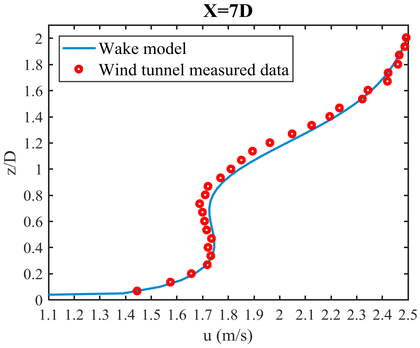

Fig. 3 Comparisons of the measurement data and results predicted by the 3-D wake model at 7D downstream distance

4. Comparisons of three-dimensional wake model and one-dimensional wake model

After the verification of the accuracy of the 3-D wake model, it is worthy to investigate its advancement and its applicable conditions. Therefore, the comparison of the proposed 3-D wake model and the Jensen wake model is demonstrated.

Fig. 4 demonstrates the vertical profiles of wind tunnel measured data, 3-D wake model and Jensen wake model. **Error! Reference source not found.** (a) is the incoming wind distribution, the red circles are the wind tunnel measured data, the blue line is the incoming wind predicted by the 3-D wake model and the pink line is that predicted by the Jensen wake model. The limitation of the Jensen wake model is obvious, it cannot account the wind speed variation at the height direction. While the prediction of the 3-D wake model meets well with the measured data. **Error! Reference source not found.** (b) is the results at $x = 7D$ position. Similar conclusion can be drawn at the 7D downwind distance. Furthermore, the linear characteristic of the Jensen wake model also makes it not proper to estimate the wind deficit.

(a) (b)

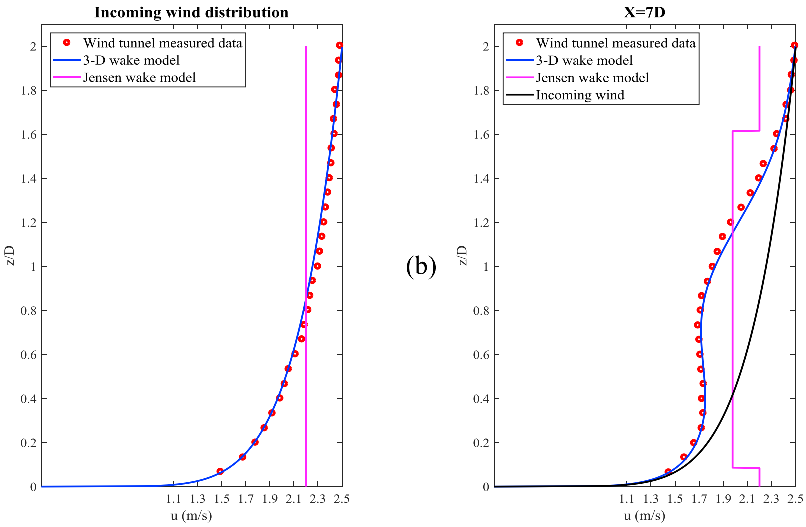

Fig. 4 Vertical profiles of wind tunnel measured data, 3-D wake model and Jensen wake model, (a) incoming wind distribution; (b) at $x = 7D$ position.

Table 1 lists the numerical analysis of the two wake models, in which the predictions are compared to the measured wind speed. A total number of 10 points are selected.

Table 1 Numerical analysis

Point Number	z/D	Measured wind speed/(m/s)	Prediction of 3-D wake model/(m/s)	Relative errors	Prediction of Jensen wake model/(m/s)	Relative errors
1	0.068	1.465	1.461	-0.31%	2.200	50.14%
2	0.266	1.736	1.725	-0.61%	1.977	13.88%
3	0.466	1.655	1.758	6.22%	1.977	19.47%
4	0.669	1.571	1.735	10.43%	1.977	25.85%
5	0.867	1.568	1.769	12.81%	1.977	26.07%
6	1.068	1.725	1.910	10.69%	1.977	14.60%
7	1.268	1.993	2.108	5.77%	1.977	-0.81%
8	1.469	2.302	2.283	-0.81%	1.977	-14.11%
9	1.669	2.418	2.399	-0.81%	2.200	-9.03%
10	1.867	2.486	2.466	-0.78%	2.200	-11.50%

From the table, the largest error of 3-D wake model is 12.81%, while that of Jensen wake model is 50.14%. There are 5 points of 3-D wake model are within 1% error, however, with Jensen wake model, only 1 point is within 1%.

5. Conclusion

In this study, most of the present analytical wake models are reviewed including 1-D and 2-D wake models. The advantages and the limitations of the 1-D Jensen wake model is discussed in detail. A newly developed 3-D wake model is then presented. The accuracy of the 3-D wake model has been verified with existing wind tunnel measurement data. Then the 3-D wake model is compared to the Jensen wake model. It is concluded that the new 3-D wake model has a better predictability of spatial wake effect than the 1-D wake model.

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