Turbo-TCM Performance under AWGN and Rayleigh Fading Channels

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Abstract:　　A simple algorithm for Turbo-TCM decoding was given in this paper. With this algo-rithm, Turbo-TCM can easily be used to real systems with various code rates and modulations of QP-SK，8PSK，16QAM or 64QAM. The bit error ratio performance was studied under AWGN and fad-ing channels. The simulation results were also given in this paper.

Key words:　Turbo codes; TCM; AWGN channel; Rayleigh fading channel

Turbo-TCM　AWGN　Rayleigh　　　　　　　　　　 .　　　　，　　　　，　　　　 .

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　　　　：　　　　　　　　　Turbo-TCM　　　　　　　，　　　　　　　　　　Turbo

　　　　　QPSK, 8PSK, 16QAM　64QAM　　　　　　　　　TCM

　　　　　　Turbo-TCM　AWGN　Rayleigh　　　　　　　　　，

　　　　：Turbo　; TCM; AWGN　　 ; Rayleigh

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Turbo codes[1-3] are one kind of error-correct-ing codes introduced in 1993 by Berrou, *et al*. Its excellent error-correcting performance of only 0. 7dB away from Shannon Limit has drawn plenty of research. But, it was not suitable for bandwidth limited communication systems when proposed. Trellis coded modulation (TCM) and multilevel coding are bandwidth-efficient coding techniques and widely used. Therefore, it is worthwhile to merge TCM and Turbo codes together in order to get large coding gain and high bandwidth efficien-cy. That is called Turbo-TCM (or TTCM). Dif-ferent approaches for Turbo-TCM have been pro-posed in Refs. [4, 5]. The structure in Ref. [5] is not programmatic. In other words, different structures are for different rates $m/m+1$, then different decoders. This does not fit DVB(Digital Video Broadcast) system whose code rate ranges from 1/2 to 7/8, because the decoder will be much more complex. The structure in Ref. [4] is pro-grammatic. It can be used in DVB system. But the decoding algorithm is only suited for 2^{2p} QAM, where p　**N**. In DVB(ETSI)[6] system, the con-volution codes act as the channel codes. And the modulation type will be QPSK, 16QAM or 64QAM except for 8PSK, which the algorithm in Ref. [4] is not suited for. So, in this paper a uni-versal algorithm is given for the Turbo-TCM. And with this algorithm, the performance of Turbo-TCM under the AWGN and Rayleigh fading chan-nels was studied.

1　Structure of Turbo-TCM and Decoding Algorithm

　　Fig. 1 depicts the structure of Turbo TCM used. In Fig. 1, the Turbo codes encoder is a stan-

dard case[1-3]. It has two RSC (Recursive Systematic Convolution) encoders separated by an interleaver. Each RSC produces one parity bit for each information bit. So the code rate is 1/3. After punctured, the desired rate (1/2, 2/3, 3/4, 5/6, 7/8) can be gotten. Though the systematic bits are much more important in Turbo codes, only parity bits are punctured. The parity bits are punctured uniformly. For example, the puncture matrix for rate 3/4 is $\begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{P}_1 \\ \boldsymbol{P}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ and the serial output is $B_1 P_{11} B_2 B_3 B_4 P_{24} B_5 B_6$.
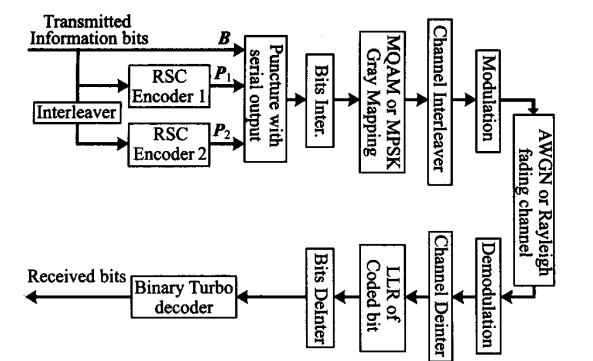

Fig. 1   The structure of Turbo-TCM

     Each q bits of the serial output bits were grouped into one symbol and Gray mapped into one point of the MQAM (or MPSK) constellation, $M = 4, 8, 6, 64$. The Gray mapping constellation of 16QAM is shown in Fig. 2.
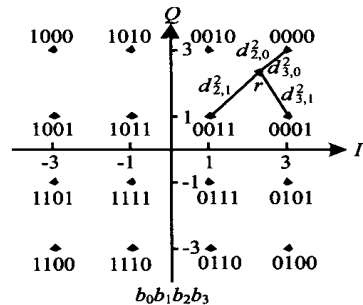

Fig. 2   The Gray mapping constellation and
the demodulation of 16QAM

     A channel interleaver is used in order to scatter burst errors induced by the fading channel.

     After transmitted over a channel, the signal is demodulated at the receiver side. Then various points on the I-Q (In-phase and Quadrature-phase) plane are obtained, for example, the point 'r' in Fig. 2. The LLR (Log likelihood ratio) of each bit of a symbol is then calculated independently in order to make use of the binary Turbo decoder.

     The additive white Gaussian noise is assumed to be zero-mean and σ^2 variance. Let $L(b_i)$ be the LLR of b_i. Then

$$L(b_i) = K \ln \frac{\sum\limits_{s_j \ (b_i=1) \ S} \exp\left(-\frac{1}{2\sigma^2}(r-s_j)^2\right)}{\sum\limits_{s_j \ (b_i=0) \ S} \exp\left(-\frac{1}{2\sigma^2}(r-s_j)^2\right)} =$$

$$K \ln \frac{\sum\limits_{s_j \ (b_i=1) \ S} \exp\left(-\frac{1}{2\sigma^2}d_{i,0,j}^2\right)}{\sum\limits_{s_j \ (b_i=0) \ S} \exp\left(-\frac{1}{2\sigma^2}d_{i,1,j}^2\right)} \quad (1)$$

where S is the set of all the constellation points. K is a constant. In the numerator, only the probabilities of the constellation points whose b_i is 1 are considered. While in the denominator, the points whose b_i is 0 are considered. $d_{i,c,j}^2 = (r-s_j)^2$, $c = 0$ or 1. It can be considered as the distance between the received point and the constellation point. Certainly, the calculation of Eq. (1) is much complex. So in Ref. [4], the author gives an approximation of $L(b_i)$, where it assumes that $q = 2p$, p **N** and separates the q bits (a symbol) into two subgroups based on I and Q coordinates. This does not suit the 8PSK modulation type.

     There is the approximation Eq. (2)[7],

$$\ln(e^{x_1} + e^{x_2}) \quad \max(x_1, x_2) \quad (2)$$

where $\max(x_1, x_2)$ takes the maximum between the x_1 and x_2.

     So $L(b_i)$ can be rewritten as Eq. (3),

$$L(b_i) \quad K[\max_j(-d_{i,1,0}^2, -d_{i,1,1}^2, \ldots,$$
$$-d_{i,1,j}^2, \ldots, -d_{i,1,n-1}^2) -$$
$$\max_j(-d_{i,0,0}^2, -d_{i,0,1}^2, \ldots,$$
$$-d_{i,0,j}^2, \ldots, -d_{i,0,n-1}^2)] =$$
$$K[\min_j(d_{i,0,0}^2, d_{i,0,1}^2, \ldots, d_{i,0,j}^2, \ldots, d_{i,0,n-1}^2) -$$
$$\min_j(d_{i,1,0}^2, d_{i,1,1}^2, \ldots, d_{i,1,j}^2, \ldots, d_{i,1,n-1}^2)] \ (3)$$

where $n = M/2 = 2^{q-1}$, $j = 0, 1, \ldots, n-1$ and $\min_j(x_j)$ takes the minimum in x_j. For example, the LLR of b_2 and b_3 in Fig. 2 can be calculated by

Eq. (4) and Eq. (5),

$$L(b_2) \quad K[d_{20}^2 - d_{21}^2] \quad (4)$$

$$L(b_3) \quad K[d_{30}^2 - d_{31}^2] \quad (5)$$

The punctured bits are set to 0 (null), which means that the bits have the same probability of 1 and 0. Those values and the calculated LLR are fed into the binary Turbo decoder. The algorithm of the Turbo decoder can be selected as MAP, log-MAP, MAX-log-MAP or SOVA[7-9].

2 The Demodulation and Decoding of Turbo-TCM under Rayleigh Fading Channels

2.1 Channel models

Let d_i be the signal input to the channel, and b_i be the received signal. The Rayleigh fading channel introduces two kinds of distortion: multiplicative and additive,

$$b_i = \alpha_i d_i + n_i \quad (6)$$

where n_i is additive white Gaussian noise with zero mean and power spectral density $N_0/2$. Fading attenuation α is a Rayleigh random variable, the multiplicative distortion. By assuming $E(\alpha^2) = 1$, the probability distribution of is

$$p(\alpha) = 2\alpha e^{-\alpha^2}, \quad \alpha \quad 0 \quad (7)$$

The autocorrelation of α is given as $J_0(2\pi B T_s)$, where B is the Doppler frequency and T_s is the symbol interval. And the parameter of BT_s is often used together.

$J_0(x) = \dfrac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos(x \cos\varphi) d\varphi$, is the modified zero-order Bessel function of the first kind.

α is produced by the Jakes model[10] in the simulation.

b_i can be considered as conditional Gaussian distribution.

$$b_i \quad N(\alpha d_i, N_0/2) \quad d_i = -1 \text{ or } 1 \quad (8)$$

y_i has the Gaussian distribution with mean of $\alpha_i d_i$ and the variance of $N_0/2$.

But in the real situation, there are two types depending on knowing α or not: with CSI(channel side information), without CSI.

2.2 Decoding with CSI

When the receiver knows well the channel side information (CSI) α_i for each bit b_i, the α should be considered when calculating the LLR,

$$L(b_i) = K \ln \frac{\sum_{s_i \ (b_i=1)} \ S \ \exp\left\{-\dfrac{1}{2\sigma^2}(r - \alpha_i s_i)^2\right\}}{\sum_{s_j \ (b_i=0)} \ S \ \exp\left\{-\dfrac{1}{2\sigma^2}(r - \alpha_i s_j)^2\right\}}$$

$$(9)$$

So, it can be calculated as Eq. (10) and Eq. (11)

$$d_{ij}^2 = (r - \alpha s_j)^2 \quad j = 0, ..., n-1 \quad (10)$$

Using the approximate way to calculate the LLR of b_i

$$L(b_i) \quad K[\min(d_{i,0,0}^2, d_{i,0,1}^2, ..., d_{i,0,n-1}^2) - \min(d_{i,1,0}^2, d_{i,1,1}^2, ..., d_{i,1,n-1}^2)] \quad (11)$$

$L(b_i)$ is then fed into the standard binary Turbo codes decoder.

2.3 Decoding without CSI

When CSI is unavailable to the receiver, the effect of Rayleigh fading noise will be removed by the expectation of α. As $E(\alpha) = 0.8862$,

$$L(b_i) = K \ln \frac{\sum_{s_i \ (b_i=1)} \ S \ \exp\left\{-\dfrac{1}{2\sigma^2}(r - E(\alpha) s_i)^2\right\}}{\sum_{s_j \ (b_i=0)} \ S \ \exp\left\{-\dfrac{1}{2\sigma^2}(r - E(\alpha) s_j)^2\right\}}$$

$$(12)$$

so,

$$d_{i,c,j}^2 = (r - 0.8862 s_j)^2 \quad j = 0, ..., n-1 \quad (13)$$

3 The Simulation and Results

The simulations were done by Monte Carlo method.

The parameters of Turbo codes are as follows: two identical RSC encoders with generator polynomial (1, 15/13); interleaver length being 1648; Log-MAP as decoding algorithm; 8 iterations.

3.1 Simulation results under AWGN channel

The simulations of several kinds of modulation and the code rates have been done under AWGN channel. And comparing with the convolution TCM used in the DVB standard of ETSI[6] gives Table 1.

Table 1   **Required *C/N* (dB) to achieve a Bit Error Rate= 2× 10⁻⁴ after the inner decoder for all combinations of coding rates and modulation types, the comparison between TCM and Turbo TCM**

TCM TTCM	Rate= 1/2		2/3		3/4		5/6		7/8	
QPSK	3.1	1.3	4.9	3.4	5.9	4.5	6.9	5.6	7.7	8.9
8PSK	—	4.4	—	7.2	—	8.5	—	9.9	—	14
16QAM	8.8	6.6	11.1	9.3	12.5	10.5	13.5	11.9	13.9	15.7
64QAM	14.4	10.9	16.5	14.6	18.0	16	19.3	17.5	20.1	21.8

Through Table 1, it can be seen that Turbo-TCM will gain 1–3dB to convolution TCM at achieving BER of 2× 10⁻⁴ under AWGN channel except for 7/8 rate. The reason may be that much more parity bits were punctured. So the 7/8 Turbo-TCM is not recommended here.

3.2   Simulation results under Rayleigh channel

The Rayleigh noise was generated by Jakes model. "BT_s= 0.01, 0.005, 0.0001, rate 1/2, with CSI and without CSI, all kinds of modulation" were simulated. Figs. 3, 4, 5, 6, 7 depict the result. The solid lines denote the simulation with CSI and the dash lines denote the simulation without CSI.
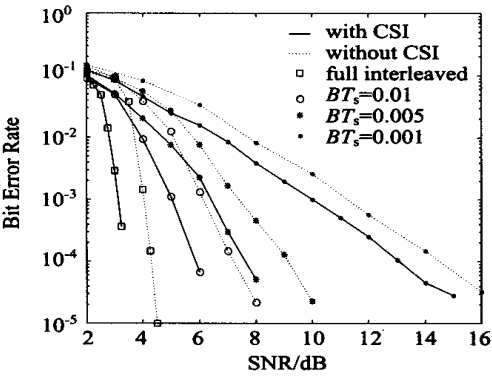

Fig. 3   The BER performance of Turbo-TCM-BPSK under fading channel
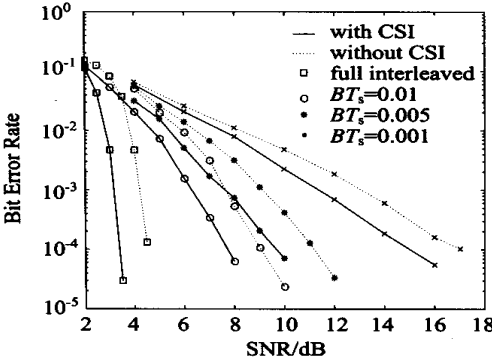

Fig. 4   The BER performance of Turbo-TCM-QPSK under fading channel
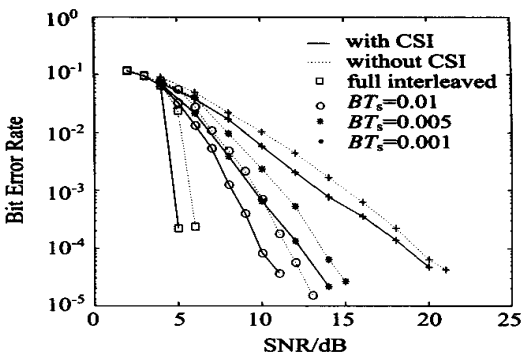

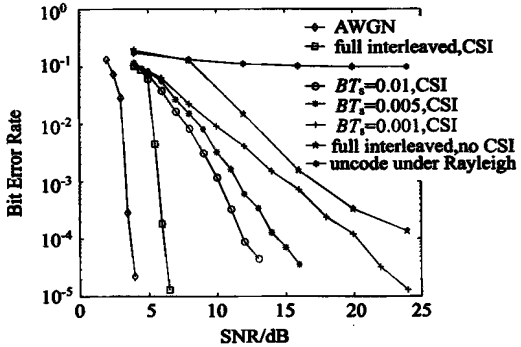Fig. 5   The BER performance of Turbo-TCM-8PSK under fading channel

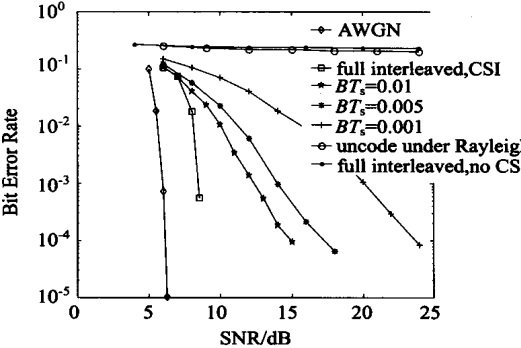Fig. 6   The BER performance of Turbo-TCM-16QAM under fading channel

Fig. 7   The BER performance of Turbo-TCM-64QAM under fading channel

Some conclusions can be drawn from these figures:

(1) The BER performance of Turbo codes under full interleaved Rayleigh fading channel (de-

coding with CSI) is about 2–3dB degradation to that under AWGN channel.

(2) With the increasing of the BT_s under correlated Rayleigh fading channel, the BER performance becomes better, no matter decoding with CSI or without CSI. The reason is that the smaller of BT_s, the greater probability of long bust errors.

(3) There is about 2–3dB gain when decoding with CSI compared with without "CSI". But this conclusion is only suited for BPSK, QPSK and 8PSK modulation, which belong to the uniform amplitude modulation types. When 16QAM, the performance degrades greatly when decoding without CSI. Even worse for 64QAM, the Turbo coded system is worse than the system without coding. So it is unbearable for the Turbo coded 64QAM modulation system without CSI.

(4) Similar with convolution codes, the channel interleaver affects greatly the BER performance of the Turbo codes. So the symbols should be interleaved as far as possible.

4   Conclusions

In this paper, a universal decoding algorithm of Turbo-TCM was presented. The simulations were done under AWGN and Rayleigh fading channel. The advantage of Turbo-TCM can be seen through the comparison between the Turbo-TCM and convolution TCM. The proposed structure and the results will be greatly useful for the real system like DVB system. Despite Turbo-TCM being not applied in the DVB system now, one can not lose sight of its advantage. It is believed that the Turbo-TCM will be applied in the future. So the work above will be useful.

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