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To cite this article: Haidong Yuan *et al* 2014 *New J. Phys.* **16** 105013

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Reduced coupling with global pulses in quantum registers

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Abstract

Decoupling is an important tool used to prolong the coherence time of quantum systems. However, most decoupling schemes have assumed selective controls on the system, and it is believed that with global pulses one can only decouple systems with certain coupling terms like secular dipole-dipole coupling. In this article, we show that with global pulses it is possible to reduce the coupling strength of other types of coupling, which we demonstrate with Ising coupling. The complexity of such pulses is independent of the size of the system.

Keywords: global pulse, decoupling, quantum control

1. Introduction

Quantum systems suffer from decoherence due to interactions with environments. The task of decoupling is to remove unwanted couplings between systems and environments [1, 11, 12]. Many decoupling schemes have been proposed and demonstrated in experiments [2–7, 9, 10, 19, 21–28]. For example, randomized dynamical decoupling [19] uses randomly selected pulses at regular intervals, Uhrig dynamical decoupling (UDD) [2] can cancel dephasing of a single qubit up to order n by using a minimal number of n pulses, and concatenated dynamical decoupling (CDD) constructs decoupling pulse sequences recursively

[6]. There are also studies on using pulses to remove the internal couplings of quantum systems [21, 29, 37] and to engineer Hamiltonians [13, 14].

A common feature of these decoupling schemes is that they all assume selective controls on the system. For many quantum systems, selectively addressing each qubit could be very demanding, especially of those systems whose environment consists of the same physical objects as the system. For example, in some solid state devices, the system and the environment can be the spin of the same nuclear species. For such systems, selectively controlling the system is very hard, as the pulses usually affect all the spins, which means the pulses will be global. The known examples of decoupling with global pulses are WAHUHA [27], MREV-8, and MREV-16 [28, 29] in nuclear magnetic resonance, which exploit the symmetry of homonuclear secular dipole-dipole coupling to decouple the system. Such decoupling schemes rely on the symmetry of secular dipole-dipole coupling and therefore do not apply to other types of coupling. Recently, applications of global pulses in Hamiltonian engineering were also studied [15]. In this paper, we examine the use of global pulses to decouple the system with Ising coupling between the qubits, and show that, contrary to existing beliefs, it is possible to decouple systems with couplings other than secular dipole-dipole coupling. The advantage of global pulses is that the number of pulses needed for decoupling will be independent of the number of qubits (i.e., the complexity of global pulses is $O(1)$).

2. Average Hamiltonian

The principle of decoupling can be illustrated by the average Hamiltonian theory [32, 33]. That is, the propagator can be written as a single exponential relying on some average Hamiltonian, $\overline{H}$, which has the same effect as a time-varying Hamiltonian. The full advantage of this theory is often realized in an interaction frame of a *period* and *cyclic* Hamiltonian. Assume that in an appropriate interaction frame, the Hamiltonian is piecewise constant, $H_1, H_2, \ldots H_m$, in corresponding time intervals, $t_1, t_2, \ldots, t_m$. Then,

$$e^{-i\overline{H}t} = e^{-iH_m t_m} \cdots e^{-iH_1 t_1}.$$

Here, $H_1 \cdots H_m$ are Hamiltonians transformed from the physical Hamiltonian by applying pulses on the system (i.e., $H_i = U_i^\dagger H U_i$, where U_i represents the propagator generated by the pulses). The first few orders of the average Hamiltonian are

$$\overline{H} = \overline{H}^{(0)} + \overline{H}^{(1)} + \overline{H}^{(2)} \cdots,$$

$$\overline{H}^{(0)} = \frac{1}{t}\left(H_1 t_1 + H_2 t_2 + \cdots + H_m t_m\right),$$

$$\overline{H}^{(1)} = -\frac{i}{2t}\left\{\left[H_2 t_2, H_1 t_1\right] + \left[H_3 t_3, H_1 t_1\right] \right.$$
$$\left. + \left[H_3 t_3, H_2 t_2\right] + \cdots\right\},$$

$$\overline{H}^{(2)} = \frac{1}{12t}\left\{\left[H_2 t_2, \left[H_2 t_2, H_1 t_1\right]\right] - \left[H_1 t_1, \left[H_2 t_2, H_1 t_1\right]\right] + \cdots\right\} \tag{1}$$
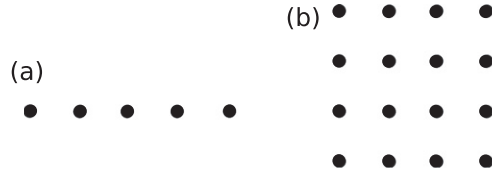
where $t = \sum_{i=1}^{m} t_i$.

Figure 1. Spin topology: (a) linear chain, (b) square lattice.

A global pulse transforms an initial Hamiltonian, H, to

$$H_i = (U \otimes U \otimes \cdots \otimes U)^\dagger H U \otimes U \otimes \cdots \otimes U,$$

where $U \in SU(2)$ represents the propagator generated by the global pulse on each qubit. Global pulses have been used to decouple systems with secular dipole-dipole coupling. In this case, the coupling Hamiltonian takes the following form:

$$H_1^{dd} = \sum_{jk} d^{jk} \left(2I_{jz}I_{kz} - I_{jx}I_{kx} - I_{jy}I_{ky} \right). \tag{2}$$

Here, $I_x := \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $I_y := \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $I_z := \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are the Pauli spin matrices, and we denote $I_{\ell\nu}$ as the operator that acts as I_ν on the ℓth spin (see [34]).

Applying a global $(x)_{\frac{\pi}{2}}$ pulse and $(y)_{\frac{\pi}{2}}$ pulse on the system, one gets

$$\begin{aligned} H_2^{dd} &= \sum_{jk} d^{jk} \left(2I_{jy}I_{ky} - I_{jx}I_{kx} - I_{jz}I_{kz} \right), \\ H_3^{dd} &= \sum_{jk} d^{jk} \left(2I_{jx}I_{kx} - I_{jz}I_{kz} - I_{jy}I_{ky} \right). \end{aligned} \tag{3}$$

It is easy to see that averaging these three Hamiltonians decouples the dipole-dipole coupling; that is,

$$e^{-iH_1^{dd}\delta t}e^{-iH_2^{dd}\delta t}e^{-iH_3^{dd}\delta t} = e^{-i3\delta t\overline{H}},$$

where $\overline{H} = 0$. This is the basic building block of WAHUHA, MREV-8, and MREV-16 [26], and the effectiveness of such a decoupling scheme has been experimentally demonstrated [30]. However, this decoupling scheme only works for systems with coupling $\alpha I_x I_x + \beta I_y I_y + \gamma I_z I_z$, where $\alpha + \beta + \gamma = 0$.

3. Reduce Ising coupling with global pulses

At first look, it may seem impossible to decouple Ising coupling with global pulses, as global pulses cannot change the signs of Ising coupling. We will show that it is indeed possible by extending our previous study on homonuclear decoupling [31].

Consider a system consisting of N qubits connected by Ising coupling. The coupling topology can take various shapes. For example, it can be a spin chain or spin lattice, as shown in figure 1. A gradient magnetic field is added to the system, which induces Zeeman splitting on the qubits. The magnetic field and its gradient are large enough that the differences of Zeeman splitting between coupled qubits are much larger than the coupling strength between the qubits. Under such a gradient magnetic field, the Hamiltonian of the system takes the form

$$H = \sum_{j=1}^{N} \omega_j I_{jz} + \sum_{(jk) \in G} J_{jk} I_{jz} I_{kz} \qquad (4)$$

where $\omega_j = -\overrightarrow{\mu} \cdot \overrightarrow{B}_j$, $\overrightarrow{\mu}$ is the magnetic moment and B_j indicates the magnetic field at site j. G is a graph indicating the coupling topology of the system (i.e., if the edge $(jk) \in G$, then the qubits at site j and k are coupled). We assume $\omega_i \gg J_{jk}$, $|\omega_j - \omega_k| \gg J_{jk}$, $\forall$ i, j, k.

We will first use a two-qubit system to illustrate the decoupling strategy, then generalize it to N qubits with various coupling topologies.

For a two-qubit system, the Hamiltonian takes the form

$$H_0 = \omega_1 I_{1z} + \omega_2 I_{2z} + J I_{1z} I_{2z}. \qquad (5)$$

The basic pulse sequence consists of four periods; within each period it evolves according to the following Hamiltonians:

$$
\begin{aligned}
H_1 &= \omega_1 I_{1z} + \omega_2 I_{2z} + J I_{1z} I_{2z} + A(I_{1x} + I_{2x}), \\
H_2 &= -\omega_1 I_{1z} - \omega_2 I_{2z} + J I_{1z} I_{2z} + A(I_{1x} + I_{2x}), \\
H_3 &= -\omega_1 I_{1z} - \omega_2 I_{2z} + J I_{1z} I_{2z} - A(I_{1x} + I_{2x}), \\
H_4 &= \omega_1 I_{1z} + \omega_2 I_{2z} + J I_{1z} I_{2z} - A(I_{1x} + I_{2x}),
\end{aligned}
\qquad (6)
$$

where H_1 is obtained simply by applying a magnetic field in x direction with effective amplitude A, H_2 is obtained from H_1 by conjugating a π-pulse along the x direction (i.e., $H_2 = e^{i\pi(I_{1x}+I_{2x})} H_1 e^{-i\pi(I_{1x}+I_{2x})}$), H_4 and H_3 are obtained with a control field along the $-x$ direction, and conjugation with π-pulses along the x direction. The π-pulses here are assumed to be infinitely narrow pulses.

Each of the four Hamiltonians is maintained for a period of Δt. To keep the terms up to the second order of the average Hamiltonian theory, we obtain the following average Hamiltonian over an interval of $4\Delta t$:

$$
\begin{aligned}
H_{1eff} = {}& J I_{1z} I_{2z} + \frac{A\Delta t}{2}(\omega_1 I_{1y} + \omega_2 I_{2y}) + A\Delta t J(I_{1y} I_{2z} + I_{1z} I_{2y}) \\
& + \frac{(A\Delta t)^2}{2}(\omega_1 I_{1z} + \omega_2 I_{2z}) + \frac{4}{3}(A\Delta t)^2 J(I_{1y} I_{2y} - I_{1z} I_{2z}) + O\big((A\Delta t)^3\big).
\end{aligned}
\qquad (7)
$$

Next, we apply π pulses along y direction to flip the signs of the third and fourth terms in H_{1eff}. The pulse sequence is shown in figure 2.

As a consequence, we create H_{2eff} such that

$$e^{-iH_{2eff}8\Delta t} = e^{-iH_{1eff}4\Delta t} e^{i\pi(I_{1y}+I_{2y})} e^{-iH_{1eff}4\Delta t} e^{-i\pi(I_{1y}+I_{2y})}. \qquad (8)$$

It is straightforward to see that

$$H_{2eff} = \frac{\theta}{2}(\omega_1 I_{1y} + \omega_2 I_{2y}) + J I_z S_z + \frac{4}{3}\theta^2 J(I_{1y} I_{2y} - I_{1z} I_{2z}) + O(\theta^3). \qquad (9)$$

Here we denote $\theta = \Delta t A$ and choose Δt such that $\theta \ll 1$, but $\theta |\omega_1 - \omega_2| \gg J$. The newly created Zeeman-like terms along the y direction are orthogonal to the Ising coupling terms. Since we have $\theta |\omega_1 - \omega_2| \gg J$, we can use rotating wave approximation to reduce the effective Hamiltonian to
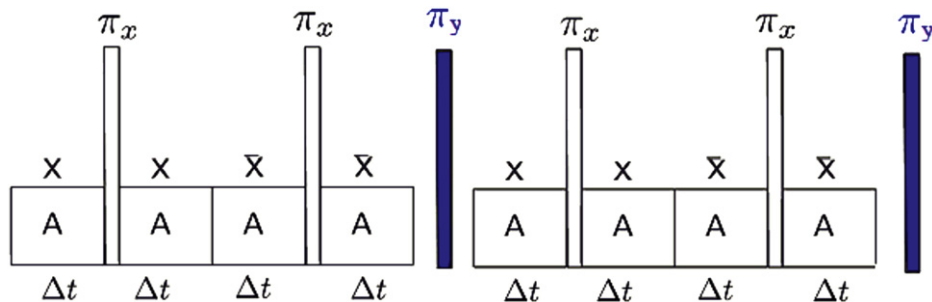
Figure 2. Basic pulse sequence for decoupling.

$$H_{3eff} = \frac{\theta}{2}\left(\omega_1 I_{1y} + \omega_2 I_{2y}\right) + \frac{4}{3}\theta^2 J I_{1y} I_{2y} + O\left(\theta^3\right). \tag{10}$$

That is, if we repeat the procedure k times such that $8k\Delta t \gg \frac{1}{\omega_1 - \omega_2}$, then $[e^{-iH_{2eff}8\Delta t}]^k = e^{-iH_{3eff}8k\Delta t}$.

Compared with the original Hamiltonian, H_0, we have effectively reduced the coupling strength by a factor of $\frac{4}{3}\theta^2$. If we can create $|\omega_1 - \omega_2| \approx 10^3 J$, then θ can be taken $\approx \frac{1}{20}$, so in this case $\frac{4}{3}\theta^2 \approx \frac{1}{300}$ (i.e., the coupling strength is reduced by ≈ 300 times). Further reduction of the coupling strength can be achieved by more iterations of the above procedure. Note that the local terms, such as $\frac{\theta}{2}(\omega_1 I_{1y} + \omega_2 I_{2y})$, can be canceled by Hahn echo pulses (i.e., inserting π-pulse along the x direction) [35].

This decoupling strategy can be generalized to N qubits with various coupling topologies, which can be easily seen by substituting the Hamiltonian for the N-qubit system in equation (6):

$$H_1 = \sum_{j=1}^{N} \omega_j I_{jz} + \sum_{(jk)\in G} J_{jk} I_{jz} I_{kz} + \sum_j A I_{jx},$$

$$H_2 = -\sum_{j=1}^{N} \omega_j I_{jz} + \sum_{(jk)\in G} J_{jk} I_{jz} I_{kz} + \sum_j A I_{jx},$$

$$H_3 = -\sum_{j=1}^{N} \omega_j I_{jz} + \sum_{(jk)\in G} J_{jk} I_{jz} I_{kz} - \sum_j A I_{jx},$$

$$H_4 = \sum_{j=1}^{N} \omega_j I_{jz} + \sum_{(jk)\in G} J_{jk} I_{jz} I_{kz} - \sum_j A I_{jx}. \tag{11}$$

Again by inserting π-pulses along σ_y direction and using rotating wave approximation, we create an effective Hamiltonian:

$$H_{eff}^N = \frac{\theta}{2} \sum_{j=1}^{N} \omega_j I_{jy} + \frac{4}{3}\theta^2 \sum_{(jk)\in G} J_{jk} I_{jy} I_{ky} + O\left(\theta^3\right). \tag{12}$$

The subsequent steps are similar to the ones outlined in the case of a two-qubit system.

Since at every step of the analysis the precise knowledge of the coupling strength is not required as long as it is small compared to the Zeeman splitting, this decoupling scheme is actually robust against random fluctuation of the coupling strength, J_{jk}. For example, on the

two-qubit system, the coupling strengths during the four periods of the basic pulse sequence in equation (6) can be fluctuating. We assume the fluctuation has a steady distribution and in each period the coupling strength is constant drawn from steady distribution. This assumption holds when the frequency of fluctuation is small compared to $\frac{1}{\Delta t}$. In this case, the basic pulse sequence becomes

$$
\begin{aligned}
H_1 &= \omega_1 I_{1z} + \omega_2 I_{2z} + J_1 I_{1z} I_{2z} + A(I_{1x} + I_{2x}), \\
H_2 &= -\omega_1 I_{1z} - \omega_2 I_{2z} + J_2 I_{1z} I_{2z} + A(I_{1x} + I_{2x}), \\
H_3 &= -\omega_1 I_{1z} - \omega_2 I_{2z} + J_3 I_{1z} I_{2z} - A(I_{1x} + I_{2x}), \\
H_4 &= \omega_1 I_{1z} + \omega_2 I_{2z} + J_4 I_{1z} I_{2z} - A(I_{1x} + I_{2x}).
\end{aligned}
\tag{13}
$$

By keeping the terms up to the second order of the average Hamiltonian theory, we obtain the following average Hamiltonian over an interval of $4\Delta t$:

$$
\begin{aligned}
H'_{1eff} = \; & \frac{J_1 + J_2 + J_3 + J_4}{4} I_{1z} I_{2z} + \frac{A\Delta t}{2}\left(\omega_1 I_{1y} + \omega_2 I_{2y}\right) \\
& + \frac{A\Delta t(J_1 + 3J_2 + 3J_3 + J_4)}{8}\left(I_{1y} I_{2z} + I_{1z} I_{2y}\right) + \frac{(A\Delta t)^2}{2}\left(\omega_1 I_{1z} + \omega_2 I_{2z}\right) \\
& + \left(\frac{J_1 + J_2 + J_3 + J_4}{3} + \frac{J_1 - J_2 - J_3 + J_4}{4}\right)(A\Delta t)^2\left(I_{1y} I_{2y} - I_{1z} I_{2z}\right) \\
& + O\left((A\Delta t)^3\right),
\end{aligned}
\tag{14}
$$

which reduces to equation (7) when the four coupling strengths are the same. Again, by inserting π pulses along y-direction, we can flip the signs of the third and fourth terms in H'_{1eff}, and create an effective Hamiltonian:

$$
e^{-iH'_{2eff}\, 8\Delta t} = e^{-iH'_{1eff}\, 4\Delta t} e^{i\pi\left(I_{1y}+I_{2y}\right)} e^{-iH'_{1eff}\, 4\Delta t} e^{-i\pi\left(I_{1y}+I_{2y}\right)}.
\tag{15}
$$

Here, the two H'_{1eff} may have different coupling strengths. Assume the coupling strengths for the four periods of basic pulse sequences for each H'_{1eff} are J_1, J_2, J_3, J_4 and J_5, J_6, J_7, J_8, respectively. Then

$$
\begin{aligned}
H'_{2eff} = \; & \frac{J_1 + J_2 + J_3 + J_4 + J_5 + J_6 + J_7 + J_8}{8} I_{1z} I_{2z} + \frac{\theta}{2}\left(\omega_1 I_{1y} + \omega_2 I_{2y}\right) \\
& + \frac{J_1 + 3J_2 + 3J_3 + J_4 - J_5 - 3J_6 - 3J_7 - J_8}{8}\theta\left(I_{1y} I_{2z} + I_{1z} I_{2y}\right) \\
& + \frac{7J_1 + J_2 + J_3 + 7J_4 + 7J_5 + J_6 + J_7 + 7J_8}{24}\theta^2\left(I_{1y} I_{2y} - I_{1z} I_{2z}\right) \\
& + O\left((A\Delta t)^3\right),
\end{aligned}
\tag{16}
$$

where $\theta = A\Delta t$. Similar to the case with constant coupling strength, we choose Δt such that $\theta |\omega_1 - \omega_2| \gg J_i$. By rotating wave approximation, the yz, zy, and zz couplings are effectively averaged out, as they do not commute with $\omega_1 I_{1y} + \omega_2 I_{2y}$. Only the yy coupling remains, whose strength is reduced by the order of θ^2 compared to the original coupling strength. The generation to N-qubit system is straightforward, similar to the case with constant coupling strength.
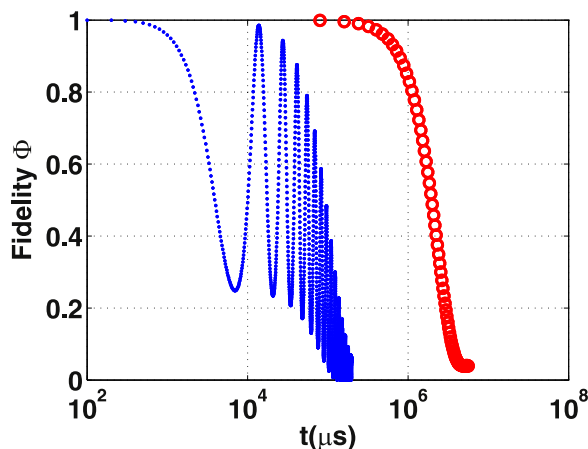
Figure 3. (Color online) The simulation was on a square lattice with four qubits. The coupling strengths between these qubits fluctuate at each instant of time. The coupling strengths between neighboring qubits have means equal to $J = [17.3, 17.9, 18.5, 19.2]$ Hz, and variances equal to $\bar{J} = 9$ Hz. The coupling strengths between diagonal qubits have means equal to $J = [6.1, 6.6]$ Hz, and variances equal to 3 Hz. The Zeeman splittings of the four qubits are $[62.8, 95.9, 120.1, 153.18]$ kHz, respectively. $\Delta t = 10^{-7}$, $\theta = \frac{1}{20}$, and $A \approx 2\pi 8 \times 10^3 = 8\,kHz$. The initial state of the qubit is $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$. The blue line shows the fidelity without the global pulses; the red circles show the fidelity with the application of the decoupling pulses.

This decoupling pulse sequence can also reduce the dephasing effect caused by the environment. Suppose each qubit in the system is coupled to the environment where the coupling Hamiltonian is modeled as $H_{SB} = \sum_k \hbar \sigma_z (g_k b_k^\dagger + g_k^\dagger b_k)$, where $b_k^\dagger$, b_k are bosonic operators for the kth field mode of the environment, characterized by a generally complex coupling parameter, g_k [1]. The π pulses along the x and y directions in our decoupling scheme also average out the net effect of H_{SB}.

Finally, we present a numerical simulation to illustrate the effects of our decoupling strategy. As shown in figure 3, the simulation was done with four spins on a square lattice, with one iteration of the pulses. We simulate the unitary evolution of the system without and with the global pulses, and see how it deviates from the identity operator. The vertical axis represents the fidelity of U with respect to the identity operator, where fidelity measurement between two unitary operators, U_1 and U_2, is defined as

$$\phi = \frac{\left| tr\left(U_1 U_2^\dagger\right)\right|^2}{\left| tr\left(U_2 U_2^\dagger\right)\right| \left| tr\left(U_1 U_1^\dagger\right)\right|}. \tag{17}$$

Other fidelity measures (for example, the average gate fidelity [36, 37]) can also be used, which is equivalent in our case. Assume the lattice is put at a vicinity of a dysprosium micro magnet with a length of $400\,\mu$m, a width of $4\,\mu$m, and a height of $10\,\mu$m, which can generate a field gradient of $\partial B/\partial z = 1.4\,T\mu$m^{-1} [38]. In addition, a large homogeneous field, B_0 of ~ 7 T, is superposed. The distance of two neighboring nuclei spin is ~ 1 nm; for the simulation we take the distance as 1 nm and the nuclear spin as ^{29}Si [38]. If the magnetic field gradient is put along the direction of $y = \frac{\sqrt{3}}{3}x$, then the Zeeman splittings for the four spins are 62.8 kHz, 95.9 kHz,

120.1 kHz, and 153.18 kHz, respectively. The secular component of the dipolar Hamiltonian which couples the *i*th spin to the *j*th spin is written as [39]

$$H_{ij} = \frac{\mu_0}{4\pi} \gamma^2 \hbar^2 \frac{1 - 3\cos^2\theta_{ij}}{r_{ij}^3} I_z^i I_z^j = J_{ij} I_z^i I_z^j,$$

where r_{ij} is the length of the vector connecting the spins and θ_{ij} is its angle with the applied field. With chosen parameters, the coupling strengths between adjacent spins are ~ 17.3 HZ, and ~ 6.1 Hz between diagonal spins. Due to the vibration of the atoms, the couplings strength can fluctuate, so we assume that the coupling strengths at each instant of time follow independent normal distributions with mean values $J = [17.3, 17.9, 18.5, 19.2, 6.1, 6.6]$ Hz and variances equal to 10 Hz for the couplings between adjacent spins and 5 Hz for the couplings between diagonal spins. Δt is taken to be 10^{-7}s and $A \approx 8$ kHz, $\theta = \frac{1}{20}$. One can see that the global pulses reduce the decoherence rate by about two orders of magnitude in one iteration. Note that the results do not depend much on precise numerical values, as long as the condition $\omega_i \gg J_{jk}$, $|\omega_j - \omega_k| \gg J_{jk}$, $\forall i, j, k$ is satisfied.

4. Conclusions

We presented a method that reduces the Ising coupling strength of register qubits using global pulses. This method can be used to reduce the residual coupling of quantum memories, where selective addressing may be difficult or undesirable. The advantage of these global pulses is that the number of pulses does not grow with the number of qubits (i.e., the complexity of these pulses is only O(1)). This opens new directions for using global pulses for decoupling.

Acknowledgments

References

[1] Viola L and Lloyd S 1998 *Phys. Rev.* A **58** 2733
[2] Uhrig G S 2007 *Phys. Rev. Lett.* **98** 100504
[3] Biercuk M, Uys H, VanDevender A, Shiga N, Itano W and Bollinger J 2009 *Nature* **458** 996
[4] Biercuk M, Uys H, VanDevender A, Shiga N, Itano W and Bollinger J 2009 *Phys. Rev.* A **79** 062324
[5] Khodjasteh K and Lidar D A 2005 *Phys. Rev. Lett.* **95** 180501
[6] Khodjasteh K and Lidar D A 2007 *Phys. Rev.* A **75** 062310
[7] West J R, Fong B H and Lidar D A 2010 *Phys. Rev. Lett.* **104** 130501
[8] Yao W, Liu R-B and Sham L J 2007 *Phys. Rev. Lett.* **98** 077602
[9] Witzel W M and das Sarma S 2007 *Phys. Rev. Lett.* **98** 077601
[10] Ryan C A, Hodges J S and Cory D G 2010 *Phys. Rev. Lett.* **105** 200402
[11] Viola L, Knill E and Lloyd S 1999 *Phys. Rev. Lett.* **82** 2417
[12] Viola L, Lloyd S and Knill E 1999 *Phys. Rev. Lett.* **83** 4888
[13] Borneman T W, Granade C E and Cory D G 2012 *Phys. Rev. Lett.* **108** 140502
[14] Bookatz A D, Wocjan P and Viola L arXiv:quant-ph 1310.4153
[15] Ajoy A and Cappellaro P 2013 *Phys. Rev. Lett.* **110** 220503
[16] Viola L and Knill E 2005 *Phys. Rev. Lett.* **94** 060502

[17] Janzing D, Wocjan P and Beth T 2002 *Phys. Rev.* A **66** 042311

[18] Wocjan P 2006 *Phys. Rev.* A **73** 062317

[19] Santos L F and Viola L 2006 *Phys. Rev. Lett.* **97** 150501

[20] Yang W and Liu R B 2008 *Phys. Rev. Lett.* **101** 180403

[21] Uhrig G S and Lidar D A 2010 *Phys. Rev.* A **82** 012301

[22] Jenista E R, Stokes A M, Branca R T and Warren W S 2009 *J. Chem. Phys.* **131** 204510

[23] Szwer D J, Webster S C, Steane A M and Lucas D M 2011 *J. Phys. B: At. Mol. Opt. Phys.* **44** 025501

[24] Ajoy A, Alvarez G A and Suter D 2011 *Phys. Rev.* A **83** 032302

[25] Wang Z Y and Liu R B 2011 *Phys. Rev.* A **83** 022306

[26] Duer M J (ed) 2001 *Solid-State NMR Spectroscopy: Principles and Applications* (Oxford: Wiley-Blackwell)

[27] Waugh J S, Huber L M and Haeberlen U 1968 *Phys. Rev. Lett.* **20** 180

[28] Mansfield P 1971 Symmetrized pulse sequences in high resolution NMR in solids *J. Phys.* C **4** 1444–52

[29] Rhim W-K, Elleman D D and Vaughan R W 1973 Analysis of multiple pulse NMR in solids *The Journal of Chemical Physics* **59** 3740–9

[30] Krojanski H G and Suter D 2006 *Phys. Rev. Lett.* **97** 150503

[31] Koroleva V D M and Khaneja N 2012 *The Joural of Chemical Physics* **137** 094103

[32] Magnus W 1954 *Commun. Pure Appl. Math.* **7** 649

[33] Haeberlen U and Waugh J S 1968 *Phys. Rev.* **175** 453

[34] Ernst R R, Bodenhausen G and Wokaun A 1987 *Principles of Nuclear Magnetic Resonance in One and Two Dimensions* (Oxford: Clarendon Press)

[35] Hahn E L 1950 *Phys. Rev.* **80** 580

[36] Nielsen M A 2002 *Phys. Lett.* A **303** 249

[37] Fortunato E M, Pravia M A, Boulant N, Teklemariam G, Havel T F and Cory D G 2002 *J. Chem. Phys.* **116** 7599

[38] Ladd T D, Goldman J R, Yamaguchi F, Yamamoto Y, Abe E and Itoh K M 2002 All-silicon quantum computer *Phys. Rev. Lett* **89** 017901

[39] Abragam A 1961 *Principles of Nuclear Magnetism* (Oxford, U.K: Oxford University Press)